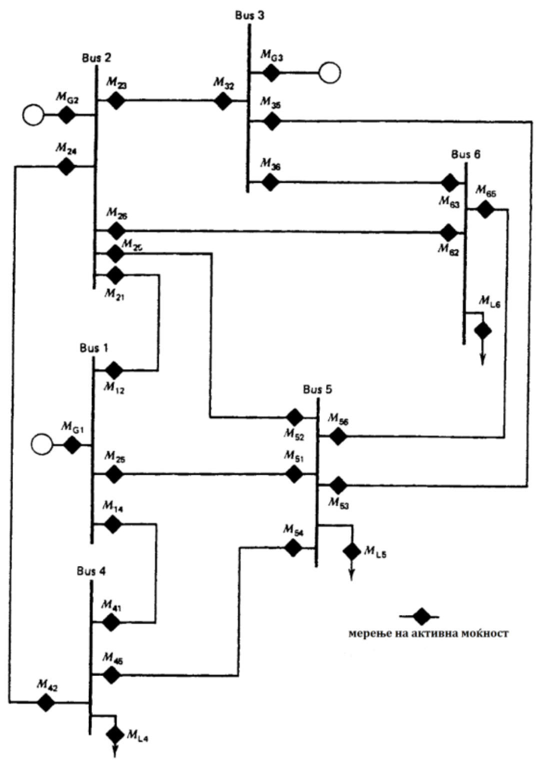
R E P O R T

"*Assessment of the state of the power system using the DC network model*"



Filip Nikolov
120/2014

15.12.2021

In this report are explained the requirements that were set for us in "Homework No.1", the ways in which they are resolved and what results are obtained. Also, in the continuation of this text is the data on the reactance of the branches and the other necessary measured data that is needed.

1. Reactance of the branches

Data on the reactance of the branches expressed in *p.u.* given in **Table 1**.

No.	Start node	End node	*X* (p.u.)
1	1	2	0.2
2	1	4	0.2
3	1	5	0.3
4	2	3	0.25
5	2	4	0.1
6	2	5	0.3
7	2	6	0.2
8	3	5	0.26
9	3	6	0.1
10	4	5	0.4
11	5	6	0.3

Table 1. Data for the reactance of the branches

2. Measured data

For the system given in Figure 1, active power measurements can be made at 28 locations. The values of the measurements (active power in MW) are given in Table 2 for each student. For each case, the data is collected for only a part of the possible 28 measurements, and their values are arranged in columns for each case separately. If there is an empty field that means that there is no measured data for that line/generator/consumer.

With the assessment of the state of the power system using the DC network model, it is necessary to calculate the estimated values of the measurements ($Z_m^{est} = f(X^{est}) = H \cdot X^{est}$), as well as the measured residual value $J(X^{est}) = [Z_m - Z_m^{est}]' \cdot R^{-1} \cdot [Z_m - Z_m^{est}]$. The balance node is the node marked with No. 6 on Figure 1. ($\theta_6 = 0$).

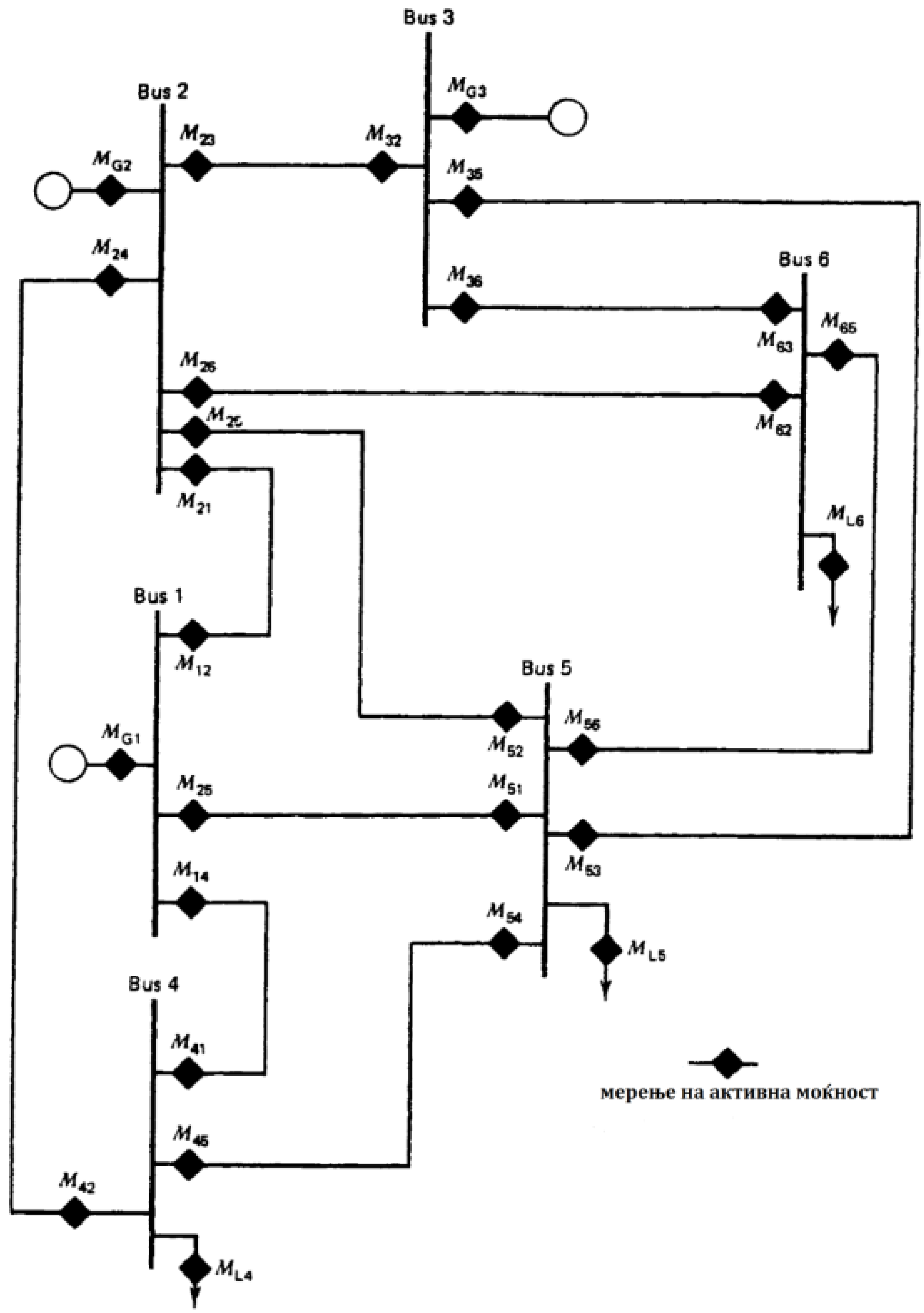


Figure 1. The system in question

Measurement	$\sigma\ (p.u.)$	Z_m (12 – Filip Nikolov)
M12	0.02	
M14	0.01	37.0
M15	0.01	
M23	0.02	
M24	0.02	39.0
M25	0.03	
M26	0.03	
M35	0.01	
M36	0.02	48.0
M45	0.02	
M56	0.03	
M21	0.01	
M32	0.02	-0.5
M41	0.01	
M42	0.02	-40.0
M51	0.02	
M52	0.01	-17.0
M53	0.03	
M54	0.01	-3.5
M62	0.01	-26.0
M63	0.03	
M65	0.02	
MG1	0.01	
MG2	0.02	
MG3	0.01	
ML4	0.02	72.0
ML5	0.01	
ML6	0.03	

Table 2 2. Measured data for the power system

3. Solution to the problem:

Firstly we start by reducing the values of Z_m to $p.u.$ shown in Table 2 and additionaly remove the extra rows for which our case has no measurements.

Measurement	$X\ (p.u.)$	$Z_m\ (p.u.)$
M14	0.2	0.37
M24	0.1	0.39
M36	0.1	0.48
M32	0.25	-0.005
M42	0.1	-0.40
M52	0.3	-0.17
M54	0.4	-0.035
M62	0.2	-0.26
ML4	/	0.72

Table 3. Measured data in p.u.

Now we will write all the necessary formulas that are needed to calculate the measuring functions so that we can determine the matrix H. The basic formula is a follows:

$$M_{ij} = \frac{\theta_i - \theta_j}{X_{i-j}}$$

$$M_{14} = \frac{\theta_1-\theta_4}{X_{1-4}} = \frac{\theta_1-\theta_4}{0.2} = 5\theta_1 - 5\theta_4$$

$$M_{24} = \frac{\theta_2-\theta_4}{X_{2-4}} = \frac{\theta_2-\theta_4}{0.1} = 10\theta_2 - 10\theta_4$$

$$M_{36} = \frac{\theta_3-\theta_6}{X_{3-6}} = \frac{\theta_3-0}{0.1} = 10\theta_3$$

$$M_{32} = \frac{\theta_3-\theta_2}{X_{3-2}} = \frac{\theta_3-\theta_2}{0.25} = 4\theta_3 - 4\theta_2$$

$$M_{42} = \frac{\theta_4-\theta_2}{X_{4-2}} = \frac{\theta_4-\theta_2}{0.1} = 10\theta_4 - 10\theta_2$$

$$M_{52} = \frac{\theta_5-\theta_2}{X_{5-2}} = \frac{\theta_5-\theta_2}{0.3} = 3.333\theta_5 - 3.333\theta_2$$

$$M_{54} = \frac{\theta_5-\theta_4}{X_{5-4}} = \frac{\theta_5-\theta_4}{0.4} = 2.5\theta_5 - 2.5\theta_4$$

$$M_{62} = \frac{\theta_6-\theta_2}{X_{6-2}} = \frac{0-\theta_2}{0.2} = -5\theta_2$$

Since in this case we do not have a measuring function for a generator, we only need to calculate one measuring function for a load, namely ML4.

$$M_{L4} = M_{14} + M_{54} + M_{24} = \frac{\theta_1 - \theta_4}{X_{1-4}} + \frac{\theta_5 - \theta_4}{X_{5-4}} + \frac{\theta_2 - \theta_4}{X_{2-4}} =$$

$$= \frac{\theta_1 - \theta_4}{0.2} + \frac{\theta_5 - \theta_4}{0.4} + \frac{\theta_2 - \theta_4}{0.1} =$$

$$= 5\theta_1 - 5\theta_4 + 2.5\theta_5 - 2.5\theta_4 + 10\theta_2 - 10\theta_4 =$$

$$= 5\theta_1 + 10\theta_2 - 17.5\theta_4 - 2.5\theta_5$$

According to the obtained data for the measuring functions we can present how the matrix H will look like in the following Table 4. After that we can present the matrix H in its natural form.

$\boldsymbol{H}$	θ_1	θ_2	θ_3	θ_4	θ_5
M14	5	0	0	-5	0
M24	0	10	0	-10	0
M36	0	0	10	0	0
M32	0	-4	4	0	0
M42	0	-10	0	10	0
M52	0	-3.333	0	0	3.333
M54	0	0	0	-2.5	2.5
M62	0	-5	0	0	0
ML4	5	10	0	-17.5	-2.5

Table 4. Values of the *H* matrix

$$H = \begin{bmatrix} 5 & 0 & 0 & -5 & 0 \\ 0 & 10 & 0 & -10 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & -4 & 4 & 0 & 0 \\ 0 & -10 & 0 & 10 & 0 \\ 0 & -3.333 & 0 & 0 & 3.333 \\ 0 & 0 & 0 & -2.5 & 2.5 \\ 0 & -5 & 0 & 0 & 0 \\ 5 & 10 & 0 & -17.5 & -2.5 \end{bmatrix}$$

In this next step we are going to calculate and present the covariance matrix and its inverse matrix. This matrix is calculated according to the given standard deviations from the beginning of the problem for the case we are examining. All of the calculations are made with MATLAB and the results are shown directly in the following two matrices.

$$R = \begin{bmatrix} 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0004 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0004 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0004 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0004 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0004 \end{bmatrix}$$

$$R^{-1} = \begin{bmatrix} 10000 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2500 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2500 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2500 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2500 \end{bmatrix}$$

We proceed by calculating the estimated values of the state variables that are obtained by substituting the above calculated matrices in the following equation:

$$X^{est} = (H^T \cdot R^{-1} \cdot H)^{-1} \cdot H^T \cdot R^{-1} \cdot Z^m$$

Where by appropriately replacing the matrices in the equation and using MATLAB to calculate it, we get the following result:

$$X^{est} = \begin{bmatrix} 0.0866 \\ 0.0519 \\ 0.0484 \\ 0.0129 \\ 0.00037304 \end{bmatrix}$$

In this last step we will calculate the matrix $Z_{m,est}$ and also present it in a table where we will compare it with the given values of Z_m in Table 3.

$$Z_{m,est} = H \cdot X^{est}$$

$$Z_{m.est} = \begin{bmatrix} 0.3687 \\ 0.3899 \\ 0.4837 \\ -0.0141 \\ -0.3899 \\ -0.1717 \\ -0.0314 \\ -0.2595 \\ 0.7253 \end{bmatrix}$$

$Z_{m,est}$ $(p.u.)$	Z_m $(p.u.)$
0.3687	0.37
0.3899	0.39
0.4837	0.48
-0.0141	-0.005
-0.3899	-0.40
-0.1717	-0.17
-0.0314	-0.035
-0.2595	-0.26
0.7253	0.72

Table 5. Comparison of estimated and given values

From the obtained results it can be noticed that in this problem the static assessment of the situation gives approximately equal results and we are satisfied with them. Lastly we will calculate the sum of squares of the deviations of the measured values from the estimated ones J:

$$J = (Z_m - Z_{m,est})^T \cdot R^{-1} \cdot (Z_m - Z_{m,est})$$

The calculated values is $J = 0.7533\ p.u.$ which tells us that we will have approximately equal values between the estimated ones and the measured ones, with some small deviations.

4. MATLAB code:

The next window shows the code used to calculate all of the matrices in MATLAB.

```
%ENTERING MATRIX "H"
H = [
    5       0       0      -5          0
    0      10       0     -10          0
    0       0      10       0          0
    0      -4       4       0          0
    0     -10       0      10          0
    0  -3.333       0       0      3.333
    0       0       0    -2.5        2.5
    0      -5       0       0          0
    5      10       0   -17.5       -2.5
    ];
 %VECTOR ROW OF STANDARD DEVIATIONS, CONVERSION IN DIAGONAL MATRIX
 D=[0.01 0.02 0.02 0.02 0.02 0.01 0.01 0.01 0.02];
 X=diag(D);

 %CALCULATING THE INVERSE MATRIX "R"
 R=(X.^2);
 Rinverz=inv(R);

 %VECTOR ROW OF MEASURED VALUES, TRANSPOSITION IN A VECTOR COLUMN
 Z=[0.37 0.39 0.48 -0.005 -0.40 -0.17 -0.035 -0.26 0.72];
 Zm=transpose(Z);

 %CALCULATING THE ESTIMATED PHASE ANGLES
 Xest=(inv(transpose(H)*Rinverz*H))*transpose(H)*Rinverz*Zm;

 %CALCULATING THE ESTIMATED VALUES
 Zmest= H*Xest;

 %SUM OF SQUARES OF DEVIATIONS OF THE MEASURED ONES FROM ESTIMATED ONES
 J=(transpose(Zm-Zmest))*(Rinverz)*(Zm-Zmest);
```

5. Conclusion:

It can be concluded that the procedure for assessing the situation using a DC network model, despite all of the disregards we make in the process, it will still give us satisfactory results that are approximately within the same range as the measured values.