Applications of Laplace Transforms

Circuits

1.) $Li' + Ri = E(t)$

$\frac{1}{10}i' + 2i = \cos t \quad i(0) = 2$

$i' + 20i = 10\cos t$

$s\mathcal{L}\{i\} - i(0) + 20\mathcal{L}\{i\} = 10\mathcal{L}\{\cos t\}$

$s\mathcal{L}\{i\} - 2 + 20\mathcal{L}\{i\} = 10 \cdot \frac{s}{s^2+1}$

$(s+20)\mathcal{L}\{i\} = \frac{10s}{s^2+1} + 2$

$(s+20)\mathcal{L}\{i\} = \frac{10s + 2s^2 + 2}{s^2+1}$

$\mathcal{L}\{i\} = 2\left(\frac{s^2+5s+1}{(s+20)(s^2+1)}\right)$ ✱

$\mathcal{L}\{i\} = 2\left(\frac{301}{401}\left(\frac{1}{s+20}\right) + \frac{100}{401}\left(\frac{s}{s^2+1}\right) + \frac{5}{401}\left(\frac{1}{s^2+1}\right)\right)$

$$\boxed{i = 2\left[\frac{301}{401}e^{-20t} + \frac{100}{401}\cos t + \frac{5}{401}\sin t\right]}$$

✱ $\frac{s^2+5s+1}{(s+20)(s^2+1)} = \frac{A}{s+20} + \frac{Bs+C}{s^2+1}$

$s^2+5s+1 = As^2 + A + Bs^2 + 20Bs + Cs + 20C$

$A+B=1 \quad 20B+C=5 \quad A+20C=1$

$20B = 5 - C \quad A = 1 - 20C$

$B = \frac{1}{4} - \frac{1}{20}C$

$1 - 20C + \frac{1}{4} - \frac{1}{20}C = 1$

$-20C - \frac{1}{20}C = -\frac{1}{4}$

$+400C + C = 5$

$401C = 5$

$C = \frac{5}{401}$

$B = \frac{1}{4} - \frac{1}{20}\left(\frac{5}{401}\right)$

$B = \frac{1}{4} - \frac{1}{4}\left(\frac{1}{401}\right)$

$B = \frac{400}{4(401)}$

$B = \frac{100}{401}$

$A = 1 - 20\left(\frac{5}{401}\right)$

$A = 1 - \frac{100}{401}$

$A = \frac{301}{401}$

2. $Li' + Ri = E(t)$

$6i' + 3i = \sin t - \sin t\, u(t-\pi) \qquad i(0)=1$

$6(s\mathcal{L}\{i\} - i(0)) + 3\mathcal{L}\{i\} = \mathcal{L}\{\sin t\} - \mathcal{L}\{\sin t\, u(t-\pi)\}$ ✱

$6(s\mathcal{L}\{i\} - 1) + 3\mathcal{L}\{i\} = \dfrac{1}{s^2+1} - e^{-\pi s}\left(\dfrac{-1}{s^2+1}\right)$

$6s\mathcal{L}\{i\} - 6 + 3\mathcal{L}\{i\} = \dfrac{1}{s^2+1} + e^{-\pi s}\left(\dfrac{1}{s^2+1}\right)$

$(6s+3)\mathcal{L}\{i\} = \dfrac{1}{s^2+1} + 6 + e^{-\pi s}\left(\dfrac{1}{s^2+1}\right)$

$(6s+3)\mathcal{L}\{i\} = \dfrac{1+6s^2+6}{s^2+1} + e^{-\pi s}\left(\dfrac{1}{s^2+1}\right)$

$\mathcal{L}\{i\} = \underbrace{\dfrac{6s^2+7}{(s^2+1)(6s+3)}}_{①} + e^{-\pi s}\underbrace{\left(\dfrac{1}{(s^2+1)(6s+3)}\right)}_{②}$

$\mathcal{L}\{i\} = \dfrac{-2}{15}\left(\dfrac{s}{s^2+1}\right) + \dfrac{1}{15}\left(\dfrac{1}{s^2+1}\right) + \dfrac{34}{5}\left(\dfrac{1}{6s+3}\right)$

$+ e^{-\pi s}\underbrace{\left(\dfrac{-2}{15}\left(\dfrac{s}{s^2+1}\right) + \dfrac{1}{15}\left(\dfrac{1}{s^2+1}\right) + \dfrac{4}{5}\left(\dfrac{1}{6s+3}\right)\right)}_{✱✱}$

$i = -\dfrac{2}{15}\cos t + \dfrac{1}{15}\sin t + \dfrac{34}{5}\cdot\dfrac{1}{6}e^{-t/2}$

$+ \left[-\dfrac{2}{15}\cos(t-\pi) + \dfrac{1}{15}\sin(t-\pi) + \dfrac{2}{15}e^{-\frac{(t-\pi)}{2}}\right]u(t-\pi)$

✱ $f(t) = \sin t$

$f(t+\pi) = \sin(t+\pi)$

$= -\sin t$

$\mathcal{L}\{-\sin t\} = \dfrac{-1}{s^2+1}$

$e^{-\pi s}\mathcal{L}\{-\sin t\} = e^{-\pi s}\left(-\dfrac{1}{s^2+1}\right)$

① $\dfrac{6s^2+7}{(s^2+1)(6s+3)} = \dfrac{As+B}{s^2+1} + \dfrac{C}{6s+3}$

$6s^2+7 = 6As^2 + 3As + 6Bs + 3B + Cs^2 + C$

$6A + C = 6 \qquad 3A + 6B = 0 \qquad 3B + C = 7$

$-12B + 7 - 3B = 6 \qquad 3A = -6B \qquad C = 7 - 3B$

$-15B = -1 \qquad A = -2B \qquad C = 7 - \dfrac{1}{5}$

$B = \dfrac{1}{15} \qquad A = \dfrac{-2}{15} \qquad C = \dfrac{34}{5}$

② $\dfrac{1}{(s^2+1)(6s+3)} = \dfrac{As+B}{s^2+1} + \dfrac{C}{6s+3}$

$1 = 6As^2 + 3As + 6Bs + 3B + Cs^2 + C$

$6A + C = 0 \qquad 3A + 6B = 0 \qquad 3B + C = 1$

$-12B + 1 - 3B = 0 \qquad A = -2B \qquad C = 1 - 3B$

$-15B = -1 \qquad A = -\dfrac{2}{15} \qquad C = 1 - \dfrac{1}{5}$

$B = \dfrac{1}{15} \qquad C = \dfrac{4}{5}$

✱✱ $F(s) = \dfrac{-2}{15}\left(\dfrac{s}{s^2+1}\right) + \dfrac{1}{15}\left(\dfrac{1}{s^2+1}\right) + \dfrac{4}{5}\cdot\dfrac{1}{6}\left(\dfrac{1}{s+\frac{1}{2}}\right)$

$f(t) = -\dfrac{2}{15}\cos t + \dfrac{1}{15}\sin t + \dfrac{2}{15}e^{-t/2}$

$f(t-\pi) = \dfrac{-2}{15}\cos(t-\pi) + \dfrac{1}{15}\sin(t-\pi)$

$+ \dfrac{2}{15}e^{-(t-\pi)/2}$

3. $Li' + Ri = E(t)$

$6i' + 3i = 20 + 60e^{-3t}u(t-2) \qquad i(0) = 1$

$6(s\mathcal{L}\{i\} - i(0)) + 3\mathcal{L}\{i\} = \mathcal{L}\{20\} + 60\mathcal{L}\{e^{-3t}u(t-2)\}$ ①

$$6s\mathcal{L}\{i\} - 6 + 3\mathcal{L}\{i\} = \frac{20}{s} + 60e^{-6}\cdot e^{-2s}\left(\frac{1}{s+3}\right)$$

$$(6s+3)\mathcal{L}\{i\} = \frac{20}{s} + 6 + 60e^{-6}e^{-2s}\left(\frac{1}{s+3}\right)$$

$$(6s+3)\mathcal{L}\{i\} = \frac{20+6s}{s} + 60e^{-6}\cdot e^{-2s}\left(\frac{1}{s+3}\right)$$

$$\mathcal{L}\{i\} = \underbrace{\frac{6s+20}{s(6s+3)}}_{②} + 60e^{-6}\cdot e^{-2s}\underbrace{\left(\frac{1}{(s+3)(6s+3)}\right)}_{③}$$

$$\mathcal{L}\{i\} = \frac{20}{3}\left(\frac{1}{s}\right) - 34\left(\frac{1}{6s+3}\right) + 60e^{-6}\left[e^{-2s}\underbrace{\left(\frac{-1}{15}\left(\frac{1}{s+3}\right) + \frac{2}{5}\left(\frac{1}{6s+3}\right)\right)}_{④}\right]$$

$$i = \frac{20}{3} - 34\cdot\frac{1}{6}e^{-t/2} + 60e^{-6}\left[\frac{-1}{15}e^{-3(t-2)} + \frac{1}{15}e^{-\frac{(t-2)}{2}}\right]u(t-2)$$

$$\boxed{i = \frac{20}{3} - \frac{17}{3}e^{-t/2} + 4e^{-6}\left(e^{-\frac{(t-2)}{2}} - e^{-3(t-2)}\right)u(t-2)}$$

① $f(t) = e^{-3t}$ $\qquad a = 2$

$f(t+2) = e^{-3(t+2)} = e^{-6}\cdot e^{-3t}$

$\mathcal{L}\{f(t+2)\} = e^{-6}\left(\frac{1}{s+3}\right)$

$e^{-2s}\mathcal{L}\{f(t+2)\} = e^{-2s}\cdot e^{-6}\left(\frac{1}{s+\ldots}\right)$

② $\frac{6s+20}{s(6s+3)} = \frac{A}{s} + \frac{B}{6s+3}$

$6s + 20 = 6As + 3A + Bs$

$6A + B = 6 \qquad 3A = 20$

$6\left(\frac{20}{3}\right) + B = 6 \qquad A = \frac{20}{3}$

$40 + B = 6$

$B = -34$

③ $\frac{1}{(s+3)(6s+3)} = \frac{A}{s+3} + \frac{B}{6s+3}$

$1 = 6As + 3A + Bs + 3B$

$6A + B = 0 \qquad 3A + 3B = 1$

$B = -6A \qquad 3A - 18A = 1$

$B = \frac{2}{5} \qquad -15A = 1$

$A = \frac{-1}{15}$

④ $F(s) = \frac{-1}{15}\left(\frac{1}{s+3}\right) + \frac{2}{5}\cdot\frac{1}{6}\left(\frac{1}{s+1/2}\right)$

$f(t) = \frac{-1}{15}e^{-3t} + \frac{1}{15}e^{-t/2}$

$f(t-2) = \frac{-1}{15}e^{-3(t-2)} + \frac{1}{15}e^{-(t-2)/2}$

4. $Lq'' + Rq' + \frac{q}{C} = E(t)$

$q'' + 2q' + 9q = \cos t \qquad i(0)=0 \quad q(0)=0$

$s^2\mathcal{L}\{q\} - sq(0) - q'(0) + 2(s\mathcal{L}\{q\} - q(0)) + 9\mathcal{L}\{q\} = \mathcal{L}\{\cos t\}$

$s^2\mathcal{L}\{q\} + 2s\mathcal{L}\{q\} + 9\mathcal{L}\{q\} = \frac{s}{s^2+1}$

$(s^2+2s+9)\mathcal{L}\{q\} = \frac{s}{s^2+1}$

$\mathcal{L}\{q\} = \frac{s}{(s^2+1)(s^2+2s+9)}$ ①

① $\frac{s}{(s^2+1)(s^2+2s+9)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2s+9}$

$s = As^3 + 2As^2 + 9As + Bs^2 + 2Bs + 9B + Cs^3 + Cs + Ds^2 + D$

$9B + D = 0 \quad A + C = 0 \quad 2A + B + D = 0 \quad 9A + 2B + C = 1$

$D = -9B \quad A = -C \quad -9C + 2B + C = 1$

$D = -\frac{9}{2} - 36C \quad -8C + 2B = 1$

$D = -9\left(\frac{33}{34}\right) \quad 2B = 1 + 8C$

$\boxed{D = -\frac{297}{34}} \quad B = \frac{1}{2} + 4C$

$-2C + \frac{1}{2} + 4C - \frac{9}{2} - 36C = 0$

$-34C = 4$

$\boxed{C = \frac{2}{17}}$

$\boxed{A = -\frac{2}{17}}$

$B = \frac{1}{2} + \frac{8}{17}$

$B = \frac{17+16}{34}$

$\boxed{B = \frac{33}{34}}$

$\mathcal{L}\{q\} = \frac{-2}{17}\left(\frac{s}{s^2+1}\right) + \frac{33}{34}\left(\frac{1}{s^2+1}\right) + \frac{\frac{2}{17}s - \frac{297}{34}}{s^2+2s+9}$

$= \frac{-2}{17}\left(\frac{s}{s^2+1}\right) + \frac{33}{34}\left(\frac{1}{s^2+1}\right) + \frac{2}{17}\left(\frac{s+1-1}{(s+1)^2+8}\right) - \frac{297}{34}\left(\frac{1}{(s+1)^2+8}\right)$

$= \frac{-2}{17}\left(\frac{s}{s^2+1}\right) + \frac{33}{34}\left(\frac{1}{s^2+1}\right) + \frac{2}{17}\left(\frac{s+1}{(s+1)^2+8}\right) - \frac{2}{17}\left(\frac{1}{(s+1)^2+8}\right) - \frac{297}{34}\left(\frac{1}{(s+1)^2+8}\right)$

$$\boxed{q = -\frac{2}{17}\cos t + \frac{33}{34}\sin t + \frac{2}{17}e^{-t}\cos(\sqrt{2}t) - \frac{2}{17\sqrt{2}}e^{-t}\sin(\sqrt{2}t) - \frac{297}{34}e^{-t}\sin(\sqrt{2}t)}$$

5. $q'' + 5q' + 6q = 20t - 20t\,u(t-3) \qquad i(0) = q(0) = 0$

$$s^2\mathcal{L}\{q\} - sq(0) - q'(0) + 5\big(s\mathcal{L}\{q\} - q(0)\big) + 6\mathcal{L}\{q\} = 20\mathcal{L}\{t\} - 20\mathcal{L}\{t\,u(t-3)\}$$

$$s^2\mathcal{L}\{q\} + 5s\mathcal{L}\{q\} + 6\mathcal{L}\{q\} = 20\mathcal{L}\{t\} - 20\underset{①}{\mathcal{L}\{t\,u(t-3)\}}$$

$$(s^2+5s+6)\mathcal{L}\{q\} = \frac{20}{s^2} - 20e^{-3s}\left(\frac{3s+1}{s^2}\right)$$

$$\mathcal{L}\{q\} = \underset{②}{\frac{20}{s^2(s+3)(s+2)}} - 20e^{-3s}\underset{③}{\left(\frac{3s+1}{s^2(s+3)(s+2)}\right)}$$

① $f(t) = t \qquad a = 3$

$f(t+3) = t+3$

$\mathcal{L}\{f(t+3)\} = \frac{1}{s^2} + \frac{3}{s}$

$e^{-3s}\mathcal{L}\{t+3\} = e^{-3s}\left(\frac{1+3s}{s^2}\right)$

② $\frac{20}{s^2(s+3)(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3} + \frac{D}{s+2}$

$20 = As(s^2+5s+6) + B(s^2+5s+6) + C(s^3+2s^2)$
$+ D(s^3+3s^2)$

$20 = As^3 + 5As^2 + 6As + Bs^2 + 5Bs + 6B$
$+ Cs^3 + 2Cs^2 + Ds^3 + 3Ds^2$

$A + C + D = 0 \qquad 5A + B + 2C + 3D = 0$

$-\frac{25}{9} + C + D = 0 \qquad -\frac{125}{9} + \frac{10}{3} + 2C + 3D = 0$

$C = \frac{25}{9} - D \qquad -\frac{125}{9} + \frac{30}{9} + \frac{50}{9} - 2D + 3D = 0$

$C = \frac{-20}{9} \qquad D = \frac{45}{9} \qquad D = 5$

$6A + 5B = 0 \qquad 6B = 20$

$6A + \frac{50}{3} = 0 \qquad B = \frac{10}{3}$

$6A = \frac{-50}{3}$

$A = \frac{-25}{9}$

③ $\frac{3s+1}{s^2(s+3)(s+2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+3} + \frac{D}{s+2}$

$3s+1 = As(s^2+5s+6) + B(s^2+5s+6) + Cs^2(s+2)$
$+ Ds^2(s+3)$

$3s+1 = As^3 + 5As^2 + 6As + Bs^2 + 5Bs + 6B + Cs^3$
$+ 2Cs^2 + Ds^3 + 3Ds^2$

$A + C + D = 0 \qquad 5A + B + 2C + 3D = 0$

$\frac{13}{36} + C + D = 0 \qquad \frac{65}{36} + \frac{1}{6} + 2\left(\frac{-13}{36} - D\right) + 3D = 0$

$C = \frac{-13}{36} - D \qquad \frac{65}{36} + \frac{6}{36} - \frac{26}{36} - 2D + 3D = 0$

$C = \frac{32}{36} = \frac{8}{9} = C \qquad D = \frac{-45}{36} = \frac{-5}{4} = D$

$6A + 5B = 3 \qquad 6B = 1$

$6A = \frac{18}{6} - \frac{5}{6} \qquad B = \frac{1}{6}$

$6A = \frac{13}{6}$

$A = \frac{13}{36}$

$$\mathcal{L}\{q\} = \frac{-25}{9}\left(\frac{1}{s}\right) + \frac{10}{3}\left(\frac{1}{s^2}\right) - \frac{20}{9}\left(\frac{1}{s+3}\right) + 5\left(\frac{1}{s+2}\right) - 20e^{-3s}\underset{④}{\left[\frac{13}{36}\left(\frac{1}{s}\right) + \frac{1}{6}\left(\frac{1}{s^2}\right) + \frac{8}{9}\left(\frac{1}{s+3}\right) - \frac{5}{4}\left(\frac{1}{s+2}\right)\right]}$$

$$q = \frac{-25}{9} + \frac{10}{3}t - \frac{20}{9}e^{-3t} + 5e^{-2t} - 20\left[\frac{13}{36} + \frac{1}{6}(t-3) + \frac{8}{9}e^{-3(t-3)} - \frac{5}{4}e^{-2(t-3)}\right]u(t-3)$$

④ $f(t) = \frac{13}{36} + \frac{1}{6}t + \frac{8}{9}e^{-3t} - \frac{5}{4}e^{-2t}$

Springs

1.) $mx'' + bx' + kx = f(t)$

$x'' + 9x = 2t \qquad x(0) = 1 \quad x'(0) = 0$

$s^2\mathcal{L}\{x\} - s\,x(0) - x'(0) + 9\mathcal{L}\{x\} = 2\mathcal{L}\{t\}$

$s^2\mathcal{L}\{x\} - s + 9\mathcal{L}\{x\} = \frac{2}{s^2}$

$s^2\mathcal{L}\{x\} + 9\mathcal{L}\{x\} = \frac{2}{s^2} + s$

$(s^2+9)\mathcal{L}\{x\} = \frac{2+s^3}{s^2}$

$\mathcal{L}\{x\} = \frac{s^3+2}{s^2(s^2+9)}$ ⊛

⊛ $\frac{s^3+2}{s^2(s^2+9)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2+9}$

$s^3+2 = As(s^2+9) + B(s^2+9) + s^2(Cs+$

$s^3+2 = As^3 + 9As + Bs^2 + 9B + Cs^3 + Ds^2$

$A+C=1 \quad B+D=0 \quad 9A=0 \quad 9B=2$

$C=1 \quad D=-\frac{2}{9} \quad A=0 \quad B=\frac{2}{9}$

$\mathcal{L}\{x\} = \frac{2}{9}\left(\frac{1}{s^2}\right) + \frac{s}{s^2+9} - \frac{2}{9}\left(\frac{1}{s^2+9}\right)$

$\mathcal{L}\{x\} = \frac{2}{9}\left(\frac{1}{s^2}\right) + \frac{s}{s^2+9} - \frac{2}{27}\left(\frac{3}{s^2+9}\right)$

$$\boxed{x = \frac{2}{9}t + \cos(3t) - \frac{2}{27}\sin(3t)}$$

2.) $x'' + 9x = 5 - 5u(t-10) \qquad x'(0) = 0 \quad x(0) = 1$

$$s^2\mathcal{L}\{x\} - sx(0) - x'(0) + 9\mathcal{L}\{x\} = \mathcal{L}\{5\} - 5\mathcal{L}\{u(t-10)\}$$

$$s^2\mathcal{L}\{x\} + 9\mathcal{L}\{x\} = \frac{5}{s} - \frac{5e^{-10s}}{s}$$

$$(s^2+9)\mathcal{L}\{x\} = \frac{5}{s} - \frac{5e^{-10s}}{s}$$

$$\mathcal{L}\{x\} = \frac{5}{s(s^2+9)} - e^{-10s}\left(\frac{5}{s(s^2+9)}\right)$$

$$\frac{5}{s(s^2+9)} = \frac{A}{s} + \frac{Bs+C}{s^2+9}$$

$$5 = As^2 + 9A + Bs^2 + Cs$$

$A + B = 0 \quad C = 0 \quad 9A = 5$

$B = \frac{-5}{9} \qquad A = \frac{5}{9}$

$$\mathcal{L}\{x\} = \frac{5}{9}\left(\frac{1}{s}\right) - \frac{5}{9}\left(\frac{s}{s^2+9}\right) - e^{-10s}\left[\frac{5}{9}\left(\frac{1}{s}\right) - \frac{5}{9}\left(\frac{s}{s^2+9}\right)\right]$$

$$x = \frac{5}{9} - \frac{5}{9}\cos(3t) - \left[\frac{5}{9} - \frac{5}{9}\cos(3(t-10))\right]u(t-10)$$

$$\boxed{x = \frac{5}{9}\left(1 - \cos(3t) - \left[1 - \cos(3t-30)\right]u(t-10)\right)}$$

Newtonian Mechanics

1.) $mv' + kv = F$

$5v' + v = (5)(9.8)$

$5v' + v = \frac{98}{2}$

$5(s\mathcal{L}\{v\} - v(0)) + \mathcal{L}\{v\} = \mathcal{L}\{\frac{98}{2}\}$

$5(s\mathcal{L}\{v\} + \mathcal{L}\{v\} = \frac{98}{2}(\frac{1}{s})$

$(5s+1)\mathcal{L}\{v\} = \frac{98}{2}(\frac{1}{s})$

$\mathcal{L}\{v\} = \frac{98}{2}\left(\frac{1}{s(5s+1)}\right)$ ⊛

⊛ $\frac{1}{s(5s+1)} = \frac{A}{s} + \frac{B}{5s+1}$

$1 = 5As + A + Bs$

$A = 1 \quad 5A + B = 0$

$5 + B = 0$

$B = -5$

$\mathcal{L}\{v\} = \frac{98}{2}\left(\frac{1}{s} - \frac{5}{5s+1}\right)$

$\mathcal{L}\{v\} = \frac{98}{2}\left(\frac{1}{s} - \frac{1}{s+1/5}\right)$

$\boxed{v = \frac{98}{2}\left(1 - e^{-t/5}\right)}$

2.) $mv' + kv = F$

$100v' + 5v = 500 - 500u(t-20)$

$20v' + v = 100 - 100u(t-20)$

$20(s\mathcal{L}\{v\} - v(0)) + \mathcal{L}\{v\} = \mathcal{L}\{100\} - \mathcal{L}\{100u(t-20)\}$

$20(s\mathcal{L}\{v\} - 4 0) + \mathcal{L}\{v\} = \frac{100}{s} - \frac{100e^{-20s}}{s}$

$20s\mathcal{L}\{v\} + 80 + \mathcal{L}\{v\} = \frac{100}{s} - \frac{100e^{-20s}}{s}$

$(20s+1)\mathcal{L}\{v\} = \frac{100}{s} - 80 - \frac{100e^{-20s}}{s}$

$\mathcal{L}\{v\} = \underbrace{\frac{100 - 80s}{s(20s+1)}}_{①} - \underbrace{\frac{100e^{-20s}}{s(20s+1)}}_{②}$

① $$\frac{100-80s}{s(20s+1)} = \frac{A}{s} + \frac{B}{20s+1}$$

$$100 - 80s = 20As + A + Bs$$

$$20A + B = -80 \qquad A = 100$$

$$2000 + B = -80$$

$$B = -2080$$

② $$\frac{100}{s(20s+1)} = \frac{A}{s} + \frac{B}{20s+1}$$

$$100 = 20As + A + Bs$$

$$20A + B = 0 \qquad A = 100$$

$$2000 + B = 0$$

$$B = -2000$$

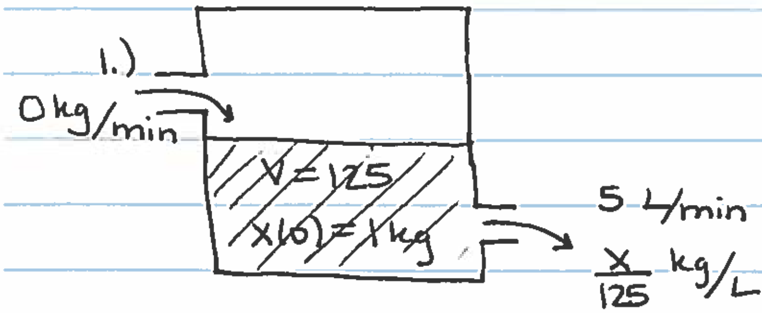
$$\mathcal{L}\{v\} = \frac{100}{s} - \frac{2080}{20s+1} - e^{-20s}\left(\frac{100}{s} - \frac{2000}{20s+1}\right)$$

$$\mathcal{L}\{v\} = \frac{100}{s} - \frac{2080}{20}\left(\frac{1}{s+1/20}\right) - e^{-20s}\left(\frac{100}{s} - \frac{2000}{20}\left(\frac{1}{s+1/20}\right)\right)$$

$$\boxed{v = 100 - 104e^{-t/20} - \left[1 - 100e^{-(t-20)/20}\right] u(t-20)}$$

Compartmental

1.)



$$\frac{dx}{dt} = \text{rate in} - \text{rate out}$$

$$x' = (0 + 4\delta(t-5)) - \frac{5x}{125}$$

$$x' = 4\delta(t-5) - \frac{x}{25}$$

$$x' + \frac{1}{25}x = 4\delta(t-5)$$

$$25x' + x = 100\delta(t-5)$$

$$25(s\mathcal{L}\{x\} - x(0)) + \mathcal{L}\{x\} = 100\mathcal{L}\{\delta(t-5)\}$$

$$25(s\mathcal{L}\{x\} - 1) + \mathcal{L}\{x\} = 100e^{-5s}$$

$$25s\mathcal{L}\{x\} - 25 + \mathcal{L}\{x\} = 100e^{-5s}$$

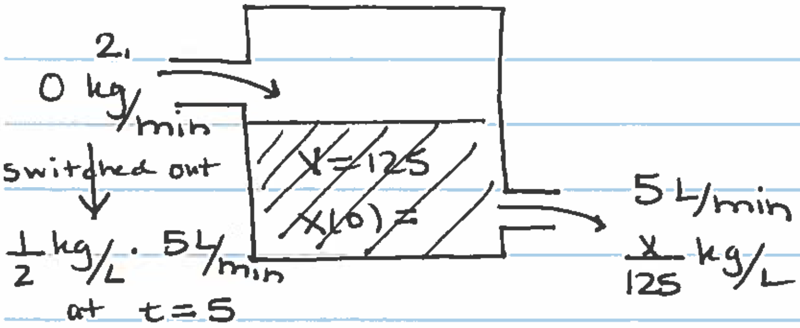
$$(25s+1)\mathcal{L}\{x\} = 10e^{-5s} + 25$$

$$\mathcal{L}\{x\} = 10e^{-5s} \cdot \frac{1}{25s+1} + \frac{25}{25s+1}$$

$$\mathcal{L}\{x\} = \frac{10}{25} e^{-5s}\left(\frac{1}{s+1/25}\right) + \frac{1}{s+1/25}$$

$$\boxed{x = \frac{2}{5} e^{-(t-5)/25} u(t-5) + e^{-t/25}}$$

2.



$$x' = \frac{5}{2}u(t-5) - \frac{5x}{125}$$

$$x' + \frac{x}{25} = \frac{5}{2}u(t-5)$$

$$25x' + x = \frac{125}{2}u(t-5)$$

$$50x' + 2x = 125\,u(t-5)$$

$$50(s\mathcal{L}\{x\} - x(0)) + 2\mathcal{L}\{x\} = 125\,\mathcal{L}\{u(t-5)\}$$

$$50(s\mathcal{L}\{x\} - 1) + 2\mathcal{L}\{x\} = \frac{125e^{-5s}}{s}$$

$$50s\mathcal{L}\{x\} - 50 + 2\mathcal{L}\{x\} = \frac{125e^{-5s}}{s}$$

$$(50s+2)\mathcal{L}\{x\} = \frac{125e^{-5s}}{s} + 50$$

$$\mathcal{L}\{x\} = 125e^{-5s}\left(\frac{1}{s(50s+2)}\right) + \frac{50}{50s+2}$$

$$\mathcal{L}\{x\} = \frac{125}{50}e^{-5s}\left(\frac{1}{s(s+1/25)}\right) + \frac{1}{s+1/25}$$

⊛

$$\mathcal{L}\{x\} = \frac{5}{2}e^{-5s}\left(25\left(\frac{1}{s}\right) - 25\left(\frac{1}{s+1/25}\right)\right) + \frac{1}{s+1/25}$$

$$\boxed{x = \frac{5}{2}\left(25 - 25e^{-(t-5)/25}\right)u(t-5) + e^{-t/25}}$$

⊛ $$\frac{1}{s(s+1/25)} = \frac{A}{s} + \frac{B}{s+1/2}$$

$$1 = As + \frac{1}{25}A + Bs$$

$$A + B = 0 \qquad \frac{1}{25}A = 1$$

$$B = -25 \qquad A = 25$$