1. The body mass index (BMI) of an adult human is given by the function $B = w/h^2$ where w is the mass measured in kilograms and h is height measured in meters. Find the total differential that approximates the BMI when weight increases from 50 to 52 kilograms and the height increases from 1 to 1.25 meters.

Integral – 0
ΔY – 0

$B = \frac{w}{h^2}$ $\quad dw = 2 \quad dh = 0.25 = \frac{1}{4}$

(3) $dB = \frac{1}{h^2}\,dw - \frac{2w}{h^3}\,dh$

$\frac{\partial B}{\partial w} = \frac{1}{h^2}$ (2)

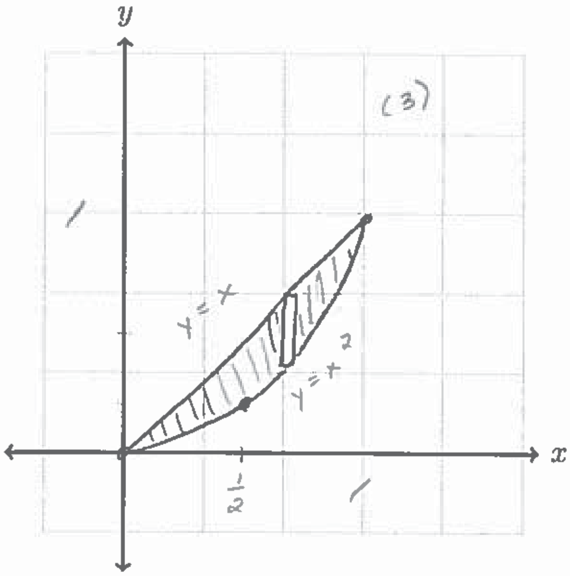
$dB = \frac{1}{(1)^2}(2) - \frac{2(50)}{(1)^3}\left(\frac{1}{4}\right)$

$\frac{\partial B}{\partial h} = \frac{2w}{h^3}$ (2)

$dB = 2 - 25$

(3) $dB = -23$

2. Find the volume of the solid below the plane $z = 4x + 3y + 1$ and above the xy-plane over the region bounded by $y = x$, $y = x^2$. You MUST sketch the region (shaded and labeled) with its typical rectangle and element of integration.



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$$\int_0^1 \int_{x^2}^{x} (4x + 3y + 1)\, dy\, dx$$

$$\int_0^1 \left(4xy + \frac{3y^2}{2} + y\right)\Big]_{x^2}^{x} dx$$

$$\int_0^1 \left[\left(4x^2 + \frac{3}{2}x^2 + x\right) - \left(4x^3 + \frac{3}{2}x^4 + x^2\right)\right] dx$$

$$\int_0^1 \left(\frac{9}{2}x^2 + x - 4x^3 - \frac{3}{2}x^4\right) dx$$

$$\frac{3}{2}x^3 + \frac{x^2}{2} - x^4 - \frac{3}{10}x^5 \Big]_0^1$$

(6) $\frac{3}{2} + \frac{1}{2} - 1 - \frac{3}{10} = \frac{7}{10}$

3. An 1kg object is dropped from the top of a building and experiences air resistance numerically equal to twice the velocity. Find the velocity of the object as a function of time. You MUST solve this equation using separation of variables.

$$m\frac{dv}{dt} = mg - kv$$

wrong method - 0

$$1 \cdot \frac{dv}{dt} = (1)(9.8) - 2v$$

$$\frac{dv}{dt} \quad 9.8 - 2v$$

(5) $$\int \frac{dv}{9.8 - 2v} = \int dt$$

(4) $$-\frac{1}{2} \ln |9.8 - 2v| = t + C$$

$$(9.8 - 2v)^{-1/2} = Ce^{t}$$

(4) $$9.8 - 2v = Ce^{-2t}$$

$$v = Ce^{-2t} + 4.9$$

$$v(0) = 0$$

$$c = -4.9$$

(2) $$v = 4.9 - 4.9e^{-2t}$$

4. A simple RL-circuit has inductance of 2H, resistance 4Ω and a battery source of $E(t) = e^{-t}$V. Find the current as a function of time. You MUST solve this equation using the first order linear integrating factor. $i(0) = 0$

$$L\frac{di}{dt} + Ri = E$$

$$2\frac{di}{dt} + 4i = e^{-t}$$

(2) $$\frac{di}{dt} + 2i = \frac{1}{2}e^{-t} \qquad e^{\int 2dt} = e^{2t}$$

(3) $$e^{2t}\left[\frac{di}{dt} + 2i\right] = \frac{1}{2}e^{-t} \cdot e^{2t}$$

$$\frac{d}{dt}\left[i \cdot e^{2t}\right] = \frac{1}{2}e^{t}$$

(3) $$\int d\left[i \cdot e^{2t}\right] = \int \frac{1}{2}e^{t}\,dt$$

(3) $$i \cdot e^{2t} = \frac{1}{2}e^{t} + C$$

$$i = \frac{1}{2}e^{-t} + Ce^{-2t} \qquad i(0) = 0$$

(2) $$0 = \frac{1}{2} + C \qquad C = -\frac{1}{2}$$

(2) $$i = \frac{1}{2}e^{-t} - \frac{1}{2}e^{-2t}$$

5. A mass of $\frac{1}{2}$kg is attached to a spring, stretched 1 meter past equailibrium and then released. The strength of the spring is 2N/m. No external force is applied to the spring until 4 seconds later when an electromagnet is switched on and applies a continual force of 5N. Find the equation of motion of the spring, assuming no damping.

$$mx'' + bx' + kx = f(t)$$ $x(0) = 1$ $x'(0) = 0$

$$\frac{1}{2}x'' + 2x = 5u(t-4)$$ T.10 $a = 4$

(2) $$x'' + 4x = 10u(t-4)$$

(3) $$s^2\mathcal{L}\{x\} - s\,x(0) - x'(0) + 4\mathcal{L}\{x\} = \frac{10e^{-4s}}{s}$$

(2) $$\mathcal{L}\{x\}(s^2+4) - s = \frac{10e^{-4s}}{s}$$

$$\mathcal{L}\{x\} = e^{-4s}\left(\frac{10}{s(s^2+4)}\right) + \frac{s}{s^2+4}$$

$$\frac{10}{s(s^2+4)} = \frac{A}{s} + \frac{Bs+C}{s^2+4}$$

$$10 = A(s^2+4) + (Bs+C)s$$

$$10 = As^2 + 4A + Bs^2 + Cs$$

$$A = \frac{5}{2} \qquad B = -\frac{5}{2} \qquad 0 = C$$

(4) $$\mathcal{L}\{x\} = e^{-4s}\left[\frac{5}{2}\cdot\frac{1}{s} - \frac{5}{2}\cdot\frac{s}{s^2+4}\right] + \frac{s}{s^2+4}$$

(4) $$x = \left[\frac{5}{2} - \frac{5}{2}\cos[2(t-4)]\right]u(t-4) + \cos(2t)$$

6. A 10m long beam is embedded at $x = 0$ and free at the other end. Find the equation of the deflection of the beam if a load of $w(x) = 24EI$ is uniformly distributed along its length. You MUST solve this equation using the Method of Undetermine Coefficients.

$$EIy^{(4)} = w(x)$$

$y(0)=0$ $\quad y''(10)=0$

$y'(0)=0$ $\quad y'''(10)=0$

$y^{(4)} = 24$

$y_c = c_1 + c_2 x + c_3 x^2 + c_4 x^3$ (4)

$y_p = Ax^4$ (4)

$y_p' = 4Ax^3$

$y_p'' = 12Ax^2$

$y_p''' = 24Ax$

$y_p^{(4)} = 24A$

$24A = 24$

$A = 1$

$y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + x^4$

$y(0)=0$ $\quad y'(0)=0$

$c_1 = 0$ (2) $\quad c_2 = 0$ (2)

$y = c_3 x^2 + c_4 x^3 + x^4$

$y' = 2c_3 x + 3c_4 x^2 + 4x^3$

$y'' = 2c_3 + 6c_4 x + 12x^2$

$y''(10) = 0$

$0 = 2c_3 + 60c_4 + 1200$

$0 = c_3 + 30c_4 + 600$

$c_3 = -30c_4 - 600$

$y''' = 6c_4 + 24x$

$y'''(10) = 6c_4 + 240$

$0 = 6c_4 + 240$

$-40 = c_4$ (2)

$c_3 = -30(-40) - 600$

$c_3 = 600$ (2)

$\boxed{y = 600x^2 - 40x^3 + x^4}$ (2)

7. A simple pendulum rotates around a point, Q. The pendulum rod is $l = 2$ feet long and is released from rest at $\theta(0) = \pi/2$ radians. Find the equation of angular motion of the pendulum.

$$\theta''(t) + \frac{g}{l}\theta(t) = 0$$

$\theta(0) = \frac{\pi}{2}$

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$$\theta'' + \frac{32}{2}\theta = 0$$

$$\theta'' + 16\theta = 0$$

$$s^2\mathcal{L}\{\theta\} - s\theta(0) - \theta'(0) + 16\mathcal{L}\{\theta\} = 0$$

$$\mathcal{L}\{\theta\}(s^2+16) - \frac{\pi s}{2} = 0$$

$$\mathcal{L}\{\theta\} = \frac{\pi}{2} \cdot \frac{s}{s^2+16}$$

$$\theta = \frac{\pi}{2}\cos(4t)$$

8. Find any relative extrema or saddle points for the following function.

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$$f(x, y) = x^2 + 2y^2 - 6x + 8y + 4$$

$f_x(x,y) = 2x - 6$ $f_{xx} = 2$ $f_y(x,y) = 4y + 8$ $f_{yy} = 4$

$2x - 6 = 0$ $4y + 8 = 0$

$x = 3$ Critical Point $(3, -2)$ $y = -2$

$D = f_{xx}(x,y) \cdot f_{yy}(x,y) - [f_{xy}(x,y)]^2$

$D = (2)(4) - (0)^2 = 8$

$D > 0$, $f_{xx} > 0$

$f(3,-2) = (3)^2 + 2(-2)^2 - 6(3) + 8(-2) + 4$

$= 9 + 8 - 18 - 16 + 4$

$= -13$

Relative Minimum @ $(3, -2, -13)$

Laplace Transforms

	$f(t) = \mathscr{L}^{-1}\{F(s)\}$	$F(s) = \mathscr{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{-at}	$\dfrac{1}{s+a}$
3.	$\cos(at)$	$\dfrac{s}{s^2+a^2}$
4.	$\sin(at)$	$\dfrac{a}{s^2+a^2}$
5.	$t^{n-1}e^{-at}$	$\dfrac{(n-1)!}{(s+a)^n}$
6.	$t\sin(at)$	$\dfrac{2as}{(s^2+a^2)^2}$
7.	$t\cos(at)$	$\dfrac{s^2-a^2}{(s^2+a^2)^2}$
8.	$e^{-at}\sin(bt)$	$\dfrac{b}{(s+a)^2+b^2}$
9.	$e^{-at}\cos(bt)$	$\dfrac{s+a}{(s+a)^2+b^2}$
10.	$u(t-a)$	$\dfrac{e^{-as}}{s}$
11.	$\delta(t-a)$	e^{-as}
12.	$f(t)u(t-a)$	$e^{-as}\mathscr{L}\{f(t+a)\}$
13.	$f(t-a)u(t-a)$	$e^{-as}F(s)$
14.	$y^{(n)}$	$s^n\mathscr{L}\{y\}-s^{n-1}y(0)-s^{n-2}y'(0)-\cdots-y^{(n-1)}(0)$