Name: _____

Section: _____

R·I·T School of Mathematical Sciences

3| - Higher Order Applications with Laplace Transforms

MATH 211

A beam of length 8m is embedded at $x = 0$ and simply supported at the other end. Find the deflection of the beam if a load $\omega(x) = 12EIx$ N is uniformly distributed along its length.

$$EIy^{(4)} = 12EIx$$

$$y^{(4)} = 12x$$

$$\mathscr{L}\{y^{(4)}\} = 12\mathscr{L}\{x\}$$

$$s^4\mathscr{L}\{y\} - s^3y(0) - s^2y'(0) - sy''(0) - y'''(0) = 12\left(\frac{1}{s^2}\right)$$

$$s^4\mathscr{L}\{y\} - 0 - 0 - sy''(0) - y'''(0) = 12\left(\frac{1}{s^2}\right)$$

$$s^4\mathscr{L}\{y\} = 12\left(\frac{1}{s^2}\right) + sy''(0) + y'''(0)$$

$$\mathscr{L}\{y\} = 12\left(\frac{1}{s^6}\right) + y''(0)\left(\frac{1}{s^3}\right) + y'''(0)\left(\frac{1}{s^4}\right)$$

$$\mathscr{L}\{y\} = \frac{1}{10}\left(\frac{120}{s^6}\right) + \frac{y''(0)}{2}\left(\frac{2}{s^3}\right) + \frac{y'''(0)}{6}\left(\frac{6}{s^4}\right)$$

$$y = \frac{1}{10}t^5 + \frac{y''(0)}{2}t^2 + \frac{y'''(0)}{6}t^3$$

$$y(8) = 0$$

$$y' = \frac{1}{2}t^4 + y''(0)t + \frac{y'''(0)}{2}t^2$$
$$y'' = 2t^3 + y''(0) + y'''(0)t$$
$$y''(8) = 0$$

$$0 = \frac{8^5}{10} + \frac{y''(0)\,8^2}{2} + \frac{y'''(0)\,8^3}{6}$$
$$0 = \frac{8^3}{5} + y''(0) + \frac{y'''(0)\,8}{3}$$
$$0 = \frac{512}{5} + y''(0) + \frac{8y'''(0)}{3}$$
$$0 = 1536 + 15y''(0) + 40y'''(0)$$
$$0 = 1536 + 15\left(-1024 - 8y'''(0)\right) + 40y'''(0)$$
$$0 = 1536 - 15360 - 120y'''(0) + 40y'''(0)$$
$$80y'''(0) = -13,824$$
$$y'''(0) = -\frac{864}{5}$$

$$0 = 2(8)^3 + y''(0) + 8y'''(0)$$
$$0 = 1024 + y''(0) + 8y'''(0)$$
$$y''(0) = -1024 - 8y'''(0)$$

$$y''(0) = -1024 - 8\left(-\frac{864}{5}\right)$$
$$y''(0) = -1024 + \frac{6912}{5}$$
$$y''(0) = \frac{1792}{5}$$

$$y = \frac{1}{10}t^5 + \frac{896}{5}t^2 - \frac{144}{5}t^3$$