CSE 015: Discrete Mathematics
Homework #5
Solution

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Chapter 1.8

1. **Question 8: Prove using the notion of without loss of generality that 5x + 5y is an odd integer when x and y are integers of opposite parity**

   (a) 8: Lets say that x is odd and y is even. Lets have variables a and b equal to x and y in an equation respectively as such: x = 2a+1 and y = 2b. And if we do 5x+5y, we can see that 5(2a+1) + 5(2b) as 10a+1+10b or as 10a+10b+1 which can be shown as an odd value due to that + 1.

2. **Question 18: Show that if a, b, and c are real numbers and a 0, then there is a unique solution of the equation ax + b = c**

   (a) 18: So with the equation ax+b=c, lets solve for x, which we would do the steps as ax=c-b, that would then be x=(c-b)/a and a 0. Lets have x1 and x2 as the solutions to ax+b=c. We would have ax1+b=c and ax2+b=c as a result. We would have x1=x2 as the end result if we were to plug them in since ax1=c-b and ax2=c-b and dividing both by a is x1=x2. We know that a cannot be 0, hence the unique solution is (c-b)/a.

3. **Question 26: The quadratic mean of two real numbers x and y equals $\sqrt{x^2 + y^2}/2$. By computing the arithmetic and quadratic means of different pairs of positive real numbers, formulate a conjecture about their relative sizes and prove your conjecture.**

   (a) 26: The arithmetic mean is shown as (x+y)/2 and the quadratic mean is shown as $\sqrt{x^2 + y^2}/2$. So lets try x = 1 and y = 2. So for the arithmetic mean, we would have (1+2)/2 = 1.5, and for the quadratic mean, we would have $\sqrt{1^2 + 2^2}/2$ as $\sqrt{1 + 4}/2$ or $\sqrt{5}/2$ as 1.58113883. And lets try x = 3 and y = 2. So for the arithmetic mean, we would have (3+2)/2 = 2.5, and for the quadratic mean, we would have $\sqrt{3^2 + 2^2}/2$ as $\sqrt{9 + 4}/2$ or $\sqrt{13}/2$ as 2.549509757. From these examples, we can say that the quadratic mean is greater than the arithmetic mean. We can set an equality as $\sqrt{x^2 + y^2}/2 >$ (x+y)/2. Then we can simplify it as such. $(x^2+y^2)/2 > (x+y)^2/4$. This is after we square both sides. After this, we can simplify it further like this with $2x^2+2y^2 > x^2+y^2+2xy$ by multiplying by 4 and finding the result of $(x+y)^2$. Simplified further, we get $(x-y)^2 > 0$. This is the statement that will always be true. Hence, the required result is found.