4 - Second Partials Test

MATH 211

What can be said about the derivative of a function $y = f(x)$ on an interval where the function is increasing? Decreasing?

$f(x)$ **is said to be increasing when** $f'(x) > 0$**.**

$f(x)$ **is said to be decreasing when** $f'(x) < 0$**.**

How do we determine relative extrema on the graph of a function $y = f(x)$?

We look for values of x where $f'(x) = 0$ or $f'(x)$ is undefined on the domain of f. Wemay use the Extreme Value Theorem or First or Second Derivative Tests.

The graph of a a function $y = f(x)$ is said to be concave up on an interval (a, b) when $f''(x) > 0$ for all x on that interval.

The graph of a a function $y = f(x)$ is said to be concave down on an interval (a, b) when $f''(x) < 0$ for all x on that interval.

What might be true about the graph of a function when $f''(x) = 0$?

There may be a point of inflection at the value for x that satifies $f''(x) = 0$.

Use the Second Partials Test to find any extrema and saddle points for the surface.

$$f(x, y) = x^3 - 3xy + y^3$$

$f_x(x, y) = 3x^2 - 3y$ $0 = 3x^2 - 3y$ $y = x^2 \rightarrow$ $y = 0, \ y = 1$	$f_x(x, y) = -3x + 3y^2$ $0 = -3x + 3y^2$ $0 = -x + x^4$ $0 = x(x-1)(x^2 + x + 1)$ $\leftarrow x = 0, \ x = 1$

Critical Points: (0,0) (1,1)

$f_{xx}(x, y) = 6x$	$f_{yy}(x, y) = 6y$	$f_{xy}(x, y) = -3$
$f_{xx}(0, 0) = 0$	$f_{yy}(0, 0) = 0$	$f_{xy}(0, 0) = -3$
$f_{xx}(1, 1) = 6$	$f_{yy}(1, 1) = 6$	$f_{xy}(1, 1) = -3$

$D_{(0,0)} = f_{xx}(0,0)f_{yy}(0,0) - [f_{xy}(0,0)]^2$ $= (0)(0) - (-3)^2$ $= -9$ $f(0,0) = 0 - 0 + 0 = 0$ $D < 0$ There is a saddle point at $(0, 0, 0)$.	$D_{(1,1)} = f_{xx}(1,1)f_{yy}(1,1) - [f_{xy}(1,1)]^2$ $= (6)(6) - (-3)^2$ $= 27$ $f(1,1) = 1 - 3 + 1 = -1$ $D > 0, \ f_{xx}(1,1) > 0$ There is a relative minimum of $z = -1$ at $(1, 1)$.