Name: Key

Section:

R·I·T School of Mathematical Sciences

Homework 3

MATH 211

1. Verify that $y = \frac{1}{2}x^2e^x$ is a solution to the differential equation.

$$y'' - 2y' + y = e^x$$

$y' = \frac{1}{2}x^2e^x + xe^x$

$y'' = \frac{1}{2}x^2e^x + xe^x + xe^x + e^x$

$= \frac{1}{2}x^2e^x + 2xe^x + e^x$

$\frac{1}{2}x^2e^x + 2xe^x + e^x - 2(\frac{1}{2}x^2e^x + xe^x) + \frac{1}{2}x^2e^x =$

$\cancel{\frac{1}{2}x^2e^x} + \cancel{2xe^x} + e^x - \cancel{x^2e^x} - \cancel{2xe^x} + \cancel{\frac{1}{2}x^2e^x} = e^x$

$e^x = e^x$

2. Find the general solution to the separable equation.

$$e^x y \frac{dy}{dx} = e^{-y} + e^{-2x-y}$$

$e^x y\, dy = (e^{-y} + e^{-2x}e^{-y})dx$

$e^x y\, dy = e^{-y}(1 + e^{-2x})dx$

$e^y y\, dy = \frac{1+e^{-2x}}{e^x}dx$

$* \int ye^y dy = \int (e^{-x} + e^{-3x})dx$

$ye^y - \int e^y dy = \int (e^{-x} + e^{-3x})dx$

$ye^y - e^y = -e^{-x} - \frac{1}{3}e^{-3x} + C$

$* \quad u = y \qquad dv = e^y dy$

$du = dy \qquad v = e^y$

3. Solve the given initial value problem.

$$x^2\,\frac{dy}{dx} = y - xy,\; y(-1) = -1$$

$x^2\,dy = (y - xy)\,dx$

$x^2 dy = y(1-x)\,dx$

$\frac{1}{y} dy = \frac{1-x}{x^2} dx$

$\int \frac{1}{y} dy = \int \left(x^{-2} - \frac{1}{x}\right) dx$

$\ln|y| = -\frac{1}{x} - \ln|x| + C$

$\ln|-1| = \frac{-1}{-1} - \ln|-1| + C$

$0 = 1 - 0 + C$

$-1 = C$

$\ln|y| = \frac{-1}{x} - \ln|x| - 1$

4. Solve the given initial value problem.

$$\sec^2 x\; dy + \csc y\; dx = 0,\; y\left(\frac{\pi}{4}\right) = 0$$

$\sec^2 x\, dy = -\csc y\, dx$

$\frac{1}{\csc y} dy = \frac{-1}{\sec^2 x} dx$

$\int \sin y\, dy = -\int \cos^2 x\, dx$

$\int \sin y = -\frac{1}{2}\int (1 + \cos(2x))\, dx$

$-\cos y = -\frac{1}{2}\left(x + \frac{1}{2}\sin(2x)\right) + C$

$-\cos 0 = -\frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4}\sin\left(\frac{\pi}{2}\right) + C$

$-1 = -\frac{\pi}{8} - \frac{1}{4} + C$

$C = \frac{\pi}{8} - \frac{3}{4}$

$-\cos y = \frac{-1}{2}\left(x + \frac{1}{2}\sin(2x)\right) + \frac{\pi}{8} - \frac{3}{4}$