CSE 015: Discrete Mathematics
Homework #11 Solutions

Chapter 5.1

- 6:The basis step is clear, since 1.1! = 2!-1. Assuming the inductive hypothesis, we then have 1.1! + 2.2! + ... + k.k! + (k+1).(k+1)! = (k+1)! - 1 + (k + 1).(k + 1)! = (k+1)! (1+k+1)-1 = (k+2)!-1, as desired.
- 16: The basis step reduces to 6 = 6. Assuming the inductive hypothesis we have 1.2.3 + 2.3.4 + ... + k(k + 1)(k + 2) + (k + 1)(k + 2)(k + 3)

  $= \frac{k(k+1)(k+2)(k+3)}{4}$ + (k+1)(k+2)(k+3)

  = k(k+1)(k+2)(k+3)($\frac{k}{4}$ + 1)

  $= \frac{(k+1)(k+2)(k+3)(k+4)}{4}$

- 20: The basis step is n = 7, and indeed $3^7 < 7!$, since 2187 < 5040. Assume the statement for k. Then $3^{k+1} = 3.3^k <$ (k + 1)$.3^k <$ (k + 1).k! = (k + 1)!, the statement for k+1.

Chapter 5.2

- 12: The basis step is to note that $1 = 2^0$. Notice for subsequent steps that $2 = 2^1$, $3 = 2^1 + 2^0$, $4 = 2^2, 5 = 2^2 + 2^0$, and so on. Indeed this is simply the representation of a number in binary form (base two). Assume the inductive hypothesis, that every positive integer up to k can be written as a sum of distinct powers of 2. We must show that k + 1 can be written as a sum of distinct powers of 2. If k + 1 is odd, then k is even, so 2^0 was not part of the sum for k . Therefore the sum for k+1 is the same as the sum for k with the extra term 2^0 added. If k+1 is even, then (k+1)=2 is a positive integer, so by the inductive hypothesis (k + 1)=2 can be written as a sum of distinct powers of 2. Increasing each exponent by 1 doubles the value and gives us the desired sum for k + 1.

Chapter 5.3

- 4(a): f(2) = f(1)-f(0) = 1-1 = 0, f(3) = f(2)-f(1) = 0-1 = -1, f(4) = f(3)-f(2) = -1-0 = -1, f(5) = f(4)-f(3) = 1-1 = 0
- 4(b): Clearly f(n) = 1 for all n, since 1.1 = 1.
- 12: The basis step (n=1) is clear. since $f_1^2 = f_1 f_2 = 1$. Assume the inductive hypothesis. Then $f_1^2 + f_2^2 + ... + f_n^2 + f_(n+1)^2 = f_n.f_{n+1} + f_{n+1}^2 = f_{n+1}(f_n + f_{n+1}) = f_{n+1}f_{n+2}$, as desired.

- 26(a): The basis step is the observation that $1 \equiv 1 \pmod 4$. For the inductive step, if $n \equiv 1$ (mod 4), then $3n + 2 \equiv 3.1 + 2 = 5 \equiv 1 \pmod 4$ and $n^2 \equiv 12 = 1 \pmod 4$.
- 26(b): One example is that $9 \notin S$. Because 9 is not of the form $3n + 2$, the only way 9 could have gotten into S would be via $9 = 3^2$, but $3 \notin S$ because $3 \not\equiv 1 \pmod 4$.