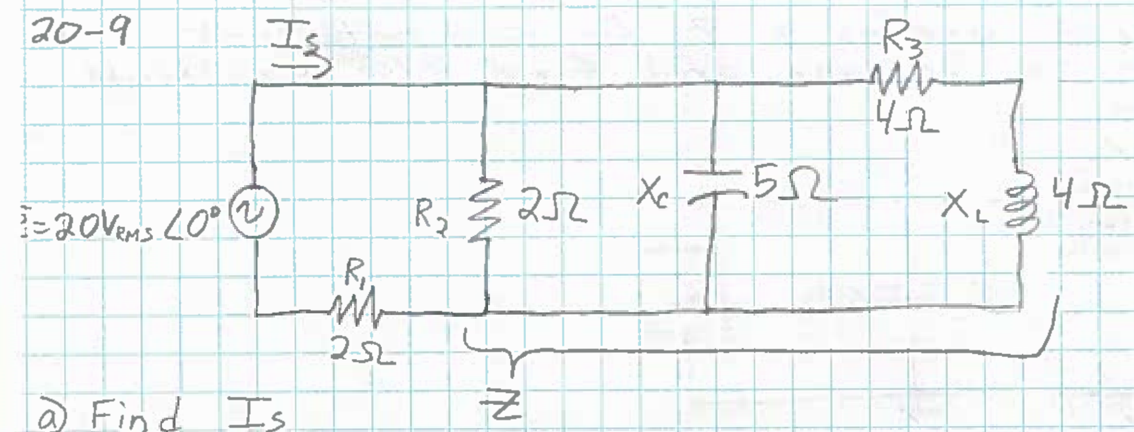
20-9



a) Find I_S

$$Z_T = 2\Omega\angle 0 + 2\Omega\angle 0 // 5\Omega\angle{-90} // (4+j4)$$

$$= 2\Omega\angle 0 + 1.589\angle{-6.843°} = 3.582\Omega\angle{-3.029}$$

$$I_S = \frac{E}{Z_T} = \frac{20V_{RMS}\angle 0°}{3.582\Omega\angle{-3.029}} = \boxed{5.58A_{RMS}\angle 3.03°}$$

b) Find the average power delivered to each element

$$P_{R1} = I_S^2 R_1 = 5.58A_{RMS}^2 \cdot 2\Omega = \boxed{62.27\,W}$$

$$V_{R2} = E - V_{R1}$$
$$= E - I_S R_1 = 20V\angle 0 - 5.58A\angle 3.03 \cdot 2\Omega\angle 0$$
$$= 20V\angle 0 - 11.16\angle 3.03$$
$$= 8.88V_{RMS}\angle{-3.81}$$

$$P_{R2} = \frac{V_{R2}^2}{R_2} = \frac{8.88V_{RMS}^2}{2\Omega} = \boxed{39.43\,W}$$

$$I_{R3} = \frac{8.88V_{RMS}\angle{-3.81}}{4+j4} = 1.57A_{RMS}\angle{-48.81}$$

$$P_{R3} = I_{R3}^2 \cdot R_3 = 1.57A_{RMS}^2 \cdot 4\Omega = \boxed{9.86\,W}$$

c) Find the reactive power for each element

$$Q_{XC} = \frac{V_{R2}^2}{X_C} = \frac{8.88V_{RMS}^2}{5\Omega} = \boxed{15.771\,VAR(C)}$$

$$Q_{XL} = I_{R3}^2 \cdot X_L = 1.57A_{RMS}^2 \cdot 4\Omega = \boxed{9.86\,VAR(L)}$$

d) Find the apparent power for each element

R_1: $S_T = P_{R1} =$ 62.27 VA
R_2: $S_T = P_{R2} =$ 39.43 VA
R_3: $S_T = P_{R3} =$ 9.86 VA
C: $S_T = Q_{XC} =$ 15.771 VA
L: $S_T = Q_{XL} =$ 9.86 VA

e) Find P_T, Q_T, S_T, and F_p for the system

$P_T = P_{R1} + P_{R2} + P_{R3} = 62.27\,W + 39.43\,W + 9.86\,W$

$\boxed{= 111.56\,W}$

$Q_T = Q_{XC} - Q_{XL} = 15.771\,VAR(C) - 9.86\,VAR(L)$

$= \boxed{5.911\ VAR(C)}$

$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{111.56^2 + 5.911^2} = \boxed{111.716\ VA}$

$F_p = \dfrac{P_T}{S_T} = \dfrac{111.56}{111.716} = \boxed{0.999\ \text{leading}}$

f) Sketch the power triangle

20-12 An electrical system is rated 10kVA, 200V at a 0.5 leading power factor.

a) Find the impedance of the system in rectangular coordinates

$$I_S = \frac{S_T}{E} = \frac{10k\,VA}{200V_{RMS}} = 50A_{RMS}$$

$F_p = 0.5$

$\cos\theta = 0.5$

$\theta = \cos^{-1} 0.5 = 60°$ I_S leads E by $60°$

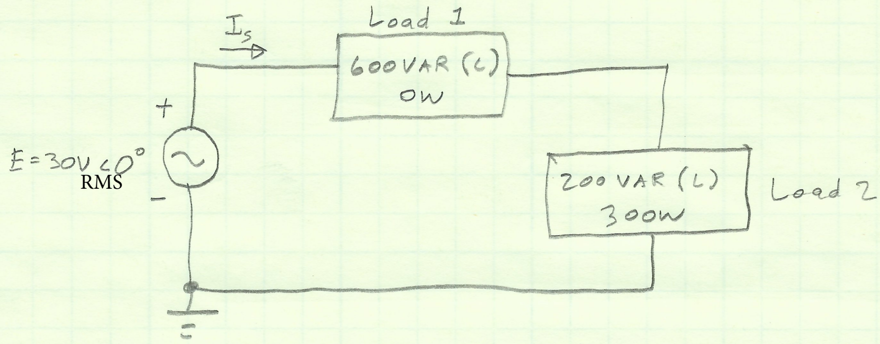
$\therefore I_S = 50A_{RMS} \angle 60°$

$$Z_T = \frac{E}{I_S} = \frac{200V\angle 0}{50A\angle 60} = \boxed{2 - j\,3.464} \text{ Ohms}$$

b) Find the verage power delivered to the system

$$F_P = \frac{P_T}{S_T} \Rightarrow P_T = F_P S_T = 0.5 \cdot 10kVA = \boxed{5kW}$$

(14) For the system:



a) Find P_T, Q_T, S_T, and F_p

$$P_T = 0W + 300W = \boxed{300\text{ W}}$$

$$Q_T = 600\text{ VAR}(c) + 200\text{ VAR}(L) = \boxed{400\text{ VAR}(c)}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \boxed{500\text{ VA}}$$

$$F_p = \frac{P_T}{S_T} = \frac{300\text{ W}}{500\text{ VA}} = \boxed{0.6\text{ leading}}$$

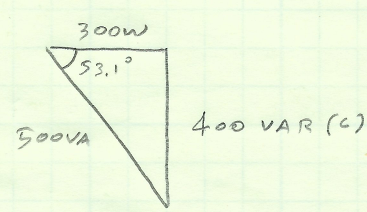
b) Find I_s

$$E \cdot I_s \cdot F_p = P_T \qquad |I_s| = \frac{P_T}{E \cdot F_p} = \frac{300\text{ W}}{(30V_{RMS})(0.6)} = 16.7\text{ A}$$

$$\theta_I = \theta_E + \cos^{-1}(F_p) = 0° + \cos^{-1}(0.6) = 53.1°$$

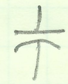
$$I_s = \boxed{16.7\,A_{RMS} \angle 53.1°}$$

c) Draw the power triangle



d) Find the types of elements and their impedance within each load

Load 1

600 VAR (C) $\rightarrow$ 

$$Z_C = \frac{Q}{I_s^2} = \frac{600\ \text{VAR}}{(16.67\ A_{RMS})^2} = \boxed{2.16\,\Omega \angle -90^\circ}$$

Load 2

200 VAR (L) $\rightarrow$ 

300W $\rightarrow$ 

$$Z_L = \frac{Q}{I_s^2} = \frac{200\ \text{VAR}}{(16.67\ A_{RMS})^2} = \boxed{0.7197\,\Omega \angle 90^\circ}$$

$$R = \frac{P}{I_s^2} = \frac{300\ W}{(16.67 A)^2_{RMS}} = \boxed{1.079\,\Omega \angle 0^\circ}$$

e) Verify that the result of part b is correct by calculating I_s using only E and the values from part d.

$$Z_T = (2.16\,\Omega \angle -90^\circ) + (0.7197\,\Omega \angle 90^\circ) + (1.079\,\Omega \angle 0^\circ) = (1.8\,\Omega \angle -53.2^\circ)$$

$$I_s = \frac{E}{Z_T} = \frac{(30\ V_{RMS} \angle 0^\circ)}{(1.8\,\Omega \angle -53.2^\circ)} = \boxed{(16.67\ A_{RMS} \angle 53.2^\circ)}$$

(19) The load on a 120 V, 60 Hz supply is 5 kW (resistive), 8 kVAR (inductive), and 2 kVAR (capacitive).

a) Find S_T

$$Q_T = 8\text{ kVAR}(L) + 2\text{ kVAR}(C) = 6\text{ kVAR}(L)$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{(5\text{ kW})^2 + (6\text{ kVAR})^2} = \boxed{7.81\text{ kVA}}$$

b) Find F_p

$$F_p = \frac{P_T}{S_T} = \frac{5\text{ kW}}{7.81\text{ kVA}} = \boxed{0.64\text{ lagging}}$$

c) Find I_T

$$|I_T| = \frac{P_T}{E \cdot F_p} = \frac{5\text{ kW}}{(120\text{ V})(0.64)} = 65\text{ A}_{RMS}$$

$$\theta_I = \theta_E - \cos^{-1}(F_p) = 0^\circ - \cos^{-1}(0.64) = 50.2^\circ$$

$$\boxed{I_T = (65\text{ A}_{RMS} \angle -50.2^\circ)}$$

d. $X_C = \dfrac{1}{2\pi fC}$, $Q_C = I^2 X_C = \dfrac{E^2}{X_C} = \dfrac{(120\text{ V}_{RMS})^2}{X_C}$

and $X_C = \dfrac{(120\text{ V}_{RMS})^2}{Q_C} = \dfrac{14{,}400}{6000} = 2.4\ \Omega$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{(2\pi)(60\text{ Hz})(2.4\ \Omega)} = \mathbf{1105\ \mu F}$$

e) Calculate I_T at unity power factor

$$S_T^* = \sqrt{P_T^2 + 0} = 5\text{ kVA}$$

$$I_T^* = \frac{S_T^*}{E} = \frac{5\text{ kVA}}{120\text{ V}} = \boxed{41.67\text{ A}}_{RMS}$$

21-1 Find the resonant ω_s and f_s for the series circuit with the following parameters:

a) $R = 10\Omega$ $L = 1\,H$ $C = 16\,\mu F$

$$\omega_s = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1H \cdot 16\mu F}} = \boxed{250\ rad/s}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1H \cdot 16\mu F}} = \boxed{39.789\ Hz}$$

b) $R = 300\Omega$ $L = 0.51\,H$ $C = 0.16\,\mu F$

$$\omega_s = \frac{1}{\sqrt{.51H \cdot .16\mu F}} = \boxed{3501\ rad/s}$$

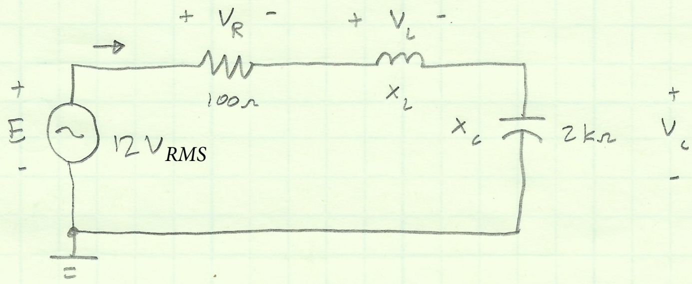
$$f_s = \frac{1}{2\pi\sqrt{.51H \cdot .16\mu F}} = \boxed{557.154\ Hz}$$

c) $R = 20\Omega$ $L = .27\,mH$ $C = 7.5\,\mu F$

$$\omega_s = \frac{1}{\sqrt{270H \cdot 7.5\mu F}} = \boxed{22{,}222\ rad/s}$$

$$f_s = \frac{1}{2\pi\sqrt{270H \cdot 7.5\mu F}} = \boxed{3{,}537\ Hz}$$

③ For the circuit:



a) Find the value of X_L for resonance

$$\boxed{X_L = 2k\Omega \angle 90^\circ}$$

b) Calculate I at resonance

$$Z_T = (100\Omega \angle 0^\circ) + (2k\Omega \angle 90^\circ) + (2k\Omega \angle -90^\circ) = 100\Omega \angle 0^\circ$$

$$I = \frac{E}{Z_T} = \frac{12\,V_{RMS} \angle 0^\circ}{100\Omega \angle 0^\circ} = \boxed{120\,mA_{RMS} \angle 0^\circ}$$

c) Calculate V_R, V_L, V_C at resonance

$$V_R = Z_R \cdot I = (100\Omega \angle 0^\circ)(120\,mA \angle 0^\circ) = \boxed{12\,V_{RMS} \angle 0^\circ}$$

$$V_L = Z_L \cdot I = (2k\Omega \angle 90^\circ)(120\,mA \angle 0^\circ) = \boxed{240\,V_{RMS} \angle 90^\circ}$$

$$V_C = Z_C \cdot I = (2k\Omega \angle -90^\circ)(120\,mA \angle 0^\circ) = \boxed{240\,V_{RMS} \angle -90^\circ}$$

d) Determine the Q-factor of the circuit

$$Q = \frac{X_L}{R} = \frac{2k\Omega}{100\Omega} = \boxed{20}$$

e) If the frequency is 5 kHz, determine the values of L and C

$$\omega = 2\pi f = 2\pi(5\text{ kHz}) = 31.42\ \frac{\text{kr}}{\text{s}}$$

$$X_L = 2\text{ k}\Omega = \omega L \qquad L = \frac{X_L}{\omega} = \frac{2\text{ k}\Omega}{31.42\ \frac{\text{kr}}{\text{s}}} = \boxed{63.7\text{ mH}}$$

$$X_C = \frac{1}{\omega C} \qquad C = \frac{1}{\omega X_C} = \frac{1}{(2\text{ k}\Omega)(31.42\ \frac{\text{kr}}{\text{s}})} = \boxed{15.9\text{ nF}}$$

f) Find the bandwidth of the response if the resonant frequency is 5 kHz

$$BW = \frac{f_R}{Q} = \frac{5\text{ kHz}}{20} = \boxed{250\text{ Hz}}$$

g) What are the low and high cut-off frequencies?

$$f_L, f_H = f_R \pm \frac{BW}{2} = 5\text{ kHz} \pm 125\text{ Hz} = \boxed{4.875\text{ kHz},\ 5.125\text{ kHz}}$$