MakerDAO Auctions Price Curve

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October 2022

Abstract

In extreme market conditions, a sudden price drop of any asset could potentially trigger liquidations; hence, it is paramount to engineer a price curve that yields fair auction dynamics for both protocol and keepers. For this reason, we analyzed the historical behavior of ETH prices during days of high volatility, concentrating on the price action right after a liquidation event happened. We discuss the underlying assumptions in the choice of *buf*, *cut*, and *step* parameters, and test the resiliency of the Auction Price Curve by simulating the price action of ETH using agnostic Geometric Brownian Motion simulations. Our simulations aim to understand how well is the Stair-Step Exponential function capturing the aftermath of a market crash. Based on these simulations we conclude that this exponential decaying process captures to very well the aftereffects of a high volatility event, and in fact tends to be more aggressive to what we have observed historically.

Contents

1 Introduction

In the context of the MakerDAO protocol [1], a liquidation is the automatic transfer of collateral from an insufficiently collateralized Vault, along with the transfer of that Vault's debt to the protocol [2]. The underlying mechanism makes use of Dutch auctions to sell off liquidated collateral and pay off the associated loan back to the protocol.

Whenever liquidations are triggered and collateral is being auctioned, the system relies on two features: 1) the Oracle Security Module (OSM) to set an initial bid price for auctions, and 2) a set of constant parameters that prevent the price from diverging to a great extent, minimizing the probability for auctions to settle too far from the collateral's market price.

For several reasons [3], an exponential price decrease function is preferred to determine the Price Curve during Auctions. The slope of the Price Curve is determined by two parameters: *cut*, and *step*; this combination also determines the time needed before the next price drop occurs and the magnitude of the drop. Another interesting parameter defining the curve function is *buf*, which marks the starting auction price. In this way there is a buffer applied to the last OSM price in case the market price rebounds between the time OSM took the last market price and until the auction is kicked.

Using Block Analitica's Maker Risk Dashboard [4] one can easily visualize the ETH-A auctions throughput as shown in Fig. (1), where $buf = 1.20$, $cut = 99$ and $step = 90$. In this example, the reason why auctions have a high *buf* parameter and a relatively slowly decreasing slope is to make sure keepers have enough reaction time before an arbitrage opportunity emerges.

In this article we focus on ETH-A [5], the oldest and most utilized Vault on MakerDAO, and analyze the historical price action of ETH during different market conditions, paying close attention to the days where liqui-

Figure 1: ETH-A vault auctions throughput. The Auction Price Curve is determined by a stair step exponential function determined by parameters: *buf*, *cut*, and *step*.

Date	Max Price Drop
2020.03.12	48.11%
2021.05.19	44.57%
2021.05.23	27.03%
2021.09.21	14.27%
2021.12.14	5.00%
2022.01.21	18.29%
2022.01.22	11.88%

Table 1: Dates analyzed. Minute-by-minute data was analyzed in periods of high volatility and considerable price drops. Maximum price drop is the percentage difference between daily maximum and daily minimum values.

dations where triggered. The aim is to understand the behavior of ETH pre- and post- liquidation, and to test the resiliency of the Auction Price Curve during these events.

This article is organized as follows. On section 2 we analyze the minute-by-minute data of 7 days where ETH price dropped considerably, and where several vaults were liquidated, not only on Maker but also in several other lending protocols.

We continue on section 3 where we take a deep dive on to the functional form of the Auctions Price Curve, and define a methodology to test its strength after liquidations have been triggered. To this end we use Geometric Brownian Motion simulations where the drift and volatility of the Stair-Step Exponential function are used to simulate the price action of ETH. Finally on section 4 we provide an overview of our work and provide a recommendation to MakerDAO.

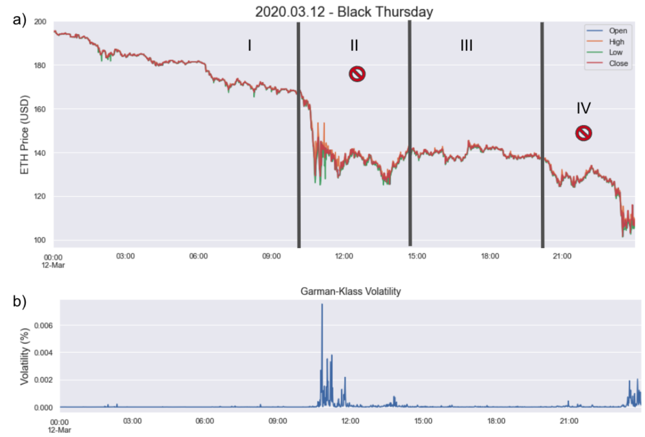


Figure 2: On black Thursday there are 4 different areas where price decayed with several different rhythms, triggering liquidations with several bid prices along the way. a) Shows the price action of ETH during this day and b) shows the Garman-Klass volatility of this asset.

2 Historical Analysis

On this section we analyze the minute-by-minute data of 7 days where ETH price dropped considerably. These dates is particularly important because where several vaults were liquidated, not only on Maker but also in several other lending protocols. The dates analyzed are shown in Tab. (1).

As seen in Tab. (1), the Maximum Price Drop during these days is considerable. Moreover, the events of the 12th of March of 2020, the so-called “Black Thursday”, forced sizable liquidations in Maker due to falling ETH prices, that is why we will first concentrate on this day and then extend our observations and hypotheses.

2.1 Black Thursday - 2020.03.12

On Thursday March 12th 2020, markets experienced a sudden collapse with oil, real state and hospitality prices getting halved in less than a day. This event was likely triggered by global fears about the spread of the coronavirus, oil price drops, and the possibility of a recession [6].

Interestingly, the price of Ethereum dropped twice by roughly 25% over a 24-hr period causing a Black Swan Event, where cryptomarkets in general exhibited extreme asymmetric volatility due to panic selling. As a result forced sizable liquidations were triggered throughout the whole ecosystem. Simultaneously, the Ethereum network transaction costs were significantly elevated creating a situation where 1461 auctions liquidated 62842.93 ETH for no collateral at a cost of 6.65M DAI to the Maker protocol.

As seen in the Fig. (2.a), Black Thursday can be divided into 4 areas where the price decayed with several different fingerprint. In area I there's a drop of 18% but nothing special happened in terms of volatility, it was just a continuous price drop. In area II the price fell significantly and we observe the clear fingerprint of a Black Swan Event, i.e., a sudden and unexpected price drop. In area III we observe what may seem like a rebound of the market with med-low volatility, followed by area IV, which is a speedy drop in prices with medium-high volatility.

In technical terms, these different areas are characterized by different volatility fingerprints. These fingerprints can be studied by means of the Garman-Klass' volatility [7] over the period of observation shown in Fig. (2.b). This figure provides a lot of insights about the volatility of this day, but most importantly it highlights the vast differences in ETH price action around 10 am and 1 pm (UTC). By looking at the GK volatility, we can distinguish multiple peaks of volatility, that can be seen as different speeds of price decay, and is nothing but the spread of the Open, High, Low and Close prices.

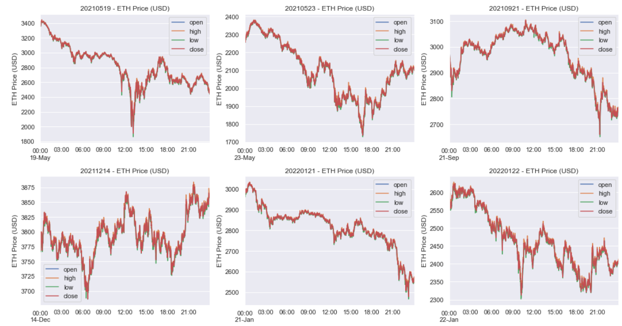
From our observation of Black Thursday we may conclude that, in a risk-off environment, a considerable sell-off will trigger a series of cascading events that result in the price violently changing two or more times in a short period of time. However, it is important to note that Black Thursday was an unexpected event and it happened in tandem with a global sell-off across markets.

2.2 Other liquidation events

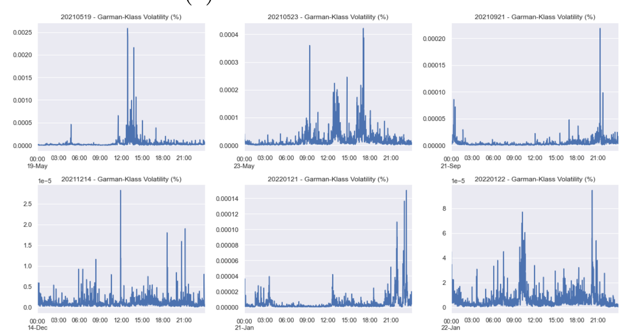
Since Black Thursday is one of the many fat-tail events observed in crypto, we should keep in mind that these events unfold in different manners, and are influenced by the existing market conditions. Therefore it is important to analyze historical price data and understand the behavior of the price during these events. Thus, we now center our attention on the following dates: 2021.05.19, 2021.05.23, 2021.09.21, 2021.12.14, 2022.01.21, and 2022.01.22.

By looking at the price action of ETH on the aforementioned dates, Fig. (3a), even though we see the price dropping significantly on different occasions, we don't observe any fingerprint of speedy price drop and high volatility in the same magnitude as in the events of Black Thursday.

To mark the stark differences between Black Thursday and 19th of May 2021, I will quickly focus on the time period between 1230hrs and 1330hrs on the 19th of May 2021. As we can observe in Fig. (4), after the first volatility event happened around 1300h, there is a second peak of volatility around 1345h, however it had



(a) Price action of ETH.



(b) Garman-Klass Volatility.

Figure 3: Price action and Garman-Klass volatility of ETH during other high volatility events: 2021.05.19, 2021.05.23, 2021.09.21, 2021.12.14, 2022.01.21, and 2022.01.22.

a lesser impact in price, since only the first event translated into a price drop of 20%.

It should be noted that the second peak of volatility highlights the fact that for a while the bid/offer spread was really wide, and thus, several keepers could make a profit. For Maker, the fact that these events unfolded in less than 1h hints that much shorter auction cycles could be established since it is clear that during market crashes arbitrage opportunities could arise and keepers might use them to profit even more on auction settlement.

Finally, our observations on price and volatility are summarized in Tab. (2). In summary, there are two common denominators that characterize these events: (i) abrupt minute and hourly price changes and (ii) high volatility, both in terms of standard deviation and GK volatility. In addition, in extreme events, prices can drop faster than expected, which in turn, can lead to under-collateralization events in the Maker Protocol. It is worth noting that two or more spikes in GK volatility for ETH prices do not necessarily result in a sudden price drop. It can, however, indicate more severe market conditions, and the emergence of arbitrage opportunities.

Since every high volatility event is different, the best metric to compare our model with the aftermath of a

	Absolute Price Change		Volatility	
Date	**1 minute**	**1 hour**	**std dev**	**GK Volatility**
2020.03.12	12.37%	23.10%	38.10%	0.751%
2021.05.19	6.75%	30.15%	36.95%	0.264%
2021.05.23	2.92%	12.16%	36.42%	0.049%
2021.09.21	2.25%	7.25%	36.36%	0.022%
2021.12.14	0.74%	2.28%	36.19%	0.007%
2022.01.21	1.54%	5.89%	36.30%	0.029%
2022.01.22	1.61%	6.12%	36.28%	0.011%

Table 2: Summary of the observations on Price Change and Volatility.

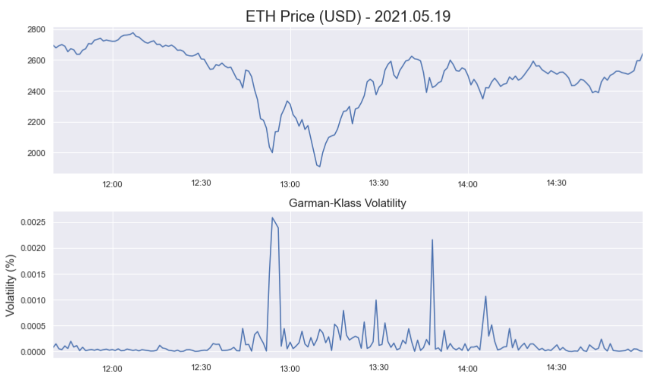


Figure 4: Price action ETH and Garman-Klass volatility between 1230hrs and 1330hrs for the 19th of May 2021.

liquidation event is through the coefficient of variation, which is the ratio between the standard deviation and the mean of the sample. The results of calculating this coefficient for this events is shown in Tab. (3).

Date	Coef. of variation
20200312	12.78
20210519	5.41
20210523	7.25
20210921	3.32
20211214	0.90
20220121	3.65
20220122	3.18

Table 3: Coefficient of variation for the observed dates.

3 Auctions Price Curve

With these observations we have set up the table to talk about the Auction Price Curve Functional Form. We want to know if the current auction price curve is able to capture sudden ETH price drops within the auction duration. In order to simulate the aftermath of the liquidation event we created a model without making any assumptions on the underlying random nature of the process by means of Geometric Brownian Motion.

3.1 Methodology

Suppose that suddenly, ETH price drops sharply and several Vaults need to be liquidated. Following this, auctions are kicked at a given OSM price, and, by means of the Stair-Step Exponential Decrease, we assume that the price decreases monotonically. The shape of this continuous decrement is set thanks to three parameters: *buf*, *cut*, and *step*.

To determine if the Stair-Step Exponential function captures real market price movements, we need to create a new time series influenced by the shape of it. To do so, we reverse engineered ETH's market price using a Geometric Brownian Motion (GBM), which is a stochastic process S_t satisfying the equation,

$$dS_t = \mu S_t dt + \sigma S_t dW_t \,, \tag{1}$$

where μ and σ are constants and are known as the percentage drift and volatility, respectively, and W_t is a standard Wiener process.

For our simulations the percentage drift μ and the percentage volatility σ are determined by the distribution of the Stair-Step Exponential function. In addition, to account for fat-tail risks, we use Laplace's probability distribution function to generate the random steps of the simulation. The results of these simulations can be seen in Fig. (5).

In detail, we created 1k GBM paths and for each simulation we generate 8400 seconds, i.e., 700 blocks (as seen in Fig. (5)). For simplicity, the initial price is set at OSM Price (100%) + *buf*. Subsequently for each random path, we aggregate prices into minute by minute Open, High, Low and Close data, resulting 1000 possible paths for ETH price. An example of a single simulation can be seen in Fig. (6).

To sum up, for every path simulated, we modeled a distinct time series characterized by their own GK volatility. The stochastic process behind each of these simulations is determined by the mean and standard de-

	Absolute Price Change		Volatility	
Simulation	**1 minute**	**1 hour**	**std dev**	**GK Volatility**
sim_5	6.22%	24.96%	10.12%	0.13%
sim_17	7.78%	21.54%	9.17%	0.12%
sim_38	7.49%	49.67%	23.37%	0.14%
sim_65	6.81%	39.18%	11.07%	0.25%
sim_92	5.94%	20.22%	11.91%	0.19%

Table 4: Selected Geometric Brownian Motion simulations.

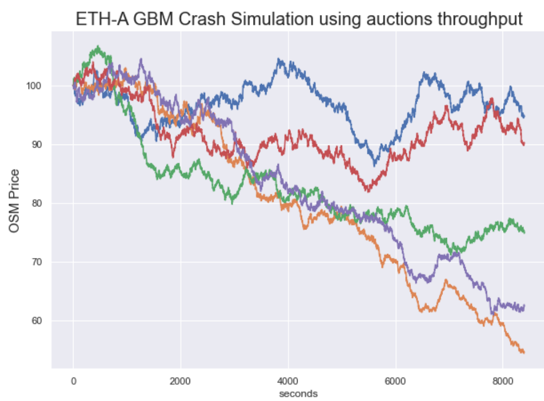


Figure 5: ETH-A Geometric Brownian Motion simulations where drift and volatility of each process are determined by the distribution of the Stair-Step Exponential function and a random step driven by Laplace's probability distribution.

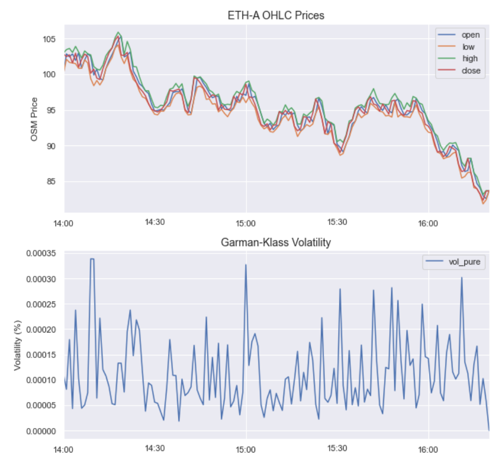


Figure 6: Result of re-sampling Geometric Brownian Motion simulation into OHLC price data. For each of these simulations Garman-Klass Volatility is calculated.

viation of the Auction Price Curve which is a Stair-Step Exponential function.

It is important to mention that in our simulations we do not make any assumptions of the particular path of each random process, and we do not apply large volatility multipliers to model interim (while the auction is running) market crashes. In this way we distance ourselves from modeler bias and truly simulate market prices as a stochastic process following the drift and volatility of the Stair-Step Exponential function.

3.2 Results

Since every path has a different volatility fingerprint, we normalize GK Volatility output to make them comparable. In Tab. (4) we show and compare some of the output from our simulations.

In the interest of having a reproducible set of results, we have open-sourced a python notebook in https://github.com/blockanalitica/research/tree/main/AuctionsPriceCurve that everyone can fork and download [1].

The metrics extracted from our simulations show qualitative agreement to the figures exhibited during the studied events in section 2. This agreement suggests that our methodology can help reproducing the aggressive price decay shown in fat-tail events.

To make comparisons clearer in Tab. (5) we show the coefficient of variations of the selected samples.

Simulation	Coef. of variation
sim_5	4.83
sim_17	4.05
sim_38	13.52
sim_65	5.26
sim_92	5.66

Table 5: Coefficient of variation of selected Geometric Brownian Motion simulations.

Since the coefficient of variation is a dimensionless unit, it can be used to compare data sets that have different sample averages. We want to highlight that our simulations suggest that the overall behavior of the Stair-Step Exponential function captures pretty well the

[1]Because of the random nature of GBM simulations actual numerical results might change.

aftermath of a market crash, and in some instances it is actually a bit more aggressive to what we have seen historically.

4 Conclusions

In this article we used qualitative and quantitative metrics to understand the events that triggered liquidations of ETH-A Vaults in the last couple of years. We observed that in most cases a major sell-off triggered a series of cascading events that resulted in price violently changing in a short period of time, giving rise to arbitrage opportunities which keepers might use to profit on auction settlement.

To test how well the Auction Price Curve behaves during these events, we reverse engineered the price action of ETH using Geometric Brownian Motion. To take into account fat-tailed events in the simulation, we used Laplace's probability distribution function to simulate a series of stochastic processes where the drift and volatility is determined by ETH-A Stair-Step Exponential function.

Our simulations aim to understand how well is the Stair-Step Exponential function capturing the aftermath of a market crash. Based on these simulations we conclude that this exponential decaying process captures to very well the aftereffects of a high volatility event, and in fact tends to be more aggressive to what we have observed historically.

There are multiple paths to follow up on this work. We strongly suggest to further analyze different auction systems across DeFi to have a better understanding of the key drivers of auction appetite throughout the ecosystem.

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