Name: _____

Section: _____

R·I·T School of Mathematical Sciences

16 - Homogeneous Equations

MATH 211

Solve the following polynomial equations using methods in algebra.

1. $x^2 + 15x + 21 = 0$

$$x = \frac{-15 \pm \sqrt{15^2 - 4(1)(21)}}{2(1)} = \frac{-15 \pm \sqrt{225 - 84}}{2} = \frac{-15 \pm \sqrt{141}}{2}$$

2. $m^2 + 12m + 36 = 0$

$$(m+6)(m+6) = 0$$
$$m_1 = m_2 = -6$$

3. $r^2 + 4 = 0$

$$r^2 = -4$$
$$r = \pm\sqrt{-4}$$
$$r = \pm 2i$$

4. $\lambda^3 - 3\lambda^2 - 2\lambda + 6 = 0$

$$\left(\lambda^3 - 3\lambda^2\right) + (-2\lambda + 6) = 0$$
$$\lambda^2(\lambda - 3) - 2(\lambda - 3) = 0$$
$$\left(\lambda^2 - 2\right)(\lambda - 3) = 0$$
$$\lambda_1 = -\sqrt{2},\ \lambda_2 = \sqrt{2},\ \lambda_3 = 3$$

Find the general solution to the higher order homogeneous equation.

$$y'' + 8y' + 16y = 0$$

$$r^2 + 8r + 16 = 0$$

$$(r+4)^2 = 0$$

$$r_1 = r_2 = -4$$

$$y_1 = e^{-4x},\ y_2 = xe^{-4x}$$

$$y = c_1 y_1 + c_2 y_2$$

$$y = c_1 e^{-4x} + c_2 x e^{-4x}$$

Solve the following IVP.

$$y'' + 25y = 0,\ y(0) = 0,\ y'(0) = 1$$

$$r^2 + 25 = 0$$

$$r^2 = -25$$

$$r = \pm\sqrt{-25}$$

$$r = \pm 5i$$

$$\alpha = 0,\ \beta = 5$$

$$y_1 = \sin(5x),\ y_2 = \cos(5x)$$

$$y = c_1 y_1 + c_2 y_2$$

$$y = c_1 \sin(5x) + c_2 \cos(5x)$$

$$0 = c_1(0) + c_2(1)$$

$$c_2 = 0$$

$$y = c_1 \sin(5x)$$

$$y' = 5c_1 \cos(5x)$$

$$1 = 5c_1(1)$$

$$c_1 = \frac{1}{5}$$

$$y = \frac{1}{5}\sin(5x)$$