R·I·T SCHOOL OF MATHEMATICAL SCIENCES

Kirchhoff's Law

MATH 211

1. Supoose that in a cimple circuit the resistance is 12 Ω and the inductance is 4 H. If the battery gives a constant voltage of 60 V and the switch is closed at $t =$ so the initial current is 0, find the current as a function of time.

$$L\frac{di}{dt} + Ri = E(t)$$

$$4\frac{di}{dt} + 12i = 60$$

$$\frac{di}{dt} + 3i = 15$$

$$e^{3t}\left[\frac{di}{dt} + 3i\right] = 15e^{3t}$$

$$\frac{d}{dt}\left[ie^{3t}\right] = 15e^{3t}$$

$$d\left[ie^{3t}\right] = 15e^{3t}\ dt$$

$$\int d\left[ie^{3t}\right] = 15\int e^{3t}\ dt$$

$$ie^{3t} = \frac{15}{3}e^{3t} + K$$

$$ie^{3t} = 5e^{3t} + K$$

$$i = 5 + Ke^{-3t}$$

$$0 = 5 + Ke^{0}$$

$$-5 = K$$

$$i = 5 - 5e^{-3t}$$

$$e^{\int P(t)\ dt}$$

$$= e^{\int 3\ dt}$$

$$= e^{3t}$$

2. Suppose that in a simple circuit the resistance is 12 Ω and the inductance is 4 H. If a battery gives a voltage of $E(t) = 60\sin(30t)$ V and the switch is closed at $t = 0$ so the initial current is 0, find the current as a function of time.

$$L\frac{di}{dt} + Ri = E(t)$$

$$4\frac{di}{dt} + 12i = 60\sin(30t)$$

$$\frac{di}{dt} + 3i = 15\sin(30t)$$

$$e^{3t}\left[\frac{di}{dt} + 3i\right] = 15e^{3t}\sin(30t)$$

$$\frac{d}{dt}\left[ie^{3t}\right] = 15e^{3t}\sin(30t)$$

$$d\left[ie^{3t}\right] = 15e^{3t}\sin(30t)\ dt$$

$$\int d\left[ie^{3t}\right] = 15\int e^{3t}\sin(30t)\ dt_{\star}$$

$$ie^{3t} = -\frac{10}{303}e^{3t}\cos(30t) + \frac{1}{303}e^{3t}\sin(30t) + K$$

$$i = -\frac{10}{303}\cos(30t) + \frac{1}{303}\sin(30t) + Ke^{-3t}$$

$$0 = -\frac{10}{303}(1) + \frac{1}{303}(0) + Ke^{0}$$

$$K = \frac{10}{303}$$

$$i = -\frac{10}{303}\cos(30t) + \frac{1}{303}\sin(30t) + \frac{10}{303}e^{-3t}$$

$$e^{\int P(t)\ dt}$$
$$= e^{\int 3\ dt}$$
$$= e^{3t}$$

$$\star\int e^{3t}\sin(30t)\ dt$$

$$\begin{aligned} u &= e^{3t} & dv &= \sin(30t)\ dt \\ du &= 3e^{3t}\ dt & v &= -\tfrac{1}{30}\cos(30t) \end{aligned}$$

$$\int e^{3t}\sin(30t)\ dt = -\frac{1}{30}e^{3t}\cos(30t) + \frac{1}{10}\int e^{3t}\cos(30t)\ dt$$

$$\begin{aligned} u &= e^{3t} & dv &= \cos(30t)\ dt \\ du &= 3e^{3t}\ dt & v &= \tfrac{1}{30}\sin(30t) \end{aligned}$$

$$\int e^{3t}\sin(30t)\,dt = -\frac{1}{30}e^{3t}\cos(30t) + \frac{1}{10}\left[\frac{1}{30}e^{3t}\sin(30t) - \frac{1}{10}\int e^{3t}\sin(30t)\,dt\right]$$

$$\int e^{3t}\sin(30t)\ dt = -\frac{1}{30}e^{3t}\cos(30t) + \frac{1}{300}e^{3t}\sin(30t) - \frac{1}{100}\int e^{3t}\sin(30t)\ dt$$

$$\frac{101}{100}\int e^{3t}\sin(30t)\ dt = -\frac{1}{30}e^{3t}\cos(30t) + \frac{1}{300}e^{3t}\sin(30t) + K$$

$$\int e^{3t}\sin(30t)\ dt = \frac{100}{101}\left[-\frac{1}{30}e^{3t}\cos(30t) + \frac{1}{300}e^{3t}\sin(30t) + K\right]$$

$$\int e^{3t}\sin(30t)\ dt = -\frac{10}{303}e^{3t}\cos(30t) + \frac{1}{303}e^{3t}\sin(30t) + K$$

3. In a simple circuit, a battery supplies a constant voltage of 40 V, the inductance is 2 H and the resistance is 10 Ω. If the initial current is 0, find the current as a function of time.

$$L\frac{di}{dt} + Ri = E(t)$$
$$2\frac{di}{dt} + 10i = 40$$
$$\frac{di}{dt} + 5i = 20$$
$$e^{5t}\left[\frac{di}{dt} + 5i\right] = 20e^{5t}$$
$$\frac{d}{dt}\left[ie^{5t}\right] = 20e^{5t}$$
$$d\left[ie^{5t}\right] = 20e^{5t}\ dt$$
$$\int d\left[ie^{5t}\right] = 20\int e^{5t}\ dt$$
$$ie^{5t} = \frac{20}{5}e^{5t} + K$$
$$i = 4 + Ke^{-5t}$$
$$0 = 4 + Ke^{0}$$
$$K = -4$$
$$i = 4 - 4e^{-5t}$$

$$e^{\int P(t)\ dt}$$
$$= e^{\int 5\ dt}$$
$$= e^{5t}$$

4. A circuit contains an electromotive force, a capacitor with capacitance of C farads and a resistor with resistance R ohms. The voltage drop across the capacitor is Q/C where Q is the charge (in coulombs). Suppose the resistance is 5 Ω, the capacitance is 0.05 F and a batter gives a constant voltage of 60 V. If the initial charge in the circuit is 0, find the charge and current as functions of time t.

$$R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

$$5\frac{dQ}{dt} + \frac{Q}{(1/20)} = 60$$

$$5\frac{dQ}{dt} + 20Q = 60$$

$$\frac{dQ}{dt} + 4Q = 12$$

$$e^{4t}\left[\frac{dQ}{dt} + 4Q\right] = 12e^{4t}$$

$$\frac{d}{dt}\left[Qe^{4t}\right] = 12e^{4t}$$

$$d\left[Qe^{4t}\right] = 12e^{4t}\ dt$$

$$\int d\left[Qe^{4t}\right] = 12\int e^{4t}\ dt$$

$$Qe^{4t} = \frac{12}{4}e^{4t} + K$$

$$Q = 3 + Ke^{-4t}$$

$$0 = 3 + Ke^{0}$$

$$K = -3$$

$$Q = 3 - 3e^{-4t}$$

$$i = \frac{dQ}{dt} = 0 - 3\left(-4e^{-4t}\right) = 12e^{-4t}$$