CSE 015: Discrete Mathematics
Homework #9
Solution

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Chapter 4.1

1. **Question 18: Suppose that a and b are integers, a $\equiv$ 11 (mod 19), and b $\equiv$ 3 (mod 19). Find the integer c with 0 $\leq$ c $\leq$ 18 such that**

   (a) 18a: c $\equiv$ 13a (mod 19). c $\equiv$ 13 (11) (mod 19), which is c = 143 (mod 19), which results in c = 10 (mod 19), because 143 is divisible by 19, and we have 10 as the result. So c is 10.

   (b) 18f: c $\equiv$ a^3 + $4b^3$ (mod 19). c $\equiv$ $(11)^3$ + $4(3)^3$ (mod 19), 1331 + 108 (mod 19), 1439 (mod 19), which gives us 14 (mod 19), because we have 1439 divided by 19 gives us 75 with 14 as the remainder. So c is 14.

2. **Question 22: Let m be a positive integer. Show that a mod m = b mod m if a $\equiv$ b (mod m).**

   (a) 22: By congruent modulo: m divides a and b, then we have a - b = mc, a = mc + b, and with modulo on both sides, we have mod m = (b + mc) mod m, which can be rewritten as mod m = b mod m + mc mod m. We'll end up with mod m = b mod m + 0, which is mod m = b mod m. This is proven to be true.

3. **Question 38: Find each of these values.**

   (a) 38a: (19^2 mod 41) mod 9, 19 squared is 361, which with mod 41 would have 33 (mod 41) since 361 / 41 would have 8 with 33 as the remainder. And 33 with mod 9 would give us 3 with the remainder of 6. Hence the solution is 6.

   (b) 38d: (21^2 mod 15$)^3$ mod 22, 21 squared is 441, and that divided by 15 is 29 with 6 as the remainder. However, that part of the expression is cubed so it would be $(6)^3$ mod 22 or 216 mod 22. 216 divided by 22 is 9, and the result of that is 18, which is the solution.

Chapter 4.2

1. **Question 26: Use Algorithm 5 to find 11^{644} mod 645.**

   (a) 26: i = 0, a_0 = 0; with power = 11^2 mod 645 = 121 mod 645 = 121, a_1 = 0; with power = 121^2 mod 645 = 14641 mod 645 = 451, a_2 = 1; with power = 451^2 mod 645 = 203401 mod 645 = 226, a_3 = 0; with power = 226^2 mod 645 = 51076 mod 645 = 121, a_4 = 0; with power = 121^2 mod 645 = 14641 mod 645 = 451, a_5 = 0; with power = 451^2 mod 645 = 203401 mod 645 = 226, a_6 = 0; with power = 226^2 mod 645 = 51076 mod 645 = 121, a_7 = 1; with power = 121^2 mod 645 = 14641 mod 645 = 451, a_8 = 0; with power = 451^2 mod 645 = 203401 mod 645 = 226, a_9 = 1; with power = 226^2 mod 645 = 203401 mod 645 = 226. We will have 1 being returned. So 11^{644} mod 645 has 1 as the result.

Chapter 4.3

1. **Question 4: Find the prime factorization of each of these integers.**

   (a) 4e: 289, with prime factorization, we will have 17 x 17, which each can not be simplified further.

   (b) 4f: 899, with prime factorization, we will have 29 x 31, which each can not be simplified further.

2. **Question 16: Determine whether the integers in each of these sets are pairwise relatively prime.**

   (a) 16c: 25, 41, 49, 64, prime factorization of each: 25 = 5^2, 41 = 41, 49 = 7^2, and 64 = 2^6. The greatest common divisor of each pair is 1, because they do not share any factors except for 1. Hence, the integers in this set are pairwise relatively prime.

   (b) 16d: 17, 18, 19, 23, prime factorization of each: 17 = 17, 18 = 2 x 3^2, 19 = 19, and 23 = 23. The greatest common divisor of each pair is 1, because there are no factors being shared except for 1. Hence, the integers in this set are pairwise relatively prime.