CSE 015: Discrete Mathematics
Homework #7
Solution

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Lab CSE-015-07L

March 29, 2022

Chapter 2.2

1. **Question 16: Let A and B be sets. Show that**

   (a) 16a: (A $\cap$ B) $\subseteq$ A. Let's set the sets A and B as [1,2,3,4] and [1,3,5,7] respectively. We know that the (A $\cap$ B) is what is shared between the two sets, which is 1 and 3. This (A $\cap$ B) is a subset of A. ([1,3], the subset, whose values is in the set of A.)

   (b) 16c: A - B $\subseteq$ A. Let's set the sets A and B as [1,2,3,4] and [1,3,5,7] respectively. We know that A - B has the values 2 and 4 since A - B means that every value of B shared with A should not be in the result, which is [2,4]. Hence we know that A - B is indeed a subset of A. ([2,4], the subset, whose values is in the set of A.)

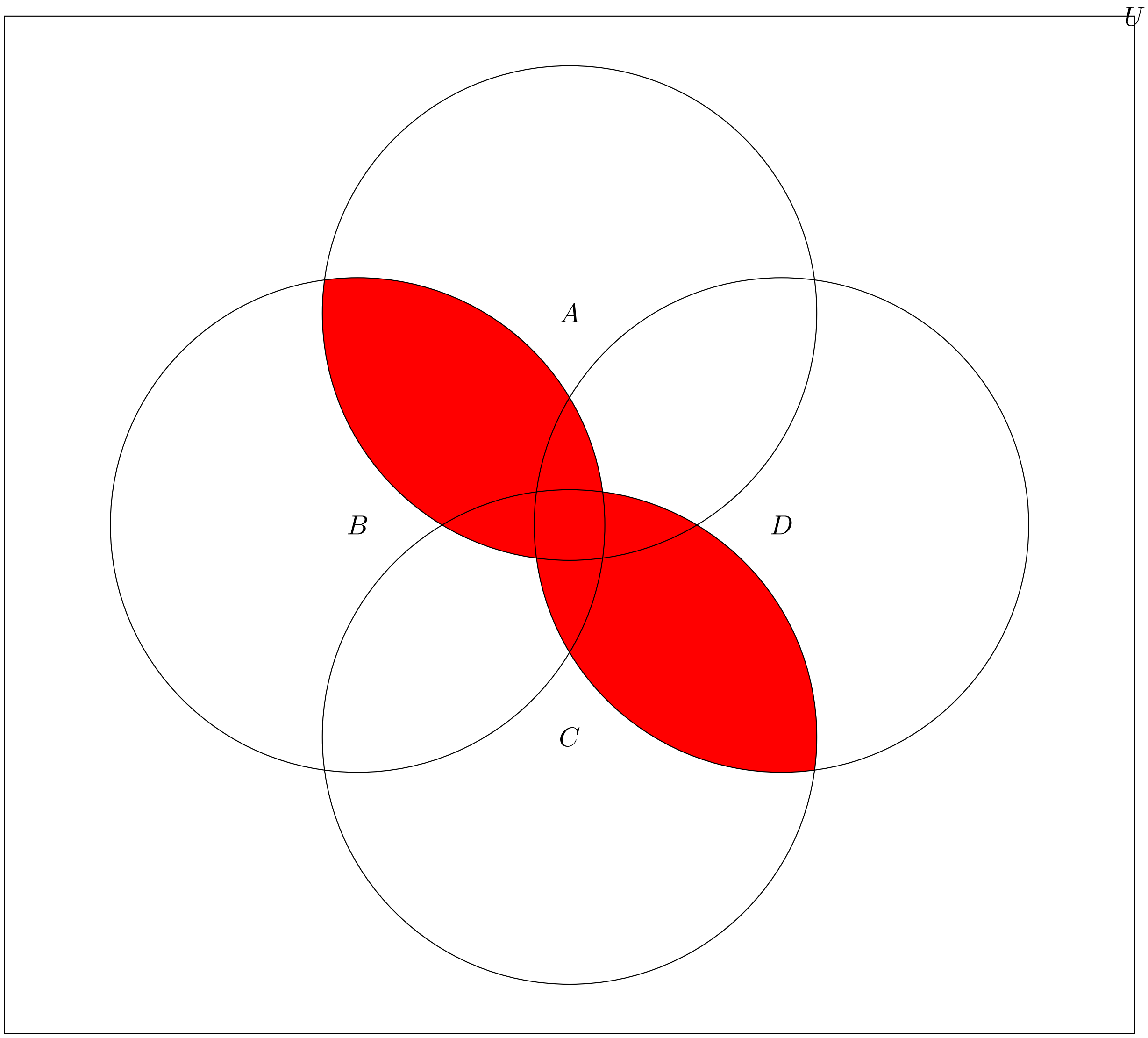
2. **Question 22: Show that if A and B are sets with A $\subseteq$ B, then**

   (a) 22a: A $\cup$ B = B. Let's set the sets A and B as [1,2] and [1,2,3,4] respectively. We know that the union of both A and B is [1,2,3,4], which is still B. Hence, we know that A $\cup$ B equals B.

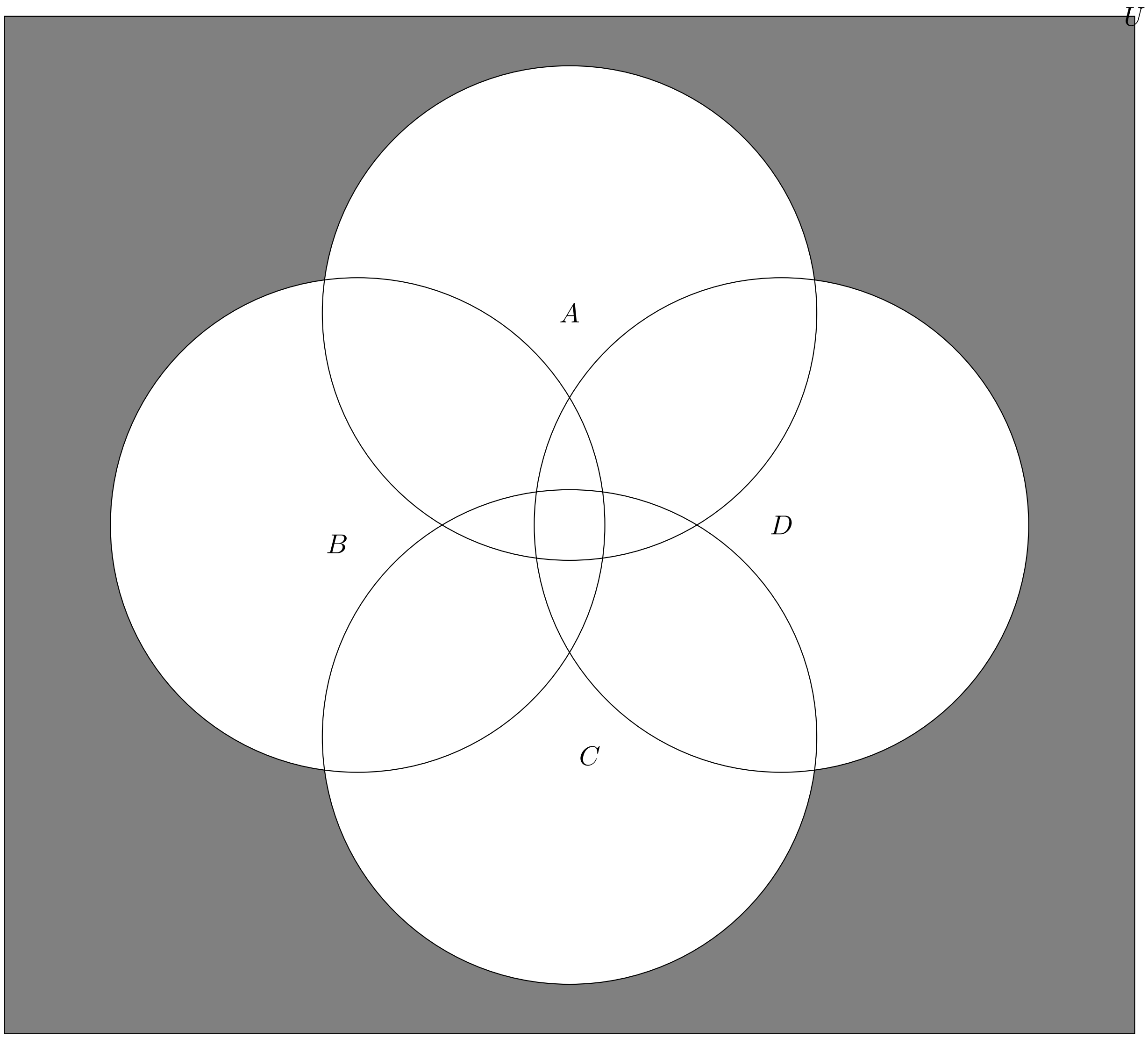
   (b) 22b: A $\cap$ B = A. Let's set the sets A and B as [1,2] and [1,2,3,4] respectively. We know that the intersection of both A and B is [1,2], which is still A. Hence, we know that A $\cap$ B equals A.

3. **Question 30: Draw the Venn diagrams for each of these combinations of the sets A, B, C, and D.**

   (a) 30a: (A $\cap$ B) $\cup$ (C $\cap$ D)



(a) 30b: $\overline{A} \cup \overline{B} \cup \overline{C} \cup \overline{D}$



4. **Question 54: Let A_i = . . . ,2,1, 0, 1, . . . , i. Find**

   (a) 54a: This is A_n, because for some value of n, we can show that since this is a union of all sets with $A_i = A_1 \cup A_2 \cup A_3$. We can say that for $A_1 \cup A_2 \cup A_3$, we can equal it to {...,-2,-1,0,1} $\cup$ {...,-2,-1,0,1,2} $\cup$ {...,-2,-1,0,1,2,3} which equals {...,-2,-1,0,1,2,3} = A_3. We can set n as some value that if $A_1 \cup A_2 \cup A_3 \cup A_n$ and that is equal to A_n, which is proven by the A_3 shown.

   (b) 54b:This is A_1, because we know that since this is a union of all sets with $A_i = A_1 \cup A_2 \cup A_3$. We can say that for $A_1 \cup A_2 \cup A_3$, we can equal it to {...,-2,-1,0,1} $\cup$ {...,-2,-1,0,1,2} $\cup$ {...,-2,-1,0,1,2,3} which equals {...,-2,-1,0,1} = A_1. We can try setting n as some value that if $A_1 \cup A_2 \cup A_3 \cup A_n$ and that is still equal to A_1, which is proven by the {...,-2,-1,0,1} shown. The A_1 is also 1, because we also know that i = 1, shown in the problem itself.

Chapter 2.3

1. **Question 2: Determine whether f is a function from Z to R if**

   (a) 2a: This is not a function, because it can not be either negative or positive at the same time.

(b) 2b: This is a function, because we know that the square root of n squared plus one is a real number that is well defined for all integers of n.

2. **Question 10: Determine whether each of these functions from a, b, c, d to itself is one-to-one.**

   (a) 10a: f(a) = b, f(b) = a, f(c) = c, f(d) = d, this function is 1 to 1, because when drawing the image map, for each f of some value, we can see that it references to a different value.

   (b) 10b: f(a) = b, f(b) = b, f(c) = d, f(d) = c, this function is not 1 to 1, because f(a) and f(b) both equal to b. A 1 to 1 function needs to have values referencing different values that are not repeated for each f of some value.

3. **Question 14: Determine whether $f : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z}$ is onto if**

   (a) 14a: f(m, n) = 2m - n, for every integer x $\in$ Z, the pair of (0,-x) $\in$ Z x Z has the image x that is shown by f(0,-x)=2(0)-(-x) = x, which means that the function f is onto.

   (b) 14b: f(m, n) = m^2 - n^2, for 3 $\in$ Z, there is no (m,n) $\in$ such that m^2 - n^2 = 3. This means that the integer 3 has no pre-image in Z x Z. This means that the function here is not onto.

4. **Question 22: Determine whether each of these functions is a bijection from R to R.**

   (a) 22a: f(x) = -3x + 4, Lets say f(x) = f(y) in R, such that -3x+4 = -3y+4. We can set x = y this way. This makes it 1 to 1. And let's suppose y = f(x), then y = f(x) = -3x+4. Then this will mean that x = (4-y) / 3). So for every element y in the co-domain R, there exists an element x = (4-y) / 3 in the domain of R such that f(x = (4-y)/3) = y. We know that the function is onto because of this, and that means that the function is a bijection from R to R.

   (b) 22b: f(x) = -3x^2 + 7, let's suppose that f(x) = f(y) in R. We can set -3x^2 + 7 = -3y^2 + 7 to get x^2 = y^2. So that will give us x = $\pm$ y. So f is not 1 to 1. Hence we know that the function is not a bijection from R to R.

5. **Question 38: Find f$\circ$g and g$\circ$f , where f(x) = x^2 + 1 and g(x) = x + 2, are functions from R to R.**

   (a) 38: f$\circ$g = f(g(x)), f(x+2), $(x+2)^2$ + 1, x^2 + 2x + 2x + 4 + 1 = [(x^2 + 4x + 5)]. And we have g$\circ$f = g(f(x)) = g(x^2 + 1). (x^2 + 1) + 2, which is x^2 + 1 + 2, or [(x^2 + 3)]. (I included the square brackets to make it easier to see.)

6. **Question 40: Let f(x) = ax + b and g(x) = cx + d, where a, b, c, and d are constants. Determine necessary and sufficient conditions on the constants a, b, c, and d so that f$\circ$g = g$\circ$f.**

   (a) 40: Let's write f$\circ$g = g$\circ$f as f(g(x)) = g(f(x)). And lets plug in the f(x) = ax + b and g(x) = cx + d. So we have f(cx+d) = g(ax+b). Now we can do it as a(cx+d)+b = c(ax+b)+d to get acx+ad+b = acx+bc+d. Get rid of acx, and we have ad+b = bc+d. Now we have ad+b = bc+d. Simplify further, and we have ad-d = bc-b. Going further, we have d(a-1) = b(c-1). This tells us that there will be an infinite set of real numbers of a,b,c, and d which satisfy the equation which allows all values of f$\circ$g = g$\circ$f.