R·I·T School of Mathematical Sciences

Pendulums

MATH 211

1. Suppose an 4 foot long simple pendulum begins from rest at an initial angle of $\frac{\pi}{6}$ radians. Find θ as a function of time. [You may use $g \approx 32$ ft/s^2.]

$$\theta'' + \frac{g}{l}\theta = 0$$
$$\theta'' + \frac{32}{4}\theta = 0$$
$$\theta'' + 8\theta = 0$$
$$r^2 + 8 = 0$$
$$r^2 = -8$$
$$r = \pm\sqrt{-8}$$
$$r = \pm 2\sqrt{2}i$$
$$\theta(t) = c_1 \sin\left(2\sqrt{2}t\right) + c_2 \cos\left(2\sqrt{2}t\right)$$
$$\frac{\pi}{6} = c_1 \sin 0 + c_2 \cos 0$$
$$\frac{\pi}{6} = 0 + c_2$$
$$c_2 = \frac{\pi}{6}$$
$$\theta(t) = c_1 \sin\left(2\sqrt{2}t\right) + \frac{\pi}{6}\cos\left(2\sqrt{2}t\right)$$
$$\theta'(t) = 2\sqrt{2}c_1 \cos\left(2\sqrt{2}t\right) - 2\sqrt{2}\left(\frac{\pi}{6}\right)\sin\left(2\sqrt{2}t\right)$$
$$0 = 2\sqrt{2}c_1 \cos 0 - 2\sqrt{2}\left(\frac{\pi}{6}\right)\sin 0$$
$$0 = 2\sqrt{2}c_1 - 0$$
$$c_1 = 0$$
$$\theta(t) = \frac{\pi}{6}\cos\left(2\sqrt{2}t\right)$$

2. Suppose an 7 meter long simple pendulum begins from rest at an initial angle of $\frac{\pi}{2}$ radians. Find θ as a function of time. [You may use $g \approx 9.8$ m/s^2.]

$$\theta'' + \frac{g}{l}\theta = 0$$

$$\theta'' + \frac{9.8}{7}\theta = 0$$

$$\theta'' + \frac{7}{5}\theta = 0$$

$$r^2 + \frac{7}{5} = 0$$

$$r^2 = -\frac{7}{5}$$

$$r = \pm\sqrt{-\frac{7}{5}}$$

$$r = \pm\frac{\sqrt{35}}{5}i$$

$$\theta(t) = c_1 \sin\left(\frac{\sqrt{35}}{5}t\right) + c_2 \cos\left(\frac{\sqrt{35}}{5}t\right)$$

$$\frac{\pi}{2} = c_1 \sin 0 + c_2 \cos 0$$

$$\frac{\pi}{2} = 0 + c_2$$

$$c_2 = \frac{\pi}{2}$$

$$\theta(t) = c_1 \sin\left(\frac{\sqrt{35}}{5}t\right) + \frac{\pi}{2} \cos\left(\frac{\sqrt{35}}{5}t\right)$$

$$\theta'(t) = \frac{\sqrt{35}}{5}c_1 \cos\left(\frac{\sqrt{35}}{5}t\right) - \frac{\sqrt{35}}{5}\left(\frac{\pi}{2}\right) \sin\left(\frac{\sqrt{35}}{5}t\right)$$

$$0 = \frac{\sqrt{35}}{5}c_1 \cos 0 - \frac{\sqrt{35}}{5}\left(\frac{\pi}{2}\right) \sin 0$$

$$0 = \frac{\sqrt{35}}{5}c_1 - 0$$

$$c_1 = 0$$

$$\theta(t) = \frac{\pi}{2} \cos\left(\frac{\sqrt{35}}{5}t\right)$$