# Higher Order Applications

1.) $mx'' + bx' + kx = f(t)$

$x'' + 6x' + 5x = 12e^t$

**Complimentary**

$x'' + 6x' + 5x = 0$

$m^2 + 6m + 5 = 0$

$(m+5)(m+1) = 0$

$m_1 = -5 \quad m_2 = -1$

$\boxed{x_c(t) = c_1 e^{-5t} + c_2 e^{-t}}$

**Particular**

$x_p = x_p' = x_p'' = Ae^t$

$x_p'' + 6x_p' + 5x_p = 12e^t$

$Ae^t + 6Ae^t + 5Ae^t = 12e^t$

$12A = 12$

$A = 1$

$\boxed{x_p = e^t}$

$x(t) = c_1 e^{-5t} + c_2 e^{-t} + e^t$

$x'(t) = -5c_1 e^{-5t} - c_2 e^{-t} + e^t$

$x(0) = 1$: $1 = c_1 + c_2 + 1$

$c_1 + c_2 = 0$

$c_1 = -c_2$

$c_1 = \frac{1}{4}$

$x'(0) = 0$: $0 = -5c_1 - c_2 + 1$

$0 = 5c_2 - c_2 + 1$

$0 = 4c_2 + 1$

$4c_2 = -1$

$c_2 = -\frac{1}{4}$

$$\boxed{x(t) = \frac{1}{4}e^{-5t} - \frac{1}{4}e^{-t} + e^t}$$

2.) $Lq'' + Rq' + \frac{q}{C} = E(t)$

$q'' + 10q' + \frac{q}{400 \times 10^{-6}} = 500\sin(100t)$

$q'' + 10q' + \frac{1000000}{400} q = 500\sin(100t)$

$q'' + 10q' + 2500q = 500\sin(100t)$

Complimentary

$q'' + 10q' + 2500q = 0$

$m^2 + 10m + 2500 = 0$

$m = \frac{-10 \pm \sqrt{10^2 - 4(2500)}}{2}$

$m = \frac{-10 \pm \sqrt{-9900}}{2}$

$m = \frac{-10 \pm 30\sqrt{11}\, i}{2}$

$m = \frac{-10}{2} \pm \frac{30\sqrt{11}}{2} i$

$m = -5 \pm 15\sqrt{11}\, i$

$q_c(t) = e^{-5t}\left[c_1\sin(5\sqrt{11}\, t) + c_2\cos(5\sqrt{11}\, t)\right]$

Particular

$q_p = A\sin(100t) + B\cos(100t)$

$q_p' = 100A\cos(100t) - 100B\sin(100t)$

$q_p'' = -10000A\sin(100t) + 10000B\cos(100t$

$q_p'' + 10q_p' + 2500q_p = 500\sin(100t)$

$-10000A\sin(100t) - 10000B\cos(100t) + 1000A\cos(100t)$

$-1000B\sin(100t) + 2500A\sin(100t) + 2500B\cos(10$

$= 500\sin(100t)$

$(-7500A - 1000B)\sin(100t) + (-7500B + 1000A)\cos$

$-7500A - 1000B = 500 \qquad -7500B + 1000A = 0$

$-75A - 10B = 5 \qquad 1000A = 7500B$

$-15A - 2B = 1 \qquad A = \frac{75}{10}B$

$-15\left(\frac{15}{2}\right)B - 2B = 1$

$-\frac{225}{2}B - \frac{4}{2}B = \frac{2}{2} \qquad A = \frac{15}{2}B$

$-229B = 2 \qquad A = \frac{-15}{229}$

$B = \frac{-2}{229}$

$q_p = \frac{-15}{229}\sin(100t) + \frac{-2}{229}\cos(100$

$$q(t) = e^{-5t}\left[c_1 \sin(5\sqrt{11}\,t) + c_2 \cos(5\sqrt{11}\,t)\right] - \frac{15}{229}\sin(100t) - \frac{2}{229}\cos(100t)$$

$q(0)=0$: $\quad 0 = c_1(0) + c_2(1) - 0 - \frac{2}{229}(1)$

$$0 = c_2 - \frac{2}{229}$$

$$c_2 = \frac{2}{229}$$

$$q(t) = e^{-5t}\left[c_1 \sin(5\sqrt{11}\,t) + \frac{2}{229}\cos(5\sqrt{11}\,t)\right] - \frac{15}{229}\sin(100t) - \frac{2}{229}\cos(100t)$$

$$q'(t) = -5e^{-5t}\left[c_1 \sin(5\sqrt{11}\,t) + \frac{2}{229}\cos(5\sqrt{11}\,t)\right] + e^{-5t}\left[5\sqrt{11}\,c_1 \cos(5\sqrt{11}\,t) - \frac{10\sqrt{11}}{229}\sin(5\sqrt{11}\,t)\right]$$
$$- \frac{1500}{229}\cos(100t) + \frac{200}{229}\sin(100t)$$

$q'(0)=0$: $\quad 0 = -5\left[0 + \frac{2}{229}\right] + \left[5\sqrt{11}\,c_1 - 0\right] - \frac{1500}{229} + 0$

$$0 = -\frac{10}{229} + 5\sqrt{11}\,c_1 - \frac{1500}{229}$$

$$5c_1\sqrt{11} = \frac{1510}{229}$$

$$c_1 = \frac{1510}{5(229)\sqrt{11}}$$

$$c_1 = \frac{302\sqrt{11}}{2519}$$

$$q(t) = e^{-5t}\left[\frac{302\sqrt{11}}{2519}\sin(5\sqrt{11}\,t) + \frac{2}{229}\cos(5\sqrt{11}\,t)\right] - \frac{15}{229}\sin(100t) - \frac{2}{229}\cos(100t)$$

3.) $EI\,y^{(4)} = w$

$y^{(4)} = \frac{w}{EI}$ (constant!)

$\boxed{y_c = c_1 + c_2x + c_3x^2 + c_4x^3}$

Particular

$y_p = Ax^4$

$y_p' = 4Ax^3$

$y_p'' = 12Ax^2$

$y_p''' = 24Ax$

$y_p^{(4)} = 24A$

$y_p^{(4)} = \frac{w}{EI}$

$24A = \frac{w}{EI}$

$A = \frac{w}{24EI}$

$\boxed{y_p = \frac{wx^4}{24EI}}$

$y = \overset{0}{\cancel{c_1}} + c_2x + \overset{0}{\cancel{c_3x^2}} + c_4x^3 + \frac{wx^4}{24EI}$

$y' = c_2 + \overset{0}{\cancel{2c_3}}x + 3c_4x^2 + \frac{wx^3}{6EI}$

$y'' = \overset{0}{\cancel{2c_3}} + 6c_4x + \frac{wx^2}{2EI}$

$y(0)=0$: $\boxed{c_1 = 0}$

$y''(0)=0$: $0 = 2c_3$ $\boxed{c_3 = 0}$

$y(L)=0$: $0 = c_2L + \overset{0}{\cancel{c_3L^2}} + c_4L^3 + \frac{wL^4}{24EI}$

$0 = c_2 + \overset{0}{\cancel{c_3L}} + c_4L^2 + \frac{wL^3}{24EI}$

$y''(L)=0$: $0 = 6c_4L + \frac{wL^2}{2EI}$

$6c_4 = -\frac{wL}{2EI}$

$\boxed{c_4 = -\frac{wL}{12EI}}$

$0 = c_2 + c_4L^2 + \frac{wL^3}{24EI}$

$c_2 = -\left(\frac{-wL}{12EI}\right)L^2 - \frac{wL^3}{24EI}$

$c_2 = \frac{2wL^3}{24EI} - \frac{wL^3}{24EI}$

$\boxed{c_2 = \frac{wL^3}{24EI}}$

$\boxed{y = \frac{wL^3}{24EI}x - \frac{wL}{12EI}x^3 + \frac{wx^4}{24EI}}$