CSE 015: Discrete Mathematics
Homework #4
Solution

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Chapter 1.7

1. **Question 16: Prove that if x, y, and z are integers and x + y + z is odd, then at least one of x, y, and z is odd.**

   (a) 16: if x is 1, y is 3, and z is 5, we know that x + y + z equals 9, which is odd. x, y, and z are also integers in this case. Furthermore, we know that x, y, and z have at least 1 odd value.

2. **Question 20: Prove that if n is an integer and 3n + 2 is even, then n is even using**

   (a) 20(a): a proof by contraposition, ¬3n+2 is ¬Q(x), which is odd, we can use k as any odd number placed in n, and plugged into that equation is still odd.

   (b) 20(b): a proof by contradiction, since when 3n+2 is even and n is an integer, and when n is even, and suppose 3n+2 is even and n is odd, we know that when n is odd and the product of two odd numbers is still odd, hence 3n is odd, and 3n+2 is also odd. Hence, assuming 3n+2 is even and n is wrong is wrong by that contradiction. Hence, if n is an integer and 3n+2 is even, then n is also even.

3. **Question 30: Prove that $m^2 = n^2$ if and only if m = n or m = n**

   (a) For the claim m=n or m+n $\neq$ 0, we can assume that m-n$\neq$0 or m+n$\neq$0. We can see with this: (m-n)(m+n)$\neq$0. The steps are as follows: m^2 + mn - nm - $n^2 \neq 0$, m^2 - $n^2 \neq 0$, $m^2 \neq n^2$. This is the contradiction to $m^2 = n^2$. And our assumption that m$\neq$ n or m $\neq$ n is false. So that means that $m^2 = n^2$ if and only if m = n or m = n.

4. **Question 44: Prove that these four statements about the integer n are equivalent:**

   (a) n^2 is odd: So lets say that n^2 is odd, then n is also odd since the product of 2 odd numbers is odd. Then lets set a value k for n = 2k+1. Then the steps: 1 - n = -2k, 1-n = 2(-k) in which case k is an integer and -k is an integer as well. Since 1-n is proved in this case, we can say that (i) $\rightarrow$ (ii) is proved.

   (b) 1 n is even: Lets say that 1 - n is even, and let's say l-n = 2k for some integer known as k. n = -2k+1. Here are the steps: n = 2(-k)+1, for -k is an integer. n is odd, and n x n is also odd. hence n^2 x n is also odd. This is proven, and we can say that (ii) $\rightarrow$ (iii) is proved.

(c) n^3 is odd: So lets say n^3 is odd. and lets have $n^2 = 2k+1$ for some integer k. Here are the steps: $n^2+1 = 2k+2$, $n^2+1 = 2(k+1)$, which is even. Hence n^2+1 is even, which is proven in that case, and we can say that (iii) $\rightarrow$ (iv) is proved.

(d) $n^2 + 1$ is even: So lets say that n^2+1 is even with the even $n^2+1=2k$ for some integer known as k. We can set $n^2 = 2k-1$ and see that as odd since 2k-1 is the odd part. Hence n^2, and (iv) and (i) is proven.

(e) With these parts, we can say that the four statements,(i), (ii), (iii), (iv) are equivalent.

Chapter 1.8

5. **Question 6: Use a proof by cases to show that min(a, min(b, c)) = min(min(a, b), c) whenever a, b, and c are real numbers.**

   (a) 6: So lets have four cases: (i): a¿b¿c, (ii): a¡b¡c, (iii):a¡b and b¿c, and (iv):a¿b and b¡c.

   (b) for case (i): steps: then min(b,c) = c, min(a,min(b,c)) = min(a,c) = c, and then min(a,b) = b, which then means that min(min(a,b),c) = min(b,c) which is equal to c. And this case shows that min(a,min(b,c)) = min(min(a,b),c)

   (c) for case (ii): steps: min(b,c) = b, min(a, min(b,c)) = min(a,b) = a, and on the other side, min(a,b) = a, and min(a, min(b,c)) = min(a,c) = a. This means that min(a, min(b,c)) = min(min(a, b),c).

   (d) for case (iii): steps: min(b,c) = c, min(a, min(b,c)) = min(a,c), min(a,b) = a, min(min(a,b),c) = min(a,c)

   (e) for case (iv): steps: min(b,c) = b, min(a, min(b,c)) = min(a,b) = b, min(min(a,b),c) = min(b,c).

   (f) We can conclude that min(a, min(b,c)) = min(min(a,b),c).

6. **Question 10: Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?**

   (a) 10: So lets set the positive integer as n. And lets have the equation: n = (n(n+1))/ 2. With the steps as follows: 0 = (n(n+1)/2) - n, $(n^2+n)/2$ - n = 0, $(n^2+n-2n)/2 = 0$, n^2-n = 0, n(n-1)=0, which makes n = 0 or n = 1. We know that n is not zero. Hence, the only positive number that equals the sum of the positive numbers not exceeding it is 1. This proof is constructive.

7. **Question 24: Use forward reasoning to show that if x is a nonzero real number, then x^2 + $1/x^2$ 2. [Hint: Start with the inequality $(x\ \ 1/x)^2$ 0, which holds for all nonzero real numbers x.]**

   (a) 24: we can have these steps: $(x\ \ 1/x)^2$ 0 $x^2 + x^-2$-2(x(x^-1) 0, $x^2 + x^-2$-2 0, $x^2 + x^-2$ 2, and this inequality holds well for all nonzero real numbers x.

8. **Question 32: Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.**

   (a) 32: Because x and y are both squared, The result is positive for the terms $2x^2$ and $5y^2$. And in this case, x can only take in values from -2 to 2. Any larger would would result in a value greater than 14. The same goes for y except that it goes from -1 to 1. And for all of these differing combinations, the left side does not equal the right hand side. Hence there are no solutions with the equation $2x^2 + 5y^2 = 14$.

9. **Question 44: Prove or disprove that you can use dominoes to tile a standard checkerboard with all four corners removed.**

   (a) 44: There are 64 squares(8 rows, 8 columns) in a checkerboard, but there are 60 since the corner squares are removed. And there are 30 openings for the dominoes to take up on the checkerboard if one were to arrange them either all vertically or horizontally. And the dominoes can cover up all the openings, which proves that we can use dominoes to tile a standard checkerboard with all four corners removed.