10 - First Order Linear Equations I

MATH 211

Differentiate ye^{x^2} to show that:

$$\frac{d}{dx}\left[ye^{x^2}\right] = e^{x^2}\left[\frac{dy}{dx} + 2xy\right]$$

The left hand side requires a product rule and implicit differentiation.

$$\frac{d}{dx}[y] \cdot e^{x^2} + y \cdot \frac{d}{dx}\left[e^{x^2}\right] = e^{x^2}\left[\frac{dy}{dx} + 2xy\right]$$

$$\frac{dy}{dx} \cdot e^{x^2} + y \cdot 2xe^{x^2} = e^{x^2}\left[\frac{dy}{dx} + 2xy\right]$$

$$e^{x^2}\left[\frac{dy}{dx} + 2xy\right] = e^{x^2}\left[\frac{dy}{dx} + 2xy\right]$$

Find the general solution to the first order linear equation.

$$\frac{dy}{dx} + xy = x$$

$$P(x) = x$$

$$e^{\int P(x)\ dx} = e^{\int x\ dx} = e^{x^2/2}$$

$$e^{x^2/2}\left[\frac{dy}{dx} + xy\right] = xe^{x^2/2}$$

$$\frac{d}{dx}\left[ye^{x^2/2}\right] = xe^{x^2/2}$$

$$d\left[ye^{x^2/2}\right] = xe^{x^2/2}\ dx$$

$$\int d\left[ye^{x^2/2}\right] = \int xe^{x^2/2}\ dx$$

$$u = x^2/2$$

$$du = x\ dx$$

$$ye^{x^2/2} = \int e^u\ du$$

$$ye^{x^2/2} = e^u + C$$

$$ye^{x^2/2} = e^{x^2/2} + C$$

$$y = 1 + Ce^{-x^2/2}$$