Name: _____

Section: _____

R·I·T School of Mathematical Sciences

Homework 2

MATH 211

1. Evaluate the iterated integral.

$$\int_0^{\pi/3} \int_0^{\sec\theta} \sec\theta \; dr \; d\theta$$

$$= \int_0^{\pi/3} r \sec\theta \Big|_0^{\sec\theta} \; d\theta$$

$$= \int_0^{\pi/3} \sec\theta \sec\theta \; d\theta$$

$$= \int_0^{\pi/3} \sec^2\theta \; d\theta$$

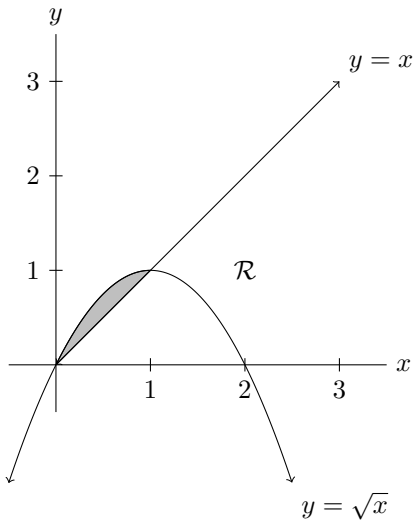
$$= \tan\theta \Big|_0^{\pi/3}$$

$$= \tan\left(\frac{\pi}{3}\right) - \tan(0)$$

$$= \sqrt{3} - 0$$

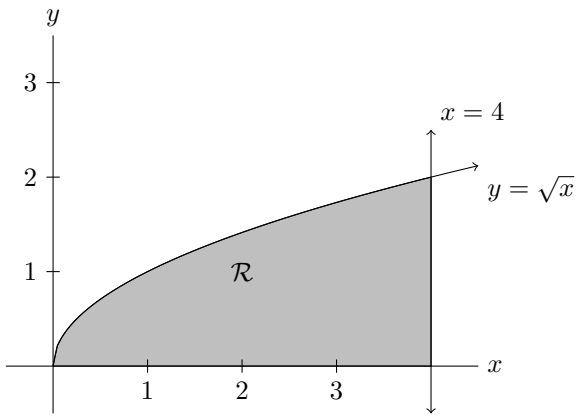
$$= \sqrt{3}$$

2. Use double integration to find the area of the region bounded by $y = 2x - x^2$ and $-x + y = 0$. You must sketch the region.



$$
\begin{aligned}
A &= \int_0^1 \int_x^{2x-x^2} \, dy \, dx \\
&= \int_0^1 y|_x^{2x-x^2} \, dx \\
&= \int_0^1 \left(2x - x^2 - x\right) \, dx \\
&= \int_0^1 \left(x - x^2\right) \, dx \\
&= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 \\
&= \frac{1}{2} - \frac{1}{3} - 0 \\
&= \frac{1}{6}
\end{aligned}
$$

3. Use double integration to find the volume of the solid bounded above by $z = 1 + 2x - 3y^2$, and below by the region bounded by $y = \sqrt{x}$, $x = 4$ and $y = 0$. You must sketch the region.



$$V = \int_0^4 \int_0^{\sqrt{x}} \left(1 + 2x - 3y^2\right)\, dy\, dx$$

$$= \int_0^4 \left[y + 2xy - y^3\right]_0^{\sqrt{x}}\, dx$$

$$= \int_0^4 \left(\sqrt{x} + 2x^{3/2} - x^{3/2}\right)\, dx$$

$$= \int_0^4 \left(x^{1/2} + x^{3/2}\right)\, dx$$

$$= \left[\frac{2x^{3/2}}{3} + \frac{2x^{5/2}}{5}\right]_0^4$$

$$= \frac{2(4)^{3/2}}{3} + \frac{2(4)^{5/2}}{5}$$

$$= \frac{2(8)}{3} + \frac{2(32)}{5}$$

$$= \frac{16}{3} + \frac{64}{5}$$

$$= \frac{80 + 192}{15}$$

$$= \frac{272}{15}$$