R·I·T School of Mathematical Sciences

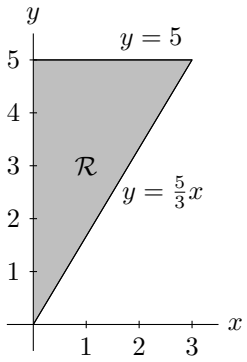
7 - Double Integrals and Volume

MATH 211

Set up but DO NOT evaluate an integral for each order of integration

$$\int_R \int xydA$$

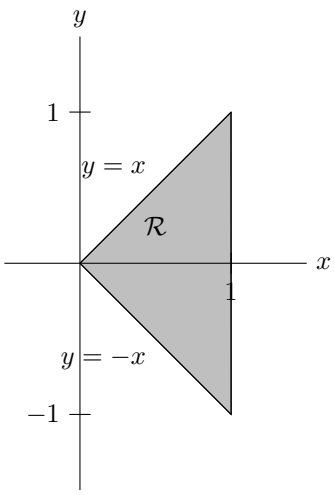
where R : {the triangular region with vertices at $(0,0),(0,5),(3,5)$}.



$$\mathcal{R}: \begin{cases} 0 \le x \le 3 \\ \frac{5}{3}x \le y \le 5 \end{cases} \qquad \mathcal{R}: \begin{cases} 0 \le y \le 5 \\ 0 \le x \le \frac{3}{5}y \end{cases}$$

$$\int_R \int xydA = \int_0^3 \int_{5x/3}^5 xy\ dy\ dx \qquad \int_R \int xydA = \int_0^5 \int_0^{3y/5} xy\ dx\ dy$$

Find the volume of the solid under the surface $z = 3 + x^2 - 2y$ and above the region in the xy-plane bounded by $y = x$, $y = -x$, $x = 0$, and $x = 1$.



$$\mathcal{R} : \begin{cases} 0 \le x \le 1 \\ -x \le y \le x \end{cases}$$

$$V = \int_0^1 \int_{-x}^{x} (3 + x^2 - 2y)\, dy\, dx$$

$$= \int_0^1 \left[3y + x^2 y - y^2\right]_{-x}^{x} dx$$

$$= \int_0^1 \left[\left(3x + x^3 - x^2\right) - \left(-3x - x^3 - x^2\right)\right] dx$$

$$= \int_0^1 \left(6x + 2x^3\right) dx$$

$$= \left[3x^2 + \frac{x^4}{2}\right]_0^1$$

$$= 3 + \frac{1}{2}$$

$$= \frac{7}{2}$$