R·I·T School of Mathematical Sciences

2 - Partial Derivatives

MATH 211

The pressure p (in Pa) of a gas as a function of its volume V and temperature T is given by $p = nRT/V$. Find the rate of change of p with respect to V, dp/dV, How did you treat T?

$$\frac{dp}{dV} = \frac{d}{dV}\left[\frac{nRT}{V}\right] = nRT \cdot \frac{d}{dV}\left[\frac{1}{V}\right] = nRT \cdot \frac{d}{dV}\left[V^{-1}\right] = nRT\left[-V^{-2}\right] = -\frac{nRT}{V^2}$$

We treated T as constant.

Now, find the rate of change of p with respect to T.

$$\frac{dp}{dT} = \frac{d}{dT}\left[\frac{nRT}{V}\right] = \frac{nR}{V} \cdot \frac{d}{dT}[T] = \frac{nR}{V} \cdot [1] = \frac{nR}{V}$$

We treated V as constant.

Find all second and mixed partial derivatives of the function.

$$z = \frac{x}{y} + e^x \sin y$$

$$z = xy^{-1} + e^x \sin y$$

$$\begin{aligned} z_x &= [1]y^{-1} + [e^x]\sin y \\ &= \frac{1}{y} + e^x \sin y \end{aligned}$$

$$\begin{aligned} z_y &= x\left[-y^{-2}\right] + e^x\left[\cos y\right] \\ &= -\frac{x}{y^2} + e^x \cos y \end{aligned}$$

$$\begin{aligned} z_{xx} &= 0 + [e^x]\sin y \\ &= e^x \sin y \end{aligned}$$

$$\begin{aligned} z_{yy} &= x\left[2y^{-3}\right] + e^x\left[-\sin y\right] \\ &= \frac{2x}{y^3} - e^x \sin y \end{aligned}$$

$$\begin{aligned} z_{xy} &= -y^{-2} + e^x\left[\cos y\right] \\ &= -\frac{1}{y^2} + e^x \cos y \end{aligned}$$

$$z_{yx} = -\frac{1}{y^2} + e^x \cos y$$