CSE 015: Discrete Mathematics
Homework #3
Solution

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Chapter 1.6

1. **Question 10: For each of these sets of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.**

   (a) 10(a):

      (i) h: Play hockey
      (ii) s: Am Sore
      (iii) w: Use Whirlpool

      (A) "If I play hockey, then I am sore the next day." - - - - - $h \rightarrow s$
      (B) "I use the whirlpool if I am sore." - - - - - - - - - - - - - - $s \rightarrow w$
      (C) "I did not use the whirlpool." - - - - - - - - - - - - - - - - - $\neg$ w

      Because the whirlpool was not used, we can infer that I was not sore. That also means that hockey was not played as well. Modus Tollens. - - - $\neg$ w (with (Modus Tollens)) - - - $s \rightarrow w$ (with (Modus Tollens)) - - - $\neg$ s - - - $h \rightarrow s$ - - - $h \rightarrow w$ (with (Hypothetical Syllogism)) - - - $\neg$ h

   (b) 10(b):

      (i) w: I Work
      (ii) s: Sunny
      (iii) p: Partly Sunny

      (A) "If I work, it is either sunny or partly sunny." - - - - $\forall w \rightarrow (s \vee p)$
      (B) "I worked last Monday or I worked last Friday." - - w(Monday) $\vee$ w(Friday)
      (C) "It was not sunny on Tuesday." - - - - - - - - - - - - - - $\neg$ s(Tuesday)
      (D) "It was not partly sunny on Friday." - - - - - - - - - - - $\neg$ p(Friday)

   Because of how the condition of work does not exist for C or D(third or fourth statement), we can not draw conclusions for that since work was not used as the prerequisite. As for B(second statement), we can conclude that it was sunny or partly sunny last Monday or last Friday. A(first statement) gives us the conditional statement in the end. We use universal instantiation in this case since Monday and Friday are aspects of x within the days as x.

2. **Question 16: For each of these arguments determine whether the argument is correct or incorrect and explain why.**

   (a) 16(a): Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore, Mia is not enrolled in the university.

      (i) D: lived in a dormitory, U: enrolled in a university, x: people

      (A) Since $\forall$ x (D(x) $\rightarrow$ U(x)) with x representing everyone from the first sentence as the domain, we can plug in Mia as x once using universal instantiation. And since she has never lived in the dormitory, we can use modus tollens to say that she indeed has not lived in the university.

   (b) 16(b): A convertible car is fun to drive. Isaac's car is not a convertible. Therefore, Isaac's car is not fun to drive.

      (i) C: is a convertible car, F: fun to drive, x: people

      (A) Since $\forall$ x (C(x) $\rightarrow$ F(x)) in terms of results, we can plug in Isaac with universal instantiation as x. We know that Isaac's car is not a convertible as $\neg$C, and although it is not part of the group with fun cars, we can not conclude it is not fun to drive. Since it also possibly denies the hypothesis, we know that the argument is incorrect for this reason.

3. **Question 24: Identify the error or errors in this argument that supposedly shows that if x(P(x) Q(x)) is true then $\forall$xP(x) $\vee$ $\forall$xQ(x) is true**

   (a) 24: The steps 3 and 5 are wrong, because they have a $\vee$ instead of a $\wedge$ for simplification to work. As for step 7, it is wrong, because the $\forall$ would be distributed to the Q(x) twice when the $\forall$ symbol is already next to the Q(x) value.

   Chapter 1.7

4. **Question 8: Prove that if n is a perfect square, then n + 2 is not a perfect square**

   (a) 8: if n is 1, we can plug that in to n+2, which means that the result is 3, which is not a perfect square. Hence that is proven.

5. **Question 16: Prove that if x, y, and z are integers and x + y + z is odd, then at least one of x, y, and z is odd.**

   (a) 16: if x is 1, y is 3, and z is 5, we know that x + y + z equals 9, which is odd. x, y, and z are also integers in this case. Furthermore, we know that x, y, and z have at least 1 odd value.

6. **Question 20: Prove that if n is an integer and 3n + 2 is even, then n is even using**

   (a) 20(a): a proof by contraposition, $\neg$3n+2 is $\neg$Q(x), which is odd, we can use k as any odd number placed in n, and plugged into that equation is still odd.

   (b) 20(b): a proof by contradiction, since when 3n+2 is even and n is an integer, and when n is even, and suppose 3n+2 is even and n is odd, we know that when n is odd and the product of two odd numbers is still odd, hence 3n is odd, and 3n+2 is also odd. Hence, assuming 3n+2 is even and n is wrong is wrong by that contradiction. Hence, if n is an integer and 3n+2 is even, then n is also even.

7. **Question 28: Prove that if n is a positive integer, then n is even if and only if 7n + 4 is even.**

(a) 28: If we assume that n is even as 2k, and plugging that in to 7n+4, we get 14k+4. We can use operations to have 2(7k+2), and set a = 7k+2. That gives us 2(a). That means for any value of a that is plugged in, it will be an even result.