

Name: _____

Section: _____

R·I·T SCHOOL OF MATHEMATICAL SCIENCES

18 - Nonhomogeneous Equations II

MATH 211

Find the form of y_p for each of the following differential equations. (Do not solve for the undetermined coefficients.)

1. $y'' - y = \sin x$

$$y_p = A \sin x + B \cos x$$

2. $y'' - y = e^{2x}$

$$y_p = Ae^{2x}$$

3. $y'' - y = x^2 + e^{2x}$

$$y_p = Ax^2 + Bx + C + De^{2x}$$

4. $y'' - y' = \cos 2t + e^{2t}$

$$y_p = A \sin 2t + B \cos 2t + Ce^{2t}$$

Solve the initial value problem.

$$y'' - 4y' + 4y = e^{2x}, \quad y(0) = 0, \quad y'(0) = 0$$

Complementary:

$$\begin{aligned} y'' - 4y' + 4y &= 0 \\ r^2 - 4r + 4 &= 0 \\ (r - 2)(r - 2) &= 0 \\ r_1 = r_2 &= 2 \\ y_c &= c_1 e^{2x} + c_2 x e^{2x} \end{aligned}$$

Particular:

$$\begin{aligned} \cancel{y_p} &= \cancel{Ae^{2x}} \\ \cancel{y_p} &= \cancel{Ax^2 e^{2x}} \\ y_p &= Ax^2 e^{2x} \\ y'_p &= 2Axe^{2x} + 2Ax^2 e^{2x} \\ y''_p &= 2Ae^{2x} + 4Axe^{2x} + 4Axe^{2x} + 4Ax^2 e^{2x} \\ y''_p - 4y'_p + 4y_p &= e^{2x} \\ 2Ae^{2x} + 8Axe^{2x} + 4Ax^2 e^{2x} - 4(2Axe^{2x} + 2Ax^2 e^{2x}) + 4Ax^2 e^{2x} &= e^{2x} \\ 2Ae^{2x} + \cancel{8Axe^{2x}} + \cancel{4Ax^2 e^{2x}} - \cancel{8Axe^{2x}} - \cancel{8Ax^2 e^{2x}} + \cancel{4Ax^2 e^{2x}} &= e^{2x} \\ 2Ae^{2x} &= e^{2x} \\ 2A &= 1 \\ A &= \frac{1}{2} \\ y_p &= \frac{1}{2} x^2 e^{2x} \end{aligned}$$

$$y = y_c + y_p$$

$$y = c_1 e^{2x} + c_2 x e^{2x} + \frac{1}{2} x^2 e^{2x}$$

$$\begin{aligned} y &= c_1 e^{2x} + c_2 x e^{2x} + \frac{1}{2} x^2 e^{2x} \\ 0 &= c_1 + 0 + 0 \\ 0 &= c_1 \rightarrow \end{aligned}$$

$$\begin{aligned} y &= c_2 x e^{2x} + \frac{1}{2} x^2 e^{2x} \\ y' &= c_2 e^{2x} + 2c_2 x e^{2x} + x e^{2x} + x^2 e^{2x} \\ 0 &= c_2 + 0 + 0 + 0 \\ c_2 &= 0 \end{aligned}$$

$$y = \frac{1}{2} x^2 e^{2x}$$