

# STATS 205: Homework Assignment 5

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## Solution to Problem 1

We say that two observations  $X_1$  and  $X_2$  are *independent* of one another with respect to a collection of events  $\mathcal{A}$  if

$$Pr \{X_1 \in A \text{ and } X_2 \in B\} = Pr \{X_1 \in A\} Pr \{X_2 \in B\}$$

where A and B are any two not necessarily distinct sets of outcomes belonging to  $\mathcal{A}^3$ .

– 2.2.1 Independent Observations; Permutation, Parametric, and Bootstrap Tests of Hypotheses; Good, Phillip I

In deciding whether your own observations are exchangeable and a permutation test applicable, the key question is the one we posed in the very first chapter: Under the null hypothesis of no differences among the various experimental or survey groups, can we exchange the labels on the observations without significantly affecting the results?

– 2.2.2 Exchangeable Observations; Permutation, Parametric, and Bootstrap Tests of Hypotheses; Good, Phillip I

## Solution to Problem 2

```
cysticerici <- c(28.9, 32.8, 12.0, 9.9, 15.0, 38.0, 12.5, 36.5, 8.6, 26.8);cysticerici
```

```
## [1] 28.9 32.8 12.0 9.9 15.0 38.0 12.5 36.5 8.6 26.8
```

```
worms_reco <- c(1.0, 7.7, 7.3, 7.9, 1.1, 3.5, 18.9, 33.9, 28.6, 25.0); worms_reco
```

```
## [1] 1.0 7.7 7.3 7.9 1.1 3.5 18.9 33.9 28.6 25.0
```

The null hypothesis is that the mean weight of introduced cysticerici *has no correlation with* the mean weight of worms recovered. That is,

$$H_0 : \tau = 0$$

The alternative hypothesis is that the mean weight of introduced cysticerici is *positively correlated with* the mean weight of worms recovered. That is,

$$H_A : \tau > 0$$

To test the null hypothesis against the alternative hypothesis, we will use the Kendall test, a distribution-free test for independence based on signs.

```
cor.test(x = cysticerici, y = worms_reco, method = "kendall", alt = "greater")
```

```
##
```

```
## Kendall's rank correlation tau
```

```
##
```

```
## data: cysticerici and worms_reco
## T = 19, p-value = 0.7578
## alternative hypothesis: true tau is greater than 0
## sample estimates:
##      tau
## -0.1555556
```

The  $p$ -value is 0.7578, which is *not* significant at the  $\alpha = 0.05$  level. There is *not enough* evidence that the mean weight of introduced cysticerici is *positively correlated with* the mean weight of worms recovered.

### Solution to Problem 3

```
cysticerici <- c(28.9, 32.8, 12.0, 9.9, 15.0, 38.0, 12.5, 36.5, 8.6, 26.8)
worms_reco <- c(1.0, 7.7, 7.3, 7.9, 1.1, 3.5, 18.9, 33.9, 28.6, 25.0)
cor.test(x = cysticerici, y = worms_reco, method = "kendall", alt = "greater")
```

```
##
## Kendall's rank correlation tau
##
## data: cysticerici and worms_reco
## T = 19, p-value = 0.7578
## alternative hypothesis: true tau is greater than 0
## sample estimates:
##      tau
## -0.1555556
```

The estimate for  $\tau = -0.1555556$ .

### Solution to Problem 4

```
brain_weight = c(515, 286, 469, 410, 461, 436, 479, 198, 389, 262, 536); length(brain_weight)

## [1] 11

fiber_count = c(32500, 26800, 11410, 14850, 23640, 23820, 29840, 21830, 24650, 22500, 26000); length(fiber_count)

## [1] 11

library(bootstrap)
theta.hat = cor(brain_weight, fiber_count); theta.hat

## [1] 0.1604644

library(partitions)
n = length(brain_weight)
allCompositions = compositions(n, n); allCompositions[,1:length(brain_weight)]

##      [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10] [,11]
## [1,]   11   10    9    8    7    6    5    4    3    2    1
## [2,]    0    1    2    3    4    5    6    7    8    9   10
## [3,]    0    0    0    0    0    0    0    0    0    0    0
## [4,]    0    0    0    0    0    0    0    0    0    0    0
## [5,]    0    0    0    0    0    0    0    0    0    0    0
## [6,]    0    0    0    0    0    0    0    0    0    0    0
## [7,]    0    0    0    0    0    0    0    0    0    0    0
```

```
## [8,] 0 0 0 0 0 0 0 0 0 0 0 0
## [9,] 0 0 0 0 0 0 0 0 0 0 0 0
## [10,] 0 0 0 0 0 0 0 0 0 0 0 0
## [11,] 0 0 0 0 0 0 0 0 0 0 0 0

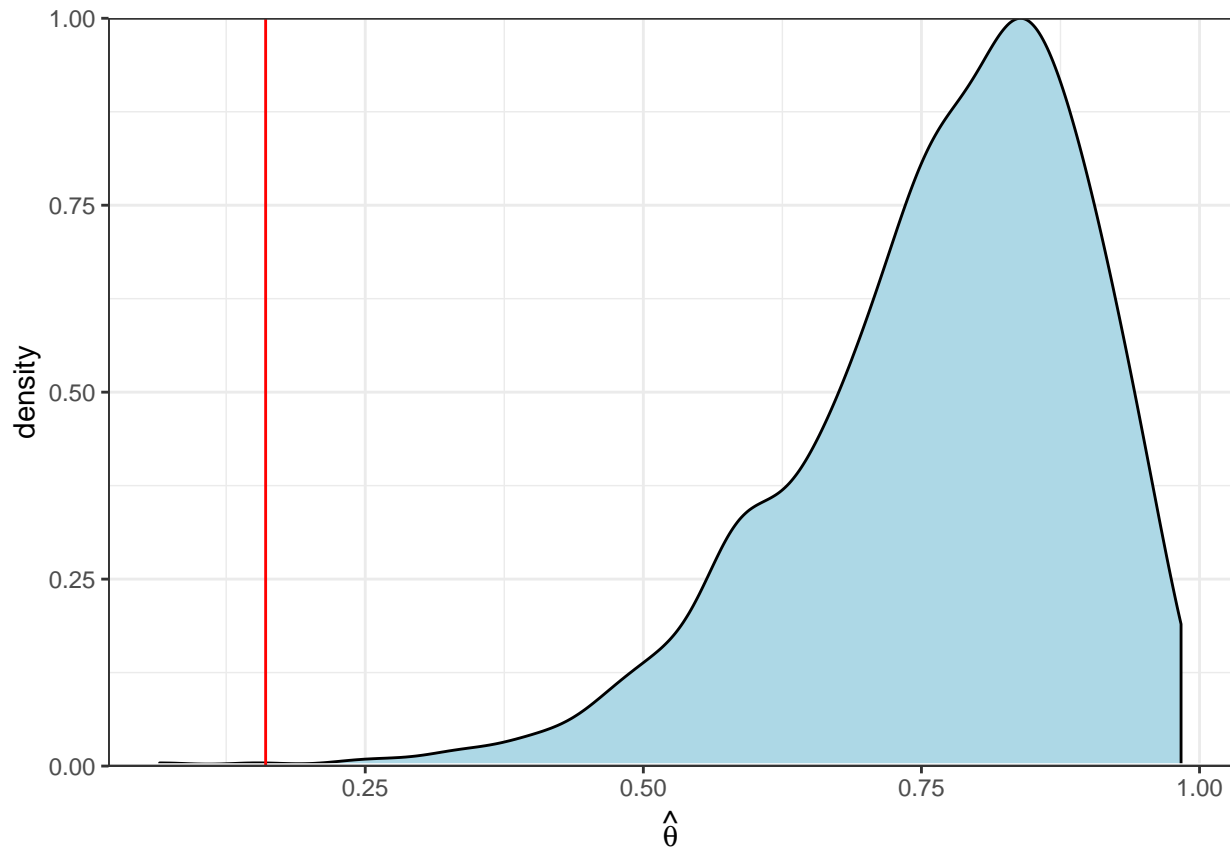
allCompositions.sub = allCompositions[, sample(1:dim(allCompositions)[2], size=1000, replace=FALSE)]

draw.bootstrap.samples = function(df){
  n = dim(df)[1]
  ind = sample(n, replace = TRUE)
  cor.bootstrap.replicate = cor(df[ind, "LSAT"], df[ind, "GPA"])
  return(cor.bootstrap.replicate)
}

R = 1000
theta.hat.star = replicate(R, draw.bootstrap.samples(law))
# make a ggplot
library(ggplot2)

## Registered S3 methods overwritten by 'ggplot2':
##   method      from
##   [.quosures   rlang
##   c.quosures   rlang
##   print.quosures rlang

theta.hat.star.df = data.frame(theta.hat.star = theta.hat.star)
ggplot(theta.hat.star.df) +
  geom_density(aes(x = theta.hat.star, y = ..scaled..),
    fill = "lightblue") +
  geom_hline(yintercept=0, colour="white", size=1) +
  theme_bw() +
  ylab("density") +
  xlab(bquote(hat(theta))) +
  geom_vline(xintercept = theta.hat, col = "red")+
  scale_y_continuous(expand = c(0,0))
```



## Solution to Problem 5

```
cysticerici <- c(28.9, 32.8, 12.0, 9.9, 15.0, 38.0, 12.5, 36.5, 8.6, 26.8)
worms_reco <- c(1.0, 7.7, 7.3, 7.9, 1.1, 3.5, 18.9, 33.9, 28.6, 25.0)
```

The null hypothesis is that the mean weight of introduced cysticerici *has no correlation with* the mean weight of worms recovered. That is,

$$H_0 : r_s < r_{s,\alpha}$$

The alternative hypothesis is that the mean weight of introduced cysticerici is *positively correlated with* the mean weight of worms recovered. That is,

$$H_A : r_s \geq r_{s,\alpha}$$

Otherwise, do not reject.

To test the null hypothesis against the alternative hypothesis, we will use the Spearman test, a distribution-free test for independence based on ranks.

```
# this method of performing the test was given in the textbook
library(SuppDists)
qSpearman(p = 0.05, r = 10)
```

```
## [1] -0.5393939
```

Since  $r_{s,\alpha} = -0.5393939$ , we will reject the null hypothesis only if  $r_s \geq -0.5393939$ .

Calculating  $r_s$ ,

```
cor(x = cysticerici, y = worms_reco, method = "spearman")
```

```
## [1] -0.2
```

Since  $r_s = -0.2$  and  $r_{s,\alpha} = -0.5393939$ , the statement  $r_s \geq r_{s,\alpha}$  is *true*. Thus, we *reject* the null hypothesis. There is *sufficient* evidence that the mean weight of introduced cysticerici is *positively correlated with* the mean weight of worms recovered.

---

**NOTE:** At this point, I tried to use `cor.test()` with `method = "spearman"` but I got a different result than I expected, and I'm not sure why. Maybe I'm interpreting the output incorrectly?

```
cor.test(x = cysticerici, y = worms_reco, method = "spearman", alternative = "greater")
```

```
##  
## Spearman's rank correlation rho  
##  
## data: cysticerici and worms_reco  
## S = 198, p-value = 0.72  
## alternative hypothesis: true rho is greater than 0  
## sample estimates:  
## rho  
## -0.2
```

The  $p$ -value is 0.72, which is *not* significant at the  $\alpha = 0.05$  level. There is *not enough* evidence that the mean weight of introduced cysticerici is *positively correlated with* the mean weight of worms recovered.

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## Solution to Problem 6

```
x = c(0, 5000, 10000, 15000, 20000, 25000, 30000, 100000)  
y = c(0.924, 0.988, 0.992, 1.118, 1.133, 1.145, 1.157, 1.357)
```

The null hypothesis is that the mean weight of introduced cysticerici *has no correlation with* the mean weight of worms recovered. That is,

$$H_0 : \beta = \beta_0$$

$$H_0 : \beta = 0$$

The alternative hypothesis is that the mean weight of introduced cysticerici is *positively correlated with* the mean weight of worms recovered. That is,

$$H_A : \beta > \beta_0$$

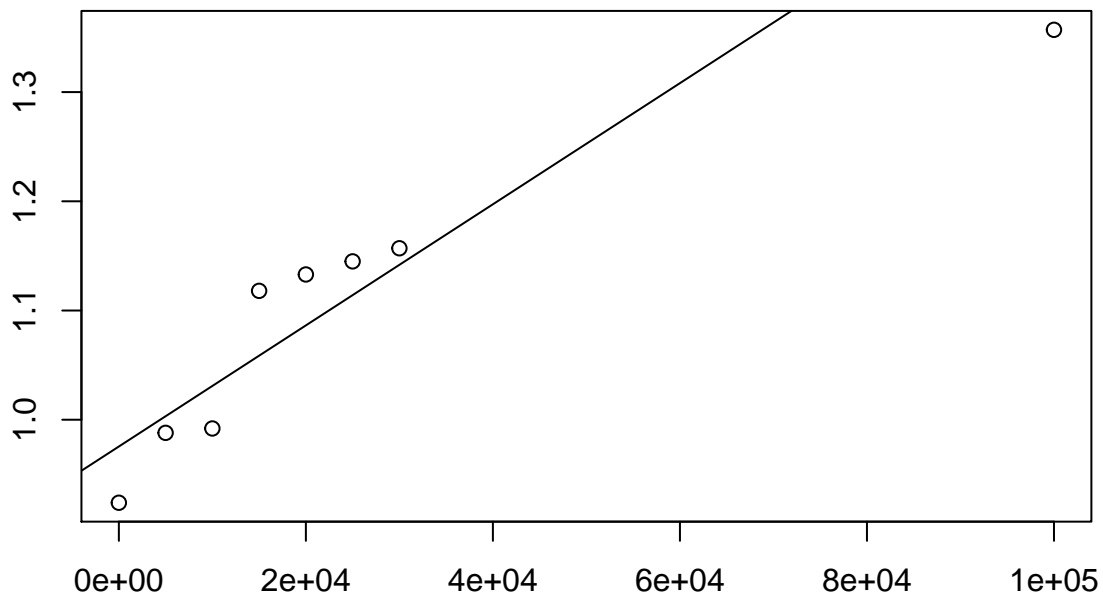
$$H_A : \beta > 0$$

To test the null hypothesis against the alternative hypothesis, we will use the Theil test, a distribution-free test for the slope of the regression line.

```
library(NSM3)
```

```
## Loading required package: combinat
##
## Attaching package: 'combinat'
## The following object is masked from 'package:utils':
##
##     combn
## Loading required package: MASS
## Loading required package: survival
## fANCOVA 0.5-1 loaded
```

```
theil(x, y, alpha=0.05, beta.0=0, type = "u")
```



```
## Alternative: beta greater than 0
## C = 28, C.bar = 1, P = 0
## beta.hat = 0
## alpha.hat = 0.975
##
## 1 - alpha = 0.95 upper bound for beta:
## -Inf, 0
```

```
theil.fit = theil (x,
  y,
  beta.0 = 0 ,
  slopes=TRUE,
  type = "u",
  doplot = FALSE)
theil.fit
```

```
## Alternative: beta greater than 0
## C = 28, C.bar = 1, P = 0
## beta.hat = 0
## alpha.hat = 0.975
##
## All slopes:
```

```
## i j      S.ij
## 1 2 1.280000e-05
## 1 3 6.800000e-06
## 1 4 1.293333e-05
## 1 5 1.045000e-05
## 1 6 8.840000e-06
## 1 7 7.766667e-06
## 1 8 4.330000e-06
## 2 3 8.000000e-07
## 2 4 1.300000e-05
## 2 5 9.666667e-06
## 2 6 7.850000e-06
## 2 7 6.760000e-06
## 2 8 3.884211e-06
## 3 4 2.520000e-05
## 3 5 1.410000e-05
## 3 6 1.020000e-05
## 3 7 8.250000e-06
## 3 8 4.055556e-06
## 4 5 3.000000e-06
## 4 6 2.700000e-06
## 4 7 2.600000e-06
## 4 8 2.811765e-06
## 5 6 2.400000e-06
## 5 7 2.400000e-06
## 5 8 2.800000e-06
## 6 7 2.400000e-06
## 6 8 2.826667e-06
## 7 8 2.857143e-06
##
##
## 1 - alpha = 0.95 upper bound for beta:
## -Inf, 0
```

```
theil.output = theil(x,
  y,
  beta.0 = 0,
  slopes=TRUE,
  type = "u", doplot = FALSE, alpha = .05)
c(theil.output$L, theil.output$U)
```

```
## [1] -Inf    0
```

TODO: Interpret these results correctly.

## Solution to Problem 7

```
height = c(42.8, 63.5, 37.5, 39.5, 45.5, 38.5, 43.0, 22.5, 37.0, 23.5, 33.0, 58.0)
weight = c(40.0, 93.5, 35.5, 30.0, 52.0, 17.0, 38.5, 8.5, 33.0, 9.5, 21.0, 79.0)
heart_catheter_length = c(37.0, 49.5, 34.5, 36.0, 43.0, 28.0, 37.0, 20.0, 33.5, 30.5, 38.5, 47.0)

cor.test(x = height, y = heart_catheter_length, method = "pearson")
```

```
##
```

```
## Pearson's product-moment correlation
##
## data: height and heart_catheter_length
## t = 5.8936, df = 10, p-value = 0.0001524
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## 0.6216270 0.9663721
## sample estimates:
## cor
## 0.8811691

cor.test(x = weight, y = heart_catheter_length, method = "pearson")
```

```
##
## Pearson's product-moment correlation
##
## data: weight and heart_catheter_length
## t = 6.3033, df = 10, p-value = 8.871e-05
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## 0.6568763 0.9700971
## sample estimates:
## cor
## 0.8938226
```

From the Pearson correlation tests, there is *strong evidence* that, individually, height and weight contribute to the determination of heart catheter length.

```
library(Rfit)

r.01 <- rfit(heart_catheter_length ~ height)
f.01 <- rfit(heart_catheter_length ~ height + weight)
first_drop_test <- drop.test(f.01, r.01)
first_drop_test
```

```
##
## Drop in Dispersion Test
## F-Statistic      p-value
## 1.55202          0.24429
```

```
r.02 <- rfit(heart_catheter_length ~ weight)
second_drop_test <- drop.test(f.01, r.02)
second_drop_test
```

```
##
## Drop in Dispersion Test
## F-Statistic      p-value
## 0.014435         0.907007
```

However, based on the large p-values from the Drop in Dispersion tests, there is *not enough evidence* to suggest that height or weight contribute significantly over each other to the determination of heart catheter length.



**Note**

Treating length of heart catheter as the *independent* variable, test for the importance of height and weight in *determining* the required catheter length.

If height and weight are the *determiners* of length of heart catheter, length of heart catheter must be the *dependent* variable.