Name: ______________________

Section: ______________________

R·I·T School of Mathematical Sciences

15 - Introduction to Higher Order Linear Equations

MATH 211

Write the n^{th} order linear differential equation in general form.

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_2(x)y'' + a_1(x)y' + a_0(x)y = f(x)$$

Now, write the equation in homogeneous form.

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$$

Write the second order, linear, homoegenous differential equation in general form.

$$a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$$

Verify that $y_1 = e^{-x}$ and $y_2 = e^{-2x}$ are both solutions to the equation

$$y'' + 3y' + 2y = 0$$

$$\begin{aligned} y_1 &= e^{-x} \\ y_1' &= -e^{-x} \\ y_1'' &= e^{-x} \\ y_1'' + 3y_1' + 2y_1 &= 0 \\ e^{-x} + 3\left(-e^{-x}\right) + 2e^{-x} &= 0 \\ e^{-x} - 3e^{-x} + 2e^{-x} &= 0 \\ 0 &= 0 \checkmark \end{aligned}$$

$$\begin{aligned} y_2 &= e^{-2x} \\ y_2' &= -2e^{-2x} \\ y_2'' &= 4e^{-2x} \\ y_2'' + 3y_2' + 2y_2 &= 0 \\ 4e^{-2x} + 3\left(-2e^{-2x}\right) + 2e^{-2x} &= 0 \\ 4e^{-2x} - 6e^{-2x} + 2e^{-2x} &= 0 \\ 0 &= 0 \checkmark \end{aligned}$$

Use Reduction of Order to find a second solution to the differential equation $y'' + 4y' + 4y = 0$ provided that $y_1 = e^{-2x}$ is a known solution.

$$y_2 = u(x)y_1 \text{ or } y_2 = uy_1$$

$$y_2 = ue^{-2x}$$
$$y_2' = u'e^{-2x} - 2ue^{-2x}$$
$$y_2'' = u''e^{-2x} - 2u'e^{-2x} - 2u'e^{-2x} + 4ue^{-2x}$$

$$y_2'' + 4y_2' + 4y_2 = 0$$
$$u''e^{-2x} - 4u'e^{-2x} + 4ue^{-2x} + 4\left(u'e^{-2x} - 2ue^{-2x}\right) + 4ue^{-2x} = 0$$
$$e^{-2x}\left(u'' - 4u' + 4u + 4u' - 8u + 4u\right) = 0$$
$$e^{-2x} \neq 0 \qquad u'' = 0$$
$$u' = C_1$$
$$u = C_1x + C_2$$
$$\text{Let } C_1 = 1 \text{ and } C_2 = 0$$
$$u = x$$
$$y_2 = uy_1$$
$$y_2 = xe^{-2x}$$