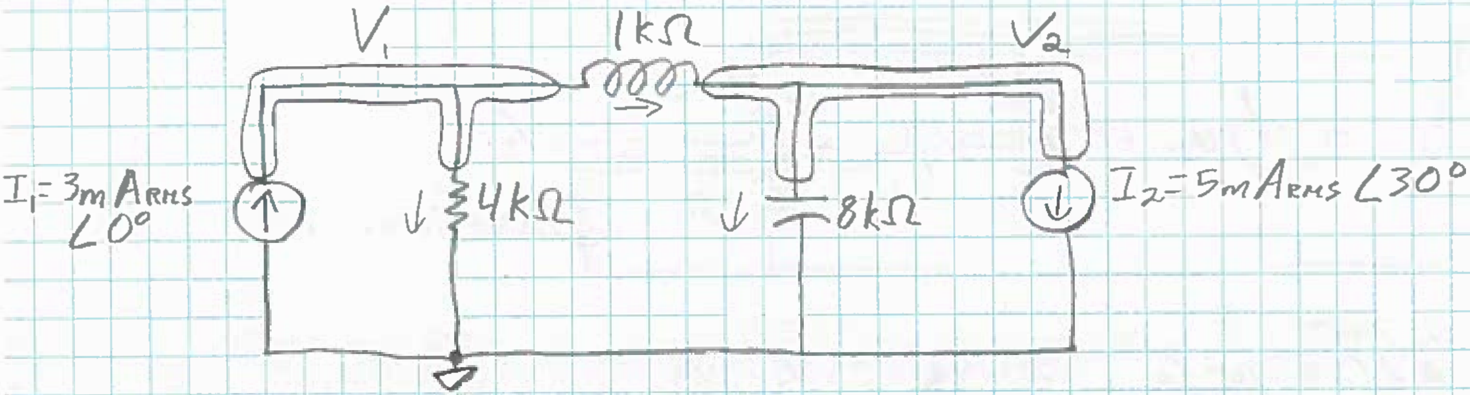
18-17 Determine the nodal voltages



IN = OUT

V_1: $I_1 = \frac{V_1 - 0}{4k\Omega} + \frac{V_1 - V_2}{1k\Omega\angle 90}$

$3mA\angle 0 = V_1\left(\frac{1}{4k\Omega} + \frac{1}{1k\Omega\angle 90}\right) + V_2\left(\frac{-1}{1k\Omega\angle 90}\right)$

V_2: IN = OUT

$\frac{V_1 - V_2}{1k\Omega\angle 90} = \frac{V_2 - 0}{8k\Omega\angle -90} + I_2$

$5mA\angle 30 = V_1\left(\frac{1}{1k\Omega\angle 90}\right) + V_2\left(\frac{-1}{1k\angle 90} + \frac{-1}{8k\Omega\angle -90}\right)$

$$V_1 = \frac{\begin{vmatrix} 3mA\angle 0° & -\frac{1}{1k\angle 90} \\ 5mA\angle 30° & \frac{-1}{1k\angle 90} + \frac{-1}{8k\angle -90} \end{vmatrix}}{\begin{vmatrix} \frac{1}{4k\Omega} + \frac{1}{1k\Omega\angle 90} & -\frac{1}{1k\angle 90} \\ \frac{1}{k\Omega\angle 90} & \frac{-1}{1k\angle 90} + \frac{-1}{8k\angle -90} \end{vmatrix}}$$

$$= \frac{\begin{vmatrix} 3mA\angle 0 & .001\angle 90 \\ 5mA\angle 30 & 875\mu\Omega\angle 90 \end{vmatrix}}{\begin{vmatrix} .002\angle -90 & .001\angle 90 \\ .001\angle -90 & 875\mu\Omega\angle 90 \end{vmatrix}}$$

$$= \frac{3mA\angle 0 \cdot 875\mu\Omega\angle 90 - 5mA\angle 30 \cdot .001\angle 90}{.002\angle 90 \cdot 875\mu\Omega\angle 90 - .001\angle 90 \cdot .001\angle -90} = 4.035\,V_{rms}\angle -34.3$$

$3mA\angle 0 = 4.035V\angle -34.3\left(\frac{1}{1k\Omega} + \frac{1}{1k\angle 90}\right) + V_2\left(\frac{-1}{1k\angle 90}\right)$

$V_2 = 10.07\,V_{rms}\ \angle -48.55$

20. Determine the nodal voltages for the network of Fig. 18.80.

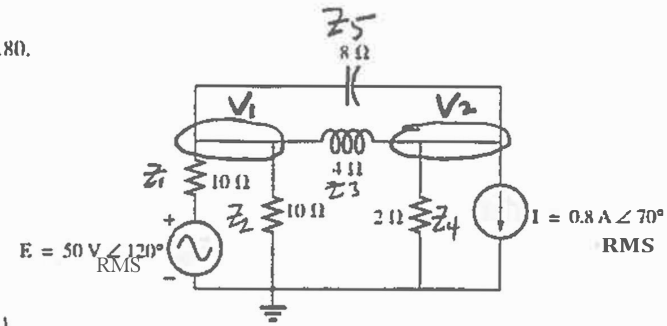
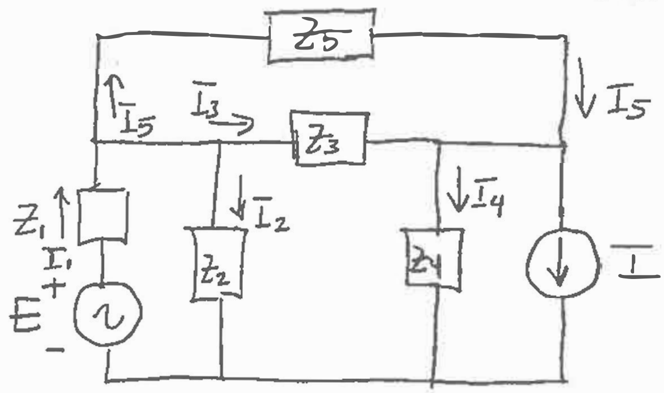


FIG. 18.80
Problem 20.



$Z_1 = 10\Omega + 0j$

$Z_2 = $ " "

$Z_3 = 4\Omega + j$

$Z_4 = 2\Omega + 0j$

$Z_5 = 8\Omega - j$

$\bar{I} = 0.8A_{rms} \angle 70°$

$E = 50V_{rms} \angle 120°$

Node V_1:

$$I_1 = I_2 + I_3 + I_5$$

$$\frac{E - V_1}{Z_1} = \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} + \frac{V_1 - V_2}{Z_5}$$

$$\therefore\ V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_5}\right] - V_2\left[1/Z_3 + 1/Z_5\right] = E/Z_1$$

Node V_2:

$$\bar{I} + I_3 + I_4 + I_5 = 0$$

$$I_3 + I_4 + I_5 = -\bar{I}$$

$$\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} + \frac{V_2 - V_1}{Z_5} = -\bar{I}$$

$$\therefore\ -V_1\left[1/Z_3 + 1/Z_5\right] + V_2\left[1/Z_3 + 1/Z_4 + 1/Z_5\right] = -\bar{I}$$

$$① \ V_1\left[0.200 - 0.125j\right] - V_2\left[-0.125j\right] = -2.5 + 4.330j$$

$$② \ -V_1\left[-0.125j\right] + V_2\left[0.5 - 0.125j\right] = \bar{0.8}A_{rms} \angle 70°$$

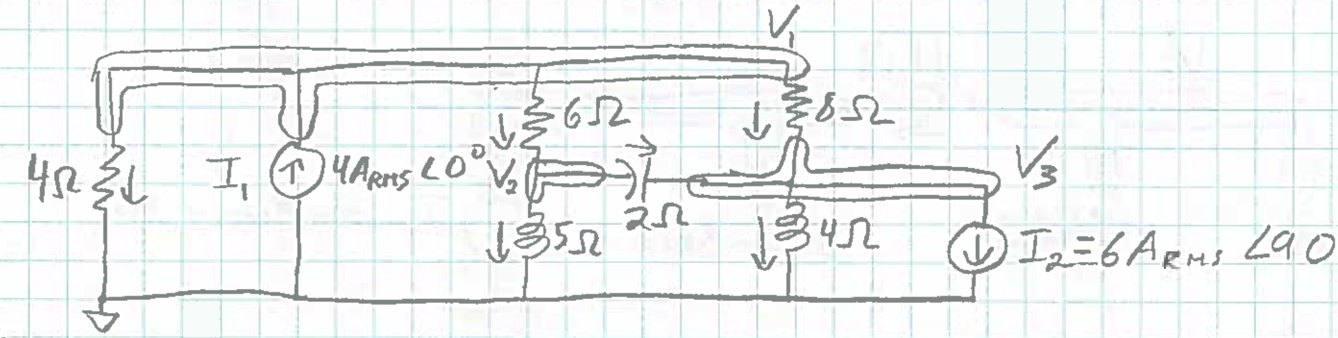
or

$$-0.274 - 0.752j$$

$$V_1 = (-16.994 + 10.248j)\,V_{rms}$$
$$= 19.85\,V_{rms} \angle 148.9^\circ$$

$$V_2 = (1.250 + 3.057j)\,V_{rms}$$
$$= 3.3\,V_{rms} \angle 67.8^\circ$$

17-24 Determine the nodal voltages



V_1:
IN = OUT

$$4A\angle 0 = \frac{V_1 - 0}{4\Omega} + \frac{V_1 - V_2}{6\Omega} + \frac{V_1 - V_3}{8\Omega}$$

$$4A\angle 0 = V_1\left(\frac{1}{4\Omega} + \frac{1}{6\Omega} + \frac{1}{8\Omega}\right) + V_2\left(\frac{-1}{6\Omega}\right) + V_3\left(\frac{-1}{8\Omega}\right)$$

V_2:
IN = OUT

$$\frac{V_1 - V_2}{6\Omega} = \frac{V_2 - 0}{5\Omega\angle 90} + \frac{V_2 - V_3}{2\Omega\angle -90}$$

$$0 = V_1\left(\frac{-1}{6\Omega}\right) + V_2\left(\frac{1}{6\Omega} + \frac{1}{5\Omega\angle 90} + \frac{1}{2\Omega\angle -90}\right) + V_3\left(\frac{-1}{2\Omega\angle -90}\right)$$

V_3:
IN = OUT

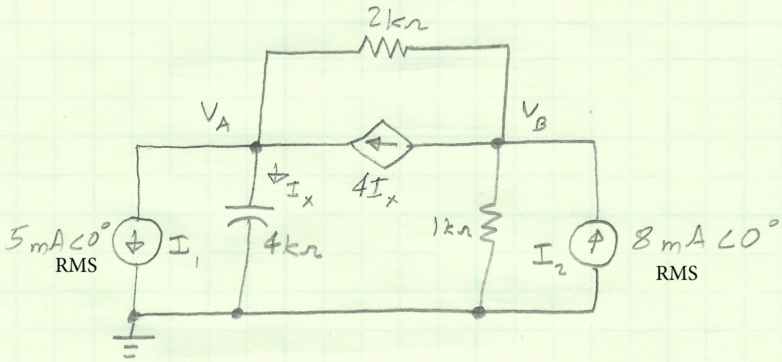
$$\frac{V_1 - V_3}{8\Omega} + \frac{V_2 - V_3}{2\Omega\angle -90} = \frac{V_3 - 0}{4\Omega\angle 90} + 6A\angle 90$$

$$6A\angle 90 = V_1\left(\frac{1}{8\Omega}\right) + V_2\left(\frac{1}{2\Omega\angle -90}\right) + V_3\left(\frac{-1}{8\Omega} + \frac{-1}{2\Omega\angle -90} + \frac{-1}{4\Omega\angle -90}\right)$$

$$X = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \qquad A = \begin{bmatrix} \frac{1}{4\Omega} + \frac{1}{6\Omega} + \frac{1}{8\Omega} & \frac{-1}{6\Omega} & \frac{-1}{8\Omega} \\ \frac{-1}{6\Omega} & \frac{1}{6\Omega} + \frac{1}{5\Omega\angle 90} + \frac{1}{2\Omega\angle -90} & \frac{-1}{2\Omega\angle -90} \\ \frac{1}{8\Omega} & \frac{1}{2\Omega\angle -90} & \frac{-1}{8\Omega} + \frac{-1}{2\angle -90} + \frac{-1}{4\Omega\angle} \end{bmatrix}$$

$$B = \begin{bmatrix} 4A\angle 0 \\ 0 \\ 6A\angle 90 \end{bmatrix} \qquad X = A^{-1}\cdot B = \begin{bmatrix} 22.96V_{RMS}\angle -35.75 \\ 39.81V_{RMS}\angle -45.68 \\ 23.28V_{RMS}\angle -59.93 \end{bmatrix}$$

(25) Find the nodal voltages for the network below, and determine the voltage across the 1kΩ resistor



$$I_x = \frac{V_A}{-j4k\Omega}$$

$$\text{①}\quad \underbrace{4I_x}_{\frac{4V_A}{-j4k\Omega}} = I_1 + \frac{V_A}{-j4k\Omega} + \frac{V_A - V_B}{2k\Omega}$$

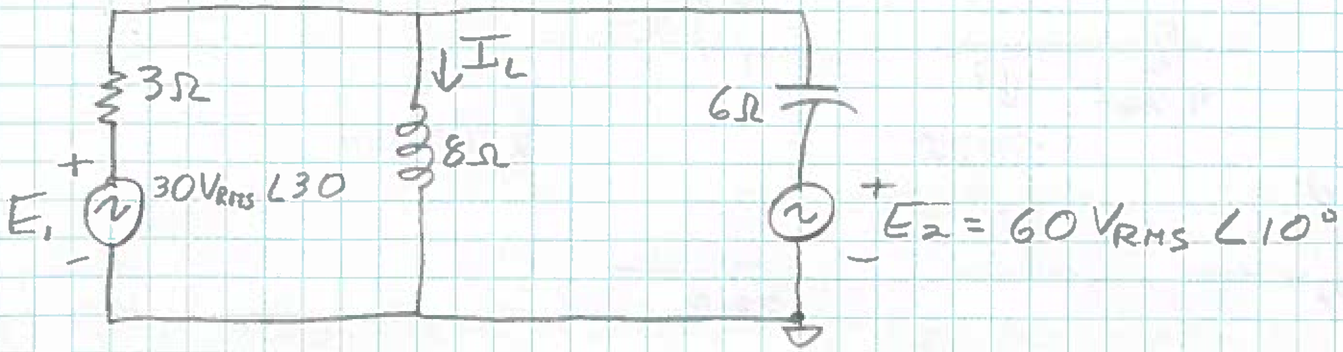
$$-I_1 = V_A\left[-\frac{4}{-j4k\Omega} + \frac{1}{-j4k\Omega} + \frac{1}{2k\Omega}\right] + V_B\left[-\frac{1}{2k\Omega}\right]$$

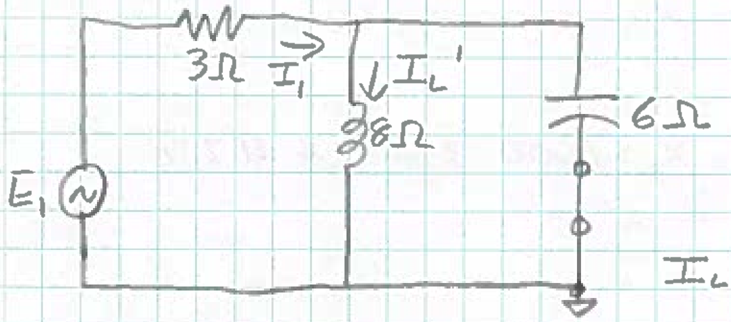
$$\text{②}\quad I_2 + \frac{V_A - V_B}{2k\Omega} = \frac{4V_A}{-j4k\Omega} + \frac{V_B}{1k\Omega}$$

$$I_2 = V_A\left[-\frac{1}{2k\Omega} + \frac{4}{-j4k\Omega}\right] + V_B\left[\frac{1}{2k\Omega} + \frac{1}{1k\Omega}\right]$$

$$\therefore\quad \boxed{\begin{aligned} V_A &= 4.373\,V_{RMS}\angle -128.7^\circ \\ V_B &= 2.252\,V_{RMS}\angle 17.65^\circ \end{aligned}}$$

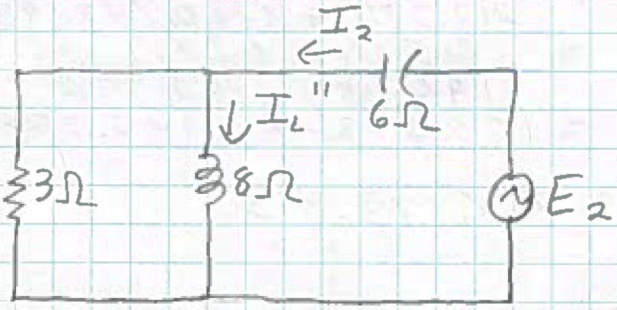
19-1 Using superposition, find I_L





$$I_1 = \frac{30V\angle 30}{3\Omega + 8\angle 90 // 6\angle -90}$$
$$= \frac{30V\angle 30}{3 - j24}$$
$$= 1.24\angle 112.875$$

$$I_L' = 1.24\angle 112.875\left(\frac{6\angle -90}{8\angle 90 + 6\angle -90}\right)$$
$$= 3.72A\angle -67.125$$

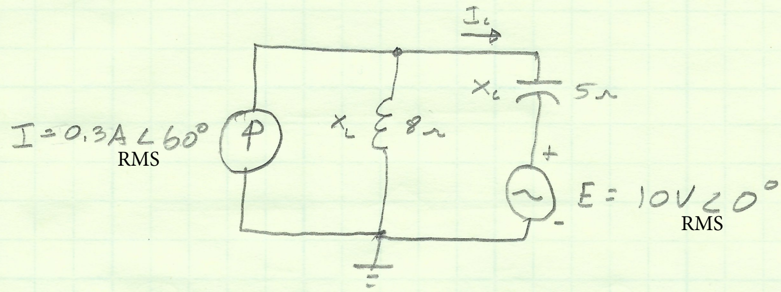


$$I_2 = \frac{60V\angle 10}{6\Omega\angle 90 + 3\angle 0 // 8\angle 90}$$
$$= \frac{60V\angle 10}{5.662\angle -62.319}$$
$$= 10.598A\angle 72.319°$$

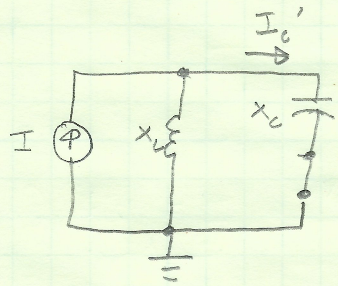
$$I_L'' = 10.598A\angle 72.319\left(\frac{3\angle 0}{3\angle 0 + 8\angle 90}\right)$$
$$= 3.721A\angle 2.875$$

$$I_L = I_L' + I_L''$$
$$= 3.72A\angle -67.125 + 3.721A\angle 2.875$$
$$\boxed{= 6.095\,A_{RMS}\angle -32.119}$$

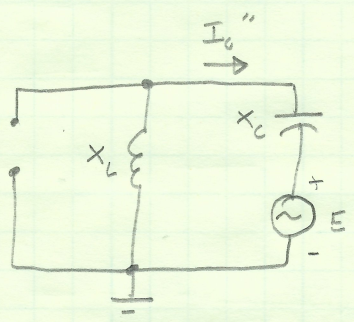
② Using superposition, determine I_c in the network:



$$I_c = I_c' + I_c''$$



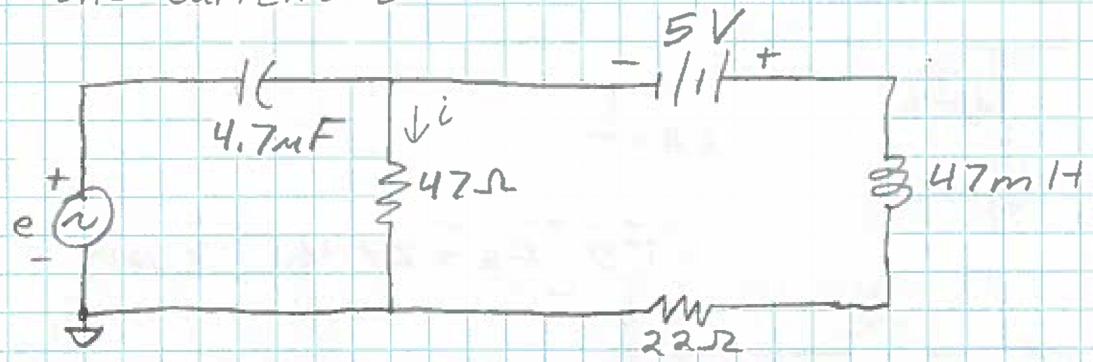
$$I_c' = \frac{X_L}{X_L + X_c} \cdot I = 0.8A\angle 60°$$



$$I_c'' = \frac{-E}{X_c + X_L} = 3.33A\angle 90°$$

$$I_c = (0.8A\angle 60°) + (3.33A\angle 90°) = \boxed{4.05A\angle 84.3°}$$
RMS

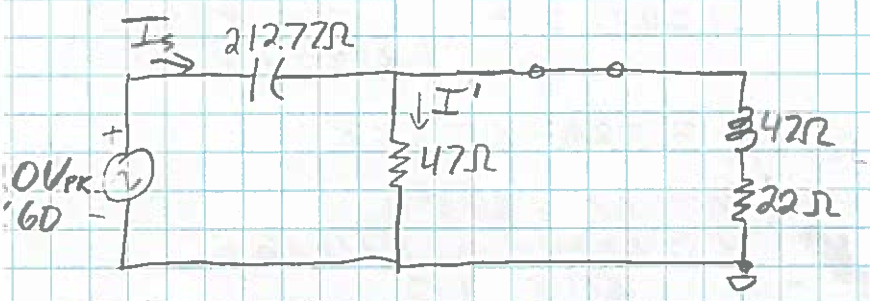
19-6 Using superposition, find the sinusoidal expression for the current i



$e = 20\sin(1000t + 60°)$

$E = 20\,V_{PK}\angle 60$

$X_C = \dfrac{1}{1000 \cdot 4.7mF} = 212.77\Omega$ $X_L = 1000 \cdot 47mH = 47\Omega$



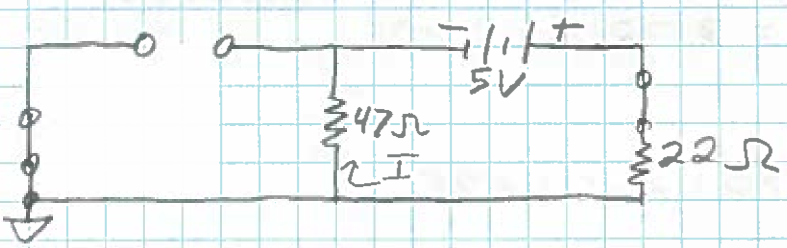
$I_s = \dfrac{20\,V_{PK}\angle 60}{212.77\Omega + 47\angle 0 // 47\angle 90 +}$

$= \dfrac{20\,V_{PK}\angle 60}{199.456\angle -82.758}$

$= 100.273mA\angle 142.758$

$I' = I_s\left(\dfrac{22+j47}{22+j47+47}\right) = 62.328mA\angle 173.413$

$i = 62.328\times10^{-3}\sin(1000t + 173.413°)$

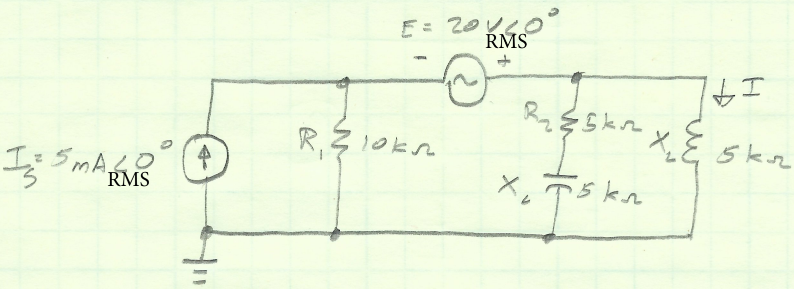


$I = \dfrac{5\,V}{47\Omega + 22\Omega}$

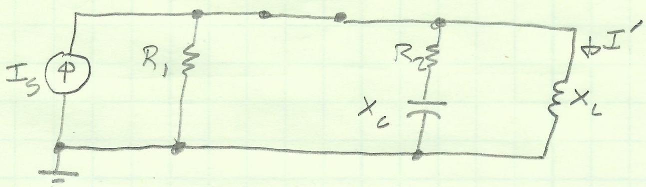
$= 72.464mA$

$i = -72.464mA + 62.328\times10^{-3}\sin(1000t + 173.413°)$

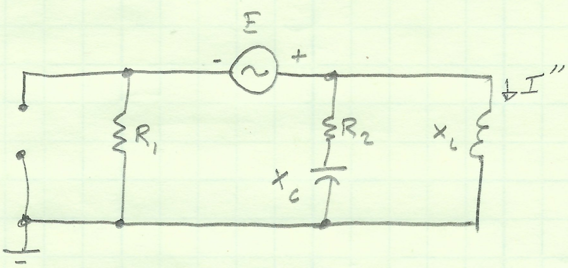
⑧ Using superposition, find I for the network below:



$I = I' + I''$



$$I' = \frac{R_1 \parallel (R_2 + Z_c)}{Z_L + (R_1 \parallel (R_2 + Z_c))} \cdot I_s = (4.472\ mA_{RMS} \angle -63.43°)$$



$$I'' = E \cdot \frac{X_L \parallel (R_2 + X_c)}{[X_L \parallel (R_2 + X_c)] + R_1} \cdot \frac{1}{X_L} = (1.789\ mA \angle -63.43°)$$

$$I = I' + I'' = \boxed{(6.26\ mA_{RMS} \angle -63.43°)}$$