Name: _____

Section: _____

R·I·T School of Mathematical Sciences

28 - First Order Applications with Laplace Transforms

MATH 211

1. A boat with a mass of 10 slugs is being towed at 3 feet per second. The tow rope is then cut and the motor, that exerts a force of 20 lb on the boat, is started one minute later. If the water exerts a retarding force that numerically equals twice the velocity, what is the velocity of the boat 3 minutes later?

   (a) Find a function for the force exerted by the motor.

   (b) Set up a differential equation modeling the scenario.

   (c) Solve the differential equation using Laplace Transforms to find velocity as a function of time.

2. Suppose that in a simple RL circuit the resistance is 10 Ω and the inductance is 2 H. If a battery gives a voltage of $E(t) = \sin t \; u(t - \pi)$ V and the switch is closed when $t = 0$ so the current starts at 3 mA, find the current as a function of time.

   (a) Set up a differential equation modeling the scenario.

   (b) Solve the differential equation using Laplace Transforms to find current as a function of time.

3. Pure water flows at a constant rate of 5 liters per minute into a large tank that initially held 125 liters of brine solution in which was dissolved 1 kilogram of salt. The solution inside the tank is kept well stirred and flows out at the same rate. At $t = 5$ minutes, 4 kilograms of salt are instantaneously dumped into the tank. Find a formula for the amount of salt in the tank as a function of time t. How much salt is in the tank after 10 minutes?

   (a) Find a function for the rate at which salt enters the tank.

   (b) Set up a differential equation modeling the scenario.

   (c) Solve the differential equation using Laplace Transforms to find the amount of salt as a function of time.