Practice Exam 4 Solutions

1. $mv' = F - kv$

$v' = 12e^{-t} - 12e^{-t}u(t-8)$ *

$s\mathcal{L}\{v\} - v(0) = \frac{12}{s+1} - 12e^{-8}\cdot\frac{1}{s+1}$

$s\mathcal{L}\{v\} - 4 = \frac{12}{s+1} - 12e^{-8}\cdot\frac{1}{s+1}$

$\mathcal{L}\{v\} = \frac{12}{s(s+1)} - 12e^{-8}\cdot\frac{e^{-8s}}{s(s+1)} + \frac{4}{s}$ *

$\mathcal{L}\{v\} = \frac{12}{s} - \frac{12}{s+1} - e^{-8}e^{-8s}\left(\frac{12}{s} - \frac{12}{s+1}\right) + \frac{4}{s}$ **

$v = 12 - 12e^{-t} - e^{-8}\left(12 - 12e^{-(t-8)}\right)u(t-8) + 4$

$v = 16 - 12e^{-t} - e^{-8}\left(12 - 12e^{-(t-8)}\right)u(t-8)$

* $f(t) = 12e^{-t}$

$a = 8$

$f(t+8) = 12e^{-(t+8)}$

$= 12e^{-8}e^{-t}$

$e^{-8s}\mathcal{L}\{f(t+8)\} = 12e^{-8}\cdot\frac{e^{-8s}}{s+1}$

* $\frac{12}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$

$As + A + Bs = 12$

$A = 12 \quad A + B = 0$

$B = -12$

** $F(s) = \frac{12}{s} - \frac{12}{s+1}$

$a = 8$

$f(t) = 12 - 12e^{-t}$

$f(t-a) = f(t-8) = 12 - 12e^{-(t-8)}$

$u(t-8)f(t-8) = \left(12 - 12e^{-(t-8)}\right)(t-8)$

2. $mx'' + bx' + kx = f(t)$

$2x'' + 18x = 18u(t-2)$

$x'' + 9x = 9u(t-2)$

$s^2\mathcal{L}\{x\} - s\cancel{x(0)}^{0} - \cancel{x'(0)}^{0} + 9\mathcal{L}\{x\} = \frac{9e^{-2s}}{s}$

$(s^2+9)\mathcal{L}\{x\} = s + \frac{9e^{-2s}}{s}$

$\mathcal{L}\{x\} = \frac{s}{s^2+9} + \frac{9e^{-2s}}{s(s^2+9)}$ *

$\mathcal{L}\{x\} = \frac{s}{s^2+9} + e^{-2s}\left(\frac{1}{s} - \frac{s}{s^2+9}\right)$ **

$x = \cos(3t) + \left[1 - \cos(3(t-2))\right]u(t-2)$

* $\frac{9}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9}$

$As^2 + 9A + Bs^2 + Cs = 9$

$A+B=0 \quad C=0 \quad 9A=9$

$B=-1 \quad A=1$

** $F(s) = \frac{1}{s} - \frac{s}{s^2+9}$

$a = 2$

$f(t) = 1 - \cos(3t)$

$f(t-2) = 1 - \cos(3(t-2))$

$u(t-2)f(t-2) = \left[1 - \cos(3(t-2))\right]u(t-2)$

3. $Lq'' + Rq' + \frac{q}{C} = E(t)$

$q'' + 6q' + 18q = 3\delta(t-2)$

$s^2\mathcal{L}\{q\} - sq(0) - q'(0) + 6(s\mathcal{L}\{q\} - q(0)) + 18\mathcal{L}\{q\} = 3e^{-2s}$

$(s^2 + 6s + 18)\mathcal{L}\{q\} = 3e^{-2s}$

$\mathcal{L}\{q\} = \frac{3e^{-2s}}{s^2 + 6s + 18}$ $+9+9$

$\mathcal{L}\{q\} = e^{-2s}\frac{3}{(s+3)^2 + 3^2}$ *

$q = e^{-3(t-2)}\sin(3(t-2))u(t-2)$

* $F(s) = \frac{3}{(s+3)^2+9}$

$a = 2$ T.8

$f(t) = e^{-3t}\sin(3t)$

$f(t-2) = e^{-3(t-2)}\sin(3(t-2)$

$u(t-2)f(t-2) = e^{-3(t-2)}\sin(3(t-2))u(t-2)$