Name: ____________________

Section: ____________________

R·I·T School of Mathematical Sciences

12 - Compartmental Analysis

MATH 211

1. (a) A brine solution of salt flows at a constant rate of 8 liters per minute into a large tank. If the concentration of the salt in the brine entering the tank is 0.05 kilograms per liter, determine the rate at which the salt is entering the tank.

$$\left(\frac{8\cancel{L}}{1\text{min}}\right)\left(\frac{5}{100}\frac{\text{kg}}{\cancel{L}}\right)=\frac{8}{20}\frac{\text{kg}}{\text{min}}=\frac{2}{5}\text{ kg/min}$$

(b) The tank initially held 100 liters of brine solution. The solution inside the tank is kept well stirred and flows out at the same rate as it flowed in. Find the rate at which the salt leaves the tank.

$$\left(\frac{8\cancel{L}}{1\text{min}}\right)\left(\frac{x}{100}\frac{\text{kg}}{\cancel{L}}\right)=\frac{8x}{100}\frac{\text{kg}}{\text{min}}=\frac{2x}{25}\text{ kg/min}$$

(c) Write a differential equation that models the rate of change in the amount of salt in the tank with respect to time.

$$\frac{dx}{dt}=\frac{2}{5}-\frac{2x}{25}$$

(d) If the tank initially held 0.5 kilograms of salt, find an equation for the amount of salt in the tank at time t.

$$\frac{dx}{dt}+\frac{2x}{25}=\frac{2}{5}$$

$$e^{\int 2/25\ dt}=e^{2t/25}$$

$$e^{2t/25}\left[\frac{dx}{dt}+\frac{2x}{25}\right]=\frac{2}{5}e^{2t/25}$$

$$\frac{d}{dt}\left[xe^{2t/25}\right] = \frac{2}{5}e^{2t/25}$$

$$\int d\left[xe^{2t/25}\right] = \int \frac{2}{5}e^{2t/25}\ dt$$

$$xe^{2t/25} = \frac{2}{5}\cdot\frac{25}{2}e^{2t/25} + C$$

$$x = 5 + Ce^{-2t/25}$$

$$\frac{1}{2} = 5 + Ce^{0}$$

$$C = -\frac{9}{2}$$

$$x = 5 - \frac{9}{2}e^{-2t/25}$$

2. Pure water flows at a constant rate of 5 liters per minute into a large tank that initially held 125 liters of brine solution in which was dissolved 1 kilogram of salt. The solution inside the tank is kept well stirred and flows out at the same rate. Find a formula for the amount of salt in the tank as a function of time t. How much salt is in the tank after 10 minutes?

$$\frac{dx}{dt} = \left(\frac{5\text{L}}{1\text{min}}\right)\left(0\frac{\text{kg}}{\text{L}}\right) - \left(\frac{5\text{L}}{1\text{min}}\right)\left(\frac{x}{125}\frac{\text{kg}}{\text{L}}\right)$$

$$\frac{dx}{dt} = -\frac{x}{25}$$

$$\frac{1}{x}\ dx = -\frac{1}{25}\ dt$$

$$\int \frac{1}{x}\ dx = -\frac{1}{25}\int\ dt$$

$$\ln|x| = -\frac{1}{25}t + C$$

$$x = e^{-t/25+C}$$

$$x = Ce^{-t/25}$$

$$1 = Ce^{0}$$

$$C = 1$$

$$x(t) = e^{-t/25}$$

$$x(10) = e^{-10/25}$$

$$= e^{-2/5}$$

3. A brine solution of salt flows at a constant rate of 7 liters per minute into a large tank that initially held 115 liters of brine solution in which was dissolved 1 kilogram of salt. The solution inside the tank is kept well stirred and flows out at the same rate. If the concentration of salt in the brine entering the tank is 0.3 kilograms per liter, find a formula for the amount of salt in the tank as a function of time t. How much salt is in the tank after 2 minutes?

$$\frac{dx}{dt} = \left(\frac{7\text{L}}{1\text{min}}\right)\left(\frac{3}{10}\frac{\text{kg}}{\text{L}}\right) - \left(\frac{7\text{L}}{1\text{min}}\right)\left(\frac{x}{115}\frac{\text{kg}}{\text{L}}\right)$$

$$\frac{dx}{dt} = \frac{21}{10} - \frac{7x}{115}$$

$$\frac{dx}{dt} + \frac{7x}{115} = \frac{21}{10}$$

$$e^{\int 7/115\ dt} = e^{7t/115}$$

$$e^{7t/115}\left[\frac{dx}{dt} + \frac{7x}{115}\right] = \frac{21}{10}e^{7t/115}$$

$$\frac{d}{dt}\left[xe^{7t/115}\right] = \frac{21}{10}e^{7t/115}$$

$$\int d\left[xe^{7t/115}\right] = \frac{21}{10}\int e^{7t/115}\ dt$$

$$xe^{7t/115} = \frac{21}{10}\cdot\frac{115}{7}e^{7t/115} + C$$

$$x = \frac{69}{2} + Ce^{-7t/115}$$

$$1 = \frac{69}{2} + Ce^{0}$$

$$C = -\frac{67}{2}$$

$$x(t) = \frac{69}{2} - \frac{67}{2}e^{-7t/115}$$

$$x(2) = \frac{69}{2} - \frac{67}{2}e^{-7(2)/115}$$

$$= \frac{69}{2} - \frac{67}{2}e^{-14/115}$$

4. A brine solution of salt flows at a constant rate of 2 liters per minute into a large tank that initially held 80 liters of pure water. The solution inside the tank is kept well stirred and flows out at the same rate. If the concentration of salt in the brine entering the tank is 1 kilograms per liter, find a formula for the amount of salt in the tank as a function of time t. How much salt is in the tank after 5 minutes?

$$\frac{dx}{dt} = \left(\frac{2\text{L}}{1\text{min}}\right)\left(\frac{1\text{kg}}{\text{L}}\right) - \left(\frac{2\text{L}}{1\text{min}}\right)\left(\frac{x\text{kg}}{80\text{L}}\right)$$

$$\frac{dx}{dt} = 2 - \frac{x}{40}$$

$$\frac{dx}{dt} + \frac{x}{40} = 2$$

$$e^{\int 1/40\ dt} = e^{t/40}$$

$$e^{t/40}\left[\frac{dx}{dt} + \frac{x}{40}\right] = 2e^{t/40}$$

$$\frac{d}{dt}\left[xe^{t/40}\right] = 2e^{t/40}$$

$$\int d\left[xe^{t/40}\right] = 2\int e^{t/40}\ dt$$

$$xe^{t/40} = 2(40)e^{t/40} + C$$

$$x = 80 + Ce^{-t/40}$$

$$0 = 80 + Ce^{0}$$

$$C = -80$$

$$x(t) = 80 - 80e^{-t/40}$$

$$x(5) = 80 - 80e^{-5/40}$$

$$= 80 - 80e^{-1/8}$$