Key

Name: ______________________

Section: ______________________

R·I·T School of Mathematical Sciences

Homework 5

MATH 211

1. Find the general solutions to the following homogeneous equations.

   (a) $y'' + 6y' + 5y = 0$

   $m^2 + 6m + 5 = 0$

   $(m+1)(m+5) = 0$

   $m_1 = -1 \quad m_2 = -5$

   $y = c_1 e^{-x} + c_2 e^{-5x}$

   (b) $y'' + 9y = 0$

   $m^2 + 9 = 0$

   $m^2 = -9$

   $m = \pm 3i$

   $y = c_1 \sin 3t + c_2 \cos 3t$

2. Solve the following initial value problem.

$$y'' + 6y' + 9y = 0,\ y(0) = 0,\ y'(0) = 1$$

$m^2 + 6m + 9 = 0$

$(m+3)(m+3) = 0$

$m_1 = m_2 = -3$

$y_c = c_1 e^{-3x} + c_2 x e^{-3x} = y$

$y' = -3c_1 e^{-3x} - 3c_2 x e^{-3x} + c_2 e^{-3x}$

$y(0) = 0$

$0 = c_1$

$y'(0) = 1$

$1 = c_2 e^{0}$

$1 = c_2$

$y = x e^{-3x}$

3. Find the general solutions to the following nonhomogeneous equations.

(a) $y'' + 5y' + 6y = e^x$

$m^2+5m+6=0$

$(m+3)(m+2)=0$

$m_1=-3 \quad m_2=-2$

$y_c = c_1 e^{-3x} + c_2 e^{-2x}$

$y_p = Ae^x$

$y_p' = Ae^x$

$y_p'' = Ae^x$

$A(e^x+5e^x+6e^x) = e^x$

$12Ae^x = e^x$

$12A = 1$

$A = \frac{1}{12}$

$y_p = \frac{1}{12}e^x$

$$\boxed{y = c_1 e^{-3x} + c_2 e^{-2x} + \frac{1}{12}e^x}$$

(b) $y''' - 2y'' = \sin x$

$m^3-2m^2=0$

$m^2(m-2)=0$

$m_1=m_2=0 \quad m_3=2$

$y_c = c_1 + c_2 x + c_3 e^{2x}$

$y_p = A\sin x + B\cos x$

$y_p' = A\cos x - B\sin x$

$y_p'' = -A\sin x - B\cos x$

$y_p''' = -A\cos x + B\sin x$

$-A\cos x + B\sin x - 2(-A\sin x - B\cos x) = \sin x + 0\cos x$

$\cos x(-A+2B) + \sin x(B+2A) = \sin x + 0\cos x$

$-A+2B=0 \quad B+2A=1$

$A=2B \quad B+2(2B)=1$

$A = \frac{2}{5} \quad 5B=1$

$B = \frac{1}{5}$

$$\boxed{y = c_1 + c_2 x + c_3 e^{2x} + \frac{2}{5}\sin x + \frac{1}{5}\cos x}$$

4. Solve the following initial value problem.

$$y'' - 5y' + 6y = e^{3x},\ y(0) = 0,\ y'(0) = 0$$

$m^2 - 5m + 6 = 0$

$\cancel{(m-6)(m+1) = 0}$

$\cancel{m_1 = 6 \quad m_2 = -1}$

$\cancel{y_c = c_1 e^{6x} + c_2 e^{-x}}$

common factoring error

$m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m_1 = 2 \quad m_2 = 3$

$y_c = c_1 e^{2x} + c_2 e^{3x}$

for linear independence with $c_2 e^{3x}$

$y_p = Axe^{3x}$

$y_p' = 3Axe^{3x} + Ae^{3x}$

$y_p'' = 3 \cdot 3Axe^{3x} + 3Ae^{3x} + 3Ae^{3x}$

$9Axe^{3x} + 6Ae^{3x} - 5(3Axe^{3x} + Ae^{3x}) + 6(Axe^{3x}) = e^{3x}$

$xe^{3x}(9A - 15A + 6A) + e^{3x}(6A - 5A) = 1e^{3x} + 0xe^{3x}$

$0A = 0$ $\qquad$ $A = 1$

$0 = 0$

$y = y_c + y_p$

$y = c_1 e^{2x} + c_2 e^{3x} + xe^{3x}$

$y(0) = 0 \Rightarrow 0 = c_1 + c_2$

$c_1 = -c_2$

$y' = 2c_1 e^{2x} + 3c_2 e^{3x} + 3xe^{3x} + e^{3x}$

$y'(0) = 0$

$0 = 2c_1 + 3c_2 + 1$

$-1 = -2c_2 + 3c_2$

$-1 = c_2$

$1 = c_1$

Final:

$y = e^{2x} - e^{3x} + xe^{3x}$