R·I·T School of Mathematical Sciences

Newtonian Mechanics

MATH 211

1. $m = 50$, $v(0) = 10$, $s(0) = 100$, $k = 5$

$$m\frac{dv}{dt} = mg - kv$$
$$50\frac{dv}{dt} = (50)(-9.81) - 5v$$
$$\frac{dv}{dt} = -9.81 - \frac{v}{10}$$
$$\frac{dv}{dt} + \frac{1}{10}v = -\frac{981}{100}$$
$$e^{t/10}\left[\frac{dv}{dt} + \frac{v}{10}\right] = -\frac{981}{100}e^{t/10}$$
$$\frac{d}{dt}[ve^{t/10}] = -\frac{981}{100}e^{t/10}$$
$$\int d[ve^{t/10}] = -\frac{981}{100}\int e^{t/10}dt$$
$$ve^{t/10} = -\frac{981}{10}e^{t/10} + C_1$$
$$v = -\frac{981}{10} + C_1e^{-t/10}$$
$$10 = -\frac{981}{10} + C_1e^{0}$$
$$\frac{1081}{10} = C_1$$
$$v(t) = -\frac{981}{10} + \frac{1081}{10}e^{-t/10}$$
$$s(t) = \int v(t)dt$$
$$s(t) = -\frac{981}{10}t \text{ - } 1081e^{-t/10} + C_2$$
$$100 = 0 + 1081 + C_2$$
$$C_2 = -1181$$
$$s(t) = -frac98110t + 1081e^{-t/10} - 1181$$
$0 = -frac98110t+1081e^{-t/10}-981$ use CAS to solve for t

$e^{\int P(t)dt} = e^{\int \frac{1}{10}dt} = e^{t/10}$

2. $m = \frac{960}{32.2}$, $v(0) = 60$, $k = 3$

$$m\frac{dv}{dt} = mg - kv$$
$$\frac{960}{32.2}\frac{dv}{dt} = \left(\frac{960}{32.2}\right)(-32.2) - 3v$$
$$\frac{dv}{dt} = -32.2 - 3\left(\frac{32.2}{960}\right)v$$
$$\frac{dv}{dt} + \frac{161}{1600}v = -\frac{161}{5}$$
$$e^{161t/1600}\left[\frac{dv}{dt} + \frac{161}{1600}v\right] = -\frac{161}{5}e^{161t/1600}$$
$$\frac{d}{dt}[ve^{161t/1600}] = -\frac{161}{5}e^{161t/1600}$$
$$\int d[ve^{161t/1600}] = -\frac{161}{5}\int e^{161t/1600}dt$$
$$ve^{161t/1600} = -\frac{161}{5}\cdot\frac{1600}{161}e^{161t/1600} + C$$
$$v = -320 + Ce^{-161t/1600}$$
$$60 = -320 + Ce^{0}$$
$$380 = C$$
$$v(t) = 320 + 380e^{-161t/1600}$$

$e^{\int P(t)dt} = e^{\int 161/1600dt} = e^{161t/1600}$

3. $m\frac{dv}{dt} = mg - kv$

$$m\frac{dv}{dt} = m(32.2) - \frac{1}{100}v$$
$$\frac{dv}{dt} = \frac{322}{10} - \frac{1}{100m}v$$
$$\frac{dv}{dt} + \frac{1}{100m}v = \frac{161}{5}$$
$$e^{t/100m}\left[\frac{dv}{dt} + \frac{1}{100m}v\right] = \frac{161}{5}e^{t/100m}$$
$$\frac{d}{dt}[ve^{t/100m}] = \frac{161}{5}e^{t/100m}$$
$$\int d[ve^{t/100m}] = \frac{161}{5}\int e^{t/100m}dt$$
$$ve^{t/100m} = \frac{161}{5}\cdot 100me^{t/100m} + C$$
$$v = 161(20)m + Ce^{-t/100m}$$
$$v = 3220m + Ce^{-t/100m}$$
$$0 = 3220\text{m} + C$$
$$C = -3220\text{m}$$
$$v(t) = 3220\text{m} - 3220\text{m}e^{-t/100m}$$