Name: ____________________

Section: ____________________

R·I·T School of Mathematical Sciences

17 - Nonhomogeneous Equations I

MATH 211

Find values for A and B such that $y = Ae^x + Be^{-x}$ is a solution to the differential equation $y'' - 5y' + 3y = e^{-x} - 2e^x$.

$$y = Ae^x + Be^{-x}$$

$$y' = Ae^x - Be^{-x}$$

$$y'' = Ae^x + Be^{-x}$$

$$\begin{gathered} y'' - 5y' + 3y = e^{-x} - 2e^x \\ Ae^x + Be^{-x} - 5\left(Ae^x - Be^{-x}\right) + 3\left(Ae^x + Be^{-x}\right) = e^{-x} - 2e^x \\ Ae^x + Be^{-x} - 5Ae^x + 5Be^{-x} + 3Ae^x + 3Be^{-x} = e^{-x} - 2e^x \\ -Ae^x + 9Be^{-x} = e^{-x} - 2e^x \end{gathered}$$

$$\begin{array}{ll} -A = -2 & 9B = 1 \\ A = 2 & B = \frac{1}{9} \end{array}$$

Solve the initial value problem.

$$y'' - 4y = x^2 + x,\ y(0) = 0,\ y'(0) = 0$$

Complementary:

$$y'' - 4y = 0$$
$$r^2 - 4 = 0$$
$$r^2 = 4$$
$$r = \pm\sqrt{4}$$
$$r = \pm 2$$
$$y_c = c_1 e^{2x} + c_2 e^{-2x}$$

Particular:

$$y_p = Ax^2 + Bx + C$$
$$y_p' = 2Ax + B$$
$$y_p'' = 2A$$
$$y_p'' - 4y_p = x^2 + x$$
$$2A - 4\left(Ax^2 + Bx + C\right) = x^2 + x$$
$$-4Ax^2 - 4Bx + 2A - 4C = x^2 + x$$

$$-4A = 1 \qquad -4B = 1 \qquad 2A - 4C = 0$$
$$A = -\tfrac{1}{4} \qquad B = -\tfrac{1}{4} \qquad 2\left(-\tfrac{1}{4}\right) - 4C = 0$$
$$-\tfrac{1}{2} = 4C$$
$$C = -\tfrac{1}{8}$$

$$y_p = -\frac{1}{4}x^2 - \frac{1}{4}x - \frac{1}{8}$$

$$y = y_c + y_p$$
$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{4}x^2 - \frac{1}{4}x - \frac{1}{8}$$

$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{4}x^2 - \frac{1}{4}x - \frac{1}{8}$$
$$0 = c_1 + c_2 - 0 - 0 - \frac{1}{8}$$
$$0 = c_1 + c_2 - \frac{1}{8}$$
$$0 = c_2 + \frac{1}{8} + c_2 - \frac{1}{8}$$
$$0 = 2c_2$$
$$c_2 = 0 \rightarrow$$

$$y' = 2c_1 e^{2x} - 2c_2 e^{-2x} - \frac{1}{2}x - \frac{1}{4}$$
$$0 = 2c_1 - 2c_2 - 0 - \frac{1}{4}$$
$$2c_1 = 2c_2 + \frac{1}{4}$$
$$\leftarrow c_1 = c_2 + \frac{1}{8}$$
$$c_1 = \frac{1}{8}$$

$$y = \frac{1}{8}e^{2x} - \frac{1}{4}x^2 - \frac{1}{4}x - \frac{1}{8}$$