CSE 015: Discrete Mathematics
Homework #12 Solutions

Chapter 5.4

- 4. First, because n = 10 is even, we use the else if clause to see that

  mpower(2; 10; 7) = mpower(2; 5; 7)2 mod 7.

  We next use the else clause to see that

  mpower(2; 5; 7) = (mpower(2; 2; 7)2 mod 7.2 mod 7) mod 7.

  Then we use the else if clause again to see that

  mpower(2; 2; 7) = mpower(2; 1; 7)2 mod 7.

  Using the else clause again, we have

  mpower(2; 1; 7) = (mpower(2; 0; 7)2 mod 7.2 mod 7) mod 7.

  Finally, using the if clause, we see that mpower(2; 0; 7) = 1. Now we work backward: mpower(2; 1; 7) = (12 mod 7.2 mod 7) mod 7 = 2, mpower(2; 2; 7) = 22 mod 7 = 4, mpower(2; 5; 7) = (42 mod 7.2 mod 7) mod 7 = 4, and finally mpower(2; 10; 7) = 42 mod 7 = 2. We conclude that 210 mod 7 = 2.

Chapter 6.1

- 16. We can subtract from the number of strings of length 4 of lower case letters from the number of strings of length 4 of lower case letters other than x. Thus the answer is $26^4 - 25^4$ = 66,351.
- 35(a) 0
- 35(b) 120
- 35(c) 720
- 35(d) 2520

Chapter 6.2

- 10 This is just a restatement of the pigeonhole principle, with k = $|T|$
- 28 Let the people be A, B, C, D, and E. Suppose the following pairs are friends: A-B, B-C, C-D D-E and E-A. The other five pairs are enemies. In this example, there are no three mutual friends and no three mutual enemies.

Chapter 6.3

- 20(a) There are C(10,3) ways to choose the positions for the 0's , and that is the only choice to be made, so the answer is C(10,3) = 120.

- 20(b) There are more 0's than 1's if there are fewer than five 1's . Using the same reasoning as in part (a), together with the sum rule, we obtain the answer C(10,0) + C(10,1) + C(10,2) + C(10,3) + C(10,4) = 1 + 10 + 45 + 120 + 210 = 386. Alternatively, by symmetry, half of all cases in which there are not five 0's have more 0's than 1's; therefore the answer is $(2^10 - C(10,5)/2 = (1024-252)/2 = 386$.