CSE 015: Discrete Mathematics
Homework #2 Solution

Chapter 1.4

- 52:
  It is enough to find a counterexample. It is intuitively clear that the first proposition is asserting much more than the second. It is saying that one of the two predicates, P or Q, is universally true; whereas the second proposition is simply saying that for every x either P(x) or Q(x) holds, but which it may well depend on x. As a simple counterexample, let P(x) be the statement that x is odd, and let Q(x) be the statement that x is even. Let the domain of discourse be the positive integers. The second proposition is true, since every positive integer is either odd or even. But the first proposition is false, since it is neither the case that all positive integers are odd nor the case that all of them are even.

- 54 (a): This is false, since there are many values of x that make x > 1 true.

- 54 (b): This is false, since there are two values of x that make $x^2 = 1$ true.

Chapter 1.5

- 12 (a): ¬I (Jerry)

- 12 (o): $\exists x \exists y (x| \neq y \wedge \forall z (C(x, z) \vee C(y.z)))$

- 24 (a): There exists an additive identity for the real numbers—a number that when added to every number does not change its value.

- 24 (b): A non-negative number minus a negative number is positive.

- 40 (a):
  There are many counterexamples. If x = 2, then there is no y among the integers such that 2 = 1/y, since the only solution of this equation is y = 1/2. Even if we were working in the domain of real numbers, x = 0 would provide a counterexample, since 0 = 1/y for no real number y.

- 40 (b):
  We can rewrite y^2 - x < 100 as y^2 < 100 + x. Since squares can never be negative, no such y exists if x is, say, -200. This x provides a counterexample.