CSE 015: Discrete Mathematics
Homework #12
Solution

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Chapter 5.4

1. **Question 4: Trace Algorithm 3 when it finds gcd(12, 17). That is, show all the steps used by Algorithm 3 to find gcd(12, 17).**

   (a) 4: We have the recursive definition that gcd(0,b) = b, gcd(a,b) = gcd(mod a,a) when a is greater than 0. If we evalue for the recursive definition at n = 5, we will be having gcd(12,17) = gcd(17 mod 12, 12) with the following steps: gcd(5,12) = gcd(12 mod 5,5) = gcd(2,5) = gcd(5 mod 2,2) = gcd(1,2) = gcd(2 mod 1, 1) = gcd(0,1) which equals 1.

Chapter 6.1

1. **Question 16: How many strings are there of four lowercase letters that have the letter x in them?**

   (a) 16: We will do the product rule to solve this. We will find the complete combination of the strings with a length of 4 which is 26*26*26*26, which is 456,976. And the strings without an x is 25*25*25*25, which is 390,625. So if we have strings with at least one x with a length of 4, we will do 456,976 - 390,625. This will give us 66,351. strings.

2. **Question 35: How many one-to-one functions are there from a set with five elements to sets with the following number of elements?**

   (a) 35a: 4, this function can not be one to one, because we know that there is are two elements that will need to have the same image.

   (b) 35b: 5, the 1st element has 5 ways, the 2nd element has 4 ways, the 3rd element has 3 ways, the 4th element has 2 ways, the 5th element has 1 way. This is all because the subsequent images have to be different. Hence there are 120 one-to-one functions that exist (5*4*3*2*1).

   (c) 35c: 6, the 1st element has 6 ways, the 2nd element has 5 ways, the 3rd element has 4 ways, the 4th element has 3 ways, the 5th element has 2 way. This is all because the subsequent images have to be different. Hence there are 720 one-to-one functions that exist (6*5*4*3*2).

   (d) 35d: 7, the 1st element has 7 ways, the 2nd element has 6 ways, the 3rd element has 5 ways, the 4th element has 4 ways, the 5th element has 3 way. This is all because the subsequent images have to be different. Hence there are 2520 one-to-one functions that exist (7*6*5*4*3).

Chapter 6.2

1. **Question 10: Show that if f is a function from S to T, where S and T are finite sets with $|S| > |T|$, then there are elements s_1 and s_2 in S such that $f(s_1) = f(s_2)$, or in other words, f is not one-to-one.**

   (a) 10: So to prove that f is not one to one, lets assume that for the sake of contradiction that f is one to one. Lets let n and m be positive integers such that |S| = n, and |T| = m. Know that S and T are finite sets. n is definitely greater than m. So if we have our sets in S and T, we can assume without loss of generality that $f(s_i) = t_i$ for i = 1,2,...,n for each element to be renamed as $t_1,...,t_m$ if the order of the elements didn't match up with the images. And since n is greater than m, we have a term s_{m+1} while we do not have a term t_{m+1}. This then implies that s_{m+1} needs to have the same image as some s_k with k as a positive integer from 1 to m. So $f(s_{m+1}) = f(s_k) = t_k$. But this is in contradiction with the fact that f is one-to-one. Since we derive a contradiction, our assumption is that f is one-to-one is incorrect. Hence f is not one-to-one.

2. **Question 28: Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.**

   (a) 28: Let's say that B and C are friends of A, and D and E are enemies of A. If B and C are enemies to each other, and D and E are friends between them, then there is no group of three people containing A have three mutual friends or enemies. So that group has to come from the rest of the four. So if D is a friend to B while an enemy to C, and E is the opposite of D, then we can not have three mutual friends or enemies from them too.

Chapter 6.3

1. **Question 20: How many bit strings of length 10 have**

   (a) 20a: exactly three 0s? We can use combination for this. We will have C(10,3) = 10!/3!(10-3)! = 10!/3!7! = 120. There are 120 bit strings.

   (b) 20b: more 0s than 1s? We can use combination for this in different ways in that we have C(10,4), C(10,3), C(10,2), C(10,1), and C(10,0). We have to add them up though, because there are more 0s than 1s is the ten bits. So for C(10,4) = 10!/(4!(10-4)!) = 10!/(4!6!) = 210. For C(10,3) = 10!/(3!(10-3)!) = 10!/(3!7!) = 120. For C(10,2) = 10!/(2!(10-2)!) = 10!/(2!8!) = 45. For C(10,1) = 10!/(1!(10-1)!) = 10!/(1!9!) = 10. For C(10,0) = 10!/(0!(10-0)!) = 10!/(0!10!) = 1. So when adding the number of bit strings up, we will have 210+120+45+10+1 = 386, which is the number that we will have when it is over. There are 386 bit strings this way.