Name: ____________________

Section: ____________________

R·I·T SCHOOL OF MATHEMATICAL SCIENCES

21 - Spring Mass Systems

MATH 211

Newton's Second Law states that the acceleration of an object as produced by a net force is directly proportional to the magnitude of the net force, in the same direction as that net force. Use F_1 to write Newton's Second Law.

$$F_1 = ma$$

A linear system, such as a spring mass system, experiences a damping force that is directly proportional to the instantaneous velocity of the spring. Use F_2 to write this force.

$$F_2 = bv$$

Hooke's Law states that the force required to stretch or compress a spring by some distance is directly proportional to that distance. Use F_3 to write Hooke's Law.

$$F_3 = kx$$

A spring with mass m attached and spring constant k experiences a damping force with damping coefficient b. If an external force $f(t)$ is applied to stretch the spring a certain distance, write $f(t)$ as the sum of forces acting on the spring to bring it back to equilibrium (balancing equation). Then, write this equation as differential equation whose solution will model the equation of motion $x(t)$.

$$F_1 + F_2 + F_3 = f(t)$$

$$ma + bv + kx = f(t)$$

$$mx''(t) + bx'(t) + kx(t) = f(t)$$

A force of 64N is required to stretch a spring 2 m past equilibrium. A mass of 2 kg is attached to the spring with an external force of $f(t) = 68e^{-2t}$ applied to the system. Find the equation of motion of the spring if the spring is stretched 1m past equilibrium length and then released. Assume the absence of damping.

Hooke's Law:

$$F = kx$$
$$64 = k(2)$$
$$k = 32$$

$$mx'' + bx' + kx = f(t)$$
$$2x'' + 0x' + 32x = 68e^{-2t}$$
$$x'' + 16x = 34e^{-2t}$$

Complementary:

$$x'' + 16x = 0$$
$$r^2 + 16 = 0$$
$$r^2 = -16$$
$$r = \pm\sqrt{-16}$$
$$r = \pm 4i$$
$$x_c = c_1 \sin(4t) + c_2 \cos(4t)$$

Particular:

$$x_p = Ae^{-2t}$$
$$x_p' = -2Ae^{-2t}$$
$$x_p'' = 4Ae^{-2t}$$
$$x_p'' + 16x_p = 34e^{-2t}$$
$$4Ae^{-2t} + 16Ae^{-2t} = 34e^{-2t}$$
$$20Ae^{-2t} = 34e^{-2t}$$
$$20A = 34$$
$$A = \frac{17}{10}$$
$$x_p = \frac{17}{10}e^{-2t}$$

$$x = x_c + x_p$$

$$x = c_1 \sin(4t) + c_2 \cos(4t) + \frac{17}{10}e^{-2t}$$

$$x = c_1 \sin(4t) + c_2 \cos(4t) + \frac{17}{10}e^{-2t}$$
$$x(0) = 1$$
$$1 = c_1(0) + c_2(1) + \frac{17}{10}(1)$$
$$1 = c_2 + \frac{17}{10}$$
$$c_2 = -\frac{7}{10}$$

$$x' = 4c_1 \cos(4t) - 4c_2 \sin(4t) - \frac{17}{5}e^{-2t}$$
$$x'(0) = 0$$
$$0 = 4c_1(1) - 4c_2(0) - \frac{17}{5}(1)$$
$$0 = 4c_1 - \frac{17}{5}$$
$$c_1 = \frac{17}{20}$$

$$x = \frac{17}{20}\sin(4t) - \frac{7}{10}\cos(4t) + \frac{17}{10}e^{-2t}$$