Name: ______________________

Section: ______________________

R·I·T School of Mathematical Sciences

23 - Beam Deflection

MATH 211

A beam of length L has a load applied in a vertical plane containing its axis of symmetry. The beam then undergoes distortion and the curve connecting the centroids of all cross sections of the beam is called its deflection (or elastic) curve. The deflection curve approximates the shape (curvature) of the beam with the load impressed.

In the theory of elasticity, the bending moment $M(x)$ at a point x along the beam is related to the load per unit length $\omega(x)$ by the equation

$$\frac{d^2M}{dx^2} = \omega(x) \qquad (1)$$

The bending moment is also proportional to the curvature of the elastic curve in the equation

$$M(x) = EI\kappa \qquad (2)$$

where E and I are constants (Young's modulus of elasticity and the moment of inertia, respectively). If the curvature is approximated by the second derivative of a function, then $\kappa \approx y''$. So,

$$M(x) = EIy'' \qquad (3)$$

Find a fourth order form of (1) whose solution models the deflection y of the beam as a function of its lengths x.

$$\frac{d^2}{dx^2}\left[EIy''\right] = \omega(x)$$

$$EI\frac{d^2}{dx^2}\left[y''\right] = \omega(x)$$

$$EIy^{(4)} = \omega(x)$$

A beam of length 8m is embedded at $x = 0$ and free at the other end. Find the deflection of the beam is a constant load $\omega(x) = 12EI$ N is uniformly distributed along its length.

$$EIy^{(4)} = \omega(x)$$
$$EIy^{(4)} = 12EI$$
$$y^{(4)} = 12$$

Complemetary:

$$y^{(4)} = 0$$
$$m^4 = 0$$
$$m_1 = m_2 = m_3 = m_4 = 0$$
$$y_c = c_1e^0 + c_2xe^0 + c_3x^2e^0 + c_4x^3e^0$$
$$y_c = c_1 + c_2x + c_3x^2 + c_4x^3$$

Particular:

$$\cancel{y_p = A}$$
$$\cancel{y_p = Ax}$$
$$\cancel{y_p = Ax^2}$$
$$\cancel{y_p = Ax^3}$$
$$y_p = Ax^4$$
$$y_p' = 4Ax^3$$
$$y_p'' = 12Ax^2$$
$$y_p''' = 24Ax$$
$$y_p^{(4)} = 24A$$
$$y_p^{(4)} = 12$$
$$24A = 12$$
$$A = \frac{1}{2}$$
$$y_p = \frac{1}{2}x^4$$

$$y = y_c + y_p$$
$$y = c_1 + c_2x + c_3x^2 + c_4x^3 + \frac{1}{2}x^4$$

$$y = c_1 + c_2x + c_3x^2 + c_4x^3 + \frac{1}{2}x^4$$
$$y(0) = 0$$
$$0 = c_1 + 0 + 0 + 0 + 0$$
$$c_1 = 0$$

$$y' = c_2 + 2c_3x + 3c_4x^2 + 2x^3$$
$$y'(0) = 0$$
$$0 = c_2 + 0 + 0 + 0$$
$$c_2 = 0$$

$$y'' = 2c_3 + 6c_4x + 6x^2$$
$$y''(8) = 0$$
$$0 = 2c_3 + 6c_4(8) + 6(8)^2$$
$$0 = c_3 + 24c_4 + 192$$
$$c_3 = -24c_4 - 192$$
$$c_3 = -24(-16) - 192$$
$$c_3 = 192$$

$$y''' = 6c_4 + 12x$$
$$y'''(8) = 0$$
$$0 = 6c_4 + 12(8)$$
$$\leftarrow c_4 = -16$$

$$y = 192x^2 - 16x^3 + \frac{1}{2}x^4$$

A beam of length 8m is embedded at $x = 0$ and simply supported at the other end. Find the deflection of the beam is a constant load $\omega(x) = 12EI$ N is uniformly distributed along its length.

$$y = c_3 x^2 + c_4 x^3 + \frac{1}{2} x^4$$

$$\begin{gathered} y = c_3 x^2 + c_4 x^3 + \frac{1}{2} x^4 \\ y(8) = 0 \\ 0 = c_3 (8)^2 + c_4 (8)^3 + \frac{1}{2} (8)^4 \\ 0 = c_3 + 8c_4 + \frac{1}{2} (8)^2 \\ 0 = c_3 + 8c_4 + 32 \\ c_3 = -8c_4 - 32 \\ c_3 = 80 - 32 \\ c_3 = 48 \end{gathered}$$

$$\begin{gathered} y' = 2c_3 x + 3c_4 x^2 + 2x^3 \\ y'' = 2c_3 + 6c_4 x + 6x^2 \\ y''(8) = 0 \\ 0 = 2c_3 + 6c_4(8) + 6(8)^2 \\ 0 = c_3 + 3c_4(8) + 3(8)^2 \\ 0 = -8c_4 - 32 + 24c_4 + 3(64) \\ 0 = -c_4 - 4 + 3c_4 + 3(8) \\ 0 = 2c_4 - 4 + 24 \\ 2c_4 = -20 \\ c_4 = -10 \end{gathered}$$

$$y = 48x^2 - 10x^3 + \frac{1}{2} x^4$$