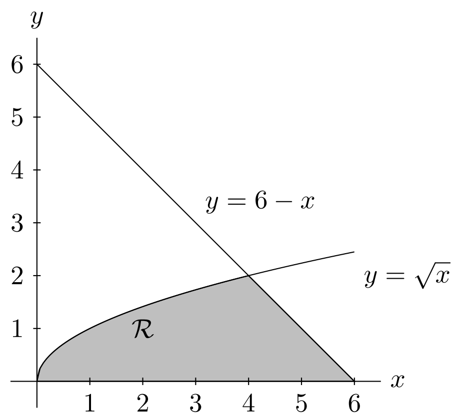
R·I·T School of Mathematical Sciences

6 - Double Integrals and Area

MATH 211

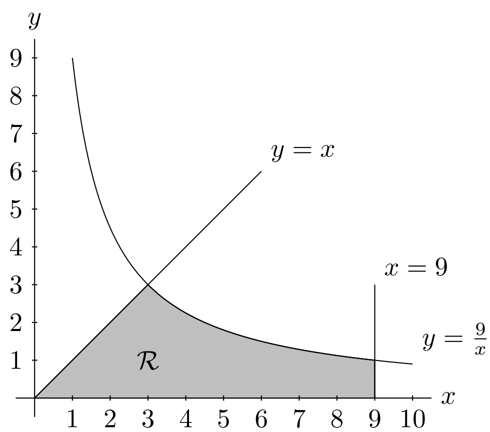
Find the area of the region bounded by $y = 6 - x$, $y = \sqrt{x}$ and $y = 0$. You MUST sketch the region and the typical rectangle, labeling the differential.



$$
\begin{aligned}
A &= \int_0^2 \left[(6-y) - \left(y^2\right)\right]\,dy \\
&= \left[6y - \frac{y^2}{2} - \frac{y^3}{3}\right]_0^2 \\
&= \left[12 - 2 - \frac{8}{3}\right] - 0 \\
&= \frac{22}{3}
\end{aligned}
$$

$$
\begin{aligned}
A &= \int_0^4 \sqrt{x}\,dx + \int_4^6 (6-x)\,dx \\
&= \left[\frac{2}{3}x^{3/2}\right]_0^4 + \left[6x - \frac{x^2}{2}\right]_4^6 \\
&= \frac{2}{3}\left[4^{3/2} - 0\right] + \left[36 - \frac{36}{2}\right] - \left[24 - \frac{16}{2}\right] \\
&= \frac{2}{3}(8) + 36 - 18 - 24 + 8 \\
&= \frac{16}{3} + 2 \\
&= \frac{22}{3}
\end{aligned}
$$

Find the area of the region bounded by $xy = 9$, $y = x$, $y = 0$ and $x = 9$ using double integration. You MUST sketch the region.



$$\mathcal{R}_1 : \begin{cases} 0 \le x \le 3 \\ 0 \le y \le x \end{cases} \qquad \mathcal{R}_2 : \begin{cases} 3 \le x \le 9 \\ 0 \le y \le \frac{9}{x} \end{cases}$$

$$\begin{aligned} A &= \int_0^3 \int_0^x \, dy \; dx + \int_3^9 \int_0^{9/x} \, dy \; dx \\ &= \int_0^3 (x - 0) \; dx + \int_3^9 \left(\frac{9}{x} - 0 \right) \; dx \\ &= \left[\frac{x^2}{2} \right]_0^3 + 9 \left[\ln |x| \right]_3^9 \\ &= \frac{9}{2} - 0 + 9 \left[\ln 9 - \ln 3 \right] \\ &= \frac{9}{2} + 9 \ln 3 \\ &= \frac{9}{2} + \ln 19683 \end{aligned}$$