Name: ____________________

Section: ____________________

Date: ____________________

R·I·T School of Mathematical Sciences

Beam Deflection

MATH 211 - 01

A beam of length 12 m is embedded at $x = 0$ and free at the other end. A constant load of $w(x) = kEI$ is distributed along the beam where $k = 0.005$.

1. Set up the differential equation whose solution models the deflection of the beam.

$$EI\frac{d^4y}{dx^4} = w(x)$$

$$\frac{d^4y}{dx^4} = k$$

2. Find all linearly independent solutions to the corresponding homogeneous equation.

$$\frac{d^4y}{dx^4} = 0$$

$$m^4 = 0$$

$$m_1 = m_2 = m_3 = m_4 = 0$$

$$y_1 = 1, y_2 = x, y_3 = x^2, y_4 = x^3$$

3. Find the linear combination of linearly independent solutions.

$$y_c = c_1 + c_2x + c_3x^2 + c_4x^3$$

4. Find the solution to the nonhomogeneous equation.

$$f(x) = k$$

$$y_p = Ax^4$$

$$y_p' = 4Ax^3$$

$$y_p'' = 12Ax^2$$

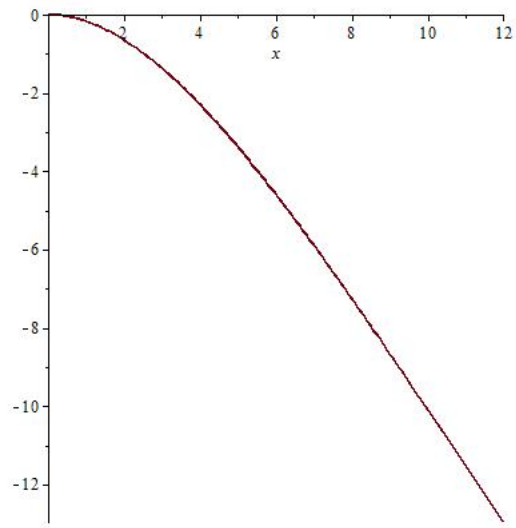
$$y_p''' = 24Ax$$
$$y_p^{(4)} = 24A$$
$$\frac{d^4 y_p}{dx^4} = k$$
$$24A = k$$
$$A = \frac{k}{24}$$
$$y_p = \frac{k}{24}x^4$$

5. Write the general solution to the differential equation.

$$y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + \frac{k}{24}x^4$$

6. Solve the BVP.

$$\text{Embedded at } x = 0$$
$$y(0) = 0$$
$$0 = c_1$$
$$y' = c_2 + 2c_3 x + 3c_4 x^2 + \frac{k}{6}x^3$$
$$y'(0) = 0$$
$$0 = c_2$$
$$\text{Free at } x = 12$$
$$y'' = 2c_3 + 6c_4 x + \frac{k}{2}x^2$$
$$y''(12) = 0$$
$$0 = 2c_3 + 72c_4 + 72k$$
$$c_3 = -36c_4 - 36k$$
$$y''' = 6c_4 + kx$$
$$y'''(12) = 0$$
$$0 = 6c_4 + 12k$$
$$c_4 = -2k$$
$$c_3 = -36(-2k) - 36k$$
$$c_3 = 72k - 36k$$
$$c_3 = 36k$$
$$y = 36kx^2 - 2kx^3 + \frac{k}{24}x^4$$
$$y = \frac{9}{50}x^2 - \frac{1}{100}x^3 + \frac{1}{4800}x^4$$



7. Now, consider a distributed load of $w(x) = kEIx$. Find the new solution to the nonhomogeneous equation.

$$\frac{d^4y}{dx^4} = kx$$

$$f(x) = kx$$

$$f'(x) = k$$

$$y_p = Ax^5 + Bx^4$$

$$y_p' = 5Ax^4 + 4Bx^3$$

$$y_p'' = 20Ax^3 + 12Bx^2$$

$$y_p''' = 60Ax^2 + 24Bx$$

$$y_p^{(4)} = 120Ax + 24B$$

$$\frac{d^4y_p}{dx^4} = kx$$

$$120Ax + 24B = kx$$

$$120A = k$$

$$A = \frac{k}{120}$$

$$B = 0$$

$$y_p = \frac{k}{120}x^5$$

8. Write the new general solution to the differential equation.

$$y = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + \frac{k}{120} x^5$$

9. Solve the BVP.

$$\text{Embedded at } x = 0$$

$$y(0) = 0$$

$$c_1 = 0$$

$$y' = c_2 + 2c_3 x + 3c_4 x^2 + \frac{k}{24} x^4$$

$$y'(0) = 0$$

$$0 = c_2$$

$$\text{Free at } x = 12$$

$$y'' = 2c_3 + 6c_4 x + \frac{k}{6} x^3$$

$$y''(12) = 0$$

$$0 = 2c_3 + 72c_4 + 288k$$

$$c_3 = -36c_4 - 144k$$

$$y''' = 6c_4 + \frac{k}{2} x^2$$

$$y'''(12) = 0$$

$$0 = 6c_4 + 72k$$

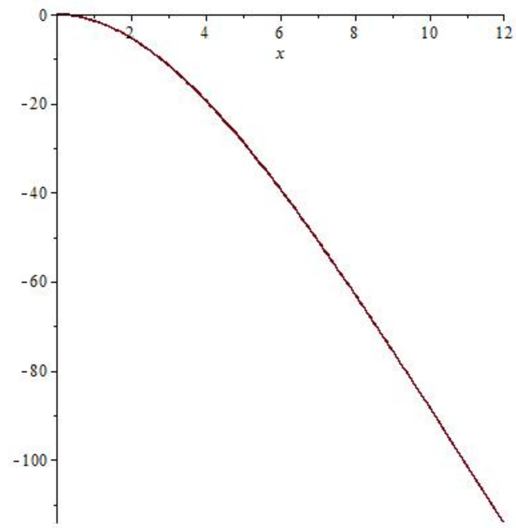
$$c_4 = -12k$$

$$c_3 = -36(-12k) - 144k$$

$$c_3 = 288k$$

$$y = 288kx^2 - 12kx^3 + \frac{k}{120} x^5$$

$$y = \frac{36}{25} x^2 - \frac{3}{50} x^3 + \frac{1}{24000} x^5$$



10. Suppose I came in and fixed the beam at $x = 12$ so that it is no longer free at either end. What would the deflection of the beam be now? (Use the load of $w(x) = kEIx$.)

$$y = c_1 + c_2x + c_3x^2 + c_4x^3 + \frac{k}{120}x^5$$

$$y' = c_2 + 2c_3x + 3c_4x^2 + \frac{k}{24}x^4$$

Embedded at $x = 0$

$$y(0) = 0$$

$$0 = c_1$$

$$y'(0) = 0$$

$$0 = c_2$$

Embedded at $x = 12$

$$y(12) = 0$$

$$0 = 144c_3 + 1728c_4 + \frac{10368}{5}k$$

$$c_3 = -12c_4 - \frac{72}{5}k$$

$$y'(12) = 0$$

$$0 = 24c_3 + 432c_4 + 864k$$

$$0 = c_3 + 18c_4 + 36k$$

$$0 = -12c_4 - \frac{72}{5}k + 18c_4 + 36k$$
$$0 = 6c_4 - \frac{108}{5}k$$
$$c_4 = \frac{18}{5}k$$
$$c_3 = -12\left(\frac{18}{5}k\right) - \frac{72}{5}k$$
$$c_3 = -\frac{288}{5}k$$
$$y = -\frac{288}{5}kx^2 + \frac{18}{5}kx^3 + \frac{k}{120}x^5$$
$$y = -\frac{36}{25}x^2 + \frac{9}{500}x^3 + \frac{1}{24000}x^5$$