Compartmental Analysis

MATH 211

1. .

$$\frac{dx}{dt} = 5 - \frac{5x}{100}$$

$$\frac{dx}{dt} + \frac{x}{20} = 5$$

$$e^{t/20}\left[\frac{dx}{dt} + \frac{x}{20}\right] = 5e^{t/20}$$

$$\frac{d}{dt}[xe^{t/20}] = 5e^{t/20}$$

$$\int d[xe^{t/20}] = 5\int e^{t/20}dt$$

$$xe^{t/20} = 5(20)e^{t/20} + C$$

$$x = 100 + Ce^{-t/20}$$

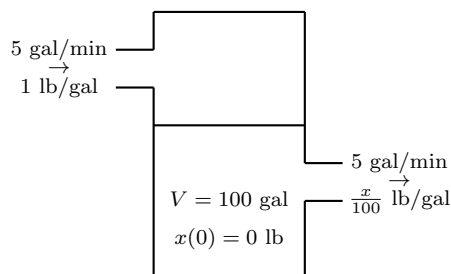$$0 = 100 + Ce^0$$

$$C = -100$$

$$x(t) = 100 - 100e^{-t/20}$$

$$x(1) = 100 - 100e^{-1/20} \approx 4.877 \text{ lb}$$

$$499 = 100 - 100e^{-t/20}$$

$$399 = -100e^{-t/20}$$

$$-3.99 = e^{-t/20}$$

Never! $e^{-t/20} > 0$

5 gal/min
$\xrightarrow{\;}$
1 lb/gal

5 gal/min
$\xrightarrow{\;}$

$V = 100$ gal $\quad \frac{x}{100}$ lb/gal

$x(0) = 0$ lb

$$e^{\int P(t)dt} = e^{\int \frac{1}{20}dt} = e^{t/20}$$

2. .

$$\frac{dx}{dt} = 5 - \frac{5x}{100}$$

$$\frac{dx}{dt} + \frac{x}{20} = 5$$

$$e^{t/20}\left[\frac{dx}{dt} + \frac{x}{20}\right] = 5e^{t/20}$$

$$\frac{d}{dt}[xe^{t/20}] = 5e^{t/20}$$

$$\int d[xe^{t/20}] = 5\int e^{t/20}\,dt$$

$$xe^{t/20} = 5(20)e^{t/20} + C$$

$$x = 100 + Ce^{-t/20}$$
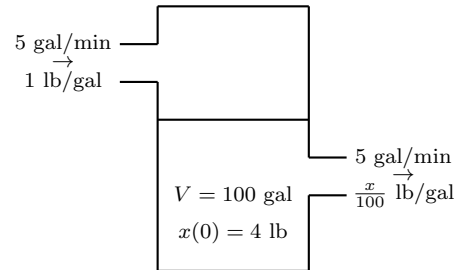
$$4 = 100 + Ce^0$$

$$C = -96$$

$$x(t) = 100 - 96e^{-t/20}$$

$$x(1) = 100 - 96e^{-1/20} \approx 8.682 \text{ lb}$$

$$499 = 100 - 96e^{-t/20}$$

$$399 = -96e^{-t/20}$$

Never! $e^{-t/20} > 0$

5 gal/min
$\rightarrow$
1 lb/gal

5 gal/min
$\rightarrow$
$\frac{x}{100}$ lb/gal

$V = 100$ gal

$x(0) = 4$ lb

$$e^{\int P(t)dt} = e^{\int \frac{1}{20}dt} = e^{t/20}$$

3. .

$$\frac{dx}{dt} = 10 - \frac{5x}{400}$$

$$\frac{dx}{dt} + \frac{x}{80} = 10$$

$$e^{t/80}\left[\frac{dx}{dt} + \frac{x}{80}\right] = 10e^{t/80}$$

$$\frac{d}{dx}[xe^{t/80}] = 10e^{t/80}$$
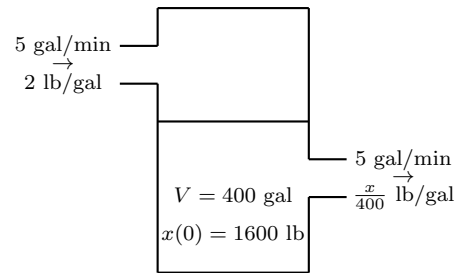
$$\int d[xe^{t/80}] = 10\int e^{t/80}dt$$

$$xe^{t/80} = 10(80)e^{t/80} + C$$

$$x = 800 + Ce^{-t/80}$$

$$1600 = 800 + Ce^0$$

$$C = 800$$

$$x(t) = 800 + 800e^{-t/80}$$

5 gal/min
$\xrightarrow{}$
2 lb/gal

5 gal/min
$\xrightarrow{}$
$\frac{x}{400}$ lb/gal

$V = 400$ gal

$x(0) = 1600$ lb

$$e^{\int P(t)dt} = e^{\int \frac{1}{80}dt} = e^{t/80}$$

4. .

$$\frac{dx}{dt} = \frac{6}{10} - \frac{6x}{1000}$$

$$\frac{dx}{dt} + \frac{3x}{500} = \frac{3}{5}$$

$$e^{3t/500}\left[\frac{dx}{dt} + \frac{3x}{500}\right] = \frac{3}{5}e^{3t/500}$$

$$\frac{d}{dt}[xe^{3t/500}] = \frac{3}{5}e^{3t/500}$$

$$\int d[xe^{3t/500}] = \frac{3}{5}\int e^{3t/500}\,dt$$

$$xe^{3t/500} = \frac{3}{5}\cdot\frac{500}{3}e^{3t/500} + C$$

$$x = 100 + Ce^{-3t/500}$$

$$0 = 100 + Ce^{0}$$
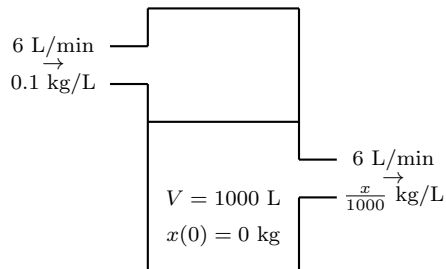
$$C = -100$$

$$x(t) = 100 - 100e^{-3t/500}$$

$$\frac{1}{20} = 100 - 100e^{-3t/500}$$

$$-\frac{1999}{20} = -100e^{-3t/500}$$

$$\frac{1999}{2000} = e^{-3t/500}$$

$$\ln\left(\frac{1999}{2000}\right) = -\frac{3t}{500}$$

$$t = -\frac{500}{3}\ln\left(\frac{1999}{2000}\right) \approx 0.08335 \text{ min}$$

6 L/min
$\xrightarrow{}$
0.1 kg/L

$V = 1000$ L
$x(0) = 0$ kg

6 L/min
$\xrightarrow{}$
$\frac{x}{1000}$ kg/L

$$e^{\int P(t)dt} = e^{\int \frac{3}{500}dt} = e^{3t/500}$$