# CSE 015: Discrete Mathematics
# Homework #6
# Solution

## Chapter 2.1

12. Determine whether these statements are true or false.

    (a) $\emptyset \in \{\emptyset\}$
    **Solution**. True

    (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$
    **Solution**. True

    (d) $\{\emptyset\} \in \{\{\emptyset\}\}$
    **Solution**. True

    (f) $\{\{\emptyset\}\} \subset \{\emptyset, \{\emptyset\}\}$
    **Solution**. True. The one element in the set on the left is an element of the set on the right, and the sets are not equal.

20. Find two sets $A$ and $B$ such that $A \in B$ and $A \subseteq B$.

    **Solution**. Since the empty set is a subset of every set, we just need to take a set $B$ that contains $\emptyset$ as an element. Thus we can let $A = \emptyset$ and $B = \{\emptyset\}$ as the simplest example.

26. Determine whether each of these sets is the power set of a set, where $a$ and $b$ are distinct elements.

    (a) $\emptyset$
    **Solution**. The power set of every set includes at least the empty set, so the power set cannot be empty. Thus $\emptyset$ is not the power set of any set.

    (d) $\{\emptyset, \{a\}, \{b\}, \{a, b\}\}$
    **Solution**. This is the power set of $\{a, b\}$.

34. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

    (a) $A \times B \times C$
    **Solution**.
    $\{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$

    (b) $C \times A \times B$
    **Solution**.
    $\{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$

## Chapter 2.2

4. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find
   (c) $A - B$
   **Solution**. Note that $A \subset B$. There are no elements in $A$ that are not in $B$, so the answer is $\emptyset$.
   (d) $B - A$
   **Solution**. $\{f, g, h\}$

8. Prove the idempotent laws in Table 1 by showing that
   (a) $A \cup A = A$
   **Solution**. $A \cup A = \{x | (x \in A) \vee (x \in A)\} = \{x | x \in A\} = A$
   (b) $A \cap A = A$
   **Solution**. $A \cap A = \{x | (x \in A) \wedge (x \in A)\} = \{x | x \in A\} = A$

10. Show that
    (a) $A - \emptyset = A$
    **Solution**.
    $A - \emptyset = \{x | (x \in A) \wedge (x \notin \emptyset)\} = \{x | (x \in A) \wedge True\} = \{x | x \in A\} = A$
    (b) $\emptyset - A = \emptyset$
    **Solution**.
    $\emptyset - A = \{x | (x \in \emptyset) \wedge (x \notin A)\} = \{x | False \wedge (x \in A)\} = \{x | False\} = \emptyset$