Assignment 3

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2/1/2017

Problem 1 *Consider the function* $f : \mathbb{R} \to \mathbb{R} : f(x) = 3x + 3$
Prove the function is both injective and surjective. In other words, prove that the function is bijective.

Answer 1

Proof 1
Let $\mathbb{R} \to \mathbb{R} : f(x) = 3x + 3$ *be a function*
Let $h, k \in \mathbb{R}$
Assume $f(h) = f(k)$

$$3h + 3 = 3k + 3$$

$$3h = 3k$$

$$h = k$$

$\therefore f$ *is injective*
Let $b \in \mathbb{R}$

$$3x + 3 = b$$

$$3x = b - 3$$

$$x = ((b - 3)/3) \in \mathbb{R}$$

$\therefore f$ *is surjective*
$\therefore f$ *is bijective*
$\blacksquare$

Problem 2
Consider the function $f : \mathbb{Z} \to \mathbb{N} : f(x) = x^4$
Prove that the function is not injective and not surjective.

Answer 2

Proof 2
Let $f : \mathbb{Z} \to \mathbb{N} : f(x) = x^4$ *be function*
Since $2^4 = 16$ *and* $-2^4 = 16$ *then* $f(2) = f(-2)$ *but* $2 \neq -2$
$\therefore f$ *is not injective*
Since $\nexists x \in \mathbb{Z} | x^4 = 2$
$\therefore f$ *is not surjective*
$\blacksquare$

Problem 3 *Define the relation* R *on* $\mathbb{Z}$ *by* $\forall a, b \in \mathbb{Z}, aRb \iff a^2 = b^2$.
Prove that R *is an equivalence relation.*

Answer 3

Proof 3
$\mathbb{Z}$ *by* $\forall a, b \in \mathbb{Z}, aRb \iff a^2 = b^2$
Let R *be defined as above*
Let $x, y, z \in \mathbb{Z}$

$$x^2 = x^2$$

$$x * x = x * x$$

$$x = x$$

$\therefore xRx$
$\Rightarrow R$ *is reflexive*
Assume xRy
Since xRy, $x^2 = y^2$
$\Rightarrow y^2 = x^2$
$\therefore yRx$
$\Rightarrow R$ *is symmetric*

Assume xRy and yRz
Since xRy, $x^2 = y^2$

$$yRx, y^2 = z^2$$

$\Rightarrow$ *Since* $x^2 = y^2$ *and* $y^2 = z^2, x^2 = z^2$
$\therefore$ *xRz*
$\Rightarrow$ *R is transitive*
$\therefore$ *R is an equivalence relation*
$\blacksquare$

Problem 4
Consider the function:
[x]=the greatest integer less than or equal to x
For example, $[2.3] = 2$ *and* $[-2.5] = -3$ *Define the relation R on* $\mathbb{R}$ *by aRb* $\iff [a] = [b]$
Prove this is an equivalence relation. Then, describe the equivalence classes.

Answer 4

Proof 4
$\exists a, b \in \mathbb{R},\ aRb \iff [a] = [b]$
Let R be defined as above
Let $x, y, z \in \mathbb{R}$

$$|x| = |x|$$

$$x = x$$

$\therefore$ *xRx*
$\Rightarrow$ *R is reflexive*
Assume xRy
Since xRy,

$$|x| = |y|$$

$$\Rightarrow |y| = |x|$$

$\therefore$*yRx*
$\Rightarrow$ *R is symmetric*
Assume xRy and yRz

Since xRy, $|x| = |y|$
yRz, $|y| = |z|$
$\Rightarrow$ *Since* $|y| = |x|$ *and* $|y| = |z|$, $|x| = |z|$
$\therefore xRz$
$\Rightarrow$ *R is transitive*
$\therefore$ *R is an equivalence relation.*
$\{x \in \mathbb{R} | \{(x >= a) \cap (x >= b)\} = |a| = |b|\}$ *is the equivalence class of R*
$\blacksquare$

Problem 5
Prove that congruence modulo n forms an equivalence relation on the set of integers.

Answer 5

Proof 5
$\forall x, y \in \mathbb{Z}, xRy \iff n/(x-y) \iff x \equiv y (\mod n)$
Let $\exists k \in \mathbb{Z}$ *s.t* $x - y = nk$
Let $a,b,c \in \mathbb{Z}$

$$a - a = nk$$
$$0 = nk$$
$$(0/k) = n$$
$$0 = n$$
$$0 = n * 0$$

$\therefore n/(a-a)$
$\therefore a \equiv a$
$\therefore aRa$
$\Rightarrow$ *R is reflexive*
Assume aRb

$$a - b = nk$$
$$-(a-b) = -(nk)$$
$$b - a = n(-k)$$

$\therefore n/(b-a)$
$\therefore b \equiv a$
$\therefore bRa$
$\Rightarrow$ *R is symmetric*
Assume aRb and bRc

$$a - b = nk$$

$$b - c = nj$$

$$-b = nk - a$$

$$b = ((nk - a)/-1)$$

$$b = -nk + a$$

$$b = n(-k) + a$$

$$(n(-k) + a) - c = nk$$

$$-nk + a - c = nk$$

$$a - c = nk + nk$$

$$a - c = n(k + k)$$

$$a - c = nk$$

$\therefore n/(a-c)$
$\therefore a \equiv c$
$\therefore aRc$
$\Rightarrow$ *R is transitive*
$\therefore$ *R is an equivalence relation*
$\blacksquare$