CSE 015: Discrete Mathematics
Homework #8
Solution

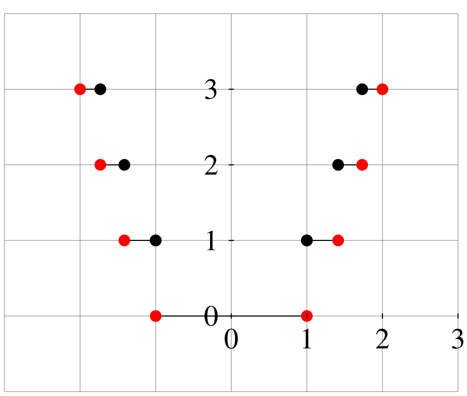
Arvind Kumar
Lab CSE-015-07L

April 5, 2022

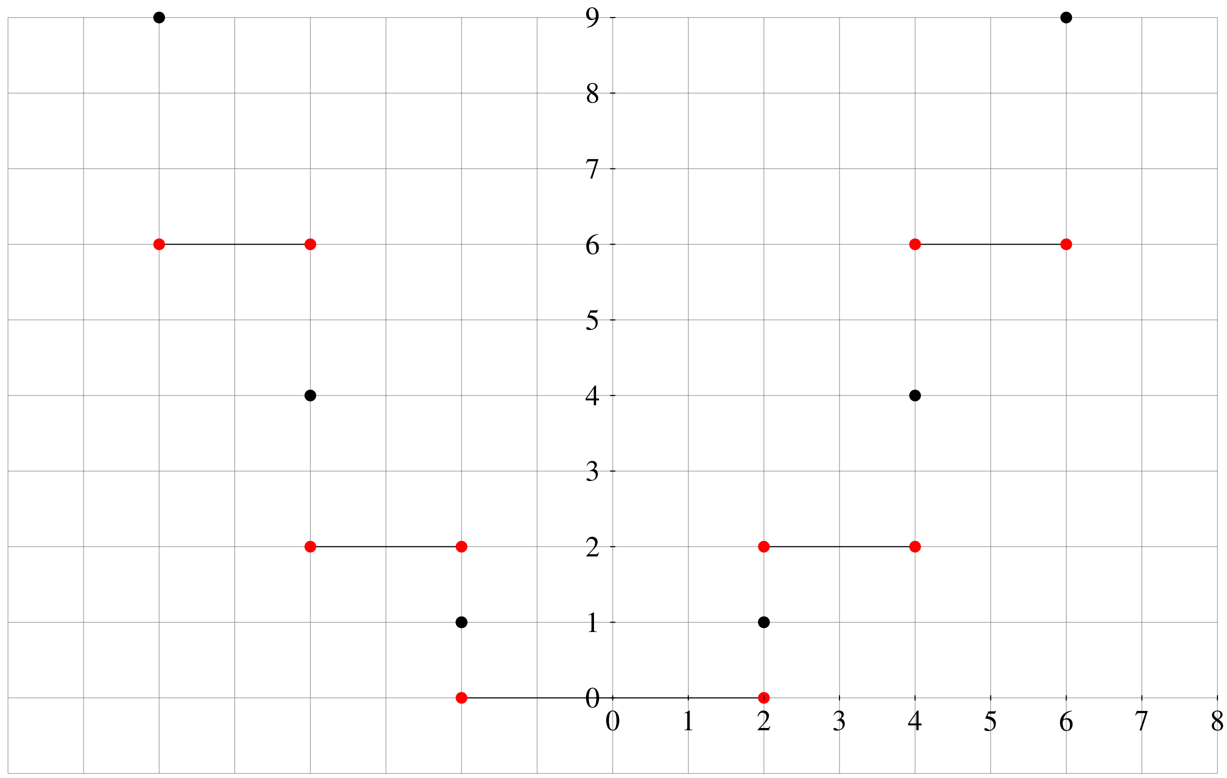
Chapter 2.3

1. **Question 54: Show that if x is a real number and n is an integer, then**
   (a) 54a: $x \leq n$ if and only if $\lceil x \rceil \leq n$. Lets set x = 3.1 and n = 4. We can see that this ($x \leq n$) is already satisfied. The ceiling of x gives us 4, and that is equal to 4. The conditions here are satisfied this way.
   (b) 54b: $n \leq x$ if and only if $n \leq \lfloor x \rfloor$. Lets set n = 1 and x = 2.4. We know that the floor of x is 2, and it is still greater than n. The conditions are satisfied here.

2. **Question 70: Draw graphs of each of these functions** The red dots are not inclusive within the graph.
   (a) 70d: $f(x) = \lfloor x^2 \rfloor$



   (a) 70e: $f(x) = \lceil x/2 \rceil + \lfloor x/2 \rfloor$



Chapter 2.4

1. **Question 10: Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.**

   (a) 10a: a_n = -2a_{n-1}, a_0 = -1, Found: a_1 = 2, a_2 = -4, a_3 = 8, a_4 = -16, a_5 = 32, a_6 = -64.

   (b) 10b: a_n = a_{n-1} - a_{n-2}, a_0 = 2, a_1 = -1, Found: a_2 = -3, a_3 = -2, a_4 = 1, a_5 = 3, a_6 = 2, a_7 = -1.

2. **Question 12: Show that the sequence a_n is a solution of the recurrence relation a_n = -3a_{n-1} + 4a_{n-2} if**

   (a) 12c: a_n = (-4)^n, lets replace n with n - 1. so we will have a_{n-1} = (-4)$^{n-1}$. And lets replace n with n - 2. so we will have a_{n-2} = (-4)$^{n-2}$. When deriving, we will have -3a_{n-1} + 4a_{n-2} = -3(-4)$^{n-1}$ + 4(-4)$^{n-2}$ to then have -3(-4)(-4)$^{n-2}$ + 4(-4)$^{n-2}$. So when we factor out (-4)$^{n-2}$, we will have (-4)$^{n-2}$(-3(-4)+4), to have (-4)$^{n-2}$(12+4), (-4)$^{n-2}$(16) or (-4)$^{n-2}$(-4)^2 to (-4)^n, which is equal to a_n.

   (b) 12d: a_n = 2(-4)^n + 3, lets replace n with n - 1. so we will have a_{n-1} = 2(-4)$^{n-1}$ + 3. And lets replace n with n - 2. so we will have a_{n-2} = 2(-4)$^{n-2}$ + 3. When deriving the function, we will have -3a_{n-1} + 4a_{n-2} = -3(2(-4)$^{n-1}$+3)+4(2(-4)$^{n-2}$+3). Using the distributive property, we would have -6(-4)$^{n-1}$ - 9 + 8(-4)$^{n-2}$ + 12. And lets group the powers of -4. We would then have (-6(-4)$^{n-1}$+8(-4)$^{n-2}$)-9+12 to then equal (-6(-4)(-4)$^{n-2}$+8(-4)$^{n-2}$)+3. So factoring (-4)$^{n-2}$, we would have (-4)$^{n-2}$(-6(-4)+8)+3. And then we would have (-4)$^{n-2}$(24+8)+3, (-4)$^{n-2}$(32)+3, (-4)$^{n-2}$(16(2))+3, (-4)$^{n-2}$(-4)^2(2)+3, (-4)^n(2)+3, 2(-4)^n+3, which is equal to a_n.

3. **Question 16: Find the solution to each of these recurrence relations with the given initial conditions. Use an iterative approach such as that used in Example 10.**

   (a) 16a: a_n = -a_n-1, a_0 = 5 is the given, a_n = -a_{n-1} = (-1)^1 a_{n-1} is the first step, (-1)^1(-a^{n-2}) = (-1)^2a_{n-2}, (-1)^2(-a^{n-3}) = (-1)^3a_{n-3}, and we have (-1)^3(-a^{n-4}) = (-1)^4a_{n-4}. We will have (-1)^na_{n-n} or (-1)^na_0 with 5(-1)^n as the final result.

(b) 16b: a_n = a_{n-1} + 3, a_0 = 1 is the given, a_{n-1} + 3 = a_{n-1} + 3(1), then we have (a_{n-2} + 3) + 3 = (a_{n-2} + 3(2), then (a_{n-3} + 3 + 3(2) = (a_{n-3} + 3(3), then (a_{n-4} + 3 + 3(3) = (a_{n-4} + 3(4). Now we have a_{n-n}+3n, a_0+3n, 1 + 3n or 3n+1 as the final result.

4. **Question 40: Find sum with 200 at the top and k=99 and k^3**

   (a) 40: With the help of table 2, we have (200 at the top and k=1 and k^3) - (98 at the top and k=1 and k^3) as (404010000)-(23532201) = 380477799, which is the result.

Chapter 2.5

1. **Question 12: Show that if A and B are sets and A $\subset$ B then $|A| \leq |B|$.**

   (a) 12: By the definition of a subset, we know that if a $\in$ A, then a $\in$ B. We can define this as f: A $\rightarrow$ B, f(a) = a. We need to check if f is 1 to 1. So if we let f(a) = f(b), , we then obtain a = b by the definition of f. So this is 1 to 1. And because of this, $|A| \leq |B|$. This is proven by that result.

2. **Question 22: Suppose that A is a countable set. Show that the set B is also countable if there is an onto function f from A to B**

   (a) 22: B is countable, because since A is countable, then it is either finite or countably infinite. Since f is a function from A to B: we know that the subsets have to be the same for every b in B, and there exists an element for a in A such that f(a) = b. Since the sets are the same, their cardinality is the same too. With this, we can say that A contains n elements, and contains at most n elements, which is f(A) being finite. But since f(A) and B have the same cardinality and since f(A) is finite, B then has to be finite as well and thus B is countable.

3. **Question 28:. Show that the set $Z^+ \times Z^+$ is countable.**

   (a) 28: $Z^+ \times Z^+$ is countable, because if we define the function f as f: $Z^+ \times Z^+ \rightarrow Z^+$, f(m,n) = 2^m x 3^n. We will need to check is they are 1 to 1. So if f(a,b) = f(m,n), then 2^a x 3^b = 2^a x 3^b. Because 2 and 3 are primes, we will have 2^a = 2^m and 3^b = 3^b. The bases are the same so the powers are the same with a = m, b = n. We have shown that f is a 1 to 1 function. And by definition, $|Z^+Z^+| \leq |Z^+|$. Hence set A is countable if and only if $|A| \leq |Z^+|$. $Z^+ \times Z^+$ is shown to be countable.