R·I·T School of Mathematical Sciences

8 - Introduction to First Order Differential Equations

MATH 211

Suppose that $y'' = 2e^x + \sin x$, $y(0) = 0$ and $y'(0) = 2$. Use your Calculus skills to find y as a function of x.

$$y'' = 2e^x + \sin x$$

$$y' = 2e^x - \cos x + C_1$$

$$2 = 2e^0 - \cos 0 + C_1$$

$$2 = 2 - 1 + C_1$$

$$C_1 = 1$$

$$y' = 2e^x - \cos x + 1$$

$$y = 2e^x - \sin x + x + C_2$$

$$0 = 2e^0 - \sin 0 + (0) + C_2$$

$$0 = 2 + C_2$$

$$C_2 = -2$$

$$y = 2e^x - \sin x + x - 2$$

Find a first order differential equation whose solution is $y = \tan^{-1} x - 2^x + 1$.

$$\frac{dy}{dx} = \frac{1}{1+x^2} - 2^x \ln 2$$

Show that $x(t) = 2\cos t - 2\sin t$ is a solution to the differential equation $x'' + x = 0$.

$$x'(t) = -2\sin t - 2\cos t$$

$$x''(t) = -2\cos t + 2\sin t$$

$$x'' + x = 0$$

$$-2\cos t + 2\sin t + 2\cos t - 2\sin t = 0$$

$$0 = 0$$

What is a general solution to the differential equation $x'' + x = 0$?

$$x(t) = c_1 \sin t + c_2 \cos t$$

Give an example of a particular solution to the differential equation $x'' + x = 0$.

$$x(t) = \sin t + \cos t$$