CSE 015: Discrete Mathematics
Homework #1
Solution

Chapter 1.1

10. Let p and q be the propositions:

p: I bought a lottery ticket this week.

q: I won the million dollar jackpot.

Express each of these propositions as an English sentence.

(f) $\neg p \rightarrow \neg q$

(g) $\neg p \wedge \neg q$

Solution

(f) If I did not buy a lottery ticket this week, then I did not win the million dollar jackpot on Friday.

(g) I did not buy a lottery ticket this week, and I did not win the million dollar jackpot on Friday.

16. Let p, q, and r be the propositions

p: You get an A on the final exam.

q: You do every exercise in this book.

r: You get an A in this class.

Write these propositions using p, q, and r and logical connectives (including negations).

(a) You get an A in this class, but you do not do every exercise in this book.

(f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Solution

(a) $r \wedge \neg q$

(f) $r \leftrightarrow (q \vee p)$

20. Determine whether each of these conditional statements is true or false.

(a) If $1 + 1 = 3$, then unicorns exist.

(b) If $1 + 1 = 3$, then dogs can fly.

Solution

(a) This is $F \to F$, which is True

(b) This is $F \to F$, which is True

34. Determine whether each of these conditional statements is true or false.

(f) Construct a truth table for each of these compound propositions.

Solution

(f)

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
0	0	1
0	1	1
1	0	1
1	1	1

Chapter 1.3

12. Show that each of these conditional statements is a tautology by using truth tables.

(a) $[\neg p \wedge (p \vee q)] \to q$

Solution

(a) We construct a truth table for each conditional statement and note that the relevant column contains only T's. For part (a) we have the following table.

p	q	$\neg p$	$p \vee q$	$\neg p \wedge (p \vee q)$	$(\neg p \wedge (p \vee q)) \to q$
0	0	1	0	0	1
0	1	1	1	1	1
1	0	0	1	0	1
1	1	0	1	0	1

32. Show that $p \leftrightarrow q$ and $\neg p \leftrightarrow \neg q$ are logically equivalent.

Solution

We know that $p \leftrightarrow q$ is true precisely when p and q have the same truth value. But this happens precisely when $\neg p$ and $\neg q$ have the same truth value, that is, $\neg p \leftrightarrow \neg q$.

Chapter 1.4

18. Suppose that the domain of the propositional function P(x) consists of the integers $-2, -1, 0, 1,$ and 2. Write out each of these propositions using disjunctions, con- junctions, and negations.

(a) $\exists x P(x)$

(f) $\neg \forall x P(x)$

Solution

Existential quantifiers are like disjunctions, and universal quantifiers are like conjunctions.

(a) We want to assert that P(x) is true for some x in the domain, so either $P(-2)$ is true or $P(-1)$ is true or $P(0)$ is true or $P(1)$ is true or $P(2)$ is true. Thus the answer is $P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2)$. The other parts of this exercise are similar.

(f) $\neg(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

24. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

(a) Every one in your class has a cellular phone.

(e) Some student in your class does not want to be rich.

Solution

In order to do the translation the second way, we let $C(x)$ be the propositional function "x is in your class." Note that for the second way, we always want to use conditional statements with universal quantifiers and conjunctions with existential quantifiers.

(a) Let $P(x)$ be "x has a cellular phone." Then we have $\forall x P(x)$ the first way, or $\forall x(C(x) \rightarrow P(x))$ the second way.

(e) Let $R(x)$ be "x wants to be rich." Then we have $\exists x \neg R(x)$ the first way, or $\exists x(C(x) \wedge \neg R(x))$ the second way.

48. Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.

(a) $(\forall x P(x)) \vee A \equiv \forall x(P(x) \vee A)$

(b) $(\exists x P(x)) \vee A \equiv \exists x(P(x) \vee A)$

Solution

(a) There are two cases. If A is true, then $(\forall x P(x)) \vee A$ is true, and since $P(x) \vee A$ is true for all x, $\forall x(P(x) \vee A)$ is also true. Thus both sides of the logical equivalence are true (hence equivalent). Now suppose that A is false. If $P(x)$ is true for all x, then the left-hand side is true. Furthermore, the right-hand side is also true (since $P(x) \vee A$ is true for all x). On the other hand, if $P(x)$ is false for some x, then both sides are false. Therefore again the two sides are logically equivalent.

(b) There are two cases. If A is true, then $(\exists x P(x)) \vee A$ is true, and since $P(x) \vee A$ is true for some (really all) x , $\exists x(P(x) \vee A)$ is also true. Thus both sides of the logical equivalence are true (hence equivalent). Now suppose that A is false. If $P(x)$ is true for at least one x, then the left-hand side is true. Furthermore, the right-hand side is also true (since $P(x) \vee A$ is true for that x). On the other hand, if $P(x)$ is false for all x , then both sides are false. Therefore again the two sides are logically equivalent.