R·I·T School of Mathematical Sciences

3 - Total Differential

MATH 211

The pressure p (in Pa) of a gas as a function of its volume V and temperature T is $p = nRT/V$ where $n = 3$ mol and $R = 8$ J/mol·K. Find the actual and approximate changes in pressure if the volume is allowed to decrease from 2 cubic meters to 1.9 cubic meter and the temperature is allowed to increase from 300 Kelvin to 301 Kelvin. Then, find the error in your approximation.

$$\Delta T = dT = 1$$

$$\Delta V = dV = -\frac{1}{10}$$

$$p(V,T) = nRTV^{-1}$$

$$p_V(V,T) = -nRTV^{-2} = -\frac{nRT}{V^2} \qquad p_T(V,T) = nRV^{-1} = \frac{nR}{V}$$

$$p_V(2,300) = -\frac{(3)(8)(300)}{2^2} = -1800 \qquad p_T(2,300) = \frac{(3)(8)}{2} = 12$$

Approximate Change

$$dp = p_V(2, 300)dV + p_T(2, 300)dT$$

$$= (-1800)\left(-\frac{1}{10}\right) + (12)(1)$$

$$= 180 + 12$$

$$= 192$$

Actual Change

$$\Delta p = p(301, 1.9) - p(300, 2)$$

$$= \frac{(3)(8)(301)}{1.9} - \frac{(3)(8)(300)}{2}$$

$$= \frac{(8)(301)(30)}{19} - (3)(4)(300)$$

$$= \frac{72240}{19} - 3600$$

$$= \frac{3840}{19}$$

$$\approx 202.11$$

$$|dp - \Delta p| = 10.11$$