Analysis of graphs generated by the number of divisors function

B.L. Mayer[a,1] and L.H.A. Monteiro[a,b,2]

(a) Universidade Presbiteriana Mackenzie,
PPGEEC, São Paulo, SP, Brazil

(b) Universidade de São Paulo,
Escola Politécnica, São Paulo, SP, Brazil

(1) bleemayer@gmail.com
(2) luizm@mackenzie.br, luizm@usp.br

Corresponding author:
Luiz Henrique Alves Monteiro
Universidade Presbiteriana Mackenzie
Escola de Engenharia
Rua da Consolação, n.896
01302-907, São Paulo, SP, Brazil
E-mail addresses: luizm@mackenzie.br, luizm@usp.br
Telephone number: (55)(11)2114-8711
Fax number: (55)(11)2114-8600

ORCID ID: 0000-0002-2309-1254

1 Introduction

This paper presents studies on two series generated using the divisor function.

The first is generated by the points $(x, |\{d : d|x\}|)$ for $n \in N$ that is: each number to its quantity of divisors. That is:

$$\{(2,2),(3,2),(4,3),(5,2),\ldots,(10000,25)\}$$

The second applies the divisor function iteratively until the result reaches 2, that is, is a prime number, this series will be called as the orbit series through this paper. For example, take the number 60:

- 60 has 12 divisors, first time
- 12 has 6 divisors, second time
- 6 has 4 divisors, third time
- 4 has 3 divisors, fourth time
- and finally, 3 has 2 divisors, fifth time

Is this case we applied the function 5 times, hence the orbit of 60 has length 5, and the point is $(60, 5)$. Generating the following sequence:

$$\{(2,1),(3,1),(4,2),(5,1),\ldots,(10000,3)\}$$

The intention is to discover for each case how the natural visibility and horizontal visibility measures relate. And in consequence, how the two series are related.

Also, for the two cases the mean shortest path was calculated at each new vertex.

2 Methodology and numerical results

This section describes the procedure to build and analyse each series. The analysis consists of calculating the natural and horizontal visibilities for the series, then fitting the model $Cx^{-\gamma}$ on the degree distribution of the generated graph and analyse the coefficients.

In the studies the numbers started at 2 to avoid the only case of 1 divisor. On the second study specifically, the orbit of 2 was set to 1 instead of 0 to avoid this special case and, as consequence, dividing the graph in two disconnected subgraphs. All links are considered undirected.

The following table summarizes the parameter values of the two studies.

series	visibility	γ	C
divisors	natural	1.21489 ± 0.18265	0.559692 ± 0.132711
divisors	horizontal	0.637136 ± 0.658635	0.190621 ± 0.174726
orbits	natural	0.556285 ± 0.414888	0.149885 ± 0.0999156
orbits	horizontal	0.353234 ± 1.56196	0.198959 ± 0.379138

(1)

In the following paragraphs all results for the two studies are displayed, for each set we show first the graph, for illustration purposes, then the plot of the mean shortest path.

The figure (1) shows a graph that is generated using the divisors series, by adding links $(x, y) \mapsto x \to y$

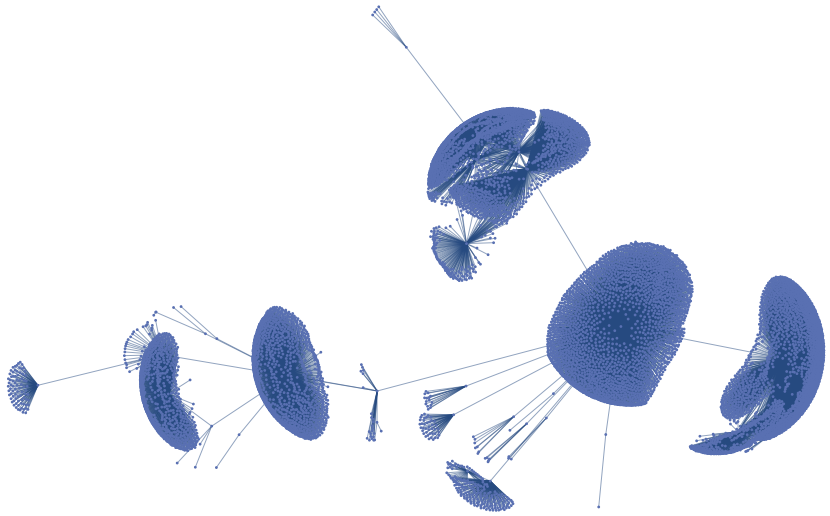

Figure 1: Graph illustrating the divisors series

The figure (2) shows the corresponding plot of mean shortest path in relation to the number of nodes for the divisors series.

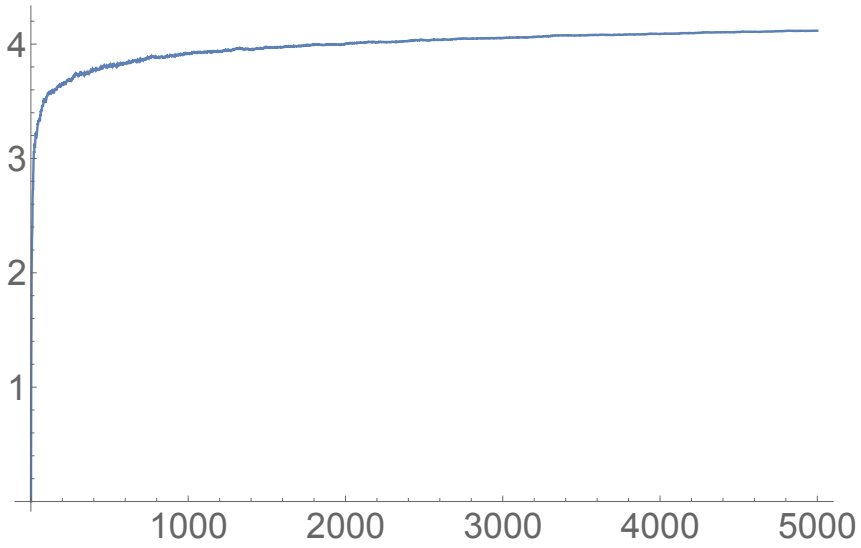

Figure 2: Graph of the mean shortest path for each new vertex up to 5000 nodes

The figure (3) shows the graph generated by the orbits series.
And figure (4), the corresponding plot of mean shortest path.

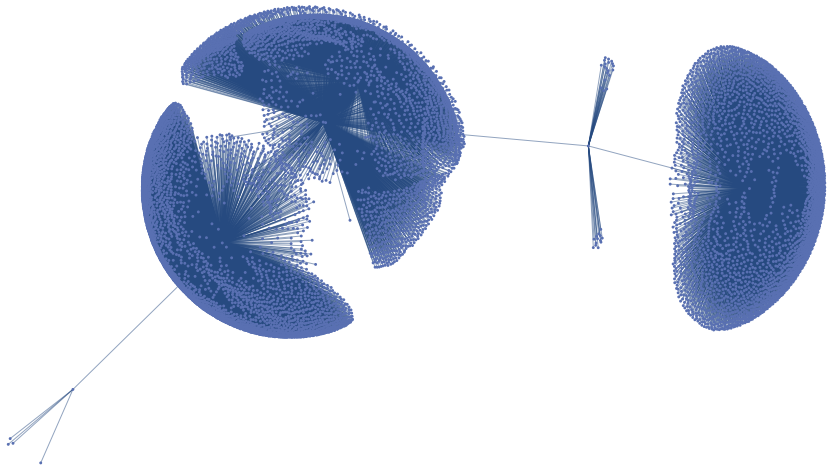

Figure 3: Graph illustrating the orbits series

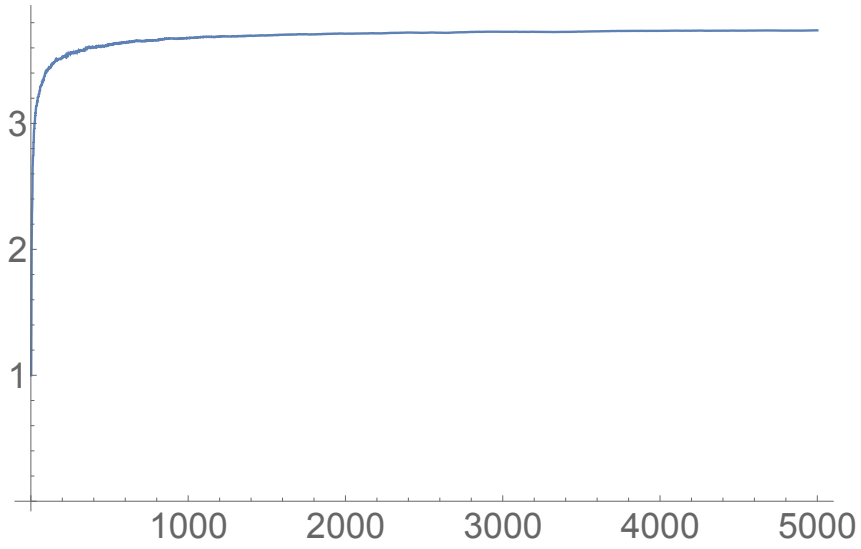

Figure 4: Graph of the mean shortest path for each new vertex up to 5000 nodes

3 Discussion and Conclusion

By visual inspection the graphs are alike, they show a fractal structure and the associated γ coefficient for the orbits series is approximately half of the divisors series, but has shown a big variance, which indicates that the model may not be the best choice.

In the two studies the mean minimum path seemed to converge, the divisors series converges to 4.11735 and the orbits series to 3.73921. Using more nodes can increase the precision, in the divisor series the value seems to take longer to converge.

In both cases the number of nodes can be increased to give more precise results, also the orbit of the orbit can be studied.

A possible further study is to determine the likelihood of a node forming a link with the new node during the construction of a graph. This can lead to the values of convergence of the mean minimum path for both graphs.