Applications of Laplace Transforms

MATH 211

Circuits

1. A simple RL circuit has a resistance of 2 Ω and inductance of $\frac{1}{10}$ H. If a battery gives off a voltage of $E(t) = \cos t$ and the initial current is 2 A, find the current as a function of time.

2. A simple RL circuit has a resistance of 3 Ω and inductance of 6 H. If a battery giving off a voltage of $E(t) = \sin t$ is switched off at $t = \pi$ seconds and the initial current is 1 A, find the current as a function of time.

3. A simple RL circuit has a resistance of 3 Ω, inductance of 6 H and constant voltage of 20 V. If a battery is switched on giving off a voltage of $E(t) = 60e^{-3t}$ at $t = 2$ seconds and the initial current is 1 A, find the current as a function of time.

4. An RLC circuit has a resistance of 2 Ω, capacitance of $\frac{1}{9}$ F and inductance of 1 H. If a battery gives off a voltage of $E(t) = \cos t$ and the initial current and charge are both 0 A, find the charge as a function of time.

5. An RLC circuit has a resistance of 5 Ω, capacitance of $\frac{1}{6}$ F and inductance of 1 H. If a battery giving off a voltage of $E(t) = 20t$ is switched off at $t = 3$ seconds and the initial current and charge are both 0 A, find the charge as a function of time.

Springs

1. A mass of 1 kg is attached to a spring with spring constant 9 N/m. An external force given by $f(t) = 2t$ initially stretches the spring 1 m. The spring is then released with no initial velocity. Find the equation of motion, assuming no damping.

2. A mass of 1 kg is attached to a spring with spring constant 9 N/m. An electromagnet gives off a constant force of 5 N on the spring. It is switched off 10 seconds after the spring is released from 1 m with no initial velocity. Find the equation of motion, assuming no damping.

Newtonian Mechanics

1. An object with a mass of 5 kg is dropped from the top of a building. If the air resistance working against the object in free fall is equal to the velocity of the object at any time, find the velocity of the object as a function of time.

2. A 100 kg car is moving along a track by means of a motor exerting a force of 500 N. The motor is shut off 20 seconds after the car begins its motion with a velocity of 40 meters per second and the car continues coasting. If the restricting force of the track against the tires is numerically equal to 5 times the instantaneous velocity, find the velocity as a function of time.

Compartmental Analysis

1. Pure water flows at a constant rate of 5 liters per minute into a large tank that initially held 125 liters of brine solution in which was dissolved 1 kilogram of salt. The solution inside the tank is kept well stirred and flows out at the same rate. At $t = 5$ minutes, 4 kilograms of salt are instantaneously dumped into the tank. Find a formula for the amount of salt in the tank as a function of time t. How much salt is in the tank after 10 minutes?

2. Pure water flows at a constant rate of 5 liters per minute into a large tank that initially held 125 liters of brine solution in which was dissolved 1 kilogram of salt. The solution inside the tank is kept well stirred and flows out at the same rate. At $t = 5$ minutes, the pure water is switched out for a brine solution containing 0.5 kg of salt per liter, and the rate at which the solution flows in remains 5 liters per minute. Find a formula for the amount of salt in the tank as a function of time t. How much salt is in the tank after 10 minutes?

Beams

1. A uniform beam of length 12 m has a constant load of $\omega(x) = 24EI$. If the beam is fixed at $x = 0$ and free at the other end, find the deflection of the beam.

2. A uniform beam of length 12 m has a load of $\omega(x) = 24EI \sin t$. If the beam is fixed at $x = 0$ and free at the other end, find the deflection of the beam.

3. A uniform beam of length 12 m has a concentrated load of $\omega_0 = 24EI$ at $x = 6$. If the beam is fixed at $x = 0$ and free at the other end, find the deflection of the beam.