Assignment 1

Jacob Paul Bryant

1/16/2017

Problem 1 *Translate the following statement into symbols of formal logic:* ***If you are not in South Korea, then you are not in Seoul or Kwangju****. Then, translate the following formal statement to English:*

$$q \to (r \land p)$$

- p = *"You are in Seoul"*
- q = *"You are in Kwangju"*
- r = *"You are in South Korea"*

Answer 1 *The statement* ***If you are not in South Korea, then you are not in Seoul or Kwangju*** *translates in symbols of formal logic to:*

$$\neg r \to \neg p \lor \neg q$$

The formal statement

$$q \to (r \land p)$$

translates in English to: ***If you are in Kwangju, then you are in South Korea and Seoul.***

Problem 2 *Let s be the following statement: "If you are studying hard, then you are staying up late at night. First, give the* **converse** *of s. Then, give the* ***contrapositive*** *of s.*

Answer 2 *The statement* ***If you are studying hard, then you are staying up late at night*** *translates to the converse:*

If you are staying up late at night, then you are studying hard

And translates to the contrapositive:

If you are not staying up late at night, then you are not studying hard

Problem 3 *Write a proof for the following formal logical statement:*

$$(p \to q) \wedge (p \wedge r) \Rightarrow (q \wedge r)$$

Proof 1

$$((p \to q) \wedge (p \wedge r)) ::: Given$$
$$((p \to q) \wedge (r \wedge p)) ::: Commutative$$
$$(p \to q) \wedge r ::: Simplification$$
$$(\neg p \vee q) \wedge r ::: Implication$$
$$\neg p \vee (q \wedge r) ::: Associative$$
$$(q \wedge r) \vee \neg p ::: Commutative$$
$$(\neg\neg q \wedge \neg\neg r) \vee \neg p ::: DoubleNegation$$
$$\neg((\neg p \vee \neg r) \wedge p) ::: DeMorgan's Laws$$
$$\neg((\neg q \vee \neg r)) ::: Simplification$$
$$\neg\neg q \wedge \neg\neg r ::: DeMorgan's Laws$$
$$q \wedge r ::: DoubleNegation$$
$$\therefore (p \to q) \wedge (p \wedge r) \Rightarrow (q \wedge r)\blacksquare$$

Problem 4 *Let the following predicates be given. The domain is all mammals, M.*

- *L(x) = "x is a lion"*
- *F(x) = "x is fuzzy"*

Translate the statement "All lions are fuzzy" into predicate logic. That is, using the for all and there exists symbols. Then, translate the statement "Some Lions are fuzzy", into predicate logic.

Answer 3 *The statement "All lions are fuzzy" translates in predicate logic to:*

$$\forall(x)(L(x) \to F(x))$$

The statement "Some Lions are fuzzy" translates in predicate logic to:

$$\exists(x)(L(x) \wedge F(x))$$

Problem 5 *Find a counterexample for the following statements:*

1. *If all the sides of a quadrilateral have equal lengths, then the diagonals of the quadrilateral have equal lengths.*
2. *For every real number $N > 0$, there is some real number x such that $Nx > x$*
3. *Let l, m, n be lines in the plane. If l is perpendicular to m and n intersects l, then n intersects m.*
4. *If p is prime, then $p^2 + 4$ is prime.*

Answer 4

1. *A rhombus is a quadrilateral where all sides have equal lengths, but the diagonals are unequal in length.*
2. *Because negatives are real numbers, consider N=2,x=-1 such that 2(-1)<-1.*
3. *Consider where line l is perpendicular to line m, line n intersects line l at 90 degrees, then both lines m and n are perpendicular to line l. Thus, lines m and n are parallel to each other.*
4. *Consider the prime number p=2, such that*

   $$2^2 + 4 = 8$$

   *which is composite and not prime because 8=2*4.*

Problem 6 *Give a direct proof to the following theorem: Let a,b,c $\in \mathbb{Z}$. If a/b and a/c then a/(b*c). Be sure to use the definition of / in your proof.*

Proof 2 ***Statement***
*If a/b and a/c then a/(b*c).*
Proof
Let a,b,c $\in \mathbb{Z}$
Assume a/b and a/c where / means a divided by b and a divided by c
Since a/b, $\exists k_1 \in \mathbb{Z}(s.t)b = k_1 a$
Since a/c, $\exists k_2 \in \mathbb{Z}(s.t)c = k_1 a$
Since $b * c = (k_1 a) * (k_2 a)$

$$= k_1 k_2 a^2$$

$$b * c = (k_1 k_2 a)a$$

$$= Ka$$

$$\therefore a|b * C\blacksquare$$

Problem 7 *Prove that the sum of two even integers is even.*

Proof 3 ***Statement***
For all integers, the sum of two even integers is even.
Proof
$Let n_1 and n_2 \in \mathbb{Z}$
Let E(n) be "is an even integer"
Let S(x,y) be "the sum of x and y"
Assume $\exists(n_1, n_2)E(n_1) \wedge E(n_2)$
Since $E(n_1), \exists k_1 \in \mathbb{Z}(s.t)n_1 = 2k_1$
Since $E(n_2), \exists k_2 \in \mathbb{Z}(s.t)n_2 = 2k_2$
Since $n_1 + n_2 = 2k_1 + 2k_2$
$= 2(k_1 + k_2)$
$n_1 + n_2 = 2K$

$$\therefore E(S(n_1, n_2))\blacksquare$$