STATS 205: Homework Assignment 5

Brian Liu 6/10/2019

Solution to Problem 1

We say that two observations X_1 and X_2 are independent of one another with respect to a collection of events A if

$$Pr\{X_1 \in A \text{ and } X_2 \in B\} = Pr\{X_1 \in A\} Pr\{X_2 \in B\}$$

where A and B are any two not necessarily distinct sets of outcomes belonging to A^3 .

- 2.2.1 Independent Observations; Permutation, Parametric, and Bootstrap Tests of Hypotheses; Good, Phillip I

In deciding whether your own observations are exchangeable and a permutation test applicable, the key question is the one we posed in the very first chapter: Under the null hypothesis of no differences among the various experimental or survey groups, can we exchange the labels on the observations without significantly affecting the results?

– 2.2.2 Exchangeable Observations; Permutation, Parametric, and Bootstrap Tests of Hypotheses; Good, Phillip I

Solution to Problem 2

```
cysticerci <- c(28.9, 32.8, 12.0, 9.9, 15.0, 38.0, 12.5, 36.5, 8.6, 26.8);cysticerci 
## [1] 28.9 32.8 12.0 9.9 15.0 38.0 12.5 36.5 8.6 26.8
worms_reco <- c(1.0, 7.7, 7.3, 7.9, 1.1, 3.5, 18.9, 33.9, 28.6, 25.0); worms_reco
```

[1] 1.0 7.7 7.3 7.9 1.1 3.5 18.9 33.9 28.6 25.0

The null hypothesis is that the mean weight of introduced cysticerci has no correlation with the mean weight of worms recovered. That is,

$$H_0: \tau = 0$$

The alternative hypothesis is that the mean weight of introduced cysticerci is *positively correlated with* the mean weight of worms recovered. That is,

$$H_A: \tau > 0$$

To test the null hypothesis against the alternative hypothesis, we will use the Kendall test, a distribution-free test for independence based on signs.

```
cor.test(x = cysticerci, y = worms_reco, method = "kendall", alt = "greater")
```

Kendall's rank correlation tau

##

```
## data: cysticerci and worms_reco
## T = 19, p-value = 0.7578
## alternative hypothesis: true tau is greater than 0
## sample estimates:
## tau
## -0.1555556
```

The p-value is 0.7578, which is not significant at the $\alpha = 0.05$ level. There is not enough evidence that the mean weight of introduced cysticerci is positively correlated with the mean weight of worms recovered.

Solution to Problem 3

```
cysticerci <- c(28.9, 32.8, 12.0, 9.9, 15.0, 38.0, 12.5, 36.5, 8.6, 26.8); cysticerci
## [1] 28.9 32.8 12.0 9.9 15.0 38.0 12.5 36.5 8.6 26.8
worms_reco <- c(1.0, 7.7, 7.3, 7.9, 1.1, 3.5, 18.9, 33.9, 28.6, 25.0); worms_reco
   [1] 1.0 7.7 7.3 7.9 1.1 3.5 18.9 33.9 28.6 25.0
cor.test(x = cysticerci, y = worms_reco, method = "kendall", alt = "greater")
##
##
   Kendall's rank correlation tau
##
## data: cysticerci and worms_reco
## T = 19, p-value = 0.7578
## alternative hypothesis: true tau is greater than 0
## sample estimates:
##
         tau
## -0.155556
The estimate for \tau = -0.1555556.
```

Solution to Problem 4