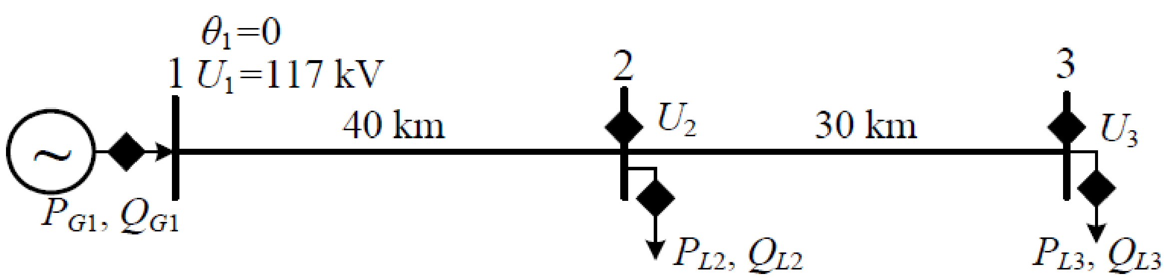
R E P O R T

"*Assessment of the state of the power system using the AC network model*"



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This report contains the solution for the problem given to us in "*Homework No.2*", the ways in which the problem was solved and the results that were obtained. Also, in continuation of this section, the data of the real and imaginary part of the matrix Y and the other necessary measured data will be listed.

1. Real and imaginary parts of the matrix Y:

Data for the real and imaginary part of the matrix Y given in *p.u.* are presented in **Table 1** and **Table 2**.

2.223	-2.223	0
-2.223	5.187	-2.964
0	-2.964	2.964

Table 1. $\boldsymbol{G(p.u.)}$

-6.840	6.840	0
6.840	-15.960	9.120
0	9.120	-9.120

Table 2. $\boldsymbol{B(p.u.)}$

2. Measured data and basic values:

For the system shown in Figure 1, the active and reactive power and voltage measurements can be performed at 8 locations. The values of the measured data (active power in MW, reactive power in MVAr, and voltage in kV) for which measurements are made, are given in Table 3 for each student. For each different values (cases) the data for only one part of the measurements is collected, and the values are arranged in columns for each case separately. If the field is empty, then there is no measured data for that measurement. Using the assessment of the state of the power system with the AC network model it is necessary to calculate the estimated values of the measured values ($Z_m^{est} = f(X^{est})$), as well as the size of the measured residual value:

$$J(X^{est}) = [Z_m - Z_m^{est}]' \cdot R^{-1} \cdot [Z_m - Z_m^{est}]$$

It is necessary to perform calculations for three iterations in the iterative process in order to obtain a significant value of X^{est}.

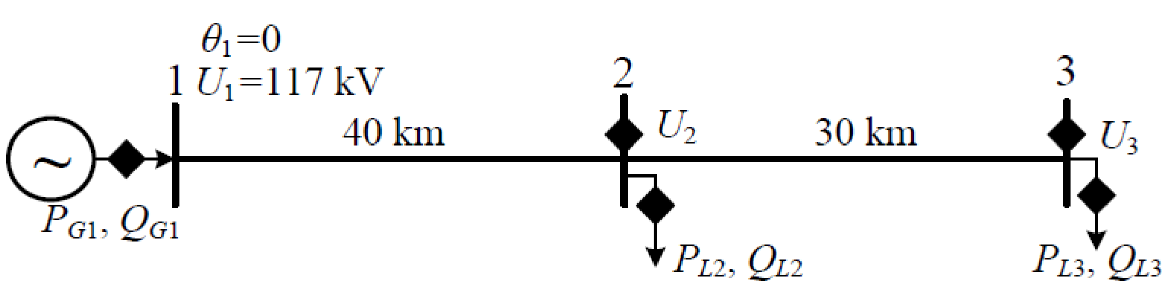

Figure 1. The network in consideration

Type of measurement	12. Filip Nikolov	$p.u.$
P_{G1} (MW)	59.3	0.593
Q_{G1} $(MVAr)$	/	/
P_{P2} (MW)	27.1	0.271
Q_{P2} $(MVAr)$	/	/
U_2 (kV)	107.7	0.979
P_{P3} (MW)	32.2	0.322
Q_{P3} $(MVAr)$	15.0	0.15
U_3 (kV)	/	/

Table 3. Measured data

It is additionally stated that the lines have the following parameters

$$r = 0.13 \left[\frac{\Omega}{km}\right] \quad x = 0.4 \left[\frac{\Omega}{km}\right]$$

while the capacitance of the lines is disregarded. Node 1 is considered the balance node and the voltage maintained is a precisely determined value of $U_1 = 117\ [kV] = 1.0636\ [p.u.]$.

It is known that the nominal voltage of the network is $U_n = U_b = 110\ [kV]$ as that is also the base value of the voltage, while the base power is $S_b = 100\ [MVA]$.

3. <u>Solution to the problem:</u>

We start by defining the values of the measurements for our problem which in this care are defined inside the following vector:

$$Z^m = [P_{G1} \quad P_{P2} \quad U_2 \quad P_{P3} \quad Q_{P3}] = [0.593 \quad 0.271 \quad 0.979 \quad 0.322 \quad 0.15]^T \; [p.u]$$

After that we define the state variables for our problem with the following vector:

$$x = [U_2 \quad U_3 \quad \theta_2 \quad \theta_3]^T$$

In the problem itself we are given the values of the matrix Y therefore we don’t need to calculate it, we will only use the necessary data directly from the tables into the formulas.

In order to be able to determine the measurement functions, we firstly need to write the equations for the injected active and reactive power in each node. They are as follows:

$$P_i = G_{ii} \cdot U_i^2 + U_i \cdot \sum_{j \in \alpha} U_j \cdot [G_{ij} \cdot \cos(\theta_i - \theta_j) + B_{ij} \cdot \sin(\theta_i - \theta_j)]$$

$$Q_i = -B_{ii} \cdot U_i^2 + U_i \cdot \sum_{j \in \alpha} U_j \cdot [G_{ij} \cdot \sin(\theta_i - \theta_j) - B_{ij} \cdot \cos(\theta_i - \theta_j)]$$

whereas α is a set of nodes that are directly connected to the node i.

<u>Remember:</u> In the nodes in which we have generators, there the formulas are used as they are. Whereas, in the nodes where we have load then the whole formula is written in a parenthesis and one minus " $-$ " is written in front of the parenthesis.

Now we can write the measurement functions, and they are as follows:

$$f_1(x) = f_1(U_2, U_3, \theta_2, \theta_3) = P_{G1} = G_{11} \cdot U_1^2 + U_1 \cdot U_2 \cdot (G_{12} \cdot cos\theta_{12} + B_{12} \cdot sin\theta_{12})$$

$$f_2(x) = f_2(U_2, U_3, \theta_2, \theta_3) = P_{P2} =$$

$$= -[G_{22} \cdot U_2^2 + U_2 \cdot U_1 \cdot (G_{21} \cdot cos\theta_{21} + B_{21} \cdot sin\theta_{21}) + U_2 \cdot U_3 \cdot (G_{23} \cdot cos\theta_{23} + B_{23} \cdot sin\theta_{23})]$$

$$f_3(x) = f_3(U_2, U_3, \theta_2, \theta_3) = U_2 = U_2$$

$$f_4(x) = f_4(U_2, U_3, \theta_2, \theta_3) = P_{P3} = -[G_{33} \cdot U_3^2 + U_3 \cdot U_2 \cdot (G_{32} \cdot cos\theta_{32} + B_{32} \cdot sin\theta_{32})]$$

$$f_5(x) = f_5(U_2, U_3, \theta_2, \theta_3) = Q_{P3} = -[-B_{33} \cdot U_3^2 + U_3 \cdot U_2 \cdot (G_{32} \cdot sin\theta_{32} - B_{32} \cdot cos\theta_{32})]$$

Now that we have written the measurement functions in their basic form, we will substitute in the functions the values we know for the corresponding G и B, the phase angle $\theta_1 = 0$ and voltage $U_1 = 1.0636\ [p.u.]$. That way we get the following matrix:

$$f(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \\ f_4(x) \\ f_5(x) \end{bmatrix} =$$

$$= \begin{bmatrix} 2.5147 - 2.3643 \cdot U_2 \cdot cos\theta_2 - 7.275 \cdot U_2 \cdot sin\theta_2 \\ -5.187 \cdot U_2^2 + 2.3643 \cdot U_2 \cdot cos\theta_2 - 7.275 \cdot U_2 \cdot sin\theta_2 + 2.964 \cdot U_2 \cdot U_3 \cdot \cos(\theta_2 - \theta_3) - 9.12 \cdot U_2 \cdot U_3 \cdot \sin(\theta_2 - \theta_3) \\ U_2 \\ -2.964 \cdot U_3^2 + 2.964 \cdot U_2 \cdot U_3 \cdot \cos(\theta_3 - \theta_2) - 9.12 \cdot U_2 \cdot U_3 \cdot \sin(\theta_3 - \theta_2) \\ -9.12 \cdot U_3^2 + 2.964 \cdot U_2 \cdot U_3 \cdot \sin(\theta_3 - \theta_2) + 9.12 \cdot U_2 \cdot U_3 \cdot \cos(\theta_3 - \theta_2) \end{bmatrix}$$

Now that we have the values of the measurement functions, we need the matrix H which contains the derivatives of the functions $f_i(x)\ i = 1 \dots N_m$, to the state variables $x_i\ i = 1 \dots N_s$ and is called Jacobian. The number of the rows in the Jacobian is equal to the number of functions N_m, whereas the number of the columns is equal to the number of the state variables N_s.

Due to the size of the results for each derivative it will not be possible to write the whole result in one matrix, therefore we will write only the base formula matrix and then the result for each element of the matrix individually.

$$H = H(x) = \begin{bmatrix} \frac{\partial f_1(x)}{\partial U_2} & \frac{\partial f_1(x)}{\partial U_3} & \frac{\partial f_1(x)}{\partial \theta_2} & \frac{\partial f_1(x)}{\partial \theta_3} \\ \frac{\partial f_2(x)}{\partial U_2} & \frac{\partial f_2(x)}{\partial U_3} & \frac{\partial f_2(x)}{\partial \theta_2} & \frac{\partial f_2(x)}{\partial \theta_3} \\ \frac{\partial f_3(x)}{\partial U_2} & \frac{\partial f_3(x)}{\partial U_3} & \frac{\partial f_3(x)}{\partial \theta_2} & \frac{\partial f_3(x)}{\partial \theta_3} \\ \frac{\partial f_4(x)}{\partial U_2} & \frac{\partial f_4(x)}{\partial U_3} & \frac{\partial f_4(x)}{\partial \theta_2} & \frac{\partial f_4(x)}{\partial \theta_3} \\ \frac{\partial f_5(x)}{\partial U_2} & \frac{\partial f_5(x)}{\partial U_3} & \frac{\partial f_5(x)}{\partial \theta_2} & \frac{\partial f_5(x)}{\partial \theta_3} \end{bmatrix}$$

Function f_1:

$$\frac{\partial f_1(x)}{\partial U_2} = -2.3643 \cdot cos\theta_2 - 7.275 \cdot sin\theta_2$$

$$\frac{\partial f_1(x)}{\partial U_3} = 0$$

$$\frac{\partial f_1(x)}{\partial \theta_2} = 2.3643 \cdot U_2 \cdot sin\theta_2 - 7.275 \cdot U_2 \cdot cos\theta_2$$

$$\frac{\partial f_1(x)}{\partial \theta_3} = 0$$

Function f_2:

$$\frac{\partial f_2(x)}{\partial U_2} = -10.374 \cdot U_2 + 2.3643 \cdot cos\theta_2 - 7.275 \cdot sin\theta_2 + 2.964 \cdot U_3 \cdot \cos(\theta_2 - \theta_3) - 9.12 \cdot U_3 \cdot \sin(\theta_2 - \theta_3)$$

$$\frac{\partial f_2(x)}{\partial U_3} = 2.964 \cdot U_2 \cdot \cos(\theta_2 - \theta_3) - 9.12 \cdot U_2 \cdot \sin(\theta_2 - \theta_3)$$

$$\frac{\partial f_2(x)}{\partial \theta_2} = -2.3643 \cdot U_2 \cdot sin\theta_2 - 7.275 \cdot U_2 \cdot cos\theta_2 - 2.964 \cdot U_2 \cdot U_3 \cdot \cos(\theta_2 - \theta_3) - 9.12 \cdot U_2 \cdot U_3 \cdot \cos(\theta_2 - \theta_3)$$

$$\frac{\partial f_2(x)}{\partial \theta_3} = 2.964 \cdot U_2 \cdot U_3 \cdot \sin(\theta_2 - \theta_3) + 9.12 \cdot U_2 \cdot U_3 \cdot \cos(\theta_2 - \theta_3)$$

Function f_3:

$$\frac{\partial f_3(x)}{\partial U_2} = 1$$

$$\frac{\partial f_3(x)}{\partial U_3} = 0$$

$$\frac{\partial f_3(x)}{\partial \theta_2} = 0$$

$$\frac{\partial f_3(x)}{\partial \theta_3} = 0$$

Function f_4:

$$\frac{\partial f_4(x)}{\partial U_2} = 2.964 \cdot U_3 \cdot \cos(\theta_3 - \theta_2) - 9.12 \cdot U_3 \cdot \sin(\theta_3 - \theta_2)$$

$$\frac{\partial f_4(x)}{\partial U_3} = -5.928 \cdot U_3 + 2.964 \cdot U_2 \cdot \cos(\theta_3 - \theta_2) - 9.12 \cdot U_2 \cdot \sin(\theta_3 - \theta_2)$$

$$\frac{\partial f_4(x)}{\partial \theta_2} = 2.964 \cdot U_2 \cdot U_3 \cdot \sin(\theta_3 - \theta_2) + 9.12 \cdot U_2 \cdot U_3 \cdot \cos(\theta_3 - \theta_2)$$

$$\frac{\partial f_4(x)}{\partial \theta_3} = -2.964 \cdot U_2 \cdot U_3 \cdot \sin(\theta_3 - \theta_2) - 9.12 \cdot U_2 \cdot U_3 \cdot \cos(\theta_3 - \theta_2)$$

Function f_5:

$$\frac{\partial f_5(x)}{\partial U_2} = 2.964 \cdot U_3 \cdot \sin(\theta_3 - \theta_2) + 9.12 \cdot U_3 \cdot \cos(\theta_3 - \theta_2)$$

$$\frac{\partial f_5(x)}{\partial U_3} = -18.24 \cdot U_3 + 2.964 \cdot U_2 \cdot \sin(\theta_3 - \theta_2) + 9.12 \cdot U_2 \cdot \sin(\theta_3 - \theta_2)$$

$$\frac{\partial f_5(x)}{\partial \theta_2} = -2.964 \cdot U_2 \cdot U_3 \cdot \cos(\theta_3 - \theta_2) + 9.12 \cdot U_2 \cdot U_3 \cdot \sin(\theta_3 - \theta_2)$$

$$\frac{\partial f_5(x)}{\partial \theta_3} = 2.964 \cdot U_2 \cdot U_3 \cdot \cos(\theta_3 - \theta_2) - 9.12 \cdot U_2 \cdot U_3 \cdot \sin(\theta_3 - \theta_2)$$

Once we have determined all of the functions and matrices that will enter the iterative equation, we will proceed with the calculations of the iterative equation. At this point in order to ease our life and the time it takes, we move on to a simpler method for calculating with the help of $MATLAB$.

The code used to solve this task is displayed on the next page, while the order in which the code is executed is as follows:

① First we determine an initial solution.

② Then we determine the value of the function $f(x)$ for the adopted values.

③ After that we determine the Jacobian in that point (for the known values of x).

④ Finally the resulting matrices are all inserted in the iterative equation.

⑤ The calculation is repeated until a solution is obtained with the desired accuracy of 3 decimal digits.

Due to the size of the data contained in the matrices f and H in the code, they are omitted from displaying. The entire code will be sent as an additional file. In the meantime, the matrices written above can be used to note what data is stored in which matrix.

```
%Filip Nikolov 120/2014 EES%

① x = [1 1 0 0]'; %%% U2--U3--Theta2--Theta3%%%

     for i=1:25
        disp('iteration'); i

        %Measurement functions%
② f = [ Omitted due to size. ];

③ H = [ Omitted due to size. ];

        %Measured values%
        Zm = [0.593   0.271    0.979    0.322    0.15]';

④ dX = inv(H'*H)*H'*(Zm-f);
⑤ x = x + dX;

        Zmest = f;
        J = (Zm-Zmest)'*(Zm-Zmest);
      end
```

In this problem, a lot of iterations were needed, precisely 25, in order to achieve the required result i.e. to satisfy the condition of accuracy to the 3rd decimal digit. However, due to the given condition in the problem which states that the calculations should be performed for three iterations, both solutions are presented in order to present the difference in the results obtained:

$$x^{(3)} = \begin{bmatrix} 0.9607 \\ 0.9280 \\ -0.0479 \\ -0.0730 \end{bmatrix} [p.u.] \qquad x^{(25)} = \begin{bmatrix} 0.9918 \\ 0.9654 \\ -0.0597 \\ -0.0874 \end{bmatrix} [p.u.]$$

$$J^{(3)} = 0.0238\ [p.u.] \qquad J^{(25)} = 0.00037924\ [p.u.]$$