CSE 015: Discrete Mathematics
Homework #11
Solution

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Lab CSE-015-07L

April 27, 2022

Chapter 5.1

1. **Question 6: Prove that 1*1! + 2*2! + + n*n! = (n + 1)! 1 whenever n is a positive integer.**

   (a) 6: Lets set n = 1, we will have 1*1!+2*2!+...+n*n! = 1*1! = 1*1! = 1*1 = 1. Then we will have (n+1)!-1 = (1+1)!-1=2!-1=2-1=1. Now lets set n = (k+1). We will have 1*1!+2*2!+...+k*k!+(k+1)*(k+1)! = 1*(k+1)!-1+(k+1)*(k+1)! = 1*(k+1)!+(k+1)*(k+1)!-1 = (k+2)(k+1)!-1 = (k+2)!-1 = ((k+1)+1)!-1. Hence, P(k+1) is true by the principle of mathematical induction.

2. **Question 16: Prove that for every positive integer n, 1 2 3 + 2 3 4 + + n(n + 1)(n + 2) = n(n + 1)(n + 2)(n + 3)/4.**

   (a) 16: Lets set n = 1. We will have 1(1 + 1)(1 + 2)(1 + 3)/4 = 1(2)(3)(4)/4 = 24/4 = 6. Hence P(1) is true. Now lets set n = k+1. We will have 1*2*3+2*3*4+...+k(k+1)(k+2)+(k+1)(k+2)(k+3) = (k(k+1)(k+2)(k+3))/4+(k+1)+ = ((k/4)+1)(k+1)(k+2)(k+3) = ((k+4)/4)(k+1)(k+2)(k+3) = ((k+1)(k+2)(k+3)(k+4))/4 = ((k+1)((k+1)+1)((k+1)+2)((k- which is denoted to be true. And by mathematical induction, P(n) is true for all positive integers n.

   Question 20: Prove that $3^n < n!$ if n is an integer greater than 6.

   (a) 20: Let's set n = 8. So 3^8 = 6561 ¡ 40320 = 8! Hence, n = 8 is true. Let's now set n = k+1. We will have $3^{k+1} = 3*3^k < 3*k! < (k+1)*k! = (k+1)!$. We now know that n = (k+1) is also true. And by mathematical induction, P(n) is true for all positive integers n greater than 6.

Chapter 5.2

1. **Question 12: Use strong induction to show that every positive integer can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers 2^0=1, 2^1=2, 2^2=4, and so on. [Hint: For the inductive step, separately consider the case where k+1 is even and where it is odd. When it is even, note that (k+1)/2 is an integer.]**

   (a) 12:We know that 2^0=1, 2^1=2, 2^2=4 are true. Now we need to show that n = k+1 is true. Lets have different cases of k+1 in that it can be odd or even. In the event that k+1 is odd, we know that k is even. This can be written as a sum of distinct powers of 2, and none of these powers can be 0. Hence, we can write k+1 as the sum of these powers and one more term, 2^0. Let's say that k = $2^{e_1} + 2^{e_2}+...+2^{e_m}$ where all e_i is not equal to 0 for all $1 \leq i \leq m$ then k+1 = $2^{e_1} + 2^{e_2}+...+2^{e_m} + 2^0$ since 1 = 2^0. Now in the event that k+1 is even, we know that (k+1)/2 is an integer. Since (k+1)/2 is an integer, we can write it as a sum of distinct powers of 2 (by assumption). And for each term in the sum for (k+1)/2, we can multiply it by 2, and their sum will be k+1. Let's say that (k+1)/2 = $2^{e_1} + 2^{e_2}$+...+2^{e_m} where all e_i are distinct, then k+1 = 2[(k+1)/2] = 2[$2^{e_1} + 2^{e_2}$+...+2^{e_m}] = $2^{e_1+1} + 2^{e_2+1}$+...+2^{e_m+1}. And regardless of k+1 being odd or even, we can write this as a sum of distinct powers of 2.

Chapter 5.3

1. **Question 4: Find f(2), f(3), f(4), and f(5) if f is defined recursively by f(0) = f(1) = 1 and for n = 1, 2, . . .**

   (a) 4a: f(n + 1) = f(n) f(n 1), f(2)=f(1)-f(0)=1-1=0, f(3)=f(2)-f(1)=0-1=-1, f(4)=f(3)-f(2)=-1-0=-1, f(5)=f(4)-f(3)=-1+1=0.

   (b) 4b: f(n + 1) = f(n)f(n 1), f(2)=f(1)f(0)=(1)(1)=1, f(3)=f(2)f(1)=(1)(1)=1, f(4)=f(3)f(2)=(1)(1)=1, f(5)=f(4)f(3)=(1)(1)=

2. **Question 12: Prove that $f_1^2 + f_2^2 + ... + f_n^2 = f_n f_{n+1}$ when n is a positive integer.**

   (a) 12: Since these are all in Fibonacci, we already have $f_0 = 0$ and $f_1 = 1$. Let's set n = 1, $f_1^2 = 1^2 = 1 = 1*1 - 1*(0+1) = f_1(f_0+f_1)=f_1f_2$. Now lets set n = k+1. We will have $f_1^2 + f_2^2 + ... + f_k^2 + f_{k+1}^2 = f_k f_{k+1} + f_{k+1}^2 = (f_k+f_{k+1})f_{k+1} = f_{k+2}f_{k+1} = f_{k+1}f_{k+2} = f_{k+1}f_{(k+1)+1}$. We know that n = k+1 is true and is proven by the principle of mathematical induction.

3. **Question 26: Let S be the set of positive integers defined by, Basis step: 1 $\in$ S, Recursive step: If n $\in$ S, then 3n + 2 $\in$ S and $n^2 \in$ S**

   (a) 26: To prove our work, if n $\in$ S, then n $\equiv$ 1 (mod 4). We will use Proof by Structural Induction in that the property p will state that n $\equiv$ 1 (mod 4). And 4 works in the base case in that 4 divides (1-1) = 0, and thus 1 = 1 (mod 4). And with the Recursive step, lets have n be a positive integer in S such that n $\equiv$ 1 (mod 4). We can now derive the positive integer 3n + 2 $\in$ S since n $\equiv$ 1 (mod 4), n mod 4 = 1. The steps: (3n+2) mod 4 = (3(n mod 4)+2) mod 4, = (3(1)+2) mod4, = (3+2) mod 4, 5 mod 4 = 1, and this implies that (3n+2) $\equiv$ 1 (mod 4). Now if we derive the $n^2 \in$ S, since n $\equiv$ 1 (mod 4), and n mod 4 = 1, we have the steps with (n^2) mod 4 = ((n mod 4))^2 mod 4, = (1)^2 mod 4, = 1 mod 4, = 1, which implies that $n^2 \equiv$ 1 (mod 4) and that all in S is satisfied. But to prove that there exists an integer o $\equiv$ 1 (mod 4) that does not belong to S, er can put down 9 $\equiv$ 1 (mod 4), because 4 divides 9-1 = 8. We know that 9 is not an integer added by the basis step as 9 not equal to 1. The integer then adds 3(1)+2=5 and 1^2 = 1 by the recursive step. The integer 5 then adds 3(5)+2=17 and 5^2 = 25 by the recursive step. This shows that the integers 17 and 25 can only add larger integers to the set of S and will thus note that 9 is not an integer in the set of S.