Sample Problem 2/1

The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 24t + 6$, where s is measured in meters from a convenient origin and t is in seconds. Determine:

(a) the time required for the particle to reach a velocity of 72 m/s from its initial condition at $t = 0$,

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We have,

$$s = 2t^3 - 24t + 6 \tag{1}$$
$$\tag{2}$$
$$v = \frac{ds}{dt} = 6t^2 - 24 \tag{3}$$
$$\tag{4}$$
$$a = \frac{d^2 s}{dt^2} = 12t \tag{5}$$

From the equation of velocity, we may substitute $v = 72 \text{ m/s}$

$$v = 6t^2 - 24 \tag{6}$$
$$\tag{7}$$
$$72 = 6t^2 - 24 \tag{8}$$
$$\tag{9}$$
$$72 + 24 = 6t^2 \tag{10}$$
$$\tag{11}$$
$$96 = 6t^2 \tag{12}$$
$$\tag{13}$$
$$\frac{96}{6} = t^2 \tag{14}$$
$$\tag{15}$$
$$16 = t^2 \tag{16}$$
$$\tag{17}$$
$$t = \pm 4 \tag{18}$$

Thus, the desired result is,

$$t = 4s$$

(b) Determine the acceleration of the particle when $v = 30 \text{ m/s}$

We may solve for the time, when $v = 30\ \mathrm{m/s}$,

$$
\begin{align}
v &= 6t^2 - 24 \tag{19}\\
\tag{20}\\
30 &= 6t^2 - 24 \tag{21}\\
\tag{22}\\
30 + 24 &= 6t^2 \tag{23}\\
\tag{24}\\
54 &= 6t^2 \tag{25}\\
\tag{26}\\
\frac{54}{6} &= t^2 \tag{27}\\
\tag{28}\\
9 &= t^2 \tag{29}\\
\tag{30}\\
t &= \pm 3 \tag{31}
\end{align}
$$

So it's at:

$t = 3s$

From the equation of acceleration, we may find the actual acceleration at $t = 3s$,

$$
\begin{align}
a &= 12t \tag{32}\\
\tag{33}\\
a &= 12(3) \tag{34}\\
\tag{35}\\
a &= 36\,\mathrm{m/s}^2 \tag{36}
\end{align}
$$

(c) Determine the net displacement of the particle during the interval from $t : (1s \to 4s)$

The net displacement is defined as:

$$\Delta s = s_4 - s_1 \tag{37}$$

Where,

$$\begin{cases} s(1) = 2(1)^3 - 24(1) + 6 = -16 \text{ m} \\ s(4) = 2(4)^3 - 24(4) + 6 = 38 \text{ m} \end{cases}$$

So

$$\begin{aligned} \Delta s &= s_4 - s_1 && (38) \\ &= 38 - (-16) && (39) \\ &= 54 \text{ m} && (40) \end{aligned}$$