Name: ______________________________

Section: ______________________________

R·I·T School of Mathematical Sciences

13 - Newtonian Mechanics

MATH 211

Consider a falling object influenced by gravity. There are two forces acting on the object.
Let F_1 be the force due to gravity. State F_1.

$$F_1 = mg$$

Let F_2 be the force due to air resistance which is directly proportional to the instantaneous velocity. State F_2.

$$F_2 = kv$$

Find an equation for the net force $F = F_1 + F_2$.

$$F = mg - kv$$

Using Newton's Second Law of Motion, rewrite your previous equation as a differential equation.

$$ma = mg - kv$$

$$m\frac{dv}{dt} = mg - kv$$

$$m\frac{dv}{dt} = F - kv$$

1. A boat with a mass of 10 slugs is being towed at 3 feet per second. The tow rope is then cut and the motor, that exerts a force of 20 lb on the boat, is started. If the water exerts a retarding force that numerically equals twice the velocity, what is the velocity of the boat 3 minutes later?

$$m\frac{dv}{dt} = F - kv$$

$$10\frac{dv}{dt} = 20 - 2v$$

$$\frac{dv}{dt} = 2 - \frac{1}{5}v$$

$$\frac{dv}{dt} + \frac{1}{5}v = 2$$

$$e^{\int {}^{1}/_{5}\ dt} = e^{{}^{t}/_{5}}$$

$$e^{{}^{t}/_{5}}\left[\frac{dv}{dt} + \frac{1}{5}v\right] = 2e^{{}^{t}/_{5}}$$

$$\frac{d}{dt}\left[ve^{{}^{t}/_{5}}\right] = 2e^{{}^{t}/_{5}}$$

$$\int d\left[ve^{{}^{t}/_{5}}\right] = 2\int e^{{}^{t}/_{5}}\ dt$$

$$ve^{{}^{t}/_{5}} = 2(10)e^{{}^{t}/_{5}} + C$$

$$v = 20 + Ce^{-{}^{t}/_{5}}$$

$$3 = 20 + Ce^{0}$$

$$-17 = C$$

$$v(t) = 20 - 17e^{-{}^{t}/_{5}}$$

$$3 \text{ minutes} = 180 \text{ seconds}$$

$$v(180) = 20 - 17e^{-{}^{180}/_{5}}$$

$$= 20 - 17e^{-36}$$

2. A 50 kg mass is shot from a cannon straight up with an initial velocity of 10 m/sec. If air resistance is equal to 5 times the instantaneous velocity of the mass, determine the velocity of the mass as a function of time.

$$m\frac{dv}{dt} = mg - kv$$

$$50\frac{dv}{dt} = 50(-9.81) - 5v$$

$$\frac{dv}{dt} = -\frac{981}{100} - \frac{1}{10}v$$

$$\frac{dv}{dt} + \frac{1}{10}v = -\frac{981}{100}$$

$$e^{\int {}^{1}/_{10}\ dt} = e^{{}^{t}/_{10}}$$

$$e^{{}^{t}/_{10}}\left[\frac{dv}{dt} + \frac{1}{10}v\right] = -\frac{981}{100}e^{{}^{t}/_{10}}$$

$$\frac{d}{dt}\left[ve^{{}^{t}/_{10}}\right] = -\frac{981}{100}e^{{}^{t}/_{10}}$$

$$\int d\left[ve^{{}^{t}/_{10}}\right] = -\frac{981}{100}\int e^{{}^{t}/_{10}}\ dt$$

$$ve^{{}^{t}/_{10}} = -\frac{981}{100}(10)e^{{}^{t}/_{10}} + C$$

$$v = -\frac{981}{10} + Ce^{-{}^{t}/_{10}}$$

$$10 = -\frac{981}{10} + Ce^{0}$$

$$\frac{1081}{10} = C$$

$$v(t) = -\frac{981}{10} + \frac{1081}{10}e^{-{}^{t}/_{10}}$$

3. A Saturn V rocket is launched with an orthogonal (perpendicular) trajectory to the earth's surface and initial velocity of 500 meters per second. If the mass of the rocket is 2,290,000 kilograms and the air resistance is negligble due to the aerodynamic design of the rocket, find its velocity as a function of time. When will the rocket leave the earth's atmosphere? [Use the FAI standard of 62 miles or 997800 meters above the earth's surface for this.]

$$m\frac{dv}{dt} = mg - kv$$

$$2,290,000\frac{dv}{dt} = 2,290,000(9.81) - 0$$

$$\frac{dv}{dt} = 9.81$$

$$v = 9.81t + C$$

$$500 = 9.81(0) + C$$

$$C = 500$$

$$v(t) = 9.81t + 500$$

$$s(t) = \frac{9.81t^2}{2} + 500t + C$$

$$997,800 = 0 + 0 + C$$

$$s(t) = \frac{9.81}{200}t^2 + 500t + 997,800$$