Name: Key

Section:

R·I·T School of Mathematical Sciences

Homework 4

MATH 211

1. Find the general solution to the first order linear equation.

$$\sec x \frac{dy}{dx} + y = 1$$

$$\frac{dy}{dx} + \frac{y}{\sec x} = \frac{1}{\sec x}$$

$$\frac{dy}{dx} + (\cos x)y = \cos x \qquad e^{\int \cos x\, dx} = e^{\sin x}$$

$$e^{\sin x}\left(\frac{dy}{dx} + (\cos x)y\right) = (\cos x)e^{\sin x}$$

$$\int dx \cdot \frac{d}{dx}\left(ye^{\sin x}\right) = \int (\cos x)e^{\sin x}\, dx$$ must multiply by dx before integral

$$ye^{\sin x} = \int e^u\, du \qquad \begin{aligned} u &= \sin x \\ du &= \cos x\, dx \end{aligned}$$

$$ye^{\sin x} = e^u + C$$

$$ye^{\sin x} = e^{\sin x} + C$$ general implicit

$$y = 1 + Ce^{-\sin x}$$ general explicit

2. Solve the following initial value problem.

$$x\frac{dy}{dx} + y = x,\ y(1) = 3$$

$$\frac{dy}{dx} + \frac{1}{x}y = 1 \qquad e^{\int \frac{1}{x}dx} = e^{\ln|x|} = x$$

$$x\left(\frac{dy}{dx} + \frac{1}{x}y\right) = x$$

$$\int dx \cdot \frac{d}{\cancel{dx}}(yx) = \int x\,dx$$

$$yx = \frac{x^2}{2} + c \quad \text{general implicit}$$

$$3(1) = \frac{1^2}{2} + c$$

$$\frac{5}{2} = c$$

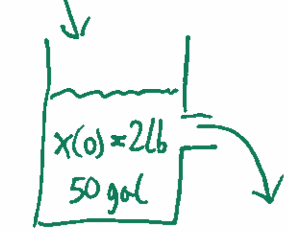
$$yx = \frac{x^2}{2} + \frac{5}{2} \quad \text{particular implicit}$$

$$y = \frac{x^2}{2x} + \frac{5}{2x}$$

$$y = \frac{x}{2} + \frac{5}{2x} \quad \text{particular explicit}$$

3. A 50 gallon tank full of a salt solution initally contains 2 pounds of salt. Pure water is added at a rate of 5 gallons per minute. If the salt solution is flowing out of the tank at the same rate, find the amount of salt in the tank as a function of time.

$$\frac{5\,gal}{min}\cdot\frac{0\,lb}{gal}=0\,\frac{lb}{min}$$



$$\frac{5\,gal}{min}\cdot\frac{x\,lb}{50\,gal}=\frac{x}{10}\,\frac{lb}{min}$$

$$\frac{dx}{dt}=0-\frac{x}{10}$$

$$e^{\frac{t}{10}}\left(\frac{dx}{dt}+\frac{1}{10}x\right)=0\cdot e^{\frac{t}{10}} \qquad e^{\int\frac{1}{10}dt}=e^{\frac{t}{10}}$$

$$\int \cancel{dx}\cdot\frac{d}{\cancel{dx}}\left(xe^{\frac{t}{10}}\right)=\int 0\cdot dx$$

$$xe^{t/10}=c$$

$$2e^{0/10}=c$$

$$2=c$$

$$xe^{t/10}=2$$

$$x(t)=2e^{-t/10}$$

4. A car of mass 500 kilograms is guided along a track by a motor exerting a force of 25 N and is subject to a resistant force numerically equal to half the velocity. If the initial velocity is 2 meters per second, find the velocity v as a function of time t.

$$m\frac{dv}{dt} = F - kv \qquad v(0) = 2$$

$$500\frac{dv}{dt} = 25 - \frac{1}{2}v$$

$$500\frac{dv}{dt} + \frac{1}{2}v = 25$$

$$e^{\frac{t}{1000}}\left(\frac{dv}{dt} + \frac{1}{1000}v\right) = \frac{1}{20}e^{t/1000} \qquad e^{\int \frac{1}{1000}dt} = e^{\frac{1}{1000}t}$$

$$\int \cancel{dt} \cdot \frac{d}{\cancel{dt}}\left(ve^{t/1000}\right) = \int \frac{1}{20}e^{\frac{t}{1000}}\,dt$$

$$ve^{t/1000} = \frac{1}{20} \cdot 1000\, e^{\frac{t}{1000}} + c$$

$$2e^{0/1000} = 50e^{\frac{0}{1000}} + c$$

$$-48 = c$$

$$ve^{\frac{t}{1000}} = 50e^{\frac{t}{1000}} - 48$$

$$v(t) = 50 - 48e^{-\frac{t}{1000}}$$

5. Suppose that in a simple circuit the resitance is 6Ω and the inductance is 2H. If a battery gives a constant voltage of $E(t) = 20e^{-3t}\sin(30t)$ volts and the switch is closed with $t = 0$ so the initial current is 0, find the current as a function of time.

no capacitance

$$L\frac{di}{dt} + Ri + \cancel{\frac{q}{C}} = E(t) \qquad i(0)=0$$

$$2\frac{di}{dt} + 6i = 20e^{-3t}\sin(30t)$$

$$e^{3t}\left(\frac{di}{dt} + 3i\right) = 10e^{-3t}\sin(30t)e^{3t} \qquad e^{\int 3dt} = e^{3t}$$

$$\int \cancel{dt}\cdot\frac{d}{\cancel{dt}}(ie^{3t}) = \int 10\cancel{e^{-3t}}\sin(30t)\cancel{e^{3t}}\,dt \qquad e^{-3t}\cdot e^{3t} = e^{0} = 1$$

$$ie^{3t} = -\frac{10}{30}\cos(30t) + K$$

$$0 = -\frac{1}{3}\cos(0) + k$$

$$\frac{1}{3} = k$$

$$i(t) = \frac{-\frac{1}{3}\cos(30t) + \frac{1}{3}}{e^{3t}} \qquad \text{not simplified}$$

$$i(t) = \frac{-\cos(30t)}{3e^{3t}} + \frac{1}{3e^{3t}}$$

$$i(t) = -\frac{\cos(30t)e^{-3t}}{3} + \frac{1}{3}e^{-3t}$$

both are acceptable forms