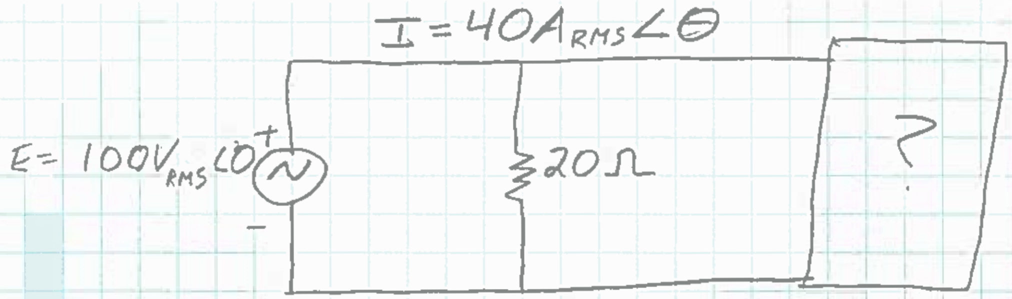
16-23



Find the element or elements that must be in the closed container to satisfy the following conditions (Find the simplest parallel circuit)

a. Average Power to the circuit = 3000 W
b. Circuit has a lagging power factor

$P = V_{RMS} I_{RMS} \cos\theta = (100V)(40A)\cos\theta = 3000\,W$

$\theta = \cos^{-1}\left(\frac{3000W}{100V \cdot 40A}\right)$

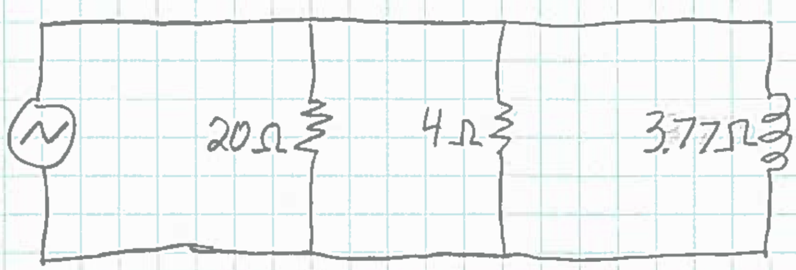
$= 41.41°$

Since parallel, use admittance:

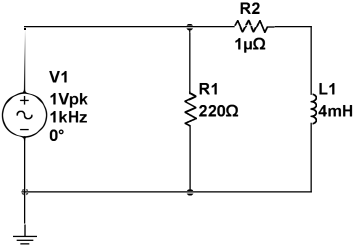
$Y = \frac{1}{Z_T} = \frac{I}{E} = \frac{40A_{RMS}\angle 41.41}{100\,V_{RMS}\angle 0} = .45\angle 41.41 = \underbrace{.3S}_{\text{Resistive}} \underbrace{-j.265S}_{\text{Inductive}}$

Resistive: $.3S = \frac{1}{R_T} = \frac{1}{20\Omega} + \frac{1}{R2}$ $\quad R2 = 4\,\Omega$

Inductive: $.265S = \frac{1}{X_L}$ $\quad X_L = 3.774\Omega$



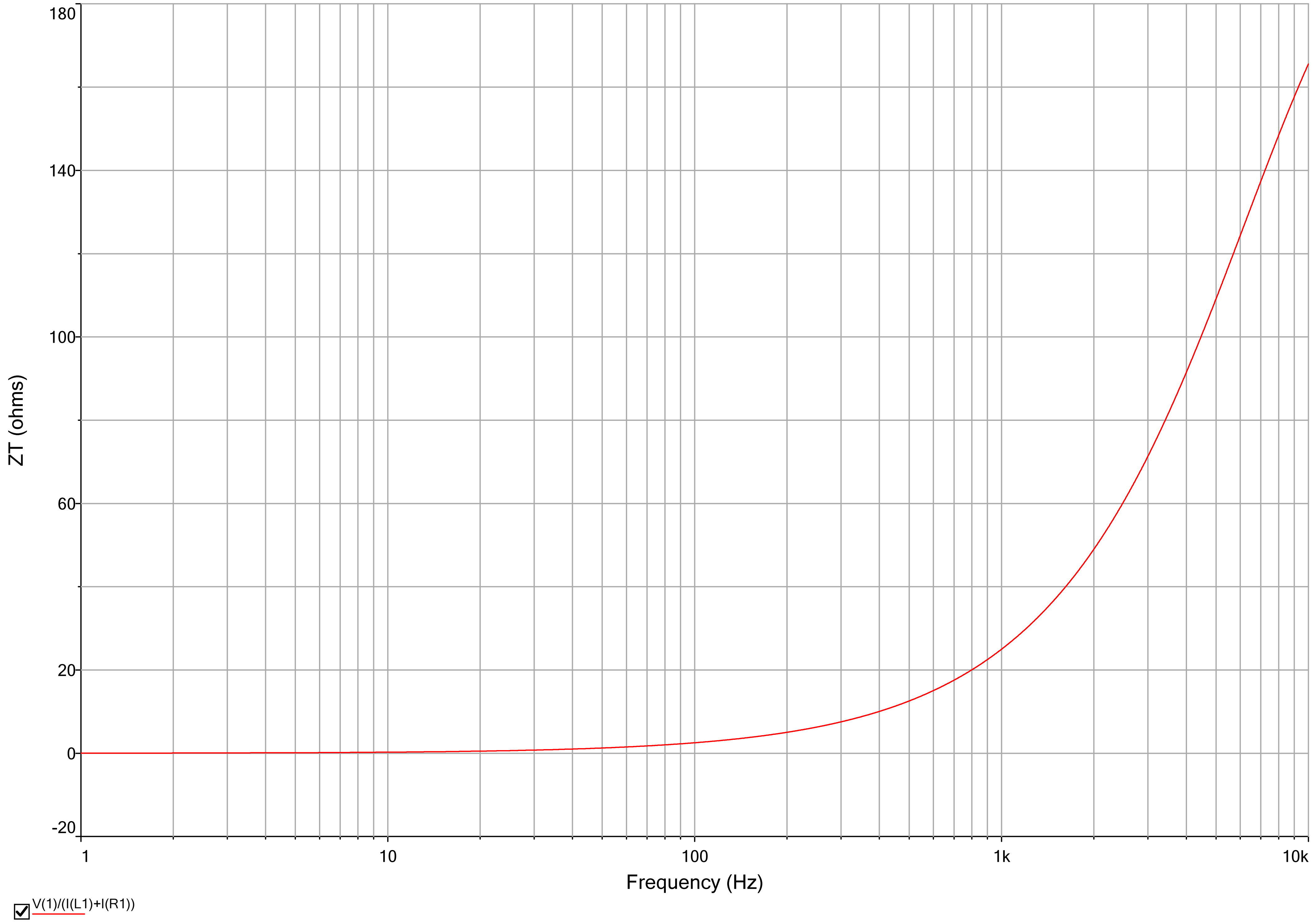
Question 16-26



AC Circuits HW

Printing Time:17 February, 2018, 20:29:25

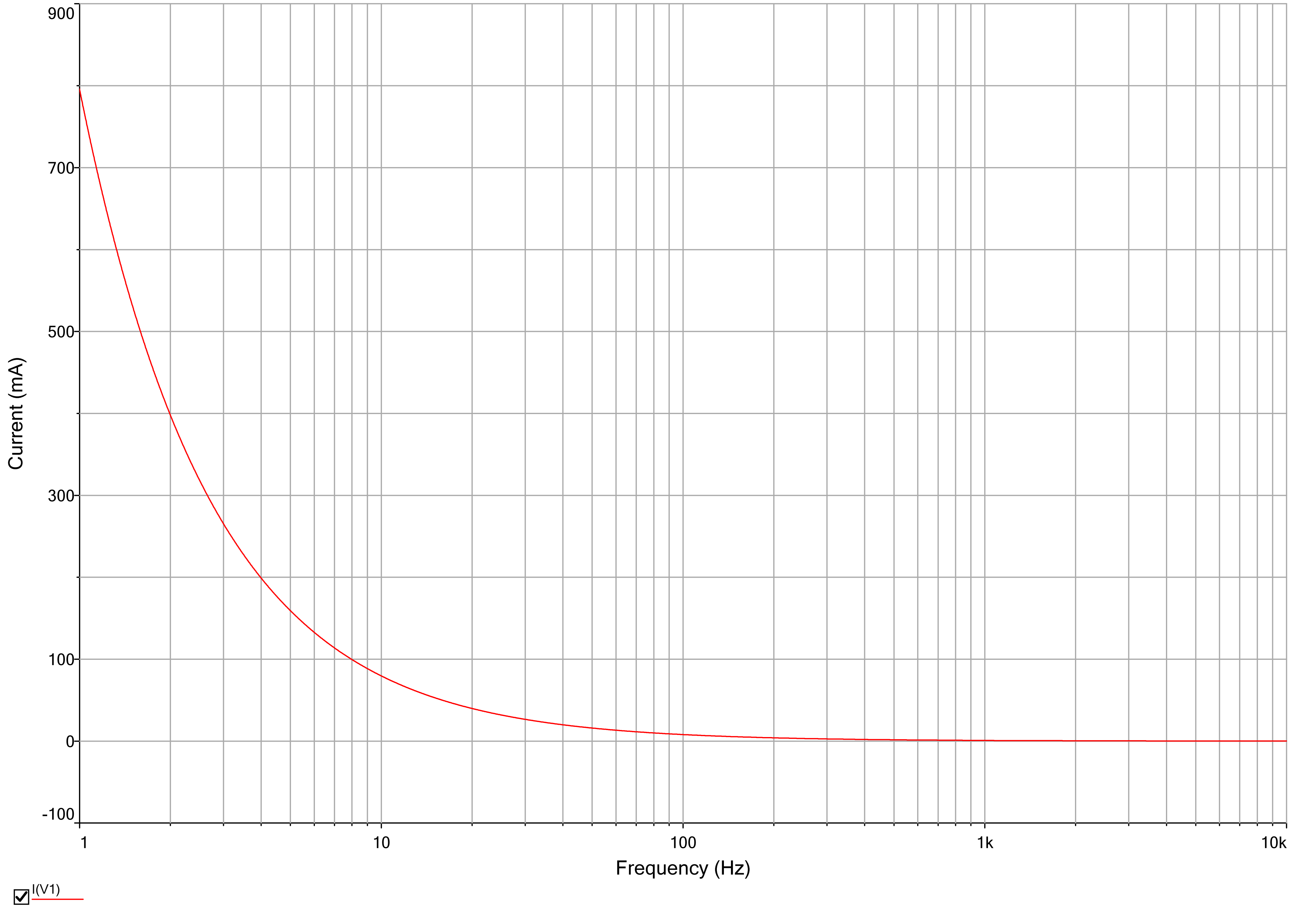
16.26 a - ZT vs Freq.



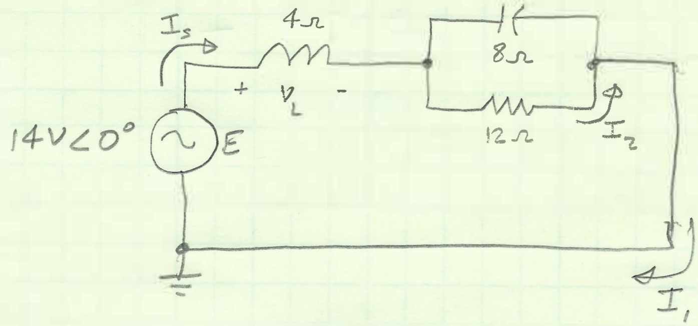
AC Circuits HW

Printing Time:17 February, 2018, 20:29:25

16.26 b - iT vs Freq.



17-1 For the series-parallel network:



a) Calculate Z_T

$$Z_T = (4\Omega\angle 90°) + \left[(8\Omega\angle -90°) \,//\, (12\Omega\angle 0°)\right] = \boxed{(4\Omega\angle -22.6)}$$

b) Determine I_s

$$I_s = \frac{E}{Z_T} = \frac{14V\angle 0°}{4\Omega\angle -22.6} = \boxed{(3.5A\angle 22.6°)}$$

c) Determine I_1

$$I_1 = I_s = \boxed{(3.5A\angle 22.6°)}$$

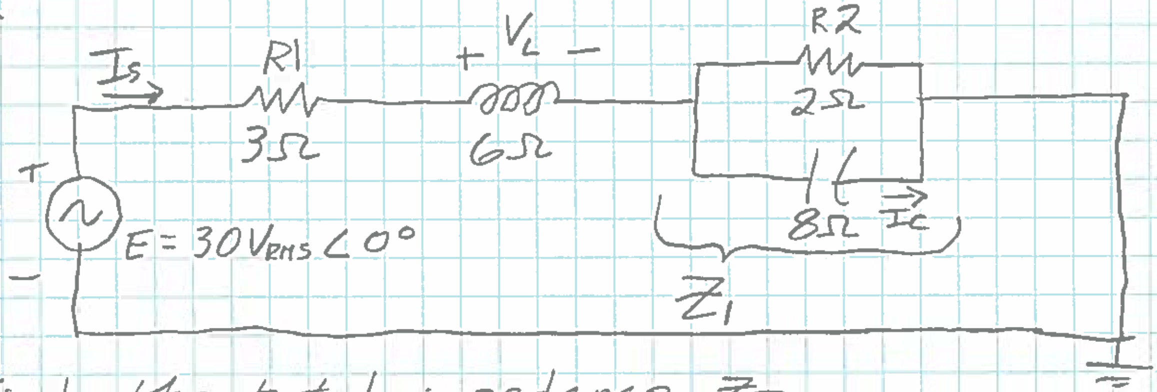
d) Find I_2

$$I_2 = I_s \frac{(8\Omega\angle -90°)}{(8\Omega\angle -90°)+(12\Omega\angle 0°)} = \boxed{(1.94A\angle -33.7°)}$$

e) Find V_L

$$V_L = X_L \cdot I_s = (4\Omega\angle 90°)(3.5A\angle 22.6°) = \boxed{(14V\angle 112.6°)}$$

17-2



a) Find the total impedance Z_T

$$Z_1 = 2\Omega \,//\, 8\Omega\angle 90 = \frac{2\angle 0 \cdot 8\angle 90}{2 + j8} = 1.94\Omega\angle 14.04$$

$$Z_T = 3\Omega\angle 0 + 6\Omega\angle 90 + 1.94\Omega\angle 14.04 = \boxed{7.38\Omega\angle 48.56°}$$

b) Find I_s

$$I_s = \frac{E}{Z_T} = \frac{30V_{RMS}\angle 0°}{7.38\Omega\angle 48.56°} = \boxed{4.067A_{RMS}\angle -48.56}$$

c) Find I_C using current divider

$$I_C = \frac{I_s \cdot R_2}{R_2 + Z_C} = 4.067A_{RMS}\angle -48.56\left(\frac{2\Omega\angle 0}{2\Omega\angle 0 + 8\Omega\angle 90}\right)$$

$$= \boxed{986.43mA_{RMS}\angle -124.52°}$$

d) Find V_L using voltage divider

$$V_L = \frac{E \cdot Z_L}{Z_T} = 30V_{RMS}\angle 0°\left(\frac{6\Omega\angle 90}{7.38\Omega\angle 48.56}\right) = \boxed{24.39V_{RMS}\angle 41.44°}$$

Problem 17-5. For the network in Fig.17.42:

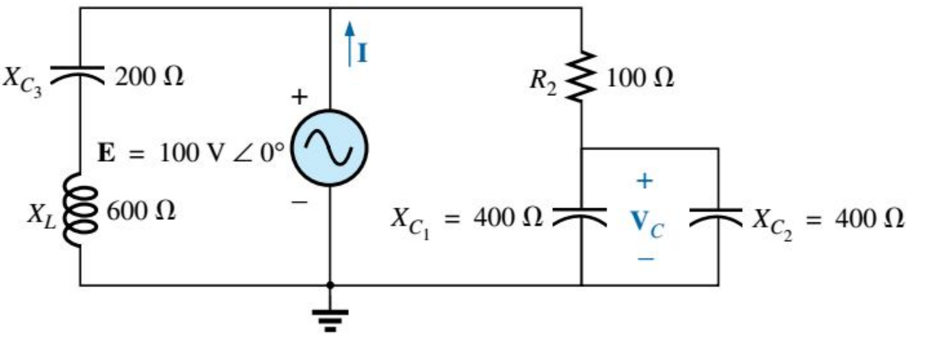


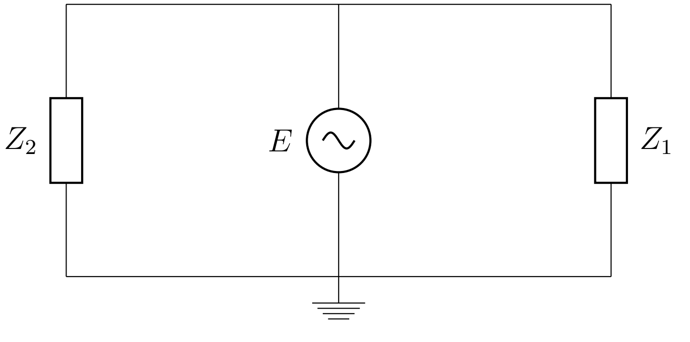
FIG. 17.42
Problem 5.

a.) Find the source current ***I***
Convert the elements to impedances and combine:

$$\begin{aligned}\boldsymbol{Z_1} &= \boldsymbol{Z_{R_2}} + \boldsymbol{Z_{C_1}}||\boldsymbol{Z_{C_2}} \\ &= 100 + (-j400)||(-j400) \\ &= 100 - \frac{(j400)(j400)}{(j400)+(j400)} \\ &= (100 - j200)\Omega = 223.6\Omega\angle - 63.4^\circ\end{aligned}$$

$$\begin{aligned}\boldsymbol{Z_2} &= \boldsymbol{Z_{C_3}} + \boldsymbol{Z_L} \\ &= -j400 + j600 \\ &= j400\Omega = 400\Omega\angle 90^\circ\end{aligned}$$

The circuit can be redrawn as follows:



$$
\begin{aligned}
\boldsymbol{Z_T} &= \boldsymbol{Z_1} \,//\, \boldsymbol{Z_2} \\
&= (223.6\ \Omega\angle-63.4^\circ) + (400\ \Omega\angle 90^\circ) \\
&= \frac{(223.6\ \Omega\angle-63.4^\circ)(400\ \Omega\angle 90^\circ)}{(223.6\ \Omega\angle-63.4^\circ) + (400\ \Omega\angle 90^\circ)} \\
&= 400\Omega\angle-36.87^\circ
\end{aligned}
$$

Therefore, $\boldsymbol{I}$ can be calculated using ohms law as follows:

$$
\begin{aligned}
\boldsymbol{I} &= \frac{\boldsymbol{E}}{\boldsymbol{Z_T}} \\
&= \frac{100\ V_{rms}\ \angle 0^\circ}{400\Omega\angle-36.87^\circ} \\
&= 250\ mA_{rms}\ \angle 36.9^\circ
\end{aligned}
$$

b.) Find the voltage $\boldsymbol{V_C}$
$\boldsymbol{V_C}$ can be found using voltage divider between R_2 and parallel combination of $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$

$$
\begin{aligned}
\boldsymbol{V_C} = \boldsymbol{E} \cdot &\frac{(\boldsymbol{Z_{C_1}}||\boldsymbol{Z_{C_2}})}{\boldsymbol{Z_{R_2}} + (\boldsymbol{Z_{C_1}}||\boldsymbol{Z_{C_2}})} \\
&= 100\ V_{rms}\ \angle 0^\circ \cdot \frac{(-j200)}{100 + (-j200)} \\
&= 89.44\ V_{rms}\ \angle-26.57^\circ = (80 - j40)V_{rms}
\end{aligned}
$$

c.) Find the average power delivered to the network

$$
\begin{aligned}
P_{ave.} &= V_{rms} \cdot I_{rms} \cdot \cos(\theta_v - \theta_i) \\
&= 100 \cdot 250 \cdot \cos(-36.9^\circ) \\
&= 20\ W
\end{aligned}
$$

Problem 17-8. For the network in Fig.17.45:

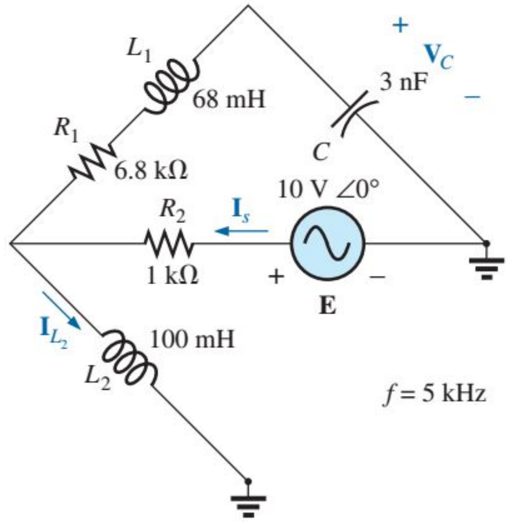


FIG. 17.45
Problem 8.

a.) Find the source current $\boldsymbol{I_s}$
Redraw the circuit before solving:

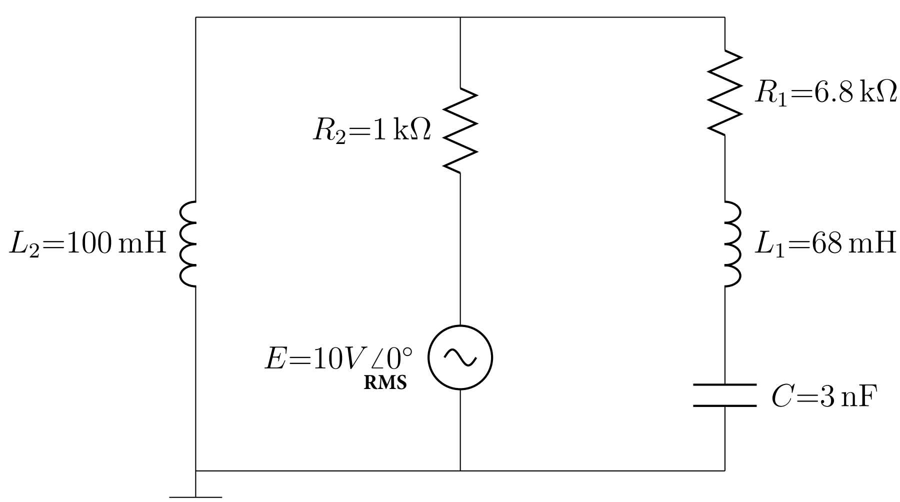
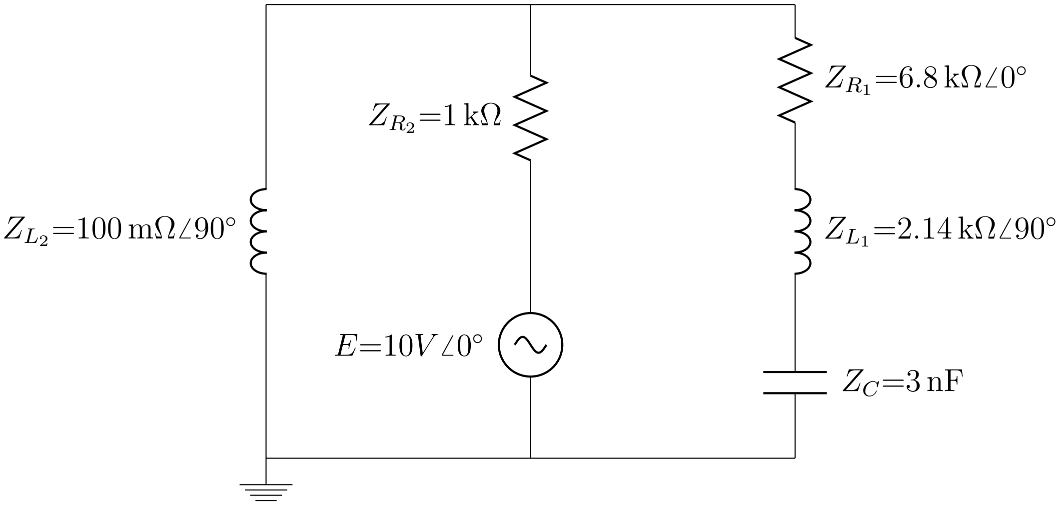


Figure 1: Parallel RLC circuit.

Convert the elements to their respective impedances



$$\begin{aligned}\boldsymbol{Z_{eq1}} &= \boldsymbol{Z_{R_1}} + \boldsymbol{Z_{L_1}} + \boldsymbol{Z_C} \\ &= 6.8\,\mathrm{k\Omega}\angle 0^\circ + 2.14\,\mathrm{k\Omega}\angle 90^\circ + 10.6\,\mathrm{k\Omega}\angle -90^\circ \\ &= 6.8\,\mathrm{k\Omega} - j8.47\,\mathrm{k\Omega} \\ &= 10.86\,\mathrm{k\Omega}\angle -51.24^\circ\end{aligned}$$

$$\boldsymbol{Z_T} = \boldsymbol{Z_{R_1}} + \boldsymbol{Z_{L_2}} || \boldsymbol{Z_{eq}}$$

$$\begin{aligned}\boldsymbol{Z_{eq2}} = \boldsymbol{Z_{L_2}} || \boldsymbol{Z_{eq}} &= \frac{(\boldsymbol{Z_{L_2}}) \cdot (\boldsymbol{Z_{eq}})}{\boldsymbol{Z_{L_2}} + \boldsymbol{Z_{eq}}} \\ &= \frac{(3.14\,\mathrm{k\Omega}\angle 90^\circ) \cdot (10.86\,\mathrm{k\Omega}\angle -51.24^\circ)}{3.14\,\mathrm{k\Omega}\angle 90^\circ + 10.86\,\mathrm{k\Omega}\angle -51.24^\circ} \\ &= 898.74\,\Omega - j3.85\,\mathrm{k\Omega}\end{aligned}$$

$$\begin{aligned}\boldsymbol{Z_T} &= 1\,\mathrm{k\Omega} + \boldsymbol{Z_{eq2}} \\ &= 1\,\mathrm{k\Omega} + 898.74\,\Omega - j3.85\,\mathrm{k\Omega} \\ &= 1898.74\,\Omega + j3846.37\,\Omega\end{aligned}$$

$$\begin{aligned}\boldsymbol{I_S} &= \frac{\boldsymbol{E}}{\boldsymbol{Z_T}} \\ &= 1.03\,\mathrm{mA}_{\textbf{RMS}} - j2.09\,\mathrm{mA}_{\textbf{RMS}}\end{aligned}$$

b.) Find the voltage across the capacitor $\boldsymbol{V_C}$
Use current divider rule to find the current that flows through the capacitor I_C:

$$\begin{aligned}\boldsymbol{I_C} &= \frac{\boldsymbol{Z_{L_2} I_s}}{\boldsymbol{Z_{L_2}} + (\boldsymbol{Z_{R_1}} + \boldsymbol{Z_{L_1}} + \boldsymbol{Z_C})} \\ &= \frac{(3.14\,\mathrm{k\Omega}\angle 90^\circ)(1.03\,\mathrm{mA} - j2.09\,\mathrm{mA})_{\textbf{RMS}}}{(3.14\,\mathrm{k\Omega}\angle 90^\circ) + (10.86\,\mathrm{k\Omega}\angle - 51.24^\circ)} \\ &= (0.848\,\underset{\textbf{RMS}}{\mathrm{mA}}\angle 64.38^\circ)\end{aligned}$$

$$\begin{aligned}\boldsymbol{V_C} &= \boldsymbol{I_C} \cdot \boldsymbol{Z_C} \\ &= (0.848\,\underset{\textbf{RMS}}{\mathrm{mA}}\angle 64.38^\circ) \cdot (10.6\,\mathrm{k\Omega}\angle - 90^\circ) \\ &= (8.99\,\underset{\textbf{RMS}}{\mathrm{V}}\angle - 25.62^\circ)\end{aligned}$$

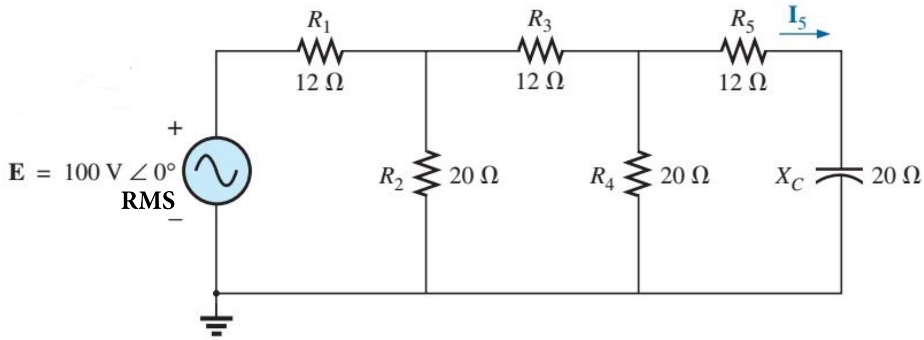
c.) Find the voltage $\boldsymbol{V_{L_2}}$
The voltage input voltage E is equal to the voltage drop over the resistor and the voltage drop over the parallel branch.

$$\boldsymbol{E} = \boldsymbol{V_{R_2}} + \boldsymbol{V_{L_2}}$$

therefore

$$\begin{aligned}\boldsymbol{V_{L_2}} &= \boldsymbol{E} - \boldsymbol{V_{R_2}} = \boldsymbol{E} - \boldsymbol{I_S} \cdot \boldsymbol{R_2} \\ &= 10\,\underset{\textbf{RMS}}{\mathrm{V}}\angle 0^\circ - (1.03\,\mathrm{mA} - j2.09\,\mathrm{mA})_{\textbf{RMS}} \cdot 1\,\mathrm{k\Omega} \\ &= 8.97\,\underset{\textbf{RMS}}{\mathrm{V}} + j2.09\,\mathrm{V}_{\textbf{RMS}}\end{aligned}$$

Problem 17-14. Find the current I_5 for the network in Fig.17.51. Note the effect of one reactive element on the resulting calculations.



Calculate the total impedance of the circuit $\boldsymbol{Z_T}$:

$$\begin{aligned}\boldsymbol{Z_1} &= \boldsymbol{Z_{R_5}} + \boldsymbol{Z_C}\\ &= 12\Omega - j20\Omega = 23.38\ \Omega\angle - 59.04^\circ\end{aligned}$$

$$\begin{aligned}\boldsymbol{Z_2} &= \boldsymbol{Z_1}||\boldsymbol{Z_{R_4}}\\ &= \frac{\boldsymbol{Z_1}\cdot\boldsymbol{Z_{R_4}}}{\boldsymbol{Z_1}+\boldsymbol{Z_{R_4}}} = \frac{(23.38\ \Omega\angle - 59.04^\circ)\cdot(20\Omega)}{(23.38\ \Omega\angle - 59.04^\circ) + (20\Omega)}\\ &= 11.01\ \Omega - 5.62\ \Omega = 12.36\ \Omega\angle - 27.03^\circ\end{aligned}$$

$$\begin{aligned}\boldsymbol{Z_3} &= \boldsymbol{Z_2} + \boldsymbol{Z_{R_3}}\\ &= 11.01\ \Omega - j5.62\ \Omega + 12\Omega\\ &= 23.01\ \Omega - j5.62\ \Omega = 23.69\ \Omega\angle - 13.72^\circ\end{aligned}$$

$$\begin{aligned}\boldsymbol{Z_4} &= \boldsymbol{Z_3}||\boldsymbol{Z_{R_2}}\\ &= \frac{(23.69\ \Omega\angle - 13.72^\circ)\cdot(20\ \Omega)}{(23.69\ \Omega\angle - 13.72^\circ) + (20\ \Omega)}\\ &= 10.86\ \Omega - j1.19\ \Omega = 10.92\ \Omega\angle - 6.28^\circ\end{aligned}$$

$$\begin{aligned}\boldsymbol{Z_T} &= \boldsymbol{Z_4} + \boldsymbol{Z_{R_1}}\\ &= 22.86\ \Omega - j1.19\ \Omega = 22.89\ \Omega\angle - 2.99^\circ\end{aligned}$$

The source current:

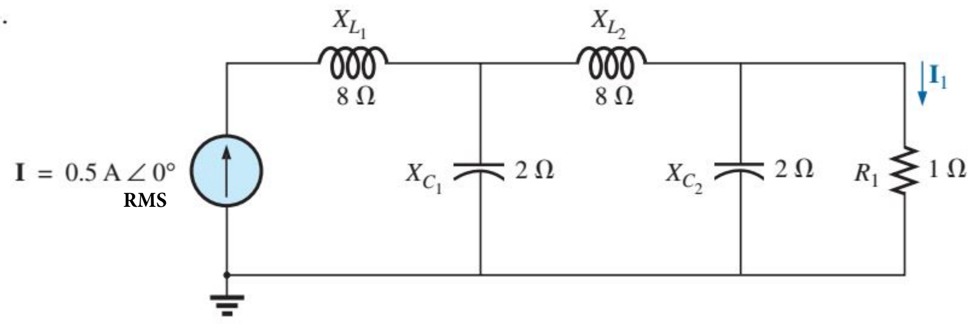
$$\begin{aligned}\boldsymbol{I_s} &= \frac{\boldsymbol{E}}{\boldsymbol{Z_T}}\\ &= \frac{100\ V\angle 0^\circ_{\text{RMS}}}{22.89\ \Omega\angle 2.99^\circ}\\ &= 4.37\ \text{A}\angle 2.991^\circ_{\text{RMS}}\end{aligned}$$

$$\boldsymbol{I_3} = \frac{\boldsymbol{Z_{R_2}} \boldsymbol{I_S}}{\boldsymbol{Z_{R_2}} + \boldsymbol{Z_3}}$$
$$= 2.02\,\underset{\textbf{RMS}}{\mathrm{A}}\angle 10.43^\circ$$

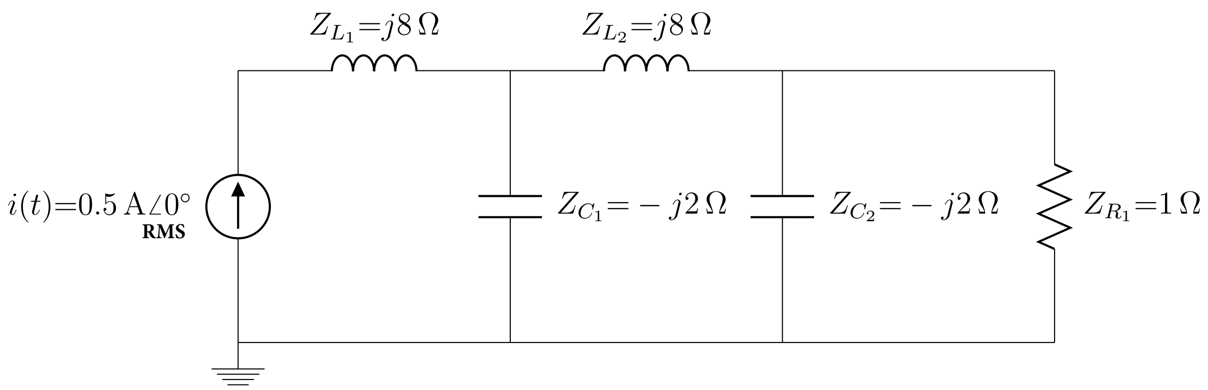
$$\boldsymbol{I_5} = \frac{\boldsymbol{Z_{R_4}} \boldsymbol{I_3}}{\boldsymbol{Z_{R_4}} + \boldsymbol{Z_1}}$$
$$= 1.07\,\underset{\textbf{RMS}}{\mathrm{A}}\angle 42.44^\circ$$

Because of the capacitor, the current leads the voltage. If the capacitor is replaced with an inductor, the current will lag the voltage.

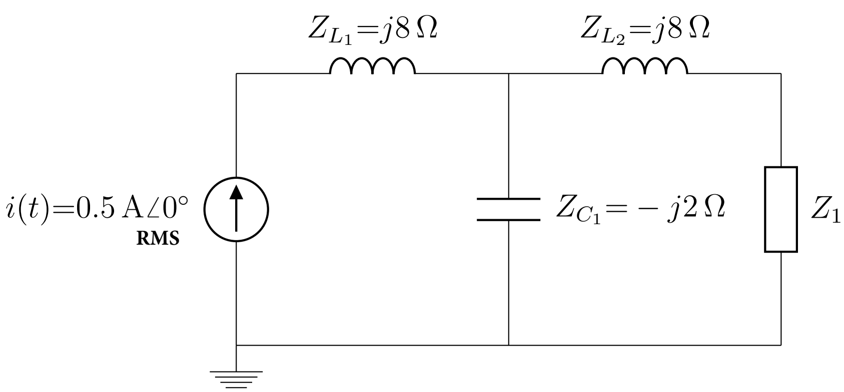
Problem 17-17.) Find the current I_1 for the network in Fig. 17.54:



Redraw the circuit replacing the circuit elements with their respective impedance



Parallel combination of impedances Z_{R_1} and Z_{C_2} to get Z_1



Series combination of Z_1 and Z_{L_2} to get Z_2

$$
\begin{aligned}
\boldsymbol{Z_1} &= \frac{(Z_{R_1})(Z_{C_2})}{(Z_{R_1})+(Z_{C_2})} = \frac{(1\angle 0^\circ)(2\angle -90^\circ)}{(1\angle 0^\circ)+(2\angle -90^\circ)} \\
&= 0.8\Omega - j0.4\Omega = 0.894\Omega\angle -26.57^\circ
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{Z_2} &= Z_1 + Z_{L_2} = (0.8\ \Omega - j0.4\ \Omega) + (0 + j8) \\
&= 0.8\ \Omega + j7.6\ \Omega = 7.64\ \Omega\angle 83.99^\circ
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{I_{L2}} &= \frac{\boldsymbol{I}(\boldsymbol{Z_{C_1}})}{(\boldsymbol{Z_{C_1}})+\boldsymbol{Z_2}} = \frac{(0.5\,\mathrm{A})(2\ \Omega\angle -90^\circ)}{(2\ \Omega\angle -90^\circ)+7.64\ \Omega\angle 83.99^\circ} \\
&= -0.175\,\mathrm{A} - j0.025\,\mathrm{A} = 0.177\,\mathrm{A}_{\mathbf{RMS}}\angle{-171.87^\circ}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{I_1} &= \frac{\boldsymbol{I_{L2}}(\boldsymbol{Z_{C_2}})}{(\boldsymbol{Z_{C_2}})+\boldsymbol{Z_{R_1}}} = \frac{(0.177\,\mathrm{A}_{\mathbf{RMS}}\angle{-171.87^\circ})(2\ \Omega\angle -90^\circ)}{(2\ \Omega\angle -90^\circ)+1\ \Omega\angle 0^\circ} \\
&= -0.15\,\mathrm{A} + j0.05\,\mathrm{A} = 158\,\mathrm{mA}\angle 161.57^\circ = 158\,\mathrm{mA}_{\mathbf{RMS}}\angle{-198.43^\circ}
\end{aligned}
$$