Assignment 2

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Problem 1
Let A = {2,3,4}, B = {3,4,5,6} and suppose that U = 1,2,3,4,5,6,7,8,9. List the elements.

1. $(A \cup B)'$
2. $(A \cap B) \times A$
3. $\wp(B \backslash A)$

Answer 1

1. $1, 7, 8, 9$
2. $\{(3,2),(3,3),(3,4),(4,2),(4,3),(4,4)\}$
3. $\{5,6\}$

Problem 2
Write down all the elements of the set $\{1,2,3\} \cap \{2,3,4,5\} \cup \{6,7\}$

Answer 2

$$\{2,3,6,7\}$$

Problem 3
Write all the elements of $\{A,B,C\} \times \{H,K\}$

Answer 3
$\{(A,H),(A,K),(B,H),(B,K),(C,H),(C,K)\}$

Problem 4
Let $S = \{a,b,c\}$ *Write down all elements in the following sets:*

1. $S \text{ x } S$
2. $\wp(S)$

Answer 4 1. $\{(a,a),(a,b),(a,c),(b,a),(b,b),(b,c),(c,a),(c,b),(c,c)\}$

2. $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

Problem 5
Let X,Y,Z be sets. Prove the following: $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$

Proof 1
Let X,Y,Z be sets
$X \cap (Y \cup Z) \subset (X \cap Y) \cup (X \cap Z)$
Let $x \in X \cap (Y \cup Z)$
$\therefore x \in X$ *and* $x \in Y \cup Z$
$\therefore x \in X$ *and* $x \in Y$ *or* $x \in Z$
$\therefore x \in X \cap Y$ *or* $x \in X \cap Z$
$\therefore x \in (X \cap Y) \cup (X \cap Z)$

So $X \cap (Y \cup Z) \subset (X \cap Y) \cup (X \cap Z)$
Let $x \in (X \cap Y) \cup (X \cap Z)$
$\therefore x \in X \cap Y$ *or* $x \in X \cap Z$
$\therefore x \in X$ *and* Y *or* $x \in X \cap Z$
$\therefore x \in X$ *and* Y *or* X *and* Z
$\therefore x \in X$ *and* $x \in Y$ *or* $x \in Z$
$\therefore x \in X$ *and* $x \in Y \cup Z$
$\therefore x \in X \cap Y \cup Z$
$\therefore (X \cap Y) \cup (X \cap Z) \subset X \cap (Y \cup Z)$
Therefore $(X \cap (Y \cup Z) = X \cap Y) \cup (X \cap Z)$
$\blacksquare$

Problem 6
Prove the following theorem:

Theorem 1 *For all sets* $A, B, C, (A \backslash C) \cap (B \backslash C) \cap (A \backslash B) = \emptyset$

Proof 2
Let A,B,C be sets
Assume $(A\backslash C) \cap (B\backslash C) \cap (A\backslash B)$
Let $x \in A$
$\Rightarrow x \in (A \cap C') \cap (B \cap C') \cap (A \cap B')$
$\Rightarrow x \in (A \cap C')$ *and* $(B \cap C')$ *and* $(A \cap B')$
$\Rightarrow x \in A$ *and* $x \notin C$ *and* $x \in B$ *and* $x \notin C$ *and* $x \in A$ *and* $x \notin B$
$\Rightarrow x \in A$ *and* $x \notin C$ *and* $x \in B$ *and* $x \notin B$
$\Rightarrow x \in B$ *but* $x \notin B \Rightarrow\Leftarrow$
$\therefore$ *This is a contradiction*
$\therefore (A\backslash C) \cap (B\backslash C) \cap (A\backslash B) = \emptyset$
$\blacksquare$

Problem 7
Prove the following theorem:

Theorem 2 *For all sets A,B,C, if* $C\subset (B\backslash A)$, *then* $A \cap C = \emptyset$

Proof 3
Let A,B,C, X be sets
Assume $C \subset (B\backslash A)$
Let $x \in C$
$\Rightarrow x \subset (B \cap A')$
$\therefore \exists x \in X, x \in C$ *then* $x \notin A$
$\Rightarrow x \not\subset A$
$\therefore A \cap C = \emptyset$
$\blacksquare$

Problem 8
Prove the following theorem:

Theorem 3 *For all sets* $A, B, C, A \times (B \cup C) = (A \times B) \cup (A \times C)$

Proof 4
Let A,B,C be sets
Let $\exists x \in A$

Assume $A \times (B \cup C)$
Since $x \in A \times (B \cup C)$
$\Rightarrow x \in A \times x \in B \cup x \in C$
$\Rightarrow x \in A \times x \in B$ *or* $x \in C$
$\Rightarrow x \in A \times x \in B$ *or* $x \in A \times x \in C$
$\Rightarrow x \in (A \times B)$ *or* $x \in (A \times C)$
$\Rightarrow x \in (A \times B) \cup x \in (A \times C)$
$\Rightarrow x \in (A \times B) \cup (A \times C)$
$\therefore A \times (B \cup C) \subset (A \times B) \cup (A \times C)$
Therefore $A \times (B \cup C) = (A \times B) \cup (A \times C)$
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Problem 9
Find a counter example to prove the following false: $(A \cup B)' = A' \cup B'$

Answer 5
Let A,B be sets
Assume $(A \cup B)' = A' \cup B'$
Let set $A = \{1, 2, 3\}$
Let set $B = \{1, 2, 3, 4, 5\}$
Let the universal set $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
Then $(A \cup B)' = \{6, 7, 8, 9\}$
Also $A' \cup B' = \{4, 5, 6, 7, 8, 9\}$
$\Rightarrow \{6, 7, 8, 9\} \neq \{4, 5, 6, 7, 8, 9\}$
$\Rightarrow (A \cup B)' \neq A' \cup B'$
$\therefore$ *This is a contradiction*
■

Problem 10 *Given sets A and B, the* ***symmetric difference*** *of A and B, denoted* $A \triangle B$ *is:*

$$A \triangle B = (A \backslash B) \cup (B \backslash A)$$

Let A = {1,2,3,4}, B = {3,4,5,6}, and C = {5,6,7,8}. Find each of the following sets:

1. $A \triangle B$

2. $B \triangle C$

3. $A \triangle C$

4. $(A \triangle B) \triangle C$

Answer 6 1. $A \triangle B \rightarrow (A \backslash B) \cup (B \backslash A)$
$\Rightarrow \{1, 2\} \cup \{5, 6\}$
$\therefore \{1, 2, 5, 6\}$

2. $B \triangle A \rightarrow (B \backslash A) \cup (C \backslash B)$
$\Rightarrow \{3, 4\} \cup \{7, 8\}$
$\therefore \{3, 4, 7, 8\}$

3. $A \triangle C \rightarrow (A \backslash C) \cup (C \backslash A)$
$\Rightarrow \{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$
$\therefore \{1, 2, 3, 4, 5, 6, 7, 8\}$

4. $(A \triangle B) \triangle C \rightarrow (\{1, 2, 5, 6\}\ C) \cup (C\ \{1, 2, 5, 6\})$
$\Rightarrow \{1, 2\} \cup \{7, 8\}$
$\therefore \{1, 2, 7, 8\}$