

# STATS 205: Homework Assignment 3

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## Solution to Problem 1

(i)

```
library(bootstrap); data(law)
t(law)
```

```
##           1      2      3      4      5      6      7      8      9     10
## LSAT 576.00 635.0 558.00 578.00 666.00 580.00 555 661.00 651.00 605.00
## GPA   3.39   3.3   2.81   3.03   3.44   3.07   3   3.43   3.36   3.13
##           11     12     13     14     15
## LSAT 653.00 575.00 545.00 572.00 594.00
## GPA   3.12   2.74   2.76   2.88   2.96
```

```
theta.hat = cor(law$LSAT, law$GPA); theta.hat
```

```
## [1] 0.7763745
```

```
library(partitions)
```

```
n = 15
```

```
allCompositions = compositions(n, n); allCompositions[,1:5]
```

```
##           [,1] [,2] [,3] [,4] [,5]
## [1,]      15   14   13   12   11
## [2,]       0    1    2    3    4
## [3,]       0    0    0    0    0
## [4,]       0    0    0    0    0
## [5,]       0    0    0    0    0
## [6,]       0    0    0    0    0
## [7,]       0    0    0    0    0
## [8,]       0    0    0    0    0
## [9,]       0    0    0    0    0
## [10,]      0    0    0    0    0
## [11,]      0    0    0    0    0
## [12,]      0    0    0    0    0
## [13,]      0    0    0    0    0
## [14,]      0    0    0    0    0
## [15,]      0    0    0    0    0
```

```
allCompositions.sub = allCompositions[, sample(1:dim(allCompositions)[2], size=10000, replace=FALSE)]
```

```
draw.bootstrap.samples = function(df){
  n = dim(df)[1]
  ind = sample(n, replace = TRUE)
  cor.bootstrap.replicate = cor(df[ind, "LSAT"], df[ind, "GPA"])
  return(cor.bootstrap.replicate)
}
```

```
R = 10000
```

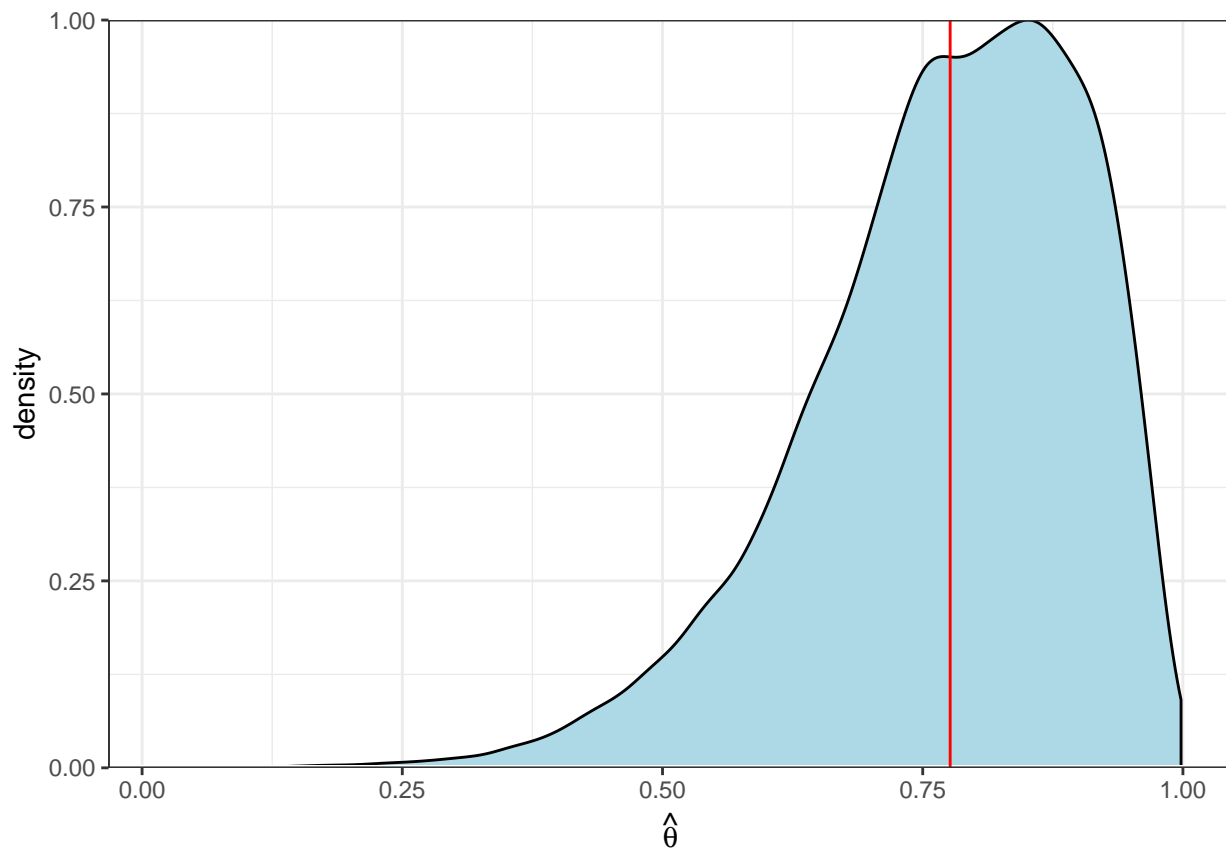
```

theta.hat.star = replicate(R, draw.bootstrap.samples(law))
# make a ggplot
library(ggplot2)

## Registered S3 methods overwritten by 'ggplot2':
##   method      from
##   [.quosures   rlang
##   c.quosures   rlang
##   print.quosures rlang

theta.hat.star.df = data.frame(theta.hat.star = theta.hat.star)
ggplot(theta.hat.star.df) +
  geom_density(aes(x = theta.hat.star, y = ..scaled..),
    fill = "lightblue") +
  geom_hline(yintercept=0, colour="white", size=1) +
  theme_bw() +
  ylab("density") +
  xlab(bquote(hat(theta))) +
  geom_vline(xintercept = theta.hat, col = "red")+
  scale_y_continuous(expand = c(0,0))

```



(ii)

```
sd(theta.hat.star)
```

```
## [1] 0.1322028
```

## Solution to Problem 2

(i)

67 runs resulting in swallowing attempts  
58 successful  
9 failed

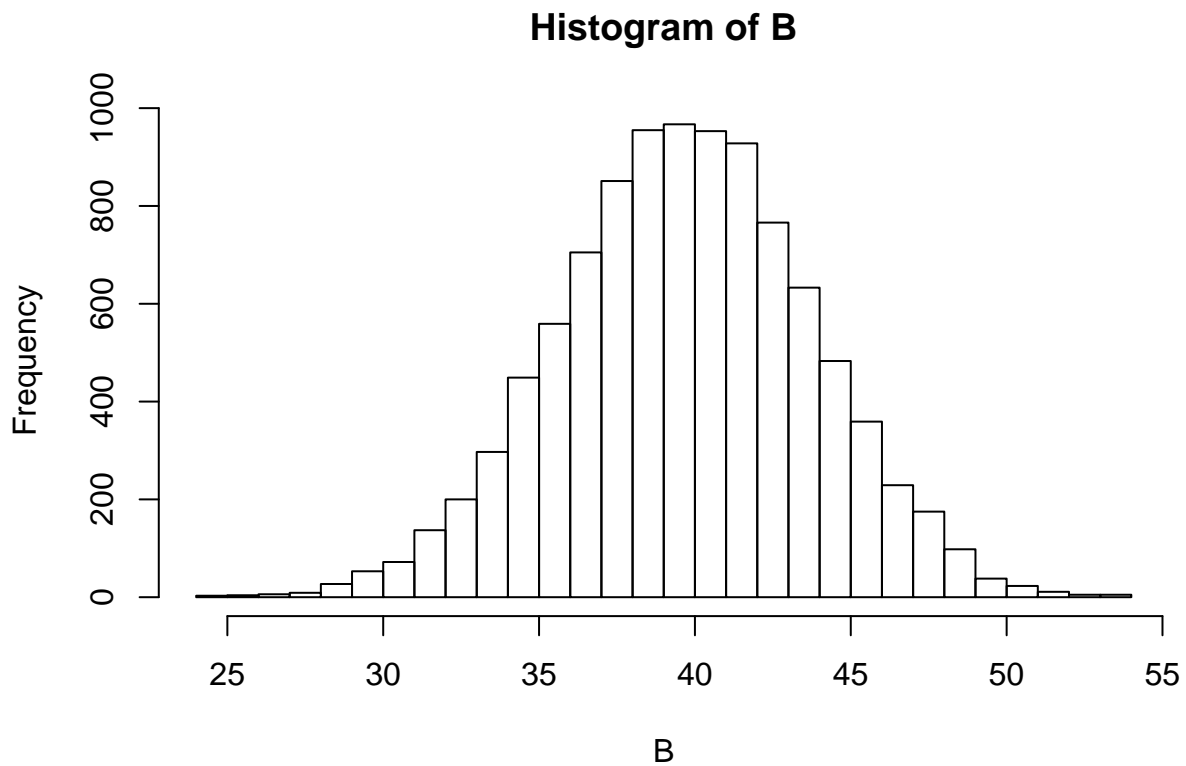
$H_0 : p = 0.6$

$H_A : p > 0.6$

```
n = 67
successes = 58
pbar = successes / n; pbar
```

```
## [1] 0.8656716
```

```
p0 = 0.6; nsim = 10000
B = rbinom(nsim, size = n, prob = p0)
hist(B, breaks = 30)
```



Test statistic  $Z$ :

$$Z_0 = \frac{B - 67(0.6)}{(67(0.6)(0.4))^{\frac{1}{2}}}$$

```
qnorm((1-0.05), mean = 0, sd = 1)
```

```
## [1] 1.644854
```

Rejection region:  $Z \geq z_{0.05} = 1.645$

Observed test statistic  $Z_o$ :

$$Z_o = \frac{58 - 67(0.6)}{(67(0.6)(0.4))^{\frac{1}{2}}} = 4.44$$

```
numerator = successes - (n * p0)
denominator = sqrt(n * p0 * (1.0 - p0))
Z.obs = numerator / denominator; Z.obs
```

```
## [1] 4.438917
```

The large sample approximation value  $Z_o = 2.5 > 1.645$  and thus we reject  $H_0 : p = 0.6$  in favor of  $p > 0.6$  at the approximate  $\alpha = 0.05$  level. Thus there is evidence that the success rate of swallowing attempts is greater than 0.6.

(ii)

Power is the probability of rejecting  $H_0$  when  $H_A$  is true. We found that test reject  $H_0$  is  $Z \geq z_{0.05} = 1.645$ . Therefore, if  $p = 0.7$ ,

$$Z_o = \frac{58 - 67(0.6)}{(67(0.6)(0.4))^{\frac{1}{2}}} = 4.44$$

is no longer standard normal.

We have

$$Z_{o7} = \frac{58 - 67(0.7)}{(67(0.7)(0.3))^{\frac{1}{2}}} = 2.96$$

```
p1 = 0.7
numerator = successes - (n * p1)
denominator = sqrt(n * p1 * (1.0 - p1))
Z.obs.seven = numerator / denominator; Z.obs.seven
```

```
## [1] 2.959211
```

$$Power = P(Z \geq 1.645 | p = 0.7)$$

$$= P_{p=0.7} \left( \frac{B - 67(0.6)}{(67(0.6)(0.4))^{\frac{1}{2}}} \geq 1.645 \right)$$

$$= P_{p=0.7} (B \geq 1.645(67(0.6)(0.4))^{\frac{1}{2}} + 67(0.6))$$

$$= P_{p=0.7} \left( \frac{B - 67(0.7)}{(67(0.7)(0.3))^{\frac{1}{2}}} \geq \frac{1.645(67(0.6)(0.4))^{\frac{1}{2}} + 67(0.6) - 67(0.7)}{(67(0.7)(0.3))^{\frac{1}{2}}} \right)$$

```
triple_product = n * p0 * (1.0 - p0)
first_term = 1.645 * sqrt(triple_product)
second_term = n * p0
third_term = n * p1
bottom_term = n * p1 * (1.0 - p1)
```

```
p7_numerator = first_term + second_term - third_term
p7_denominator = sqrt(bottom_term)
Pp_7_zvalue = p7_numerator / p7_denominator; Pp_7_zvalue
```

```
## [1] -0.02761144
```

$$P(Z^* \geq -0.0276) = 0.4890$$

```
# pvalue = pnorm(-abs(Pp_7_zvalue)); pvalue
pvalue = pnorm(Pp_7_zvalue); pvalue
```

```
## [1] 0.488986
```

If  $p = 0.8$ ,

$$Power = P(Z \geq 1.645 | p = 0.8)$$

$$= P_{p=0.8} \left( \frac{B - 67(0.8)}{(67(0.8)(0.2))^{\frac{1}{2}}} \geq \frac{1.645(67(0.6)(0.4))^{\frac{1}{2}} + 67(0.6) - 67(0.8)}{(67(0.8)(0.2))^{\frac{1}{2}}} \right)$$

```
p2 = 0.8
triple_product = n * p0 * (1.0 - p0)
first_term = 1.645 * sqrt(triple_product)
second_term = n * p0
third_term = n * p2
bottom_term = n * p2 * (1.0 - p2)
p8_numerator = first_term + second_term - third_term
p8_denominator = sqrt(bottom_term)
Pp_8_zvalue = p8_numerator / p8_denominator; Pp_8_zvalue
```

```
## [1] -2.077971
```

$$P(Z^* \geq -2.078) = 0.01886$$

```
# pvalue = pnorm(-abs(Pp_7_zvalue)); pvalue
pvalue = pnorm(Pp_8_zvalue); pvalue
```

```
## [1] 0.01885601
```

### Solution to Problem 3

**Summary:** Estimate for  $\hat{p} = 0.8615$  and estimate for standard deviation of  $\hat{p} = 0.04284$ .

Estimate for  $p$  using binomial confidence interval, `binom.confint()`:

```
library(binom)
binom.confint(x=56, n=65, conf.level=.95, methods = "asymptotic")
```

```
##      method x  n      mean      lower      upper
## 1 asymptotic 56 65 0.8615385 0.7775744 0.9455025
```

$$\hat{p} = (0.7776, 0.9455)$$

Estimate for  $p$  using 1-sample proportions test without continuity correction, `prop.test()`:

```
prop.test(x=56, n=65, p = 0.6, conf.level=0.95, alternative = c("greater"))
```

```
##
## 1-sample proportions test with continuity correction
##
## data: 56 out of 65, null probability 0.6
## X-squared = 17.452, df = 1, p-value = 1.473e-05
## alternative hypothesis: true p is greater than 0.6
## 95 percent confidence interval:
## 0.7676875 1.0000000
## sample estimates:
## p
## 0.8615385
```

$$p = 0.8615$$

Estimate for  $p$  using Exact Binomial Test:

```
binom.test(x=56, n=65, p = 0.6, alternative = c("greater"), conf.level = 0.95)
```

```
##
## Exact binomial test
##
## data: 56 and 65
## number of successes = 56, number of trials = 65, p-value =
## 4.096e-06
## alternative hypothesis: true probability of success is greater than 0.6
## 95 percent confidence interval:
## 0.7708174 1.0000000
## sample estimates:
## probability of success
## 0.8615385
```

Standard error of  $\hat{p}$  is:

$$\begin{aligned} & \sqrt{\frac{p(1-p)}{n}} \\ &= \sqrt{\frac{(0.6)(0.4)}{65}} \\ &= 0.06076 \end{aligned}$$

```
p = 0.6
n = 65
numerator = p * (1 - p)
denominator = n
answer = sqrt(numerator/denominator); answer
```

```
## [1] 0.06076436
```

and estimate is:

$$\begin{aligned} & \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ &= \sqrt{\frac{(0.8615)(1-0.8615)}{65}} \\ &= 0.04284 \end{aligned}$$

```
p.hat = 0.8615
n = 65
numerator = p.hat * (1 - p.hat)
denominator = n
answer = sqrt(numerator/denominator); answer
```

```
## [1] 0.04284458
```

## Solution to Problem 4

```
binom.confint(x = 56, n = 65, conf.level = 0.96, methods = "all")
```

```
##           method x  n      mean      lower      upper
## 1  agresti-coull 56 65 0.8615385 0.7488973 0.9301180
## 2    asymptotic 56 65 0.8615385 0.7735567 0.9495202
## 3         bayes 56 65 0.8560606 0.7655984 0.9375798
## 4      cloglog 56 65 0.8615385 0.7439982 0.9276404
## 5        exact 56 65 0.8615385 0.7480632 0.9371740
## 6         logit 56 65 0.8615385 0.7484912 0.9286194
## 7        probit 56 65 0.8615385 0.7545841 0.9312980
## 8        profile 56 65 0.8615385 0.7589798 0.9334925
## 9          lrt 56 65 0.8615385 0.7589836 0.9335307
## 10   prop.test 56 65 0.8615385 0.7483484 0.9308913
## 11         wilson 56 65 0.8615385 0.7514483 0.9275670
```

Here are the rows relevant to our problem:

```
##           method x  n      mean      lower      upper
## 1  agresti-coull 56 65 0.8615385 0.7488973 0.9301180
## 2    asymptotic 56 65 0.8615385 0.7735567 0.9495202
## 5        exact 56 65 0.8615385 0.7480632 0.9371740
## 11         wilson 56 65 0.8615385 0.7514483 0.9275670
```

where `asymptotic` is Laplace-Wald, `agresti-coull` is Agresti-Coull, `exact` is Clopper-Pearson, and `wilson` is Wilson.

It looks like `agresti-coull` and `exact` are fairly similar in terms of the location of the interval, while `asymptotic` is skewed towards the “right” side of the intervals, and `wilson` has the smallest range.

## Solution to Problem 5

```
plant_vector <- c(926, 288, 293, 104)
expected = c(9/16, 3/16, 3/16, 1/16)
```

```
# goodness-of-fit test
```

```
res <- chisq.test(x = plant_vector, p = expected); res
```

```
##
```

```
## Chi-squared test for given probabilities
```

```
##
```

```
## data: plant_vector
```

```
## X-squared = 1.4687, df = 3, p-value = 0.6895
```

```
res$expected
```

```
## [1] 906.1875 302.0625 302.0625 100.6875
```

```
res$observed
```

```
## [1] 926 288 293 104
```

The **p-value** of the test is  $p = 0.6895$ , which is more than the significance level  $\alpha = 0.05$ . We cannot conclude that the data does *not* support Mendelian theory at the  $\alpha = 0.05$  level.

## Solution to Problem 6

```
df = data.frame(Low = c(0, 2), High = c(5, 1))
```

```
rownames(df) = c("Multiple attack", "Primary attack"); df
```

```
##           Low High
```

```
## Multiple attack    0    5
```

```
## Primary attack     2    1
```

```
fisher.test(df, alternative = "less")
```

```
##
```

```
## Fisher's Exact Test for Count Data
```

```
##
```

```
## data: df
```

```
## p-value = 0.1071
```

```
## alternative hypothesis: true odds ratio is less than 1
```

```
## 95 percent confidence interval:
```

```
##  0.00000 1.75251
```

```
## sample estimates:
```

```
## odds ratio
```

```
##           0
```

There is *not enough evidence* that the unknown probability that a multiple-attack patient will have low reactivity is *less than* the unknown probability that a primary-attack patient will have low reactivity. The  $P$ -value achieved by these data if we use Fisher's exact test of  $H_0$  against the alternative  $p_1 < p_2$  is 0.1071.

```
df = data.frame(Low = c(0, 2), High = c(5, 1))
```

```
rownames(df) = c("Multiple attack", "Primary attack"); df
```

```
##           Low High
```

```
## Multiple attack    0    5
```

```
## Primary attack     2    1
```

```
fisher.test(df, alternative = "two.sided")
```



```
##
## Fisher's Exact Test for Count Data
##
## data:  df
## p-value = 0.1071
## alternative hypothesis: true odds ratio is not equal to 1
## 95 percent confidence interval:
##  0.00000 2.75184
## sample estimates:
## odds ratio
##          0
```

The  $P$ -value achieved when testing the null against a two-sided hypothesis is the same, and therefore, there is *not enough evidence* that the unknown probability that a multiple-attack patient will have low reactivity is *not equal* to the unknown probability that a primary-attack patient will have low reactivity.

## Solution to Problem 7