STATS 205: Homework Assignment 5

Brian Liu

6/10/2019

Solution to Problem 1

> We say that two observations X_1 and X_2 are *independent* of one another with respect to a collection of events $\mathcal{A}$ if
>
> $$Pr\left\{X_1 \in A \text{ and } X_2 \in B\right\} = Pr\left\{X_1 \in A\right\} Pr\left\{X_2 \in B\right\}$$
>
> where A and B are any two not necessarily distinct sets of outcomes belonging to $\mathcal{A}^3$.

– 2.2.1 Independent Observations; Permutation, Parametric, and Bootstrap Tests of Hypotheses; Good, Phillip I

> In deciding whether your own observations are exchangeable and a permutation test applicable, the key question is the one we posed in the very first chapter: Under the null hypothesis of no differences among the various experimental or survey groups, can we exchange the labels on the observations without significantly affecting the results?

– 2.2.2 Exchangeable Observations; Permutation, Parametric, and Bootstrap Tests of Hypotheses; Good, Phillip I

Solution to Problem 2

```
cysticerci <- c(28.9, 32.8, 12.0, 9.9, 15.0, 38.0, 12.5, 36.5, 8.6, 26.8);cysticerci
```

```
##  [1] 28.9 32.8 12.0  9.9 15.0 38.0 12.5 36.5  8.6 26.8
```

```
worms_reco <- c(1.0, 7.7, 7.3, 7.9, 1.1, 3.5, 18.9, 33.9, 28.6, 25.0); worms_reco
```

```
##  [1]  1.0  7.7  7.3  7.9  1.1  3.5 18.9 33.9 28.6 25.0
```

```
length(cysticerci)
```

```
## [1] 10
```

```
length(worms_reco)
```

```
## [1] 10
```

The null hypothesis is that the mean weight of introduced cysticerci *has no correlation with* the mean weight of worms recovered. That is,

$$H_0 : \tau = 0$$

The alternative hypothesis is that the mean weight of introduced cysticerci is *positively correlated with* the mean weight of worms recovered. That is,

$$H_A : \tau > 0$$

To test the null hypothesis against the alternative hypothesis, we will use the Kendall test, a distribution-free test for independence based on signs.

```
cor.test(x = cysticerci, y = worms_reco, method = "kendall", alt = "greater")
```

```
##
##  Kendall's rank correlation tau
##
## data:  cysticerci and worms_reco
## T = 19, p-value = 0.7578
## alternative hypothesis: true tau is greater than 0
## sample estimates:
##        tau
## -0.1555556
```

The p-value is 0.7578, which is *not* significant at the $\alpha = 0.05$ level. There is *not enough* evidence that the mean weight of introduced cysticerci is *positively correlated with* the mean weight of worms recovered.

Solution to Problem 3

```
cor.test(x = cysticerci, y = worms_reco, method = "kendall", alt = "greater")
```

```
##
##  Kendall's rank correlation tau
##
## data:  cysticerci and worms_reco
## T = 19, p-value = 0.7578
## alternative hypothesis: true tau is greater than 0
## sample estimates:
##        tau
## -0.1555556
```

Solution to Problem 4