Name: ____________________

Section: ____________________

R·I·T School of Mathematical Sciences

Homework 1

MATH 211

1. Evaluate the limit if it exists. If the limit does not exist, explain why.

$$\lim_{(x,y)\to(0,0)} \frac{4x^2 - 9y^2}{2x - 3y} = \frac{0}{0}$$

$$= \lim_{(x,y)\to(0,0)} \frac{\cancel{(2x - 3y)}(2x + 3y)}{\cancel{2x - 3x}}$$

$$= \lim_{(x,y)\to(0,0)} (2x + 3y)$$

$$= 0$$

2. Find all four second partial derivatives.

$$z = \frac{\sin(2x)}{3y}$$

$$z_x = \frac{2\cos(2x)}{3y} \qquad z_y = -\frac{\sin(2x)}{3y^2}$$

$$z_{xx} = -\frac{4\sin(2x)}{3y} \qquad z_{yy} = \frac{2\sin(2x)}{3y^3}$$

$$z_{xy} = -\frac{2\cos(2x)}{3y^2} \qquad z_{xy} = -\frac{2\cos(2x)}{3y^2}$$

3. The centripedal acceleration of a particle moving in a circle is $a = v^2/r$, where v is the velocity and r is the radius of the circle. Approximate the change in the acceleration if v increases by 0.1m/sec from 4m/sec and r decreases by 0.2m from 10m.

$$a(v,r) = v^2 r^{-1}$$

$$a_v(v,r) = 2vr^{-1} = \frac{2v}{r}$$

$$a_v(4,10) = \frac{2(4)}{10} = \frac{4}{5}$$

$$a_r(v,r) = -v^2 r^{-2} = -\frac{v^2}{r^2}$$

$$a_r(4,10) = -\frac{4^2}{10^2} = -\frac{16}{100} = -\frac{4}{25}$$

$$\begin{aligned} da &= a_v(4,10)\ dv + a_r(4,10)\ dr \\ &= \left(\frac{4}{5}\right)\left(\frac{1}{10}\right) + \left(-\frac{4}{25}\right)\left(-\frac{1}{5}\right) \\ &= \frac{2}{25} + \frac{4}{125} \\ &= \frac{14}{125}\ \text{m/s}^2 \end{aligned}$$

4. The temperature distribution in a square 16 foot room containing a heating element and vent fan is given by

$$T(x,y) = 72 - 2x^2 - y^2 + 8x + 2y$$

where the point (x,y) is your location on the floor of that room and temperature is given in Fahrenheit. Find the location in the room where the temperature is at its highest. What is the maximum temperature?

$$\begin{aligned} T_x(x,y) &= -4x + 8 \\ 0 &= -4x + 8 \\ 4x &= 8 \\ x &= 2 \end{aligned}$$

$$\begin{aligned} T_y(x,y) &= -2y + 2 \\ 0 &= -2y + 2 \\ 2y &= 2 \\ y &= 1 \end{aligned}$$

Critical Point: $(2,1)$

$$T_{xx}(x,y) = -4 \qquad T_{xy}(x,y) = 0 \qquad T_{yy}(x,y) = -2$$

$$T_{xx}(2,1) = -4 < 0 \qquad T_{xy}(2,1) = 0 \qquad T_{yy}(2,1) = -2$$

$$\begin{aligned} D(2,1) &= T_{xx}(2,1)T_{yy}(2,1) - [T_{xy}(2,1)]^2 \\ &= (-4)(-2) - 0^2 \\ &= 8 > 0 \end{aligned}$$

$$\begin{aligned} T(2,1) &= 72 - 2(2)^2 - 1^2 + 8(2) + 2(1) \\ &= 72 - 8 - 1 + 16 + 2 \\ &= 81 \end{aligned}$$

The maximum temperature of 81°F occurs at the point $(2,1)$.