Decision Analysis 1—Probabilistic Dominance

Dr. Dale M. Nesbitt

Management Science and Engineering
Stanford University

Huang Rm. 325, Stanford

221 State Street, Suite 1856

Los Altos, CA 94022

(650) 218-3069 mobile

dnesbitt@stanford.edu

What Are the Ex Ante Odds of This? (30 Teams; Dodger Fans Need NEVER Know!)

Midterm Redux

- There was a statement (10): *If Giovanni is extremely risk averse, he will always choose the alternative with the least down side*
  - If "extremely risk averse" were interpreted as "large but finite," the statement is not always true. (You can find a set of probabilities that refutes it.)
  - If "extremely risk averse" were interpreted as "passing to the infinitely high limit," the statement is true. This was the interpretation we intended, but…
- Clarity was lacking.
- We have credited both answers as correct.
- Everyone's score increased. Clarity is important

Decision Analysis

Dale M. Nesbitt

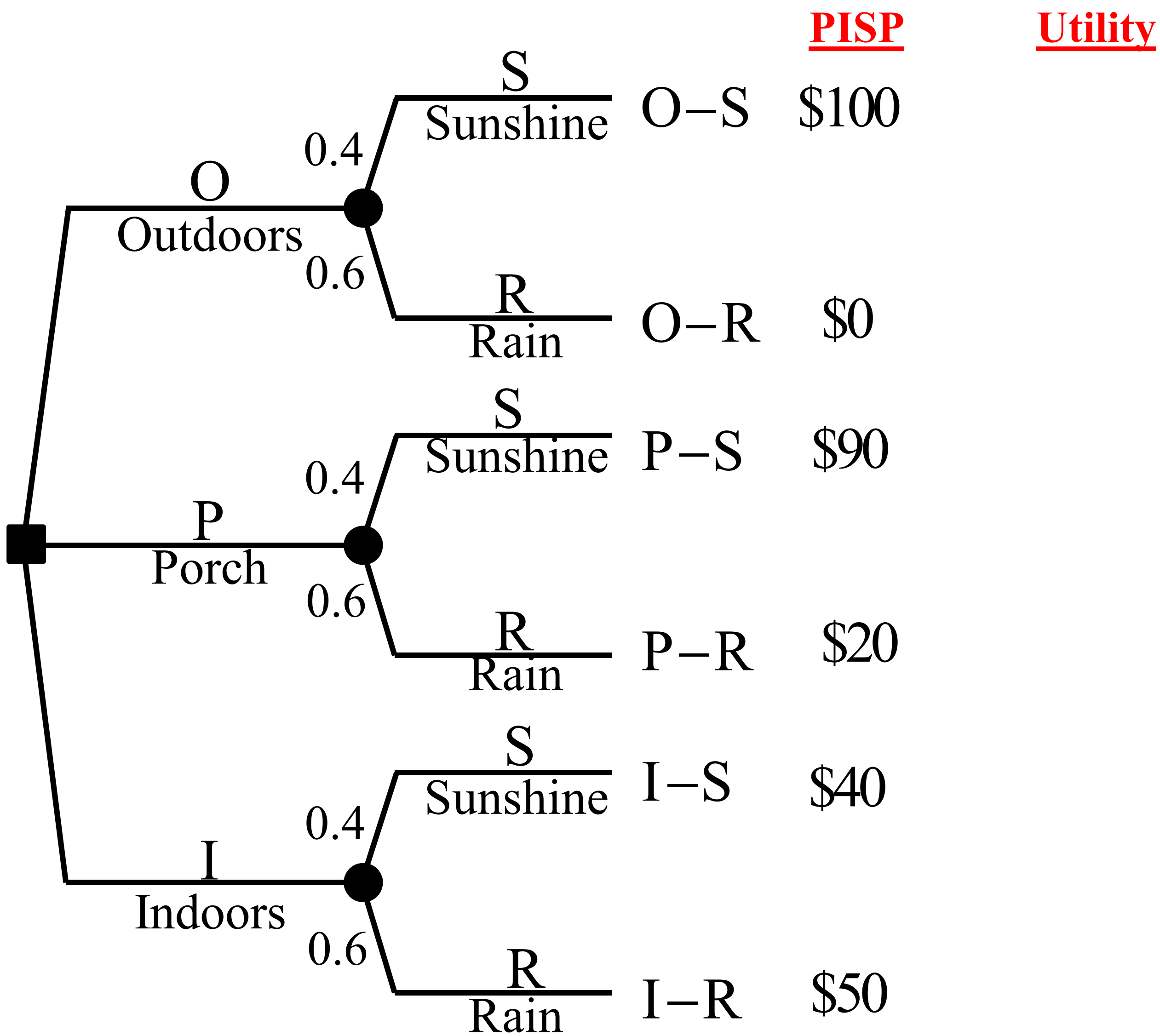
"Do We Really Need All This Risk Rigmarole?"

- How do I get Intel to share with me all their u-curve stuff?
- How would I get the Department of Education to share with me all their u-curve stuff? What is their u-curve anyway?
- This is fascinating and very insightful information.
- Probability courses don't emphasize it because they don't generally contain u-curves.

Decision Analysis

Dale M. Nesbitt

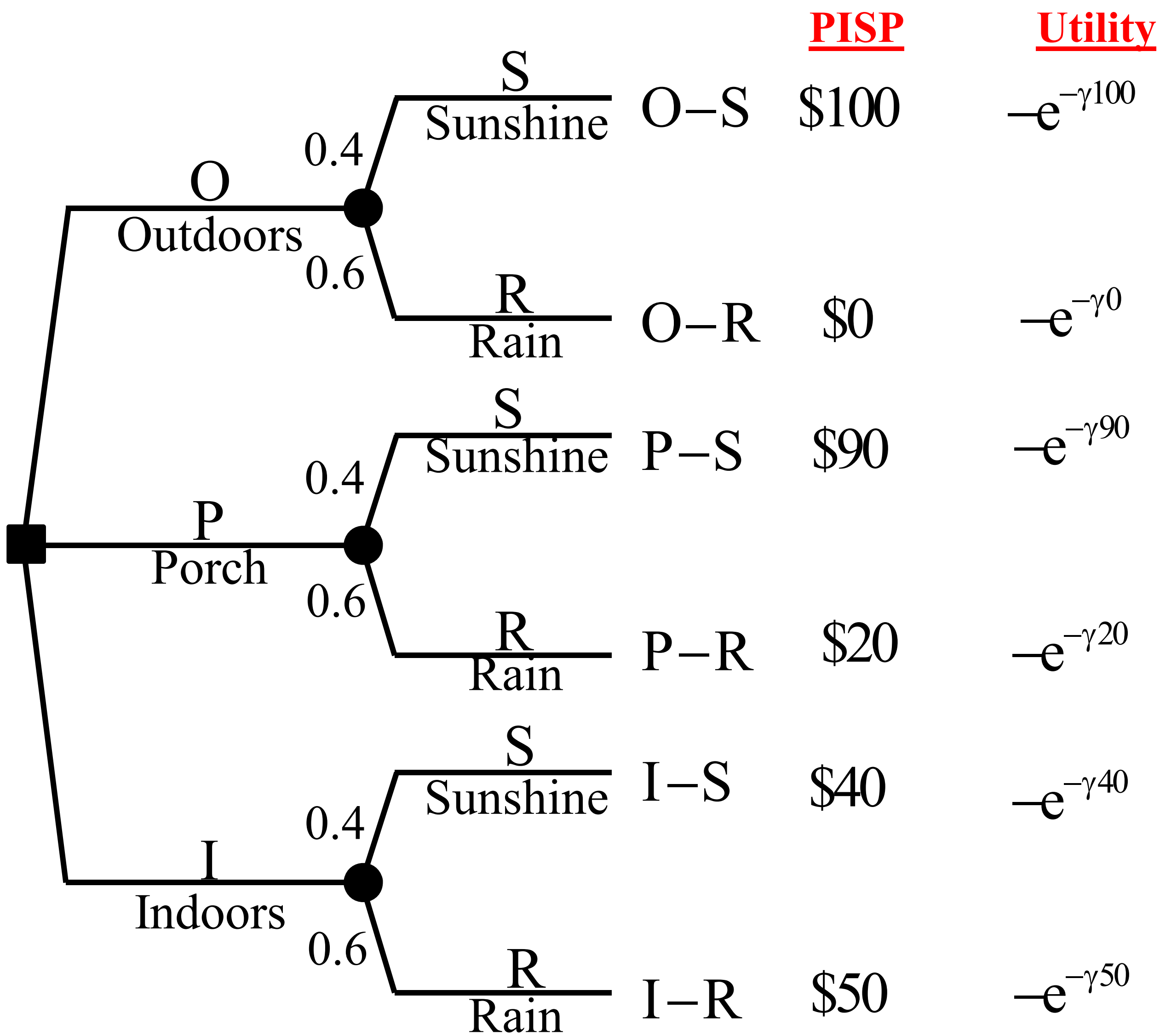
Back to the Party Problem



Delta Person with Risk Aversion Coefficient γ (Kim)

Decision Analysis

Dale M. Nesbitt



Inversion (Certain Equivalent) Formula

$$u = -e^{-\gamma x}$$

$$-u = e^{-\gamma \tilde{x}} \Rightarrow \ln(-u) = -\gamma \tilde{x} \Rightarrow \tilde{x} = -\frac{1}{\gamma}\ln(-u)$$

Can You Calculate the Expected Utility and Certain Equivalent for All Three Alternatives?

- Of course you can.
- This wont be the last time you see this!
- That has been a key point of the course—why you do it and how you do it.

Expected Utilities and Certain Equivalents

$$\langle u\rangle_O = -0.4e^{-\gamma 100} - 0.6e^{-\gamma 0}$$

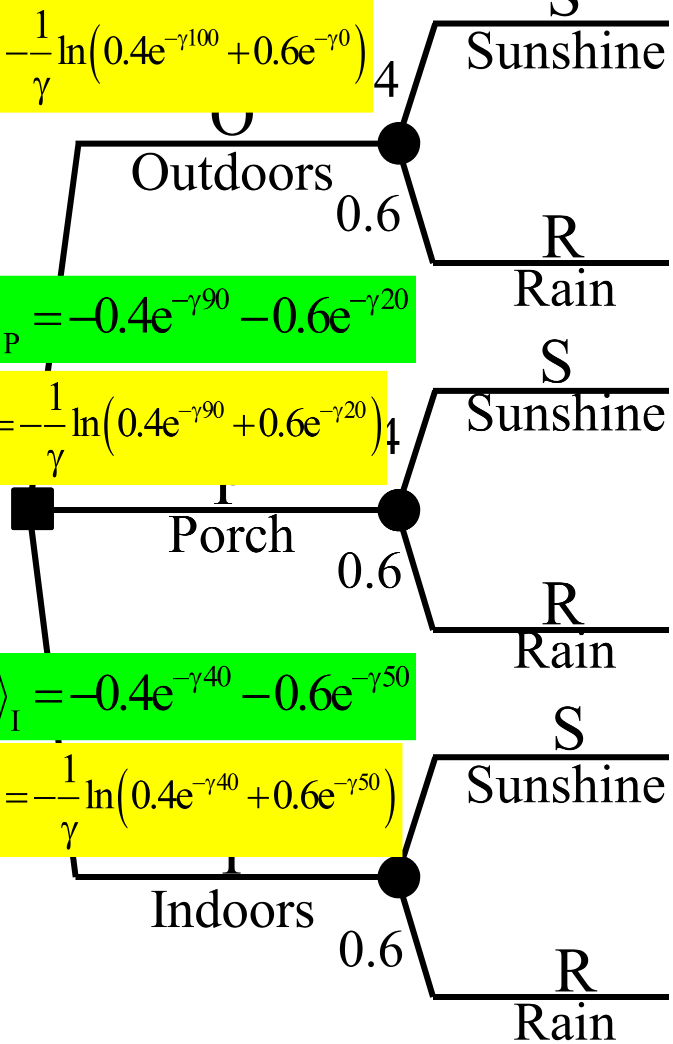
$$\tilde{x}_O = -\frac{1}{\gamma}\ln\left(0.4e^{-\gamma 100} + 0.6e^{-\gamma 0}\right)$$

$$\langle u\rangle_P = -0.4e^{-\gamma 90} - 0.6e^{-\gamma 20}$$

$$\tilde{x}_P = -\frac{1}{\gamma}\ln\left(0.4e^{-\gamma 90} + 0.6e^{-\gamma 20}\right)$$

$$\langle u\rangle_I = -0.4e^{-\gamma 40} - 0.6e^{-\gamma 50}$$

$$\tilde{x}_I = -\frac{1}{\gamma}\ln\left(0.4e^{-\gamma 40} + 0.6e^{-\gamma 50}\right)$$



	PISP	Utility
O–S	\$100	$-e^{-\gamma 100}$
O–R	\$0	$-e^{-\gamma 0}$
P–S	\$90	$-e^{-\gamma 90}$
P–R	\$20	$-e^{-\gamma 20}$
I–S	\$40	$-e^{-\gamma 40}$
I–R	\$50	$-e^{-\gamma 50}$

Kim was
γ=ln2/50
ρ=50/ln2

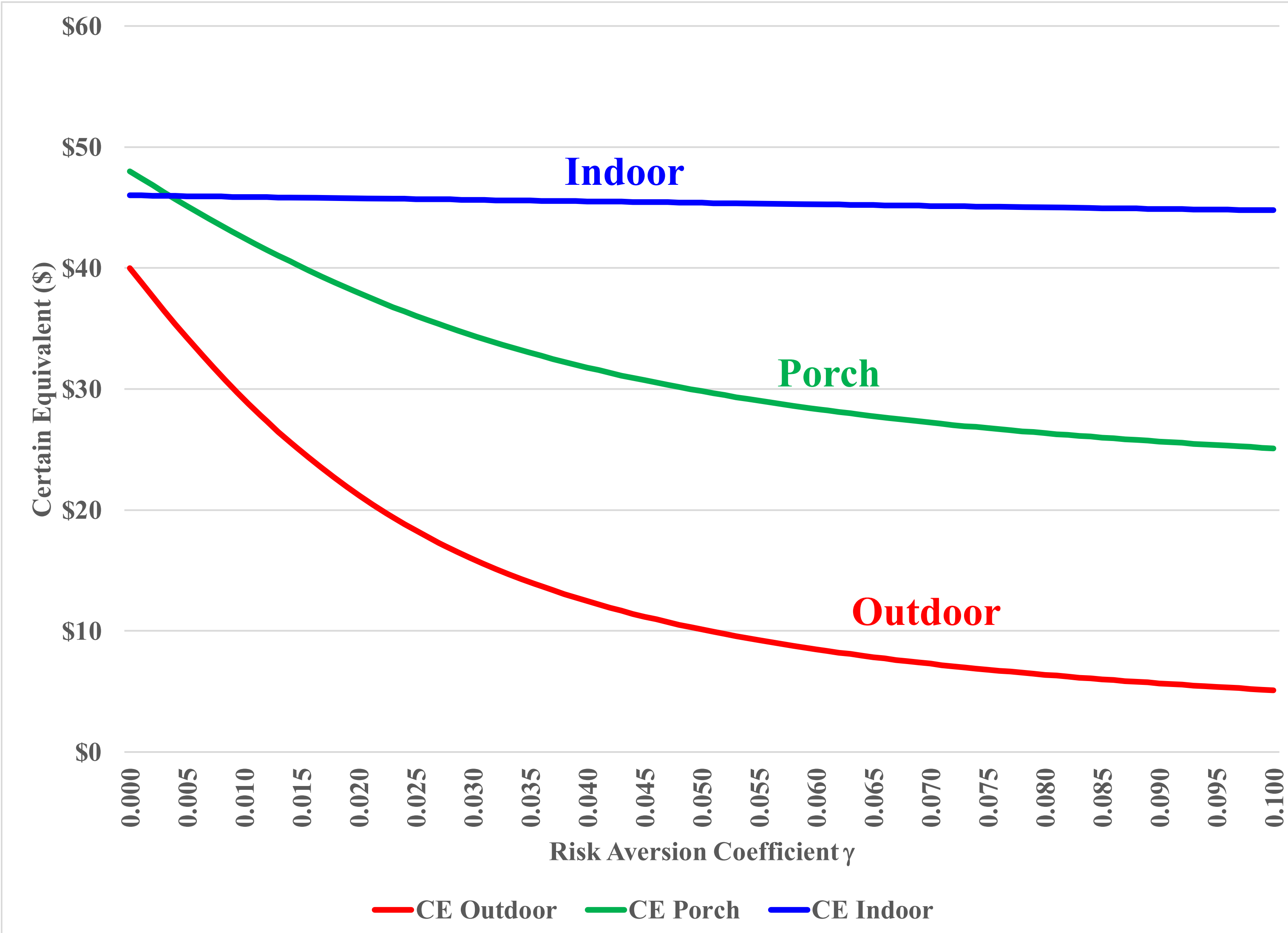
Dale M. Nesbitt

Decision Analysis

Decision
Analysis

Dale M. Nesbitt

Let’s Plot the Certain Equivalents on a Common Graph (Risk Averse and Neutral)



Decision Analysis

Dale M. Nesbitt

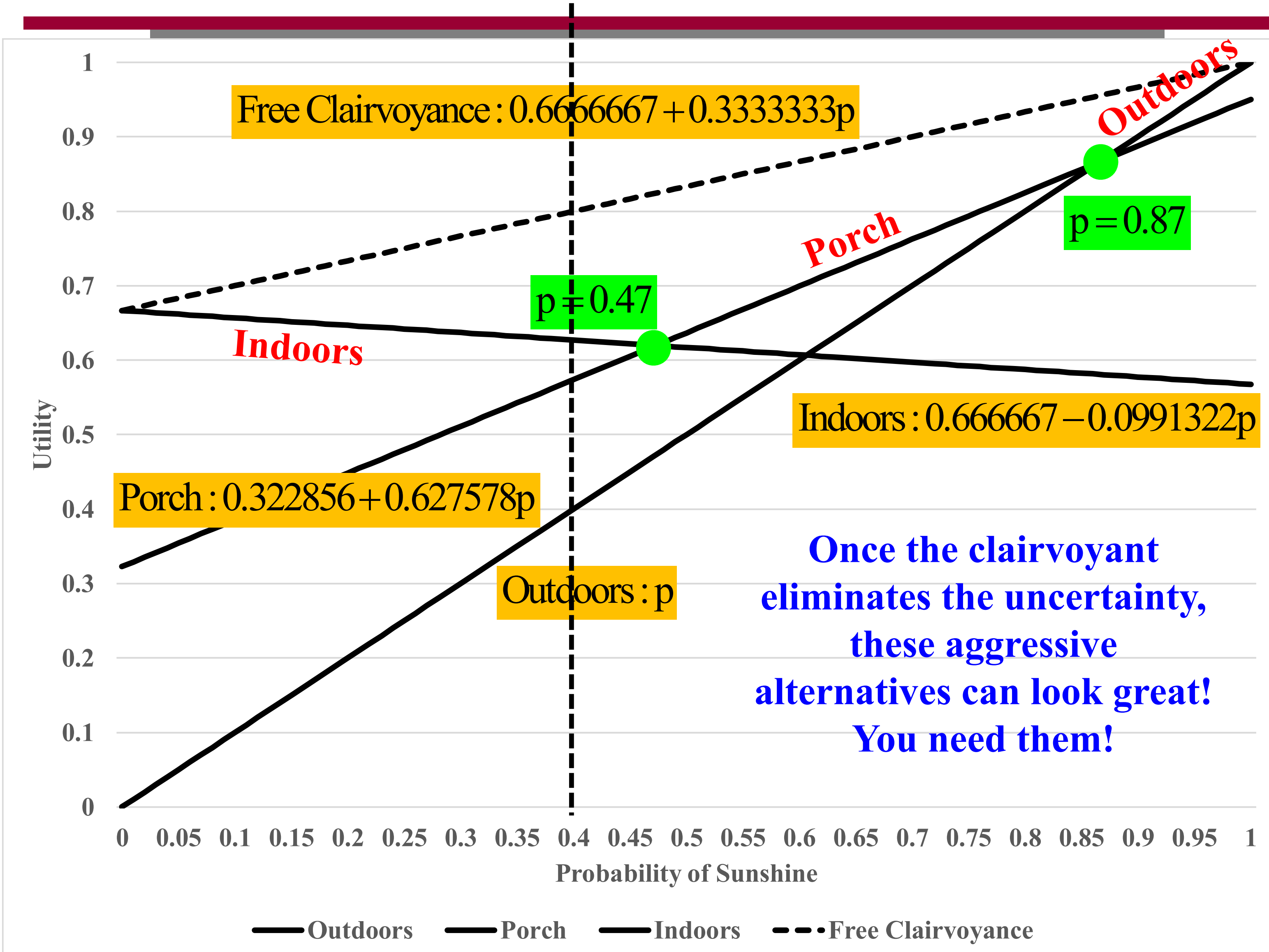
Key Question:

- What the heck do we need the outdoor alternative for?
  - Cant we just throw it away?
  - No risk neutral or risk averse delta person would ever choose it?
  - Would any risk averse person EVER choose it?
  - Not so fast……………………..
- Is there anything about a probability distribution over a measure that can guarantee us that the curves never cross?
- Do we always need all the utility/risk/u curve analysis?

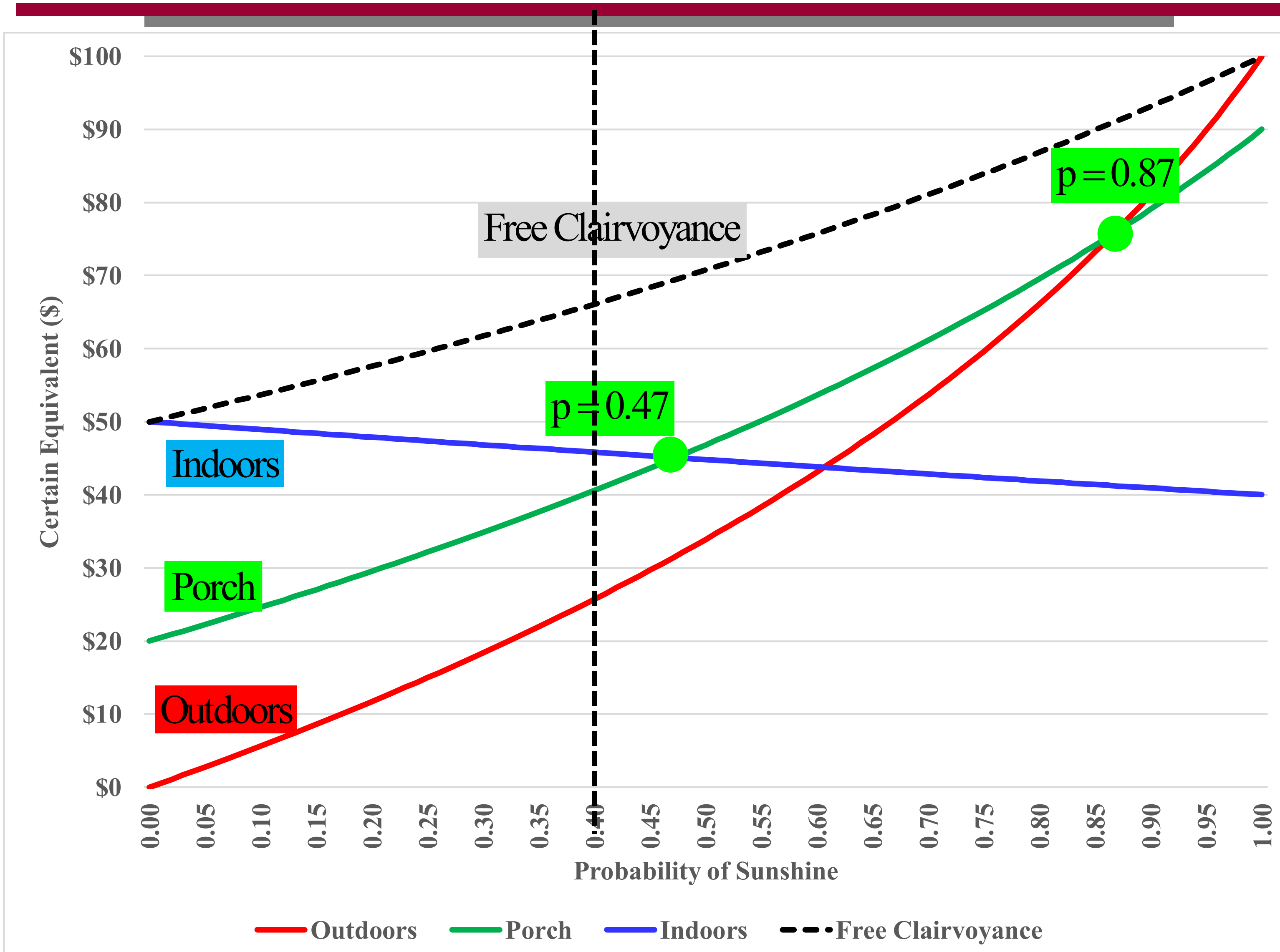
Decision Analysis

Dale M. Nesbitt

Sensitivity of <u> to Probability of Sun p



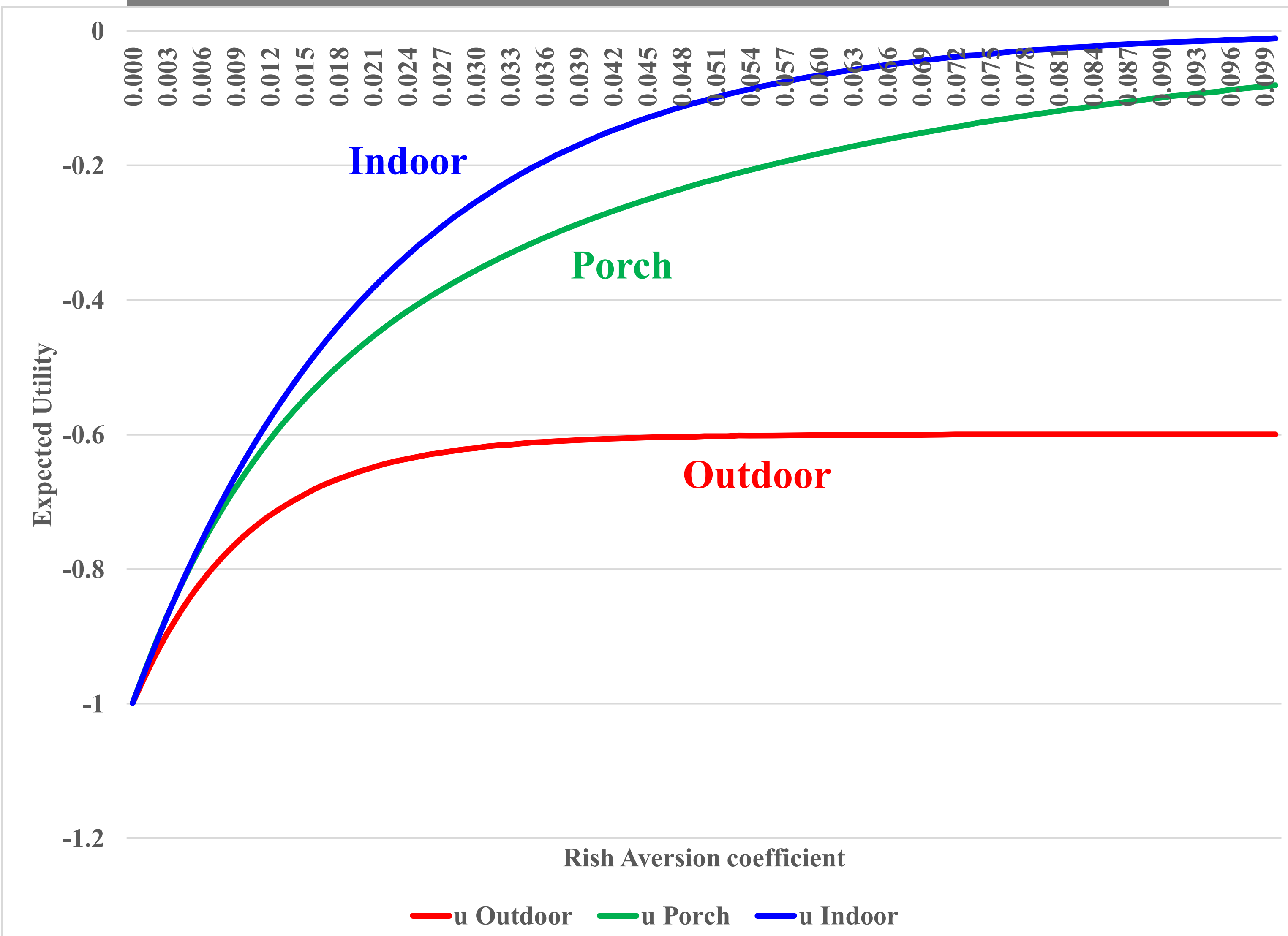
Sensitivity of Certain Equivalents



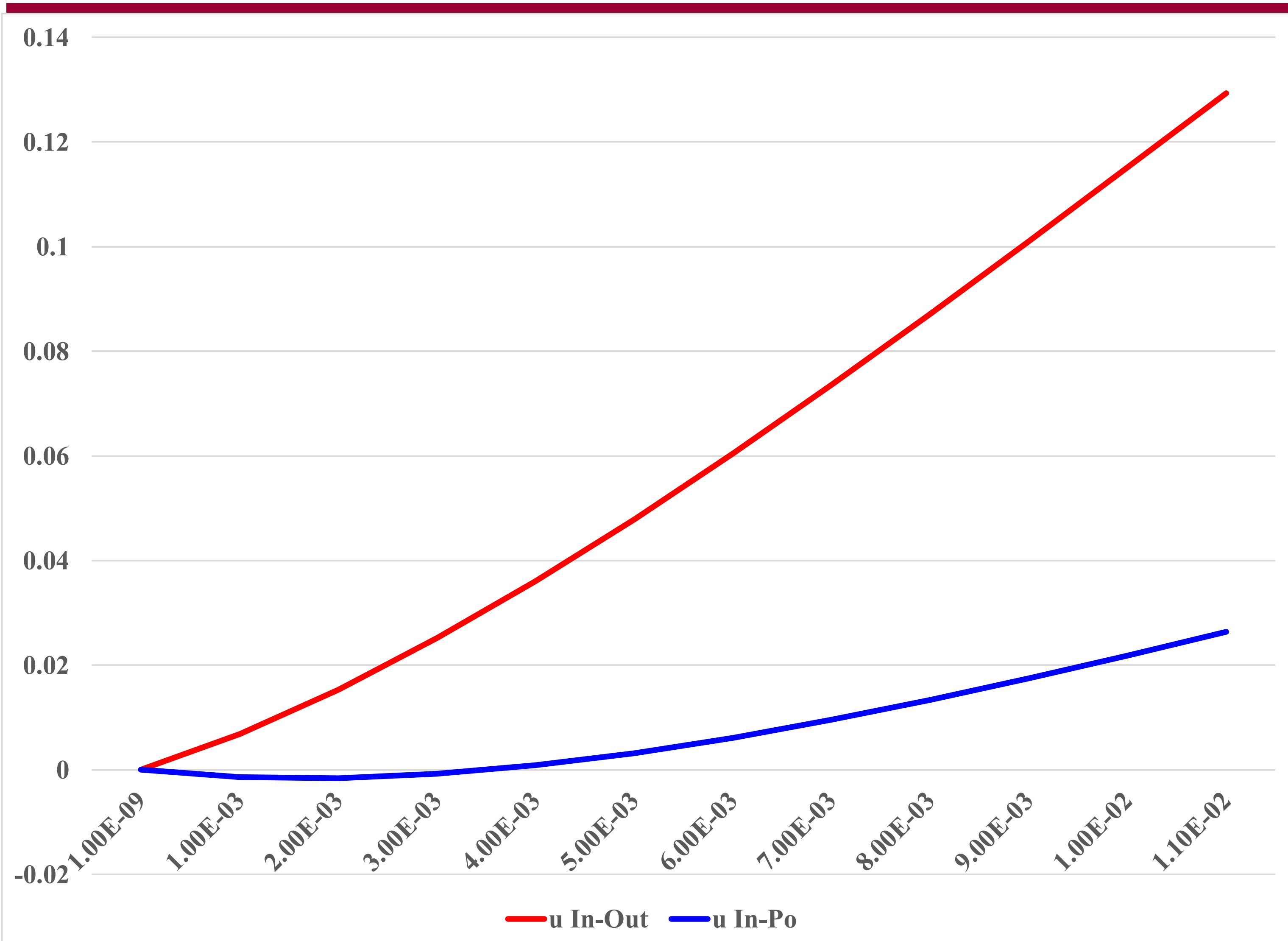
Decision Analysis

Dale M. Nesbitt

Let's Plot the Expected Utilities on a Common Graph



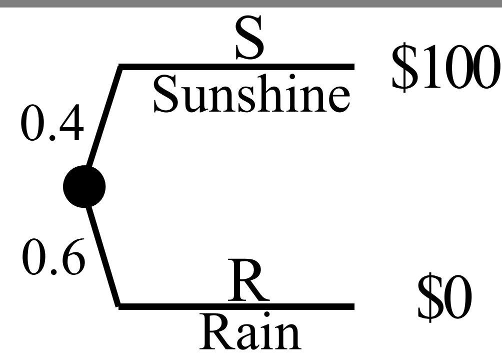
Let's Plot Expected Utility Differences from Indoor



Decision
Analysis

Dale M. Nesbitt

Calculate the Mean and Variance of Money (PISP)

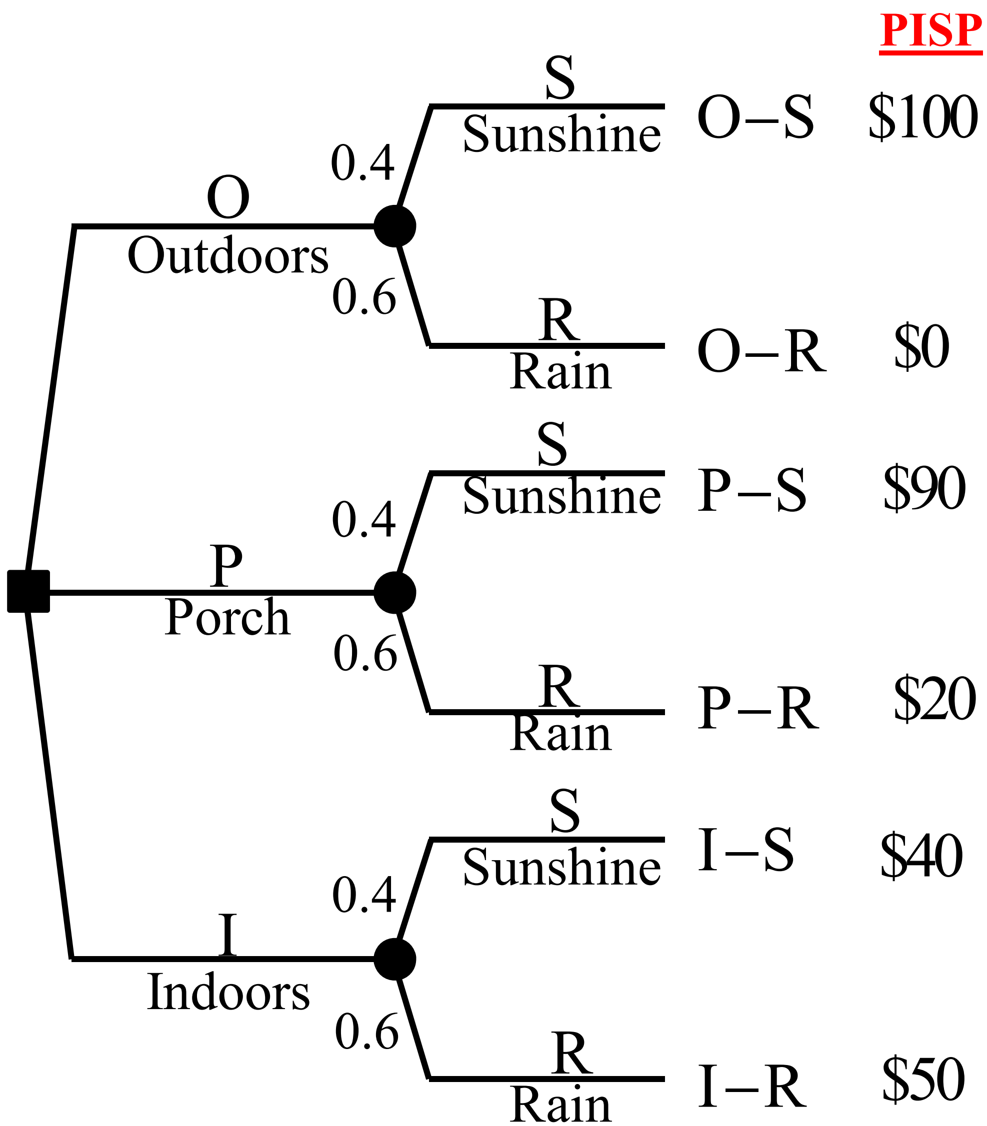


Prob	Prize	p*prize	(prize-mean)^2	p*sq
0.4	100	40	3600	1440
0.6	0	0	1600	960
		40	Variance	2400
		Mean (Sum)	Std. Deviation	48.98979

Decision Analysis

Mean, Variance, and Approximate Formula

Dale M. Nesbitt



Mean	Variance
\$40	$0.4*(100-40)^2 + 0.6*(0-40)^2 = 2400$
\$48	$0.4*(90-48)^2 + 0.6*(20-48)^2 = 865.6$
\$46	$0.4*(40-46)^2 + 0.6*(50-46)^2 = 20.8$

Decision Analysis

Dale M. Nesbitt

Approximate Formula for the Three Certain Equivalents

$$40-\frac{2400}{2\rho}=40-\gamma 1200$$

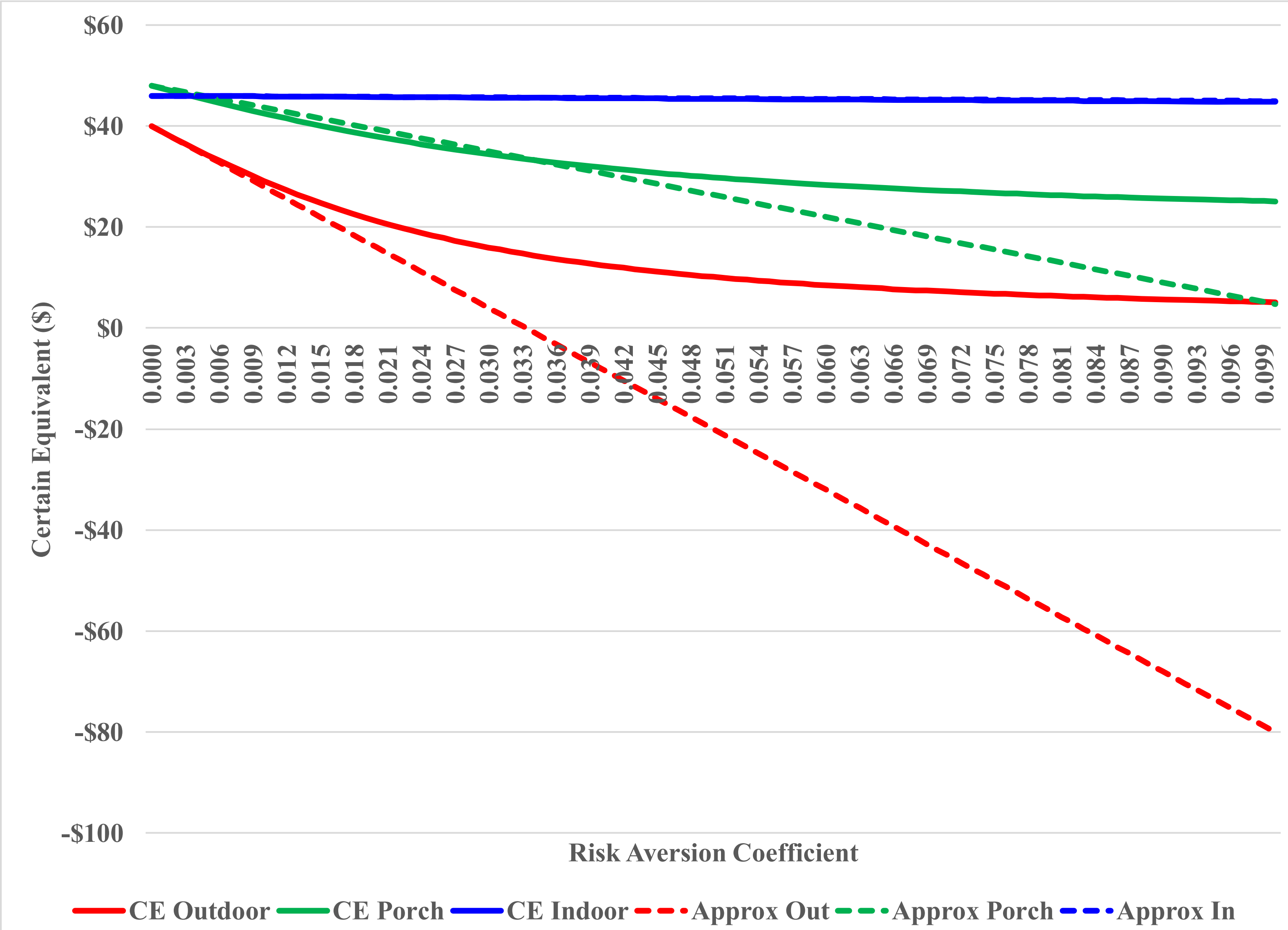
$$48-\frac{865.6}{2\rho}=48-\gamma 432.8$$

$$46-\frac{20.8}{2\rho}=46-\gamma 10.4$$

Decision Analysis

Dale M. Nesbitt

Approximate Formula Works Best for Non Extreme Risk Aversion



Decision Analysis

Dale M. Nesbitt

Are There Some Situations Where Risk Attitude Can Be Skipped?

Deterministic Dominance

- Deal 1: Receive $5
- Deal 2: Receive $10

- Deal 1 deterministically dominates Deal 2.
- You would choose Deal 1 no matter what your u-curve as long as it is upward sloping.

Decision Analysis

Dale M. Nesbitt

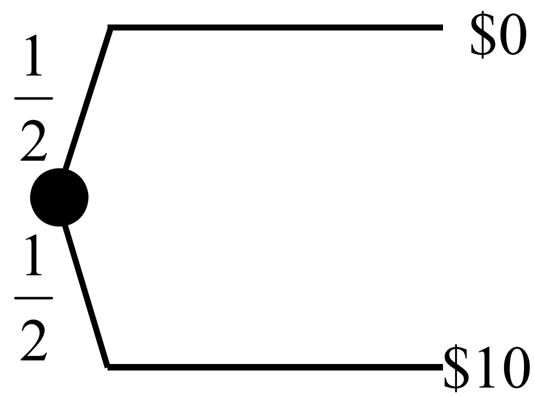
Deterministic Dominance

- Deal 1: Receive $5
- Deal 2: Receive $10

- Deal 1 deterministically dominates Deal 2.
- You would choose Deal 1 no matter what your u-curve as long as it is upward sloping.
- You don’t need this course to make decisions like this!

Deterministic Dominance

- Deal 1:

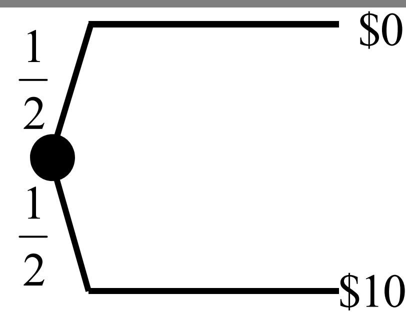


- Deal 2: $15 for sure.

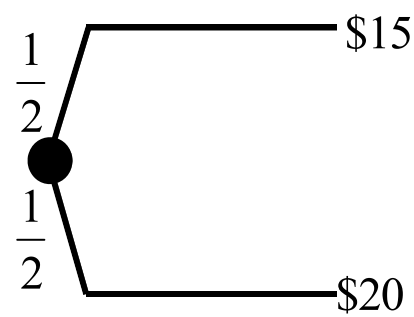
- Deal 2 deterministically dominates Deal 1.

Deterministic Dominance

- Deal 1:

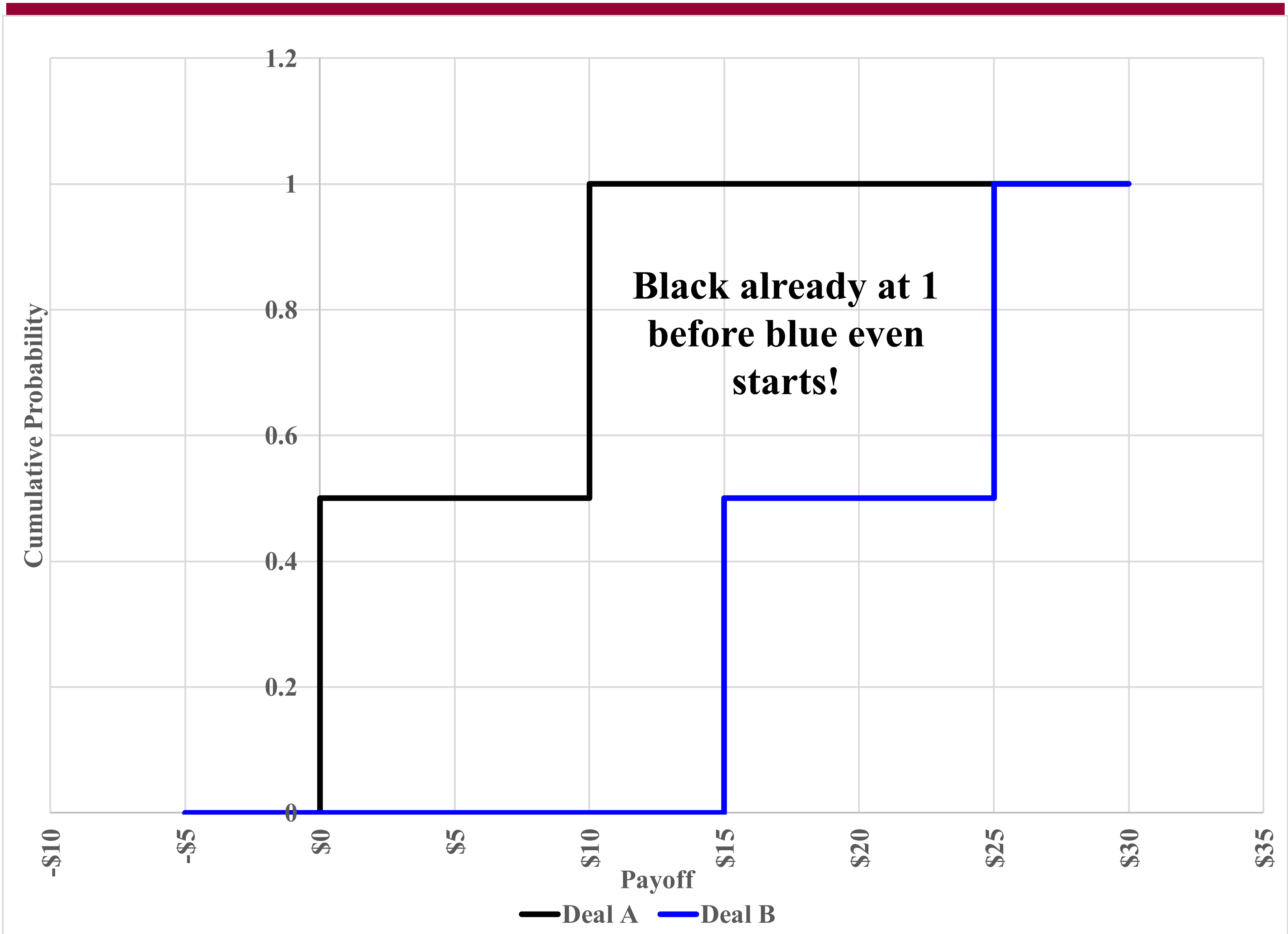


- Deal 2:

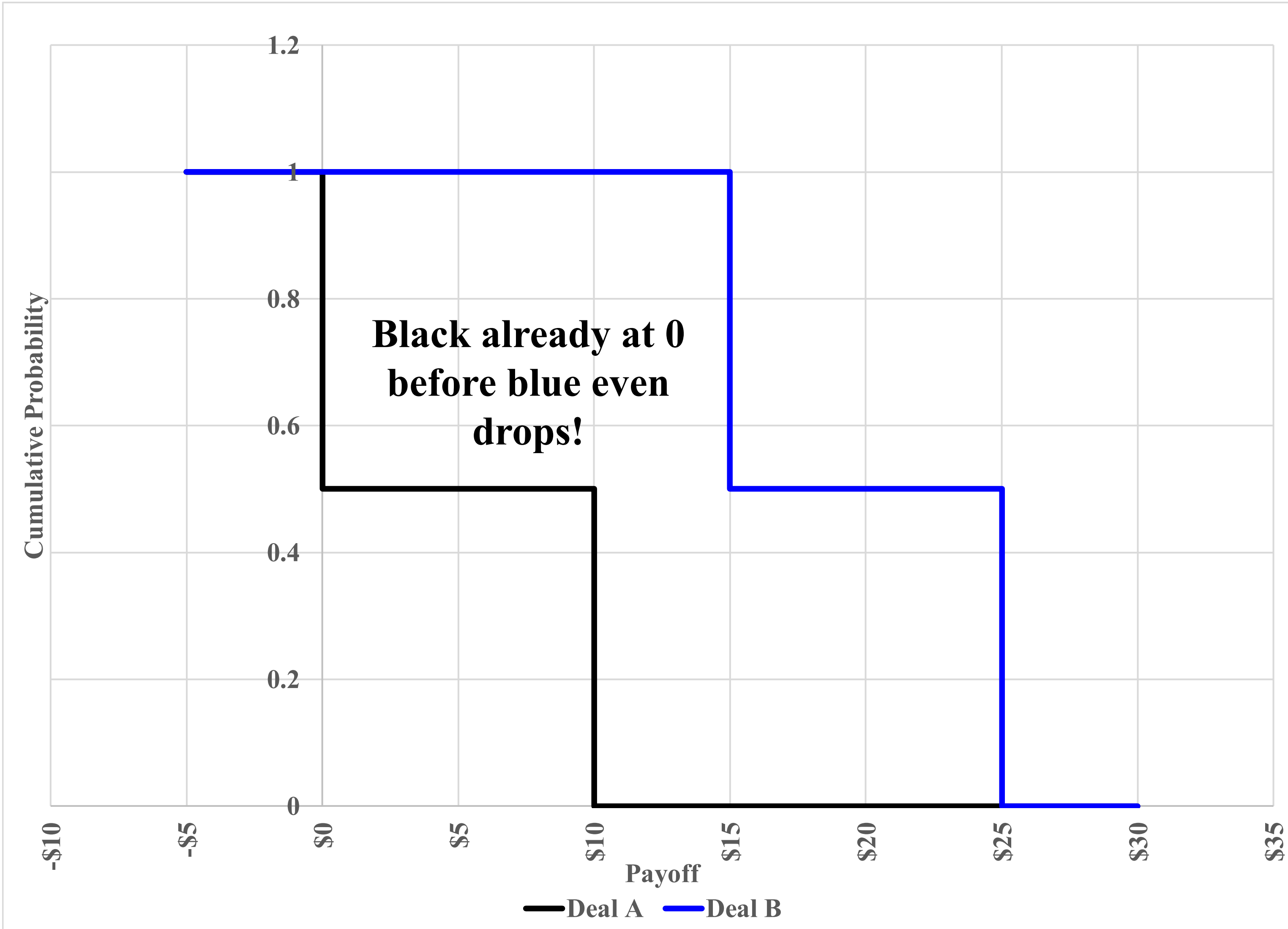


- Deal 2 deterministically dominates Deal 1.

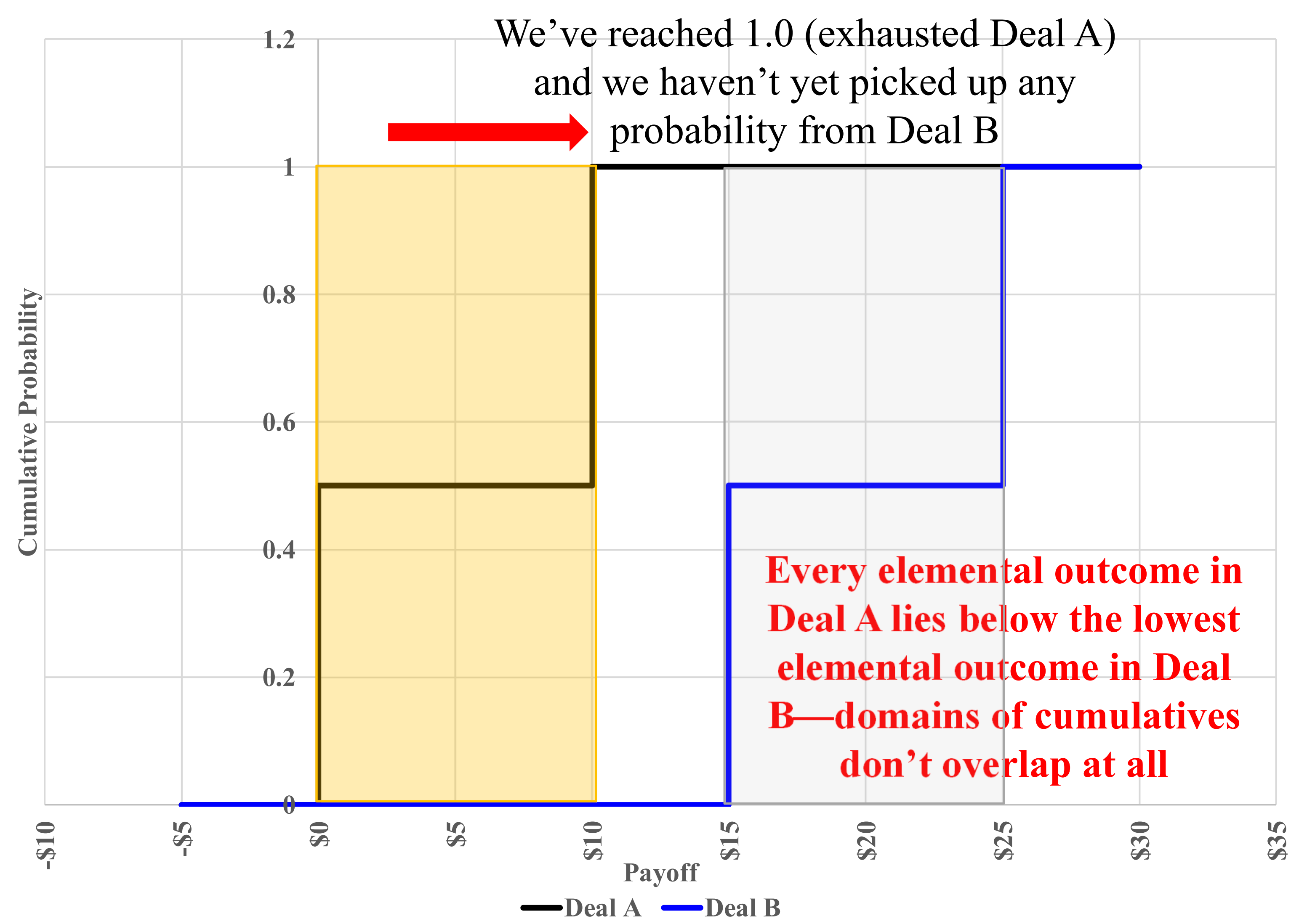
Cumulatives for Deal A and B (Cumulative Means "Less Than")



Complementary Cumulatives for Deals A and B (Means "Greater Than")



Cumulatives for Deal A and B



Decision Analysis

Dale M. Nesbitt

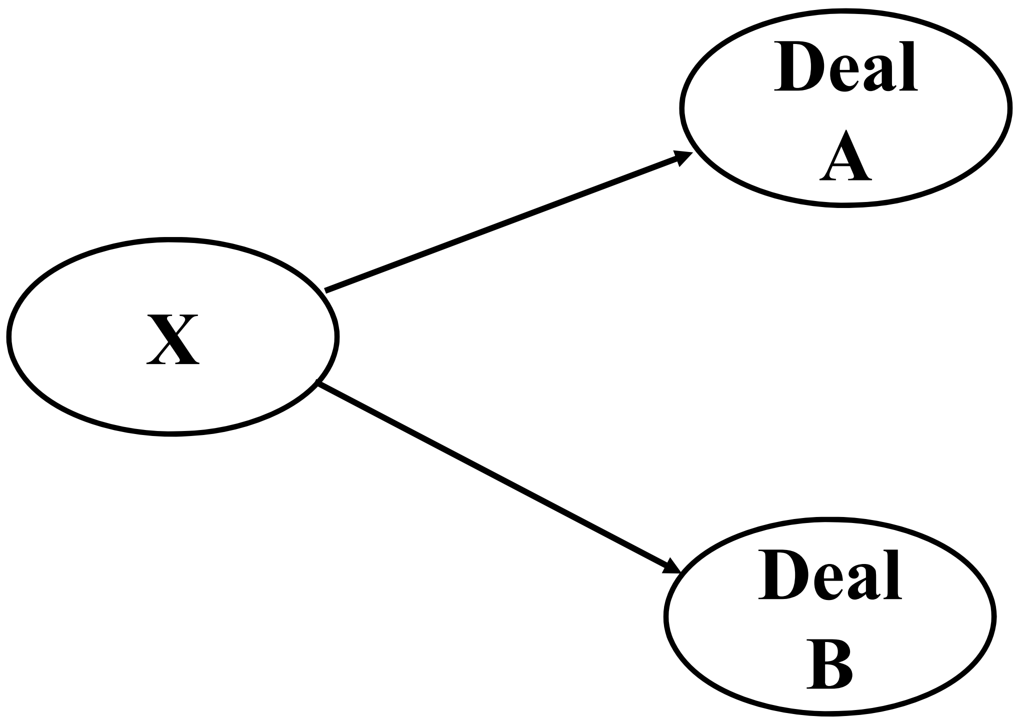
Decision Analysis

Dale M. Nesbitt

Deal B Deterministically Dominates Deal A

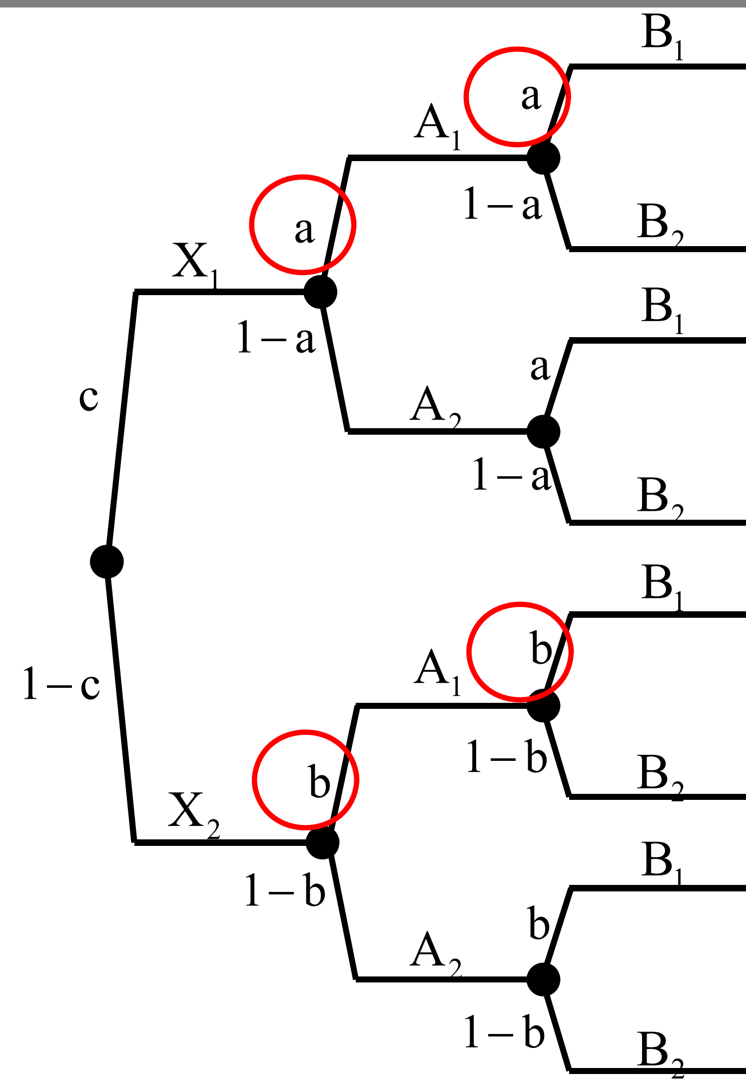
What If the Same Distinction Is Relevant to Deal A and Deal B?

- Can Deal B deterministically dominate Deal A because of relevance? Yes.



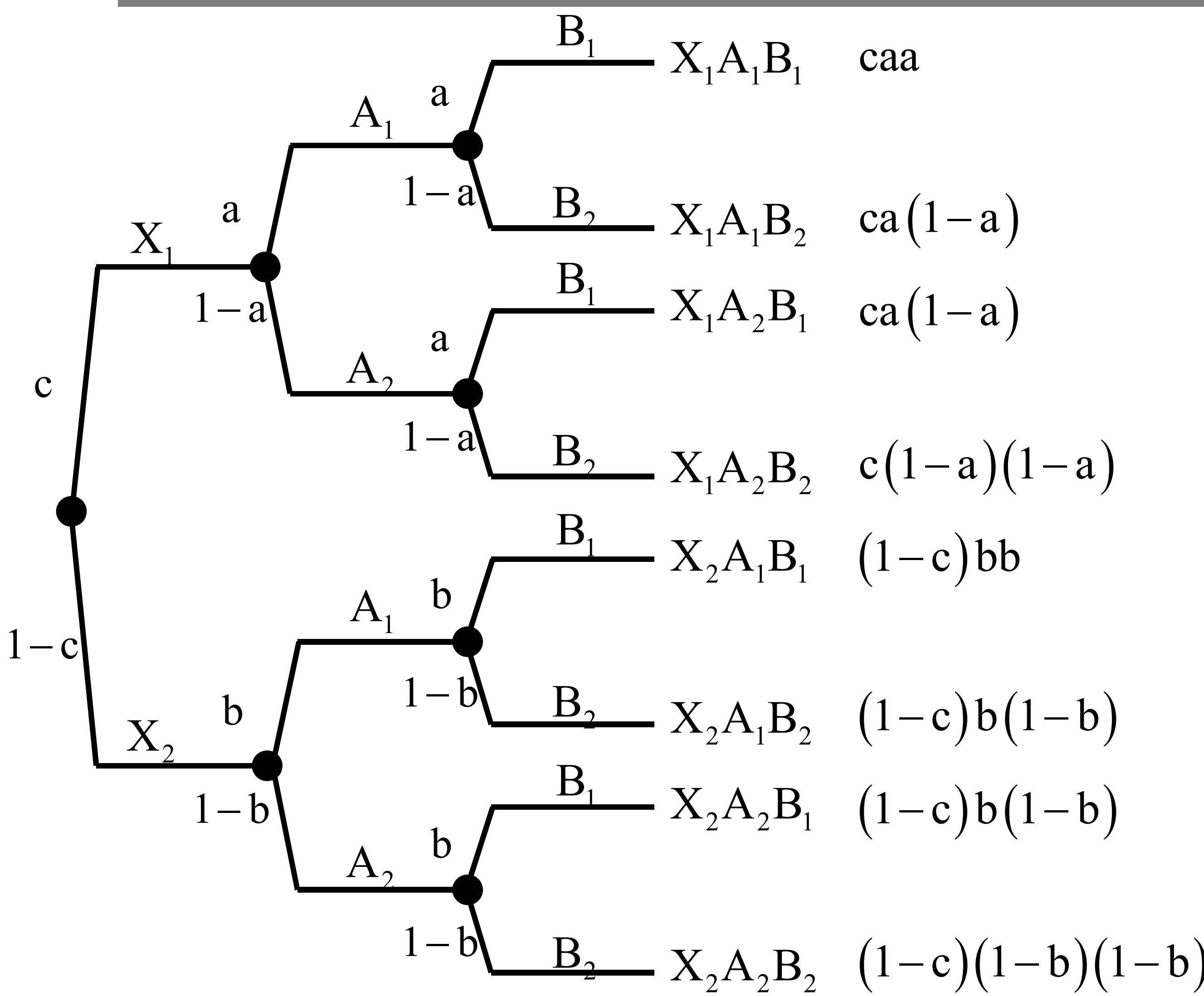
A and B Are Conditional on X

$a >> b$

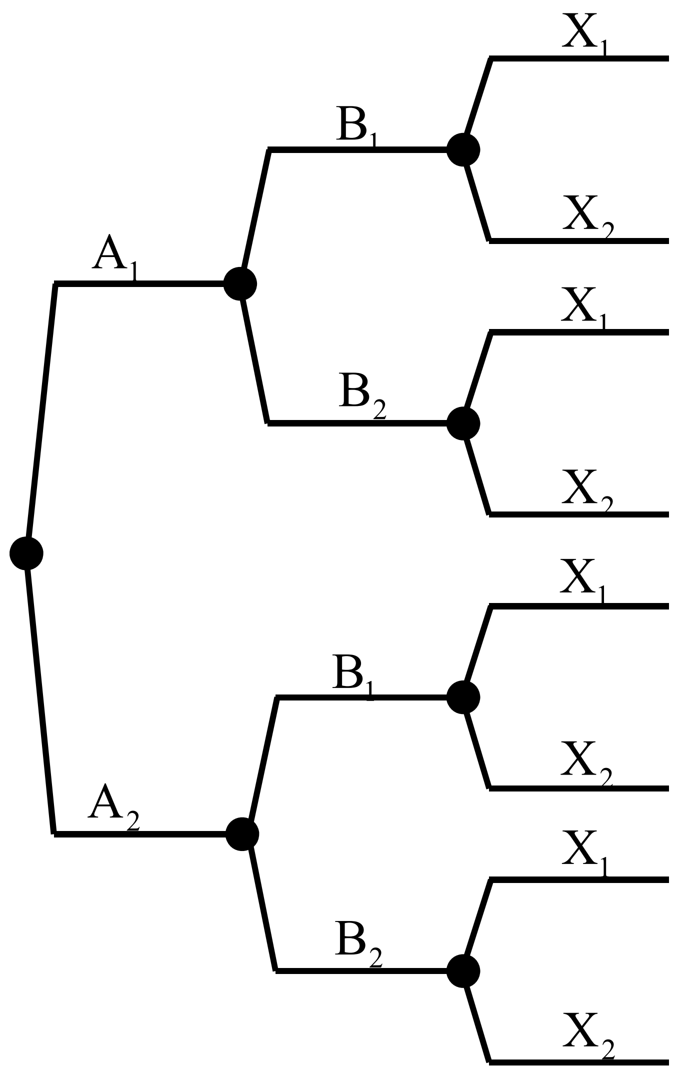


Decision Analysis
Dale M. Nesbitt

A and B Are Conditional on X



The Reversed Elemental Possibilities Tree



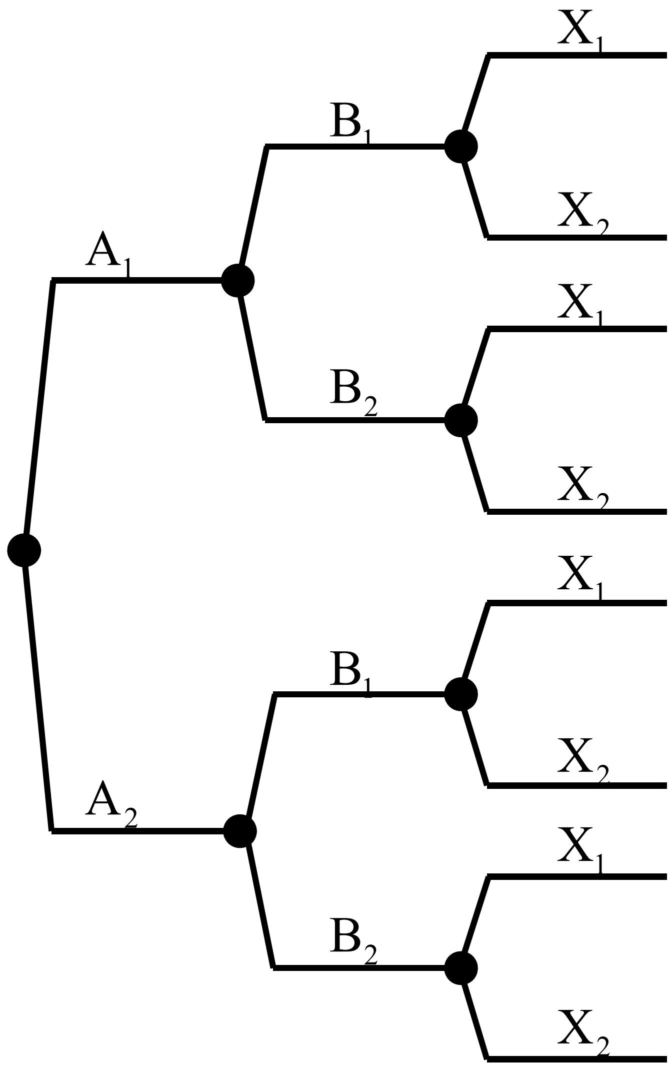
Decision Analysis

Dale M. Nesbitt

Decision Analysis

Dale M. Nesbitt

The Reversed Elemental Possibilities Tree with Elemental Probabilities

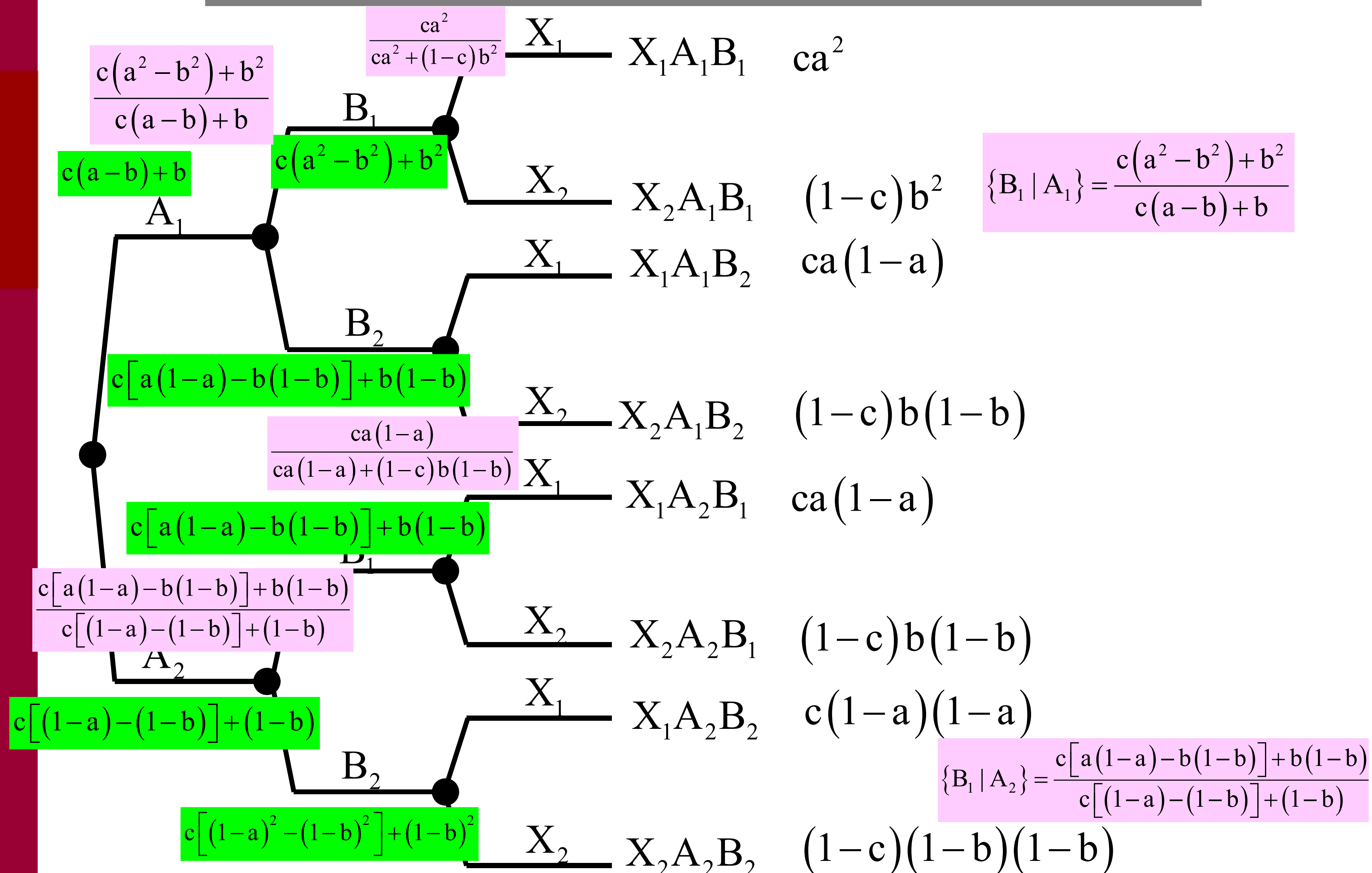


$X_1A_1B_1$	caa
$X_2A_1B_1$	$(1-c)bb$
$X_1A_1B_2$	$ca(1-a)$
$X_2A_1B_2$	$(1-c)b(1-b)$
$X_1A_2B_1$	$ca(1-a)$
$X_2A_2B_1$	$(1-c)b(1-b)$
$X_1A_2B_2$	$c(1-a)(1-a)$
$X_2A_2B_2$	$(1-c)(1-b)(1-b)$

Decision Analysis

Dale M. Nesbitt

Roll Back One and Two Levels



Decision Analysis

Dale M. Nesbitt

Let Us Look At Some Cases

- Suppose that a=1 and b=0 This means that if X1 occurs, you will get A1 and B1.
- This is full probabilistic dependency (full relevance)

$$\{B_1 \mid A_1\} = \frac{c}{c} = 1$$

$$\{A_1\} = c$$

- X determines BOTH A and B. If you know A, you know B. That is what full relevance means.

$c = 1$

- X_1 is a surrogate for A_1 and B_1.
- A_1 and B_1 are deterministically relevant, i.e., deterministically related.
- Let's look at an example

Decision Analysis

Dale M. Nesbitt

Let Us Try a=0.99, b=0.01

$$\{B_1 \mid A_1\} = \frac{0.98c + 0.0001}{0.98c + 0.01}$$

$$\{A_1\} = 0.98c + 0.01$$
$$= \frac{99}{100}\left(\frac{1}{99} + \frac{98}{99}c\right)$$

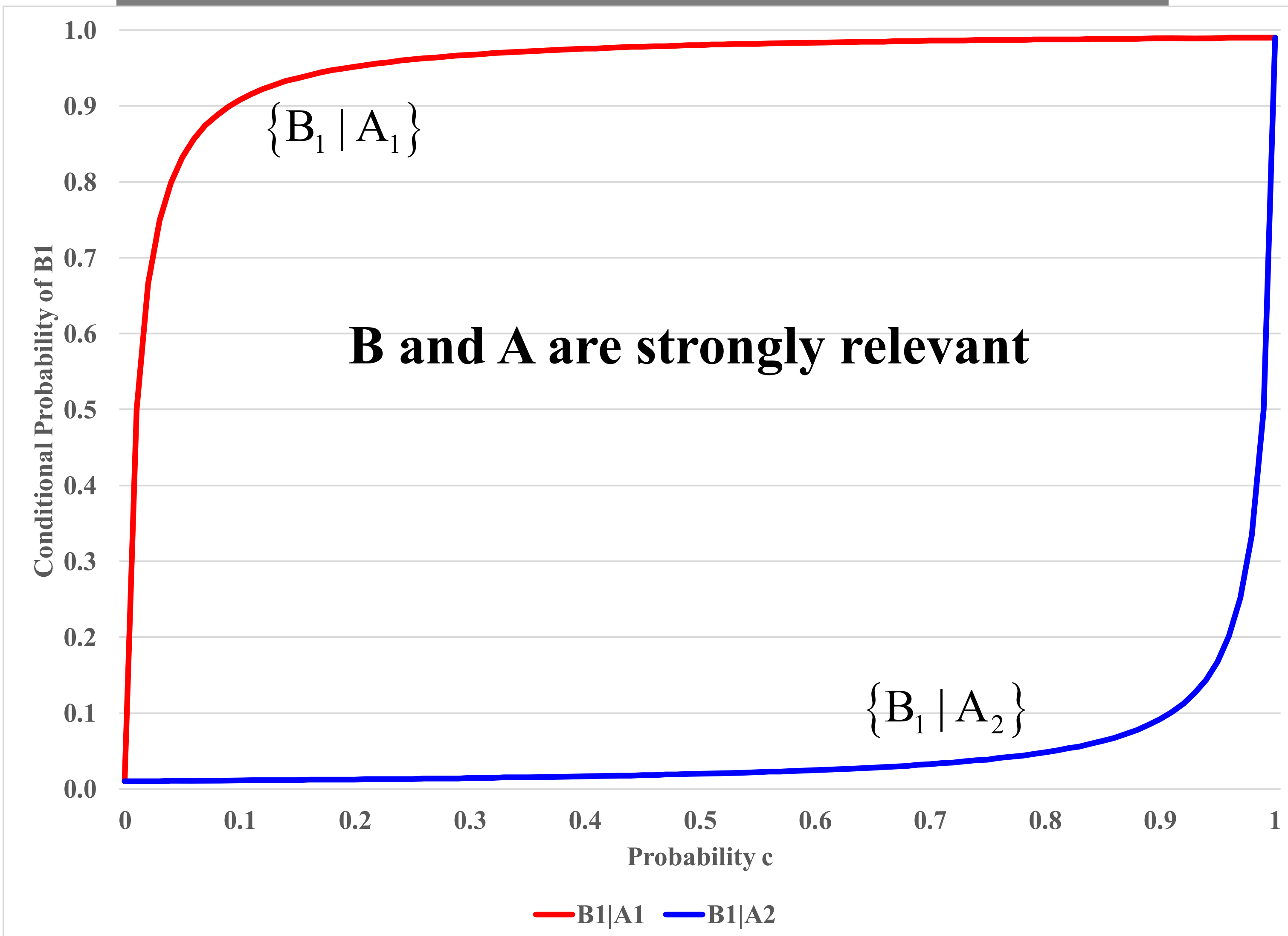
$$\{A_1\} = \frac{98}{100}c + \frac{1}{100}$$
$$c = 1 \Rightarrow \{A_1\} = \frac{99}{100} \quad c = 0 \Rightarrow \{A_1\} = \frac{1}{100}$$

$$\{B_1 \mid A_2\} = \frac{1}{100}\frac{1}{1 - \frac{98}{99}c}$$

$$\{A_2\} = -0.98c + 0.99$$
$$= \frac{99}{100}\left(1 - \frac{98}{99}c\right)$$

$$\{A_2\} = c\left[(1-a) - (1-b)\right] + (1-b)$$
$$c = 1 \Rightarrow \{A_2\} = \frac{1}{100} \quad c = 0 \Rightarrow \{A_2\} = \frac{99}{100}$$

Conditional Probabilities As a Function of Probability c of X_1 (a=0.99, b=0.1)



Let Us Try a=0.99, b=0.01, c=0.2

- Assume that c = 0.2
- $\{B_1 \mid A_1\} = 0.951942$
- $\{B_1 \mid A_2\} = 0.012469$
- Look how much relevance there is.
- The setting of A really determines the distribution of B.

Decision Analysis

Dale M. Nesbitt

Relevance Can Create Deterministic Dominance

- Deal A pays $10 if the outcome of a coin toss is heads and $0 if the outcome is tails.
- Deal B pays $15 if the outcome of that SAME coin is heads and $5 if the outcome is tails.
- The second deterministically dominates the first.
- Deterministic dominance doesn’t happen very much in the real world.

Decision Analysis

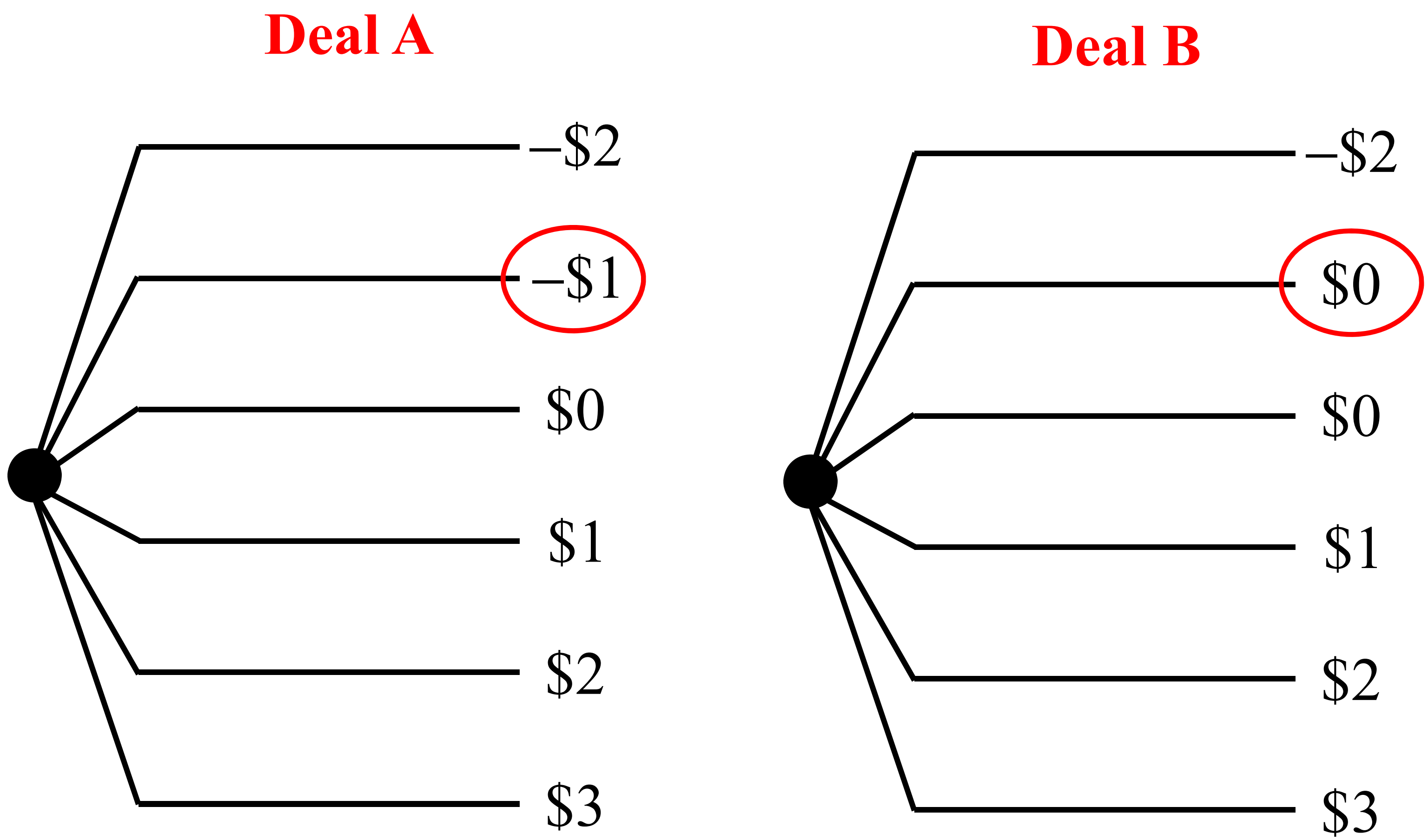
Dale M. Nesbitt

Probabilistic Dominance

Here Are Two Deals

- Deal A: Roll a die; I pay you the number on the die minus 3. (-2, -1, 0, 1, 2, 3)
- Deal B: Roll a die: I add 1 to the second number but pay you the same for every other number (-2, 0, 0, 1, 2, 3)
- The prizes are identical, but one is better.

Same Prizes Except for One



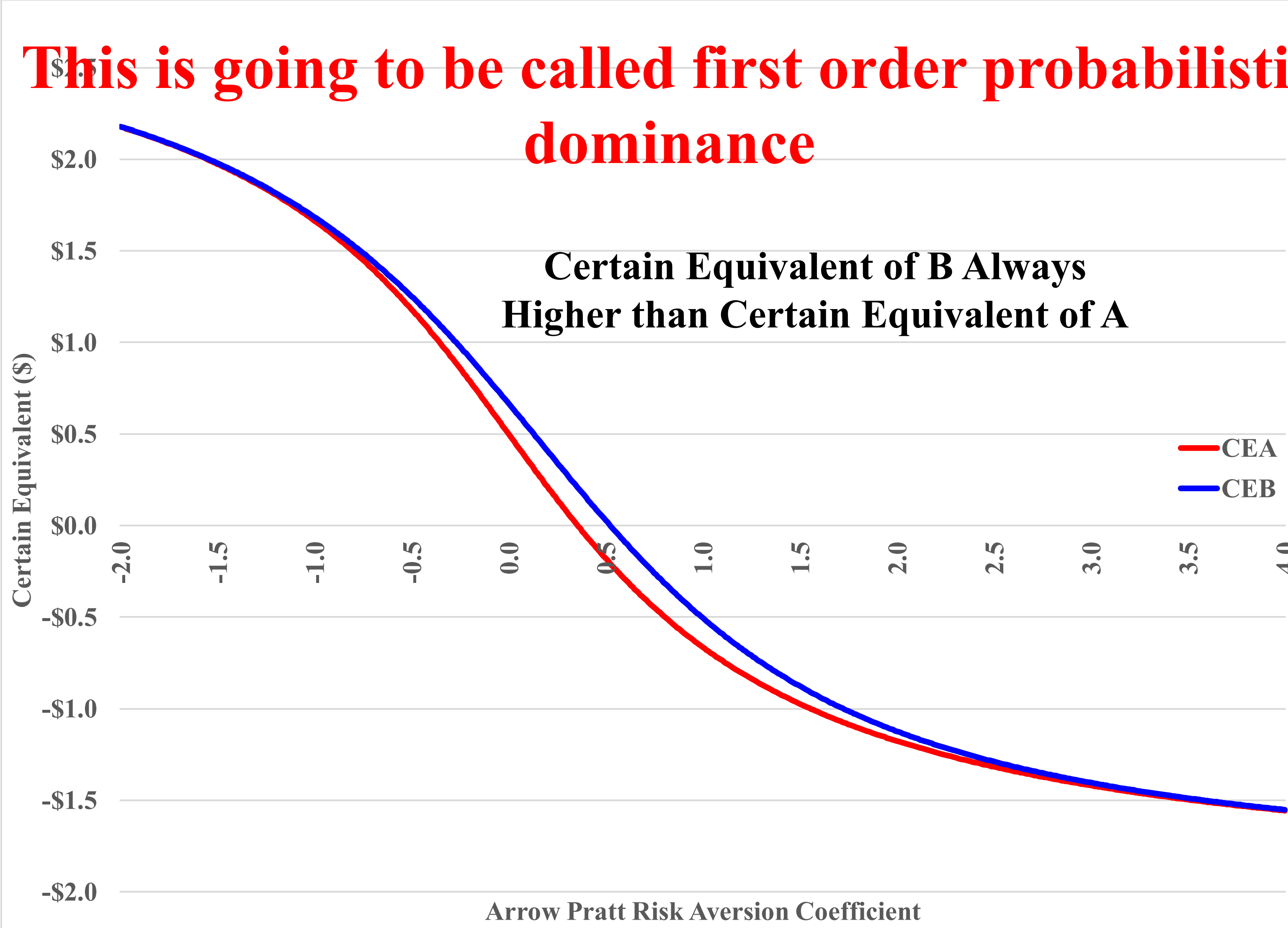
The Two Certain Equivalents—Delta Person

$$\tilde{x}_1 = -\frac{1}{\gamma}\ln\left(\frac{1}{6}e^{\gamma 2} + \frac{1}{6}e^{\gamma 1} + \frac{1}{6}e^{\gamma 0} + \frac{1}{6}e^{-\gamma 1} + \frac{1}{6}e^{-\gamma 2} + \frac{1}{6}e^{-\gamma 3}\right)$$

$$\tilde{x}_2 = -\frac{1}{\gamma}\ln\left(\frac{1}{6}e^{\gamma 2} + \frac{1}{3}e^{\gamma 0} + \frac{1}{6}e^{-\gamma 1} + \frac{1}{6}e^{-\gamma 2} + \frac{1}{6}e^{-\gamma 3}\right)$$

Certain Equivalent of B Is HIGHER for EVERY Possible Risk Attitude!

This is going to be called first order probabilistic dominance



Decision Analysis

Dale M. Nesbitt

Does This Mean You Can Omit the u-Curve Per Se?

We shall prove that the answer is YES, and we shall prove under what conditions the answer is YES.

Cumulative and Complementary Cumulative

Decision Analysis

Dale M. Nesbitt

- Cumulative probability distribution

$$F_A(x) \triangleq \int_{-\infty}^{x} f_A(\xi)\,d\xi = \{\xi \le x\}$$

- Complementary cumulative

$$1 - F_A(x) \triangleq \int_{x}^{\infty} f_A(\xi)\,d\xi = \{\xi \ge x\}$$

Decision Analysis

Dale M. Nesbitt

First Order Probabilistic Dominance

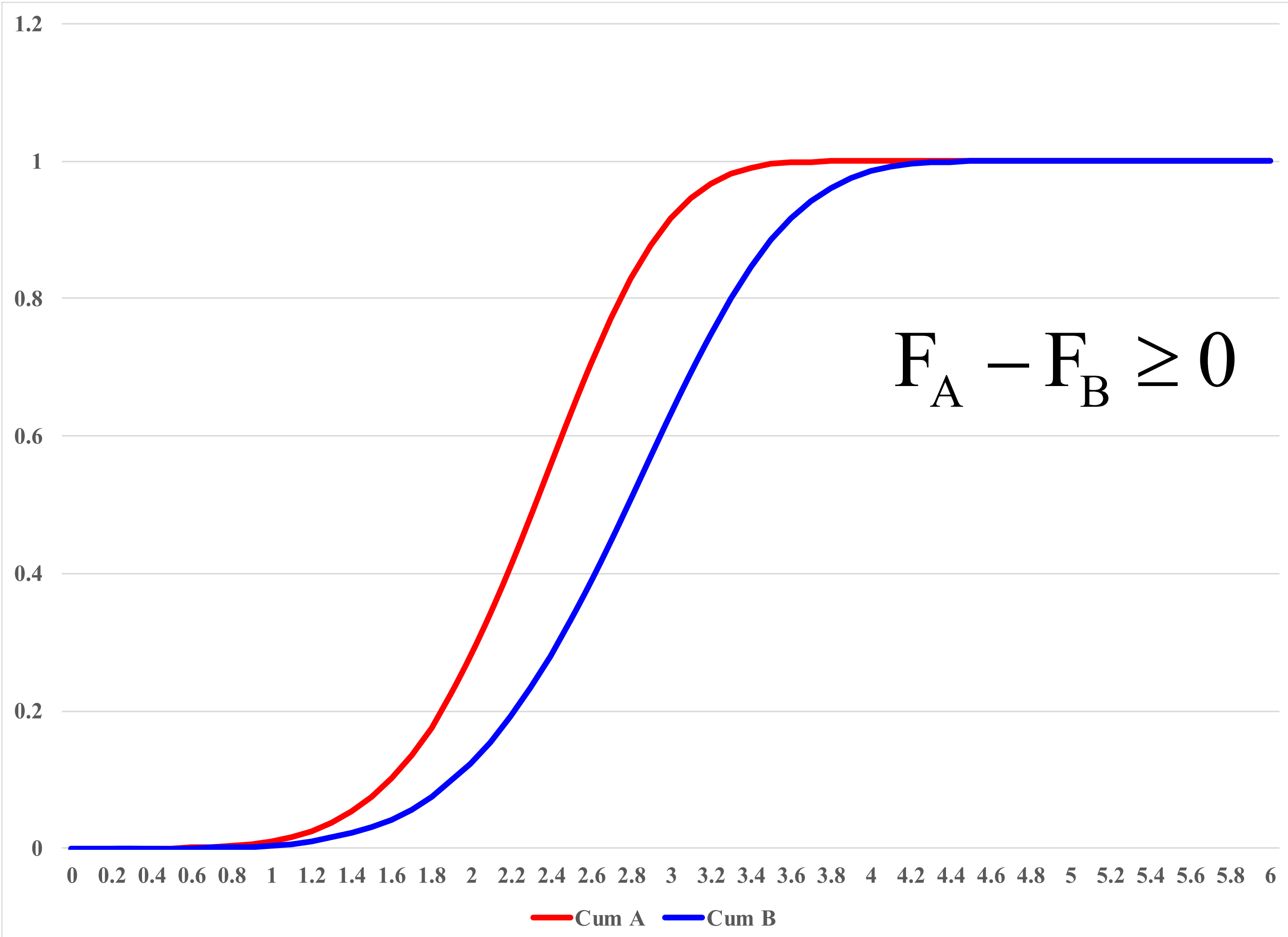
- Gamble B has first-order Probabilistic dominance over gamble A if for any outcome x, B gives at least as high a probability of receiving at least x as does A. This is a notion about the complementary cumulative

$$1-F_B(x)\geq 1-F_A(x)$$

- In terms of the cumulative distribution functions of the two gambles, B dominating A means that

$$F_B(x)\leq F_A(x)$$

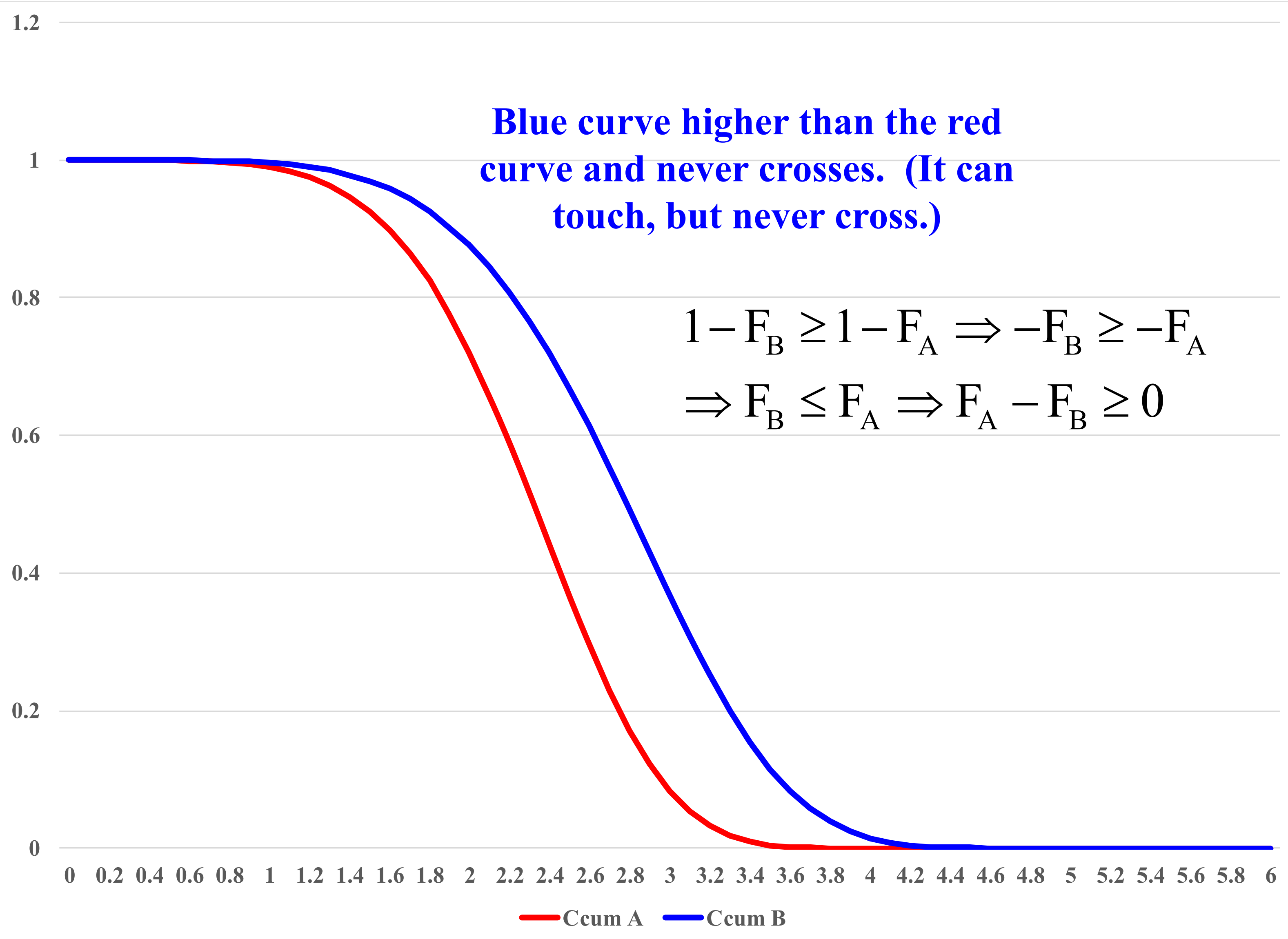
B Has First Order Probabilistic Dominance Over A



Decision Analysis

Dale M. Nesbitt

B Has First Order Probabilistic Dominance Over A—Complementary Cumulative



Blue curve higher than the red curve and never crosses. (It can touch, but never cross.)

$$1 - F_B \geq 1 - F_A \Rightarrow -F_B \geq -F_A$$

$$\Rightarrow F_B \leq F_A \Rightarrow F_A - F_B \geq 0$$

Prove That Under First Order Probabilistic Dominance

$$\lim_{z \to \infty} \int_0^z u(x) f_A(x) dx \le \lim_{z \to \infty} \int_0^z u(x) f_B(x) dx$$

- For monotonically increasing u(x). Start off integrating each side by parts

$$U = u(x) \quad dV = f_A(x) dx$$

$$dU = u'(x) dx \quad V = F_A(x)$$

$$\Rightarrow \lim_{z \to \infty} \int_{-\infty}^z u(x) f_A(x) dx = \lim_{z \to \infty} u(x) F_A(x) \Big|_{-\infty}^{z} - \lim_{z \to \infty} \int_{-\infty}^z u'(x) F_A(x) dx$$

$$\Rightarrow \lim_{z \to \infty} \int_{-\infty}^z u(x) f_B(x) dx = \lim_{z \to \infty} u(x) F_B(x) \Big|_{-\infty}^{z} - \lim_{z \to \infty} \int_{-\infty}^z u'(x) F_B(x) dx$$

Decision Analysis

Dale M. Nesbitt

Check Out the Limits

$$\lim_{z\to\infty} u(x)F_A(x)\Big|_{-\infty}^{z} = \lim_{z\to\infty} u(z)F_A(z) - u(-\infty)F_A(-\infty)$$

$$= \lim_{z\to\infty} u(z)F_A(z)$$

- Thus

$$\lim_{z\to\infty}\left[\int_0^z u(x)\left[f_B(x)-f_A(x)\right]dx\right]$$

$$= \lim_{z\to\infty} u(z)\left[F_B(z)-F_A(z)\right]$$

$$- \lim_{z\to\infty}\left[\int_0^z u'(x)F_B(x)dx - \lim_{z\to\infty}\int_0^z u'(x)F_A(x)dx\right]$$

Decision Analysis

Dale M. Nesbitt

Decision Analysis

Dale M. Nesbitt

Simplify

$$\lim_{z \to \infty} \int_0^z u(x)\left[f_B(x) - f_A(x)\right]dx$$

$$= \lim_{z \to \infty} \left\{u(z)\left[F_B(z) - F_A(z)\right]\right\} - u(0)\left[F_B(0) - F_A(0)\right]$$

$$- \lim_{z \to \infty} \int_0^z u'(x)\left[F_B(x) - F_A(x)\right]dx$$

$$= - \lim_{z \to \infty} \int_0^z u'(x)\left[F_B(x) - F_A(x)\right]dx$$

$$\int_{-\infty}^{\infty} u(x)\left[f_B(x) - f_A(x)\right]dx = -\int_{-\infty}^{\infty} u'(x)\left[F_B(x) - F_A(x)\right]dx$$

Decision Analysis

Dale M. Nesbitt

Continue to Simplify

$$\int_{-\infty}^{\infty} u(x)\left[f_B(x)-f_A(x)\right]dx = -\int_{-\infty}^{\infty} u'(x)\left[-1+1+F_B(x)-F_A(x)\right]dx$$

$$= -\int_{-\infty}^{\infty} u'(x)\left\{-\left[1-F_B(x)\right]+\left[1-F_A(x)\right]\right\}dx$$

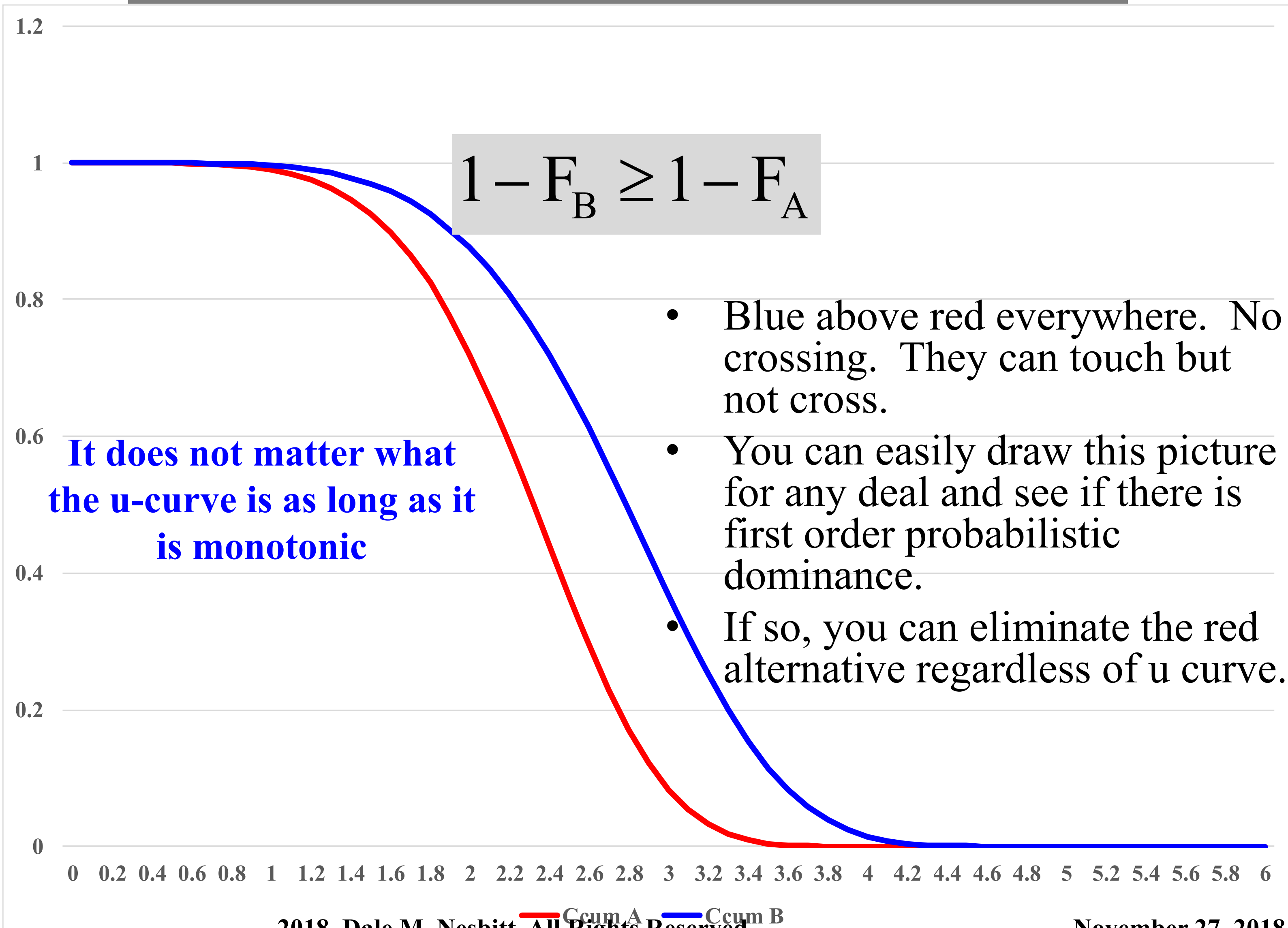
$$= \int_{-\infty}^{\infty} u'(x)\left\{\left[1-F_B(x)\right]-\left[1-F_A(x)\right]\right\}dx > 0$$

- So B dominates, requiring only that u(.) be monotonically increasing, i.e., u'(.)>0.
- **Risk averse, risk neutral, and risk preferring people all prefer the Probabilistically dominant deal B to A.**
- You don't need their utility function.

Decision Analysis

Dale M. Nesbitt

B Has First Order Probabilistic Dominance Over A—Complementary Cumulative



$$1 - F_B \geq 1 - F_A$$

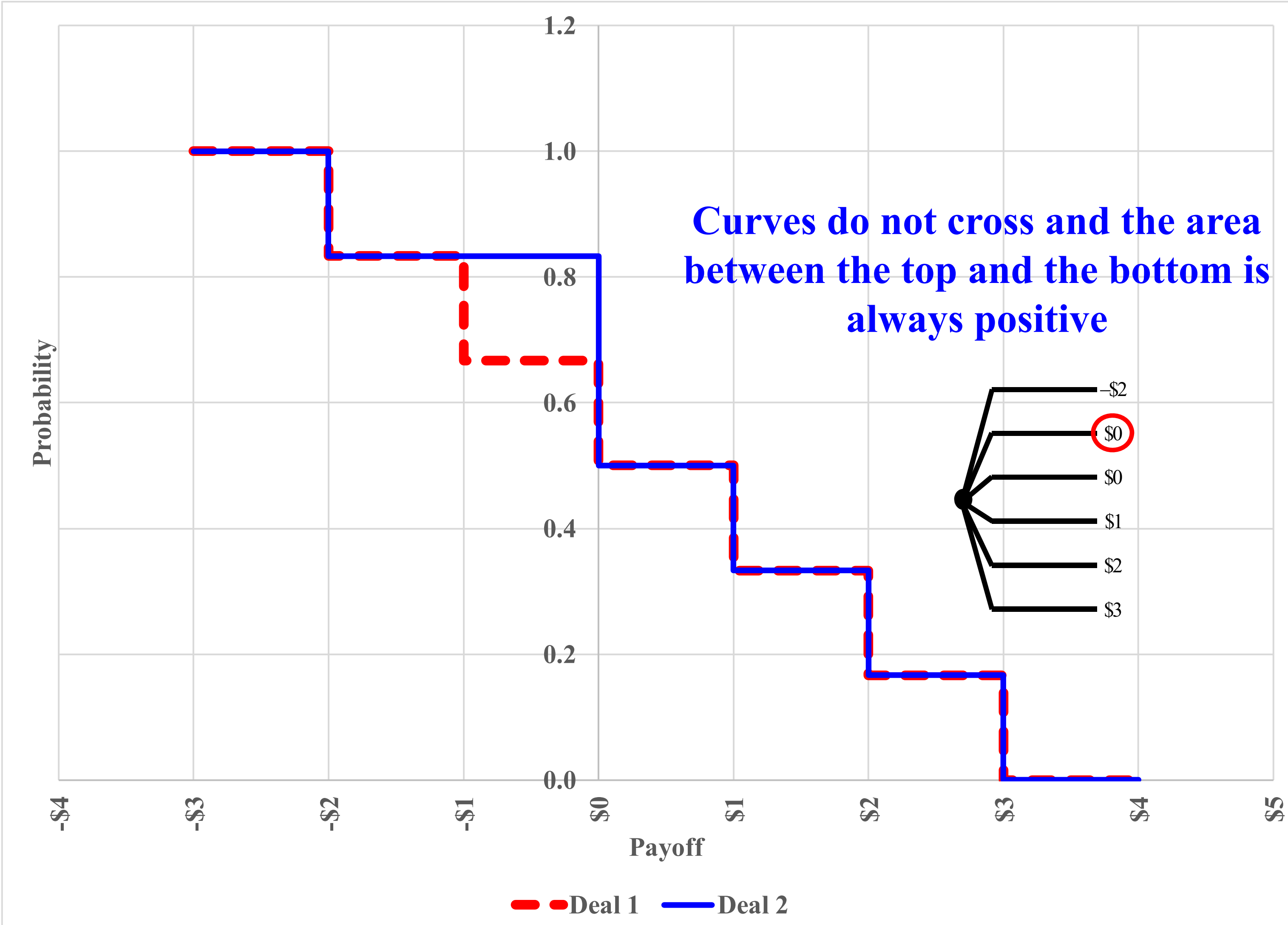
It does not matter what the u-curve is as long as it is monotonic

- Blue above red everywhere. No crossing. They can touch but not cross.
- You can easily draw this picture for any deal and see if there is first order probabilistic dominance.
- If so, you can eliminate the red alternative regardless of u curve.

Decision Analysis

Dale M. Nesbitt

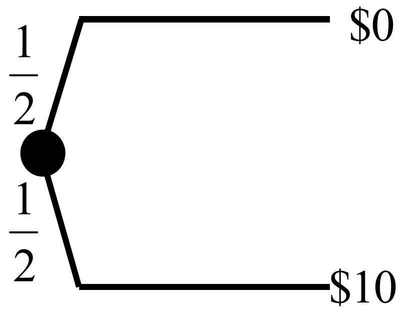
Complementary Cumulative Distributions for the Two Deals

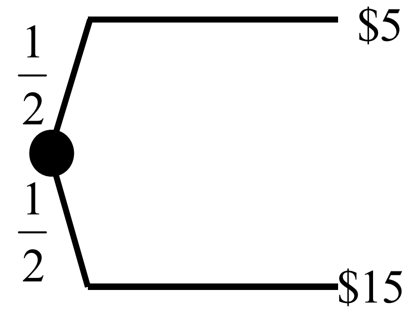


Decision Analysis

Dale M. Nesbitt

Consider These Two Deals





Mean = 5
Variance = 25

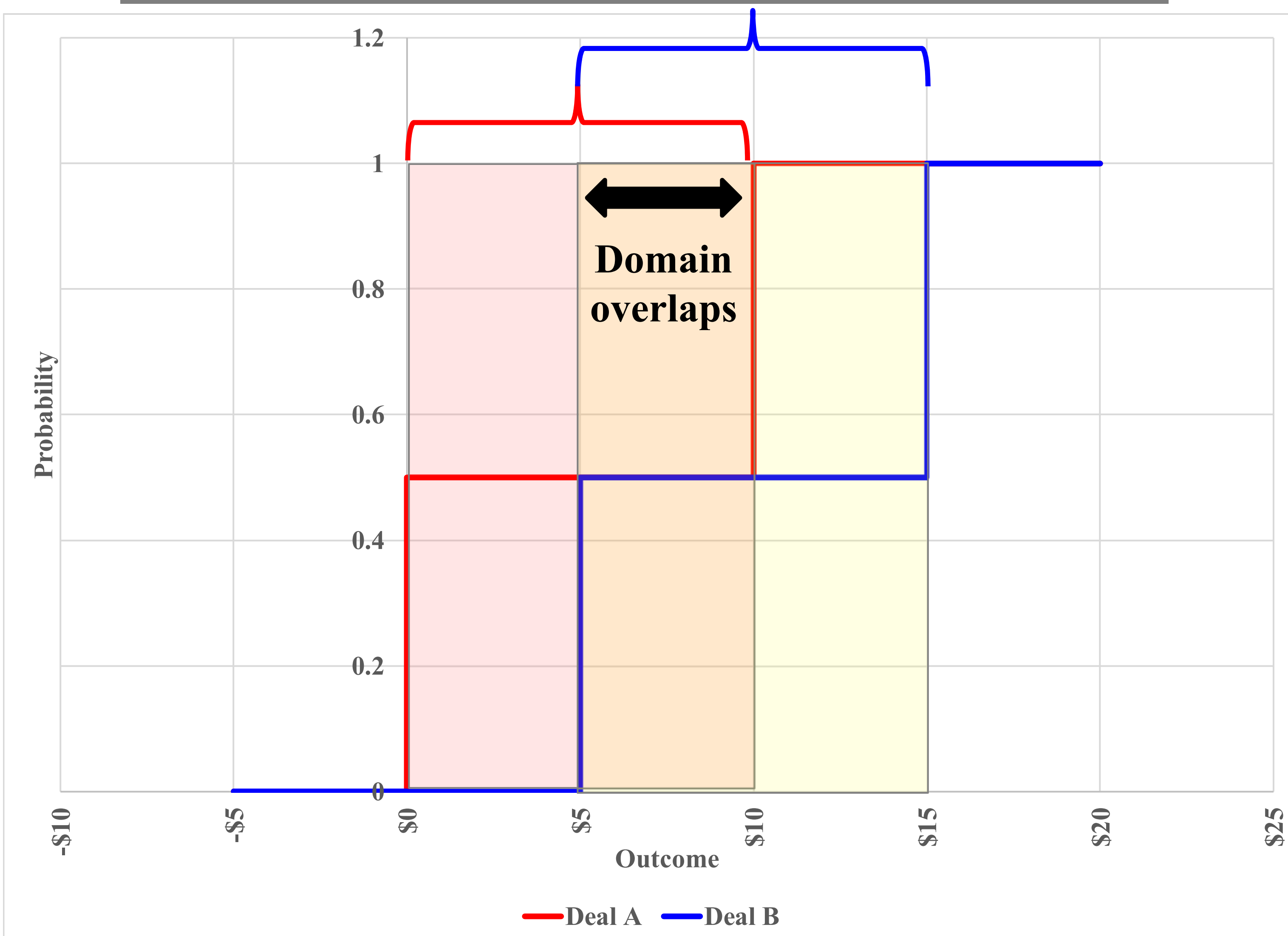
Mean = 10
Variance = 25

$$\tilde{x} = \langle x \rangle - \frac{\text{Var}}{2\rho}$$

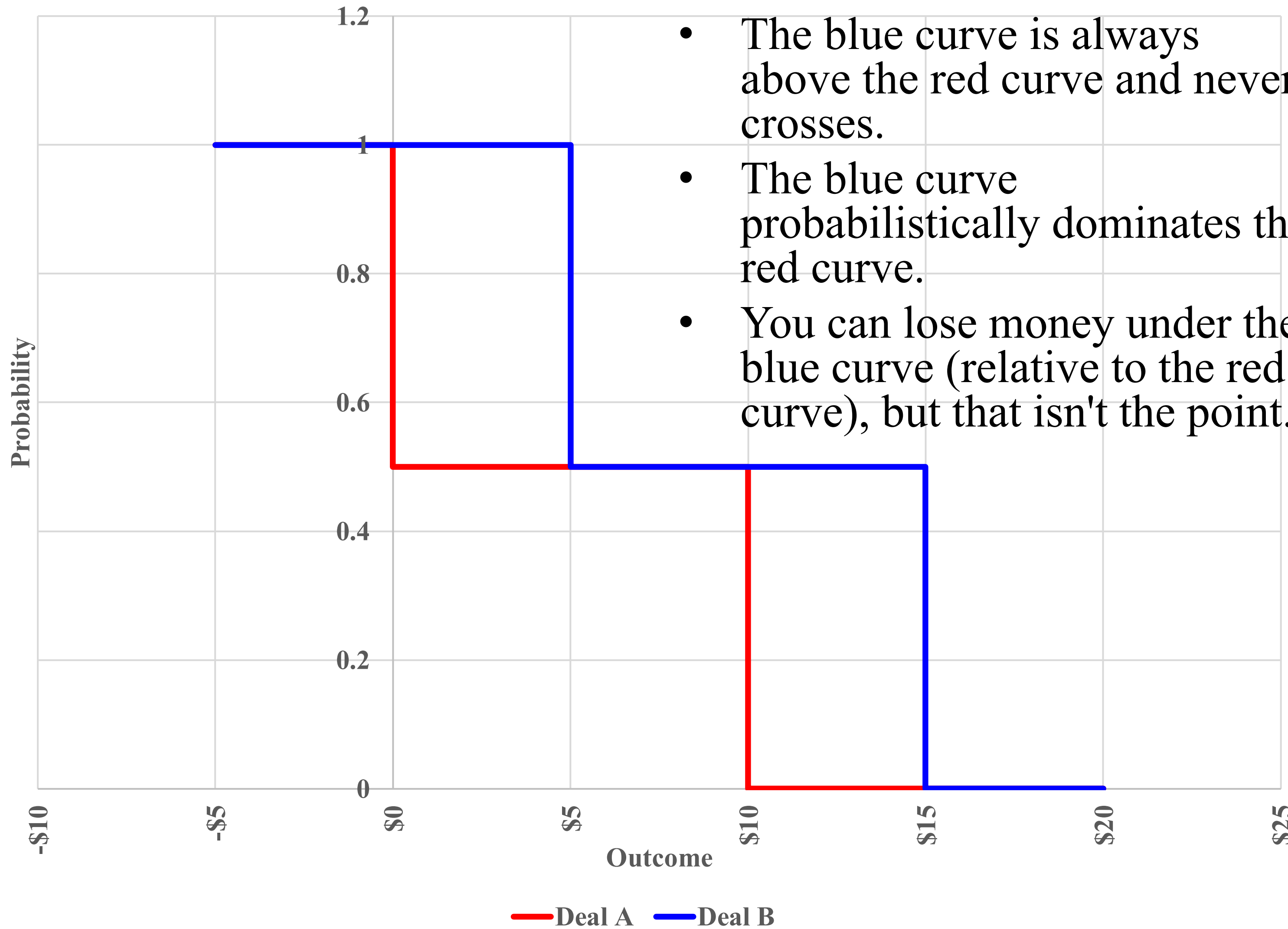
$$\tilde{x} = \langle x \rangle - \frac{\text{Var}}{2\rho}$$

- Plot their cumulatives and complementary cumulatives

Cumulatives Overlap (Domains Overlap)



Complementary Cumulatives



- The blue curve is always above the red curve and never crosses.
- The blue curve probabilistically dominates the red curve.
- You can lose money under the blue curve (relative to the red curve), but that isn't the point.

For Any Monotonic u-curve

- The certain equivalent of BLUE will always be higher than the certain equivalent of RED

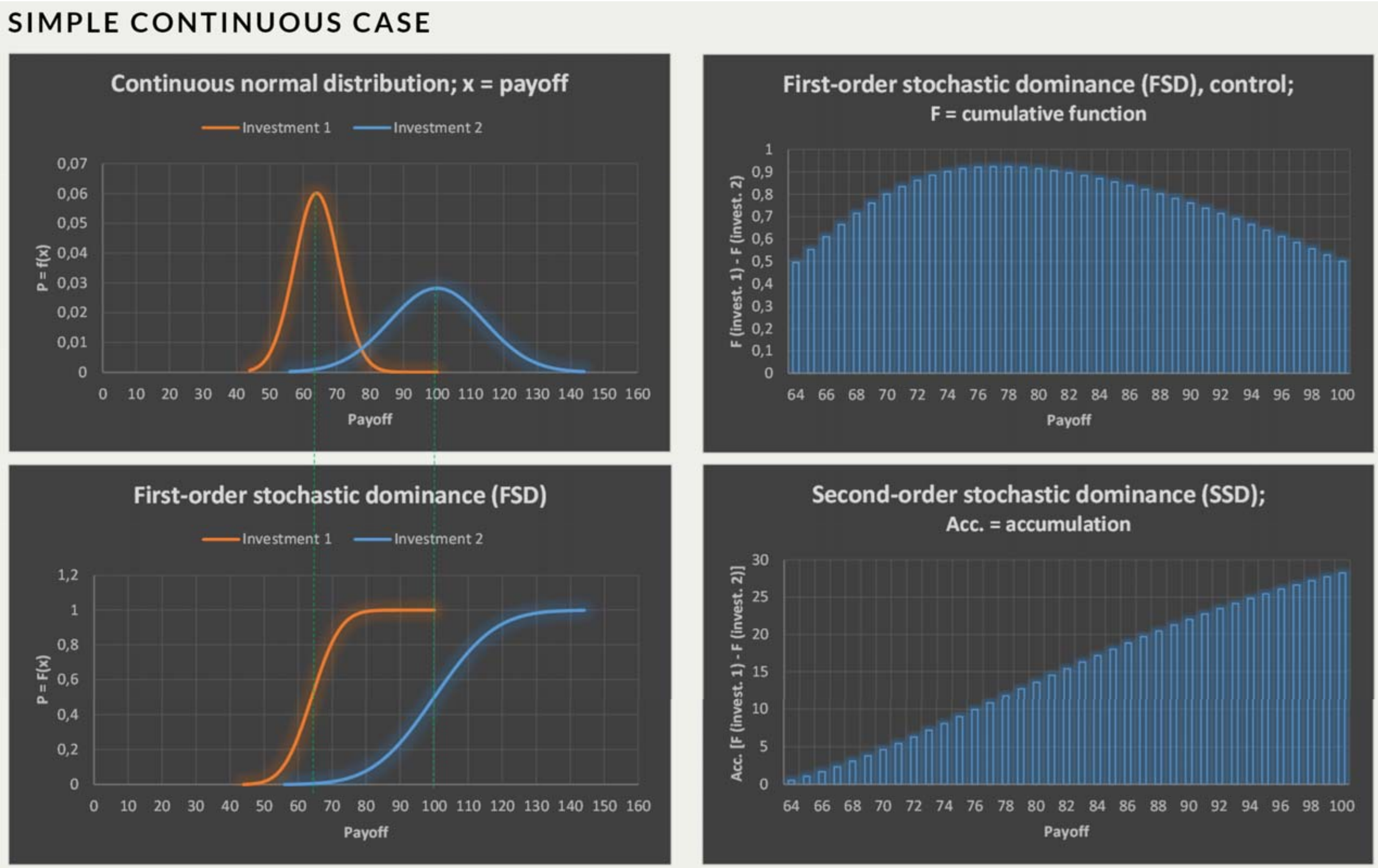
First Order Probabilistic Dominance

- The only requirement is that the u-curve be increasing.
- All increasing u curves select the probabilistically dominant deal.
- You don't have to even explore or consider the u-curve.
- The answer is "like magic."
- You always examine your deals for probabilistic dominance.

Decision Analysis

Dale M. Nesbitt

First Order Probabilistic Dominance



Decision Analysis

Dale M. Nesbitt

Second Order Probabilistic Dominance

- B probabilistically dominates A in the second order if

$$\int_{-\infty}^{z} F_A(\xi)\,d\xi \geq \int_{-\infty}^{z} F_B(\xi)\,d\xi$$

$$\Rightarrow \int_{-\infty}^{z} \left\{\left[1-F_B(\xi)\right]-\left[1-F_A(\xi)\right]\right\} d\xi \geq 0$$

- The integral under the complementary cumulative of A must everywhere exceed the integral under the complementary cumulative of b

Decision Analysis

Dale M. Nesbitt

Begin Where we Ended Up with First Order Dominance

$$\int_{-\infty}^{\infty} u(x)\left[f_B(x)-f_A(x)\right]dx = -\int_{-\infty}^{\infty} u'(x)\left[F_B(x)-F_A(x)\right]dx$$

$$\lim_{x\to\infty}\int_{-\infty}^{x} u(z)f_B(z)\,dz - \lim_{x\to\infty}\int_{-\infty}^{x} u(z)f_A(z)\,dz = \lim_{x\to\infty}\int_{-\infty}^{x} u'(z)\left[F_A(z)-F_B(z)\right]dz$$

$$U = u'(z) \quad dV = \left[F_A(z)-F_B(z)\right]dz$$

$$dU = u''(z) \quad V = \int_{-\infty}^{z}\left[F_A(\xi)-F_B(\xi)\right]d\xi$$

So Integration By Parts Once Again Yields

$$\lim_{x\to\infty}\int_{-\infty}^{x} u(z)f_B(z)dz - \lim_{x\to\infty}\int_{-\infty}^{x} u(z)f_A(z)dz$$

$$= \lim_{x\to\infty}\left\{u'(z)\int_{-\infty}^{z}\left[F_A(\xi)-F_B(\xi)\right]d\xi \,\Big|_{-\infty}^{x}\right\} - \lim_{x\to\infty}\int_{-\infty}^{\infty} u''(z)dz\int_{-\infty}^{z}\left[F_A(\xi)-F_B(\xi)\right]d\xi$$

$$= \lim_{x\to\infty} u'(x)\int_{-\infty}^{x}\left[F_A(\xi)-F_B(\xi)\right]d\xi - \lim_{x\to\infty}\int_{-\infty}^{\infty} u''(z)dz\int_{-\infty}^{z}\left[F_A(\xi)-F_B(\xi)\right]d\xi$$

$$= -\lim_{x\to\infty}\int_{-\infty}^{x} u''(z)dz\int_{-\infty}^{z}\left[F_A(\xi)-F_B(\xi)\right]d\xi$$

$$= \int_{-\infty}^{\infty} u''(z)dz\int_{-\infty}^{z}\left[-F_A(\xi)+F_B(\xi)\right]d\xi = \int_{-\infty}^{\infty} -u''(z)dz\int_{-\infty}^{z}\left\{\left[1-F_B(\xi)\right]-\left[1-F_A(\xi)\right]\right\}d\xi$$

- If u''(z)<0 everywhere (risk averse) then the final term is positive if the integral is positive

Decision Analysis

Dale M. Nesbitt

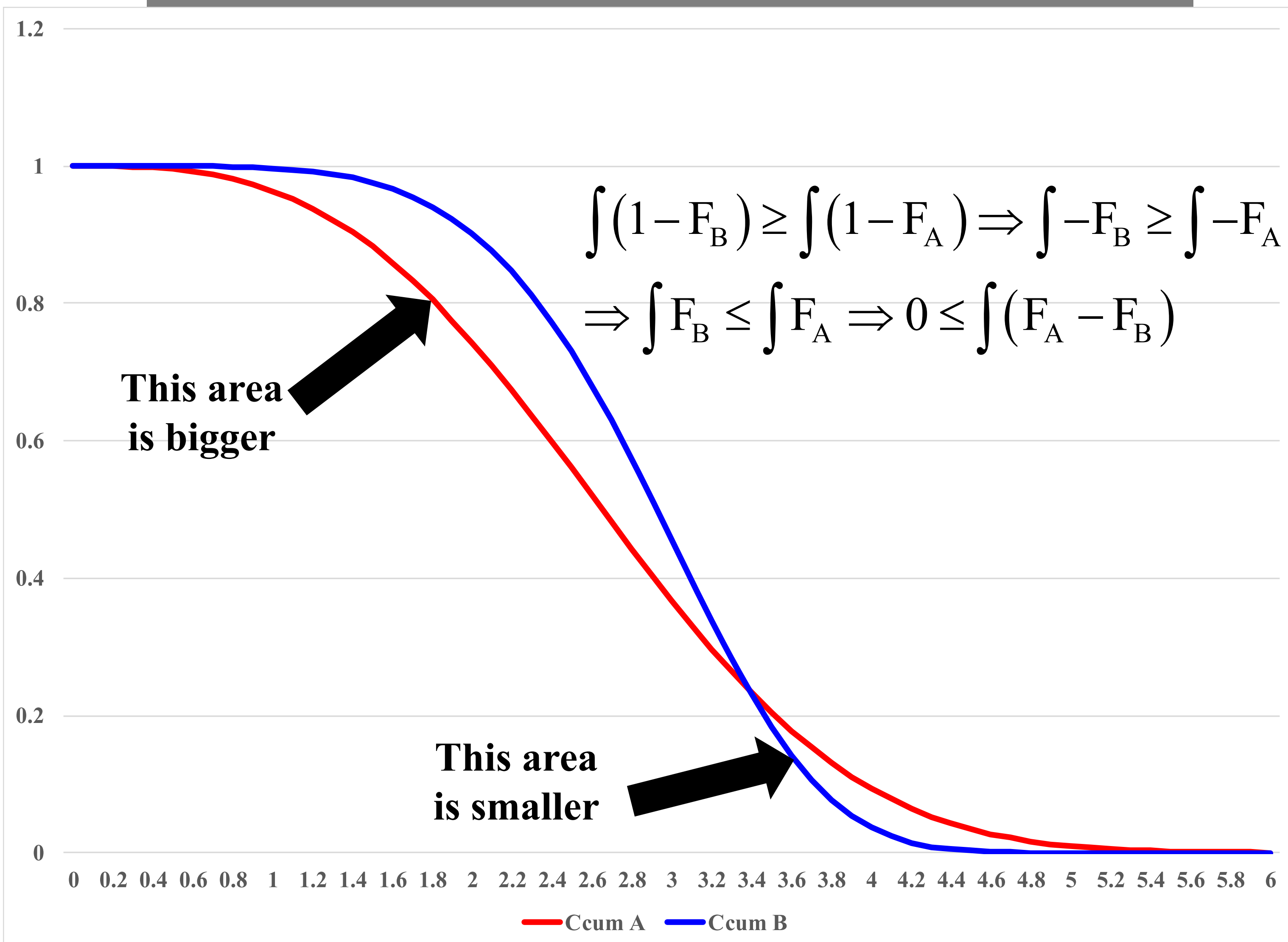
Second Order Probabilistic Dominance Is

$$\int_{-\infty}^{z} \left\{ \left[1 - F_B(\xi) \right] - \left[1 - F_A(\xi) \right] \right\} d\xi$$

B Has Second Order Probabilistic Dominance Over A

Decision Analysis

Dale M. Nesbitt



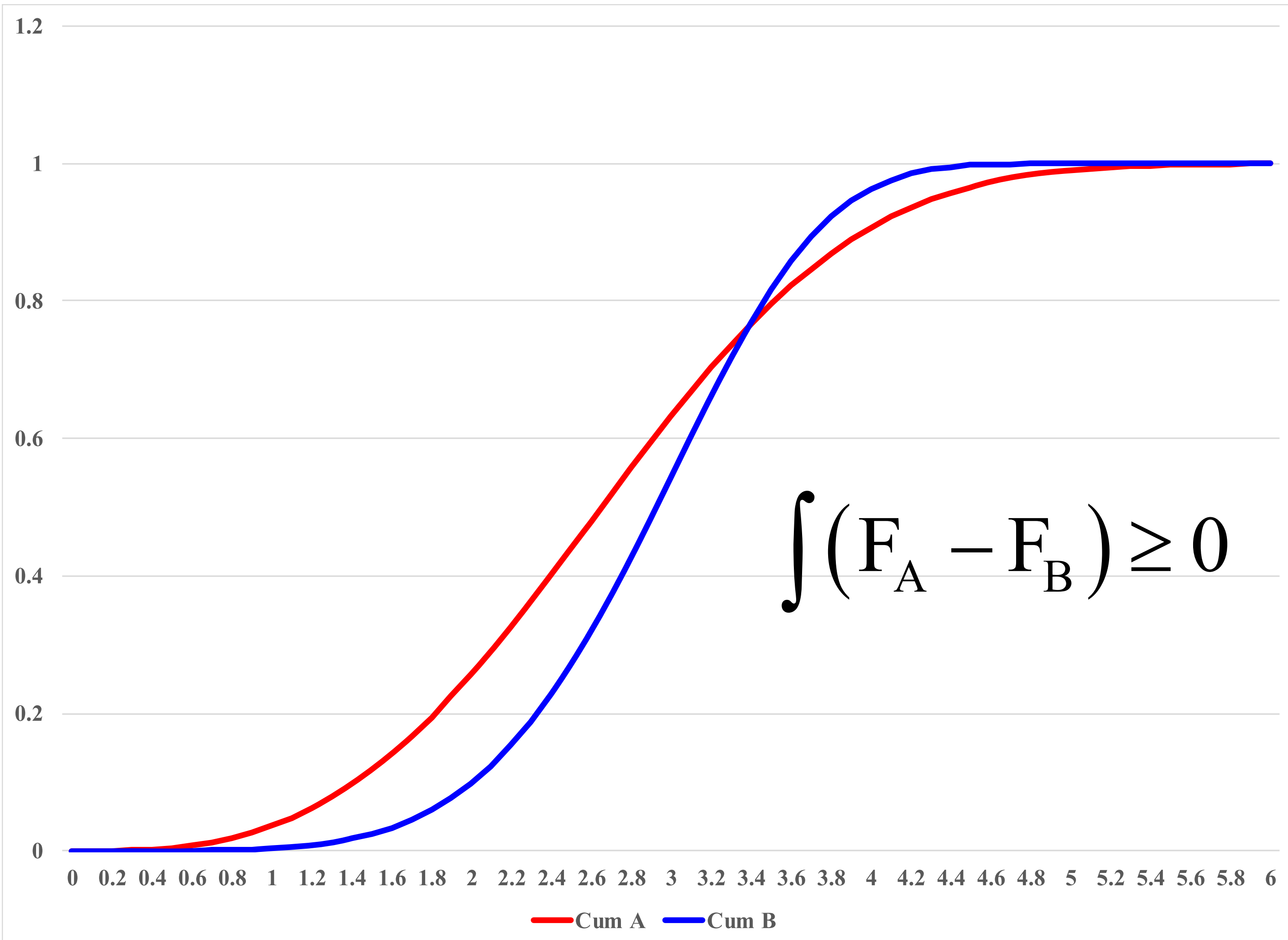
$$\int(1-F_B)\geq\int(1-F_A)\Rightarrow\int -F_B\geq\int -F_A$$

$$\Rightarrow\int F_B\leq\int F_A\Rightarrow 0\leq\int(F_A-F_B)$$

People Have Thought of This As…

- "Same mean/different variance"
- This is more general than that, but that is a good mnemonic for what second order probabilistic dominance means.
- Same mean/higher variance is not synonymous
- Second order pProbabilistic dominance isn't trivial (nor is first order)

B Dominates A in the Second Order



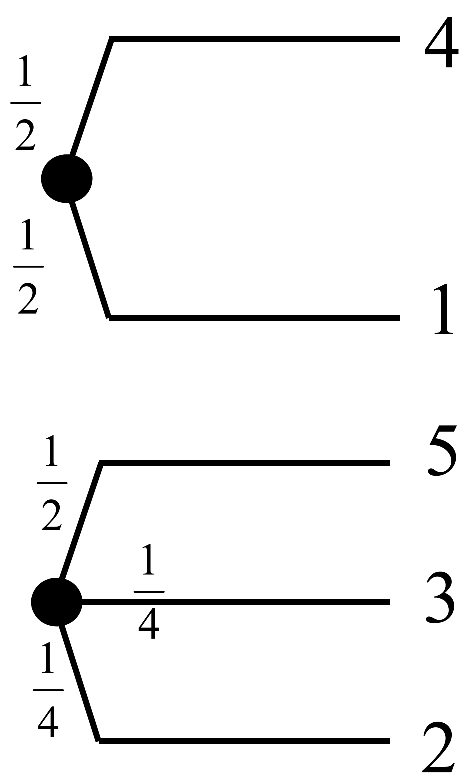
Decision Analysis

Dale M. Nesbitt

Decision
Analysis

Dale M. Nesbitt

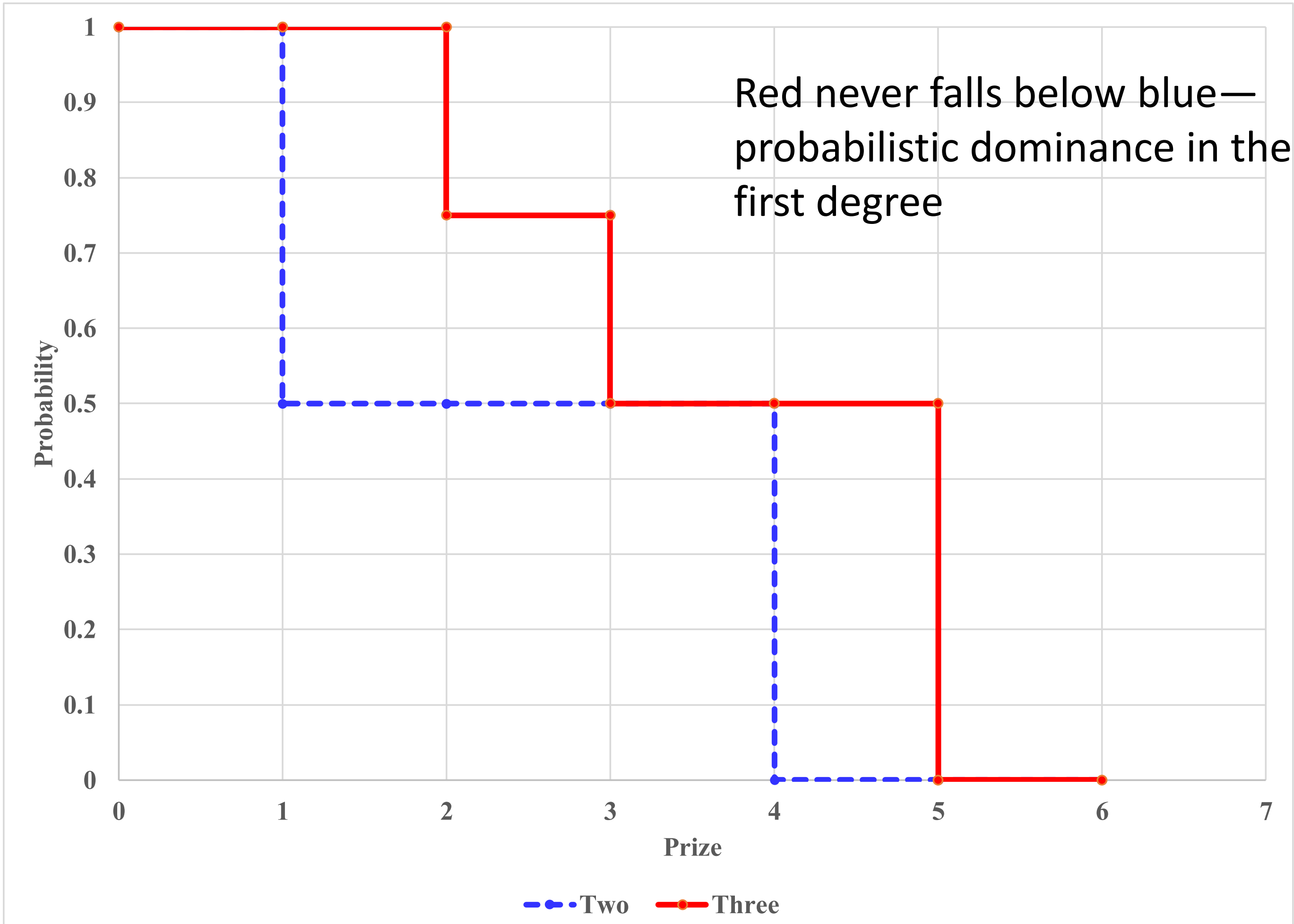
Two Simple Deals



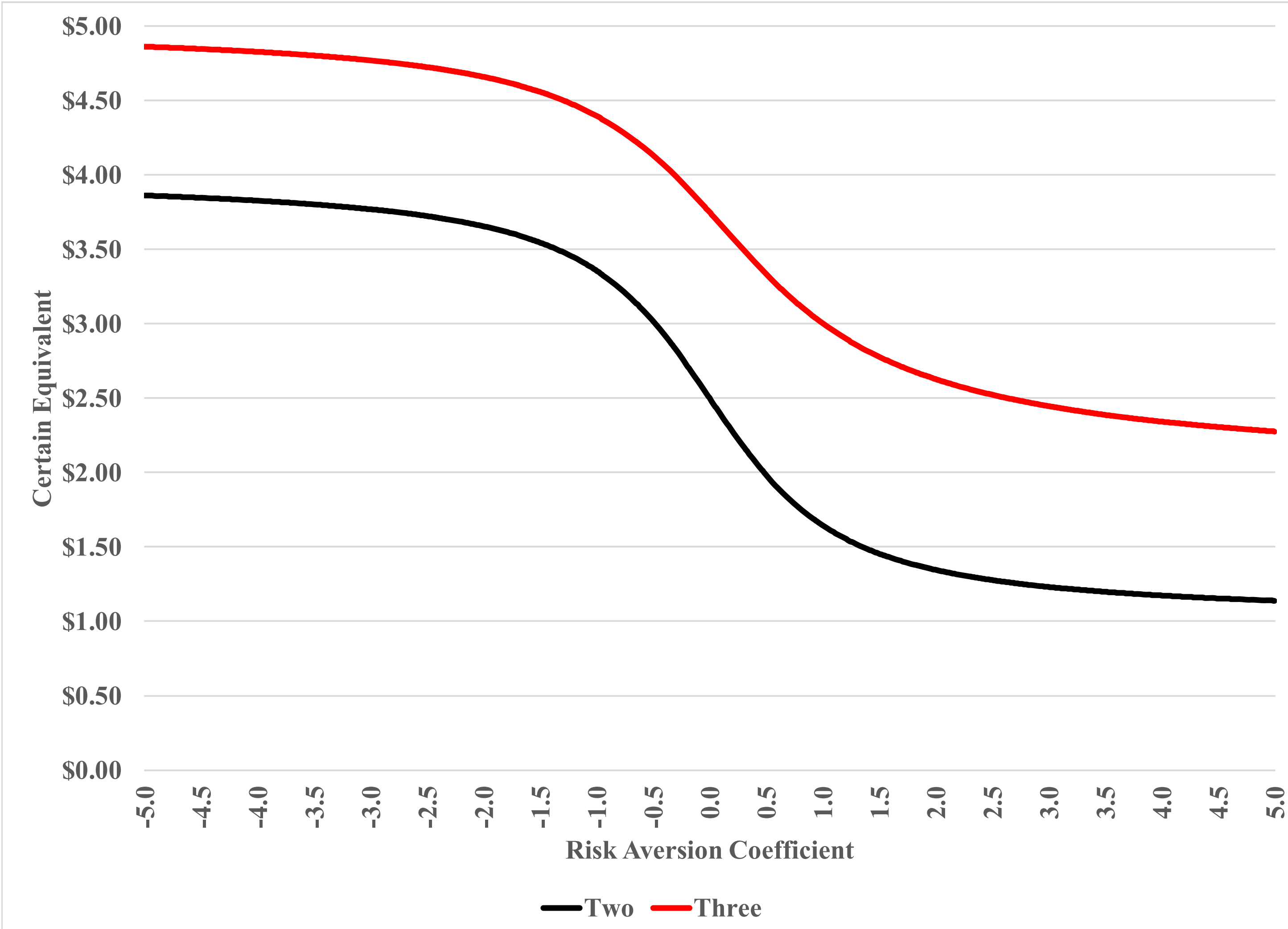
Mean=2.5
Variance=2.25

Mean=3.75
Variance=1.687
5

Complementary Cumulatives of the Two Simple Deals



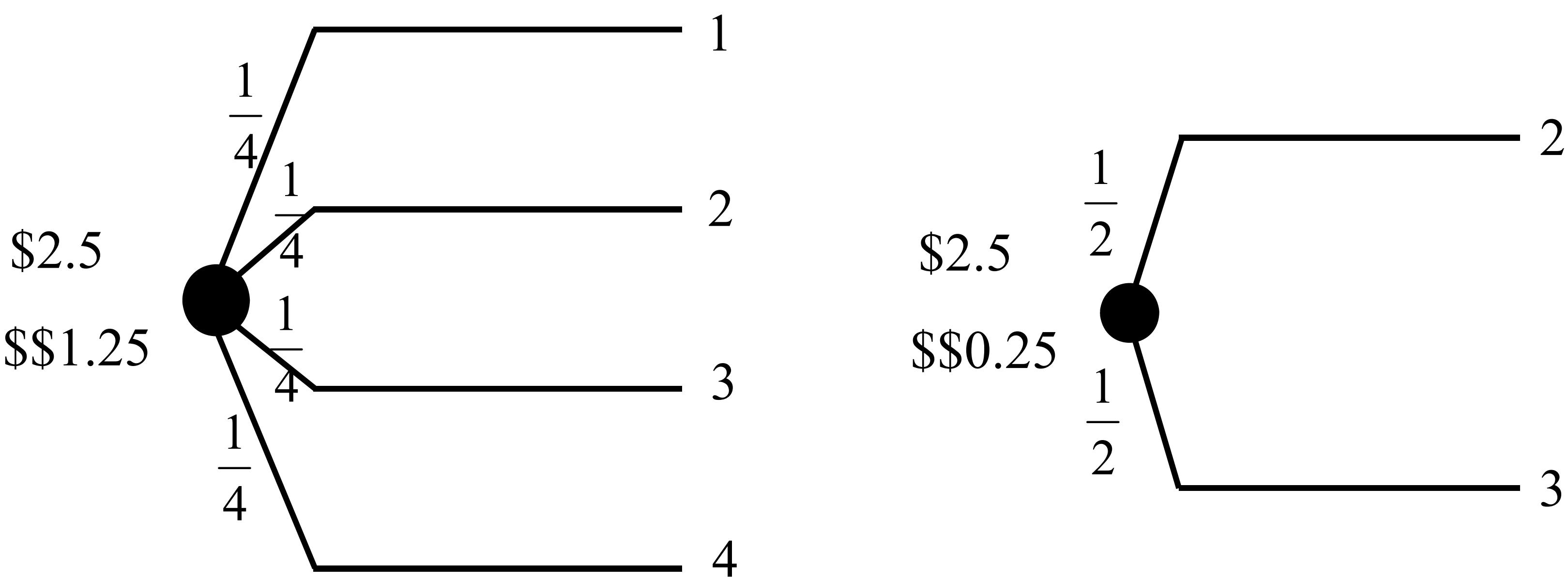
Delta Risk Aversion Sensitivity for the 2 and 3 Branch Deals



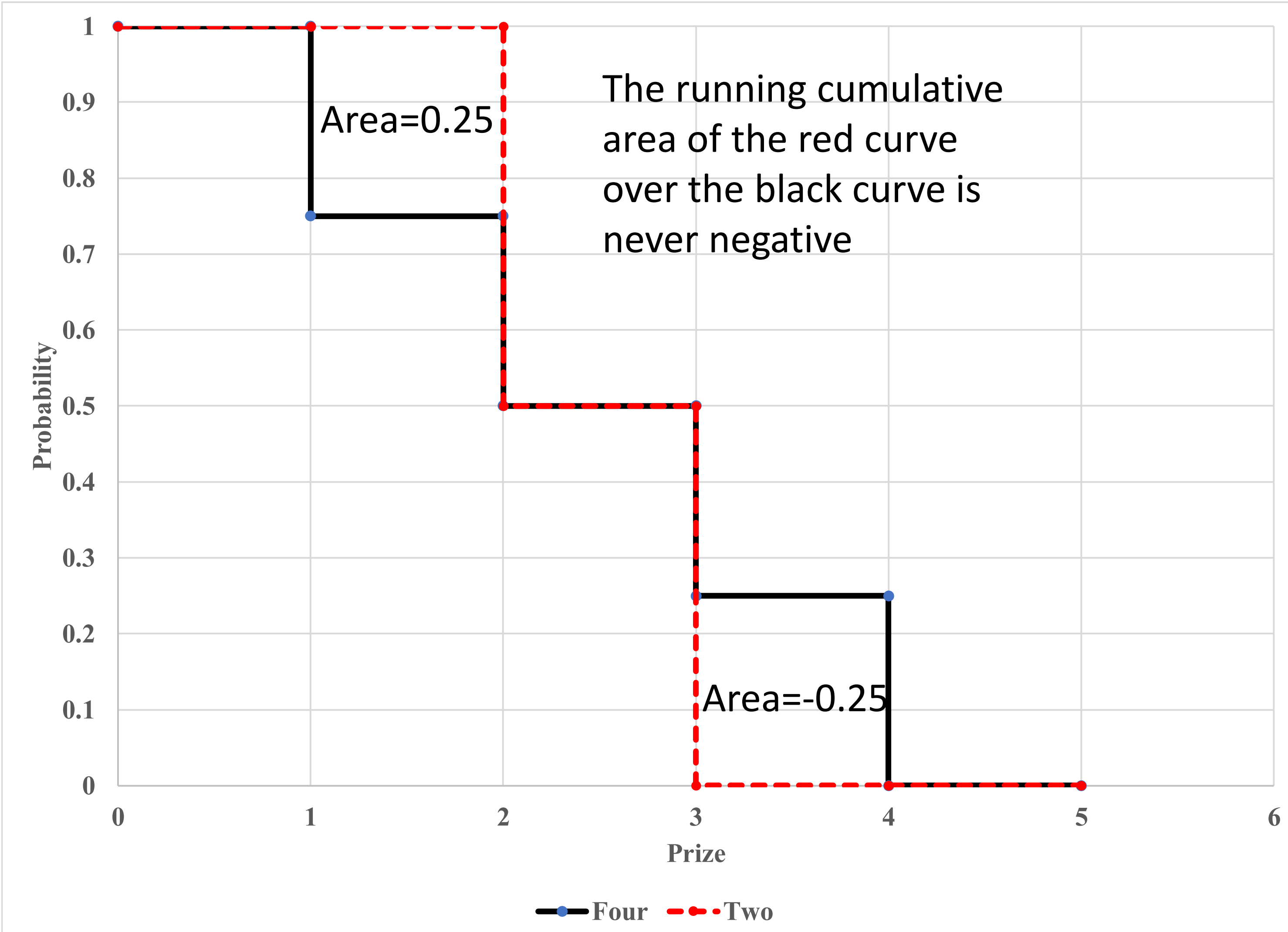
Decision Analysis

Dale M. Nesbitt

Another Simple Deal



Complementary Cumulatives of the Two Deals (2 and 4 Prong)



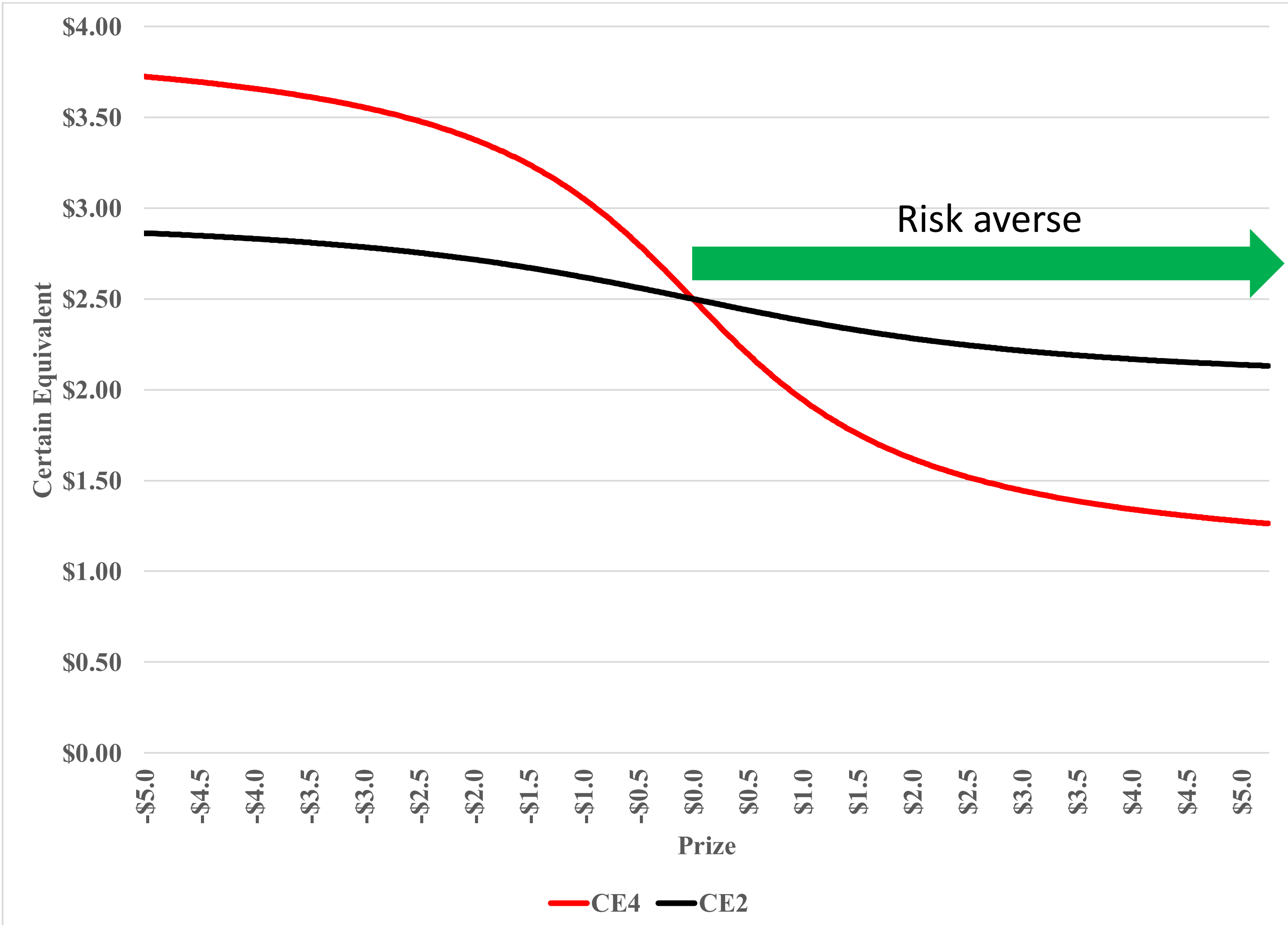
Decision Analysis

Dale M. Nesbitt

Decision Analysis

Dale M. Nesbitt

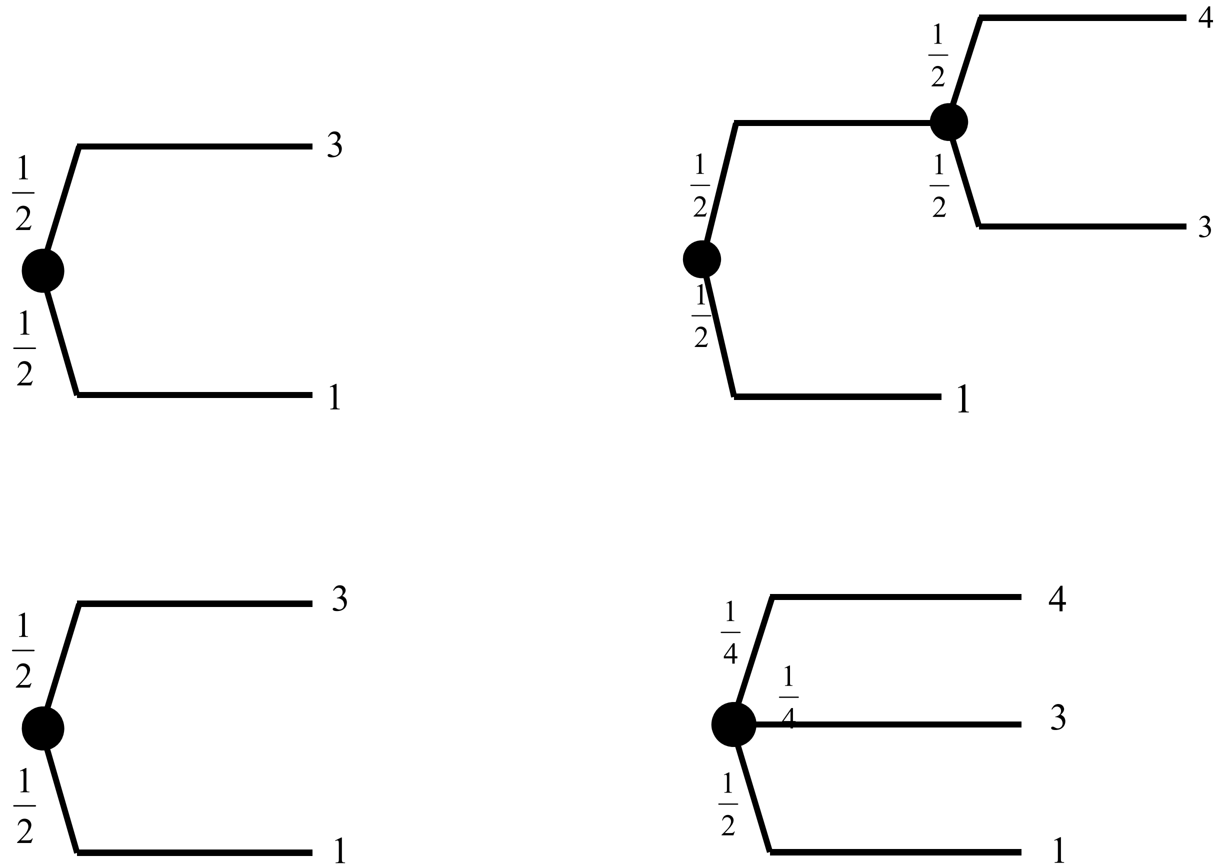
2 Prong Probabilistically Dominates 4 Prong in the Second Degreee



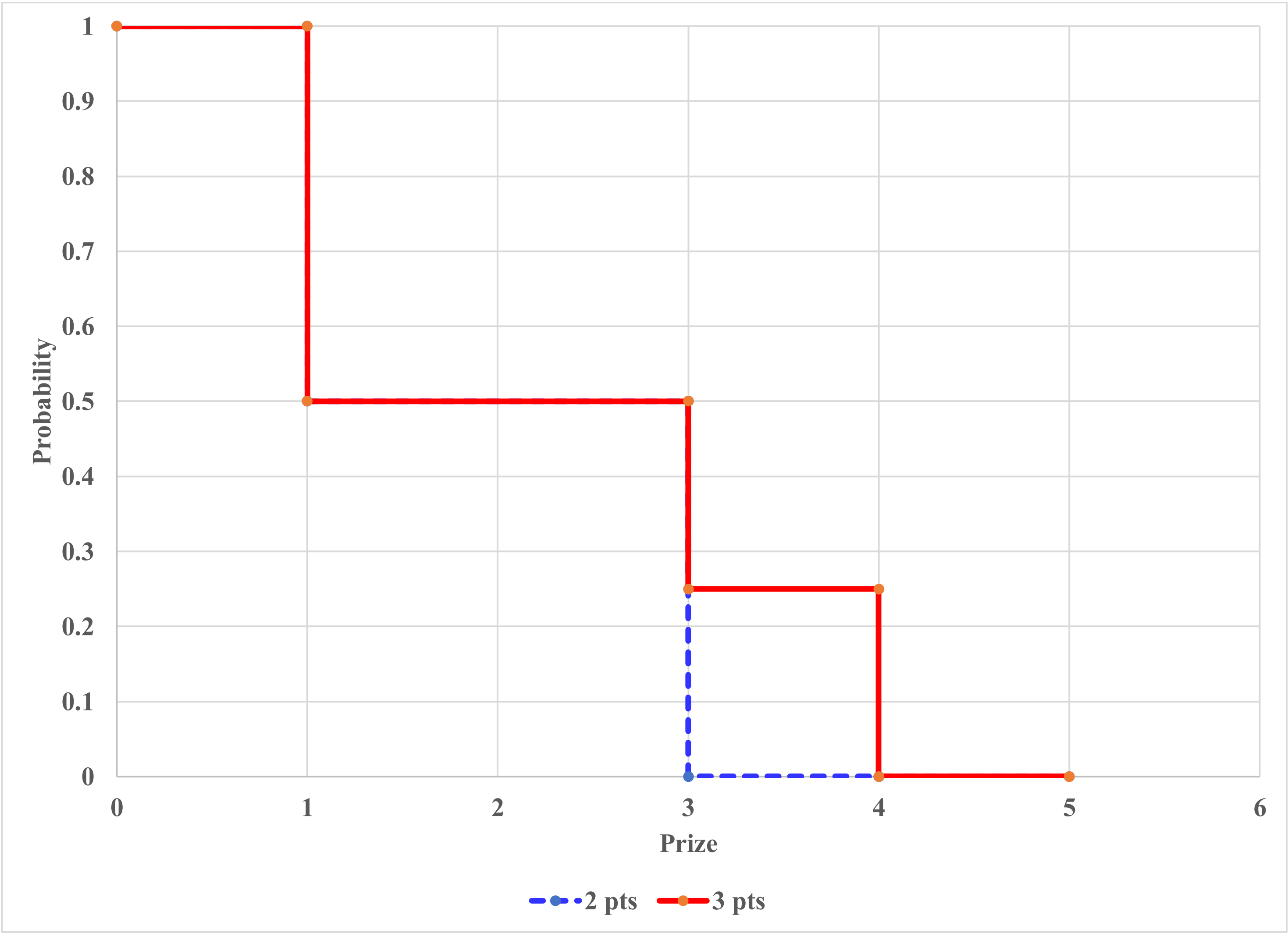
Decision Analysis

Dale M. Nesbitt

Mean Augmentation



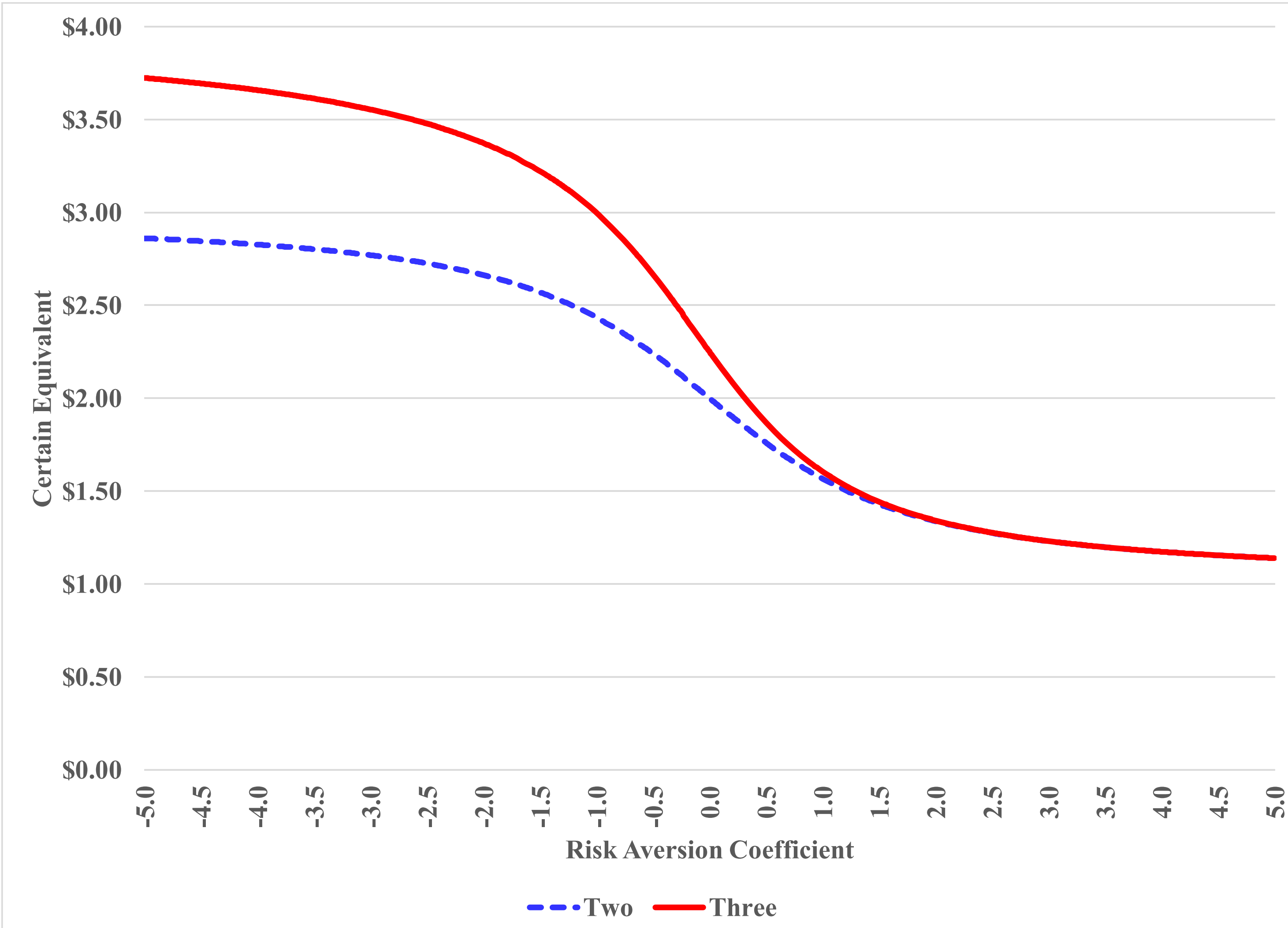
Mean Augmentation Can Lead to Probabilistic Dominance



Decision Analysis

Dale M. Nesbitt

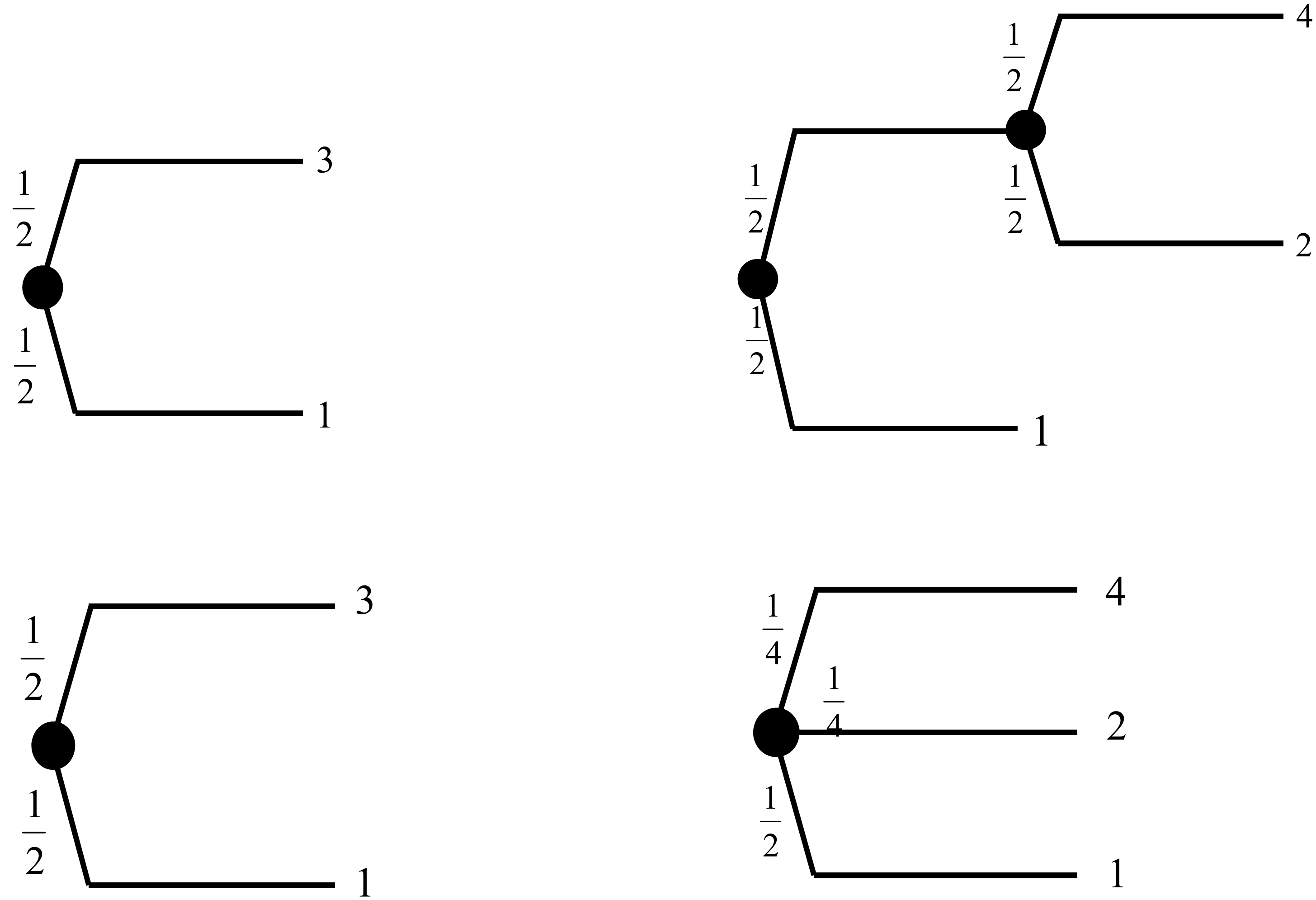
Mean Augmentation



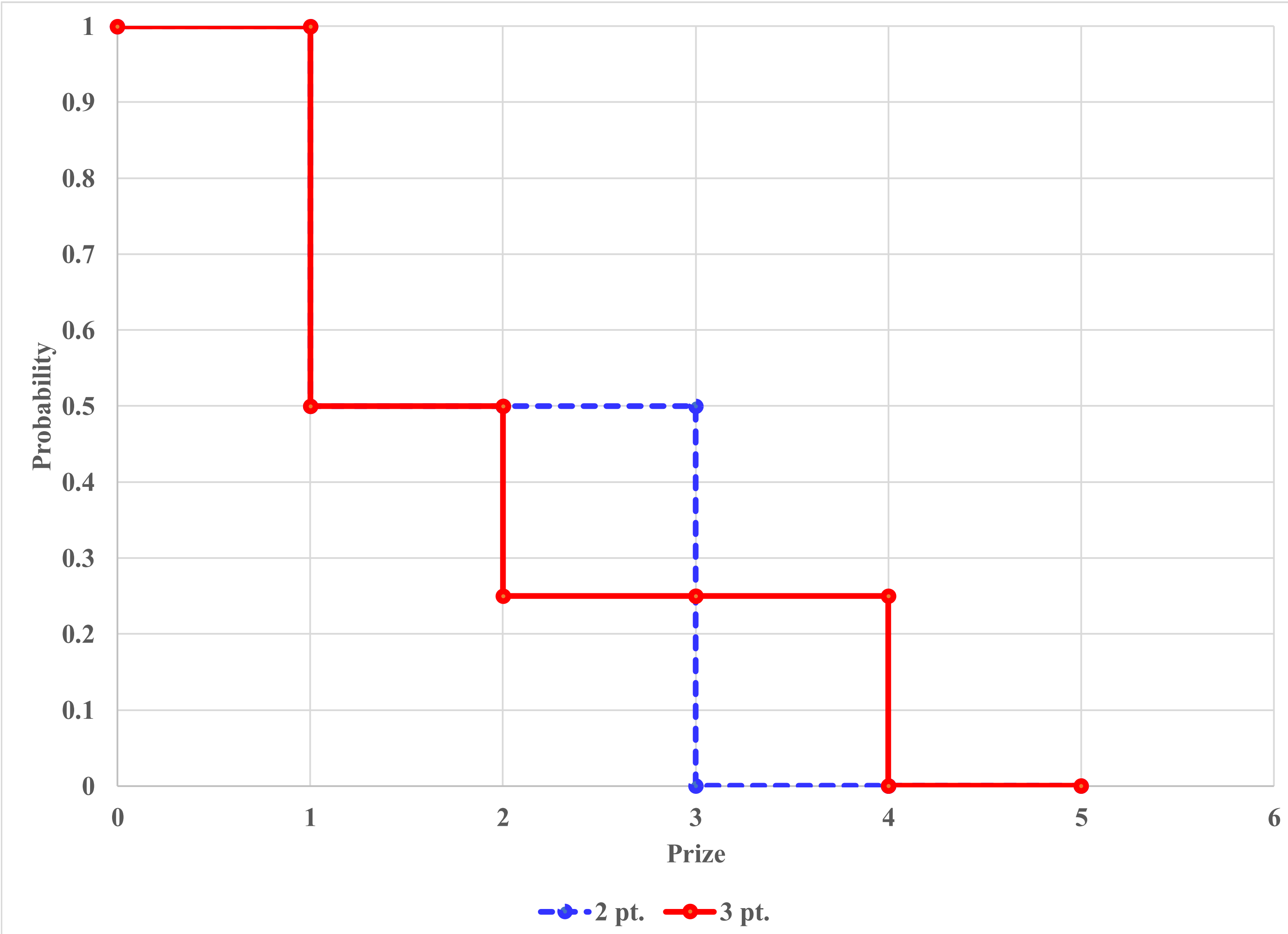
Decision Analysis

Dale M. Nesbitt

No Mean Augmentation



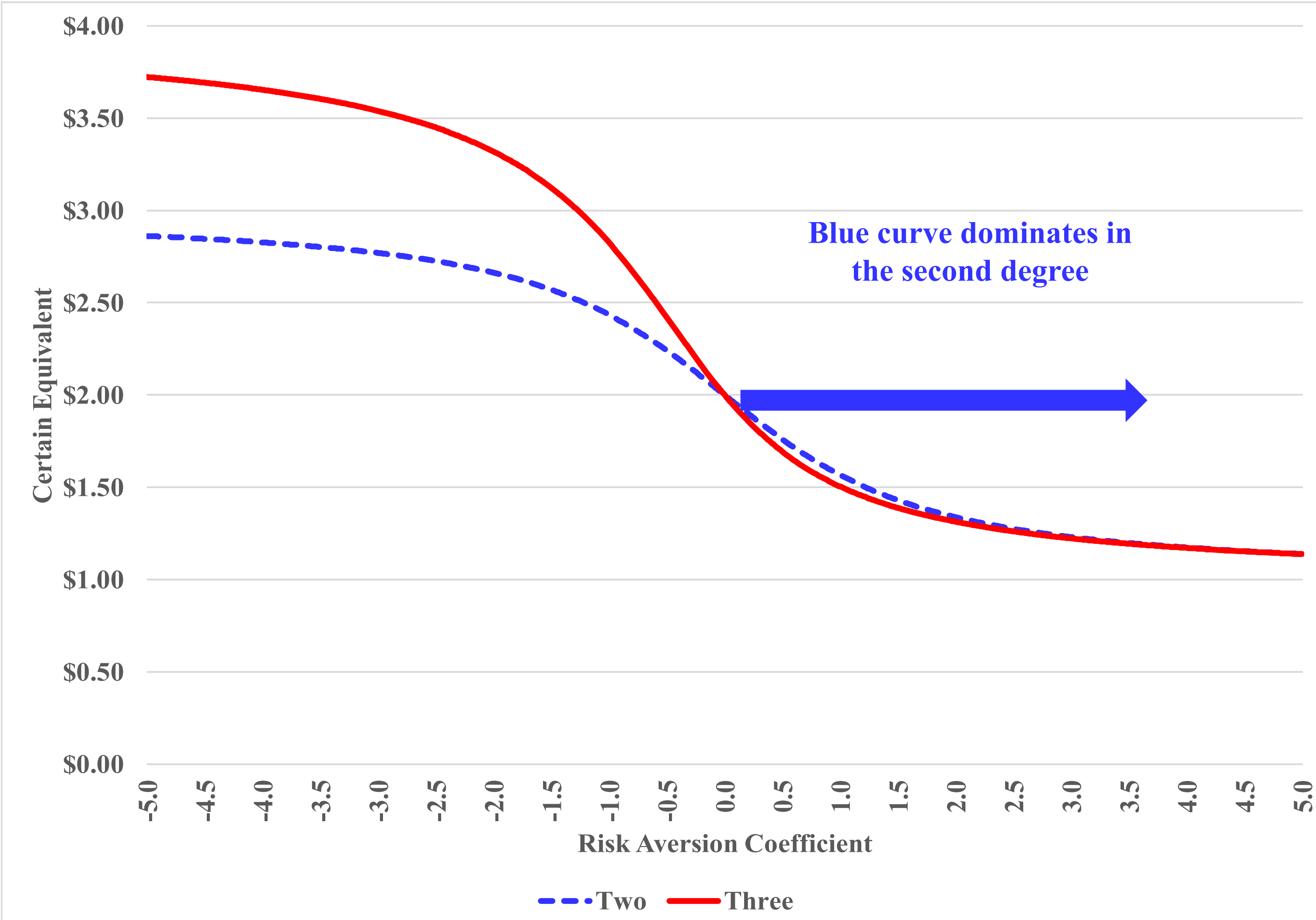
Same Mean/Different Variances



Decision Analysis

Dale M. Nesbitt

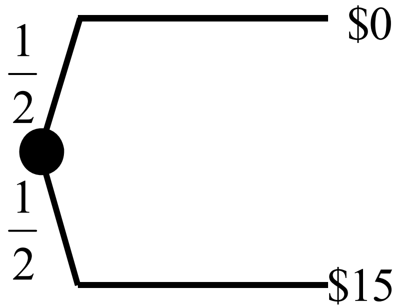
Variance Reduction Can Lead to Second Order Dominance



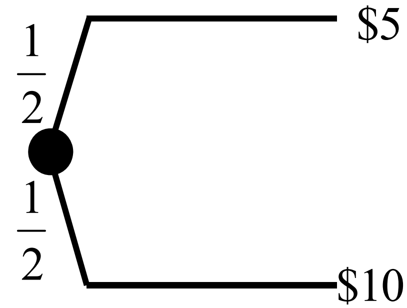
Decision
Analysis

Dale M. Nesbitt

Consider Two Deals

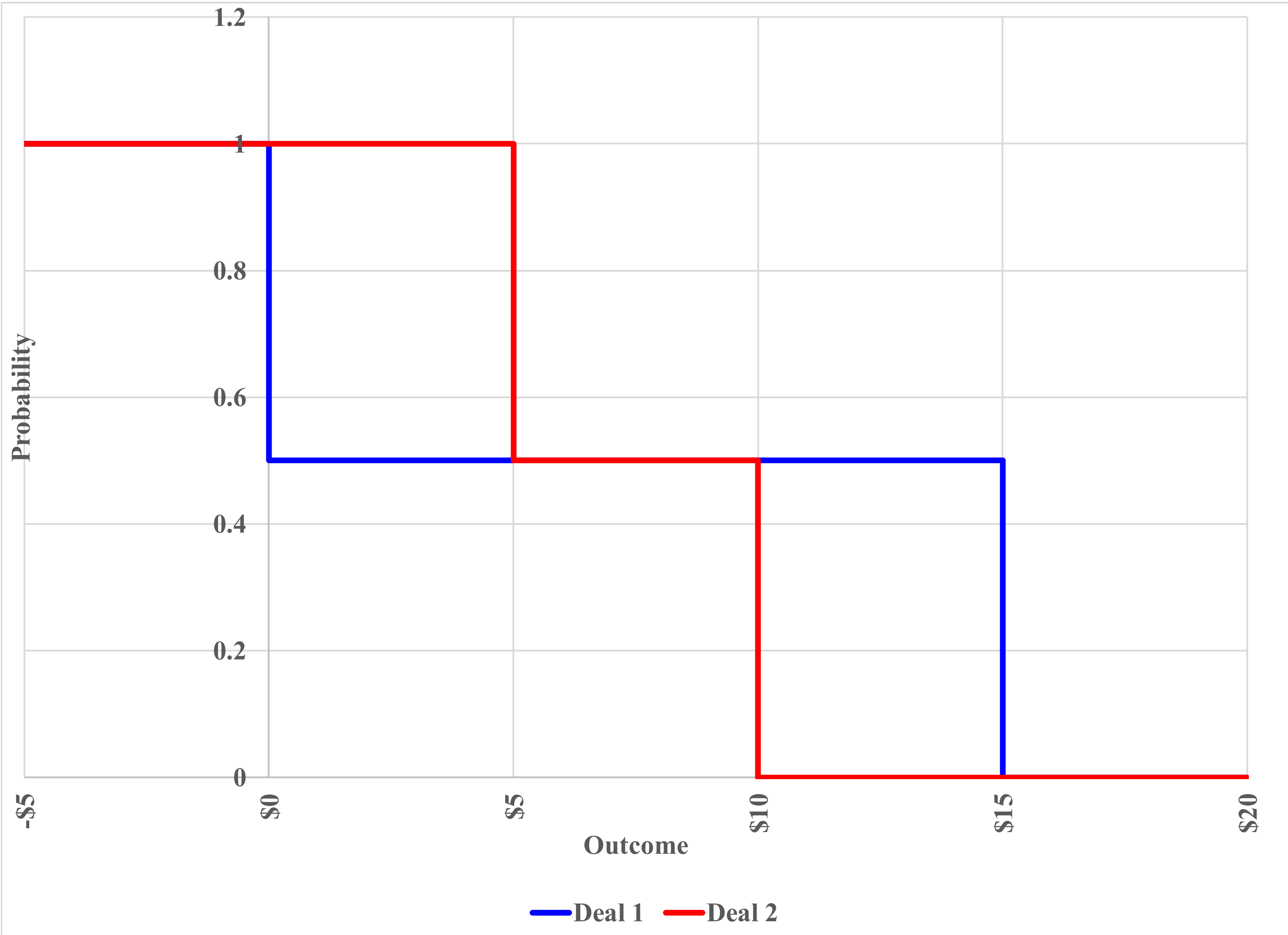


7.5
56.25



7.5
6.25

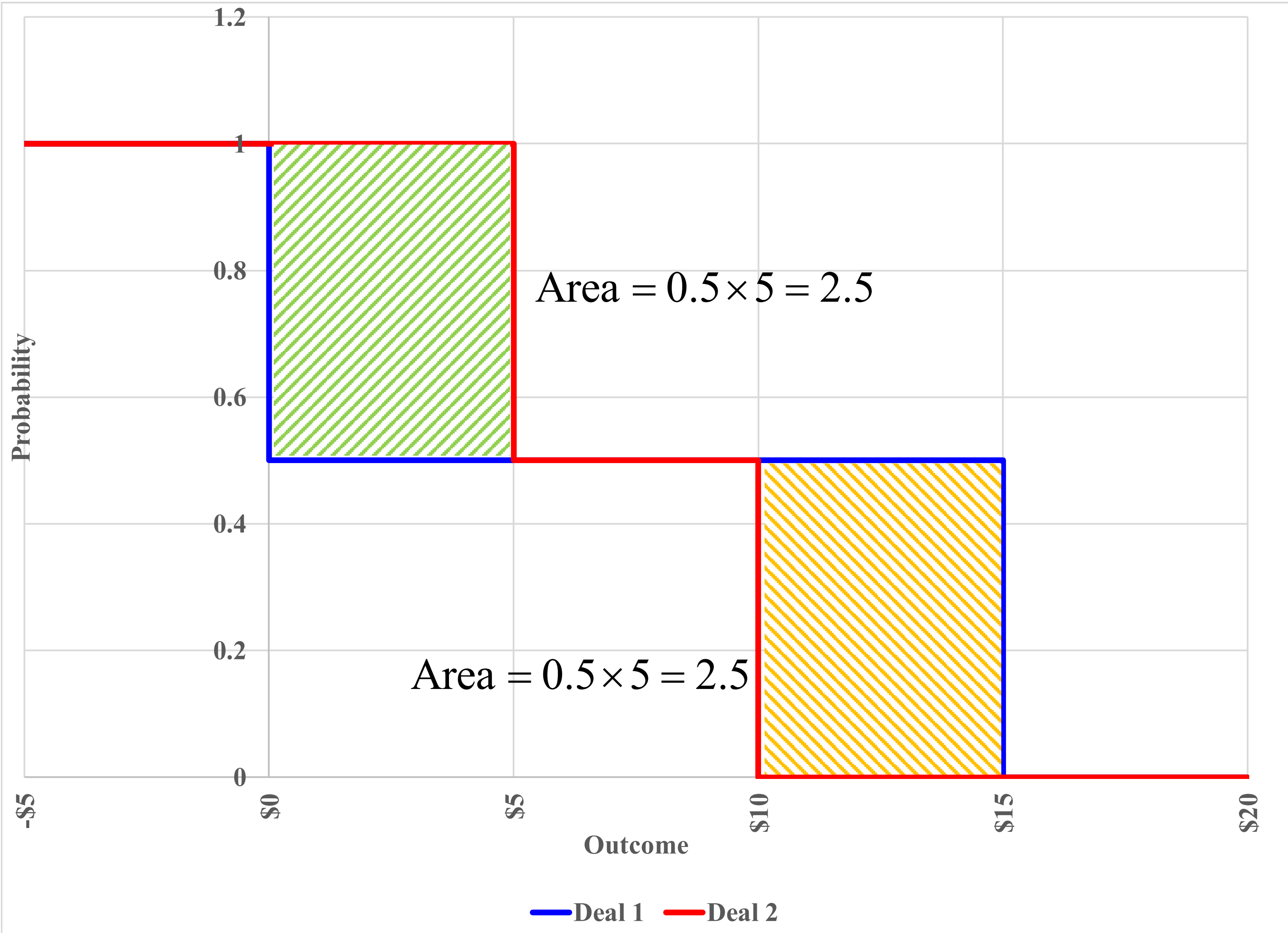
The Two Complementary Cumulatives



Decision Analysis

Dale M. Nesbitt

Second Order Probabilistic Dominance Is Established



Decision Analysis

Dale M. Nesbitt

If You Plot the Two CEs for a Risk Averse Delta Person, What Would You See?

Decision Analysis

Dale M. Nesbitt

- 0,15 and 5,10
- Good audiopedia summary
- https://www.youtube.com/watch?v=2zqmTO5Ekvs
- Dollar is added to one or more outcomes, stochastic dom.
- Lower premium and better coverage

Decision
Analysis

Dale M. Nesbitt

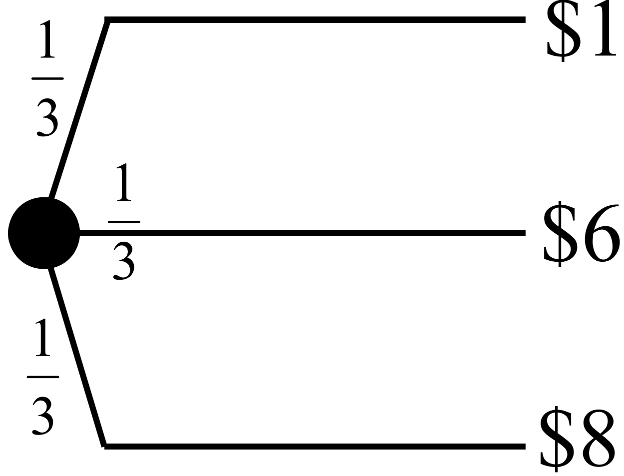
Second Order Probabilistic Dominance

Decision Analysis

Dale M. Nesbitt

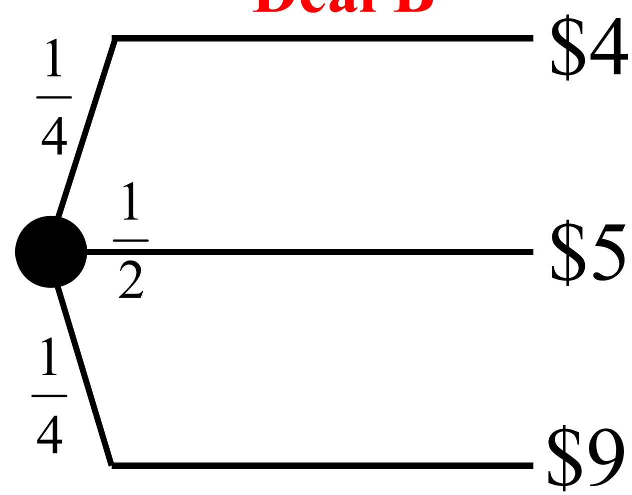
Two Deals

Deal A



Mean ($)	$5.0000
Variance ($$)	$8.6667
Std. Dev. ($)	$2.9439

Deal B

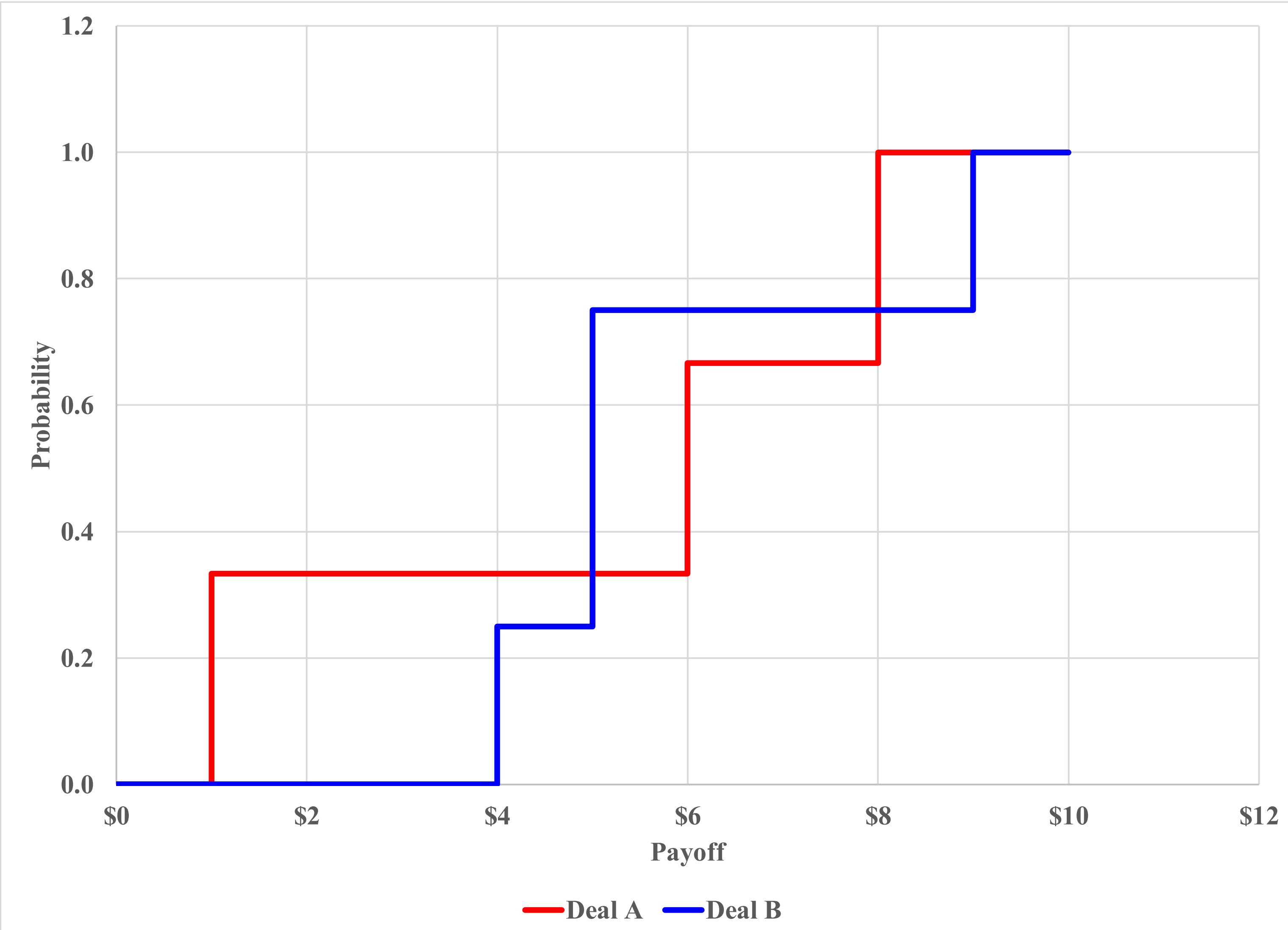


Mean ($)	$5.7500
Variance ($$)	$3.6875
Std. Dev. ($)	$1.9203

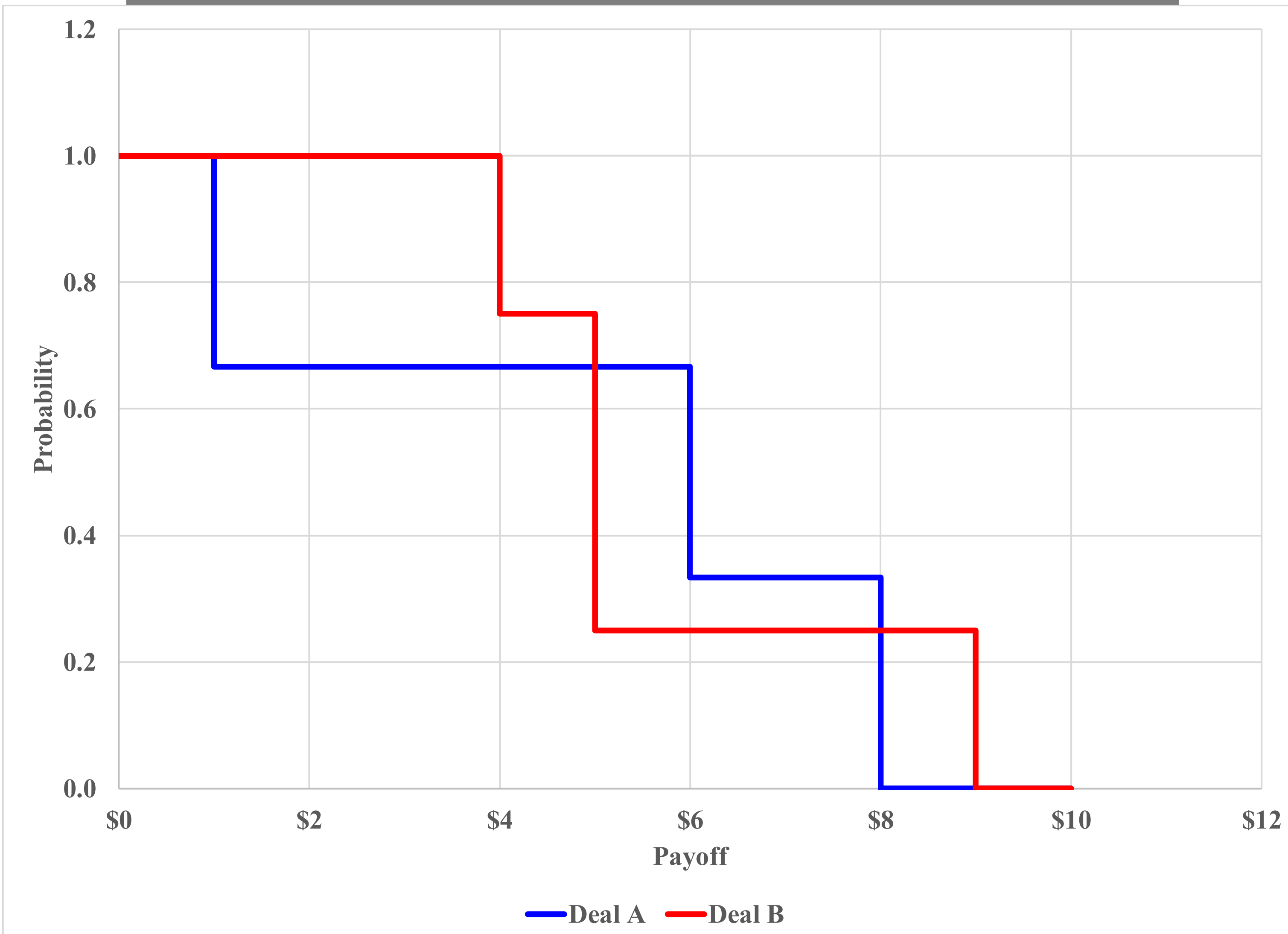
Decision Analysis

Dale M. Nesbitt

The Two Cumulatives



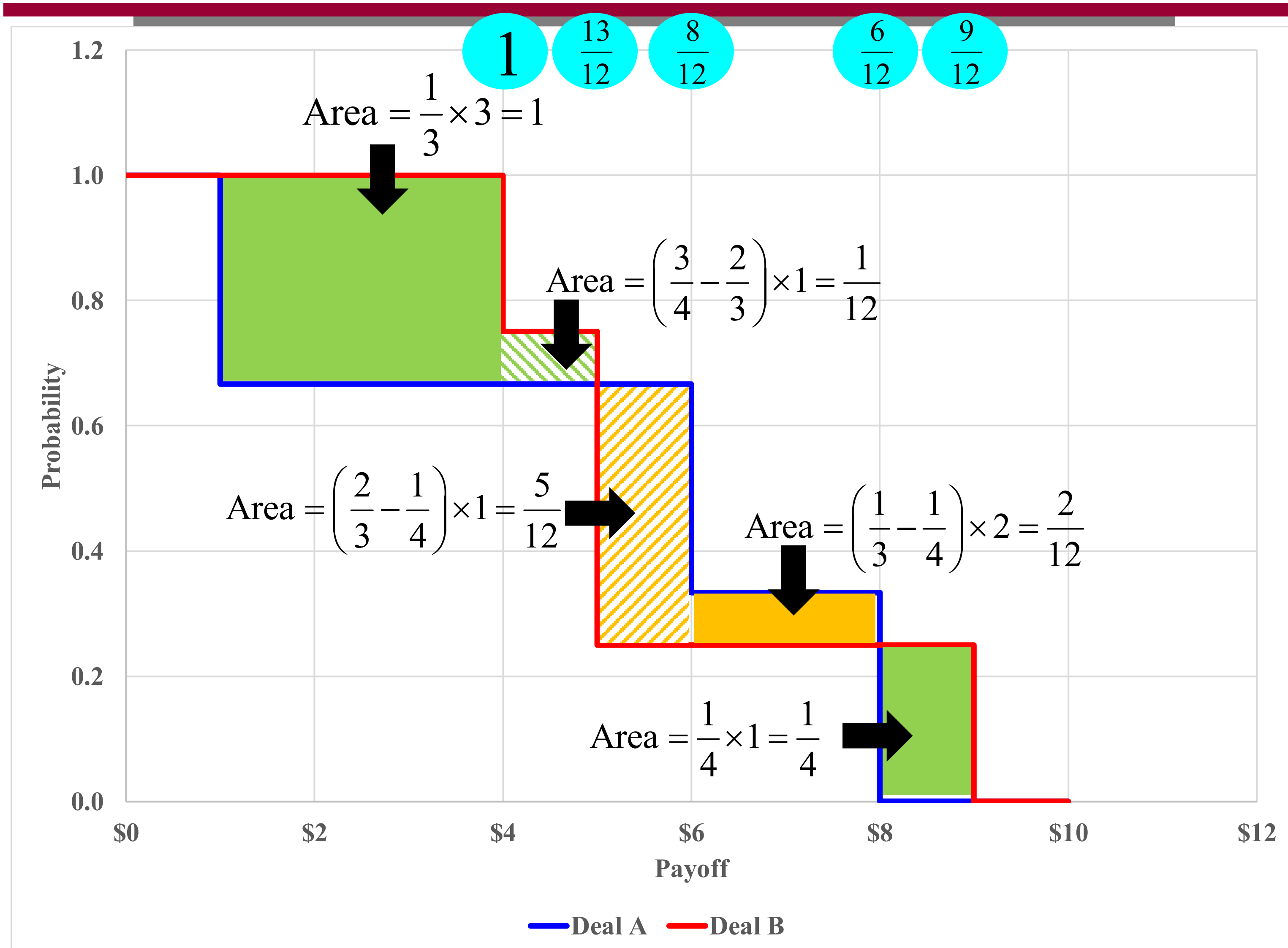
The Two Complementary Cumulatives



Decision Analysis

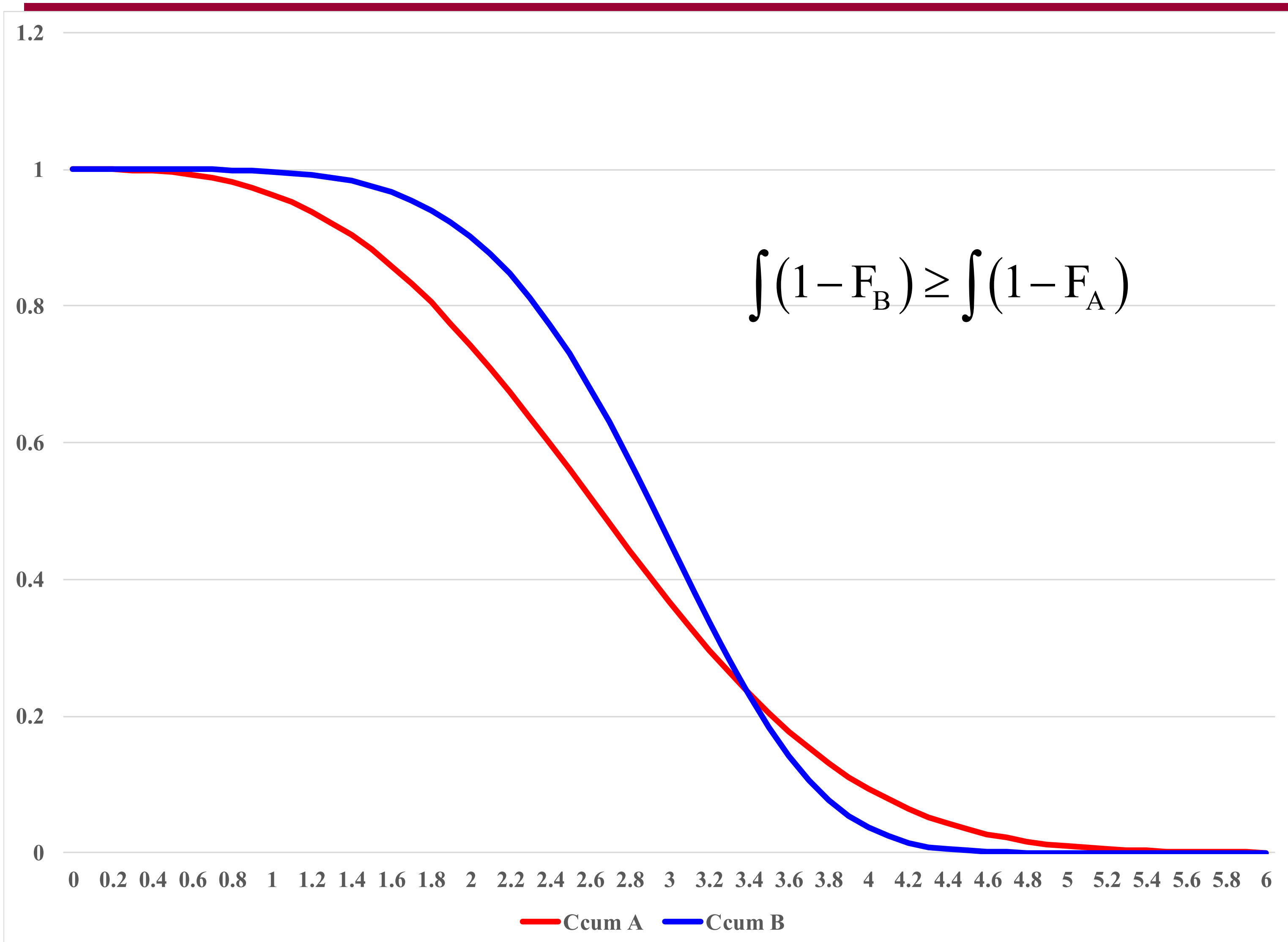
Dale M. Nesbitt

The Two Complementary Cumulatives



Decision Analysis

It Looks Like This—Second Order Probabilistic Dominance



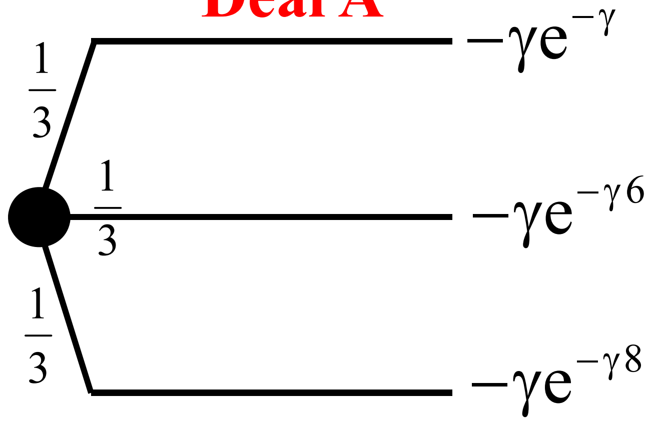
Dale M. Nesbitt

Decision Analysis

Dale M. Nesbitt

Expected Utility/Certain Equivalent for a Delta Person

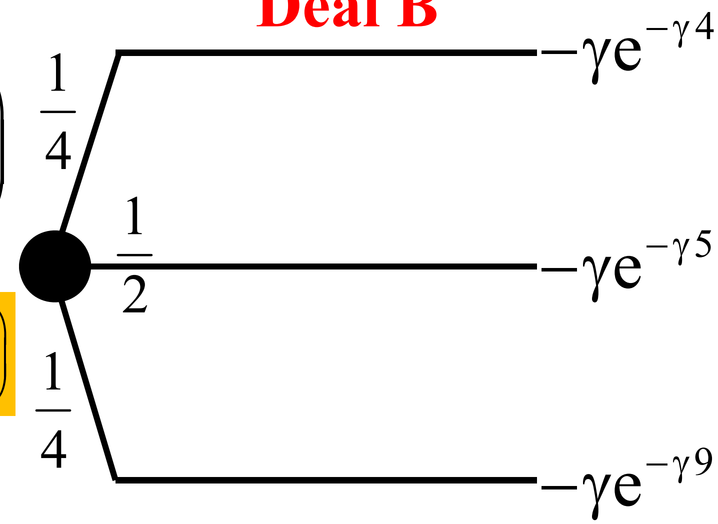
Deal A



$$\langle u \rangle = -\frac{1}{3}\gamma\left(e^{-\gamma} + e^{-6\gamma} + e^{-8\gamma}\right)$$

$$\tilde{x} = -\frac{1}{\gamma}\ln\left(-\frac{1}{\gamma}\langle u \rangle\right) = -\frac{1}{\gamma}\ln\left[\frac{1}{3}\left(e^{-\gamma} + e^{-6\gamma} + e^{-8\gamma}\right)\right]$$

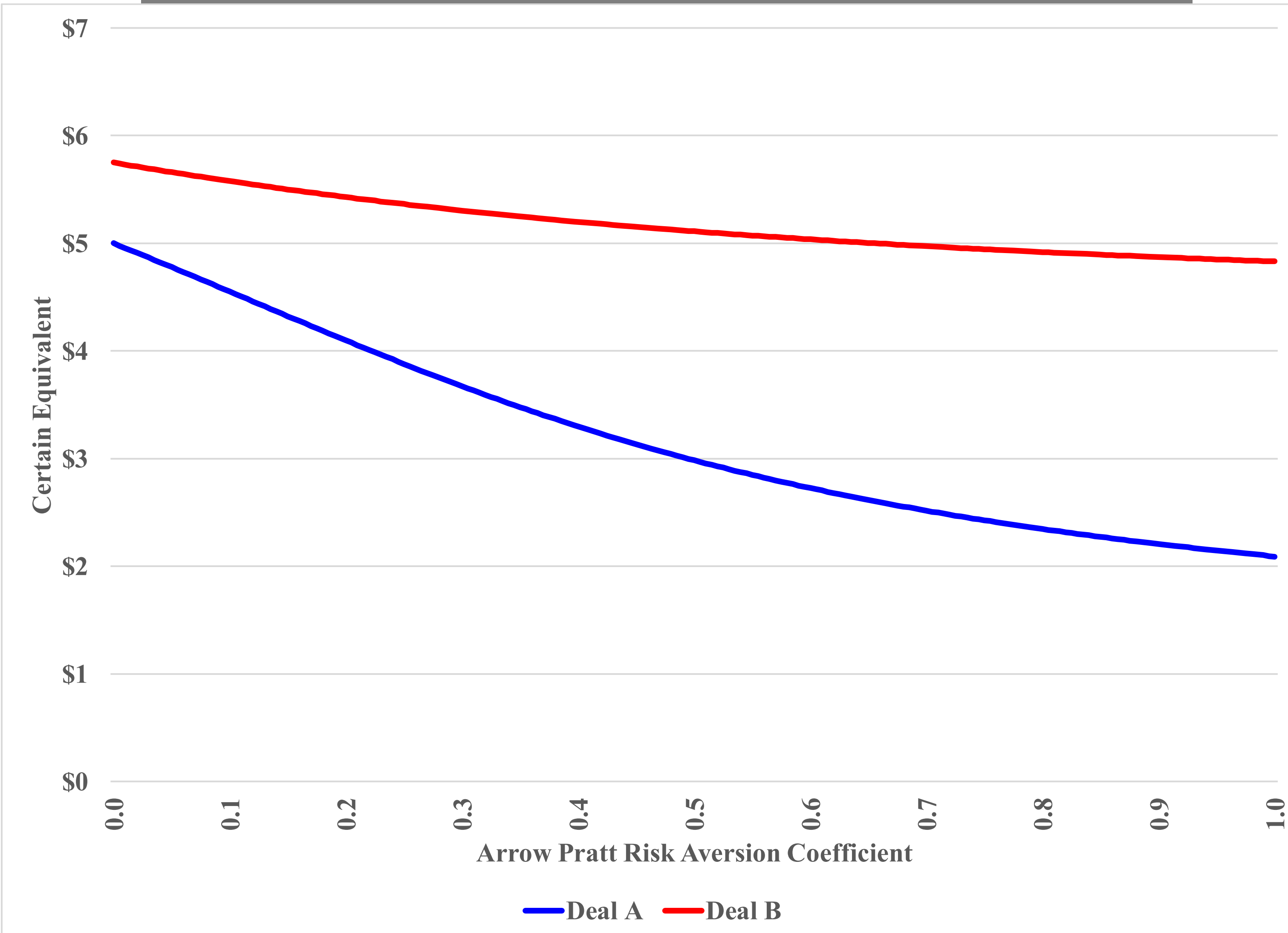
Deal B



$$\langle u \rangle = -\gamma\left(\frac{1}{4}e^{-4\gamma} + \frac{1}{2}e^{-5\gamma} + \frac{1}{4}e^{-9\gamma}\right)$$

$$\tilde{x} = -\frac{1}{\gamma}\ln\left(-\frac{1}{\gamma}\langle u \rangle\right) = -\frac{1}{\gamma}\ln\left(\frac{1}{4}e^{-4\gamma} + \frac{1}{2}e^{-5\gamma} + \frac{1}{4}e^{-9\gamma}\right)$$

Certain Equivalents—Delta Person



Decision Analysis

Dale M. Nesbitt

You Don't CARE About Their u Curve As Long As They Are Risk Averse!

- You get the exact same answer as long as they are risk averse.
- You can omit the u-curve calculation and just stick with probabilities

If Two Lotteries Have the Same Mean

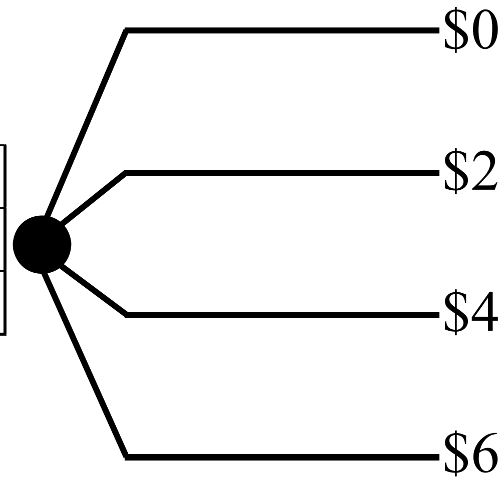
- But one has a higher variance, does the one with the lower variance always probabilistically dominate?
- People sometimes think that same mean but lower variance means probabilistic dominance.
- NO!

Decision
Analysis

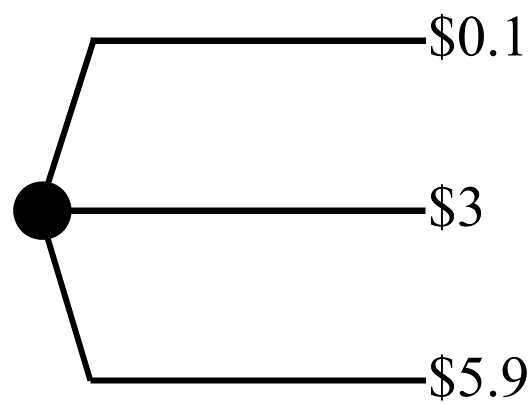
Dale M. Nesbitt

Another Example

Mean	$3.0000
Variance	$5.0000
Std. Dev.	$2.2361



Mean	$3.0000
Variance	$5.6067
Std. Dev.	$2.3678

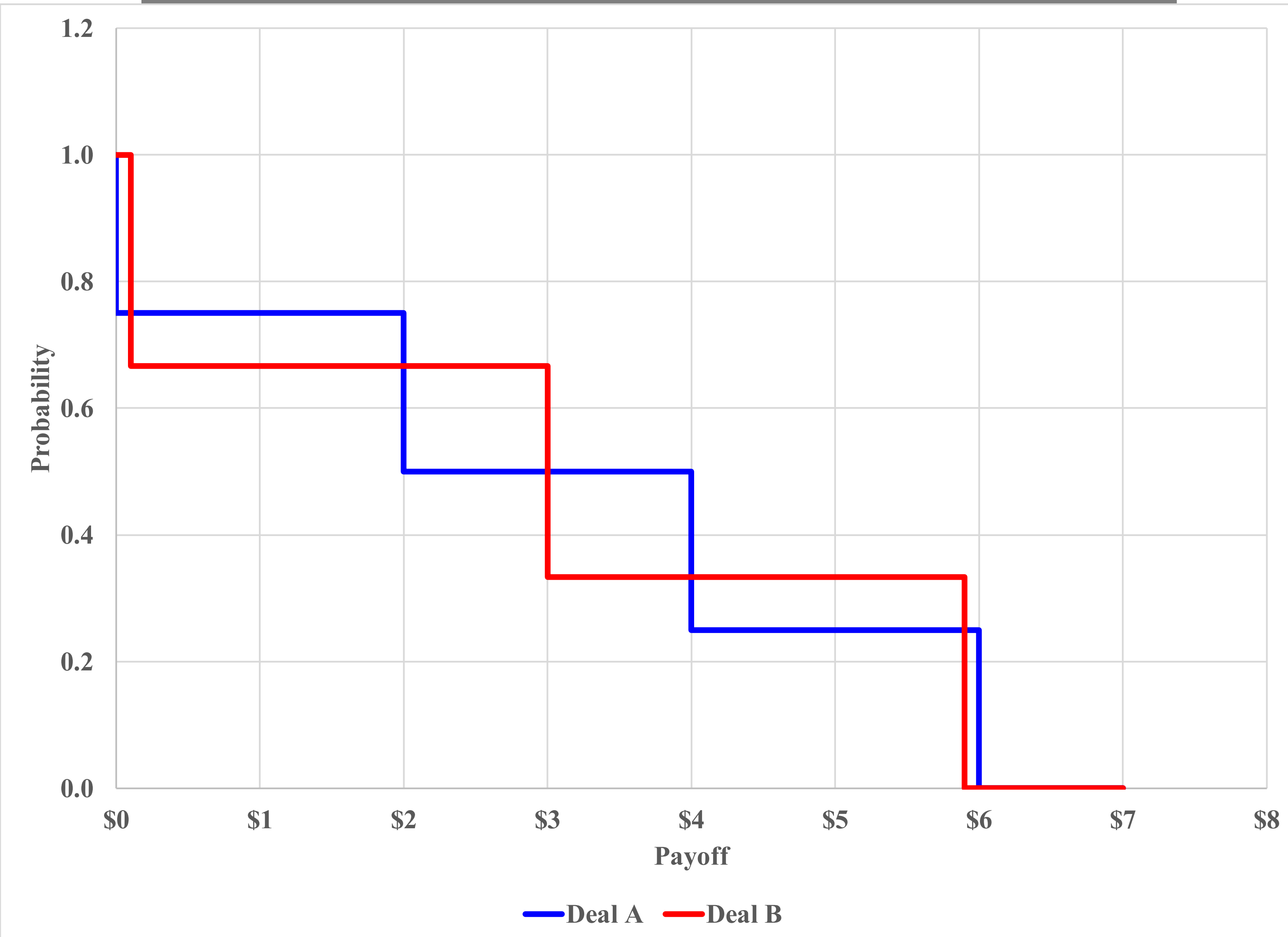


- Does Deal A probabilistically dominate Deal B in the second order?

Decision Analysis

Dale M. Nesbitt

Nope!



Decision Analysis

Dale M. Nesbitt

Same Mean/Lower Variance Is Not a Sufficient Condition for Second Order Probabilistic Dominance

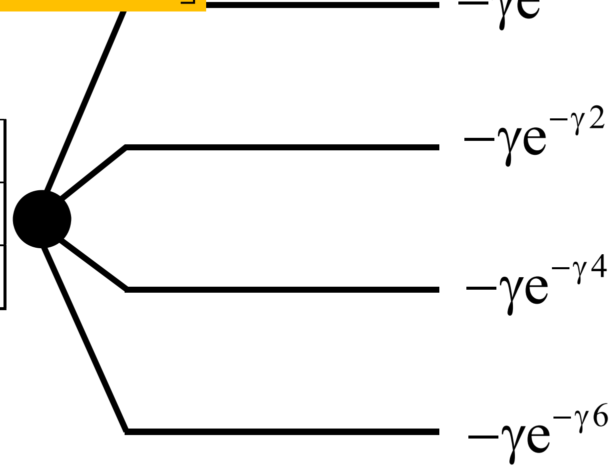
Decision Analysis

Dale M. Nesbitt

The Example

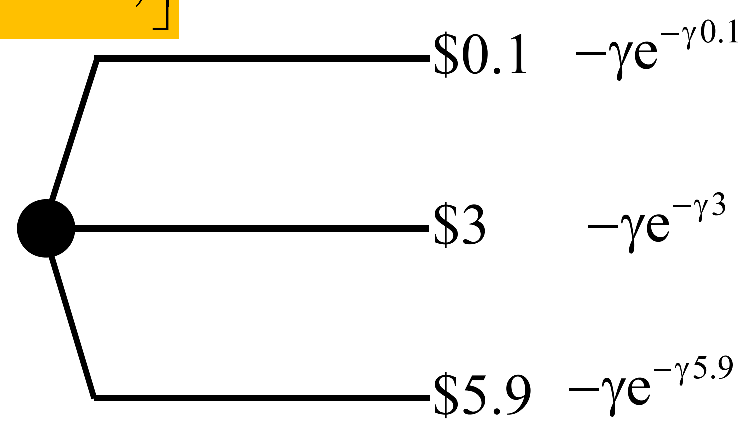
$$x = -\frac{1}{\gamma}\ln\left[\frac{1}{4}\left(e^{-\gamma 0} + e^{-\gamma 2} + e^{-\gamma 4} + e^{-\gamma 6}\right)\right]$$

Mean	\$3.0000
Variance	\$5.0000
Std. Dev.	\$2.2361

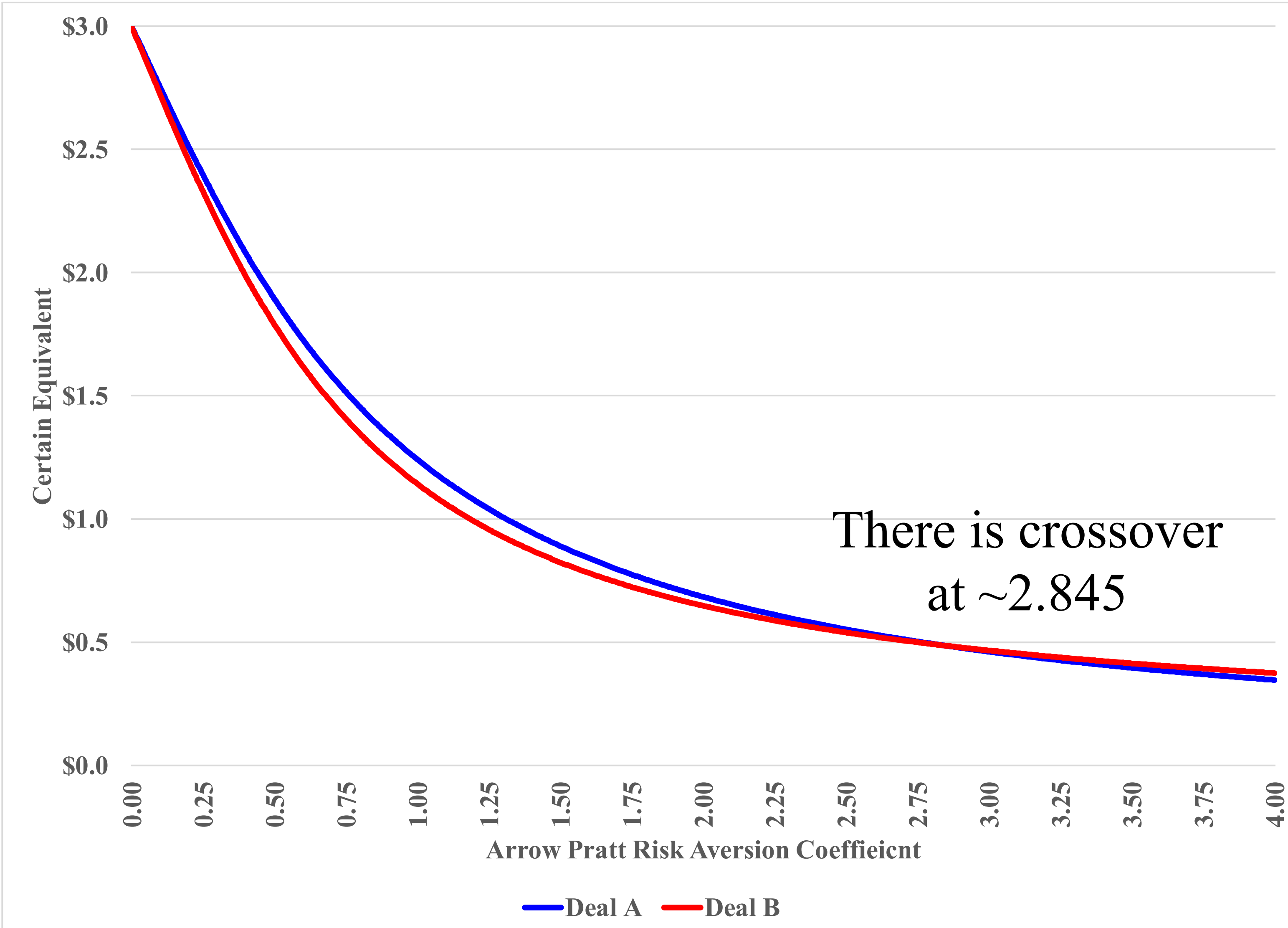


$$x = -\frac{1}{\gamma}\ln\left[\frac{1}{3}\left(e^{-\gamma 0.1} + e^{-\gamma 3} + e^{-\gamma 5.9}\right)\right]$$

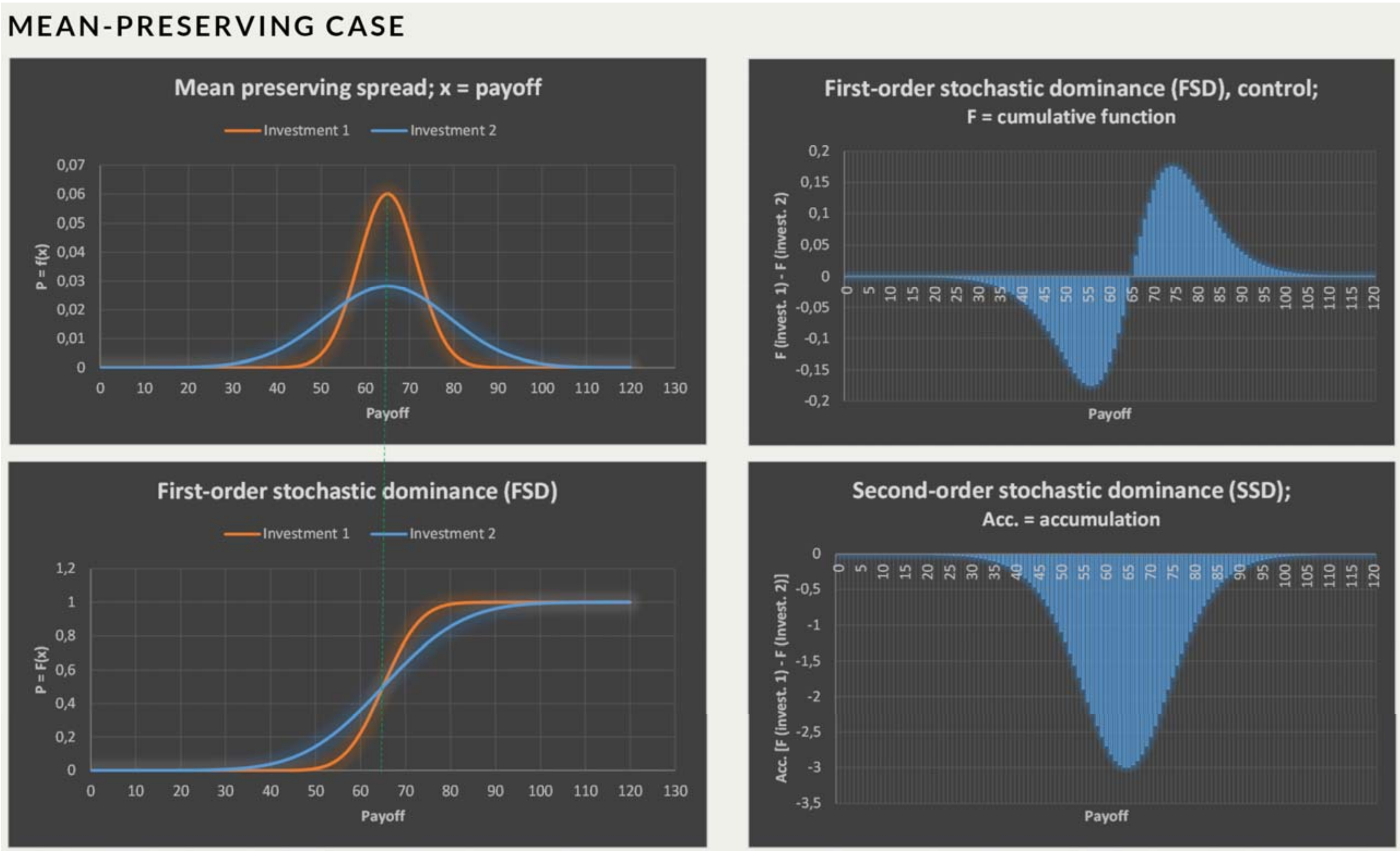
Mean	\$3.0000
Variance	\$5.6067
Std. Dev.	\$2.3678



The Best Deal Switches—No Second Order Probabilistic Dominance



Same Mean/Different Variance



Clearly When u”(x)<0 (Risk Averse)

Decision Analysis

Dale M. Nesbitt

- A risk averse decision maker will always prefer the second order probabilistically dominant lottery. You don’t have to do a bunch of utility calculations.
- These are important theorems in practice
- You should always look at your lotteries and discern if there is first or second order Probabilistic dominance.
- Ron didn’t have to drag us through the “Outdoor” alternative! We didn’t have to consider specific risk aversion questions and tons of complexity!
- There are higher orders, but they are the stuff of academic papers! (You can integrate by parts forever!)
- Clairvoyance and detectors disrupt this!!!!

Decision Analysis

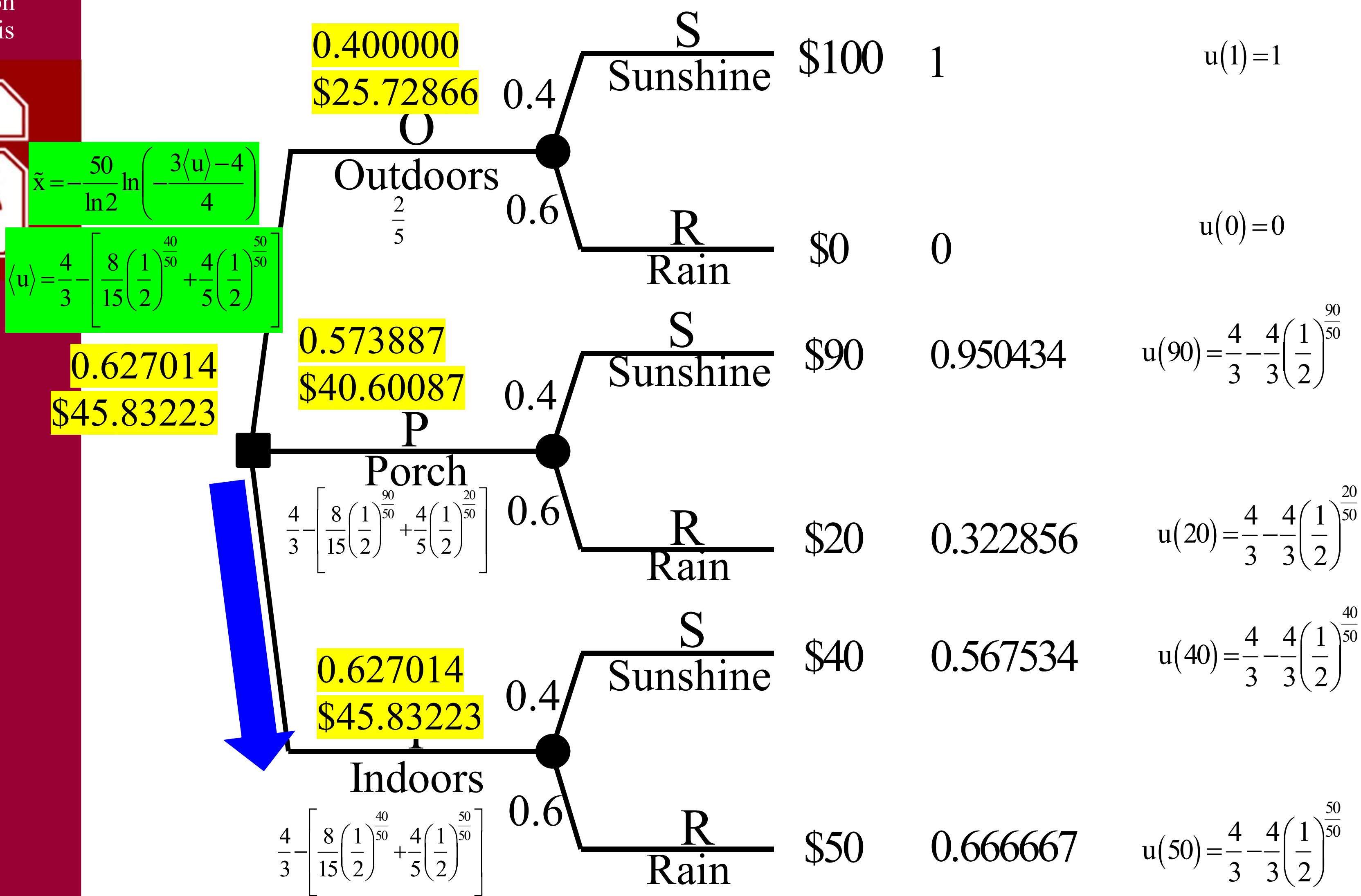
Dale M. Nesbitt

Are There Probabilistically Dominant Alternatives in the Party Problem?

Solving the Party Problem Using u Values as a Measure

Decision Analysis

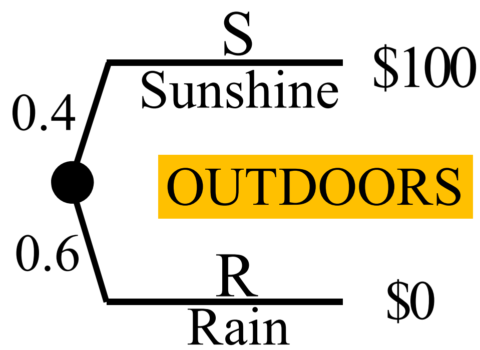
Dale M. Nesbitt

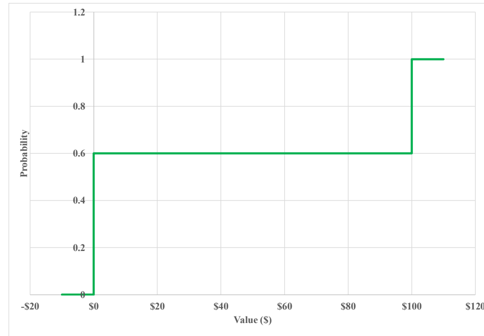


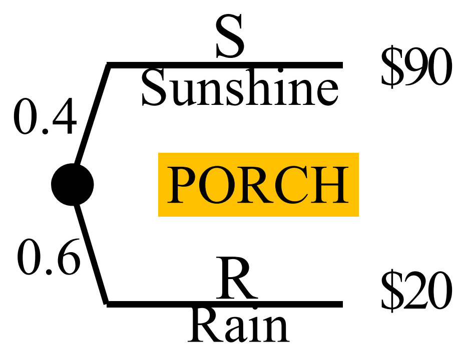
Decision Analysis

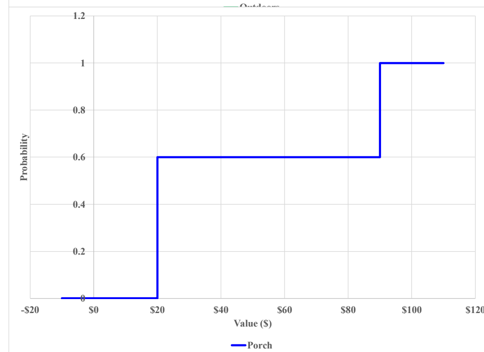
Dale M. Nesbitt

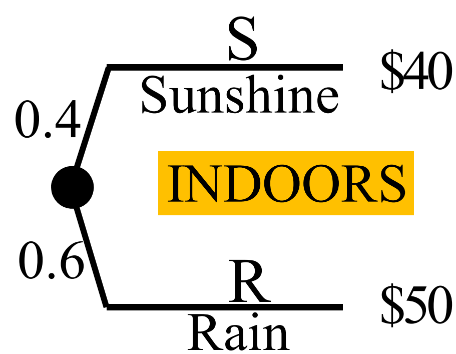
The Three Cumulatives

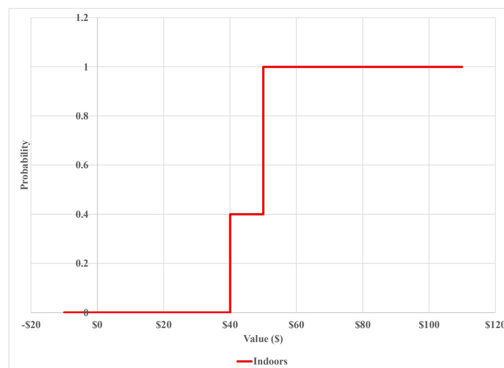




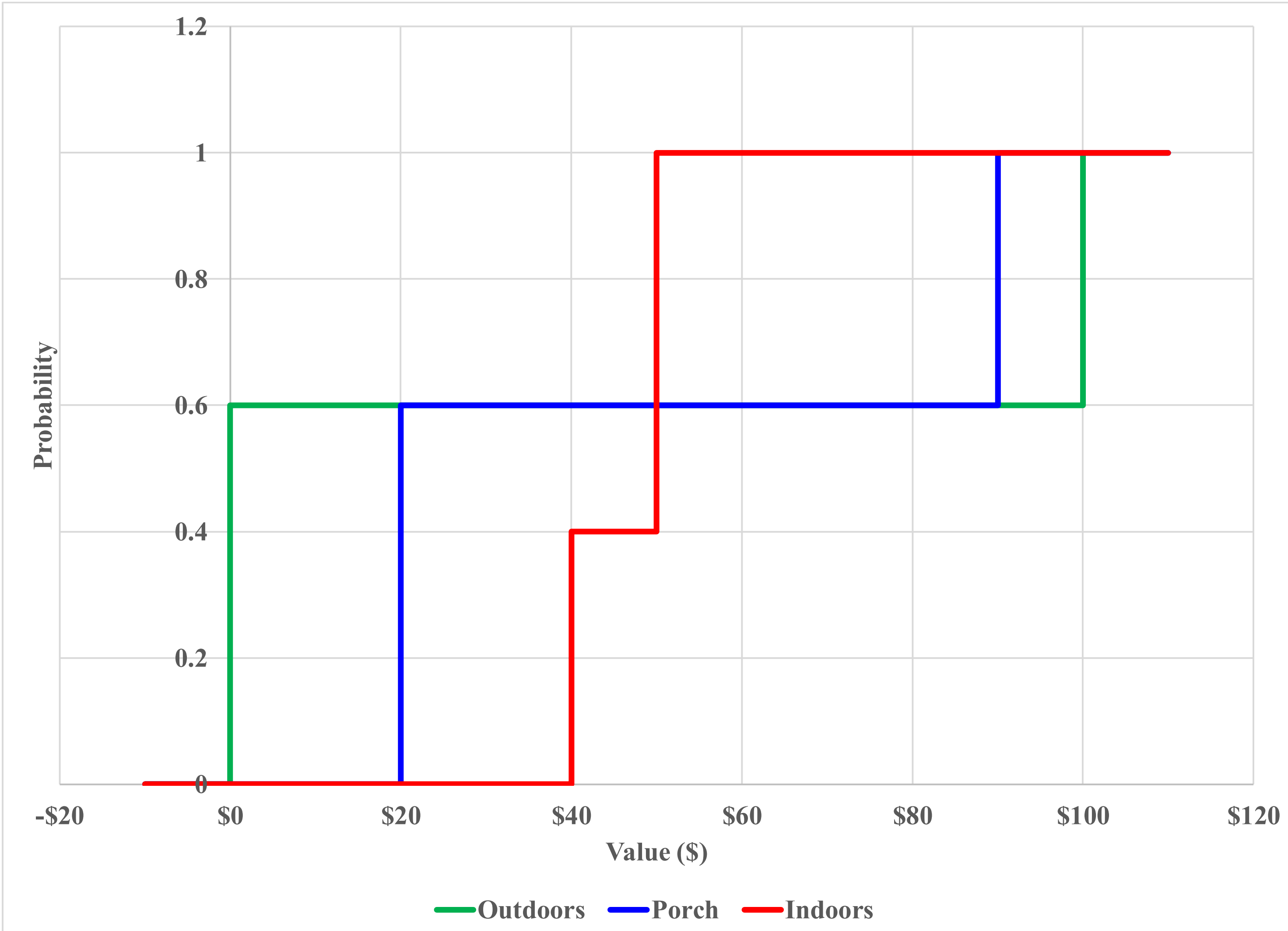




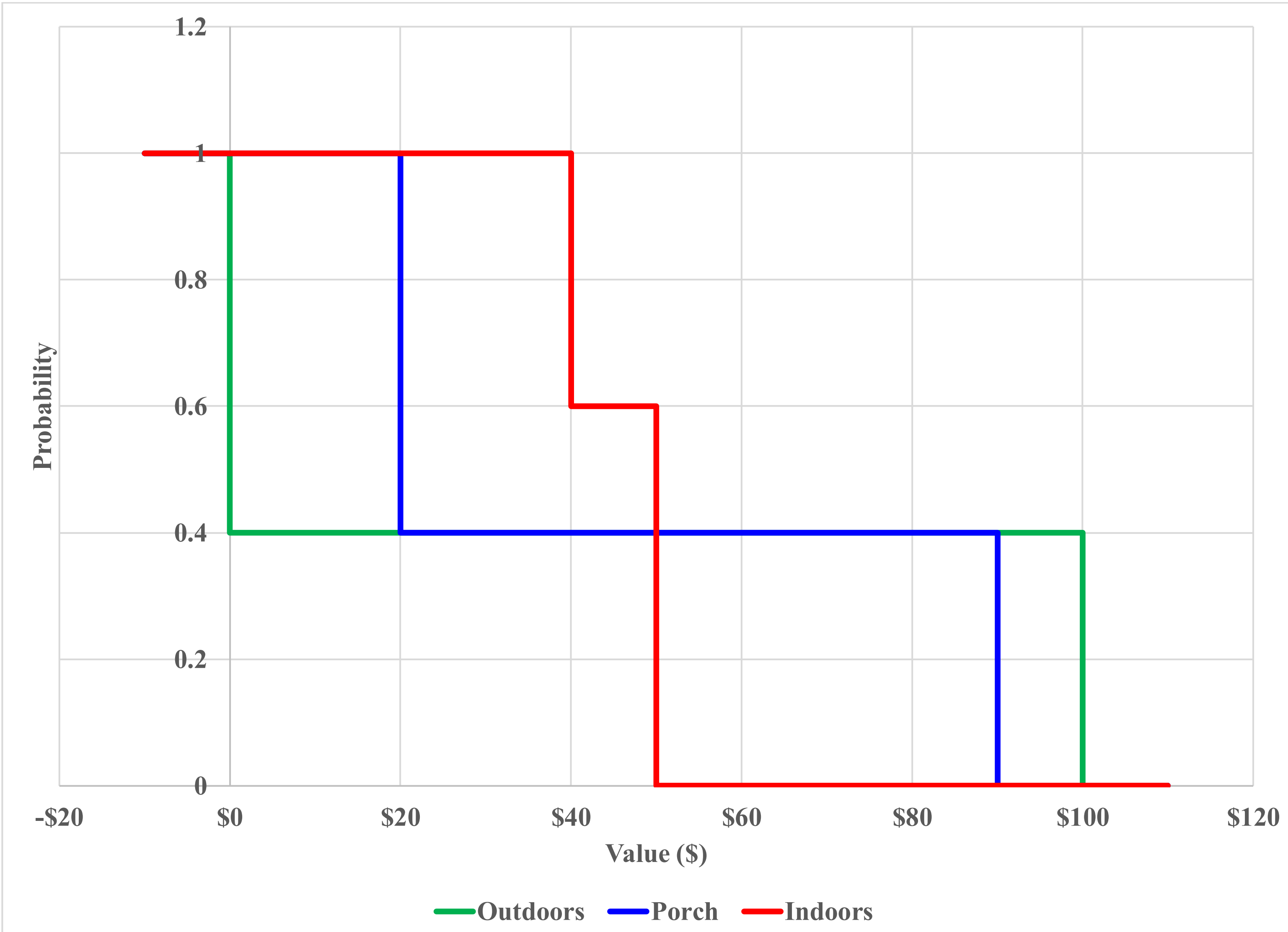




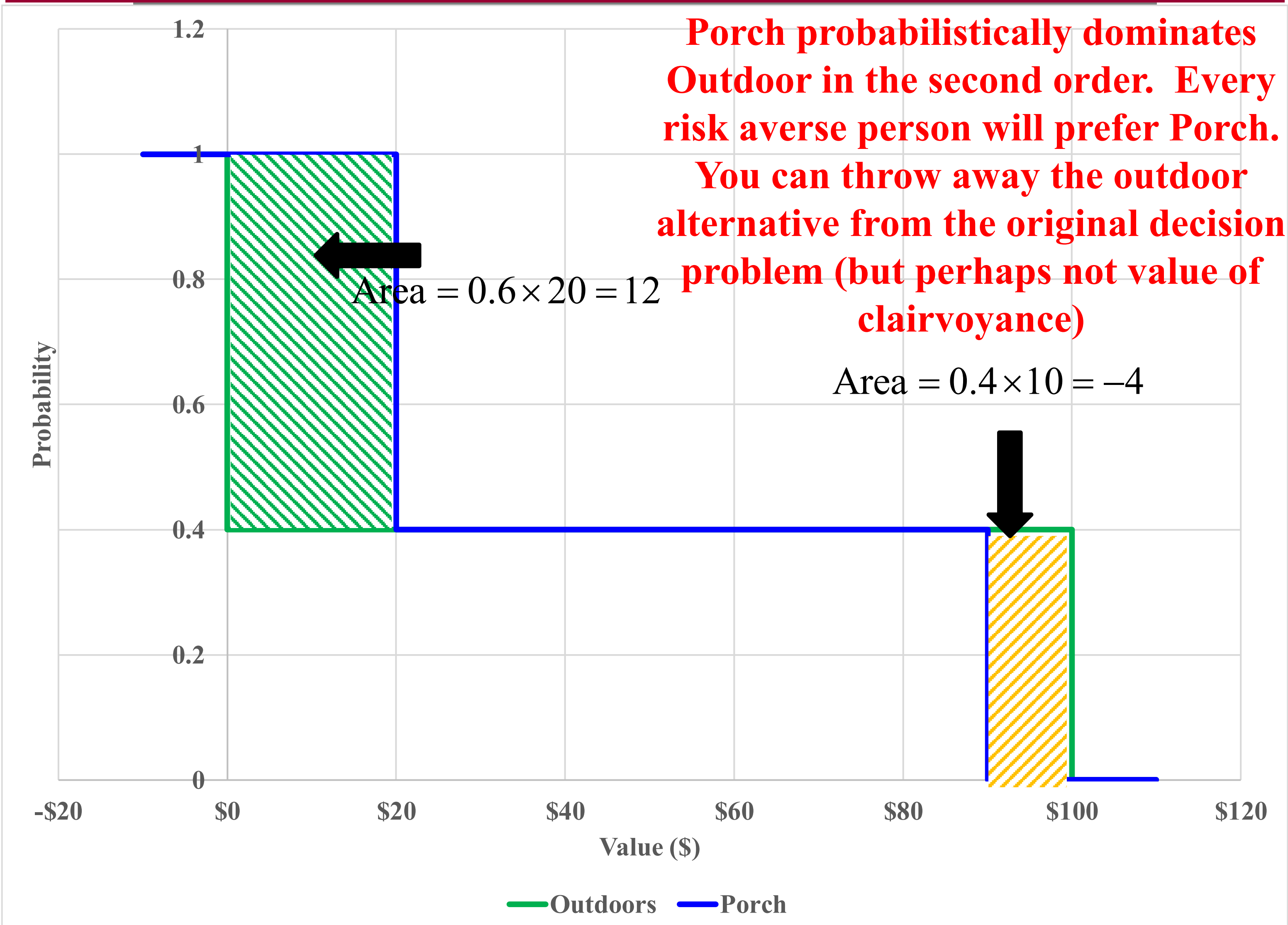
Cumulatives for Party Problem



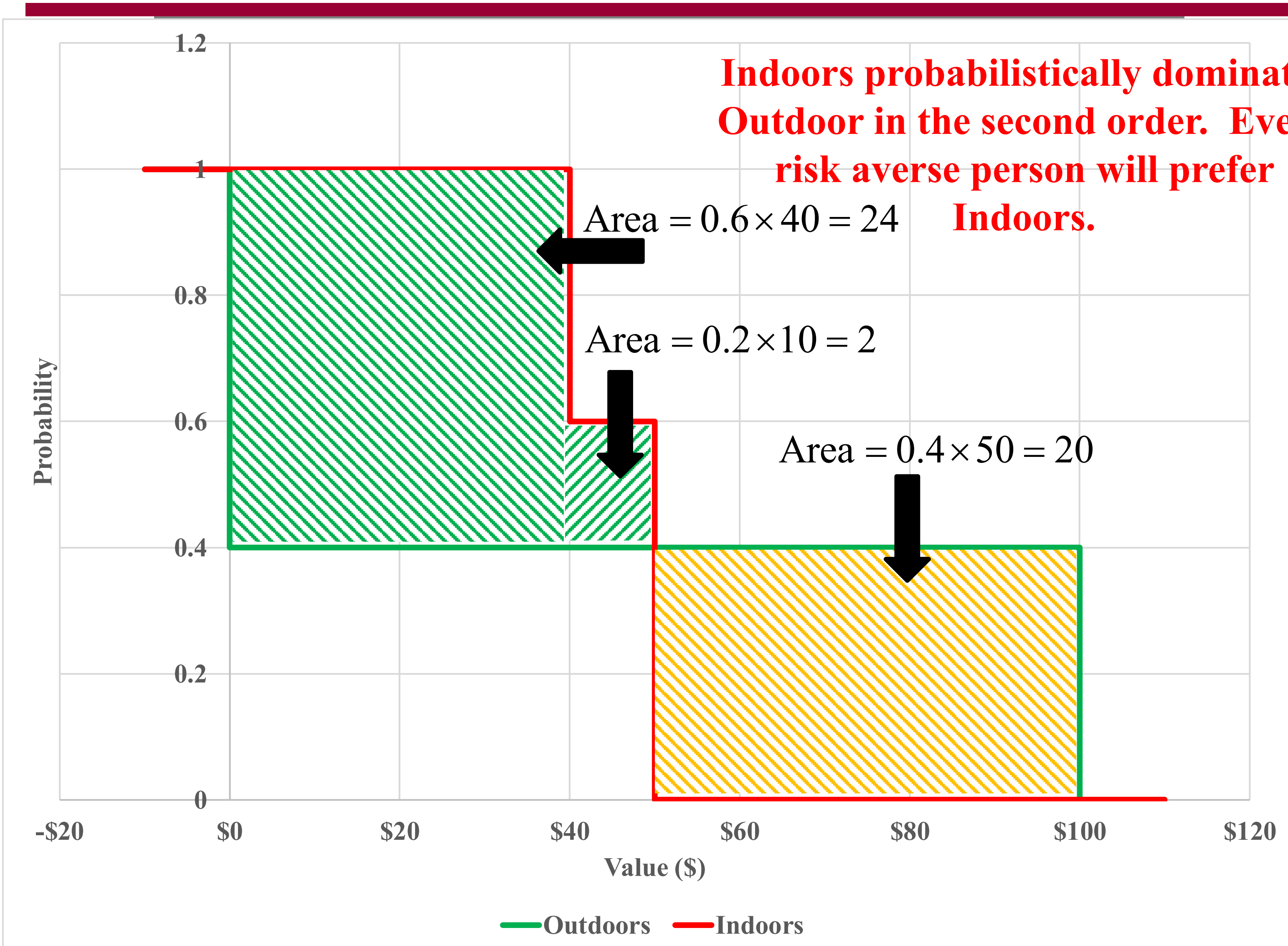
Complementary Cumulatives for Party Problem



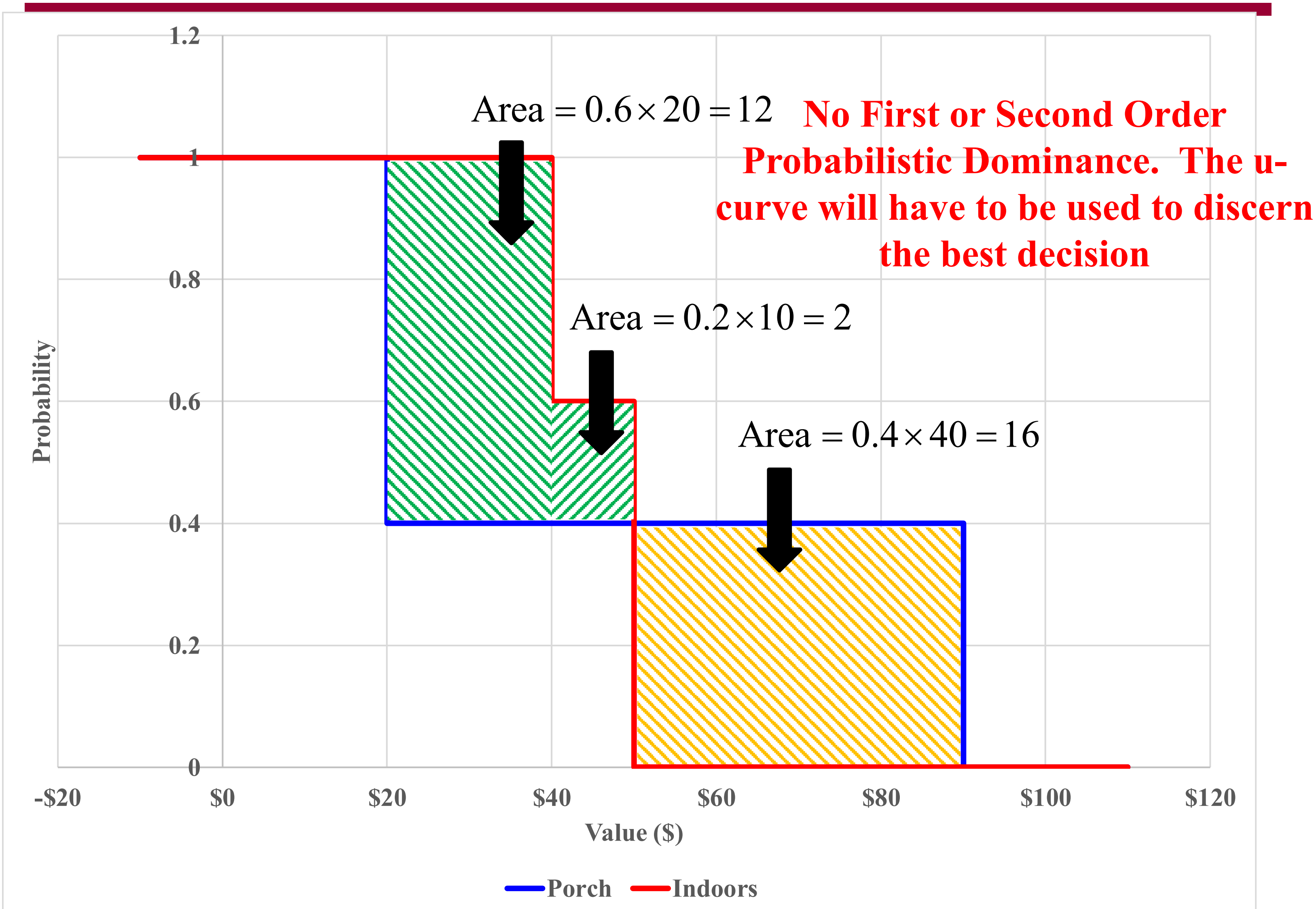
Let Us Compare Porch and Outdoor (Redact Indoor from the Graph)



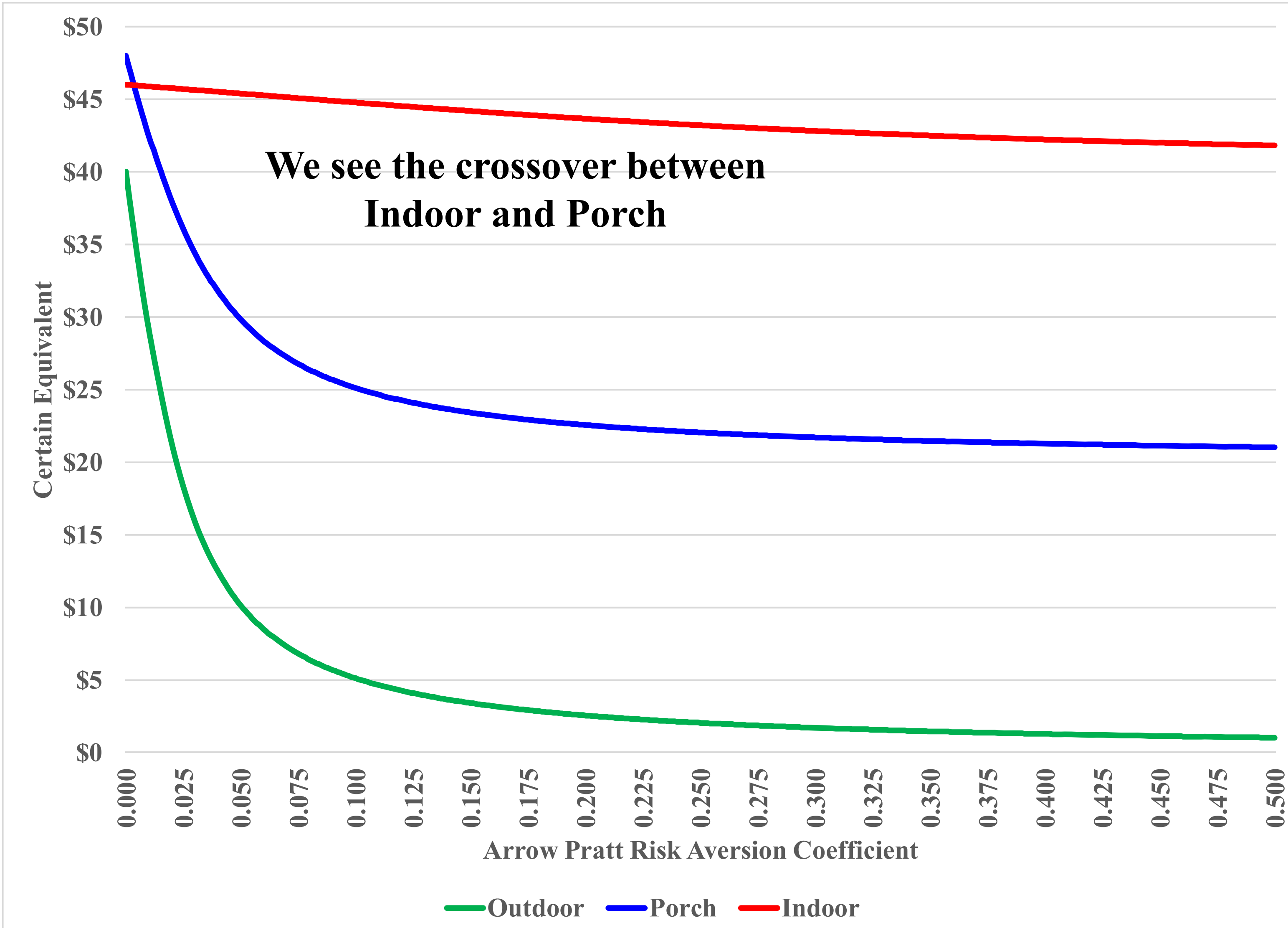
Compare Indoors and Outdoors



Compare Indoors and Porch



We See the Probabilistic Dominance by Indoor and Porch Over Outdoor



Decision Analysis

Dale M. Nesbitt

You Always Do the Probabilistic Dominance Calculations

- It provides insight
- It checks your work
- It relieves you from a lot of utility function work