KEY

CHMG-141
General and Analytical Chemistry I
with Dr. Bailey

Name_______________

Week 3

The Quantum – Mechanical Model of the Atom

(Chapter 2)

Sample and Practice Problems

Sample problem 1

A cell phone sends signals at about 850 MHz (1 MHz = 1 x 10^6 Hz or cycles per second).

(a) What is the wavelength of this radiation?
(b) What is the energy of 1.0 mol of protons with a frequency of 850 MHz?
(c) Compare the energy in part (b) with energy of a mole of photons of blue light (420 nm).
(d) Comment on the difference in energy between 850 MHz radiation and blue light.

(a) $\lambda = \frac{c}{\nu} = \frac{2.998 \times 10^8\ \text{m} \cdot \text{s}^{-1}}{850 \times 10^6\ \text{s}^{-1}} = 0.35\ \text{m}$

(b) $E = h\nu = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(850 \times 10^6\ \text{s}^{-1}) \cdot \frac{6.02 \times 10^{23}\ \text{photons}}{1.00\ \text{mol}} = 0.34\ \text{J/mol}$

(c) $E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m}\cdot\text{s}^{-1})}{4.2 \times 10^{-7}\ \text{m}} \cdot \frac{6.02 \times 10^{23}\ \text{photons}}{1.00\ \text{mol}} = 2.8 \times 10^5\ \text{J/mol}$

$$\frac{2.8 \times 10^5\ \text{J/mol}}{0.34\ \text{J/mol}} = \boxed{84{,}000}$$

(d) Blue light is 84,000 times more energetic than the radiation sent from cell phones.

Practice problem 1

Assume your eyes receive a signal consisting of blue light, $\lambda = 470$ nm. The energy of the signal is 2.50 x 10^{-14} J.
How many photons reach your eyes?

$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m}\cdot\text{s}^{-1})}{4.70 \times 10^{-7}\ \text{m}} = 4.23 \times 10^{-19}\ \text{J/photon}$ E of a photon (each photon)

$\frac{2.50 \times 10^{-14}\ \text{J}}{4.23 \times 10^{-19}\ \text{J/photon}} = \boxed{5.92 \times 10^4\ \text{photons}}$ ← Number of photons in the signal

Practice problem 2

Radiation in the ultraviolet region of the electromagnetic spectrum is quite energetic. It is this radiation that causes dyes to fade and your skin to develop a sunburn.
If you are bombarded with 1.00 mol of photons with a wavelength of 375 nm, what amount of energy, in kJ per mole of photons, are you being subject to?

$375\ \text{nm} \cdot \frac{10^{-9}\ \text{m}}{1\ \text{nm}} = 3.75 \times 10^{-7}\ \text{m}$

$E = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m}\cdot\text{s}^{-1})}{3.75 \times 10^{-7}\ \text{m}} \cdot \frac{6.022 \times 10^{23}\ \text{photons}}{1.00\ \text{mol}} \cdot \frac{1\ \text{kJ}}{10^3\ \text{J}} = 319\ \text{kJ/mol}$

Sample problem 2

A beam of electrons ($m = 9.11 \times 10^{-31}$ kg/electron) has an average speed of 1.3×10^8 m/s. What is the wavelength 0f electrone having this average speed?

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{(9.11 \times 10^{-31}\ \mathrm{kg})(1.3 \times 10^{8}\ \mathrm{m \cdot s^{-1}})} = 5.6 \times 10^{-12}\ \mathrm{m}$$

Convert to the proper units : kg

$\frac{m}{s} = m \cdot s^{-1}$

Practice problem 3

A rifle bullet (mass = 1.50 g) has a velocity 7.00×10^2 mph. What is the wavelength associated with this bullet?

$$\frac{7.00 \times 10^2\ \text{mile}}{1\ \text{hour}} \cdot \frac{1\ \text{km}}{0.6214\ \text{mile}} \cdot \frac{10^3\ \text{m}}{1\ \text{km}} \cdot \frac{1\ \text{hour}}{3600\ \text{s}} = 313\ \mathrm{m \cdot s^{-1}}$$

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{(1.50 \times 10^{-3}\ \mathrm{kg})(313\ \mathrm{m \cdot s^{-1}})} = 1.41 \times 10^{-33}\ \mathrm{m}$$

Practice problem 4

λ

Calculate the wavelength, in nanometers, associated with a 1.0×10^2 g golf ball moving at 30 m/s (about 67 mph).
How fast must the ball travel to have a wavelength of 5.6×10^{-3} nm?

= velocity

$$1.0 \times 10^2\ \text{g} \cdot \frac{1\ \text{kg}}{10^3\ \text{g}} = 0.10\ \text{kg}$$

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{(0.10\ \mathrm{kg})(30\ \mathrm{m \cdot s^{-1}})} = \boxed{2.2 \times 10^{-34}\ \mathrm{m}}$$

$$5.6 \times 10^{-3}\ \text{nm} \cdot \frac{1\ \text{m}}{10^9\ \text{nm}} = 5.6 \times 10^{-12}\ \text{m}$$

$$v = \frac{h}{m\lambda} = \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{(0.10\ \mathrm{kg})(5.6 \times 10^{-12}\ \mathrm{m})} = \boxed{1.2 \times 10^{-21}\ \mathrm{m \cdot s^{-1}}}$$

proper units !

Sample problem 3

The most prominent line in the spectrum of mercury is at 253.652 nm. Other lines are located at 365.015 nm, 404.656 nm, 435.833 nm, and 1013.975 nm.

(a) Which of these lines represents the most energetic line?
(b) What is the frequency of the most prominent line?
What is the energy of one photon with this wavelength?
(c) Are any of these lines found in the spectrum of mercury shown in the lecture slide? What colors are these lines?

(a) The most energetic line has the shortest wavelength, 253.652 nm.

(b) $253.652 \text{ nm} \cdot \dfrac{10^{-9} \text{ m}}{1 \text{ nm}} = 2.53652 \times 10^{-7} \text{ m}$

$$\nu = \frac{c}{\lambda} = \frac{2.997925 \times 10^8 \text{ m} \cdot \text{s}^{-1}}{2.53652 \times 10^{-7} \text{ m}} = 1.18190 \times 10^{15} \text{ s}^{-1}$$

$$E = h\nu = (6.626069 \times 10^{-34} \text{ J}\cdot\text{s})(1.18190 \times 10^{15} \text{ s}^{-1}) = 7.83135 \times 10^{-19} \text{ J/photon}$$

(c) The 404.656 nm line is violet, while the 435.833 nm line is blue.

Practice problem 5

The most prominent line in the spectrum of neon is found at 865.438 nm. Other lines are located at 837.761 nm, 878.062 nm, 878.375 nm, and 1885.387 nm.

(a) In what region of the electromagnetic spectrum are these lines found?
(b) Are any of these lines found in the spectrum of neon shown in the lecture slide?
(c) Which of these lines represents the most ene4rgetic light?
(d) What is the frequency of the most prominent line?
What is the energy of one photon with this wavelength?

(a) The infrared region

(b) None of the lines mentioned are in the spectrum shown in Figure 7.9. None of the lines listed are in the visible region.

(c) The most energetic line has the shortest wavelength, 837.761 nm.

(d) $865.438 \text{ nm} \cdot \dfrac{10^{-9} \text{ m}}{1 \text{ nm}} = 8.65438 \times 10^{-7} \text{ m}$

$$\nu = \frac{c}{\lambda} = \frac{2.997925 \times 10^8 \text{ m} \cdot \text{s}^{-1}}{8.65438 \times 10^{-7} \text{ m}} = 3.46406 \times 10^{14} \text{ s}^{-1}$$

$$E = h\nu = (6.626069 \times 10^{-34} \text{ J}\cdot\text{s})(3.46406 \times 10^{14} \text{ s}^{-1}) = 2.29531 \times 10^{-19} \text{ J/photon}$$

Sample problem 4

$E_{photon\,released} = -\Delta E_{hydrogen\,electron} = -(E_{final} - E_{initial})$

$$h\nu = \frac{hc}{\lambda} = -\left[\left(-2.18\times10^{-18}\,J\left(\frac{1}{n_{final}^2}\right)\right) - \left(-2.18\times10^{-18}\,J\left(\frac{1}{n_{initial}^2}\right)\right)\right]$$

Example : Calculate the wavelength of light emitted when the hydrogen electron transitions from $n = 6$ to $n = 5$

Given:	$n_i = 6$, $n_f = 5$
Find:	λ, m
Concept Plan:	$n_i, n_f \rightarrow \Delta E_{atom} \rightarrow E_{photon} \rightarrow \lambda$
Relationships:	$E = -R_H\left(\frac{1}{n^2}\right)$ $\Delta E_{atom} = -E_{photon}$ $\lambda = \frac{h\bullet c}{E}$ $E=hc/\lambda$, $E_n = -2.18 \times 10^{-18}$ J $(1/n^2)$
Solve:	$\Delta E_{atom} = -2.18\times10^{-18}\,J\left(\frac{1}{5^2}-\frac{1}{6^2}\right) = -2.66\underline{4}4\times10^{-20}\,J$ $E_{photon} = -(-2.66\underline{4}4 \times 10^{-20}\,J) = 2.66\underline{4}4 \times 10^{-20}$ J $\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\,\cancel{J\cdot s})(3.00\times10^{8}\,\frac{m}{\cancel{s}})}{(2.66\underline{4}4\times10^{-20}\,\cancel{J})} = 7.46\times10^{-6}$ m
Check:	the unit is correct, the wavelength is in the infrared, which is appropriate because less energy than 4→2 (in the visible)

$$\Delta E_{atom} = -2.18\times10^{-18}\,J\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$
$$E_{photon} = -\Delta E_{atom}$$

Practice problem 6

Calculate the wavelength and frequency of light emitted when an electron changes from n = 4 to n = 3 in the H atom. In what region of the spectrum is this radiation found?

$$\Delta E_{atom} = -2.18\times10^{-18}\,J\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 1.059\times10^{-19}\ J/photon$$

$$\nu = \frac{E}{h} = \frac{1.059\times10^{-19}\,J}{6.6261\times10^{-34}\,J\cdot s} = 1.598\times10^{14}\,s^{-1}$$

$$\lambda = \frac{c}{\nu} = \frac{3.000\times10^{8}\,m\cdot s^{-1}}{1.598\times10^{14}\,s^{-1}} = 1.876\times10^{-6}\,m$$ (infrared region)

Practice problem 7

The energy emitted when an electron moves from a higher energy state to a lower energy state in any atom can be observed as electromagnetic radiation.

(a) Which involves the emission of less energy in the H atom, an electron moving from n = 4 to n = 2 or an electron moving from n= 3 to n = 2?

(b) Which involves the emission of more energy in the H atom, an electron moving from n = 4 to n = 1 or an electron moving from n = 5 to n = 2? Explain fully.

(a) $n = 3$ to $n = 2$

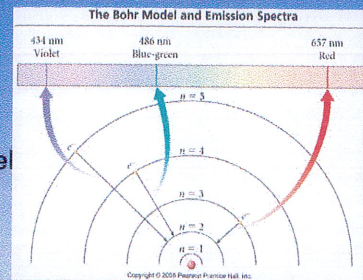
(b) $n = 4$ to $n = 1$

The energy levels are progressively closer at higher levels, so the energy difference from $n = 4$ to $n = 1$ is greater than from $n = 5$ to $n = 2$. (You can make calculations of ΔE!)

Principal Quantum Number, *n*

1

- characterizes the energy of the electron in a particular orbital
  - corresponds to Bohr's energy level
- *n* can be any integer ≥ 1



- the larger the value of *n*, the more energy the orbital has
- energies are defined as being negative
  - an electron would have E = 0 when it just escapes the atom
- the larger the value of *n*, the larger the orbital

$$E_n = -2.18 \times 10^{-18}\ J\left(\frac{1}{n^2}\right)$$

for an electron in H

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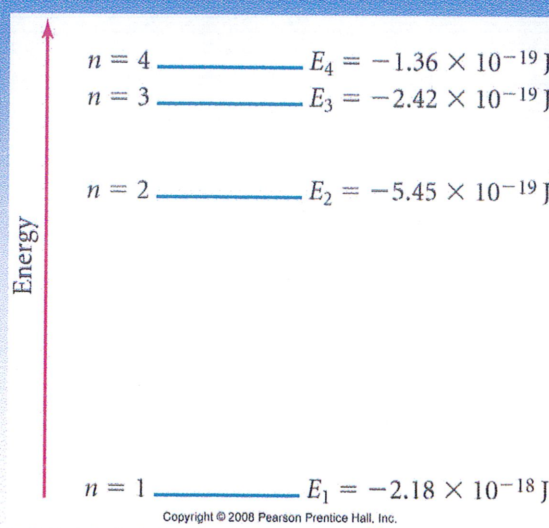
Principal Energy Levels in Hydrogen

- as *n* gets larger, the amount of <u>energy between orbitals</u> gets smaller



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Electron Transitions

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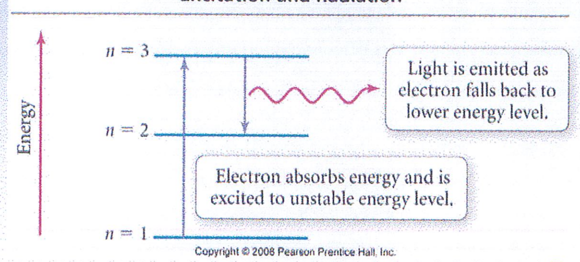
- in order to transition to a higher energy state, the electron must gain the correct amount of energy corresponding to the difference in energy between the final and initial states (absorption)
- Electrons in high energy states (exited) are unstable and tend to lose energy and transition to lower energy states
  - energy released as a photon of light (emission)
- each line in the emission spectrum corresponds to the difference in energy between two **energy states**



Predicting the Spectrum of Hydrogen

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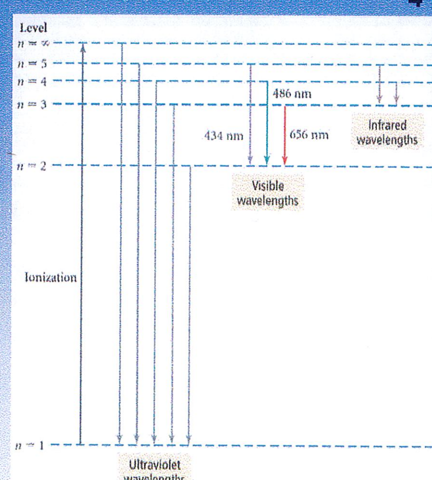
- the wavelengths of lines in the emission spectrum of hydrogen can be predicted by calculating the difference in energy between any two states
- for an electron in energy state *n*, there are (*n* – 1) energy states it can transition to, therefore (*n* – 1) lines it can generate
- both the Bohr and Quantum Mechanical Models can predict these lines very accurately



$$E_{\text{photon released}} = -\Delta E_{\text{hydrogen electron}} = -\left(E_{final} - E_{initial}\right)$$

$$h\nu = \frac{hc}{\lambda} = -\left[-2.18\times10^{-18}\ \text{J}\left(\frac{1}{n_{final}^{2}}\right) - \left(-2.18\times10^{-18}\ \text{J}\left(\frac{1}{n_{initial}^{2}}\right)\right)\right]$$

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