R·I·T School of Mathematical Sciences

Multivariable Calculus Practice

MATH 211

Functions of Several Variables:

1. Evaluate the given functions at the indicated points.
   (a) $f(x,y) = x^2 - 5y + y^2,\ (2,-2)$
   (b) $f(r,t) = r - \dfrac{r - 2t^2 - 5t}{r+t},\ (r, t+k)$
   (c) $f(x,y) = 3x^3 - x^2y + 5y^2,$ find $f(3x^2, x) - f(x,x)$
2. Evaluate the following limits, if the exist.
   (a) $\lim\limits_{(x,y)\to(2,1)} (x + 3y^2)$
   (b) $\lim\limits_{(x,y)\to(2,4)} \dfrac{x+y}{x-y}$
   (c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x+y}{x^2-y^2}$
   (d) $\lim\limits_{(x,y)\to(1,1)} \dfrac{xy-1}{1-xy}$

Partial Derivatives:

1. Find the first partial derivatives of the following functions.
   (a) $z = \dfrac{x^2}{y} - 2xy$
   (b) $z = xe^{2y}$
   (c) $f(x,y) = \dfrac{2+\cos x}{1 - \sec(3y)}$
   (d) $z = \left(x^2 + xy^3\right)^4$
   (e) $f(x,y) = \ln(x^2 + y)$
   (f) $z = e^x \cos(xy) + e^{-2x} \tan y$
2. Find the second and mixed partial derivatives of the following functions.
   (a) $z = 2xy^3 - 3x^2y$
   (b) $f(x,y) = y \ln(x + 2y)$
   (c) $z = \dfrac{x}{y} + e^x \sin y$
   (d) $z = \dfrac{1+\cos y}{1+x^2}$

3. Find the total differential for the following functions.

   (a) $f(x, y) = 2x^2 - y^2 + 3x$

   (b) $f(x, y) = xe^y - y^2$

   (c) $z = \sin(xy) - y\cos x$

4. Examine the function for any relative extrema or saddle points.

   (a) $f(x, y) = x^2 + y^2 - 2x - 6y + 10$

   (b) $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 2$

   (c) $f(x, y) = x^4 - 4x + y^2$

   (d) $f(x, y) = x - y\ln x$

5. The centripetal acceleration of a particle moving in a circle is $a = \frac{v^2}{r}$, where v is the velocity and r is the radius of the circle. Approximate the change in the acceleration if v increases by 0.1m/sec from 4m/sec and r decreases by 0.2m from 10m.

6. The temperature distribution in a square 16 foot room containing a heating element and vent fan is given by

$$T(x, y) = 72 - 2x^2 - y^2 + 8x + 2y$$

   where the point (x, y) is your location on the floor of that room and temperature is given in Fahrenheit. Find the location in the room where the temperature is at its highest. What is the maximum temperature?

7. The voltage V is across a resistance R. The current i is given by $i = V/R$. Find the approximate change in the current if the voltage changes from 220 V to 225 V and the resistance changes from 20 Ω to 21 Ω.

8. An open (no top) rectangular cargo container is to have a volume of 32 m^3. Find the dimensions that require the least material to build the container.

9. A flat plate is heated such that the temperature T at any point (x, y) on the plate is given by $T = x^2 + 2y^2 - x$. Find the temperature at the coldest point on the plate.

10. An electronic manufacturer determines that the profit P (in dollars) obtained by producing x units of a iPhone circuit board and y units of a TI-$n$spire calculator circuit board is approximated by the model

$$P(x, y) = 14x + 10y - \frac{1}{2}x^2 - \frac{1}{2}xy - \frac{1}{2}y^2 - 10$$

    Find the production level of each that produces the maximum profit.

11. A cylindrical rod made from aluminum is being heated in a vat of boiling water which compromises the dimensional and rigidity of the rod. The rod's radius increases by 0.1 mm from 2 mm and the length increases by 0.25 mm from 20 mm. Find the approximate change in the volume of the cylindrical rod.

Double Integration:

1. Evaluate the following integrals.

   (a) $\displaystyle\int_0^1 \int_2^3 \sqrt{x + 4y}\; dx\; dy$

   (b) $\displaystyle\int_0^{\sqrt{\pi/2}} \int_0^{x^2} x\cos y\; dy\; dx$

2. Use a double integral to find the area of the indicated region.

   (a) $R : \{y = x,\ y = 2x,\ x = 3\}$

   (b) $R : \{y = \sin x,\ y = \cos x,\ x = 0,\ x = \pi/4\}$

   (c) $R : \{y = 4 - x,\ y = x,\ y = 0\}$

   (d) $R : \left\{y = 4 - x^2,\ y = 3x\right\}$

3. Use a double integral to find the volume of the solid bounded

(a) above by the surface $z = 9 - x^2 - y^2$ and below by $R : \{y = x,\ y = 2x,\ x = 3\}$.

(b) above by the surface $z = e^{x+2y}$ and below by $R : \{y = \sin x,\ y = \cos x,\ x = 0,\ x = \pi/4\}$

(c) above by the surface $z = \sqrt{x + 2y}$ and below by $R : \{y = 4 - x,\ y = x,\ y = 0\}$

(d) above by the surface $z = y + 5$ and below by $R : \left\{y = 4 - x^2,\ y = 3x\right\}$