Assignment 4

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Problem 1 *Prove that* $n! + 2$ *is divisible by 2,* $\forall n \in \mathbb{N} s.t. n \geq 2$

Answer 1

Theorem 1
$\forall n \in \mathbb{N} s.t. n \geq 2$, $n! + 2$ *is divisible by 2*

Proof 1
We will proceed by mathematical induction.
Let $n \in \mathbb{N} s.t. n \geq 2$
Consider $n = 2$
$2! + 2 = 4$. *4 is divisible by 2.*
$\therefore$ *base case is shown.*
Assume true for some $k \in \mathbb{N}$
    $k! + 2$ *is divisible by 2.*
Consider $(k+1)! + 2 = (k+1) * k! + 2$
$\therefore \exists j s.t. = (k+1) * 2j$
    $= 2jk + 2j$
    $= 2(jk + j)$
$\therefore (k+1)! + 2$ *is divisible by 2.*
$\therefore k! + 2$ *is divisible by 2, by mathematical induction.*
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Problem 2 *For any real number r except 1, and any integer* $n \geq 0$ *prove the following summation. (This is the sum of a geometric sequence)*

$$\sum_{i=0}^{n} r^i = \frac{r^{n+1} - 1}{r - 1}$$

Answer 2

Proof 2
We will proceed by mathematical induction.
Let $r \in \mathbb{R}$ *s.t* $r \geq 0$ *and* $r \neq 1$
Consider $n = 0$
$\sum\limits_{i=0}^{0} r^0 = \frac{r^{0+1}-1}{r-1},\ r^0 = 1,\ \frac{r^{0+1}-1}{r-1} = 1,\ 1 = 1$
$\therefore$ *Base case is shown.*
Assume for some $k \in \mathbb{R}$
$\sum\limits_{i=0}^{k} r^i = \frac{r^{k+1}-1}{r-1}$
Consider $\sum\limits_{i=0}^{k+1} r^i = \frac{r^{k+1+1}-1}{r-1} \Rightarrow \sum\limits_{i=0}^{k+1} r^i = \frac{r^{k+2}-1}{r-1}$
$\sum\limits_{i=0}^{k+1} r^i = \sum\limits_{i=0}^{k} r^i + r^{k+1}$
$\frac{r^{k+1}-1}{r-1} + r^{k+1}$
$\frac{r^{k+1}-1}{r-1} + \frac{r^{k+1}(r-1)}{r-1}$
$\frac{(r^{k+1}-1)+r^{k+1}(r-1)}{r-1}$
$\frac{r^{k+1}-1+r^{k+2}-r^{k+1}}{r-1}$
$\frac{r^{k+2}}{r-1}$
■

Problem 3 *Prove the following:*

$$1 + 3 + 5 + 7 + 9 + ... + (2n-1) = n^2$$

Answer 3

Proof 3
We will proceed by mathematical induction.
Let $n \in \mathbb{R}$
Consider $n = 1$

$$2(1) - 1 = 1^2$$
$$2 - 1 = 1$$

$$1 = 1$$

$\therefore$ *Base case is shown.*
Assume for some $k \in \mathbb{N}$
$\sum_{i=1}^{k}(2k-1) = k^2$
Consider $\sum_{i=1}^{k+1}(2(k+1)-1) = (k+1)^2$

$$= 2(k+1) + (2(k+1)-1)$$

$$= k^2 + (2k+2-1)$$

$$= k^2 + 2k + 1$$

$$= (k+1)(k+1)$$

$$= (k+1)^2$$

$\therefore \sum_{i=1}^{k+1}(2(k+1)-1) = (k+1)^2$
$\therefore \sum_{i=1}^{k+1}(2-1) = k^2$
■

Problem 4 *Prove that* $7^n - 1$ *is divisible by 6 for all* $n \in \mathbb{N}$

Answer 4

Proof 4
We will proceed by mathematical induction.
Let $n \in \mathbb{N}$
Consider $n = 1$
$7^1 - 1 = 6$. *6 is divisible by 6.*
$\therefore$ *Base case is shown. Assume for some* $k \in \mathbb{N}$
$7^k - 1$ *is divisible by 6.*
Consider $7^{k+1} - 1$

$$= 7^k * 7 - 1$$

$$= 7 * 7^k - 1$$

$$\exists j.s.t. = 7 * 6j$$

$$= 42j$$
$$= 6(7j)$$

$\therefore 7^{k+1} - 1$ *is divisible by 6.*
$\therefore 7^k - 1$ *is divisible by 6, by mathematical induction.*
$\blacksquare$

Problem 5 *Define a sequence recursively as follows:*
$a_0 = 2$
$a_1 = 2$
$a_2 = 6$
$a_k = 3a_{k-3}, \forall k \in \mathbb{Z} s.t. k \geq 3$

Answer 5

Theorem 2
$a_k = 3a_{k-3}, \forall k \in \mathbb{Z} s.t. k \geq 3$

Proof 5
We will proceed by mathematical induction.
Let $k \in \mathbb{Z}$
Consider k=3
$a_3 = 3(2) = 6$. *6 is even.*
$\therefore$ *Base case is shown.*
Assume $\forall j \leq k, a_j = 3(a_{j-3})$ *is even for some* $k \in \mathbb{Z}$
Consider $a_{k+1} = 3(a_{k-2})$
By induction hypothesis
a_{k-2} *is even* $\rightarrow \exists w s.t. a_{k-2} = 2w$

$$\begin{aligned} a_{k+1} &= 3(a_{k-2}) \\ &= 3(2w) \\ &= 6w \\ &= 2(3w) \end{aligned}$$

$\therefore a_{k+1}$ *is even.*
$\therefore \forall n \geq 0$, a_n *is even by mathematical induction.*
$\blacksquare$