CSE 015: Discrete Mathematics
Homework #5 Solution

Chapter 1.8

- 8:
  Because x and y are of opposite parities, we can assume, without loss of generality, that x is even and y is odd. This tells us that x = 2m for some integer m and y = 2n + 1 for some integer n. Then, 5x+5y = 5(2m)+5(2n+1) = 10m+10n+1 = 10(m+n)+1 = 2·5(m+n)+1, which satisfies the definition of being an odd number.

- 18:
  We know from algebra that the following equations are equivalent: ax + b = c, ax = c - b. x = (c-b)/a. This shows, constructively, what the unique solution of the given equation is.

- 26:
  Let x = 1 and y = 10. Then their arithmetic mean is 5:5 and their quadratic mean is $\sqrt{50.5} \approx 7.11$. Similarly, if x = 5 and y = 8, p then the arithmetic mean is (5 + 8)/2 = 6.5 and the quadratic mean is $\sqrt{(5^2 + 8^2)/2} \approx 6.67$. So we conjecture that the quadratic mean is always greater than or equal to the arithmetic mean. Thus we want to prove that

  $$\sqrt{\tfrac{x^2+y^2}{2}} \geq \tfrac{x+y}{2}$$

  for all positive real numbers x and y. Doing some algebra, we find that this inequality is equivalent to the true statement that $(x-y)^2 \geq 0$:

  $$\begin{gathered} \sqrt{\tfrac{x^2+y^2}{2}} \geq \tfrac{x+y}{2} \\ 2x^2 + 2y^2 \geq x^2 + 2xy + y^2 \\ x^2 - 2xy + y^2 \geq 0 \\ (x-y)^2 \geq 0 \end{gathered}$$

  In fact, our argument also shows that equality holds if and only if x = y .