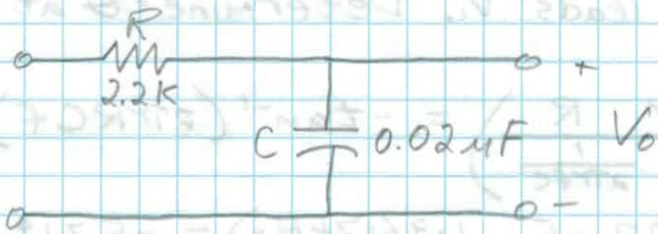
22-19 For the R-C low pass filter:



a) Sketch $A_v = V_o/V_i$ versus frequency using a log scale for the frequency axis. Determine $A_v = V_o/V_i$ at $.1f_c$, $0.5f_c$, f_c, $2f_c$, and $10f_c$

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \cdot 2.2k \cdot .02\mu F} = 3.617\,kHz$$

$$A_v = \frac{1}{\sqrt{\left(\frac{R}{X_c}\right)^2+1}} = \frac{1}{\sqrt{\left(\frac{R}{\frac{1}{2\pi fC}}\right)^2+1}} = \frac{1}{\sqrt{(2\pi RCf)^2+1}}$$

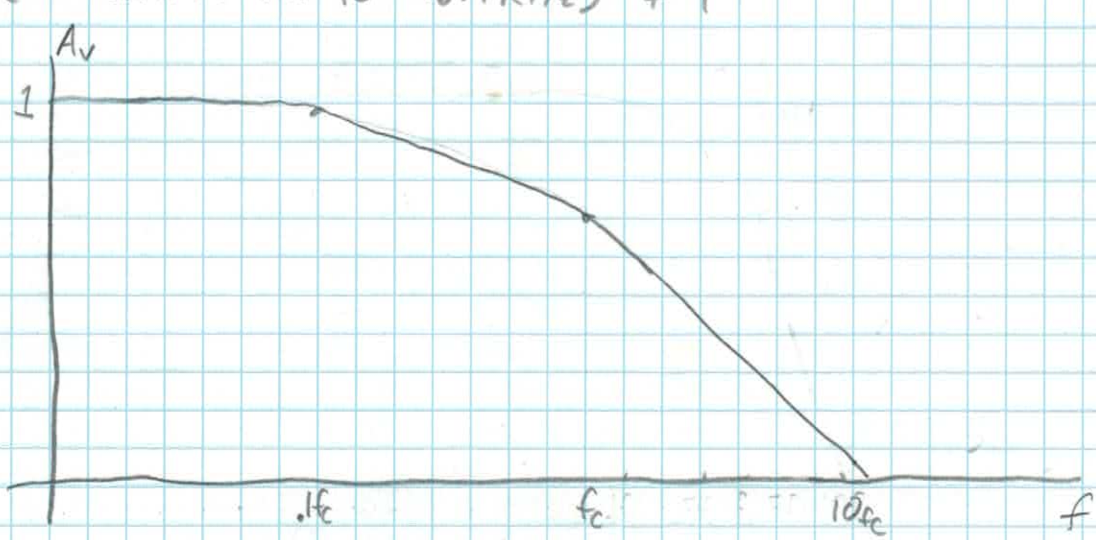
$.1f_c$: $A_v = \dfrac{1}{\sqrt{(2\pi \cdot 2.2k \cdot .02\mu F \cdot .1 \cdot 3.617kHz)^2+1}} = 0.995$

$.5f_c$: $A_v = \dfrac{1}{\sqrt{(2\pi \cdot 2.2k \cdot .02\mu F \cdot .5 \cdot 3.617kHz)^2+1}} = 0.894$

f_c: $A_v = \dfrac{1}{\sqrt{(2\pi \cdot 2.2k \cdot .02\mu F \cdot 1 \cdot 3.617kHz)^2+1}} = 0.707$

$2f_c$: $A_v = \dfrac{1}{\sqrt{(2\pi \cdot 2.2k \cdot .02\mu F \cdot 2 \cdot 3.617kHz)^2+1}} = 0.447$

$10f_c$: $A_v = \dfrac{1}{\sqrt{(2\pi \cdot 2.2k \cdot .02\mu F \cdot 10 \cdot 3.617kHz)^2+1}} = 0.0995$



b) Sketch the phase plot of θ versus frequency, where θ is the angle by which V_o leads V_i. Determine θ at $f = 0.1f_c$, $0.5f_c$, f_c, $2f_c$, and $10f_c$

$$\theta = -\tan^{-1}\left(\frac{R}{X_C}\right) = -\tan^{-1}\left(\frac{R}{\frac{1}{2\pi f C}}\right) = -\tan^{-1}(2\pi RCf)$$

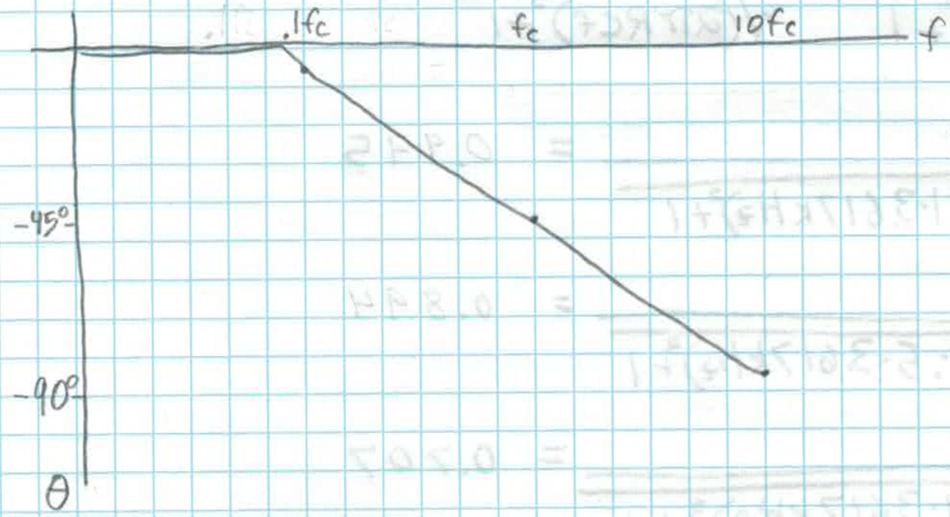
$.1f_c$: $\theta = -\tan^{-1}(2\pi \cdot 2.2k \cdot .02\,\mu F \cdot .1 \cdot 3.617\,kHz) = -5.71°$

$.5f_c$: $\theta = -\tan^{-1}(2\pi \cdot 2.2k \cdot .02\,\mu F \cdot .5 \cdot 3.617\,kHz) = -26.6°$

f_c: $\theta = -\tan^{-1}(2\pi \cdot 2.2k \cdot .02\,\mu F \cdot 1 \cdot 3.617\,kHz) = -45°$

$2f_c$: $\theta = -\tan^{-1}(2\pi \cdot 2.2k \cdot .02\,\mu F \cdot 2 \cdot 3.617\,kHz) = -78.7°$

$10f_c$: $\theta = -\tan^{-1}(2\pi \cdot 2.2k \cdot .02\,\mu F \cdot 10 \cdot 3.617\,kHz) = -84.3°$



Problem 22-20. For the network in Fig. 25.106:

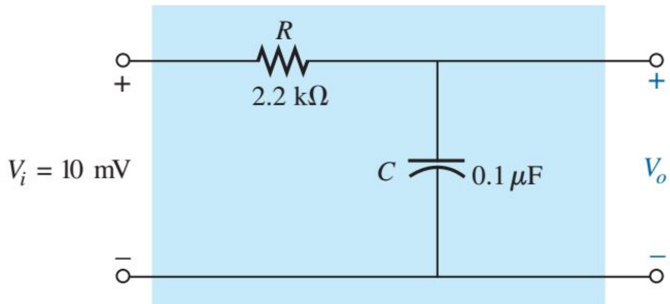


FIG. 22.106
Problem 20.

(a) Determine V_o at a frequency one octave above the critical frequency.
The critical frequency f_c for the circuit above is given by

$$\begin{aligned} f_c &= \frac{1}{2\pi RC} \\ &= \frac{1}{(2\pi)(2.2\,\mathrm{k\Omega})(0.1\,\mathrm{\mu F})} \\ &= 723.43\,\mathrm{Hz} \end{aligned}$$

One octave above the critical frequency is

$$\begin{aligned} 2f_c &= 2 \cdot \frac{1}{2\pi RC} \\ &= \frac{2}{(2\pi)(2.2\,\mathrm{k\Omega})(0.1\,\mathrm{\mu F})} \\ &= 1.45\,\mathrm{kHz} \end{aligned}$$

The reactance at one octave above the critical frequency is

$$\begin{aligned} X_c &= \frac{1}{(2\pi)(2f)(C)} \\ &= \frac{1}{(2\pi)(1.45\,\mathrm{kHz})(0.1\,\mathrm{\mu F})} = 1.1\,\mathrm{k\Omega} \end{aligned}$$

The gain at one octave above the critical frequency is

$$\begin{aligned} A_v &= \frac{V_o}{V_i} = \frac{X_c}{\sqrt{(R)^2 + (X_c)^2}} \\ &= \frac{1.1\,\mathrm{k\Omega}}{\sqrt{(2.2\,\mathrm{k\Omega})^2 + (1.1\,\mathrm{k\Omega})^2}} = 447.2 \cdot 10^{-3} \end{aligned}$$

Therefore the output voltage at one octave above the critical frequency is

$$
\begin{aligned}
A_v \cdot V_i &= \frac{V_o \cdot V_i}{V_i} = \\
&= 447.2 \cdot 10^{-3} \cdot (10\,\mathrm{mV}) \\
&= \boxed{(4.47\,\mathrm{mV})}
\end{aligned}
$$

(b) Determine V_o at a frequency one decade below the critical frequency.
The frequency at one decade below the critical frequency is

$$
\begin{aligned}
f &= \frac{1}{10} \cdot f_c \\
&= \frac{1}{10} \cdot 723.43\,\mathrm{Hz} = 72.34\,\mathrm{Hz}
\end{aligned}
$$

The reactance at one decade below the critical frequency is

$$
\begin{aligned}
X_c &= \frac{1}{(2\pi)(f)(C)} \\
&= \frac{1}{(2\pi)(723.43\,\mathrm{Hz})(0.1\,\mu\mathrm{F})} = 22\,\mathrm{k}\Omega
\end{aligned}
$$

The gain at one decade below the critical frequency is

$$
\begin{aligned}
A_v &= \frac{V_o}{V_i} = \frac{X_c}{\sqrt{(R)^2 + (X_c)^2}} \\
&= \frac{22\,\mathrm{k}\Omega}{\sqrt{(2.2\,\mathrm{k}\Omega)^2 + (22\,\mathrm{k}\Omega)^2}} = 0.995
\end{aligned}
$$

Therefore the output voltage at one octave above the critical frequency is

$$
\begin{aligned}
A_v \cdot V_i &= \frac{V_o \cdot V_i}{V_i} = \\
&= 0.995 \cdot (10\,\mathrm{mV}) \\
&= \boxed{(9.95\,\mathrm{mV})}
\end{aligned}
$$

(c) Do the levels of parts (a) and (b) verify the expected frequency plot of V_o versus frequency for the filter?

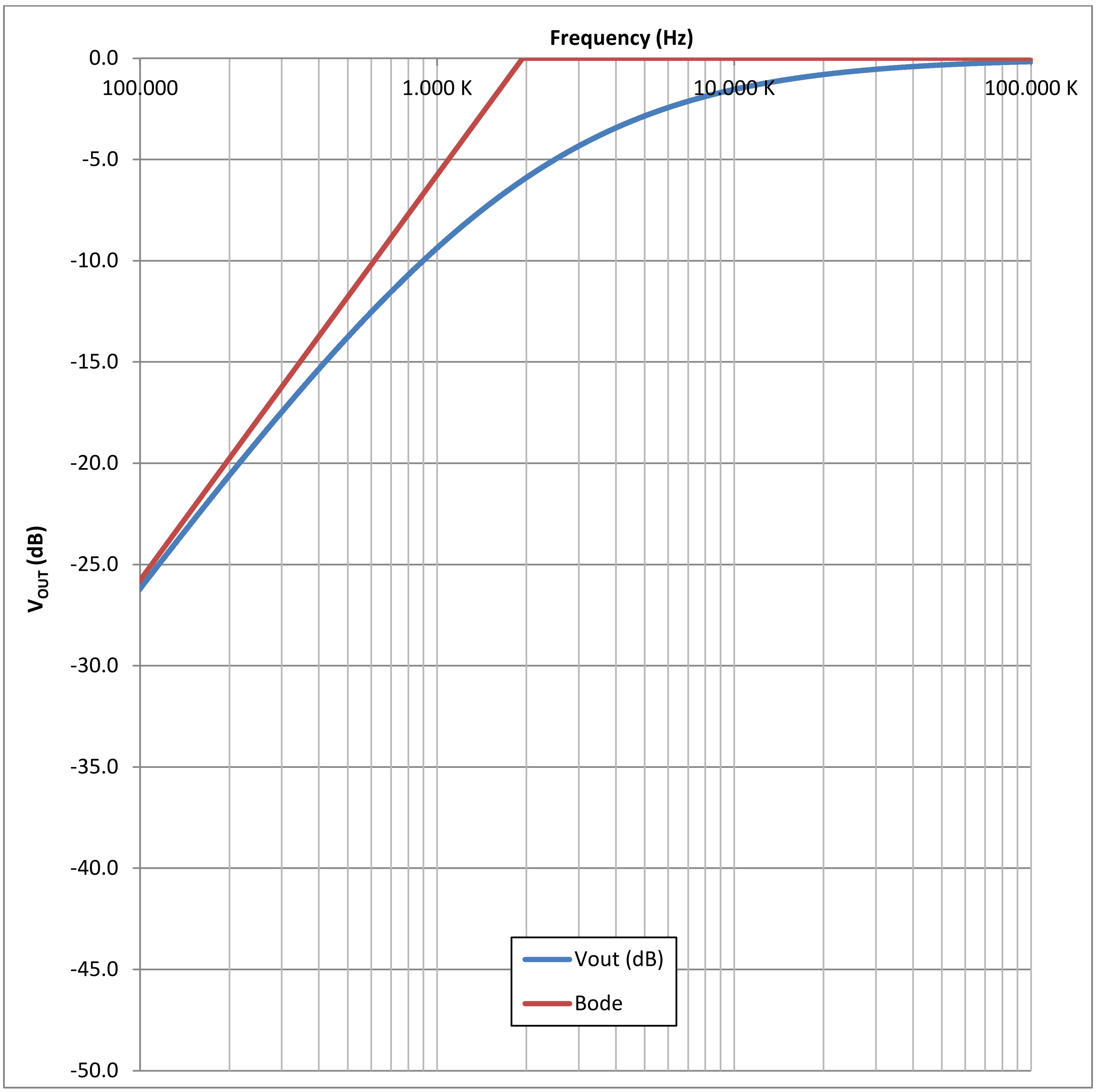
Frequency	**Calculated $V_o(f)$**	**Verification**
at $f = 2f_c$	4.47 mV	much lower
at $f = \frac{f_c}{10}$	9.95 mV	much higher

25) Design a high-pass RC filter to have a cutoff or corner frequency of 2 kHz, given a capacitor of 0.1μF. Choose the closest commercial value for R, and then recalculate the resulting corner frequency. Sketch the normalized gain $A_v = \frac{V_o}{V_i}$ for a frequency range of $0.1 f_c$ to $10 f_c$.

$$X_C = \frac{1}{(2 * \pi * f_c * C)} = 796\ \Omega$$

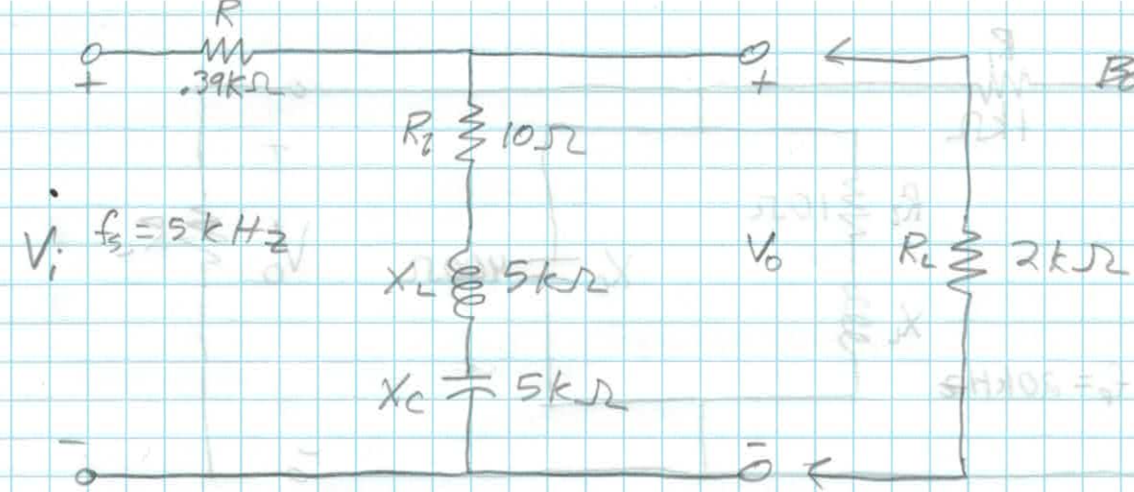
$$R = 820\ \Omega$$

$$f_C = \frac{1}{(2 * \pi * R * C)} = 1.94\ kHz$$



22-31

Band-Stop Filter



a) Determine Q_s $\quad Q_s = \dfrac{X_L}{R + R_2} = \dfrac{5\,k\Omega}{400\,\Omega} = 12.5$

b) Find the bandwidth and half-power frequencies

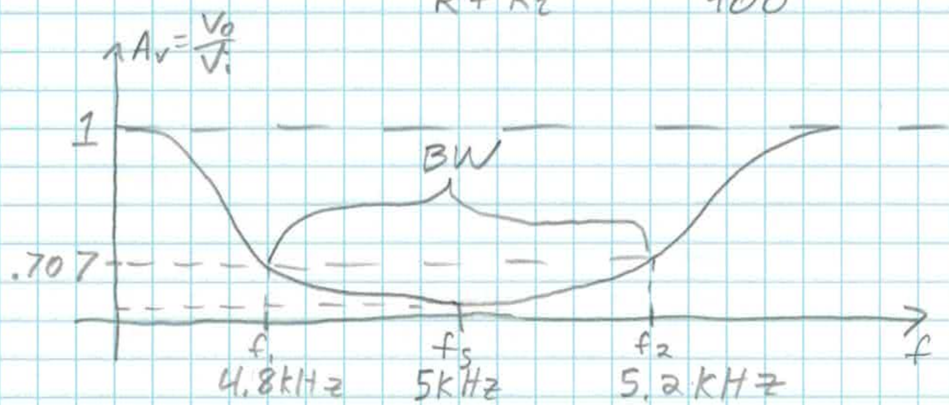
$BW = \dfrac{f_s}{Q} = \dfrac{5\,kHz}{12.5} = 400\,Hz$

$f_1 = f_s - \tfrac{1}{2}BW = 5\,kHz - 200\,Hz = 4.8\,kHz$

$f_2 = f_s + \tfrac{1}{2}BW = 5\,kHz + 200\,Hz = 5.2\,kHz$

c) Sketch the frequency characteristics of $A_v = \dfrac{V_o}{V_i}$

At resonance, $V_o = \dfrac{R_2 \cdot V_i}{R + R_2} = \dfrac{V_i\,10}{400} = .025\,V_i$



d) What is the effect on the curve of part c if a load of 2kΩ is applied?

$V_o = \dfrac{10 /\!/ 2k\ V_{in}}{10/\!/2k + 390} = .024878\,V_{in} \cong .025\,V_{in}$ as above

Problem 22-32. For the band-pass filter in Fig. 22.115:

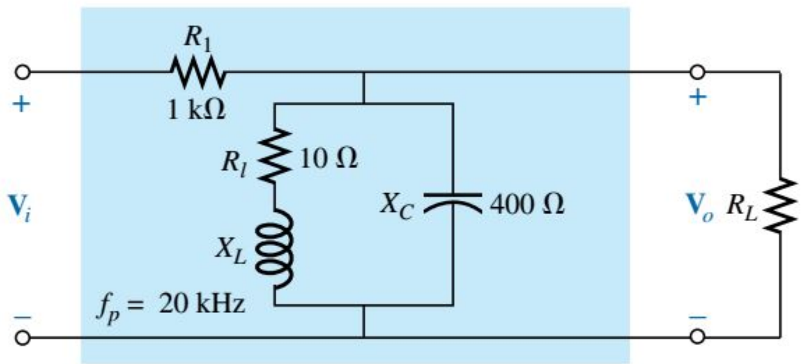


FIG. 22.115
Problem 32.

(a) Determine Q_p ($R_L = \infty\ \Omega$, an open circuit).

$$Q_l = \frac{X_L}{R_l} = \frac{400\,\Omega}{10\,\Omega} = 40$$

$$\begin{aligned} Z_{TP} &= R_s || {Q_l}^2 R_l \\ &= \frac{R_s \cdot ({Q_l}^2 R_l)}{R_s + ({Q_l}^2 R_l)} \\ &= \frac{1000 \cdot (40^2 \cdot 10)}{1000 + (40^2 \cdot 10)} = 941.18\,\Omega \end{aligned}$$

$$Q_p = \frac{|Z_{TP}|}{X_L} = \frac{941.18}{400} = \boxed{2.35}$$

(b) Sketch the frequency of $A_v = V_o/V_i$. The bandwidth is given by

$$BW = \frac{f_p}{Q_p} = \frac{20\,\text{kHz}}{2.33} = 8.5\,\text{kHz}$$

At resonance where $f_s = f_p = 20\,\text{kHz}$, the gain is given by

$$\begin{aligned} A_v &= \frac{({Q_l}^2 R_l)}{R_s + ({Q_l}^2 R_l)} \\ &= \frac{(40^2 \cdot 10)}{1000 + (40^2 \cdot 10)} = 0.94 \end{aligned}$$

see week 12, session 2 for the profile of the plot.

(c) Find Q_p (loaded) for $R_L = (100\,\text{k}\Omega)$, and indicate the effect of R_L on the characteristic of part(b).
Given that $Q_p > 10$, at resonance,

$$\begin{aligned} Z_{TP} &= (R_s || {Q_l}^2 R_l) || R_L \\ &= 941.18\,\Omega || 100\,\text{k}\Omega \\ &= \frac{941.18\,\Omega \cdot (100\,\text{k}\Omega)}{941.18\,\Omega + (100\,\text{k}\Omega)} = 932.4\,\Omega \end{aligned}$$

$$\begin{aligned} Q_p &= \frac{|Z_{TP}|}{X_L} \\ &= \frac{932.4}{400} = \boxed{2.33} \end{aligned}$$

$$\begin{aligned} BW &= \frac{f_p}{Q_p} \\ &= \frac{20\,\text{kHz}}{2.33} = \boxed{8.58\,\text{kHz}} \end{aligned}$$

(d) Repeat part (C) for $R_L = (20\,\text{k}\Omega)$.

$$\begin{aligned} Z_{TP} &= (R_s || {Q_l}^2 R_l) || R_L \\ &= 941.18\,\Omega || 20\,\text{k}\Omega \\ &= \frac{941.18\,\Omega \cdot (20\,\text{k}\Omega)}{941.18\,\Omega + (20\,\text{k}\Omega)} = 898.9\,\Omega \end{aligned}$$

$$\begin{aligned} Q_p &= \frac{|Z_{TP}|}{X_L} \\ &= \frac{898.9}{400} = \boxed{2.25} \end{aligned}$$

$$\begin{aligned} BW &= \frac{f_p}{Q_p} \\ &= \frac{20\,\text{kHz}}{2.25} = \boxed{8.9\,\text{kHz}} \end{aligned}$$

57) Using schematics, obtain the magnitude and phase response versus frequency for the network in Fig. 22.110.

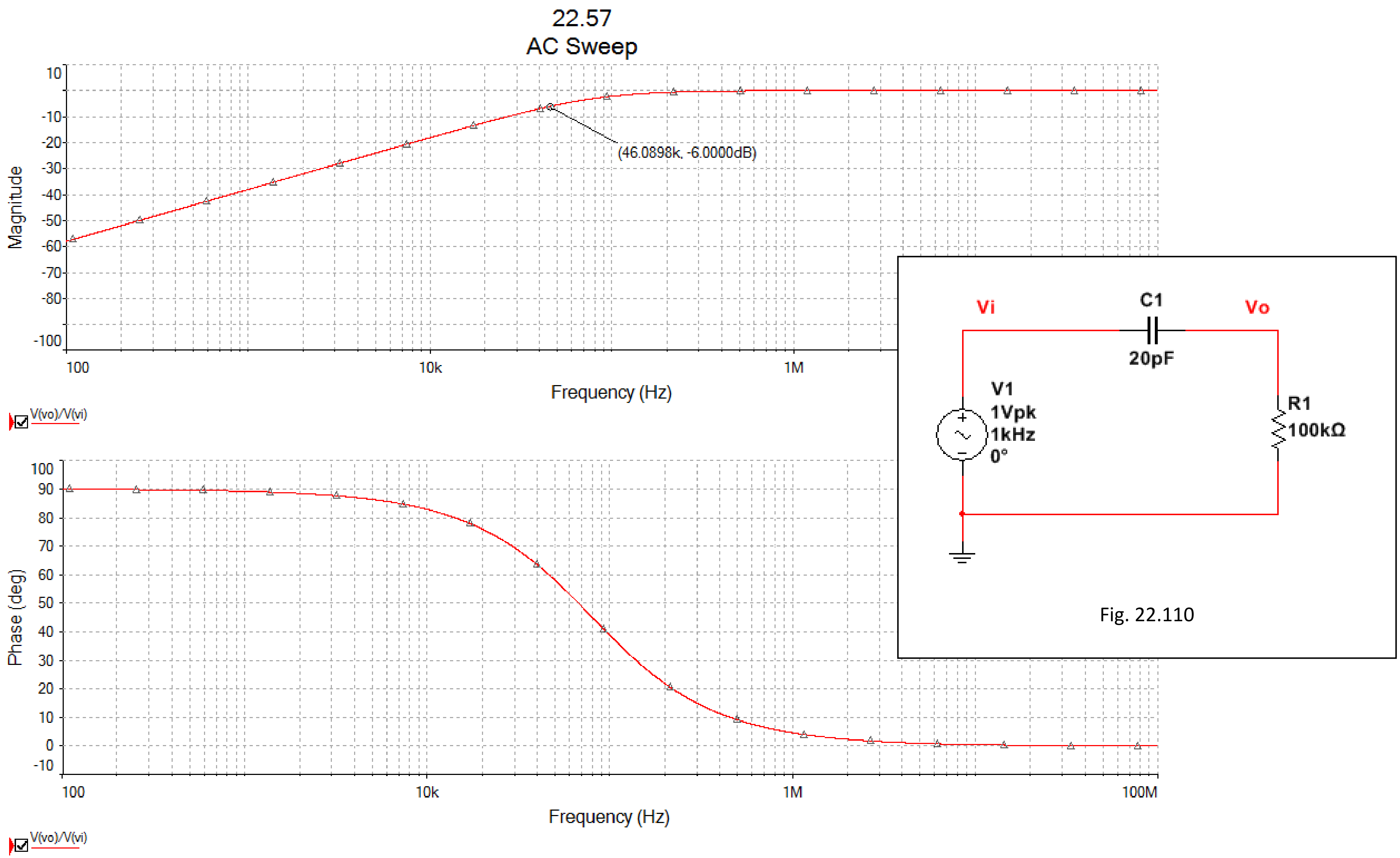
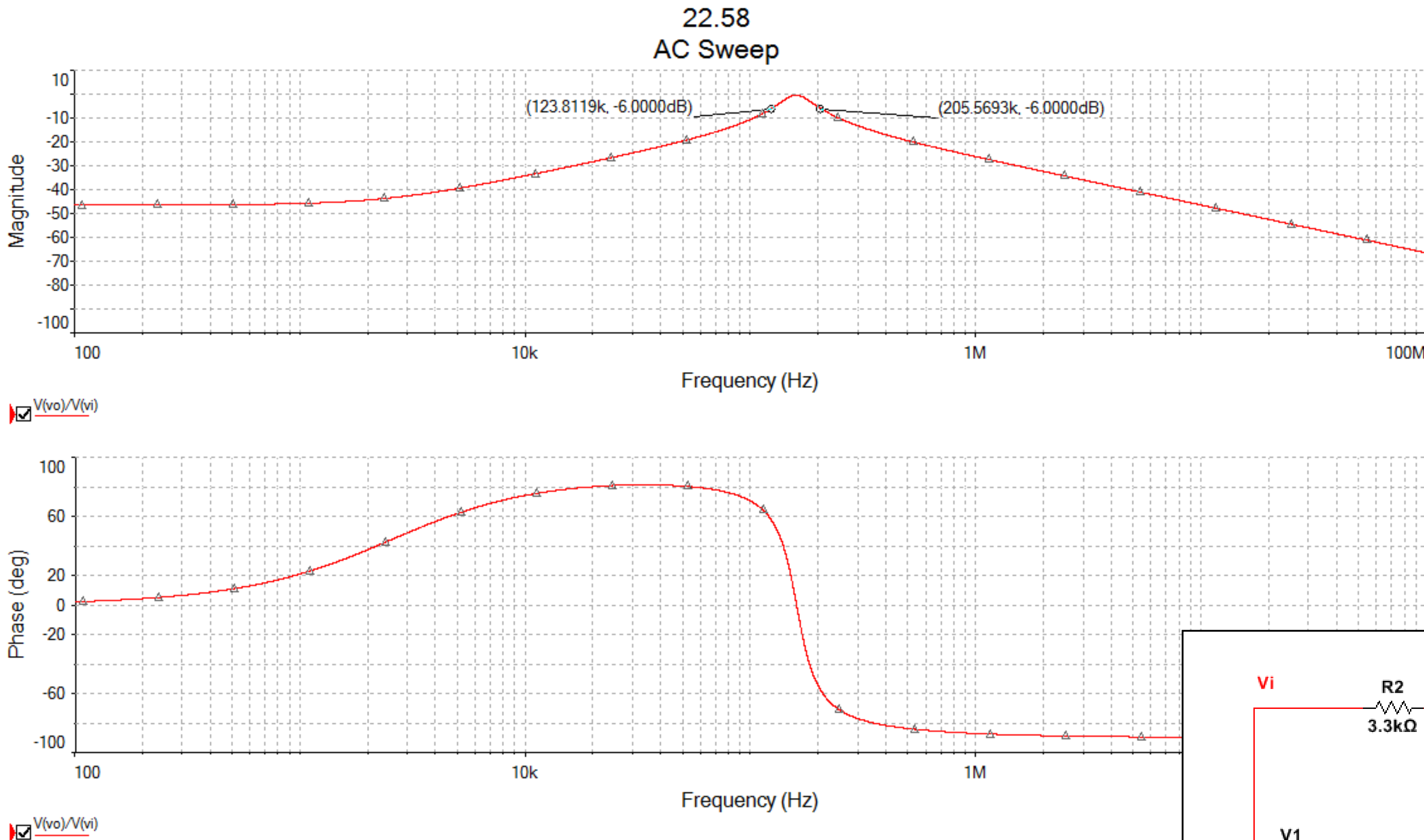


Fig. 22.110

58) Using schematics, obtain the magnitude and phase response versus frequency for the network in Fig. 22.113.



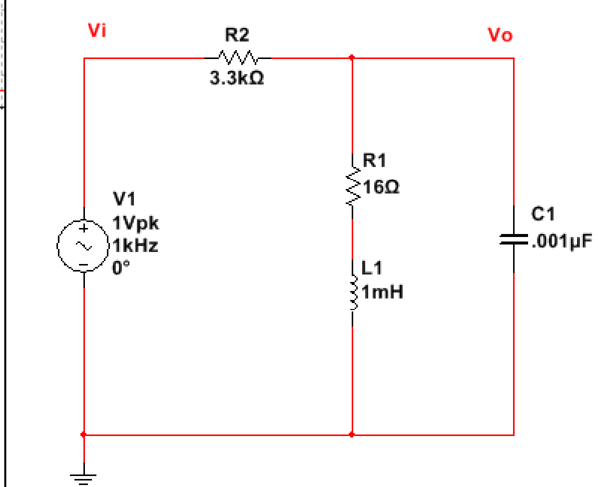


Fig. 22.113