Lecture 09: Model free learning in RL

Radoslav Neychev

Outline

1. MDP properties recap
2. Value-function and Q-function
3. Reward discounting
4. Q-learning, temporal difference
5. Approximate Q-learning

Based on: https://github.com/yandexdataschool/Practical_RL/ weeks 2, 3 and 4

References

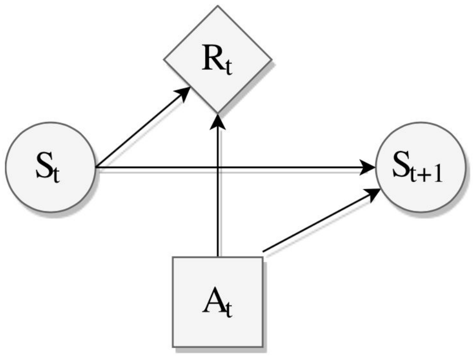
These slides are deeply based on [Practical RL course] weeks 2, 3 and 4
Special thanks to YSDA team for making them publicly available.

Given dynamics, how to find an optimal policy?

Definition of Markov Decision Process

MDP is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$, where

1. $\mathcal{S}$ – set of states of the world
2. $\mathcal{A}$ – set of actions
3. $\mathcal{P} : \mathcal{S} \times \mathcal{A} \mapsto \Delta(\mathcal{S})$ – state-transition function, giving us $p(s_{t+1} \mid s_t, a_t)$
4. $\mathcal{R} : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}$ – reward function, giving us $\mathbb{E}_R\left[\, R(s_t, a_t) \mid s_t, a_t \,\right]$.



Markov property

$$p(r_t, s_{t+1} \mid s_0, a_0, r_0, \ldots, s_t, a_t) = p(r_t, s_{t+1} \mid s_t, a_t)$$

(next state, expected reward) depend on (previous state, action)

Goal: solve an MDP by finding an optimal policy

1. What is the objective?
    a. Reward: discounting and design
    b. Expected objective: state- and action-value function

Explaining goals to agent through reward

Reward hypothesis (R.Sutton)

Goals and purposes can be thought of as the maximization of the expected value of the cumulative sum of a received scalar signal

Cumulative reward is called a **return:**

end of an episode

$$\boxed{G_t} \triangleq \boxed{R_t} + R_{t+1} + R_{t+2} + \ldots + R_{\boxed{T}}$$

immediate reward

E.g.: reward in **chess** – value of taken opponent's piece

E.g.: data center non-stop cooling system

- **S**tates – temperature measurements
- **A**ctions – different fans speed
- **R = 0** for exceeding temperature thresholds
- **R = +1** for each second system is cool

What could go wrong with such a design?

E.g.: data center non-stop cooling system

- **S**tates – temperature measurements
- **A**ctions – different fans speed
- **R = 0** for exceeding temperature thresholds
- **R = +1** for each second system is cool

What could go wrong with such a design?

Infinite return for non optimal behaviour!

$$G_t = 1 + 1 + 0 + 1 + 1 + 0 + ... = \sum_{t=1}^{\infty} R_t = \infty$$

E.g.: cleaning robot

- **S**tates – dust sensors, air
- **A**ctions – cleaning / rest / conditioning on or off
- **R = 100** for long tedious floor cleaning task done
- **R = 1** for turning air conditioning on-off
- Episode ends each day

What could go wrong with such a design?

OpenAI blog post about faulty rewards: https://openai.com/blog/faulty-reward-functions/

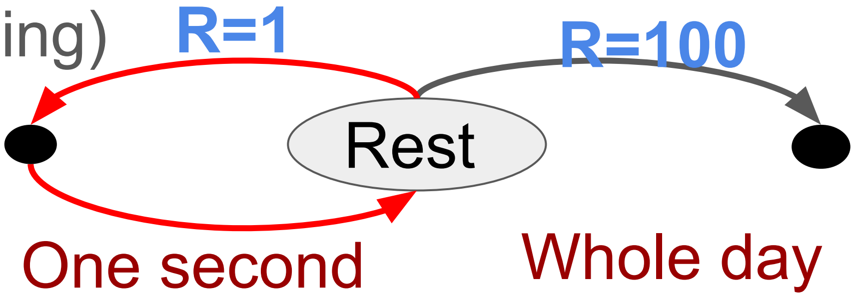
E.g.: cleaning robot

- **S**tates – dust sensors, air
- **A**ctions – cleaning / rest / conditioning on or off
- **R = 100** for long tedious floor cleaning task done
- **R = 1** for turning air conditioning on-off
- Episode ends each day

What could go wrong with such a design?

Reward(air) < Reward(cleaning)

Time(air) << Time(cleaning)



Positive feedback loop!

Explaining goals to agent through reward

Reward hypothesis (R.Sutton)

Goals and purposes can be thought of as the maximization of the expected value of the cumulative sum of a received scalar signal

Reward discounting

Get rid of infinite sum by discounting $\quad 0 \leq \gamma < 1$

$$G_t \triangleq R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \ldots = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

discount factor

The same cake compared to today's one worth

γ

- γ^2 times less tomorrow
- times less the day after tomorrow

γ will eat it day by day

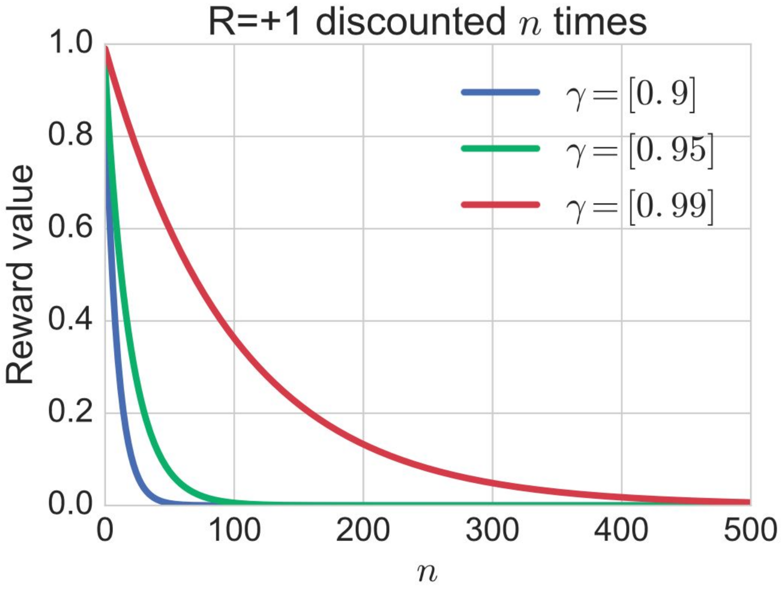
Reward discounting

Discounting makes sums finite

Maximal return for **R = +1**

γ	0.9	0.95	0.99
$\frac{1}{1-\gamma}$	10	20	100

$$G_0 = \sum_{k=0}^{\infty} \gamma^k = \frac{1}{1-\gamma}$$



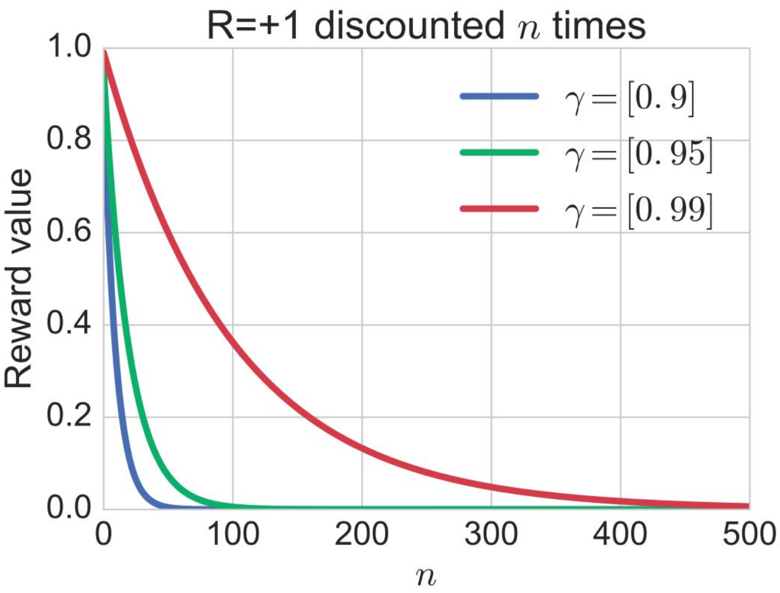
Discounting makes sums finite

Maximal return for **R = +1**

$$G_0 = \sum_{k=0}^{\infty} \gamma^k = \frac{1}{1-\gamma}$$

γ	0.9	0.95	0.99
$\frac{1}{1-\gamma}$	10	20	100

Any discounting changes optimisation task and its solution!



State- and **Action-**value functions

State-value function v(s)

v(s) is expected return conditional on state:

$$
\begin{aligned}
v_\pi(s) &\doteq \mathbb{E}_\pi\left[G_t \mid S_t = s\right] \\
&= \mathbb{E}_\pi\left[R_t + \gamma G_{t+1} \mid S_t = s\right] \\
&= \sum_a \pi(a \mid s) \sum_{r,s'} p(r, s' \mid s, a)\Big[r + \gamma \boxed{\mathbb{E}_\pi\left[G_{t+1} \mid S_{t+1} = s'\right]}\Big] \\
&= \sum_a \pi(a \mid s) \sum_{r,s'} p(r, s' \mid s, a)\left[r + \gamma \underline{v_\pi(s')}\right]
\end{aligned}
$$

By definition

Intuition: value of following policy π from state s

Action-value function q(s, a)

Is expected return conditional on state **and action**:

Intuition: value of following policy π after committing action **a** in state **s**

$$
\begin{aligned}
q_\pi(s,a) &= \mathbb{E}_\pi\left[\, G_t \,|\, S_t = s,\, A_t = a \,\right] \\
&= \mathbb{E}_\pi\left[\, R_t + \gamma G_{t+1} \,|\, S_t = s,\, A_t = a \,\right] \\
&= \sum_{r,\, s'} p(r, s' \,|\, s, a)\Big[r + \gamma \mathbb{E}_\pi\left[\, G_{t+1} | S_{t+1} = s' \,\right]\Big] \\
&= \sum_{r,\, s'} p(r, s' \,|\, s, a)\left[r + \gamma v_\pi(s')\right]
\end{aligned}
$$

Relations between v(s) and q(s,a)

We already know how to write q(s,a) in terms of v(s)

$$q_\pi(s,a) = \sum_{r,s'} p(r,s' \mid s,a)\left[r + \gamma v_\pi(s')\right]$$

What about v(s) in terms of q(s,a)?

$$\begin{aligned} v_\pi(s) &= \sum_a \pi(a \mid s) \sum_{r,s'} p(r,s' \mid s,a)\left[r + \gamma v_\pi(s')\right] \\ &= \sum_a \pi(a \mid s) q_\pi(s,a) \end{aligned}$$

So, we could now write q(s, a) in terms of q(s,a)!

$$q_\pi(s,a) = \sum_{r,s'} p(r,s' \mid s,a)\left[r + \gamma \sum_{a'} \pi(a' \mid s') q_\pi(s',a')\right]$$

Bellman expectation equation for **v(s)**

Recursive definition of v(s) is an important concept in RL

$$\begin{aligned} v_\pi(s) &= \sum_a \pi(a \,|\, s) \sum_{r,\, s'} p(r, s' \,|\, s, a) \left[r + \gamma v_\pi(s') \right] \\ &= \mathbb{E}_\pi \left[R_t + \gamma v_\pi(S_{t+1}) \,|\, S_t = s \right] \end{aligned}$$

Bellman expectation equation for **q(s,a)**

$$
\begin{aligned}
q_\pi(s,a) &= \sum_{r,\,s'} p(r, s' \mid s, a)\,[r + \gamma v_\pi(s')] \\
&= \sum_{r,\,s'} p(r, s' \mid s, a)\Big[r + \gamma \sum_{a'} \pi(a' \mid s') q_\pi(s', a')\Big]
\end{aligned}
$$

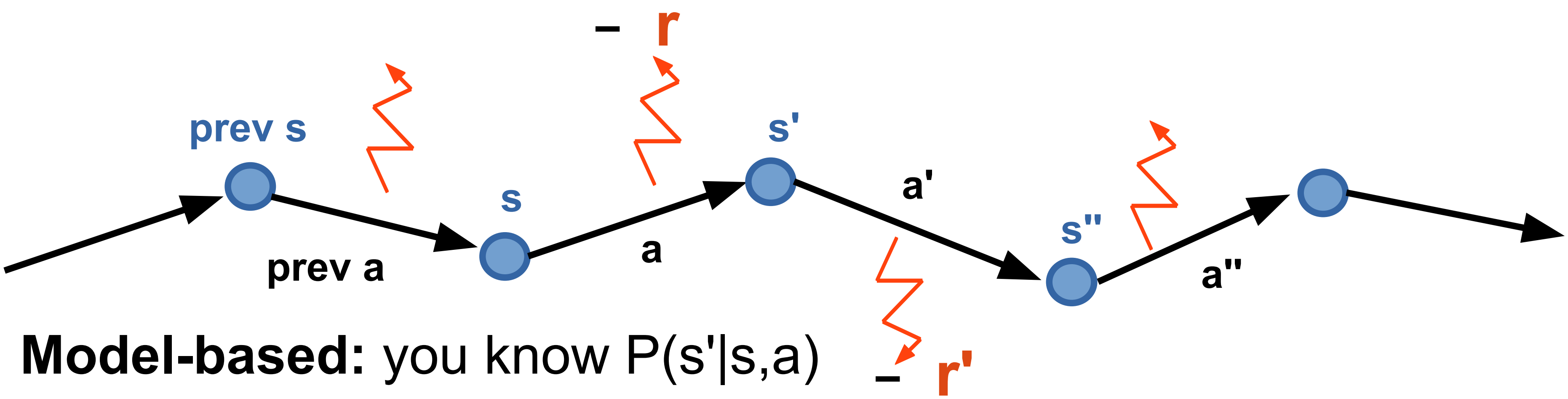
To sum up

- **Vπ(s)** – expected G from state **s** if you follow **π**
- **V*(s)** – expected G from state s if you follow **π*** – optimal
- **Qπ(s,a)** – expected G from state **s**
  - if you start by taking action **a**
  - and follow **π** from next state on
- **Q*(s,a)** – same as Qπ(s,a) where **π** = **π*** – optimal policy

$$Q^*(s,a) = \underset{s',r}{E}\, r(s,a) + \gamma \cdot V^*(s')$$

$$V^*(s) = max\, Q^*(s,a)$$

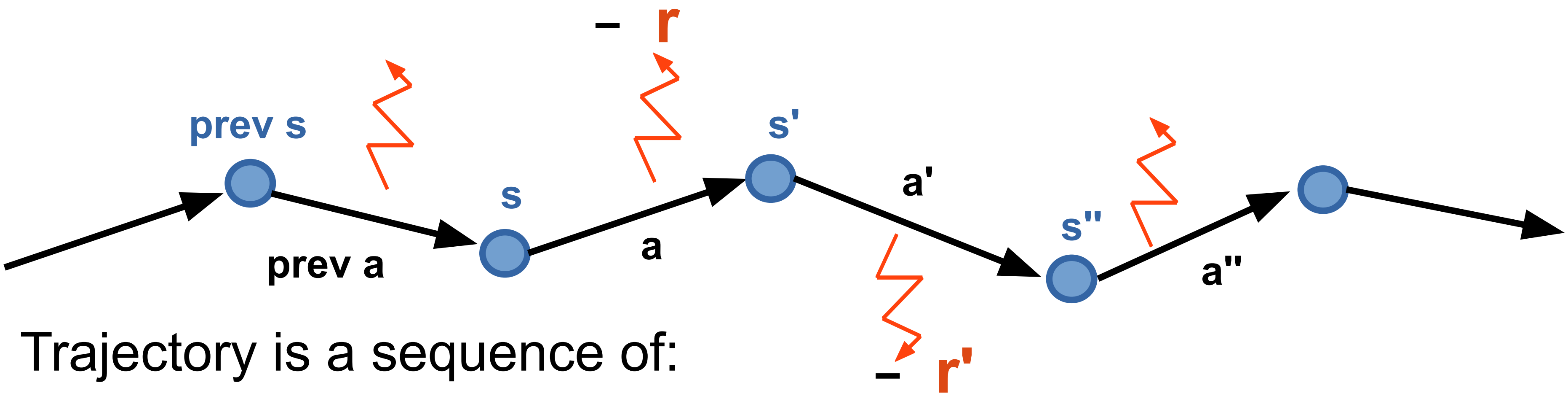
Learning from trajectories



Model-based: you know P(s'|s,a)
- can apply dynamic programming
- can plan ahead

Model-free: you can sample trajectories
- can try stuff out
- insurance not included

Learning from trajectories



Trajectory is a sequence of:

- states (s)
- actions (a)
- rewards (r)

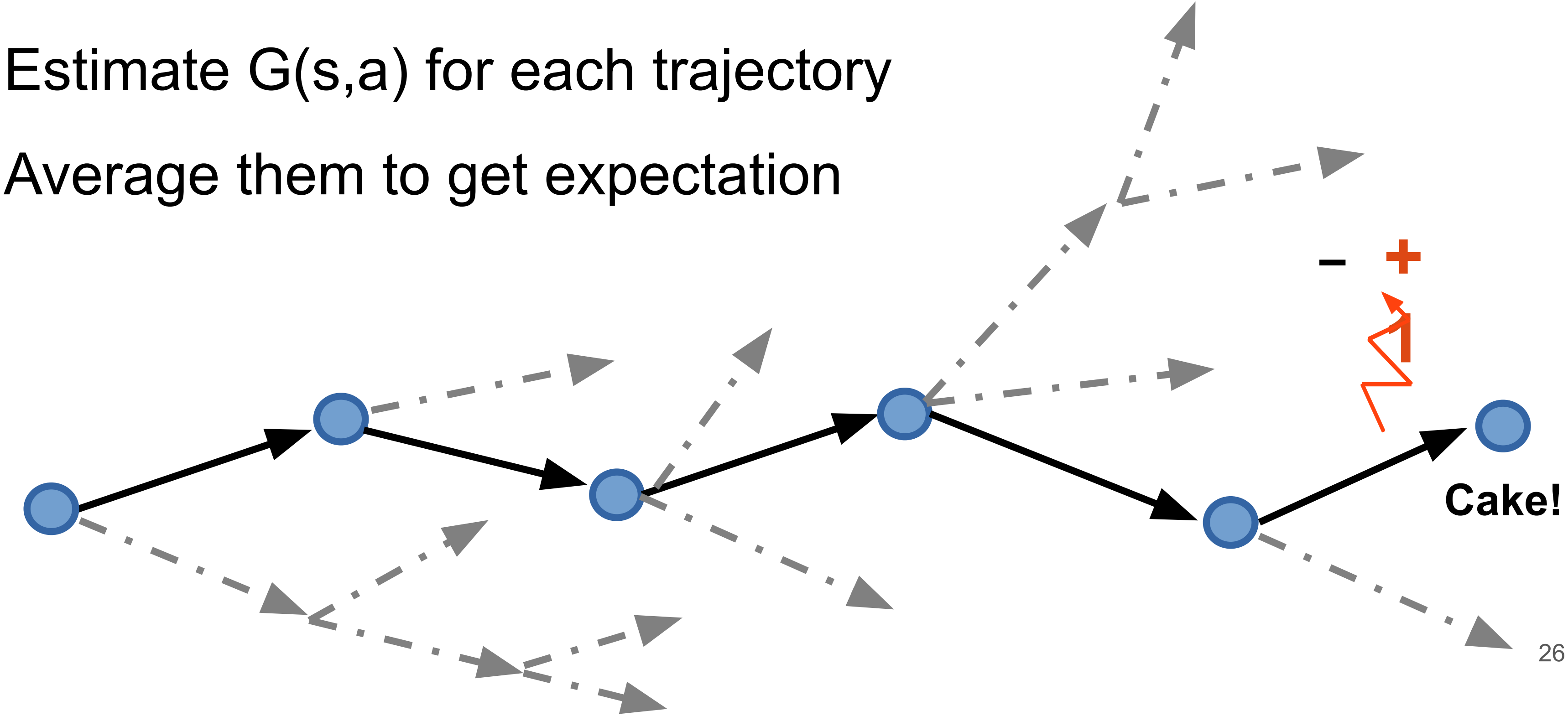
We can only sample trajectories

Q: What to learn?
V(s) or $\boxed{Q(s,a)}$

V(s) is useless
without P(s'|s,a)

Idea 1: Monte-carlo

- Get all trajectories containing particular (s,a)
- Estimate G(s,a) for each trajectory
- Average them to get expectation



Idea 2: Temporal difference

- Q(s, a) can be improved iteratively!

$$Q(s_t, a_t) \leftarrow \underset{r_t, s_{t+1}}{E} r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a')$$

That's Q*(s,a)

That's something we don't have

That's value for π*
aka optimal policy

What do we do?

Idea 2: Temporal difference

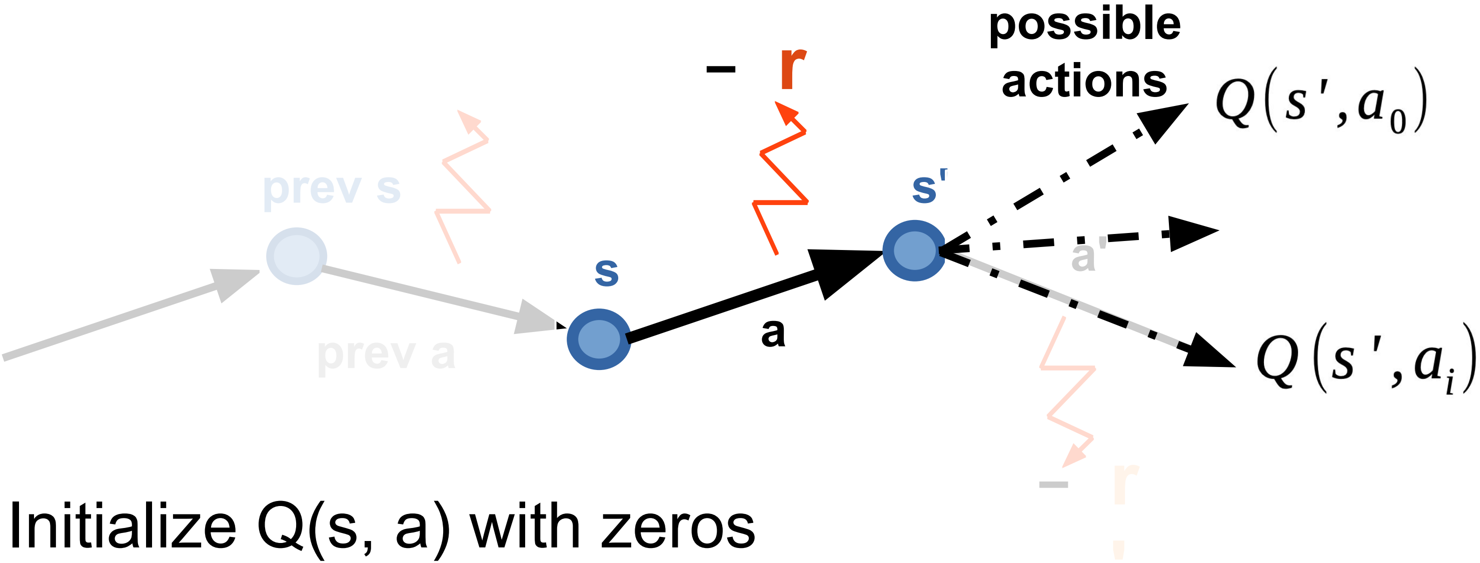
- Q(s, a) can be improved iteratively!

$$Q(s_t, a_t) \leftarrow \underset{r_t, s_{t+1}}{E} r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a')$$

$$\underset{r_t, s_{t+1}}{E} r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a') \approx \frac{1}{N} \sum_i r_i + \gamma \cdot max_{a'} Q(s_i^{next}, a')$$

$$Q(s_t, a_t) \leftarrow \alpha \cdot (r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a')) + (1 - \alpha) Q(s_t, a_t)$$

Q-learning



Initialize Q(s, a) with zeros

- Sample <s, a, r, s’> from the environment
- Compute new Q(s, a) eslimation:

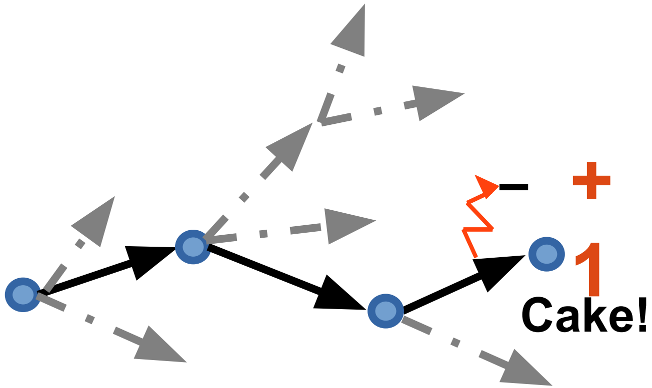
$$\hat{Q}(s,a) = r(s,a) + \gamma \max_{a_i} Q(s', a_i)$$

- Update Q(s, a):

$$Q(s,a) \leftarrow \alpha \cdot \hat{Q}(s,a) + (1-\alpha) Q(s,a)$$

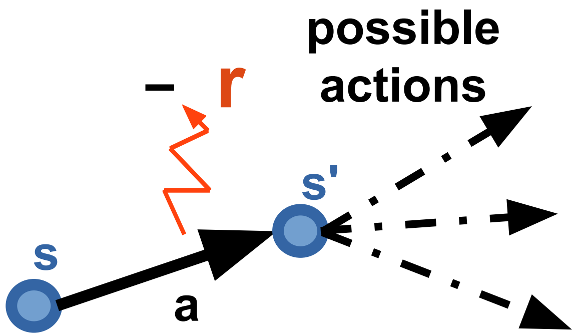
Monte-carlo

Averages Q over sampled paths



Temporal Difference

Uses recurrent formula for Q



Q-learning

$$Q^*(s,a) = \underset{s',r}{E}\, r(s,a) + \gamma \cdot V^*(s')$$

$$Q(s_t, a_t) \leftarrow \alpha \cdot (r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a')) + (1-\alpha) Q(s_t, a_t)$$

$$\pi(s) : argmax_a Q(s,a)$$

Exploration-exploitation tradeoff

Strategies:

- ε-greedy
  - With probability ε take random action; otherwise take optimal action.
- Softmax

  Pick action proportional to softmax of shifted normalized Q-values.

$$\pi(a|s) = softmax\left(\frac{Q(s,a)}{\tau}\right)$$

Last step: making it continuous

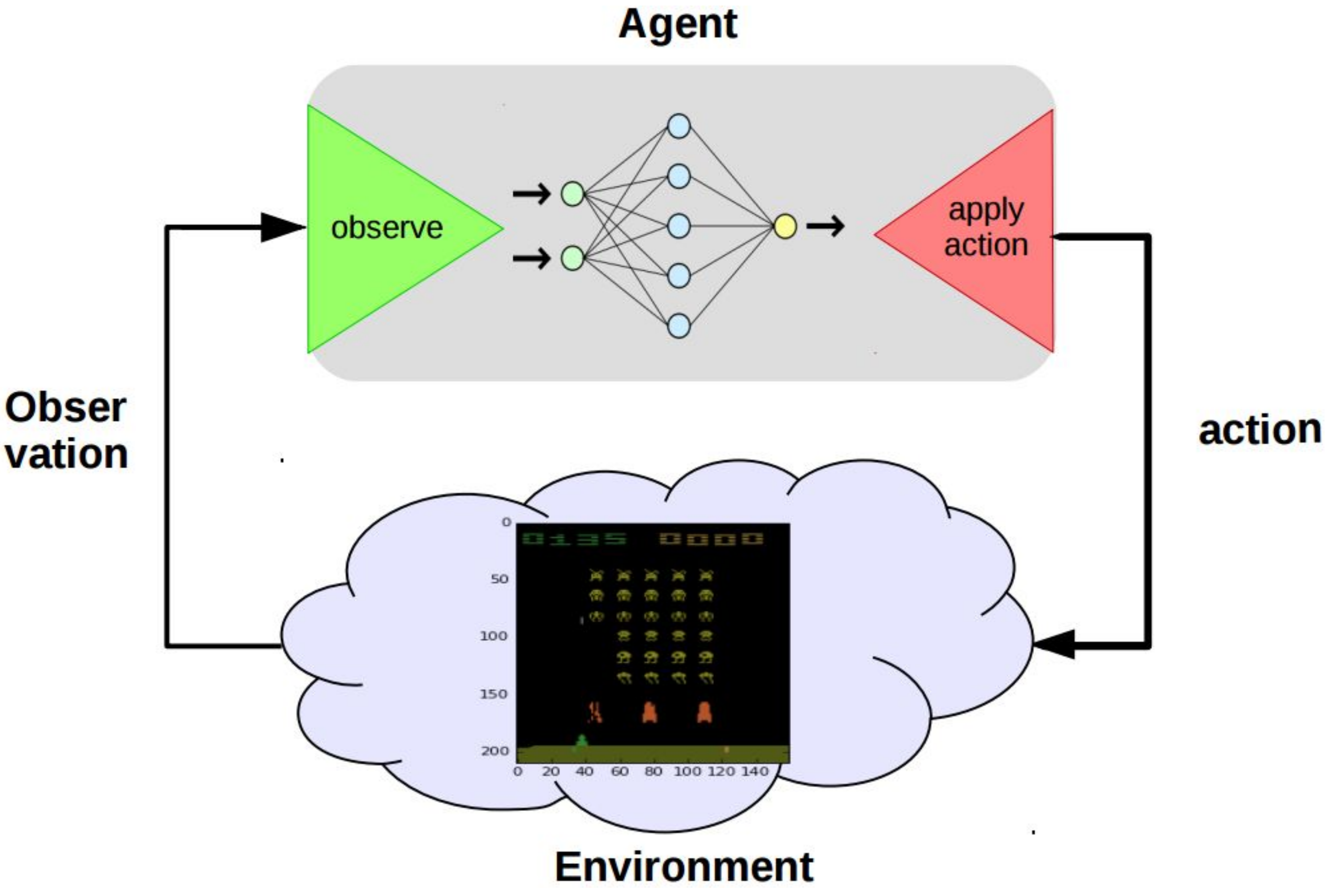
For now states and actions are discrete. What if states are **not**?

Minimize loss given <**s, a, r, s'**>

$$L = [Q(s_t, a_t) - Q^{true}(s_t, a_t)]^2$$

$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a'))]^2$$

MDP


Agent
observe
apply action
Obser vation
action
0
50
100
150
200
0 20 40 60 80 100 120 140
Environment

Basic definitions

$$G_t = \sum_{t'=t}^{T} \gamma^{(t'-t)} r_{t'}$$

$$Q^\pi(s,a) = E_\pi[G_t | s_t = s, a_t = a]$$

$$V^\pi(s) = E_\pi[G_t | s_t = s]$$

Recurrent relations

$$Q^\pi(s,a) = E_{s_{t+1}}[r_t + \gamma V^\pi(s_{t+1})]$$

$$Q^\pi(s,a) = E_{s_{t+1},a_{t+1}\sim\pi}[r_t + \gamma Q^\pi(s_{t+1}, a_{t+1})]$$

Optimal policy

For all π, s, a: $Q^{\pi^*}(s, a) \geq Q^{\pi}(s, a)$

$$\pi^*(s) = argmax_a Q^{\pi^*}(s, a)$$

Bellman optimality equation

$$Q^*(s_t, a) = E_{s_{t+1}}[r_t + \max_{a'} Q^*(s_{t+1}, a')]$$

Q-learning

Training step

$$Q(s_t, a_t) \longleftarrow Q(s_t, a_t) + \alpha(r_t + \gamma \max_{a'} Q(s_{t+1}, a') - Q(s_t, a_t))$$

Q-learning as MSE minimization

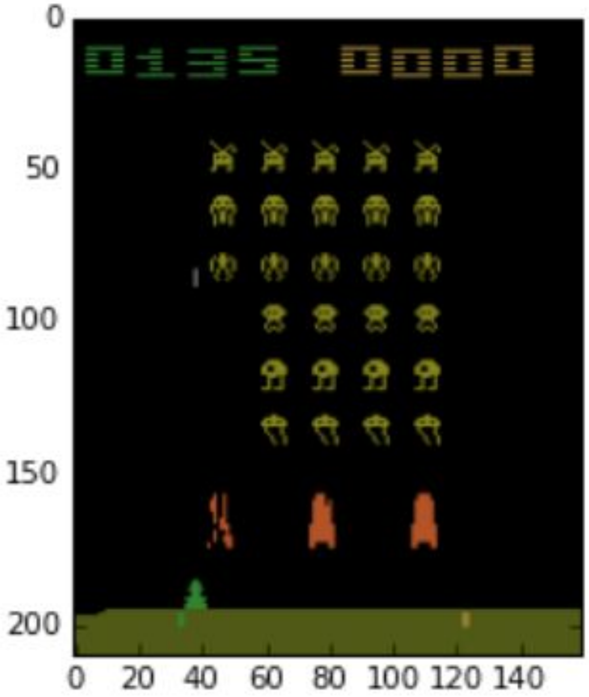
$$L = (r_t + \gamma \max_{a'} \boxed{Q(s_{t+1}, a')} - Q(s_t, a_t))^2$$

Const

$$\nabla L = 2 \cdot (r_t + \gamma \max_{a'} \boxed{Q(s_{t+1}, a')} - Q(s_t, a_t))$$

What's wrong here?

Real world



How many states are there?

approximately

$$|S| = 2^{210 \cdot 160 \cdot 8 \cdot 3}$$

Q-learning: make it continuous

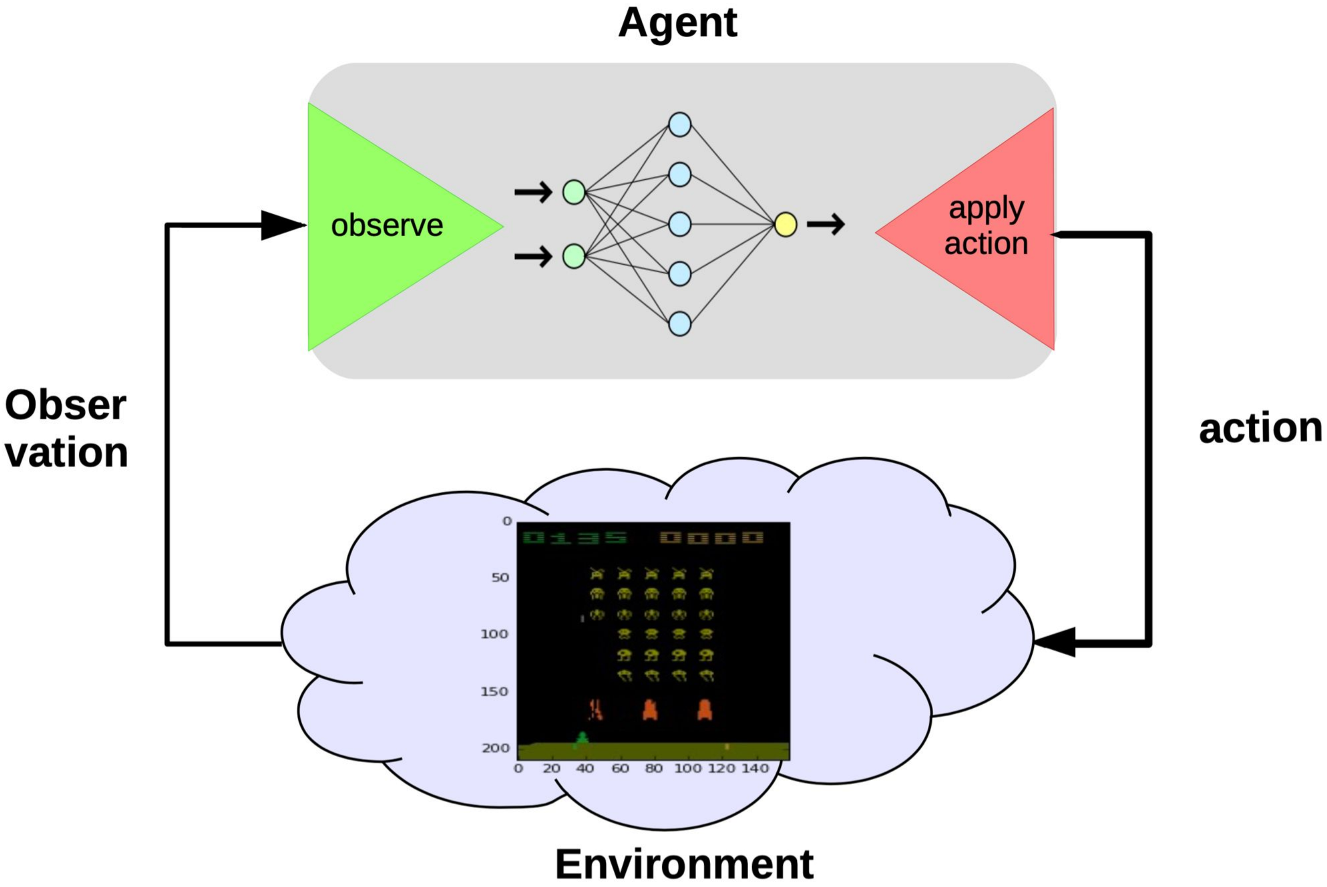
For now states and actions are discrete. What if states are **not**?

Minimize loss given <**s, a, r, s'>**

$$L = [Q(s_t, a_t) - Q^{true}(s_t, a_t)]^2$$

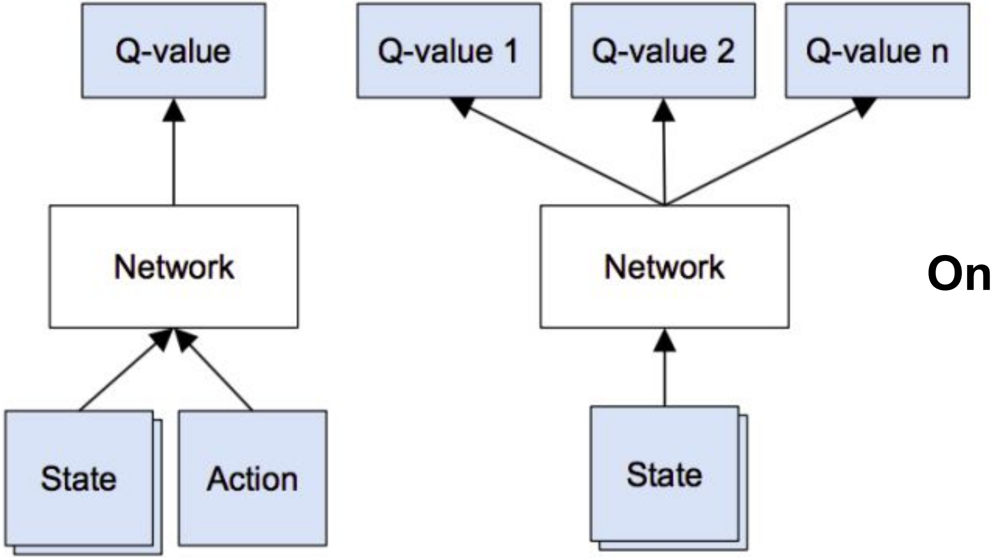
$$L \approx [Q(s_t, a_t) - (r_t + \gamma \cdot max_{a'} Q(s_{t+1}, a'))]^2$$

Approximate Q-learning



Possible architectures

Continuous control or large number of actions

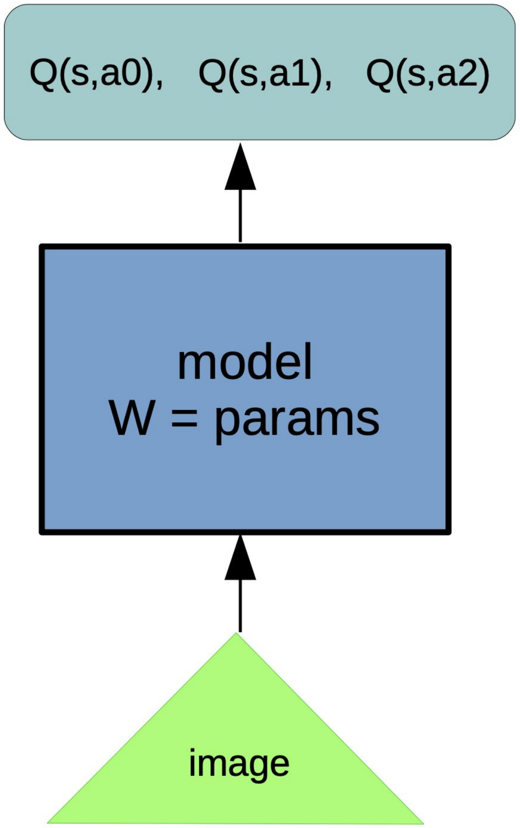


One pass for all actions

Given **(s,a)**
Predict Q(s,a)

Given **s** predict all q-values
Q(s,a0), Q(s,a1), Q(s,a2)

Approximate Q-learning



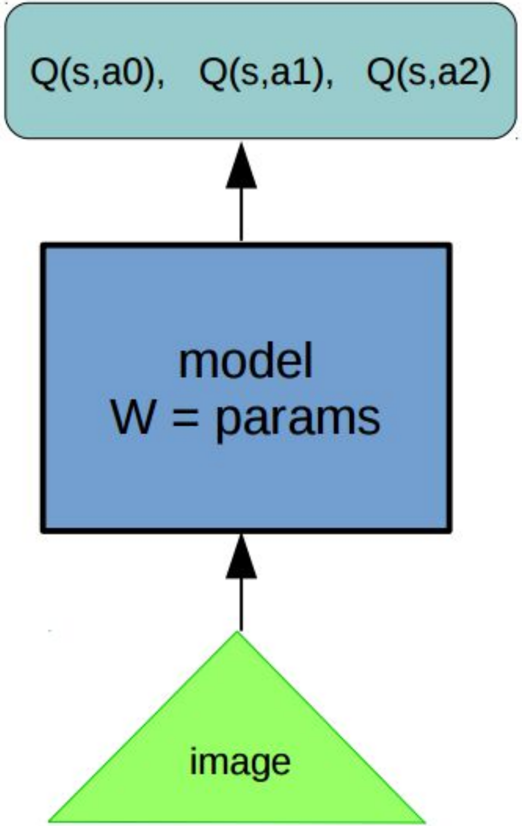
$$\hat{Q}(s_t, a_t) = r + \gamma \cdot max_{a'} \hat{Q}(s_{t+1}, a')$$

$$L = \left(Q(s_t, a_t) - \boxed{[r + \gamma \cdot max_{a'} Q(s_{t+1}, a')]}\right)^2$$

Consider const

$$w_{t+1} = w_t - \alpha \cdot \frac{\delta L}{\delta w}$$

Approximate Q-learning



Objective:

$$L = \left(Q(s_t, a_t) - \underline{\hat{Q}(s_t, a_t)}\right)^2$$

consider const

Q-learning:

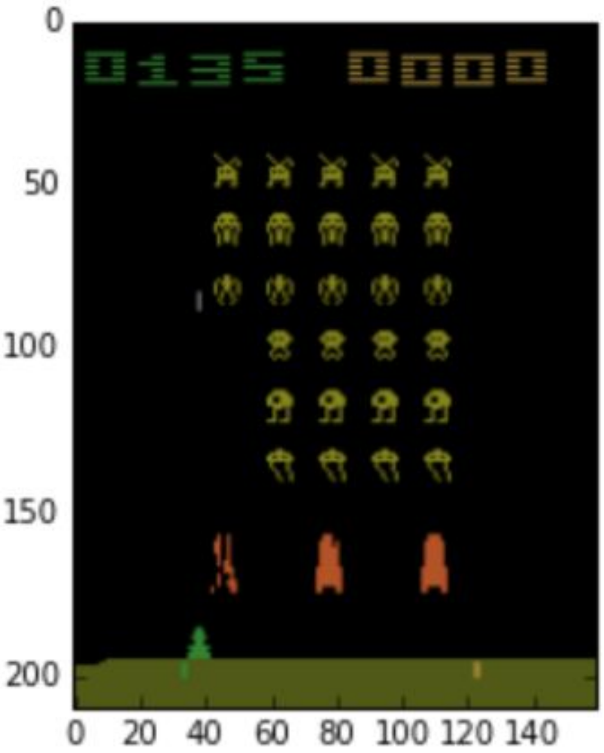
$$\hat{Q}(s_t, a_t) = r + \gamma \cdot max_{a'} Q(s_{t+1}, a')$$

SARSA:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot Q(s_{t+1}, a_{t+1})$$

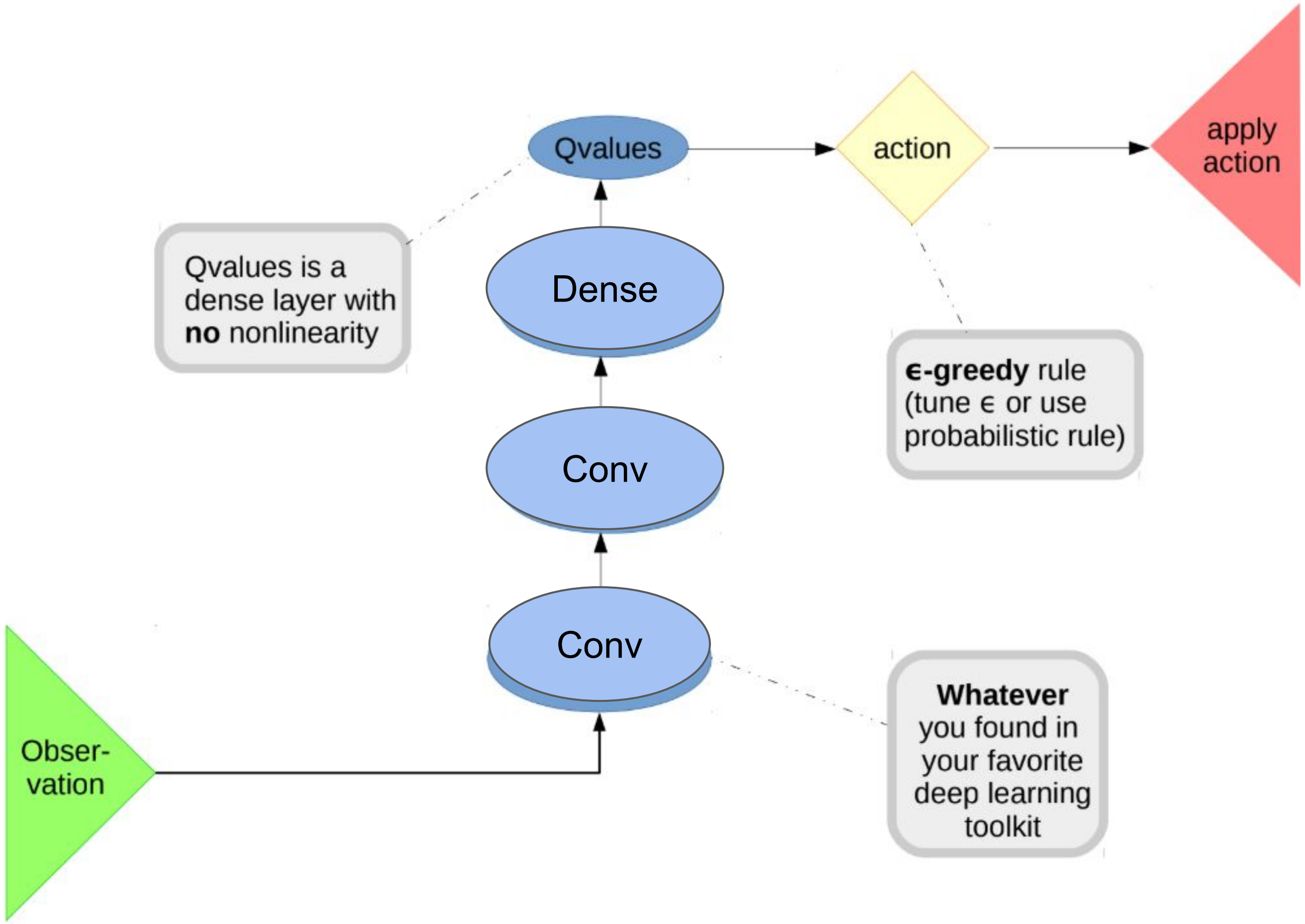
Expected Value SARSA:

$$\hat{Q}(s_t, a_t) = r + \gamma \cdot \underset{a' \sim \pi(a|s)}{E} Q(s_{t+1}, a')$$



What kind of network digests images well?

Basic deep Q-learning




WHAT IF I TOLD YOU
THAT IT'S ALL ONE GREAT BIG LIE.
imgflip.com

Outro and Q&A

- Q-learning allows to learn some approximation of the reward function and environment model
  - So we can use it to solve the desired problem

- Remember what Q(s, a) and V(s) functions do
- Remember both about exploration and exploitation
  - At least using greedy policy or softmax smoothing