Decision Analysis 1—Party Sensitivity

Decision Analysis

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Risk Odds

- Posutlate the exponential utility function

$$u(x) = a - be^{-\gamma x} = a - b\left(e^{\gamma}\right)^{-x} = a - br^{-x}$$

- where

$$r = e^{\gamma}$$

- so that

$$u(x) = a - br^{-x}$$

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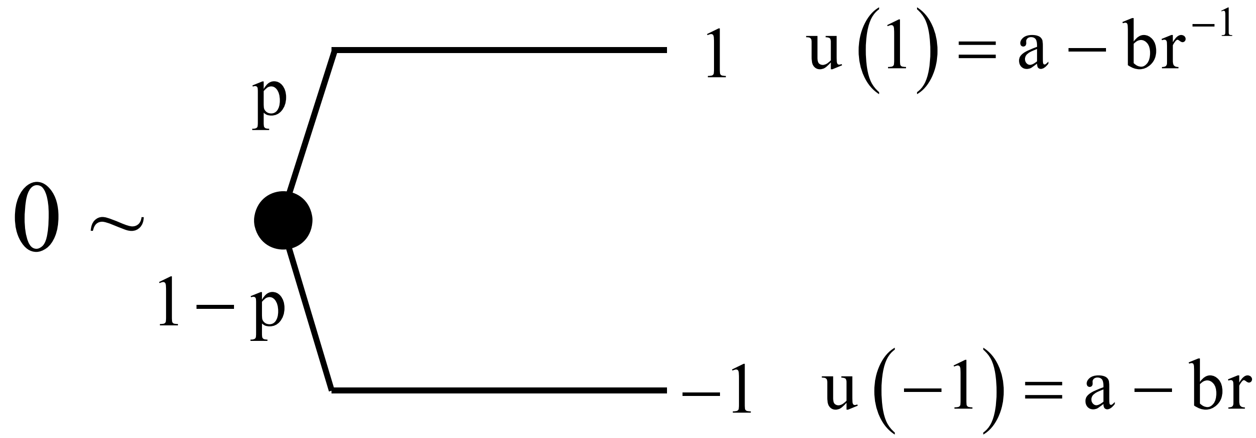
With This Utility Function

- Consider the lottery



- The risk odds of a dollar, r(1)=r, are calculated from this indifference relationship

$$\langle u \rangle = p\left(a - br^{-1}\right) + (1-p)(a - br)$$

$$u(0) = p(a-b) + (1-p)(a-b) = a - b$$

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Equating and Simplifying

$$\langle u \rangle = p\left(a - br^{-1}\right) + (1-p)(a-br) = p(a-b) + (1-p)(a-b) = a - b = u(0)$$

$$p\left(a - br^{-1}\right) + (1-p)(a-br) = a - b$$

$$pr^{-1} + (1-p)r = 1$$

- This defines a quadratic

$$(1-p)r^2 - r + p = 0$$

- This quadratic is solved for

$$r = 1 \qquad r = \frac{p}{1-p}$$

Interpretation of Risk Odds

- $r = p/(1-p)$ means that when r is equal to the odds of winning one monetary unit versus losing one monetary unit, the person is indifferent between accepting and rejecting the deal.
- r is the risk odds of a person satisfying the delta property.
- If a person satisfies the delta property then his or her entire utility curve is characterized by the risk odds.
- Risk odds focuses on the incremental $1

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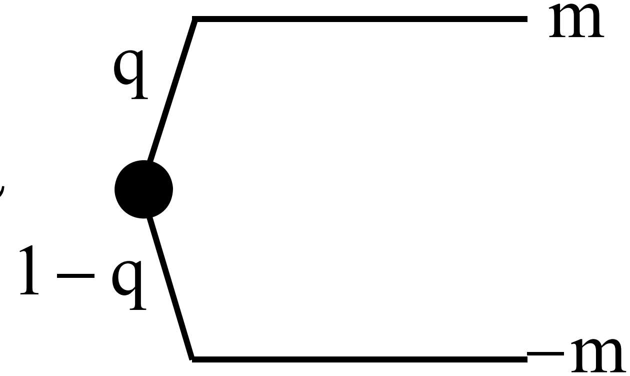
With This Utility Function

- Consider the lottery

$0 \sim$ 

$$u(m) = a - br^{-m}$$

$$u(-m) = a - br^{m}$$

$$\langle u \rangle = q\left(a - br^{-m}\right) + (1-q)\left(a - br^{m}\right) = a - b$$

$$qr^{-m} + (1-q)r^{m} = 1 \Rightarrow r^{m} = 1; \quad \frac{q}{1-q}$$

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Consider Winning or Losing m Dollars

$$r^m = \frac{q}{1-q} = \left(\frac{p}{1-p}\right)^m$$

- The risk odds for \$m are the risk odds for \$1 raised to the m power.

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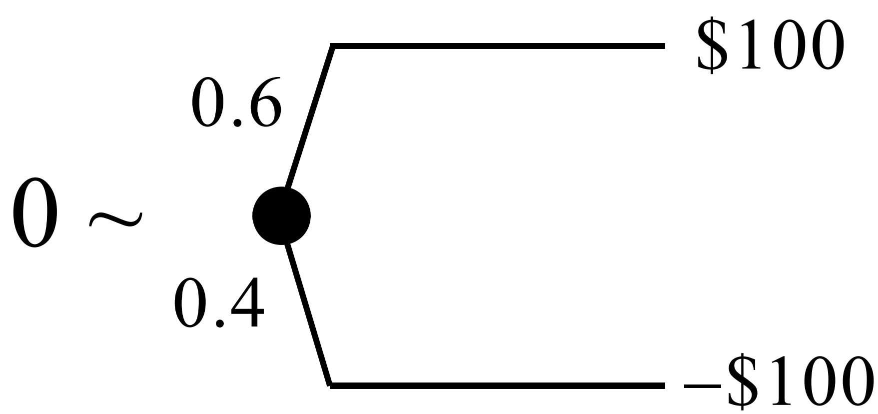
Using Risk Odds

- Suppose a person's risk odds for \$100 is $r(100)=1.5=p/(1-p)$. This means that $p=0.6$



- The risk odds of \$1000 are 1000/100 of this

$$r(1000) = r(100)^{\frac{1000}{100}} = 1.5^{10} = 57.665 = \frac{q}{1-q}$$

$$\Rightarrow q = 0.983$$

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Kim’s Risk Odds

- Kim’s utility function tells us that her risk odds for $1 are

$$r(1) = \left(\frac{1}{2}\right)^{\frac{-1}{50}} = 2^{\frac{1}{50}} = 1.014$$

- Her risk odds for $100 would be this number raised to the 100 power, or 4
- Her probability of $100 would have to be 0.8

Kim Loses a Nonchosen Alternative

- No cost. Adios.

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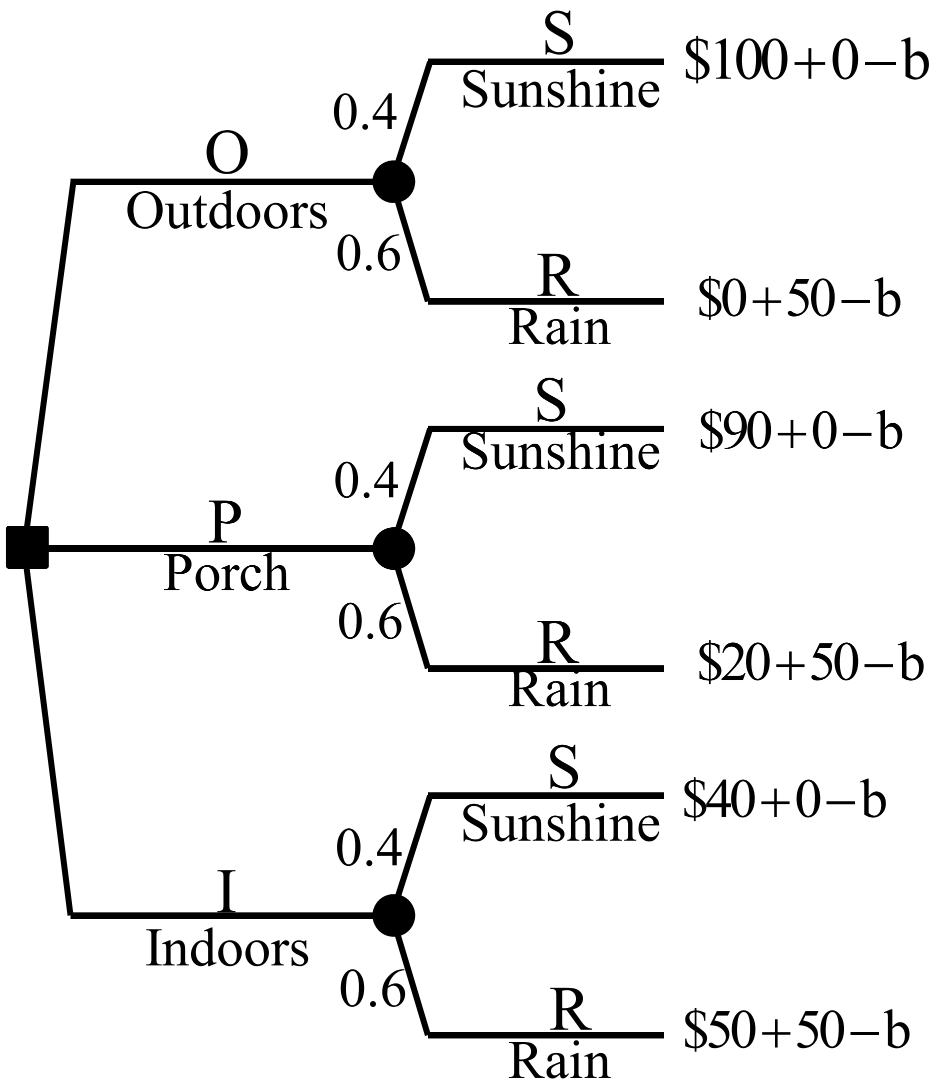
Insurance

- Suppose someone offers to write Kim a weather insurance policy.
  - The cost of the policy will be b.
  - The insurance company will pay her \$50 if it rains and \$0 if it is sunny.
  - Insurance has the nature of costing you something out front and paying you if and only if the bad outcomes occur.
- What is the maximum amount she would be willing to pay for such a policy? PIBP.
- What is insurance? A CONTRACT to pay a certain amount if a certain event occurs (or does not occur). We will return to that theme.

Kim's Party Problem with Insurance



$$u(100-b)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{100-b}{50}}$$

$$u(50-b)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{50-b}{50}}$$

$$u(90-b)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{90-b}{50}}$$

$$u(70-b)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{70-b}{50}}$$

$$u(40)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{40-b}{50}}$$

$$u(50)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{100-b}{50}}$$

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The Delta Property Makes It Easy to Solve

- Step 1: Solve with free insurance.
- Step 2: Apply the delta property, subtracting the insurance premium everywhere.
- Step 3: Subtract the insurance premium that equates the certain equivalent to 0
- Without the delta property, you would have to perform an iterative calculation.

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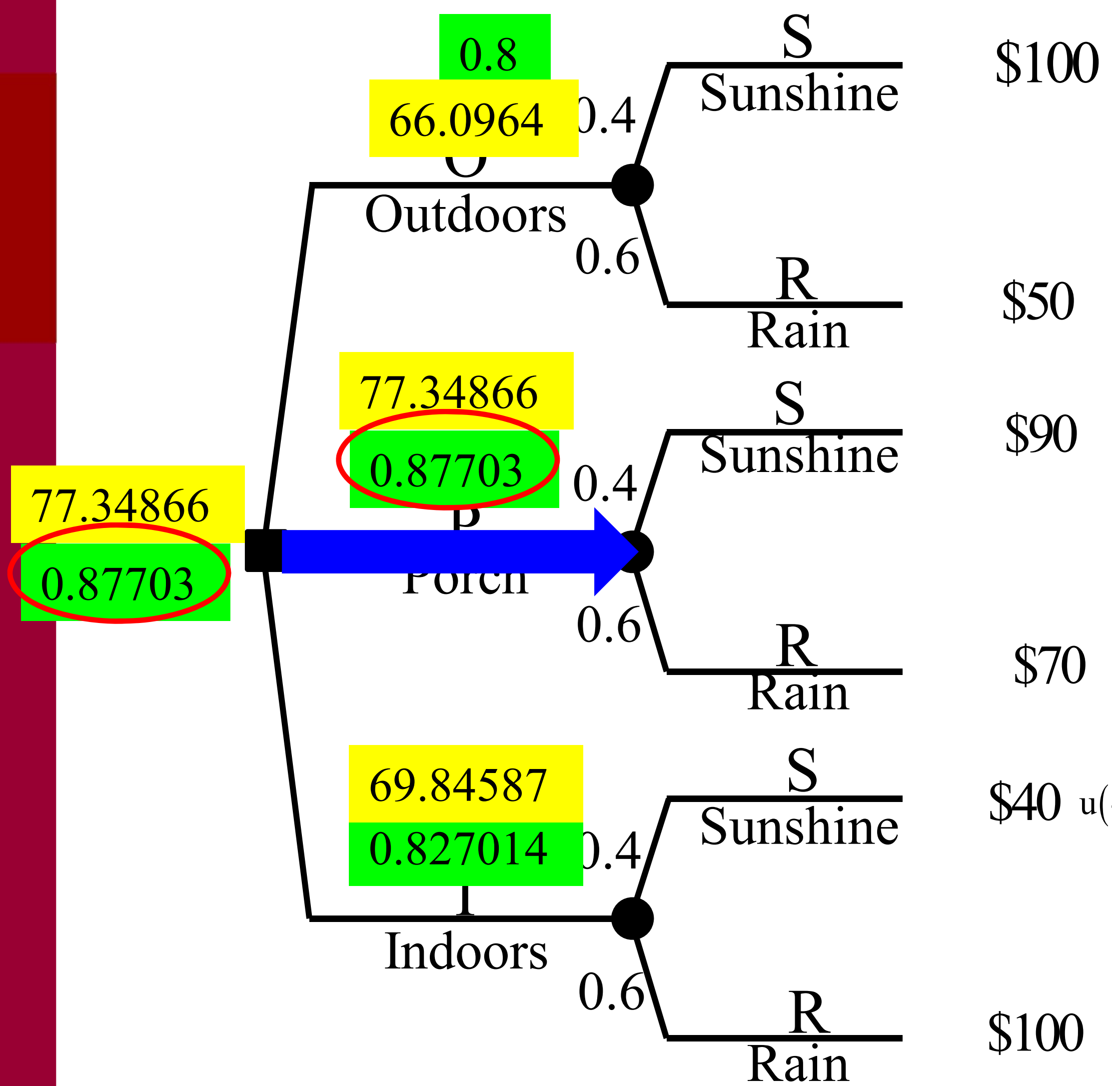
Kim's Party Problem with Zero Cost Insurance



$$u(100)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{100}{50}}=1$$

$$u(50)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{50}{50}}=\frac{2}{3}=0.666667$$

$$u(90)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{90}{50}}=0.950434$$

$$u(70)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{70}{50}}=0.828094$$

$$u(40)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{40}{50}}=0.567534$$

$$u(100)=\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{100}{50}}=1$$

With Free Insurance

- Porch party is the best
- The certain equivalent is \$77.34866
- The certain equivalent without any insurance was \$45.83223.
- If we subtract the premium b from the certain equivalent with free insurance until we get down to the certain equivalent without insurance, i.e., \$31.51643 =\$77.34866-\$45.83223.
- This is the maximum premium Kim would be willing to pay for insurance.

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Insurance

- Is buying insurance a decision?
- Does it cost you something to change your mind?
  - Yes
- Does it allocate resources irrevocably (or expensively)
  - Yes
- Why does it work? Because it offsets your bad outcomes.
- Is it always a good idea? No; you have to do the decision analysis. Insuring a $900 iPad Pro for $200 may not be a good deal.
- What if there were a $10 deductible? Doesn't that just reduce the insurance payoff?

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Sensitivity Analysis—Suppose Probability of Sun is p

Solving the Party Problem Using u Values as a Measure

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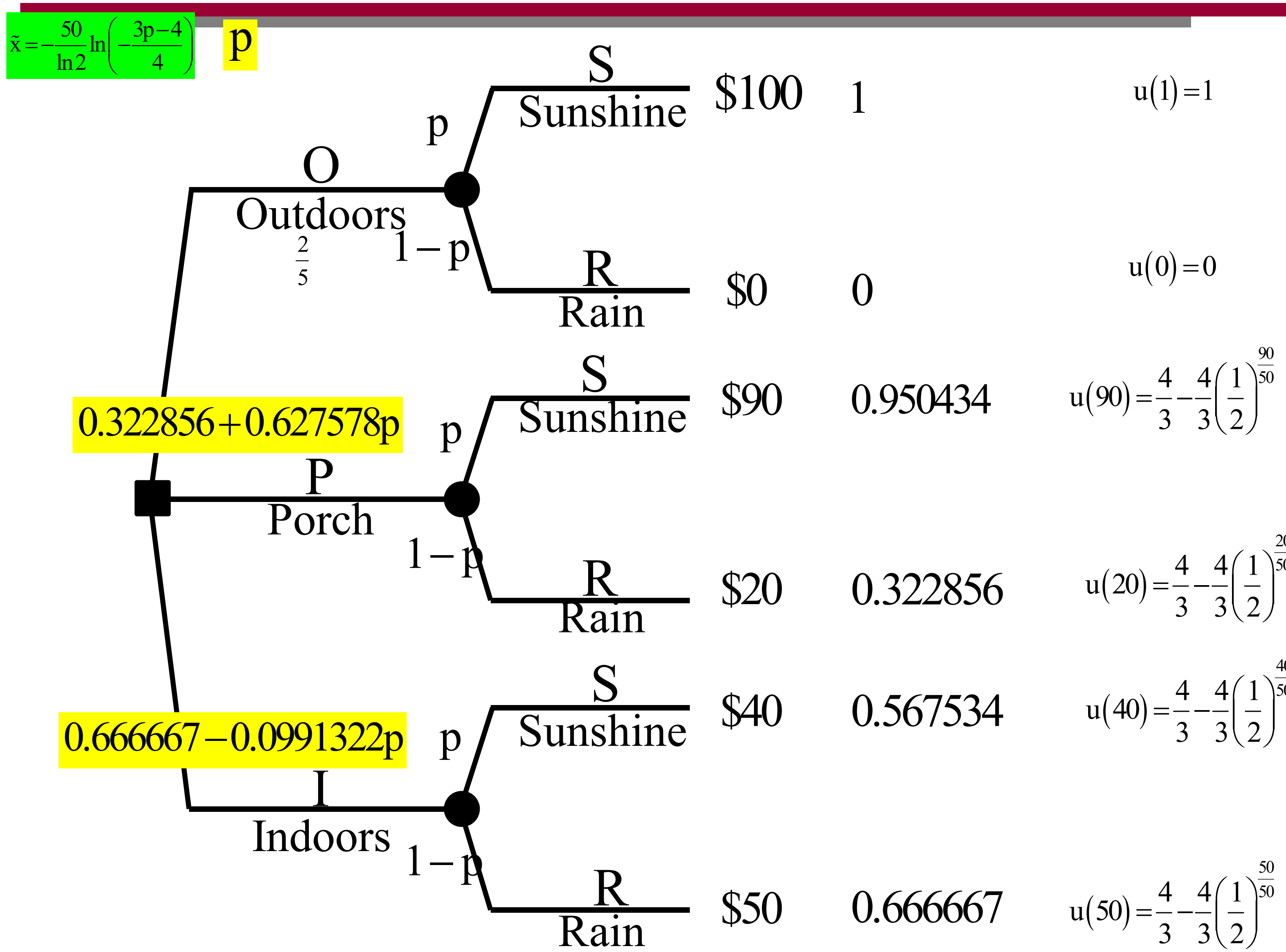

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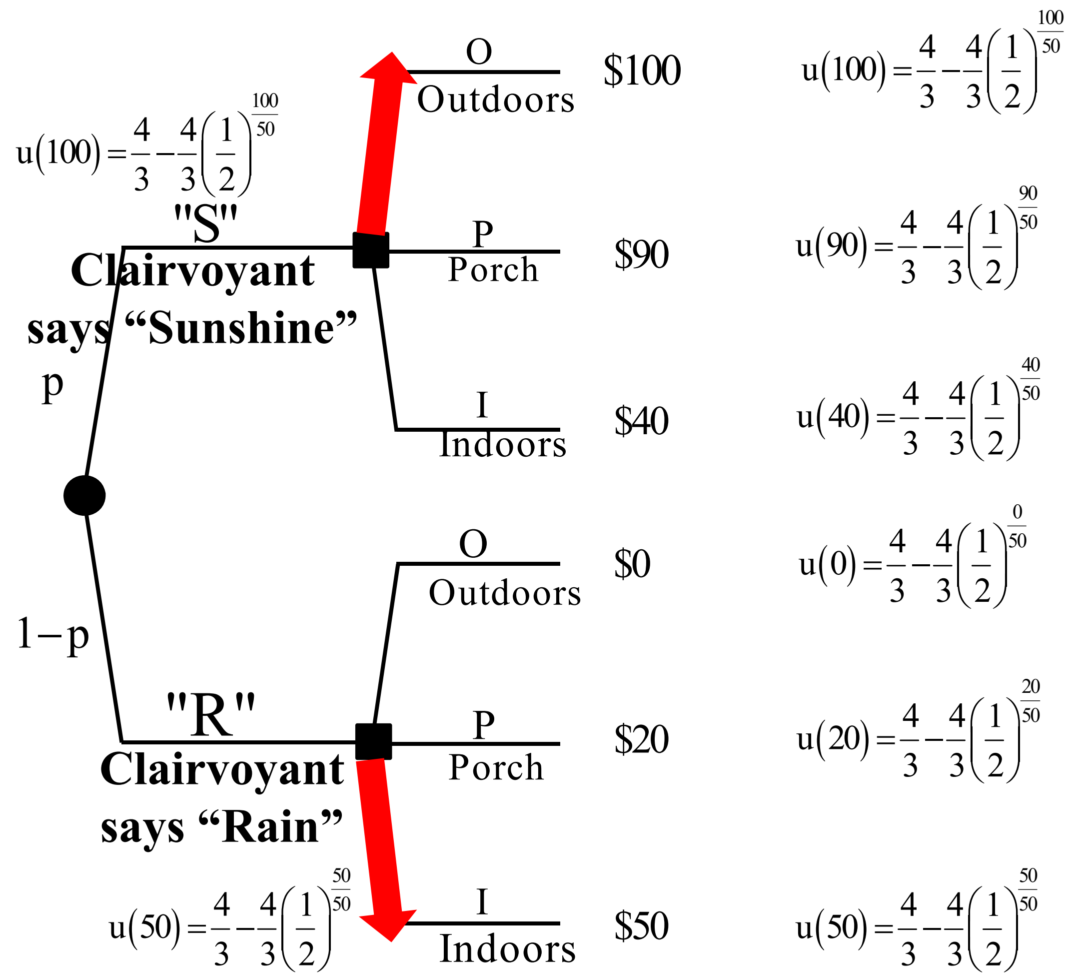
Clairvoyance without Cost

$$\langle u\rangle = p\left[\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{100}{50}}\right]+(1-p)\left[\frac{4}{3}-\frac{4}{3}\left(\frac{1}{2}\right)^{\frac{50}{50}}\right]$$

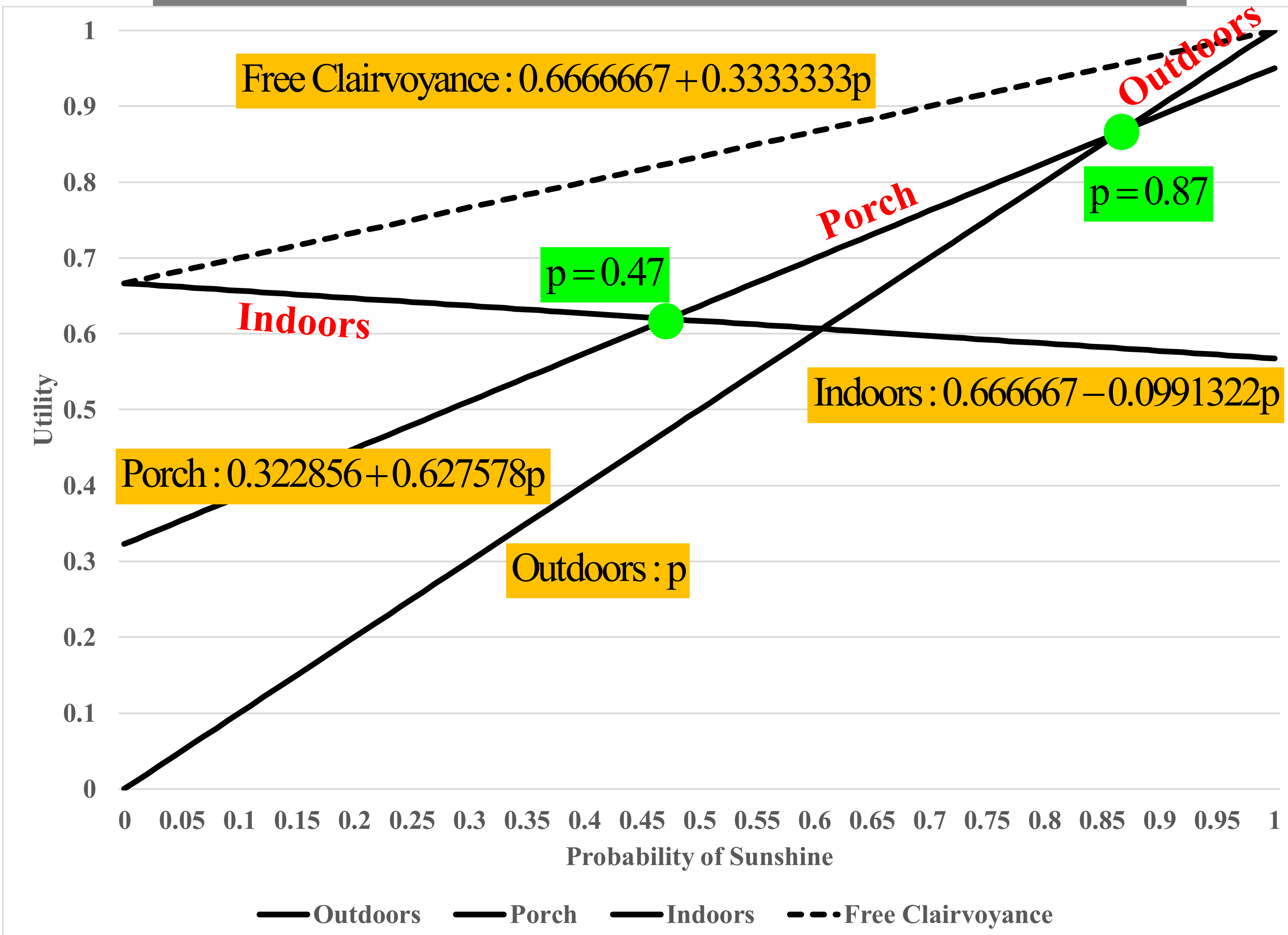
$$=\frac{4}{3}\left\{1-\left(\frac{1}{2}\right)^{\frac{50}{50}}+p\left[\left(\frac{1}{2}\right)^{\frac{50}{50}}-\left(\frac{1}{2}\right)^{\frac{100}{50}}\right]\right\}$$

$$\frac{4}{3}\left[1-\left(\frac{1}{2}\right)^{\frac{50}{50}}\right]+p\frac{4}{3}\left[\left(\frac{1}{2}\right)^{\frac{50}{50}}-\left(\frac{1}{2}\right)^{\frac{100}{50}}\right]$$

$$=0.6666667+0.3333333p$$



Sensitivity to Probability of Sun p



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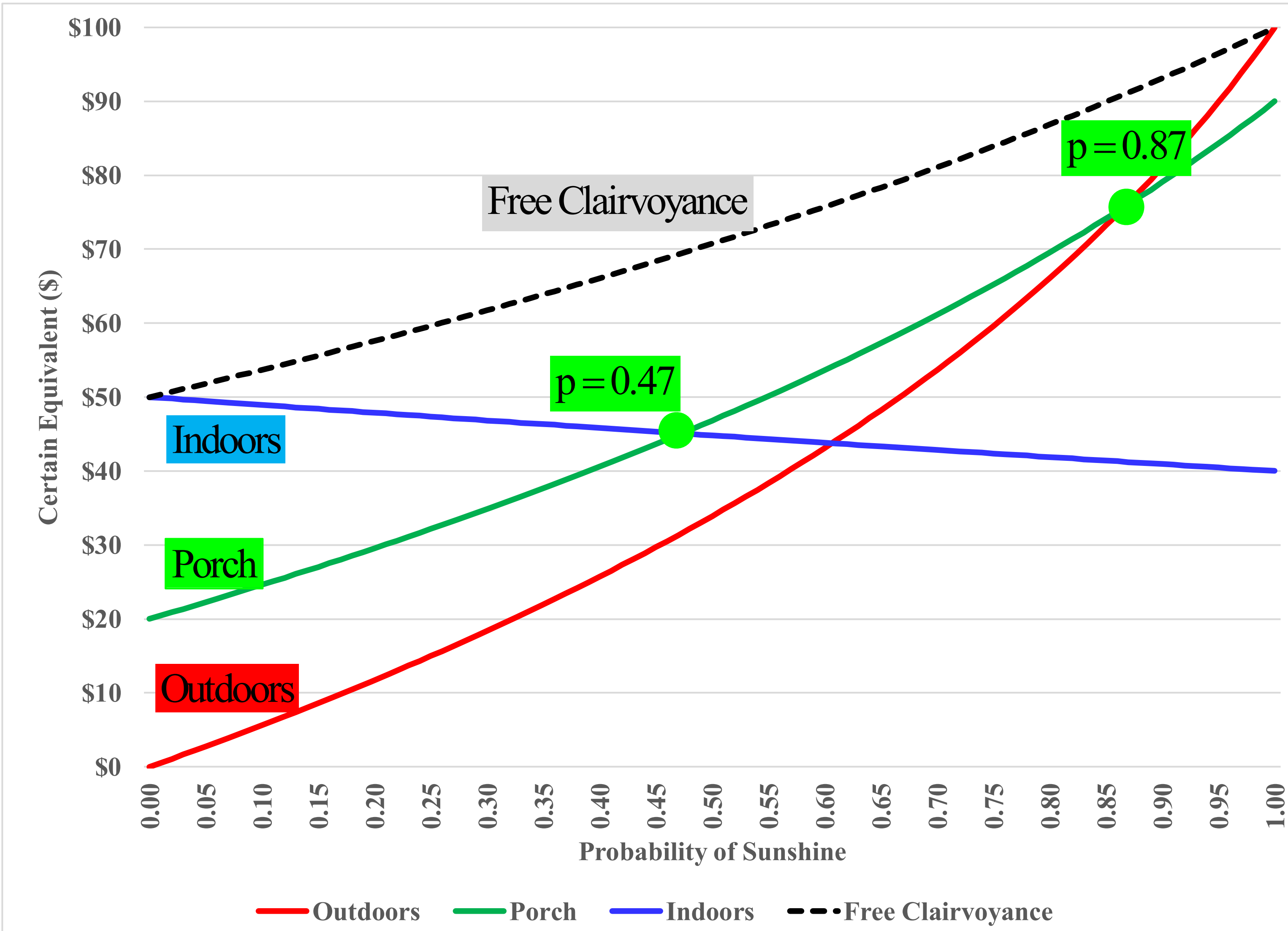
Sensitivity of Certain Equivalents



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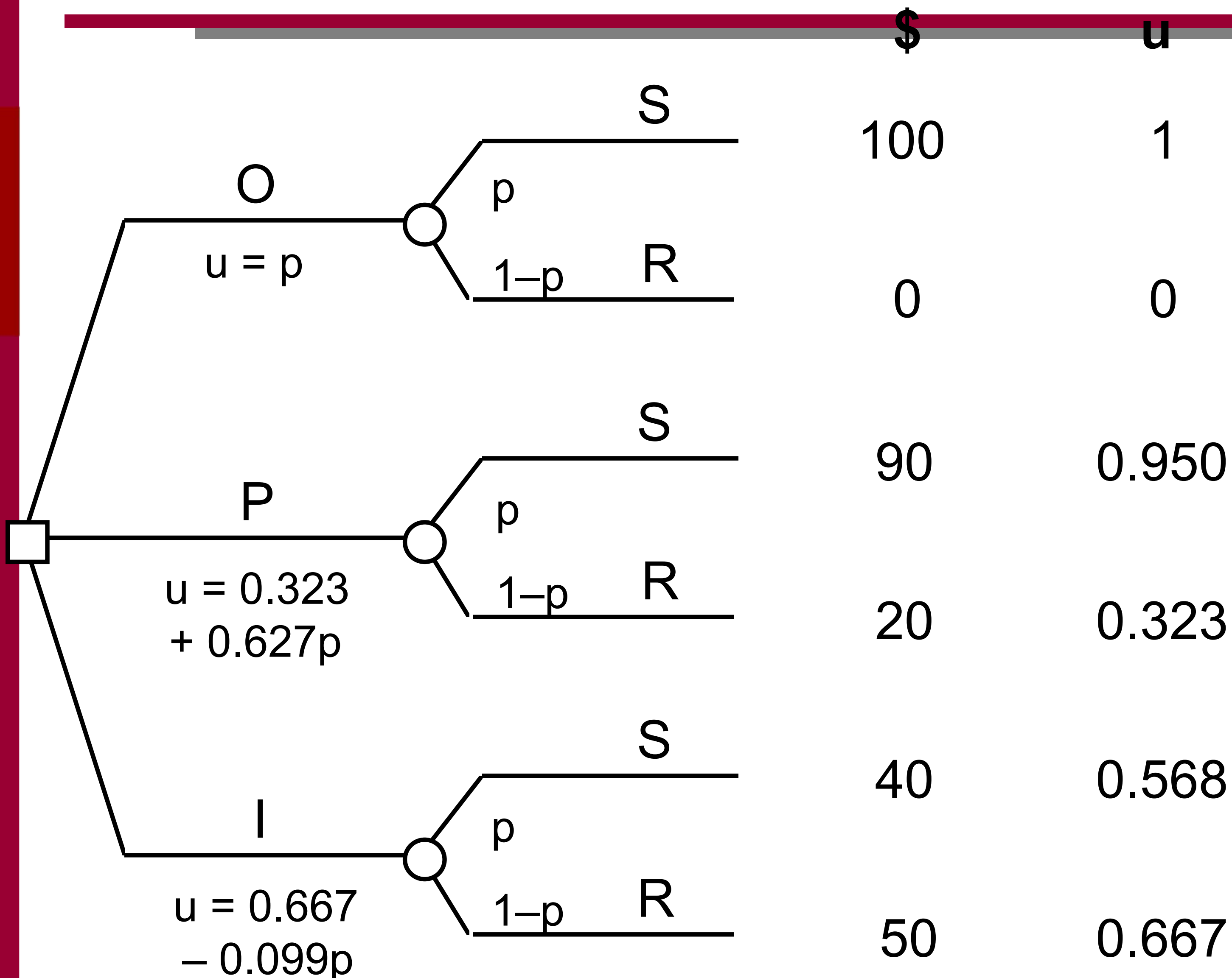
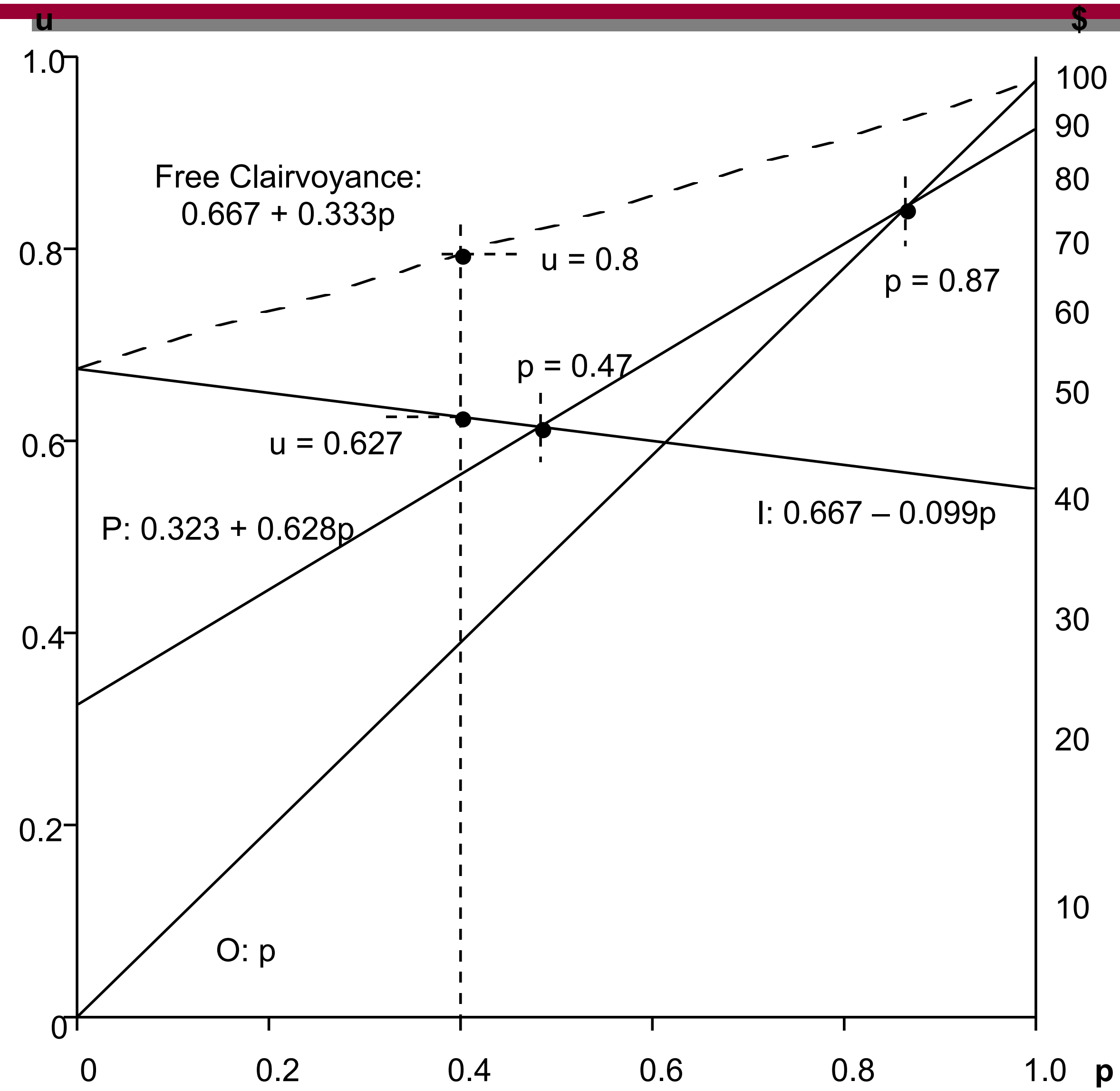
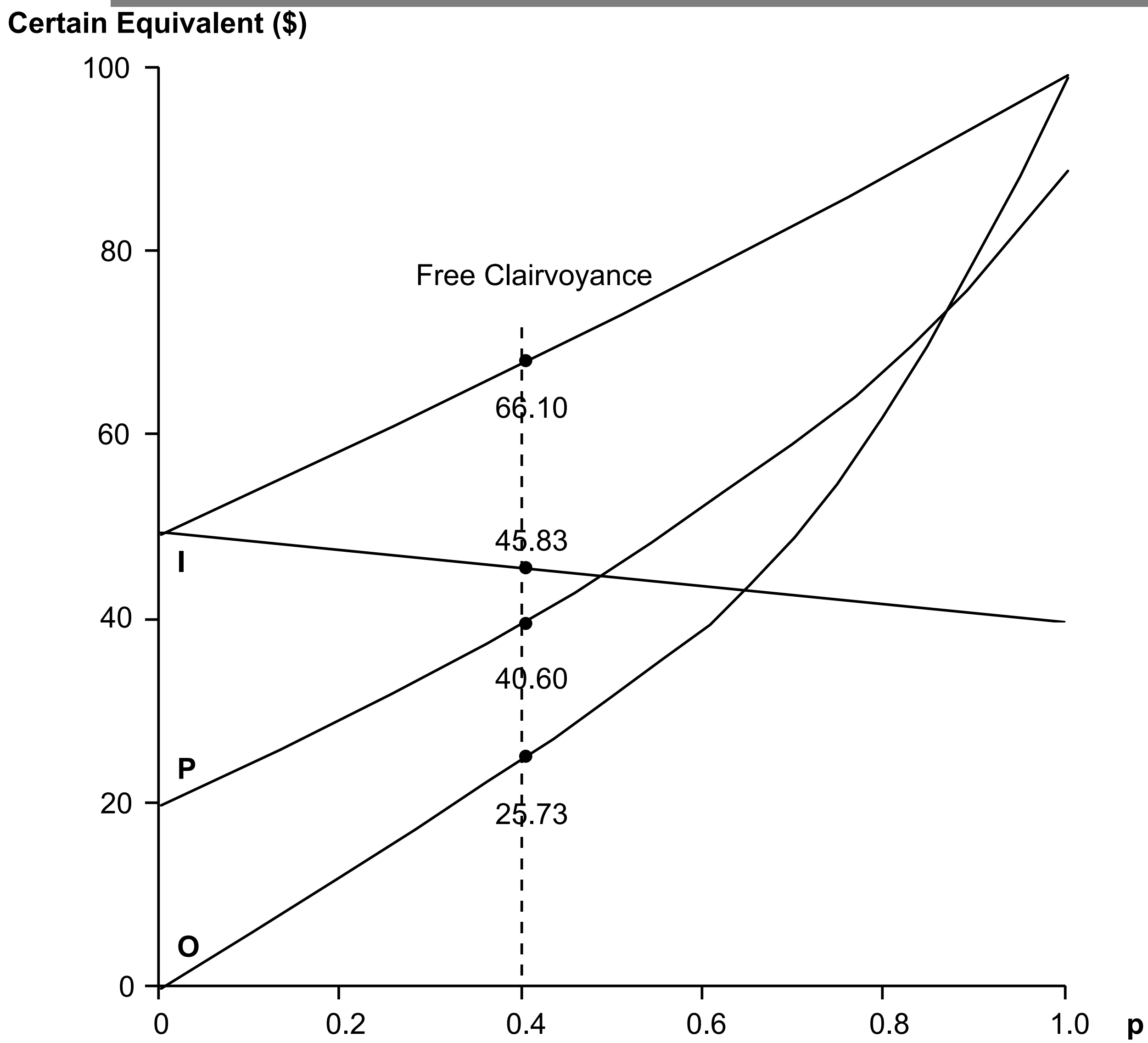
Kim's Decision Tree for General Probability of Sunshine, p



Kim’s u-value Sensitivity to Probability of Sunshine



Kim's Certain Equivalent Sensitivity to Probability of Sunshine, p



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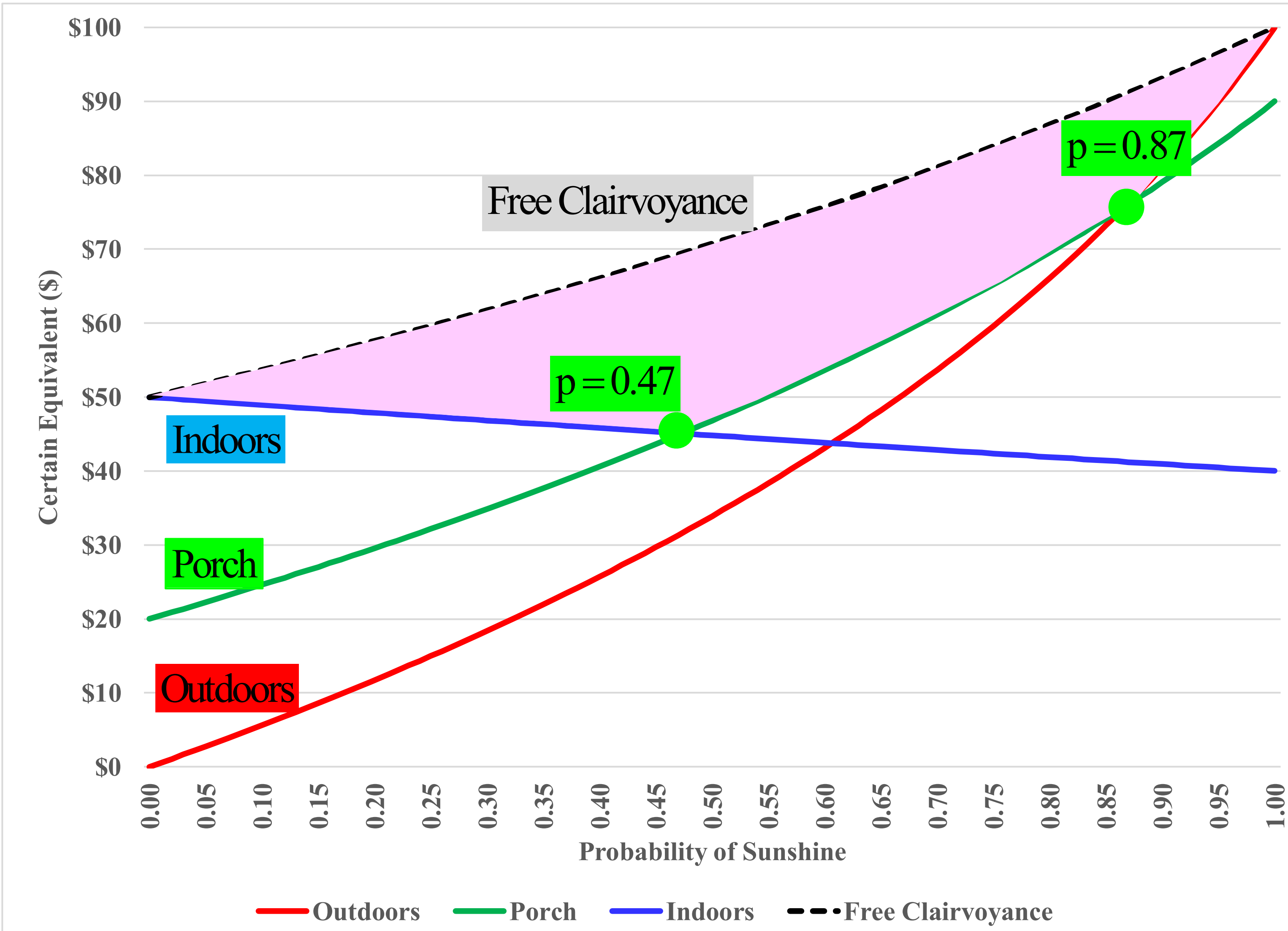
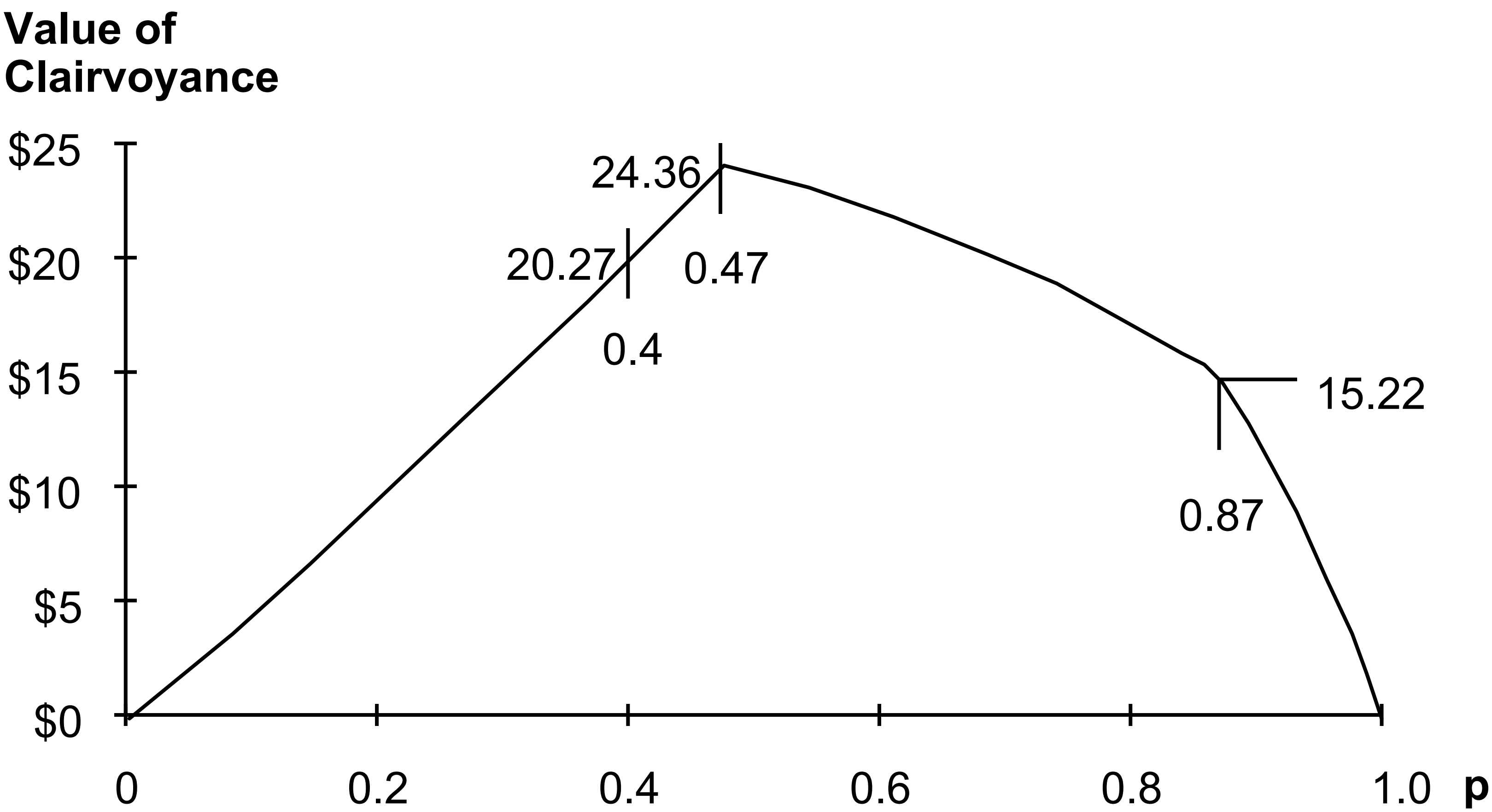
Value of Free Clairvoyance Region



Kim's Value of Clairvoyance Sensitivity to Probability of Sunshine, p



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Jane Sensitivity and Clairvoyance

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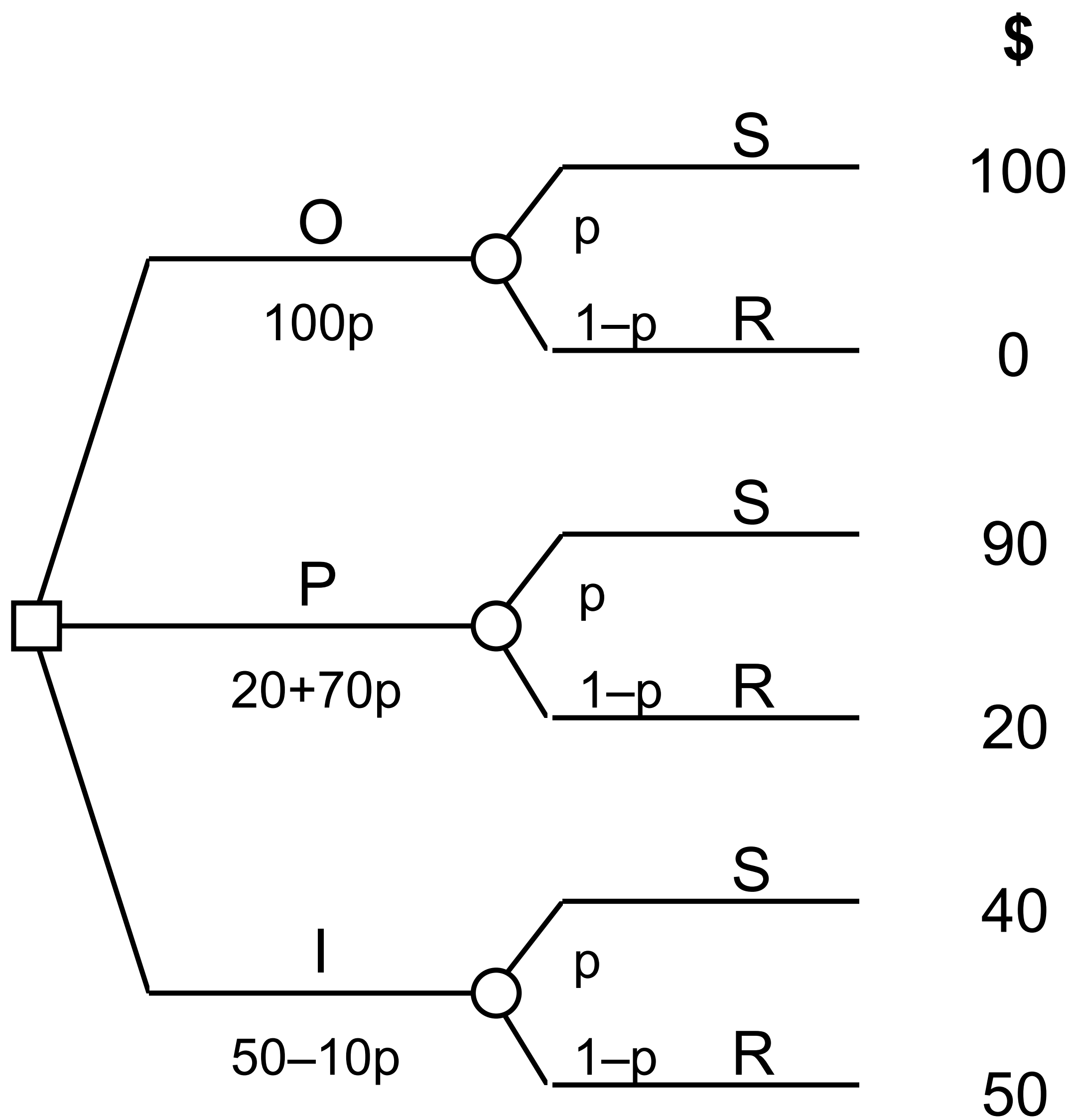
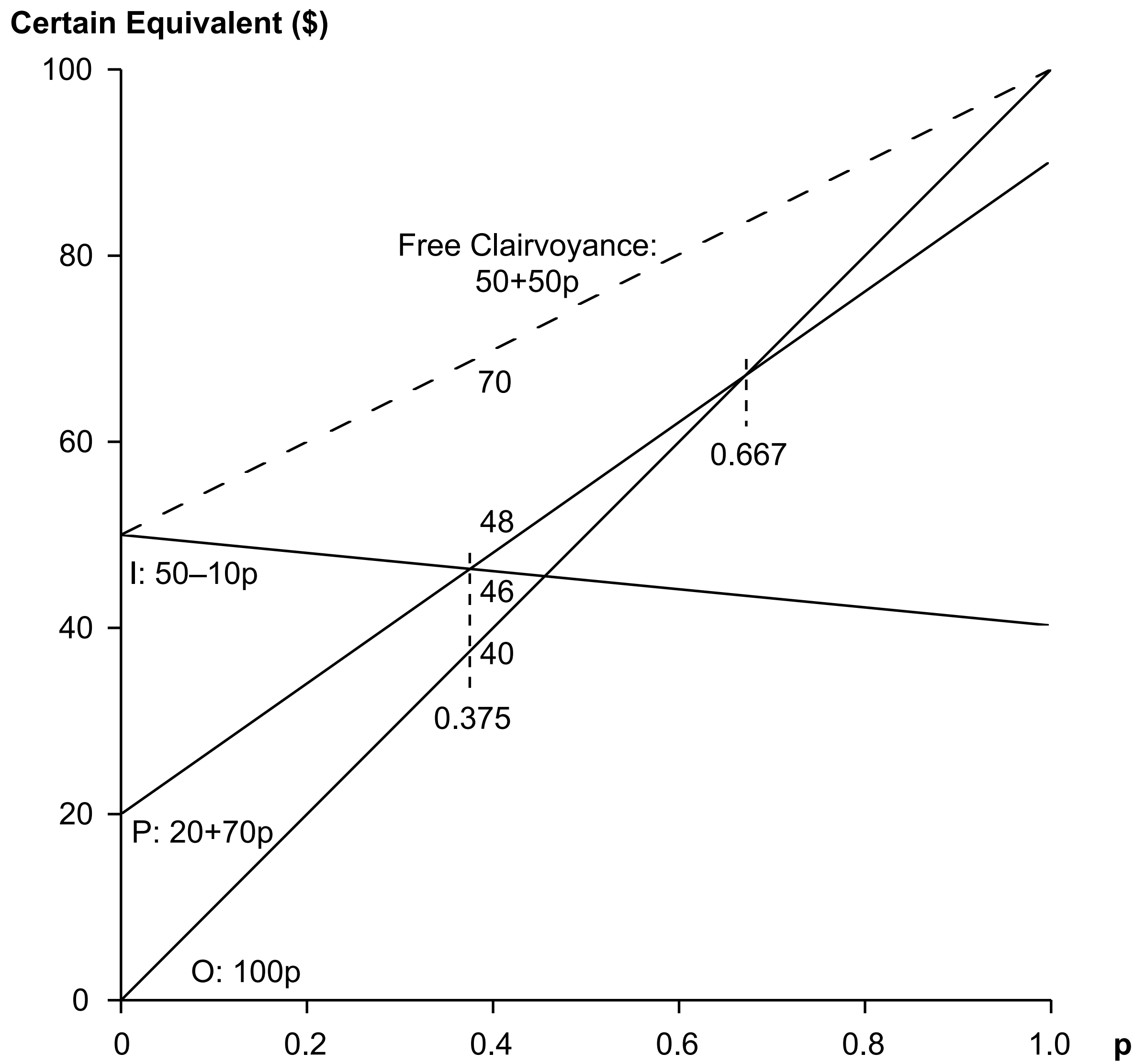
Jane's Decision Tree for General Probability of Sunshine, p



Jane's Certain Equivalent Sensitivity to Probability of Sunshine, p



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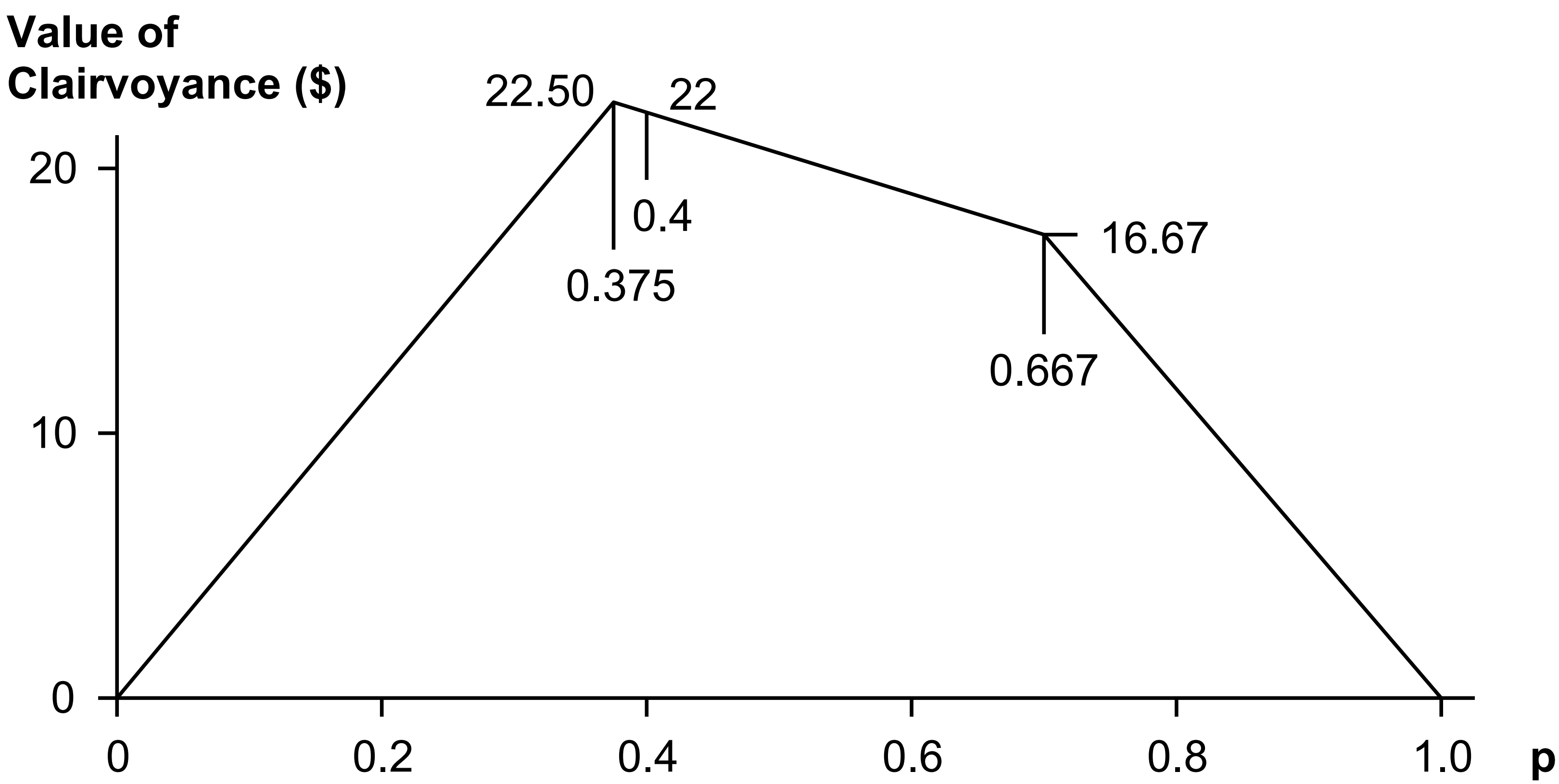
Jane's Value of Clairvoyance Sensitivity to Probability of Sunshine, p



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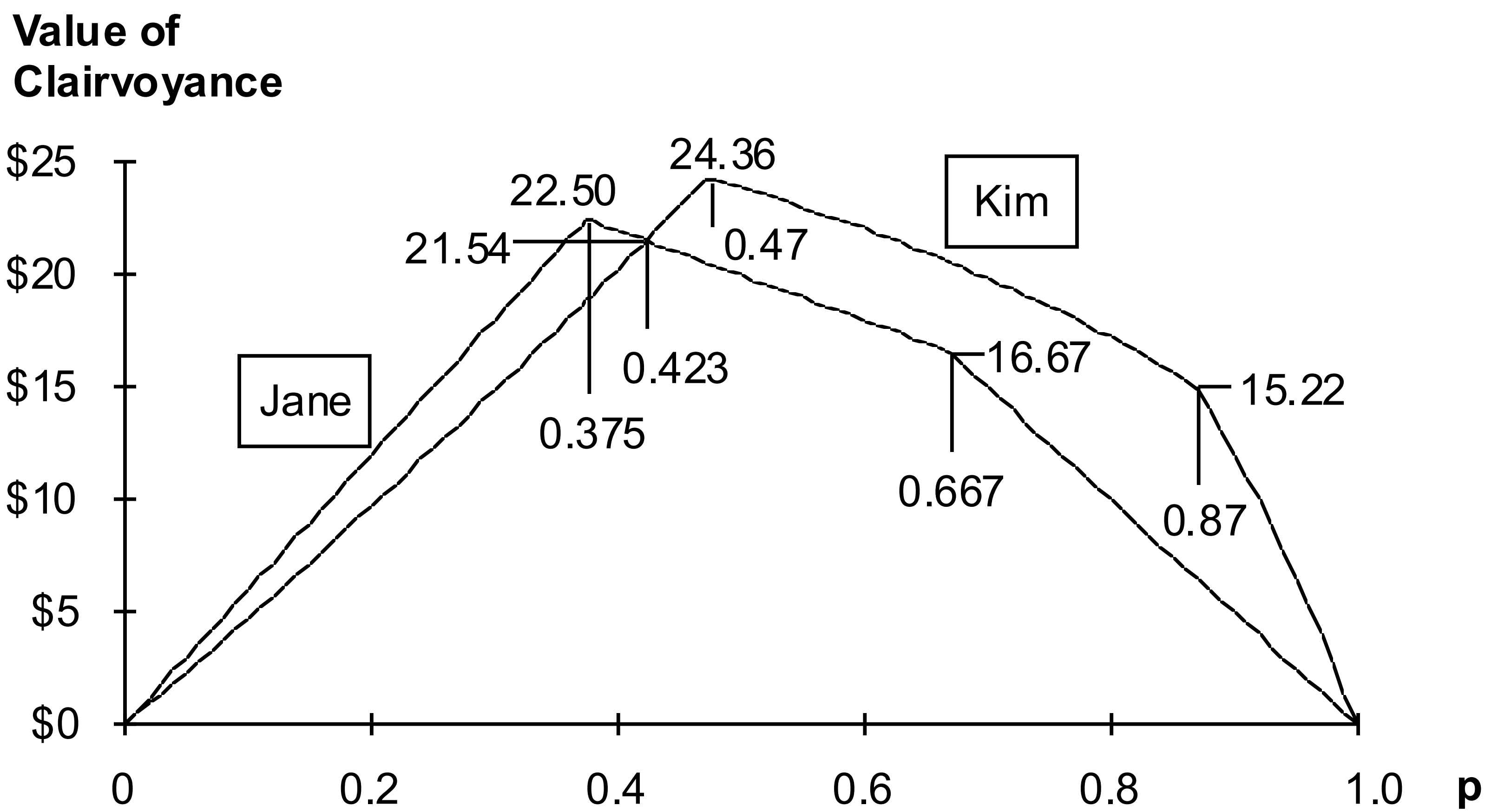
Value of Clairvoyance Sensitivity to p for Kim and Jane



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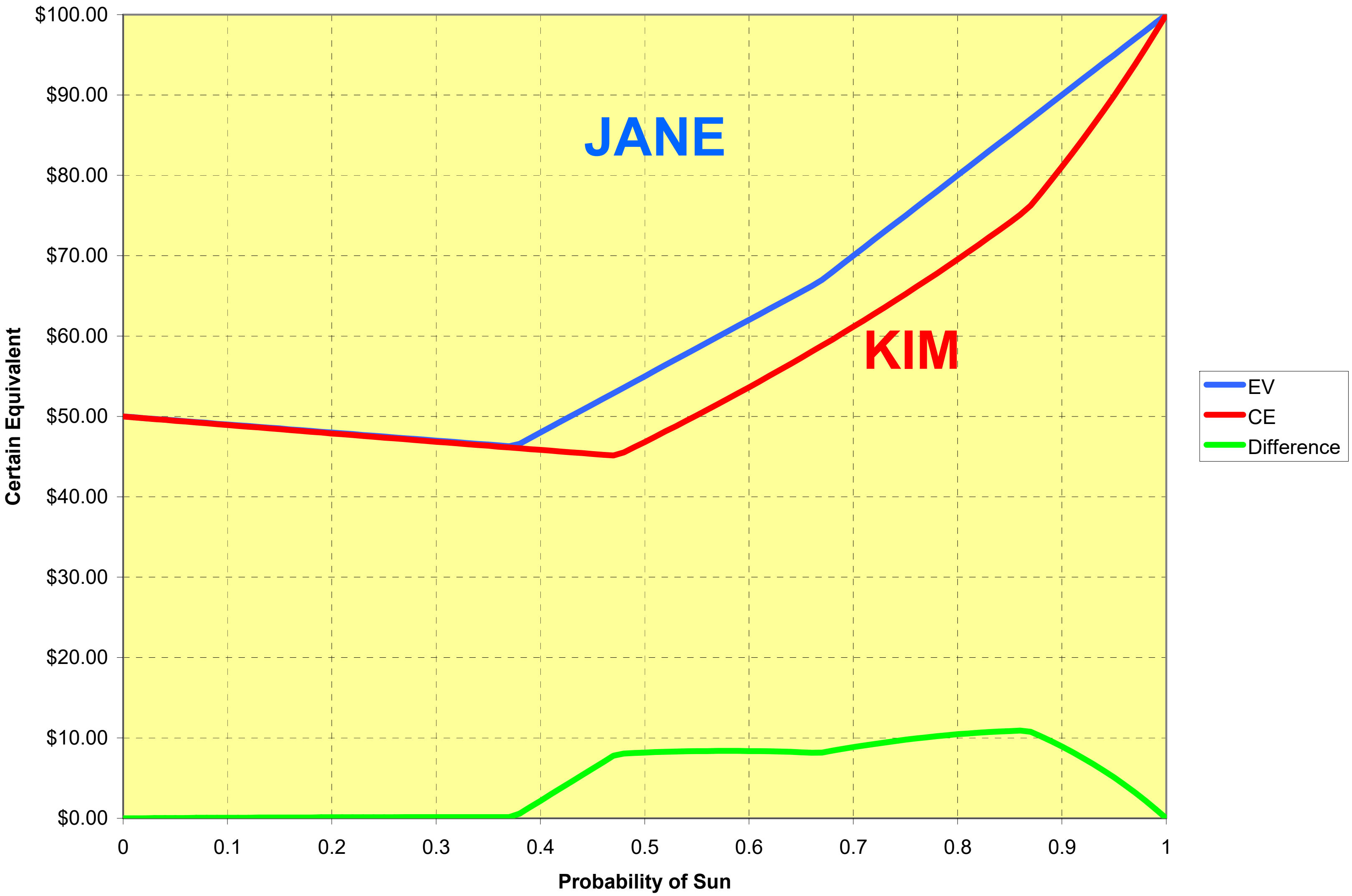
Value of Party (Without Clairvoyance) Sensitivity to p for Kim and Jane



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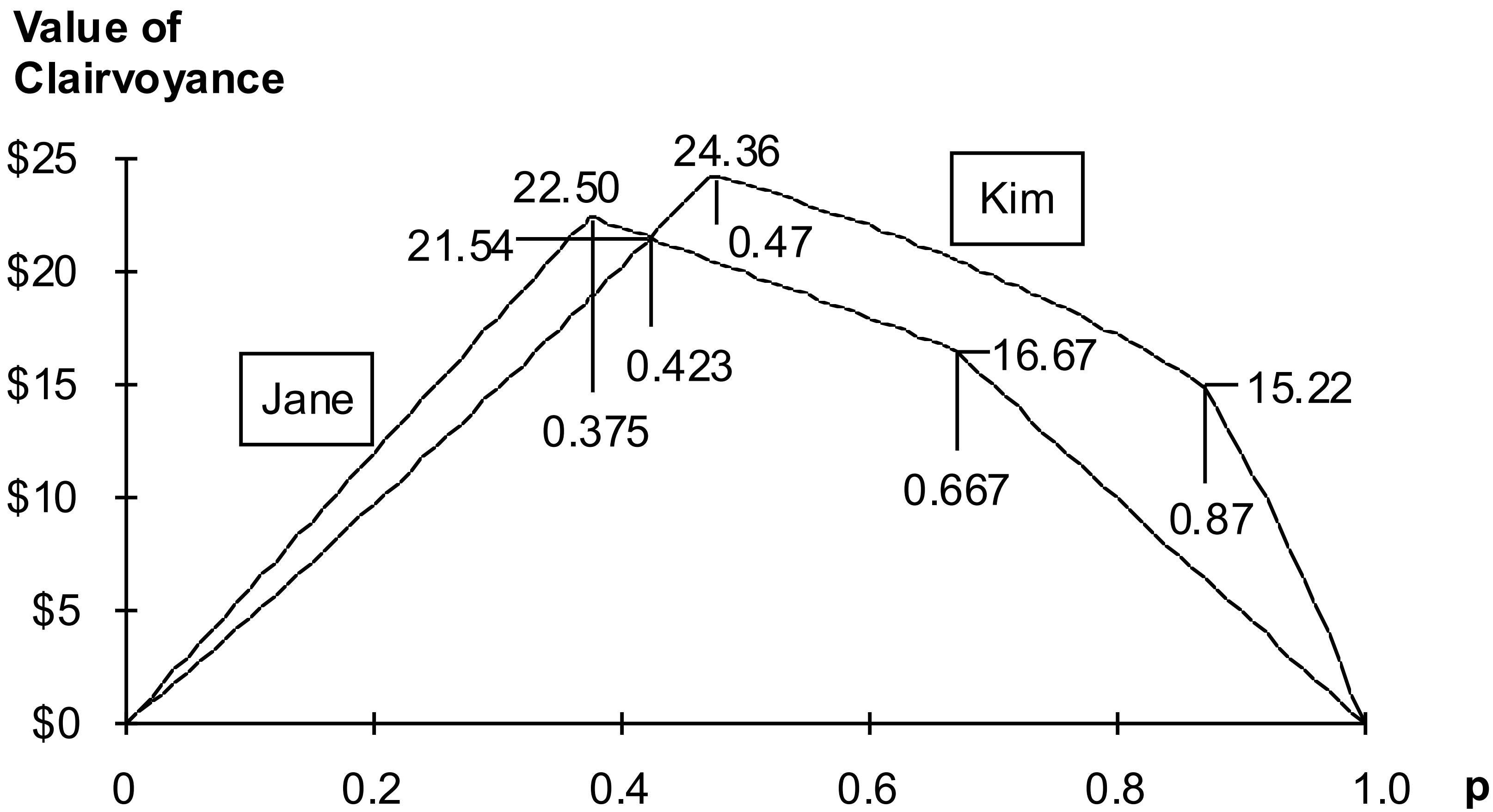
Value of Clairvoyance Sensitivity to p for Kim and Jane



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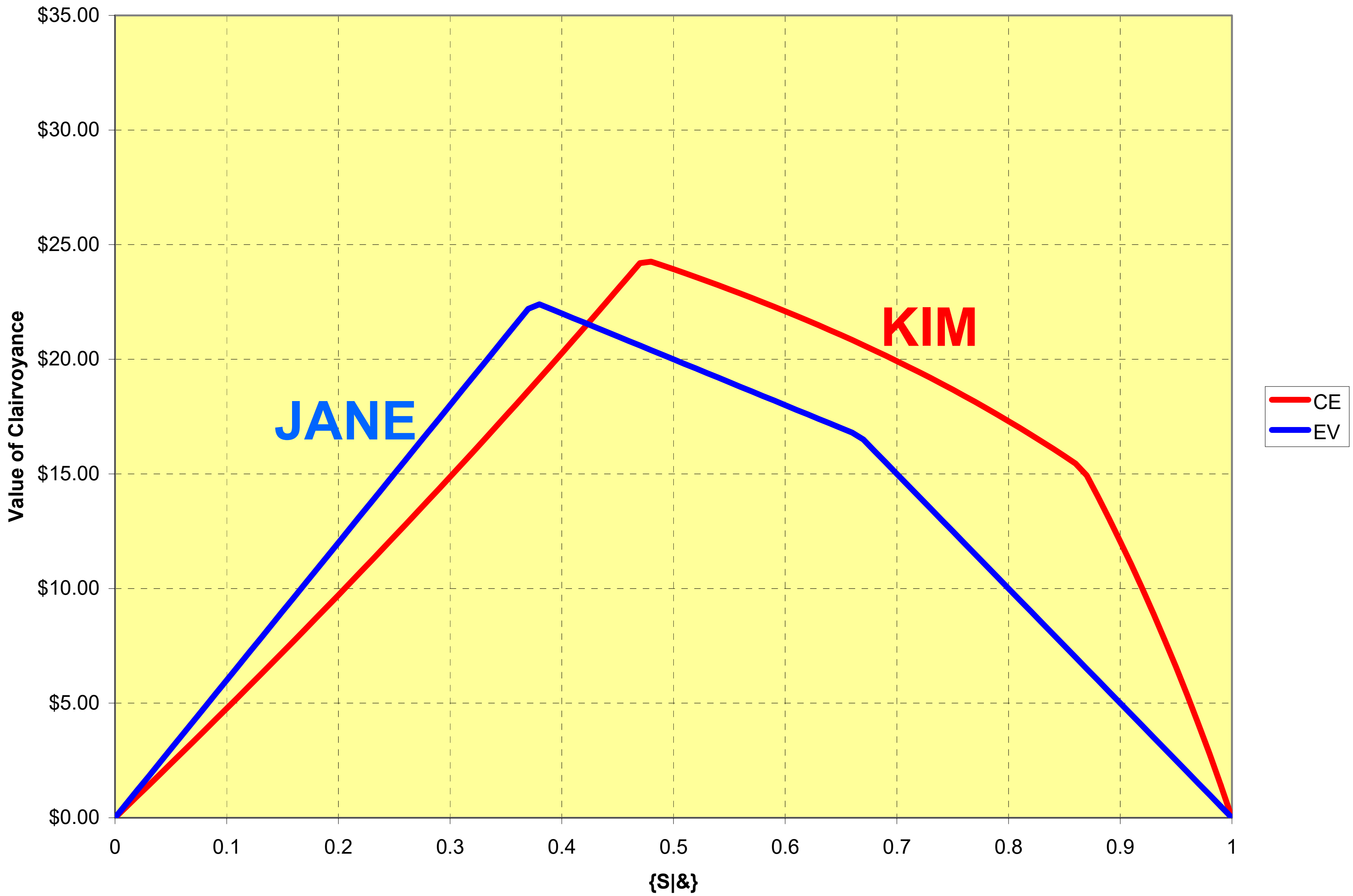
Value of Clairvoyance Sensitivity to p for Kim and Jane



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Insights

- Removal of an alternative changes the value of clairvoyance.
- The value of clairvoyance emanates from making decisions that give you MORE value.
- In the real world, clairvoyance is an abstract upper bound.
  - The real world never achieves clairvoyance.
  - The value of clairvoyance is an absolute upper bound on what you should EVER pay for information

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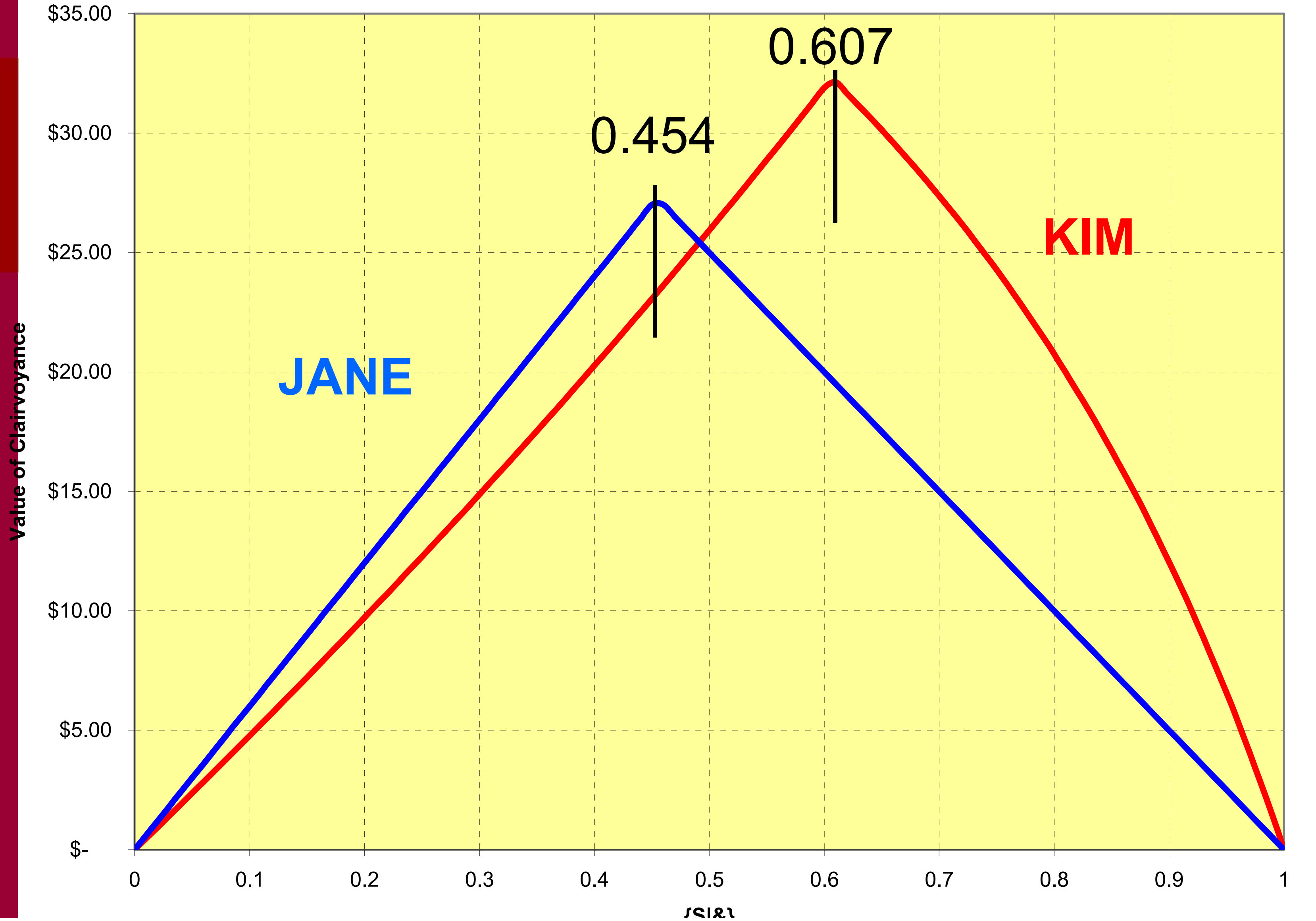
Value of Clairvoyance Sensitivity to p for Kim and Jane without Porch



Insights

- If your sunshine probability is extremely low (say 0.01), clairvoyance isnt worth much.
- The clairvoyant is highly likely to tell you it is not going to be sunny, and you already knew that, and there are no decisions to change.
- This has profound impliciations for low probability events (“black swans”)