CSE 015: Discrete Mathematics
Homework #3
Solution

Chapter 1.6

10. For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

(a) "If I play hockey, then I am sore the next day." "I use the whirlpool if I am sore." "I did not use the whirlpool."

(b) "If I work, it is either sunny or partly sunny." "I worked last Monday or I worked last Friday." "It was not sunny on Tuesday." "It was not partly sunny on Friday."

Solution

(a) If we use modus tollens starting from the back, then we conclude that I am not sore. Another application of modus tollens then tells us that I did not play hockey.

(b) We really can't conclude anything specific here.

16. For each of these arguments determine whether the argument is correct or incorrect and explain why.

(a) Every one enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

(b) A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

Solution

(a) This is correct, using universal instantiation and modus tollens.

(b) This is not correct. After applying universal instantiation, it contains the fallacy of denying the hypothesis.

24. **Solution**

Steps 3 and 5 are incorrect; simplification applies to conjunctions, not disjunctions.

Chapter 1.7

8. Solution

Let $n = m^2$. If $m = 0$, then $n + 2 = 2$, which is not a perfect square, so we can assume that $m \geq 1$. The smallest perfect square greater than n is $(m + 1)^2$, and we have $(m + 1)^2 = m^2 + 2m + 1 = n + 2m + 1 \geq n + 2 \cdot 1 + 1 > n + 2$. Therefore n+2 cannot be a perfect square.

16. Solution

Assume to the contrary that x, y, and z are all even. Then there exist integers a, b, and c such that $x = 2a$, $y = 2b$, and $z = 2c$. But then $x + y + z = 2a + 2b + 2c = 2(a + b + c)$ is even by definition. This contradicts the hypothesis that $x + y + z$ is odd. Therefore the assumption was wrong, and at least one of x, y, and z is odd.

20. Solution

(a) We must prove the contrapositive: If n is odd, then $3n + 2$ is odd. Assume that n is odd. Then we can write $n = 2k+1$ for some integer k. Then $3n+2 = 3(2k+1)+2 = 6k+5 = 2(3k+2)+1$. Thus $3n + 2$ is two times some integer plus 1, so it is odd.

(b) Suppose that $3n + 2$ is even and that n is odd. Since $3n + 2$ is even, so is $3n$. If we add subtract an odd number from an even number, we get an odd number, so $3n - n = 2n$ is odd. But this is obviously not true. Therefore our supposition was wrong, and the proof by contradiction is complete.

28. Solution

We need to prove two things, since this is an ”if and only if” statement. First let us prove directly that if n is even then $7n+4$ is even. Since n is even, it can be written as $2k$ for some integer k. Then $7n + 4 = 14k + 4 = 2(7k + 2)$. This is 2 times an integer, so it is even, as desired. Next we give a proof by contraposition that if $7n+4$ is even then n is even. So suppose that n is not even, i.e., that n is odd. Then n can be written as $2k+1$ for some integer k. Thus $7n+4 = 14k+11 = 2(7k+5)+1$. This is 1 more than 2 times an integer, so it is odd. That completes the proof by contraposition.