CSE 015: Discrete Mathematics
Homework #4
Solution

Chapter 1.7

8. Solution

Let $n = m^2$. If $m = 0$, then $n + 2 = 2$, which is not a perfect square, so we can assume that $m \geq 1$. The smallest perfect square greater than n is $(m+1)^2$, and we have $(m+1)^2 = m^2 + 2m + 1 = n + 2m + 1 \geq n + 2 \cdot 1 + 1 > n + 2$. Therefore n+2 cannot be a perfect square.

16. Solution

Assume to the contrary that x, y, and z are all even. Then there exist integers a, b, and c such that $x = 2a$, $y = 2b$, and $z = 2c$. But then $x + y + z = 2a + 2b + 2c = 2(a + b + c)$ is even by definition. This contradicts the hypothesis that $x + y + z$ is odd. Therefore the assumption was wrong, and at least one of x, y, and z is odd.

20. Solution

(a) We must prove the contrapositive: If n is odd, then $3n + 2$ is odd. Assume that n is odd. Then we can write $n = 2k+1$ for some integer k. Then $3n+2 = 3(2k+1)+2 = 6k+5 = 2(3k+2)+1$. Thus $3n + 2$ is two times some integer plus 1, so it is odd.

(b) Suppose that $3n+2$ is even and that n is odd. Since $3n+2$ is even, so is $3n$. If we add subtract an odd number from an even number, we get an odd number, so $3n - n = 2n$ is odd. But this is obviously not true. Therefore our supposition was wrong, and the proof by contradiction is complete.

30. Solution

There are two things to prove. For the "if" part, there are two cases. If $m = n$, then of course $m^2 = n^2$; if $m = -n$, then $m^2 = (-n)^2 = (-1)^2 n^2 = n^2$. For the "only if" part, we suppose that $m^2 = n^2$. Putting everything on the left and factoring, we have $(m + n)(m - n) = 0$. Now the only way that a product of two numbers can be zero is if one of them is zero. Therefore we conclude that either $m + n = 0$ (in which case $m = -n$),or else $m - n = 0$ (in which case $m = n$),and our proof is complete.

44. Solution

We show that each of these is equivalent to the statement (v) n is odd, say $n = 2k + 1$. Example 1 showed that (v) implies (i), and Example 9 showed that (i) implies (v). For $(v) \rightarrow (ii)$ we see that $1 - n = 1 - (2k + 1) = 2(-k)$ is even. Conversely, if n were even, say $n = 2m$, then we would have $1 - n = 1 - 2m = 2(-m) + 1$, so $1 - n$ would be odd, and this completes the proof by contraposition that $(ii) \to (v)$. For $(v) \to (iii)$, we see that $n^3 = (2k+1)^3 = 8k^3+12k^2+6k+1 = 2(4k^3+6k^2+3k)+1$ is odd. Conversely, if n were even, say $n = 2m$, then we would have $n^3 = 2(4m^3)$, so n^3 would be even, and this completes the proof by contraposition that $(iii) \to (v)$. Finally, for $(v) \to (iv)$, we see that $n^2 + 1 = (2k+1)2+1 = 4k^2+4k+2 = 2(2k^2+2k+1)$ is even. Conversely, if n were even, say $n = 2m$, then we would have $n^2 + 1 = 2(2m^2) + 1$, so $n^2 + 1$ would be odd, and this completes the proof by contraposition that $(iv) \to (v)$.

Chapter 1.8

6. **Solution**

There are three main cases, depending on which of the three numbers is smallest. If a is smallest (or tied for smallest), then clearly $a \le min(b, c)$, and so the left-hand side equals a. On the other hand, for the right-hand side we have $min(a, c) = a$ as well. In the second case, b is smallest (or tied for smallest). The same reasoning shows us that the right-hand side equals b; and the left-hand side is $min(a, b) = b$ as well. In the final case, in which c is smallest (or tied for smallest), the left-hand side is $min(a, c) = c$, whereas the right-hand side is clearly also c. Since one of the three has to be smallest we have taken care of all the cases.

10. **Solution**

The number 1 has this property, since the only positive integer not exceeding 1 is 1 itself, and therefore the sum is 1. This is a constructive proof.

24. **Solution**

We follow the hint. The square of every real number is non-negative, so $(x - 1/x)^2 \ge 0$. Multiplying this out and simplifying, we obtain $x^2 - 2 + 1/x^2 \ge 0$, so $x^2 + 1/x^2 \ge 2$, as desired.

32. **Solution** If $|y| \ge 2$, then $2x^2 + 5y^2 \ge 2x^2 + 20 \ge 20$, so the only possible values of y to try are 0 and ± 1. In the former case we would be looking for solutions to $2x^2 = 14$ and in the latter case to $2x^2 = 9$. Clearly there are no integer solutions to these equations, so there are no solutions to the original equation.

44. **Solution**

This is easily done, by laying the dominoes horizontally, three in the first and last rows and four in each of the other six rows.