Hash-Based Multi-Signatures for Post-Quantum Ethereum

Benedikt Wagner, Ethereum Foundation

Joint work with
Justin Drake, Dmitry Khovratovich, Mikhail Kudinov

Context: Beam Chain

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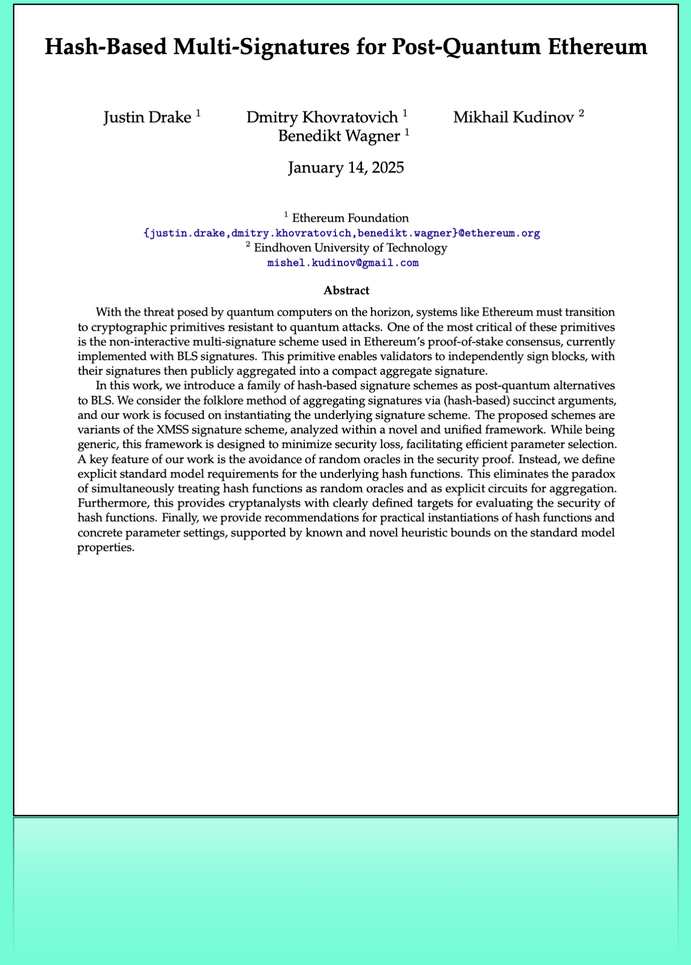
Paper



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Justin Drake [1] Dmitry Khovratovich [1] Mikhail Kudinov [2]
Benedikt Wagner [1]

January 14, 2025

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[2] Eindhoven University of Technology
mishel.kudinov@gmail.com

Abstract

With the threat posed by quantum computers on the horizon, systems like Ethereum must transition to cryptographic primitives resistant to quantum attacks. One of the most critical of these primitives is the non-interactive multi-signature scheme used in Ethereum's proof-of-stake consensus, currently implemented with BLS signatures. This primitive enables validators to independently sign blocks, with their signatures then publicly aggregated into a compact aggregate signature.

In this work, we introduce a family of hash-based signature schemes as post-quantum alternatives to BLS. We consider the folklore method of aggregating signatures via (hash-based) succinct arguments, and our work is focused on instantiating the underlying signature scheme. The proposed schemes are variants of the XMSS signature scheme, analyzed within a novel and unified framework. While being generic, this framework is designed to minimize security loss, facilitating efficient parameter selection. A key feature of our work is the avoidance of random oracles in the security proof. Instead, we define explicit standard model requirements for the underlying hash functions. This eliminates the paradox of simultaneously treating hash functions as random oracles and as explicit circuits for aggregation. Furthermore, this provides cryptanalysts with clearly defined targets for evaluating the security of hash functions. Finally, we provide recommendations for practical instantiations of hash functions and concrete parameter settings, supported by known and novel heuristic bounds on the standard model properties.

Code

Outline

The Problem

Overall Paradigm

Signature Design

Next Steps

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Multi-Signatures in Ethereum Consensus

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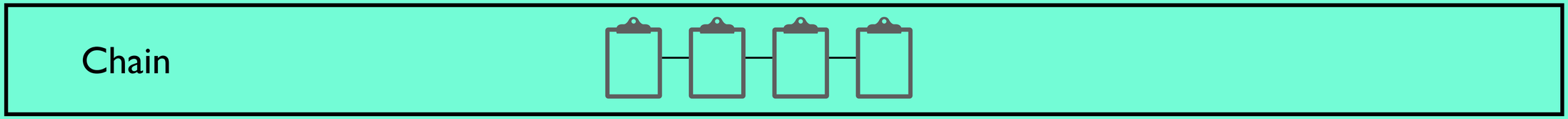
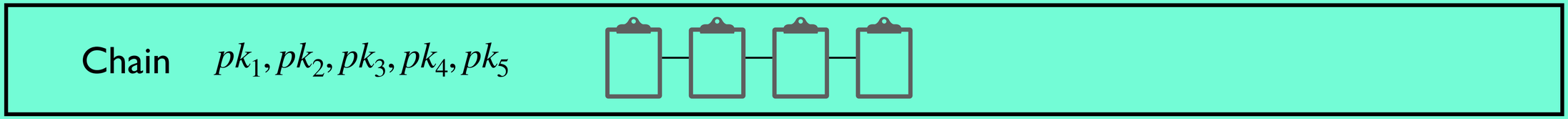


Validators

Multi-Signatures in Ethereum Consensus



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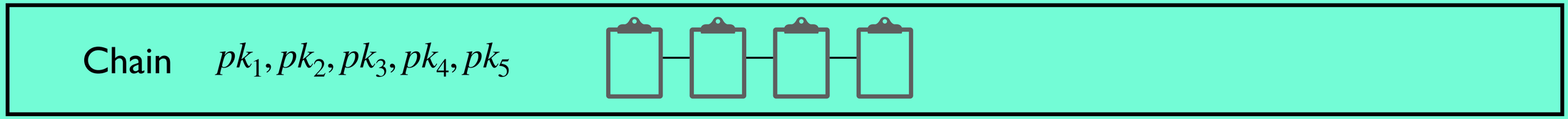

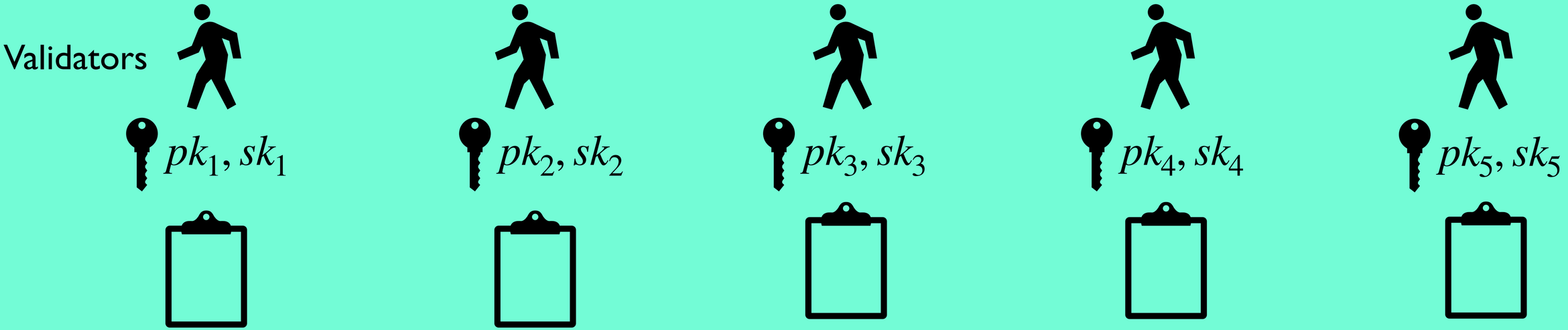

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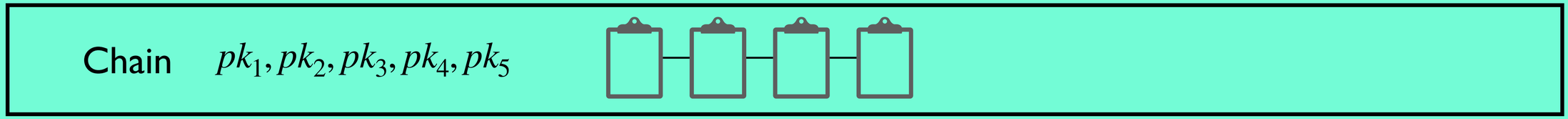

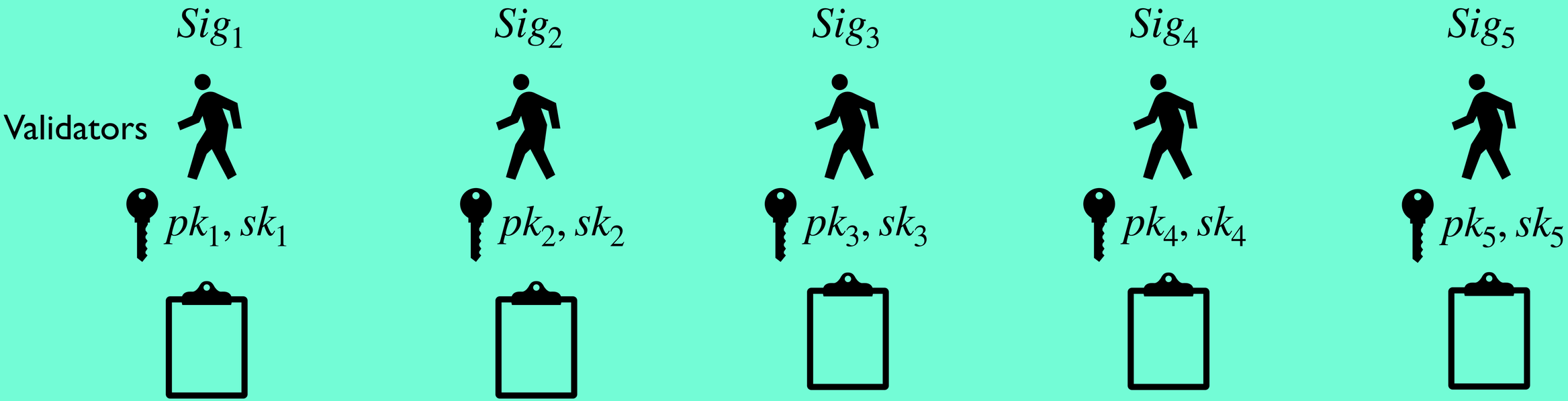

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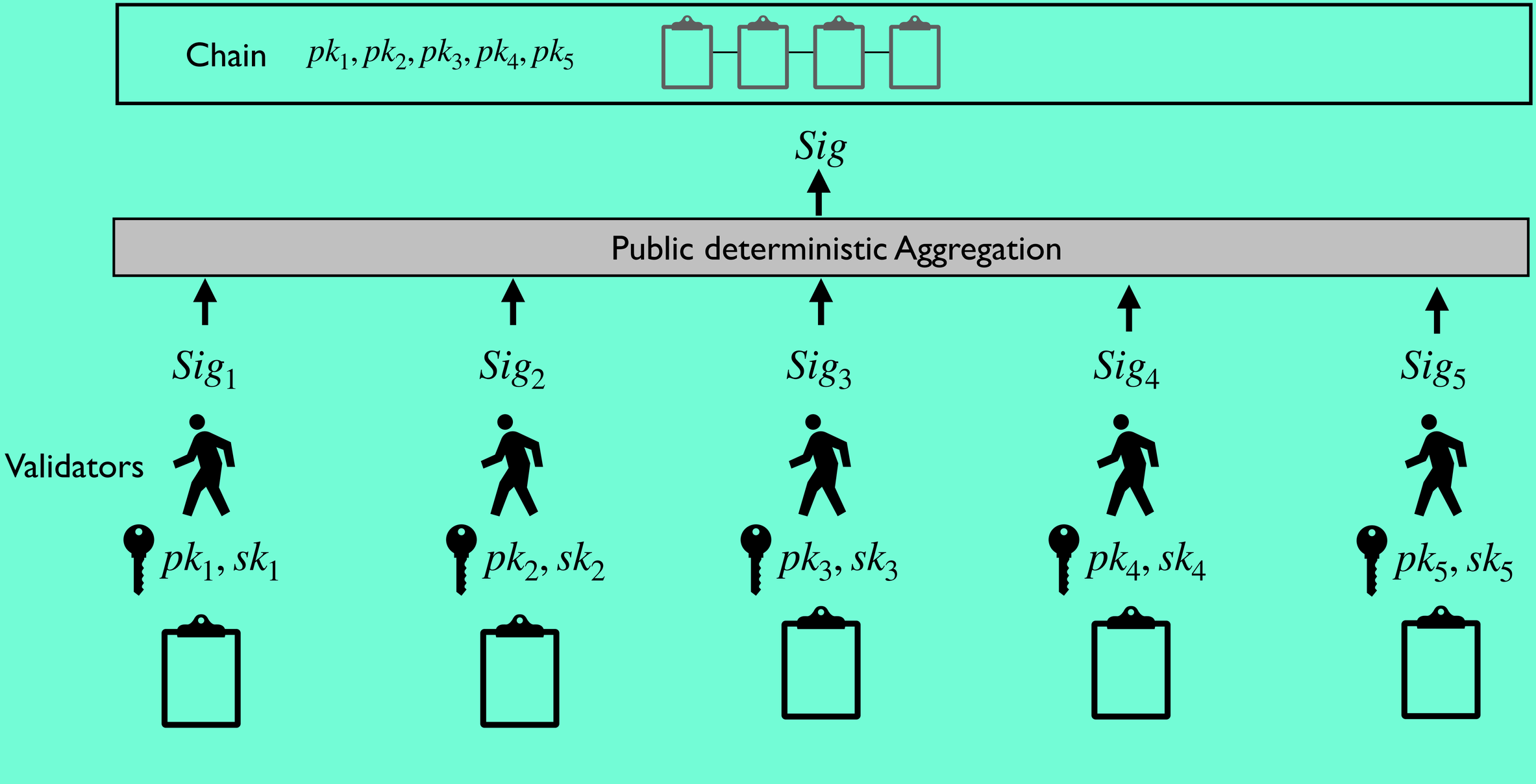

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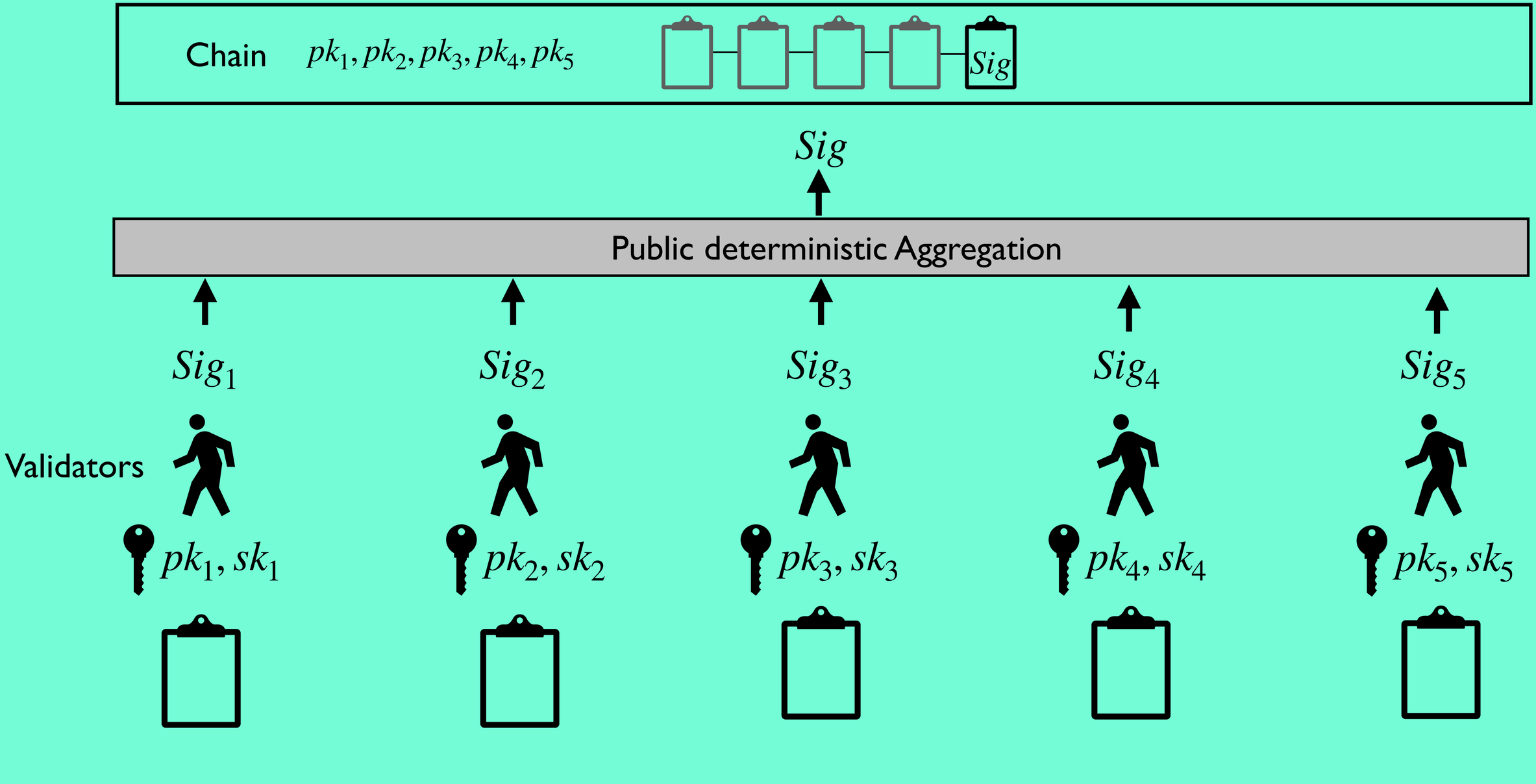

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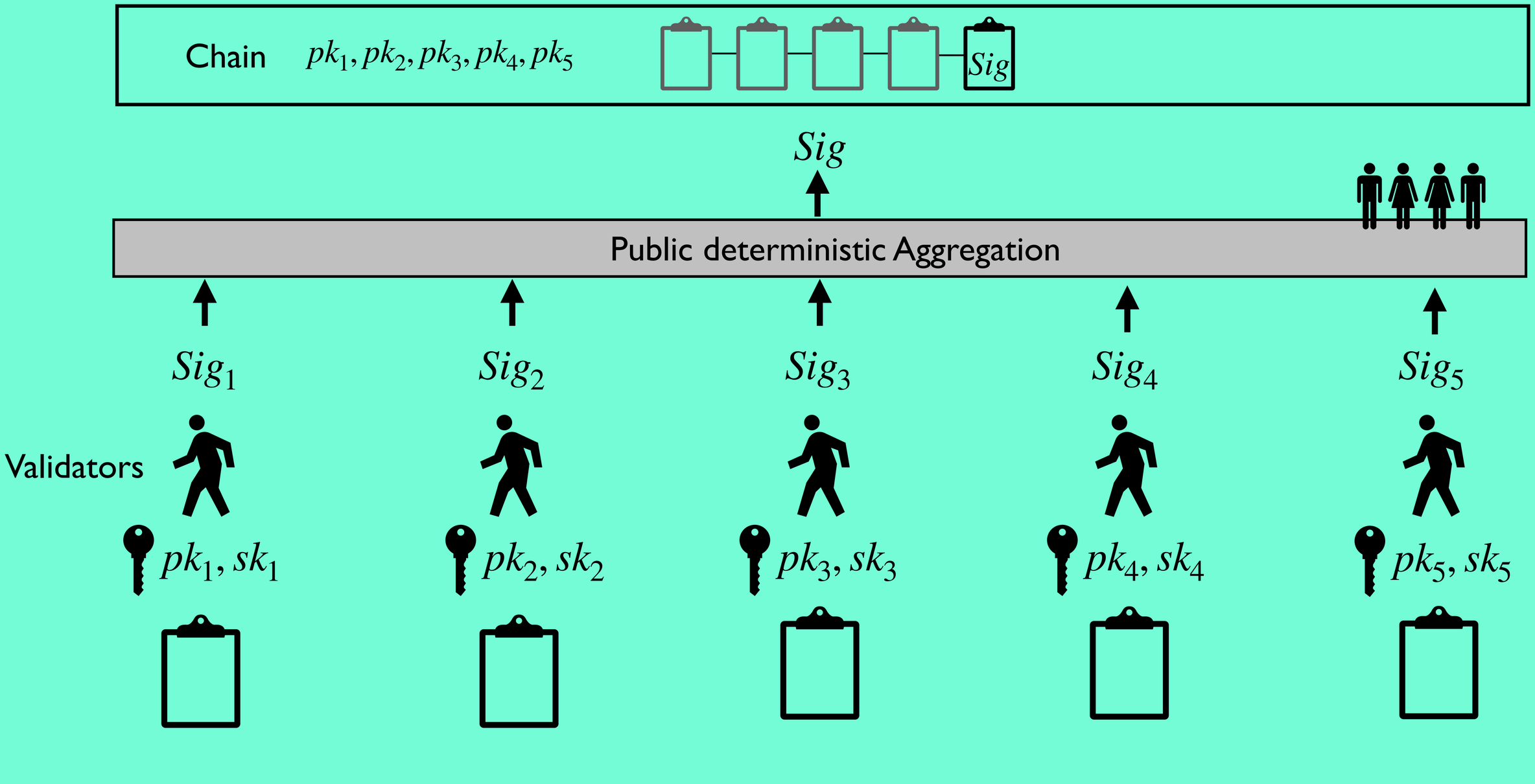

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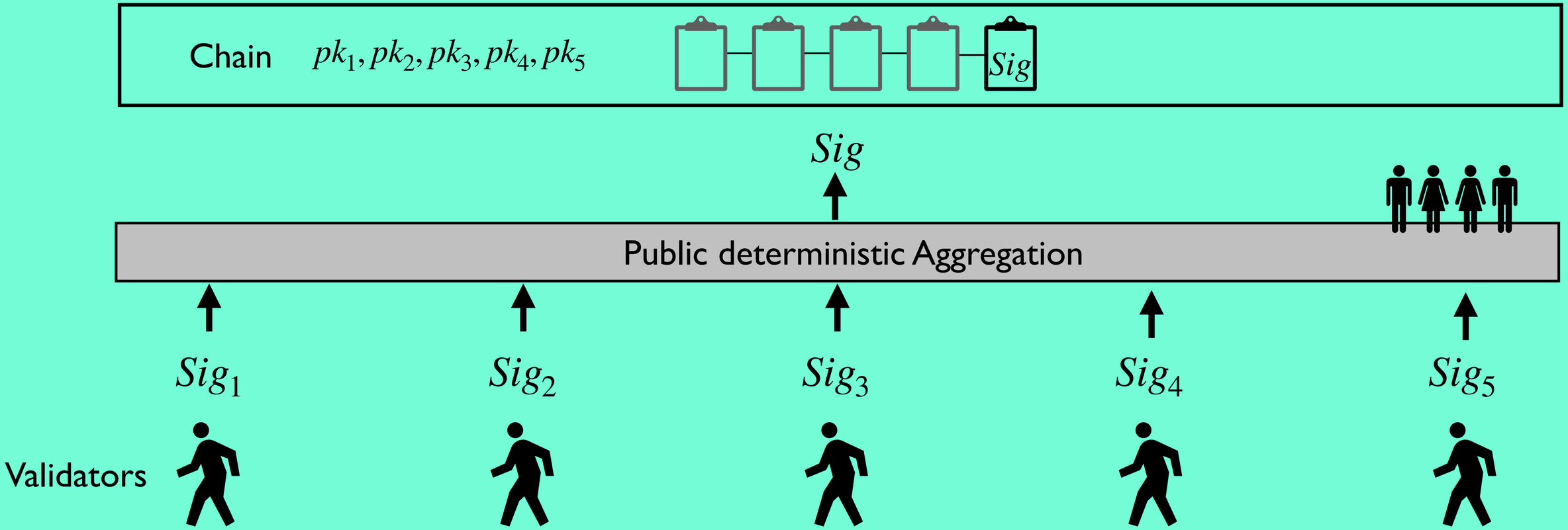

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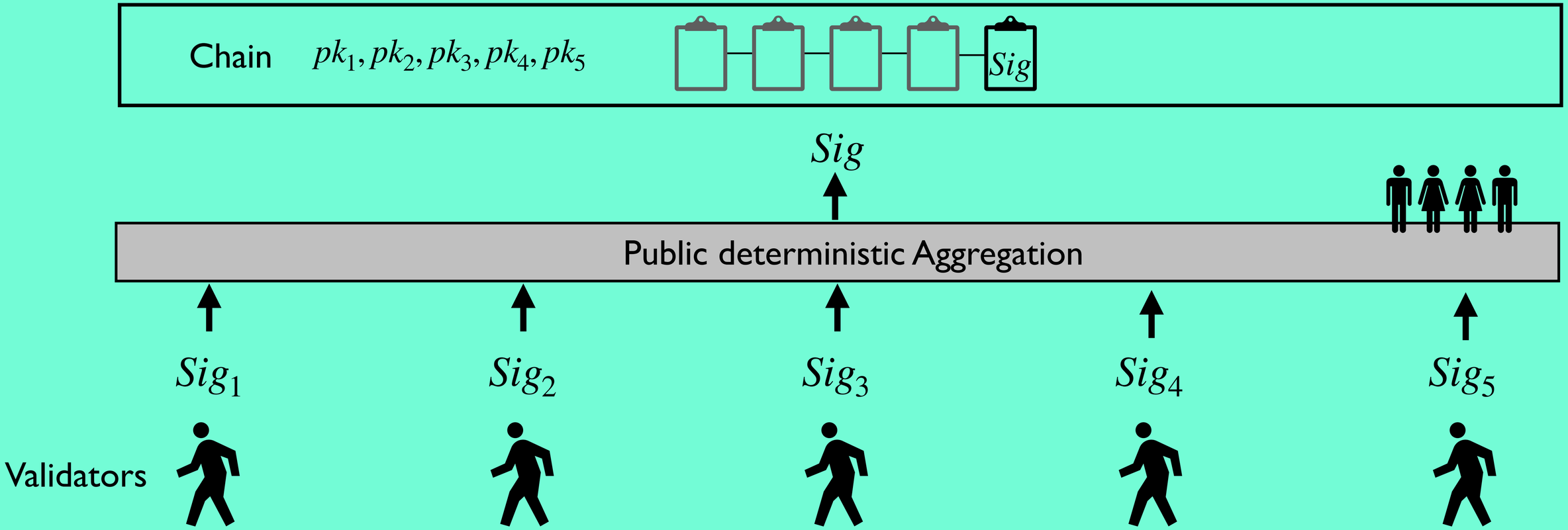
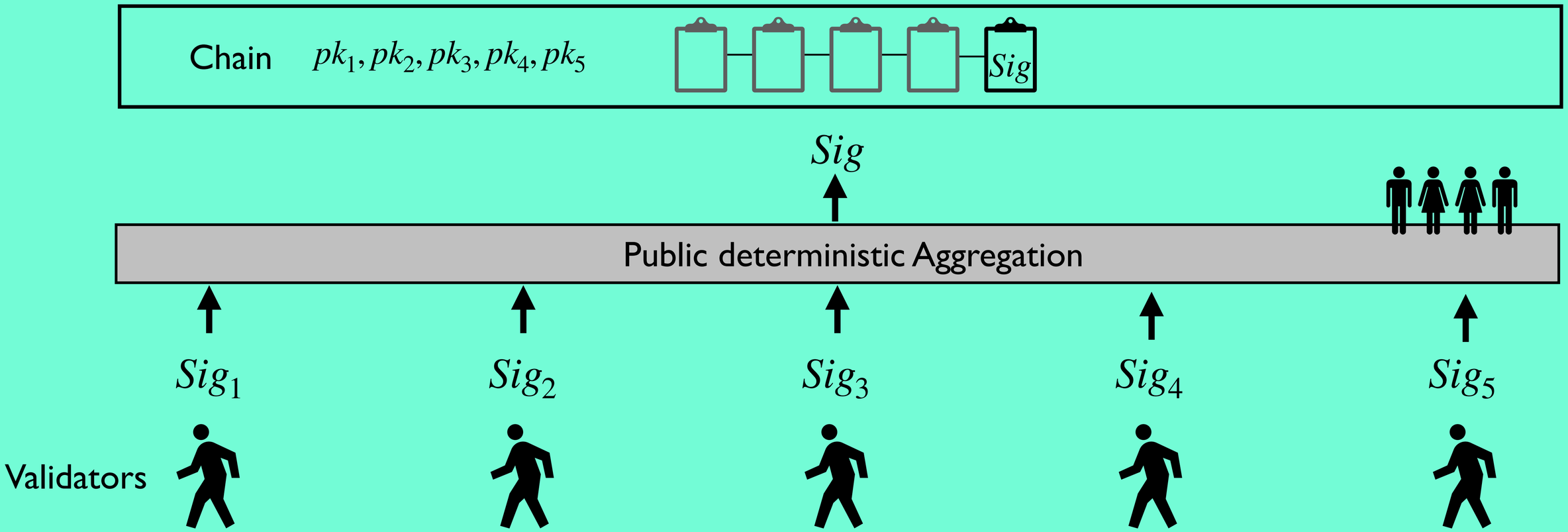


Currently: BLS Signatures

Multi-Signatures in Ethereum Consensus



Currently: BLS Signatures

Quantum Insecurity!

Our Goal

Goal: Post-Quantum Multi-Signatures for Ethereum

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Post-Quantum Cryptography

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Post-Quantum Cryptography

Lattices and Codes

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Post-Quantum Cryptography

Lattices and Codes

Isogenies

Multivariate

Our Goal

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Post-Quantum Cryptography

- Lattices and Codes
- Isogenies
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- Hash Functions

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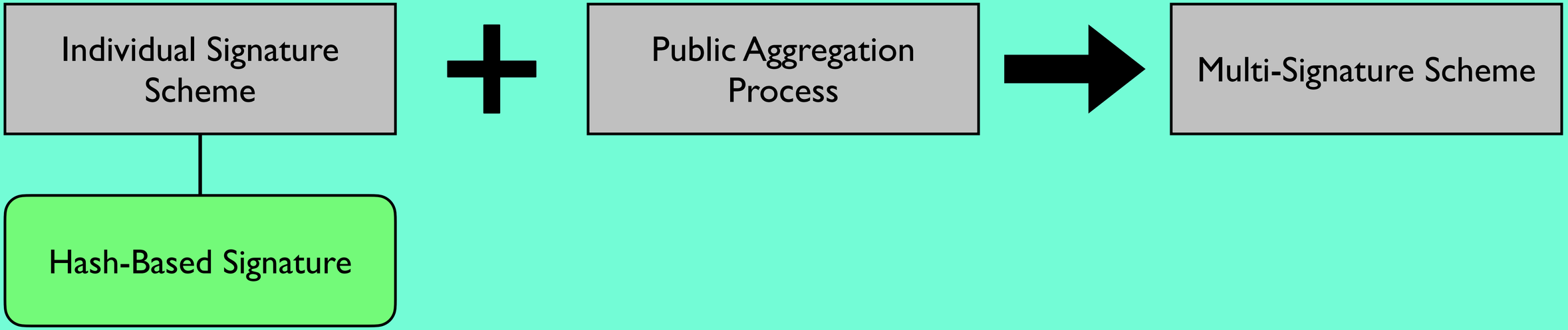

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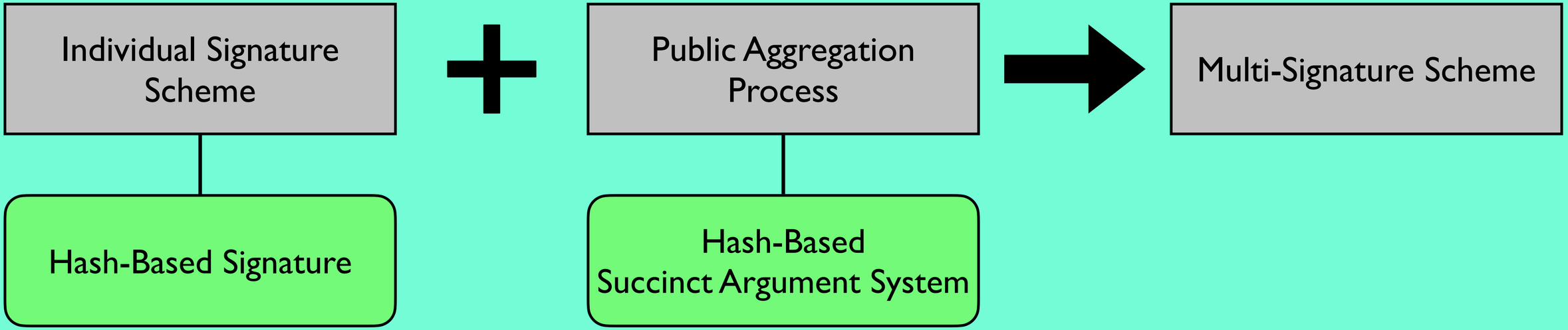

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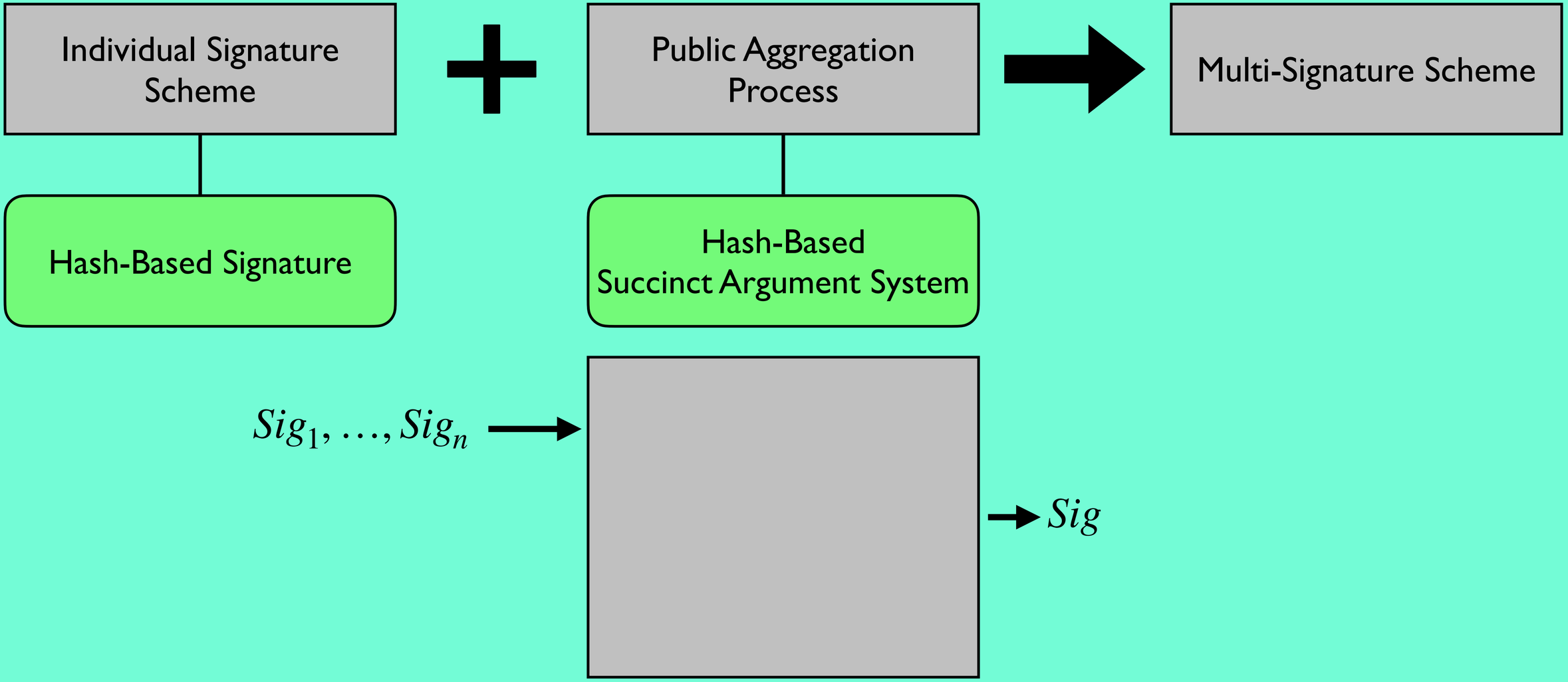

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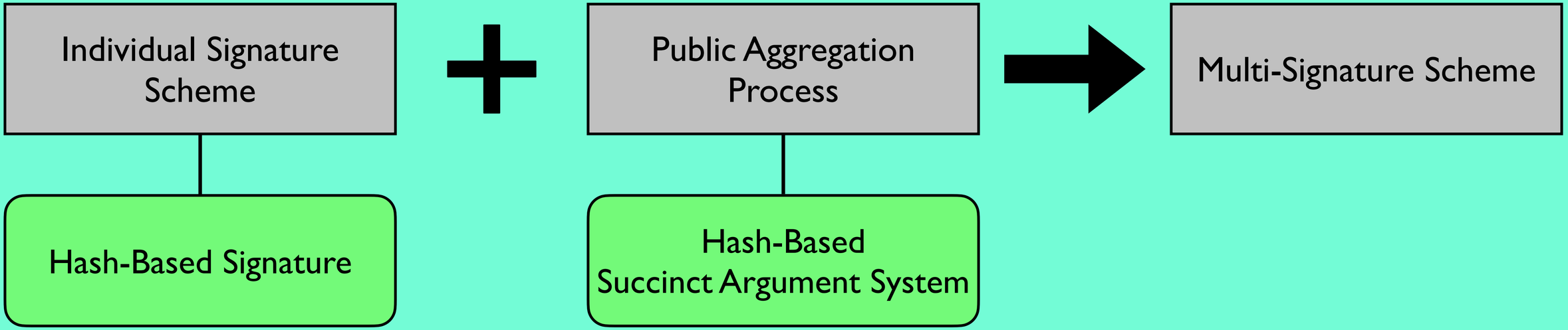

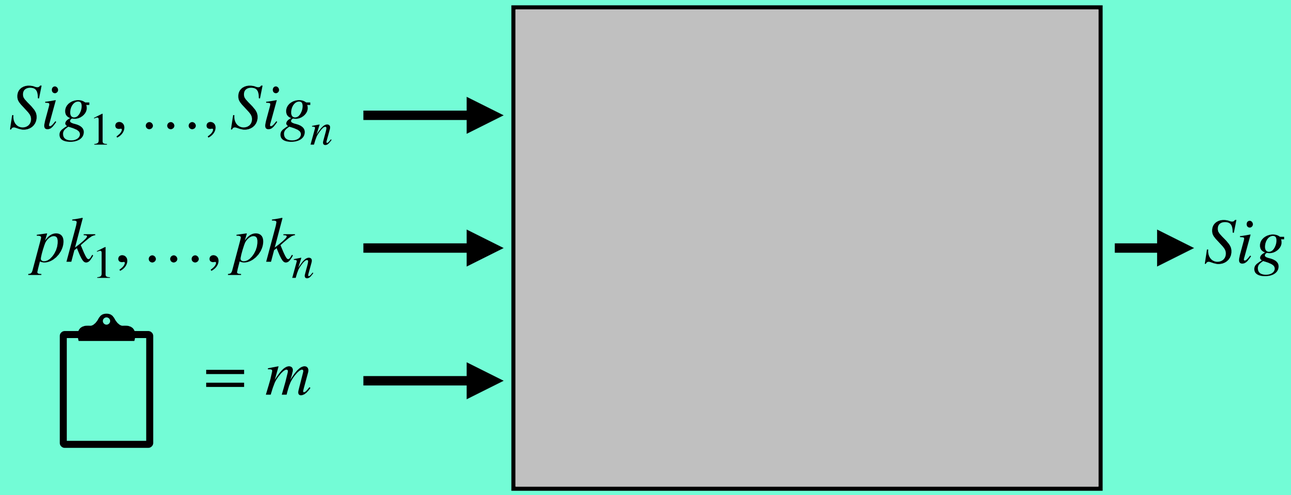

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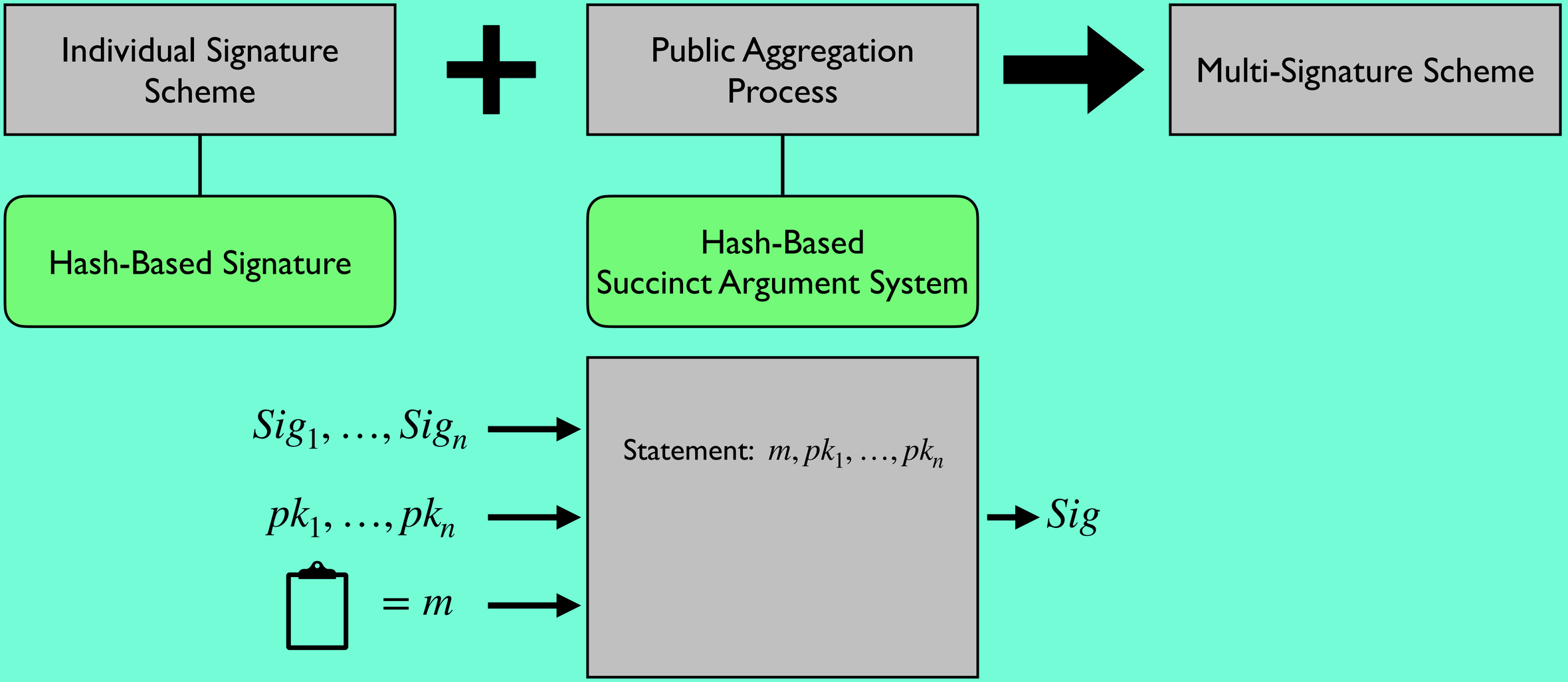

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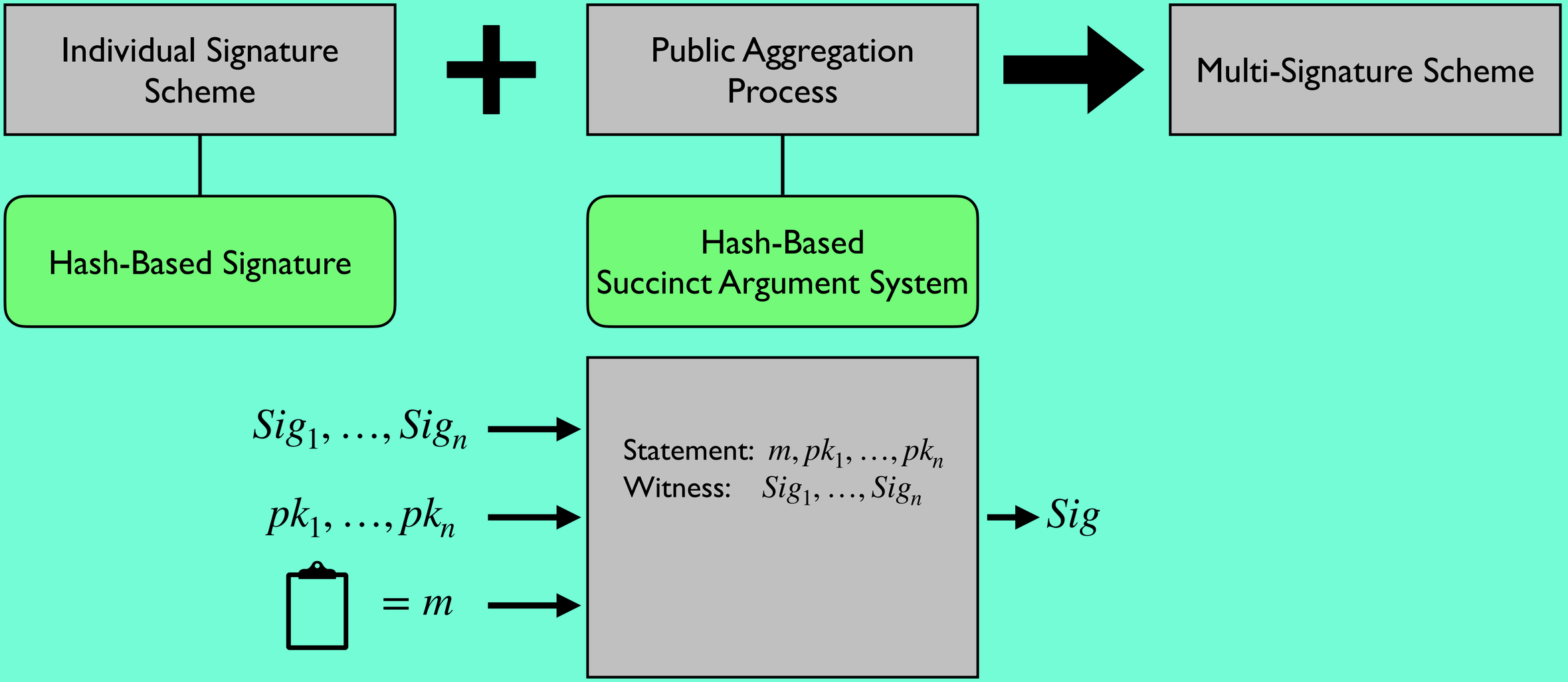

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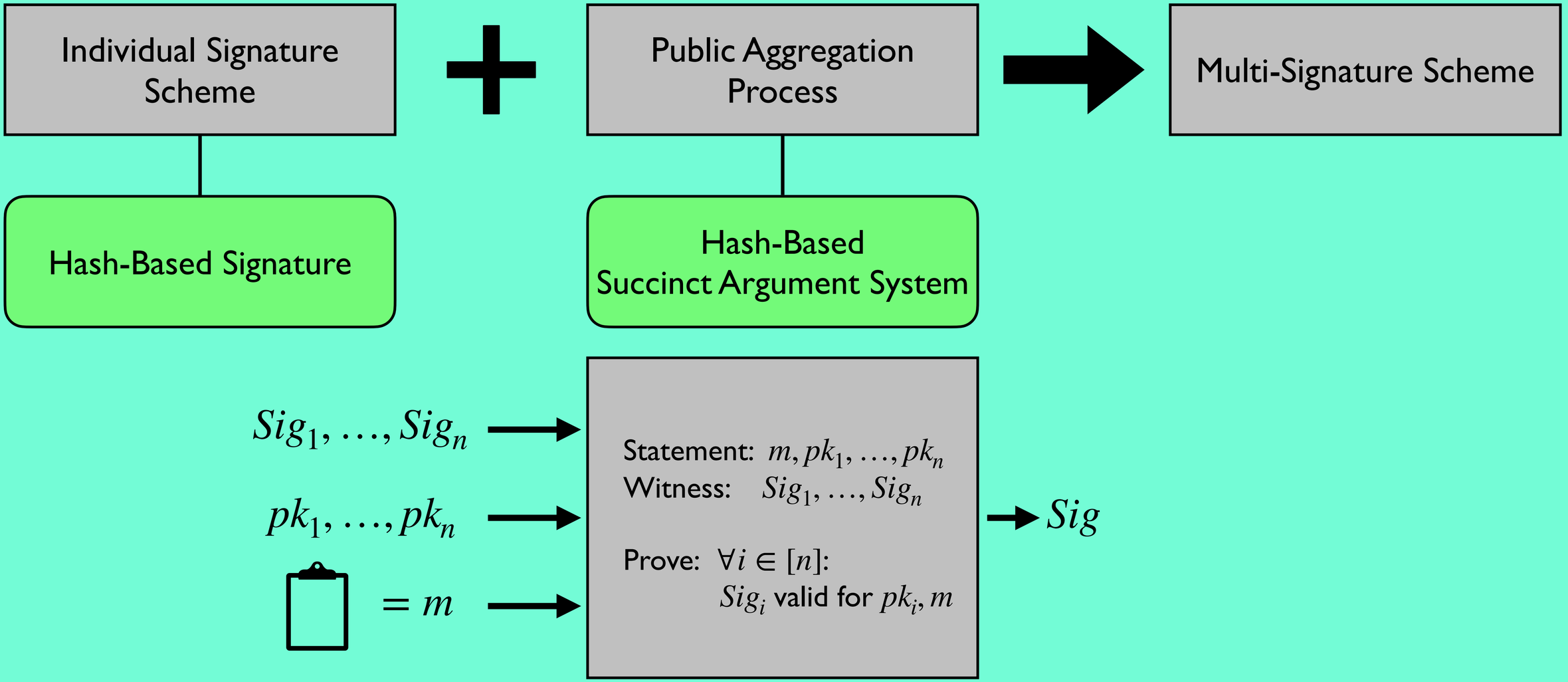


Paradigm (Folklore)

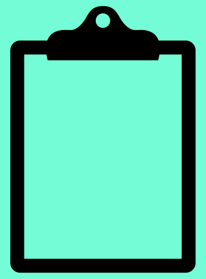
Individual Signature Scheme + Public Aggregation Process → Multi-Signature Scheme

Hash-Based Signature

Hash-Based Succinct Argument System

$Sig_1, \ldots, Sig_n$ →

$pk_1, \ldots, pk_n$ →

 $= m$ →

Statement: $m, pk_1, \ldots, pk_n$
Witness: $Sig_1, \ldots, Sig_n$

Prove: $\forall i \in [n]$:
Sig_i valid for pk_i, m

→ Sig

Need adaptive knowledge soundness

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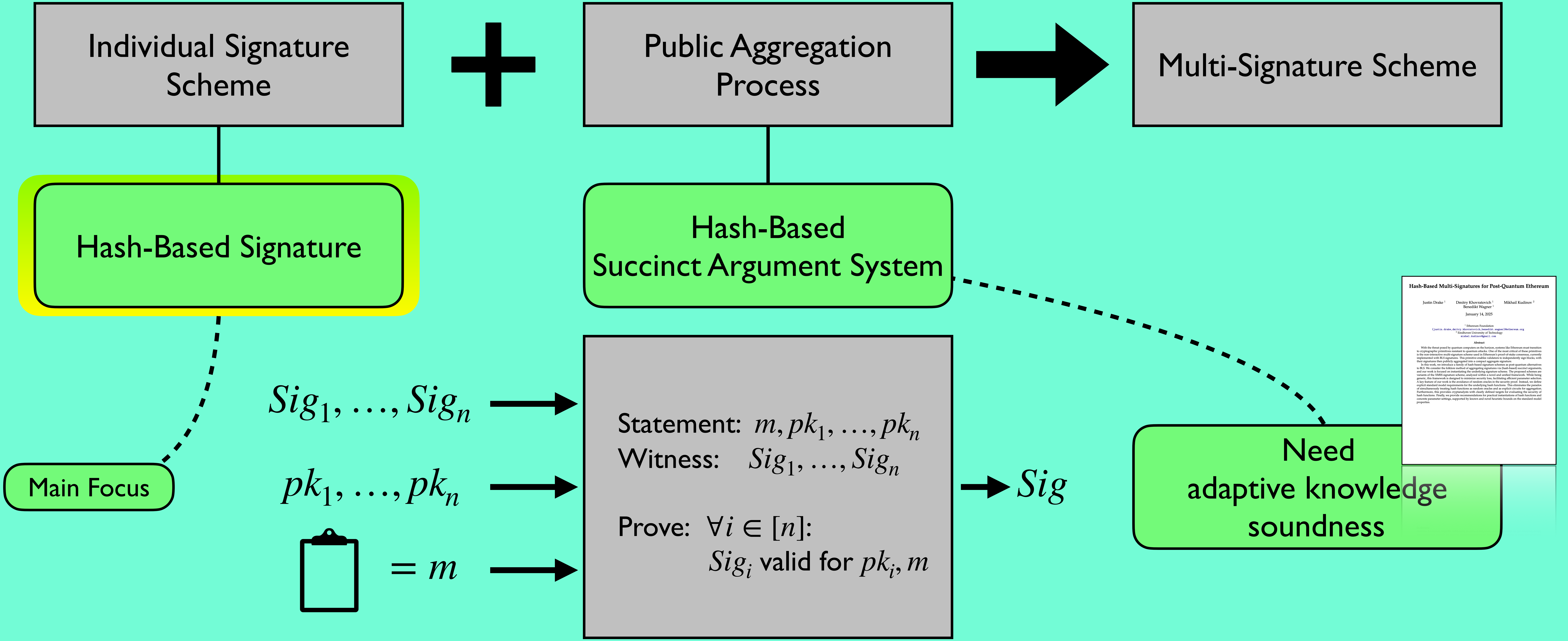

Signature Requirements

Goal: Post-Quantum Multi-Signatures for Ethereum

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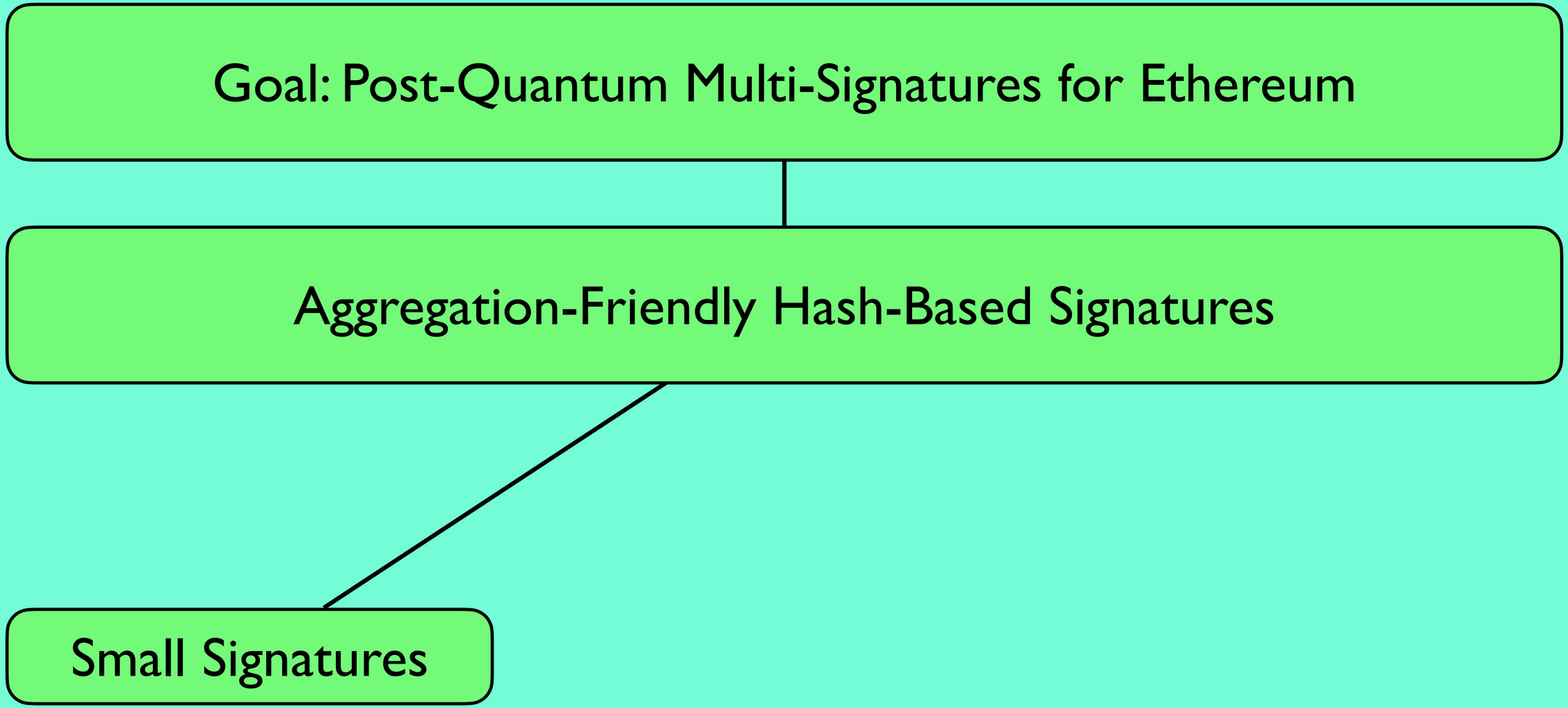
Aggregation-Friendly Hash-Based Signatures

Signature Requirements



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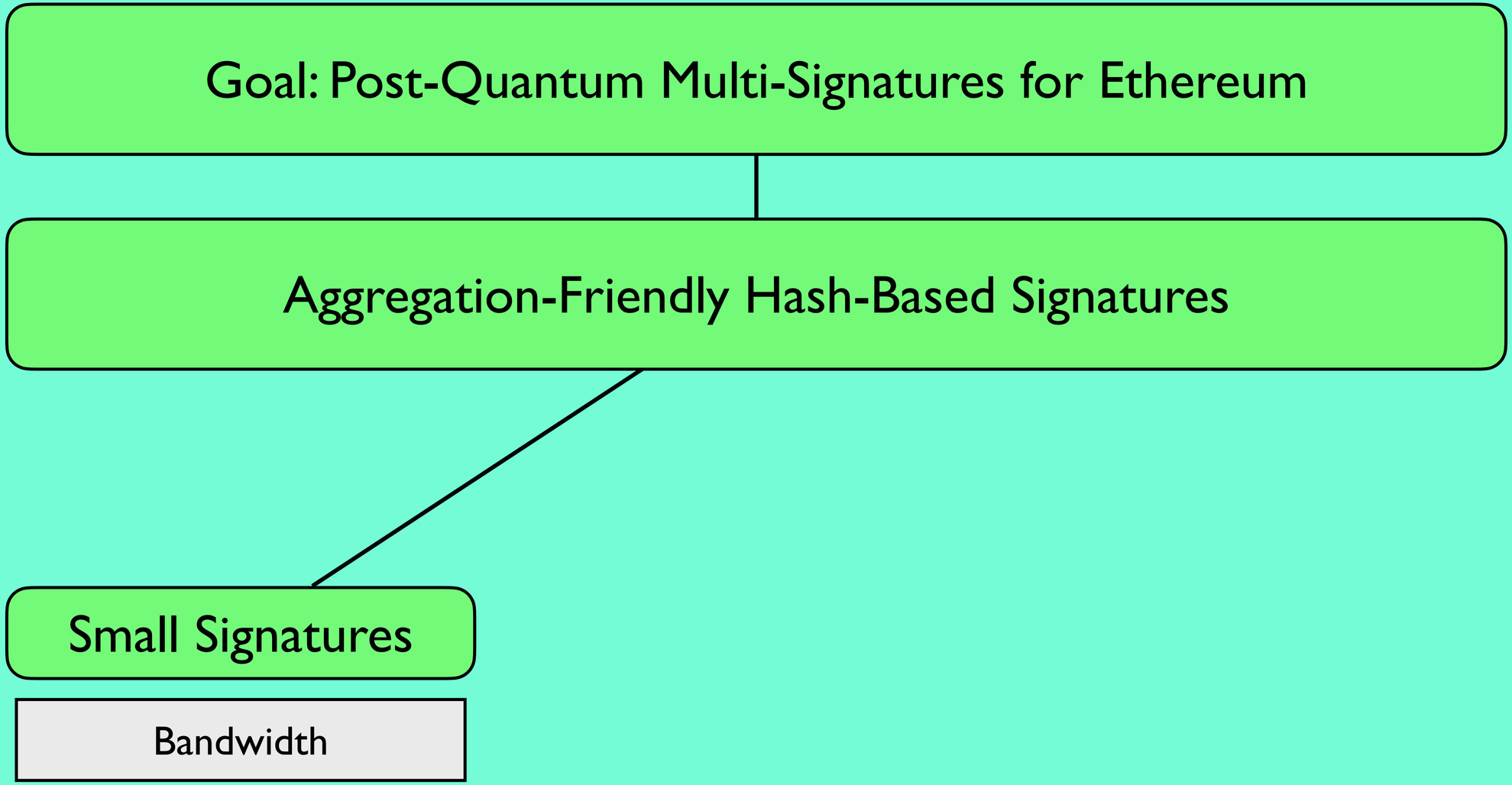

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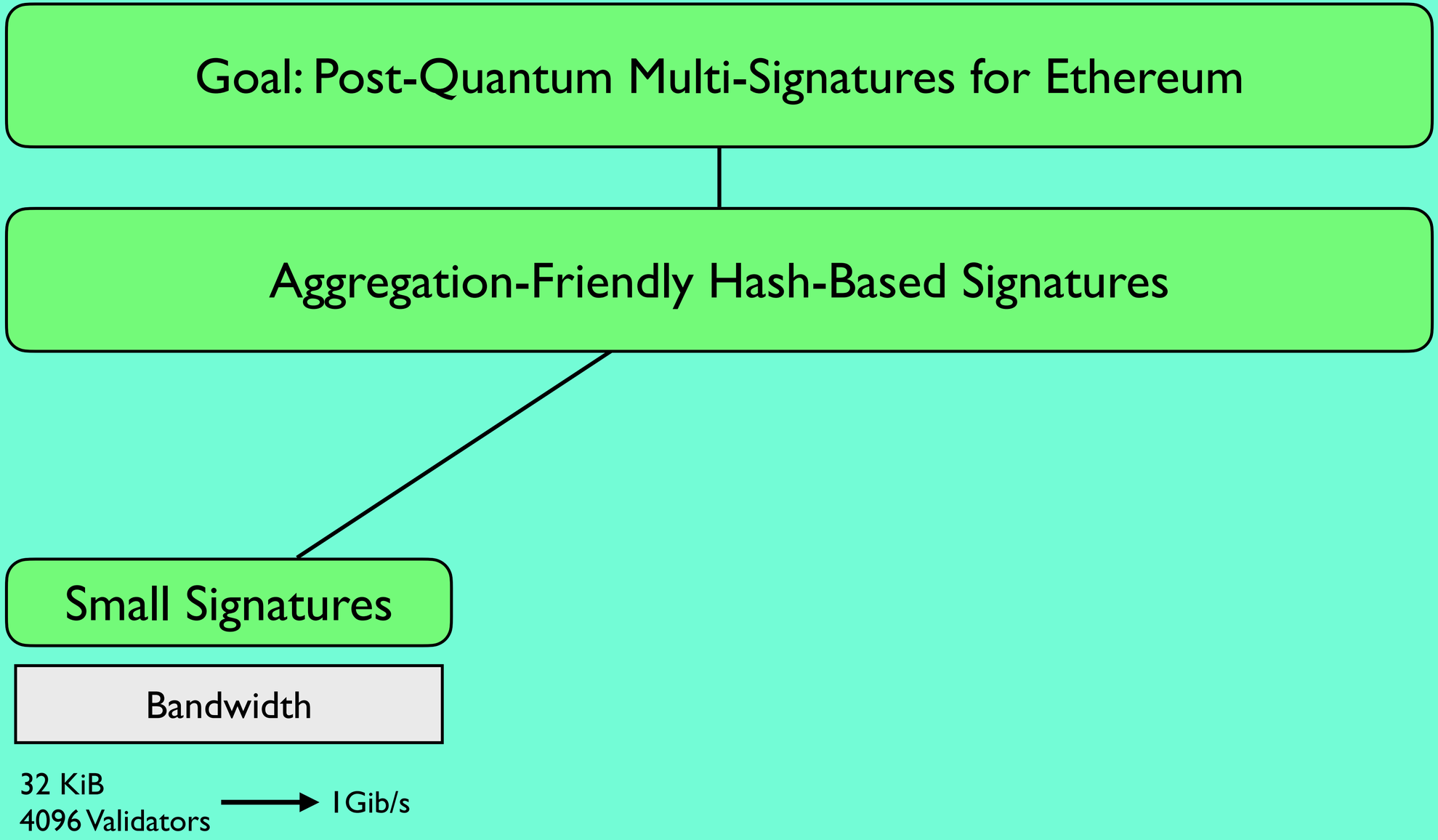

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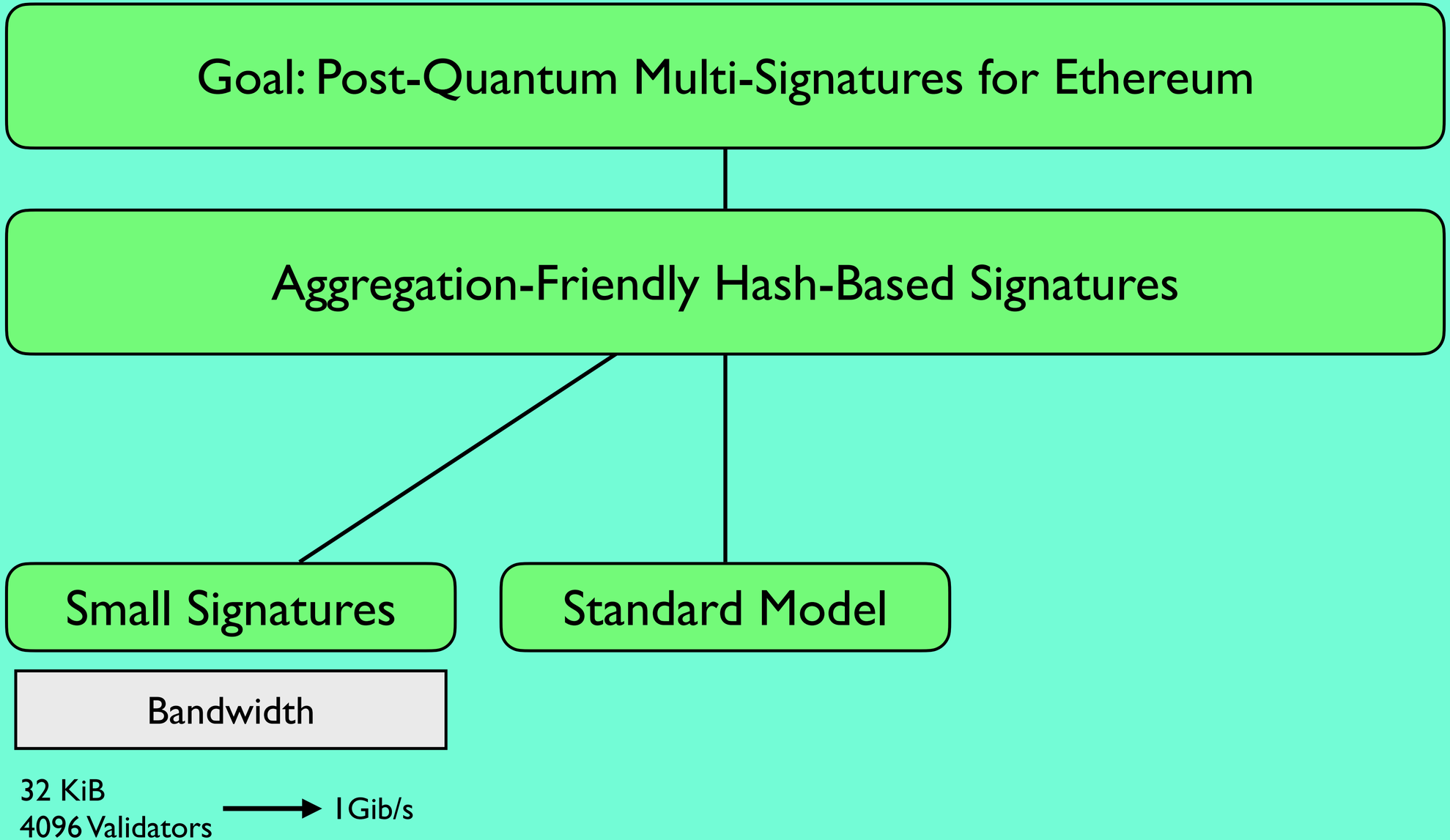

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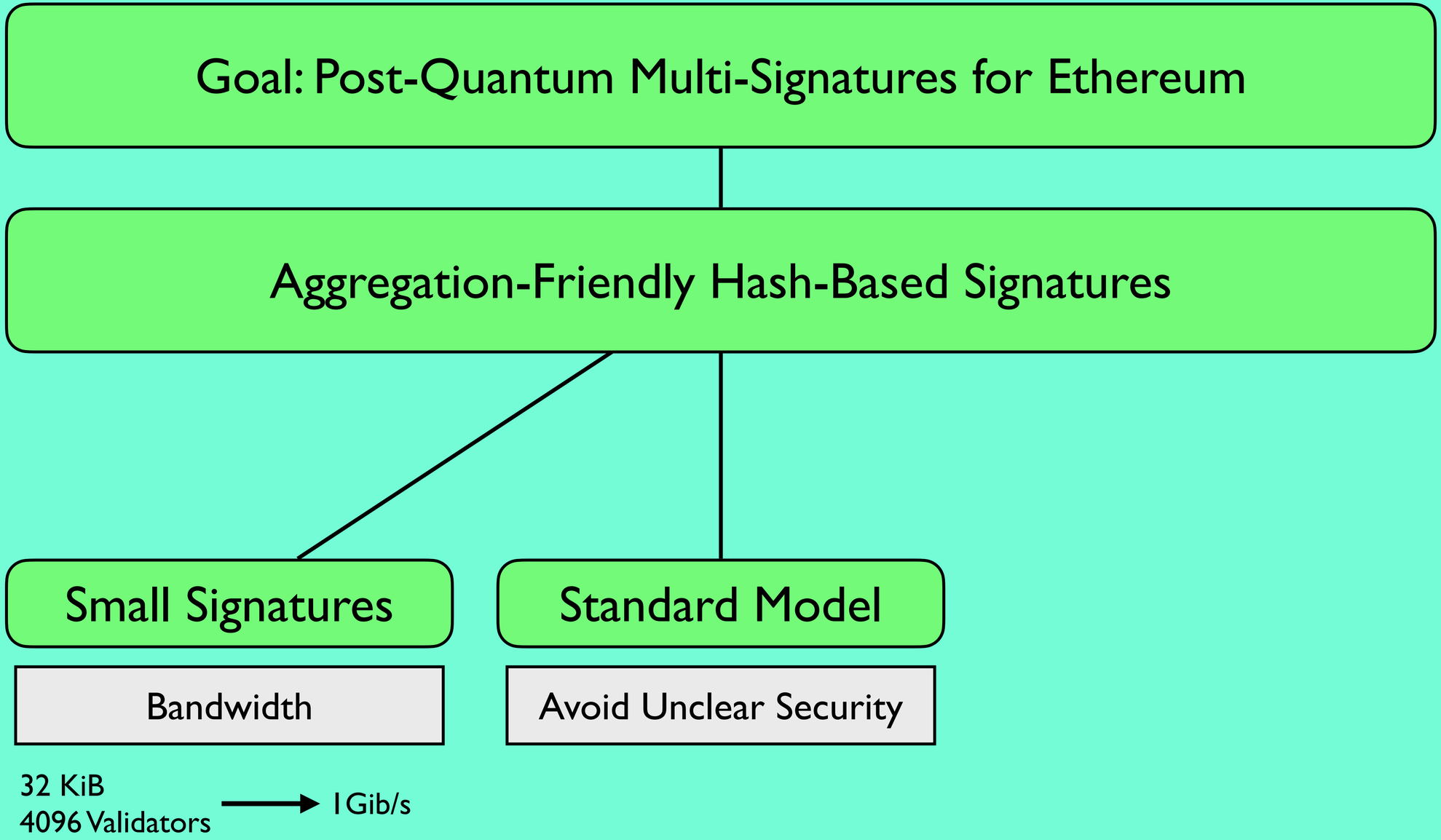

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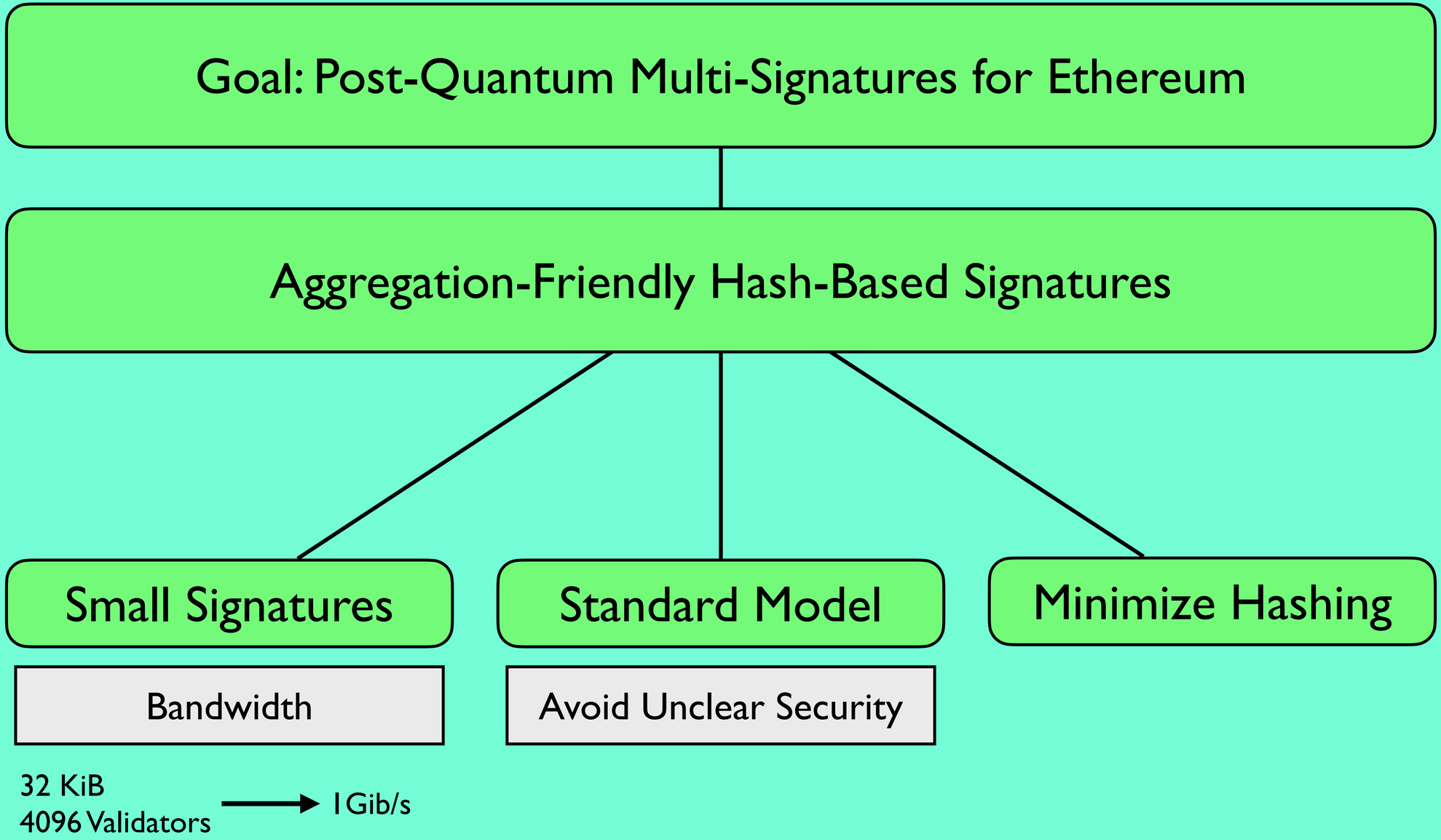

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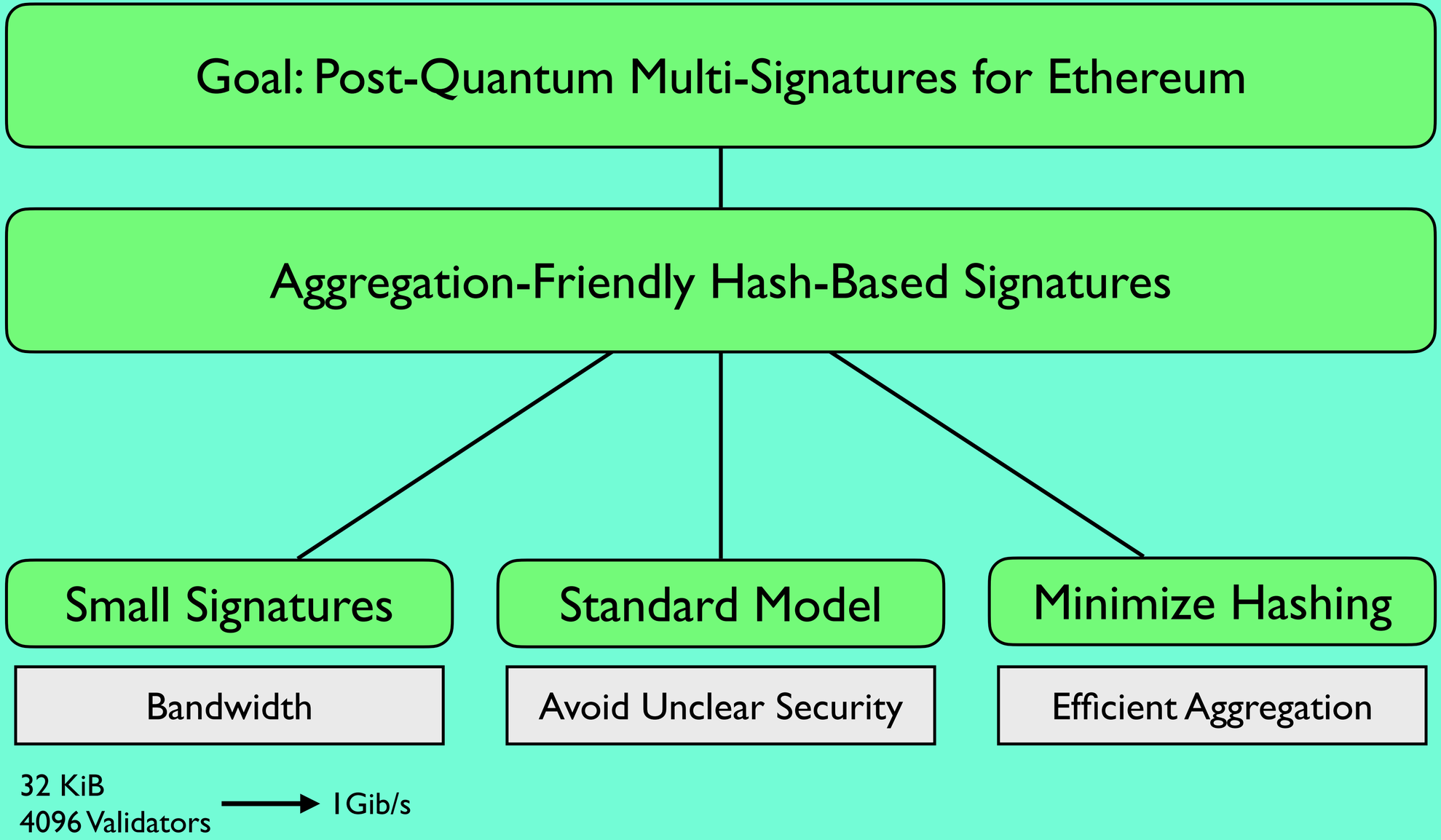

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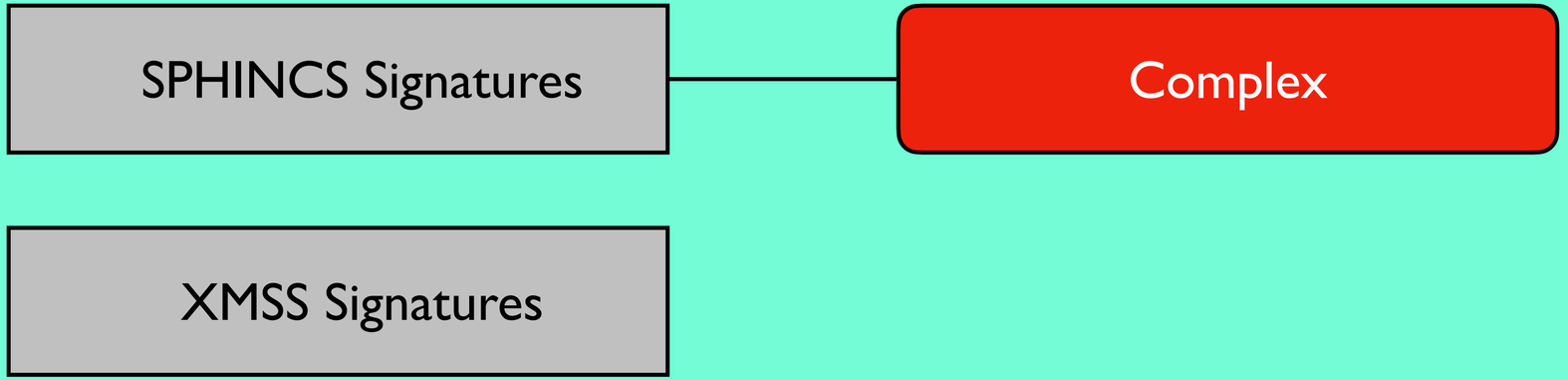
SPHINCS Signatures

XMSS Signatures

Existing Hash-Based Schemes



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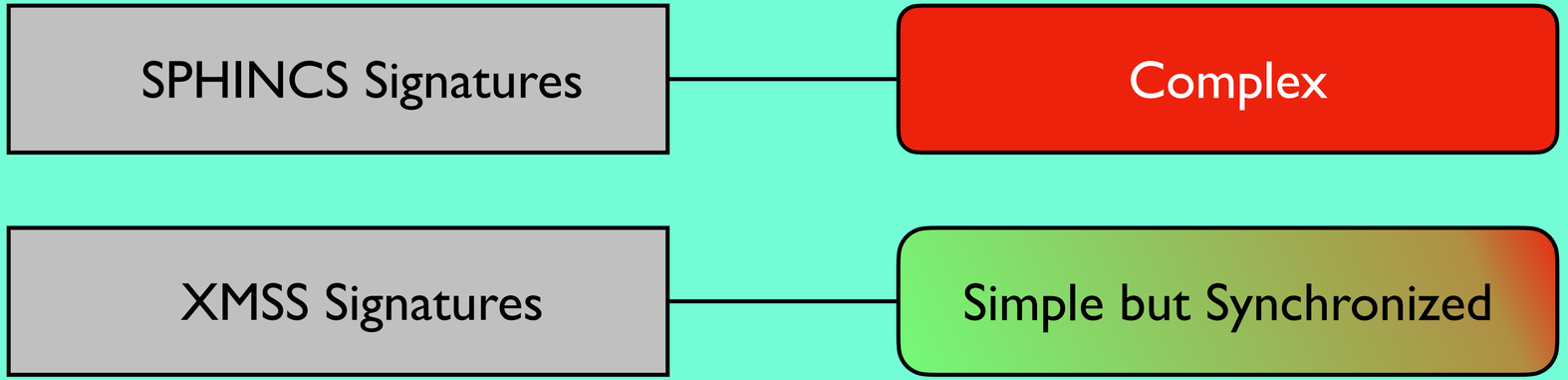

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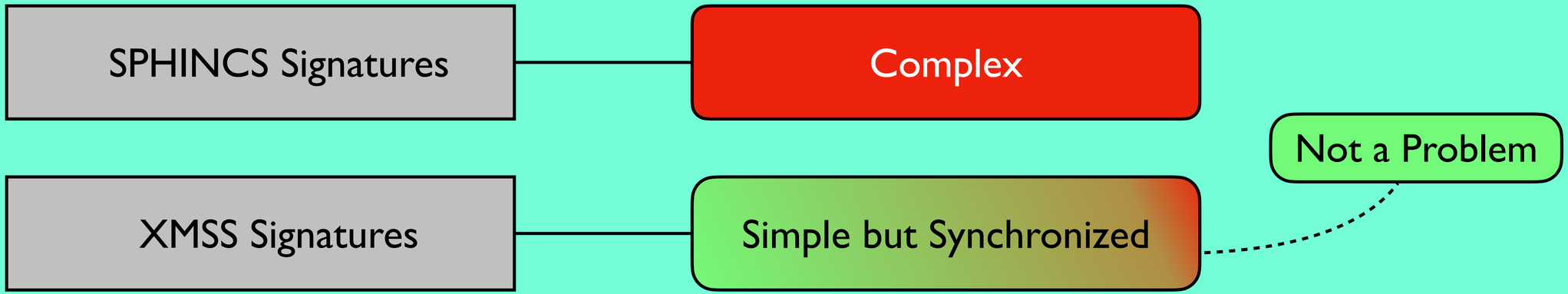

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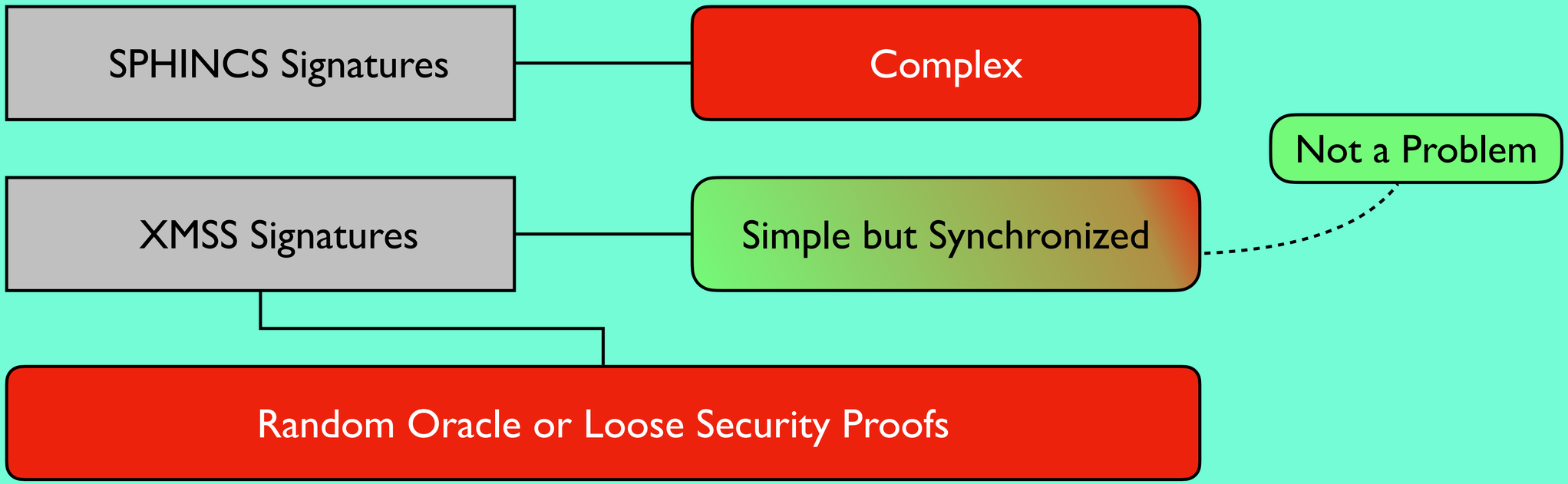

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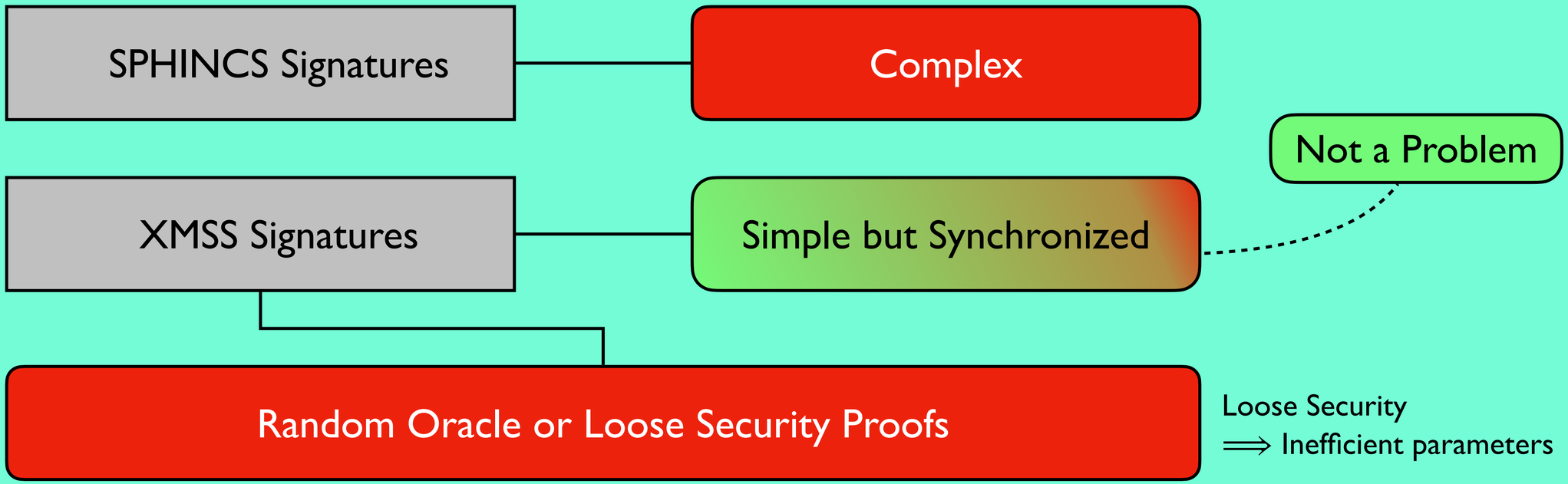

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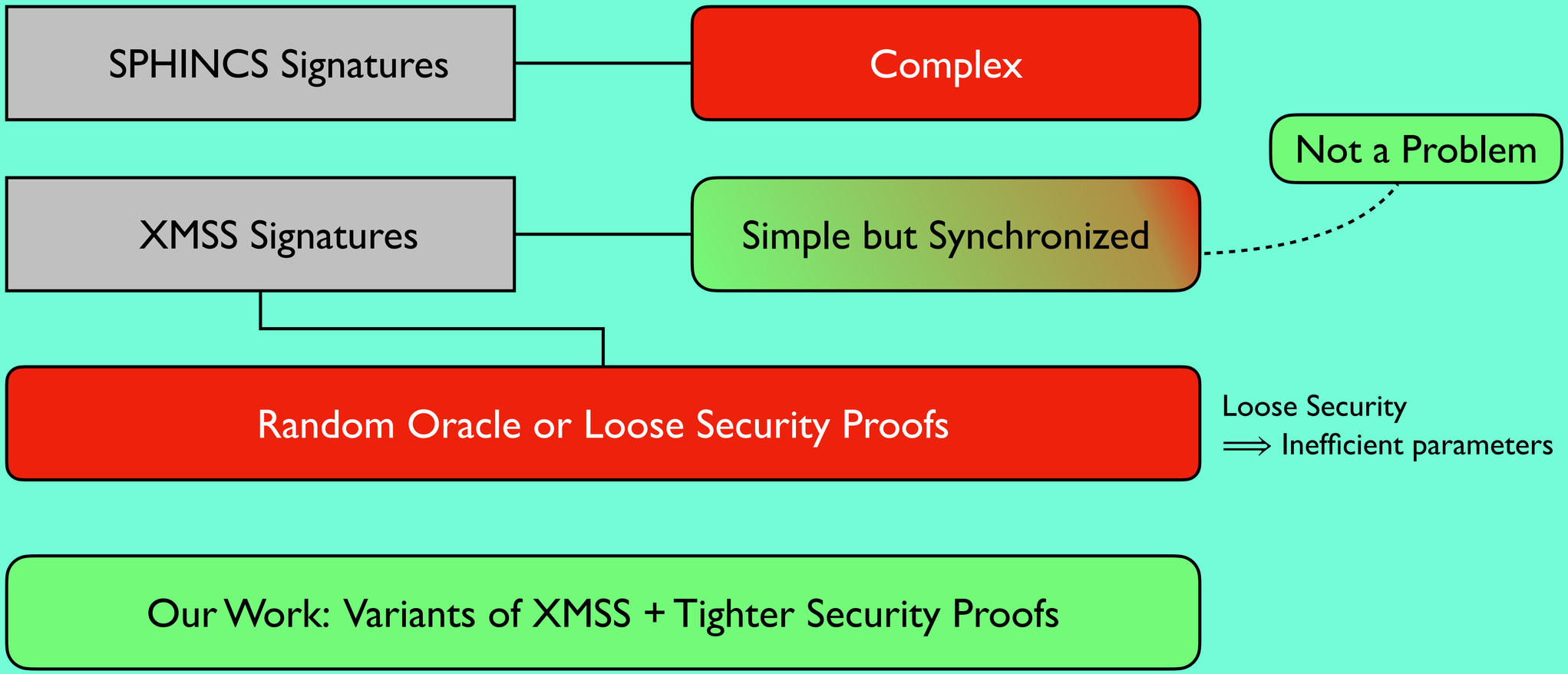

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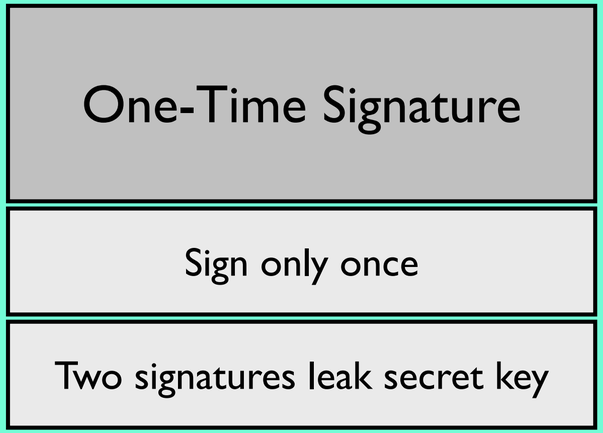

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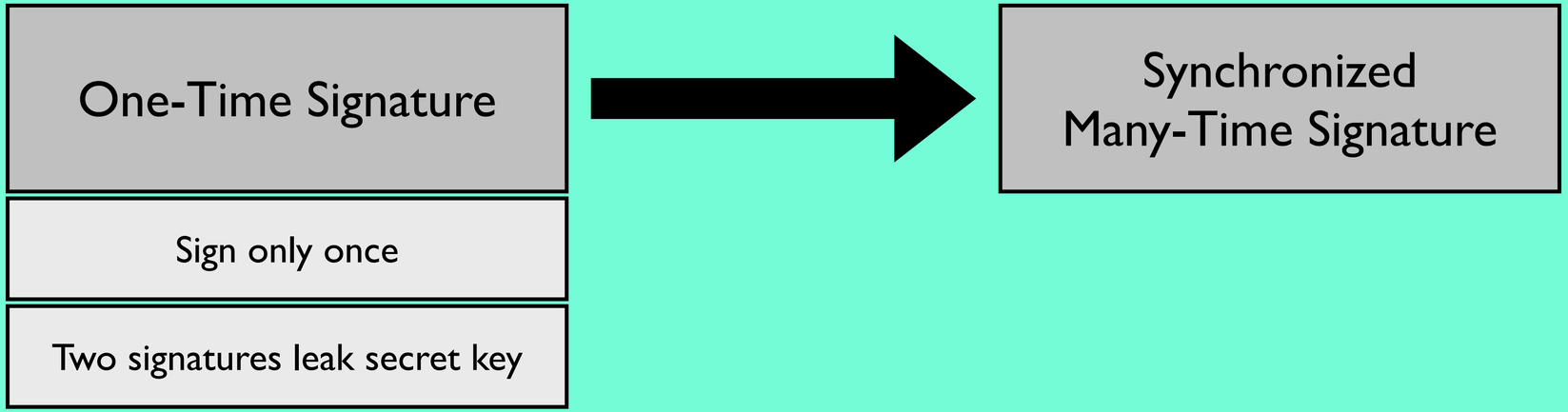

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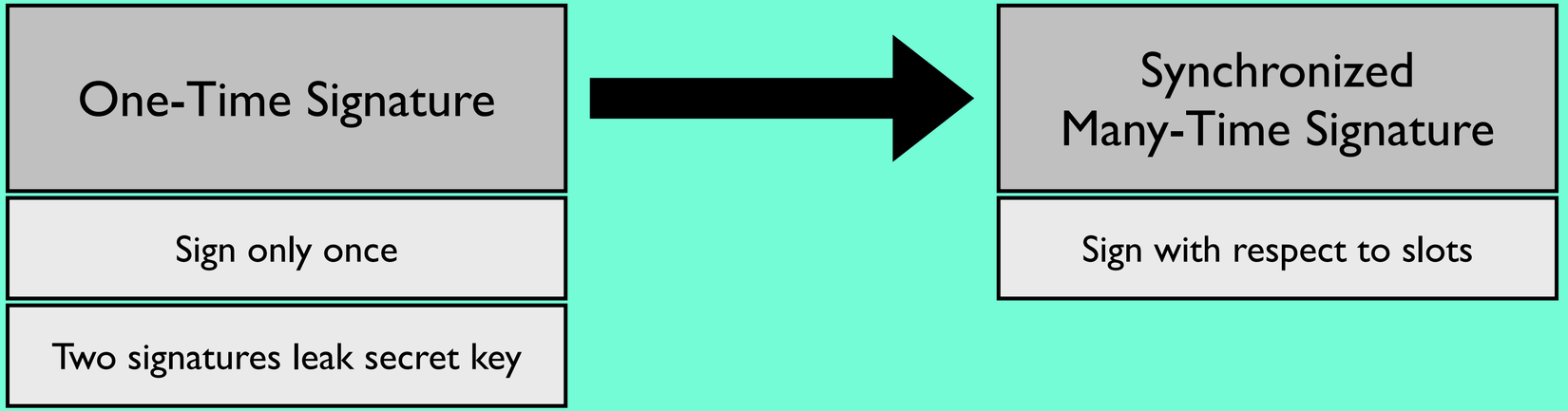

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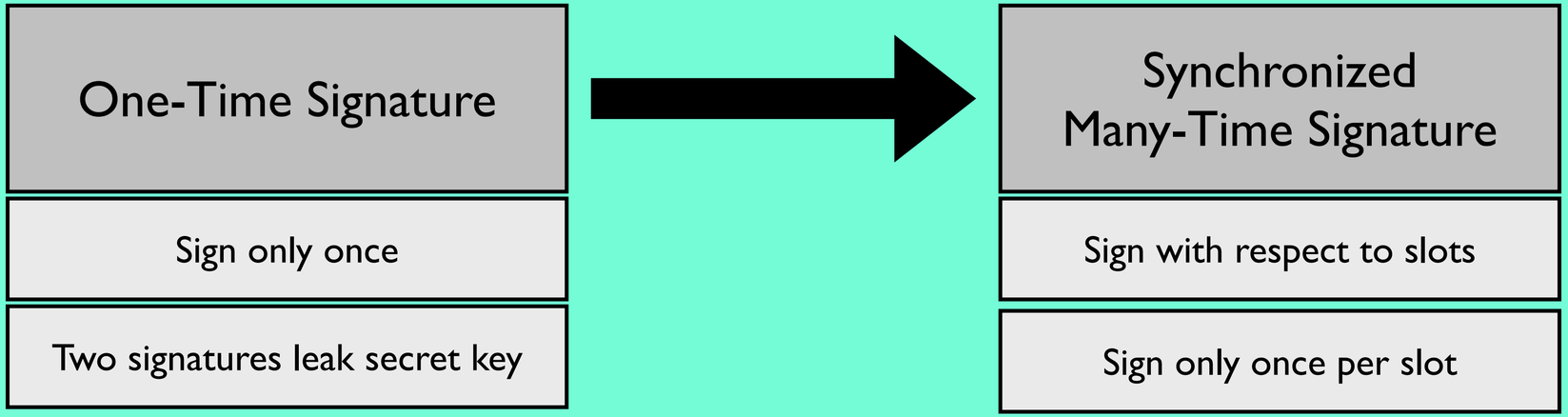

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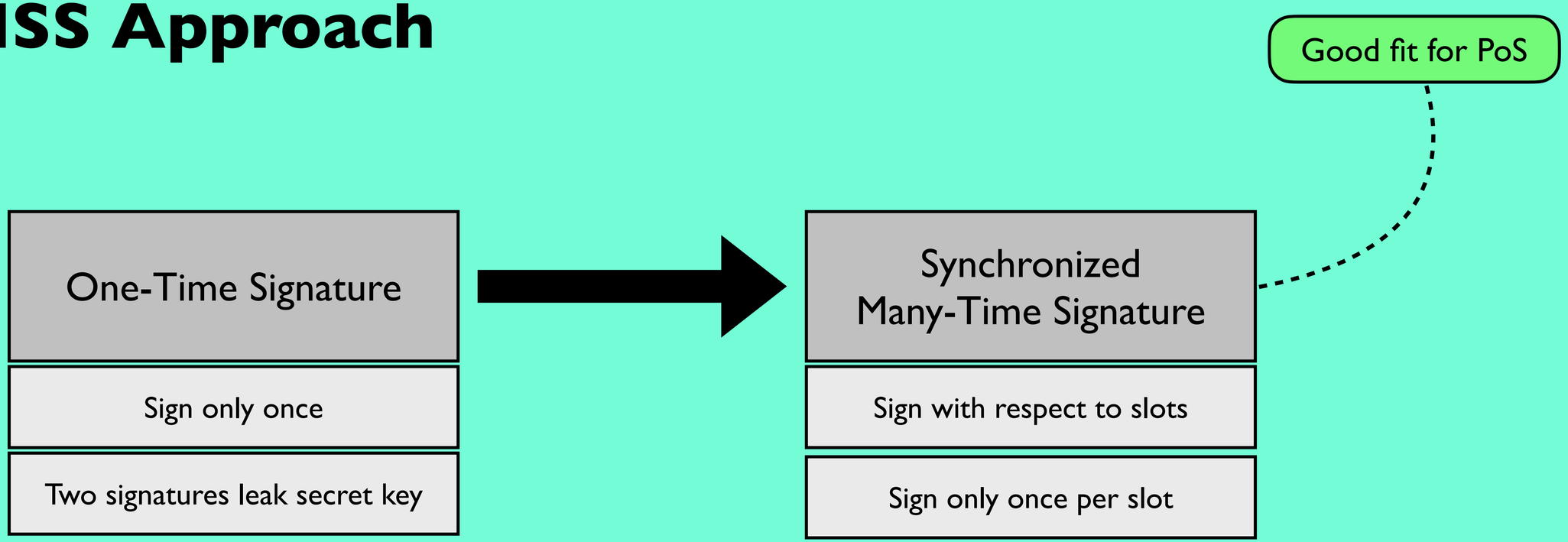

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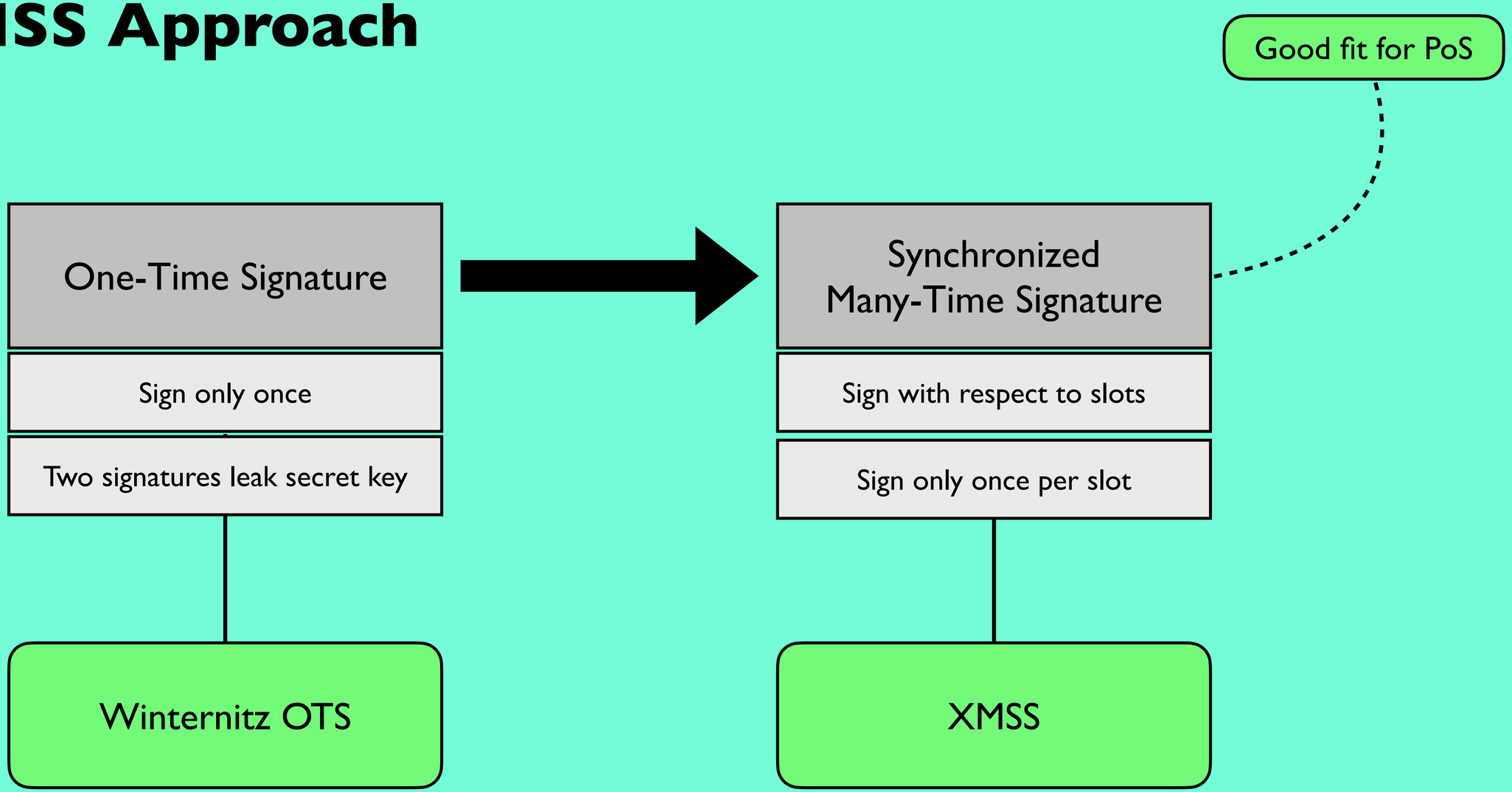

From One-Time Signing to Many-Time Signing

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$L =$ Key lifetime

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pk_1^{OTS} $\quad \cdots \quad$ pk_L^{OTS}

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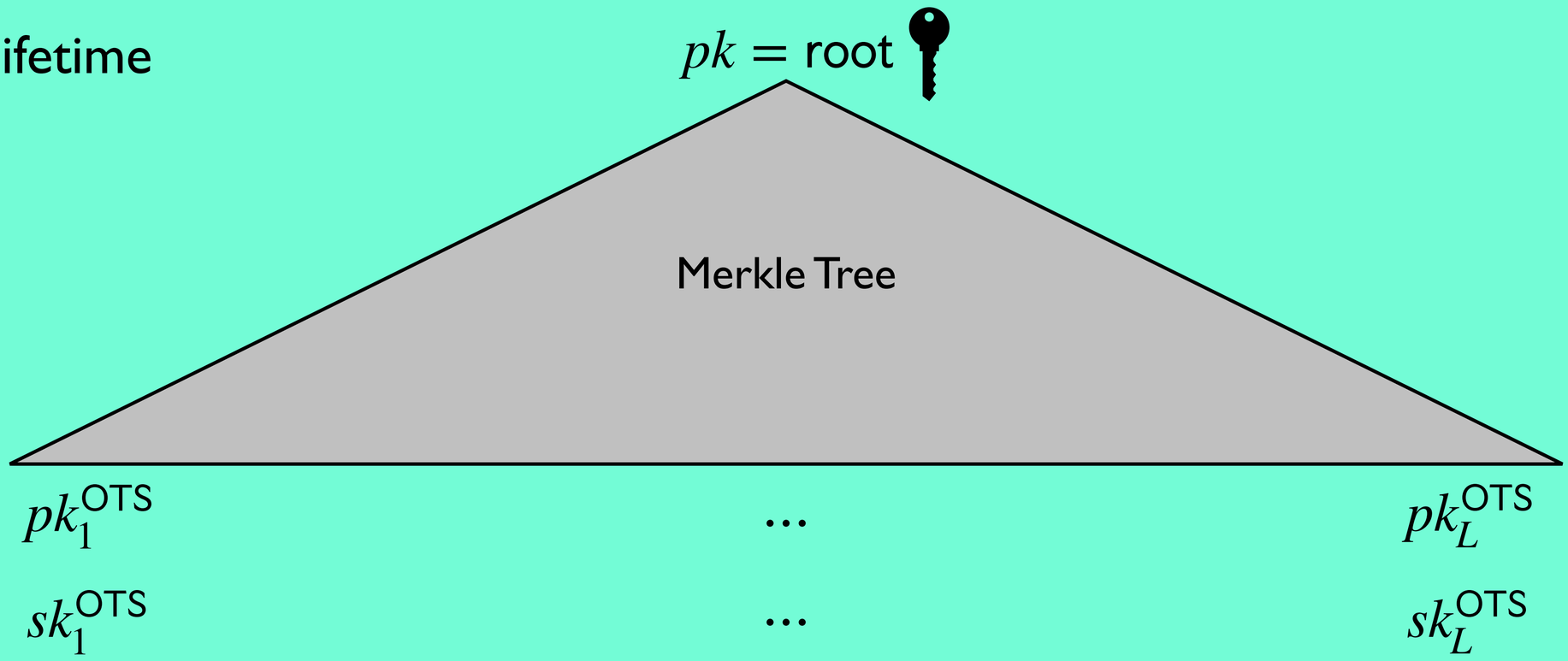

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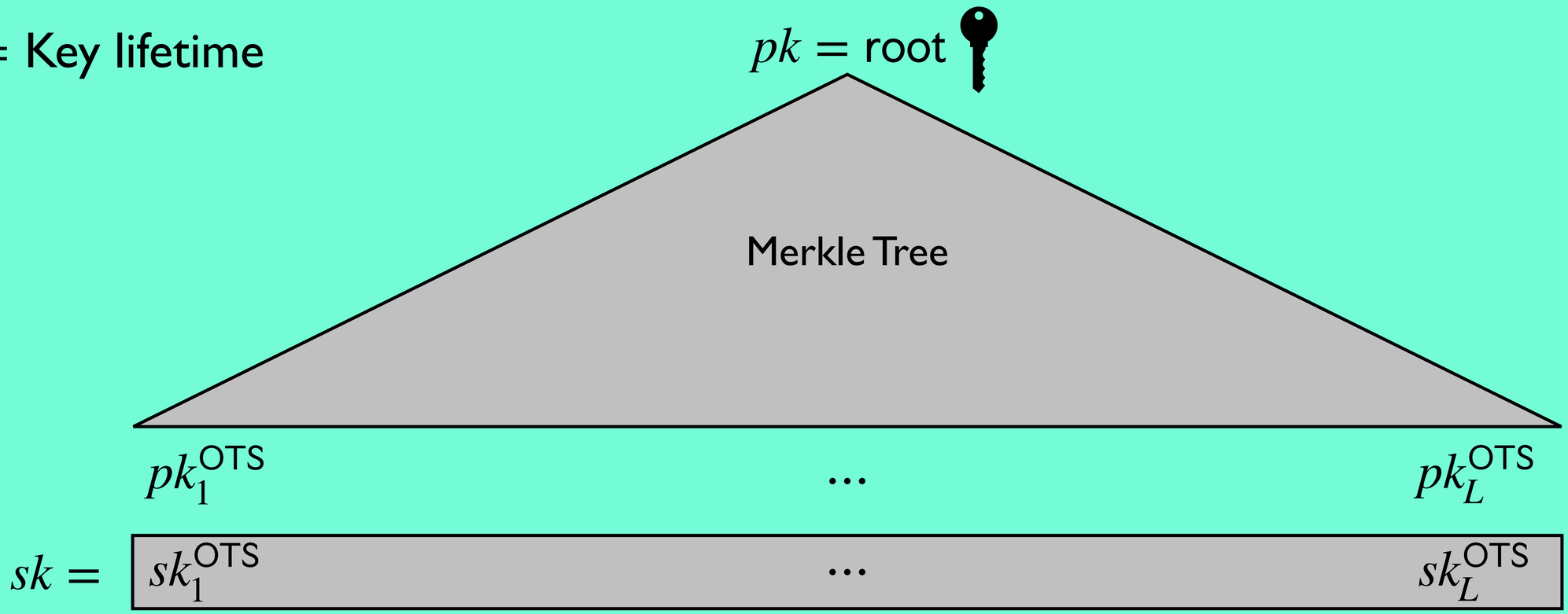

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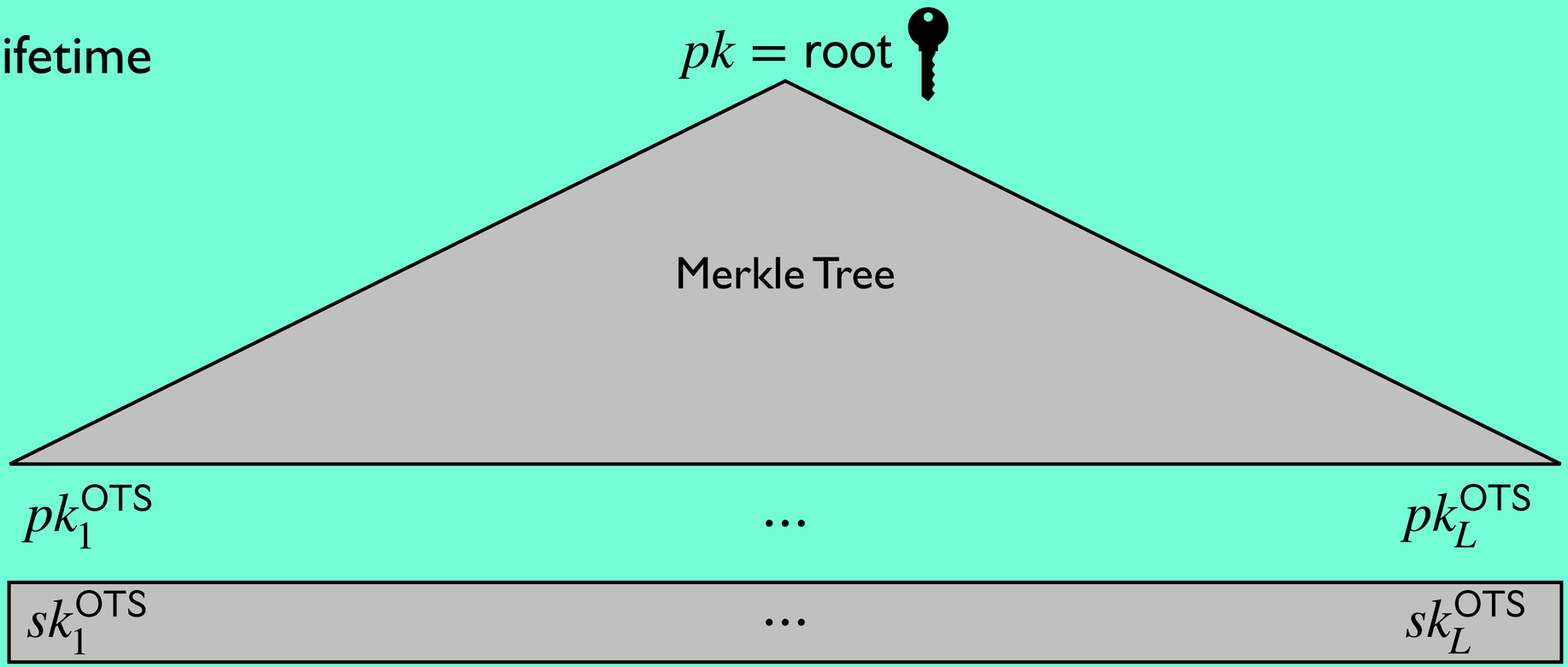

Signing message m wrt slot i

1. OT Public Key pk_i^{OTS}
2. Merkle Path
3. OT Sig $Sig(sk_i^{\text{OTS}}, m)$

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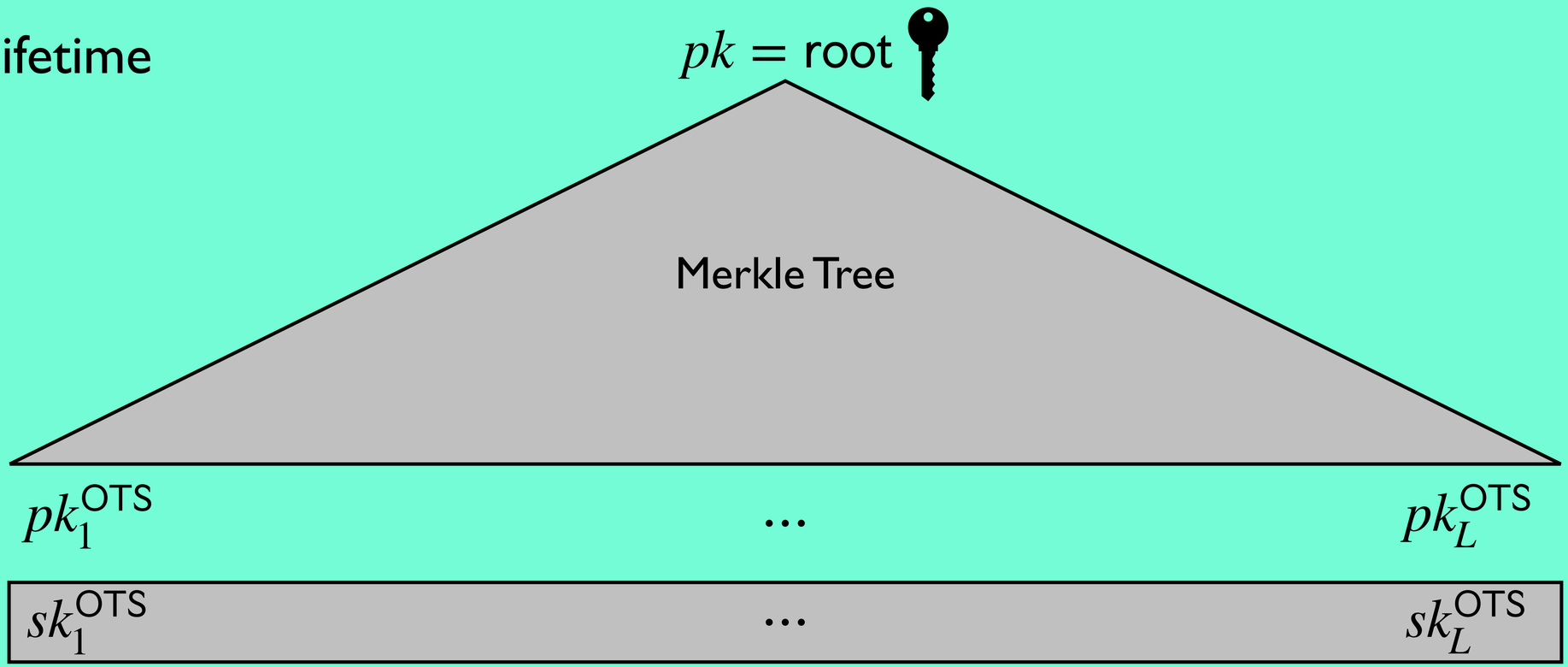

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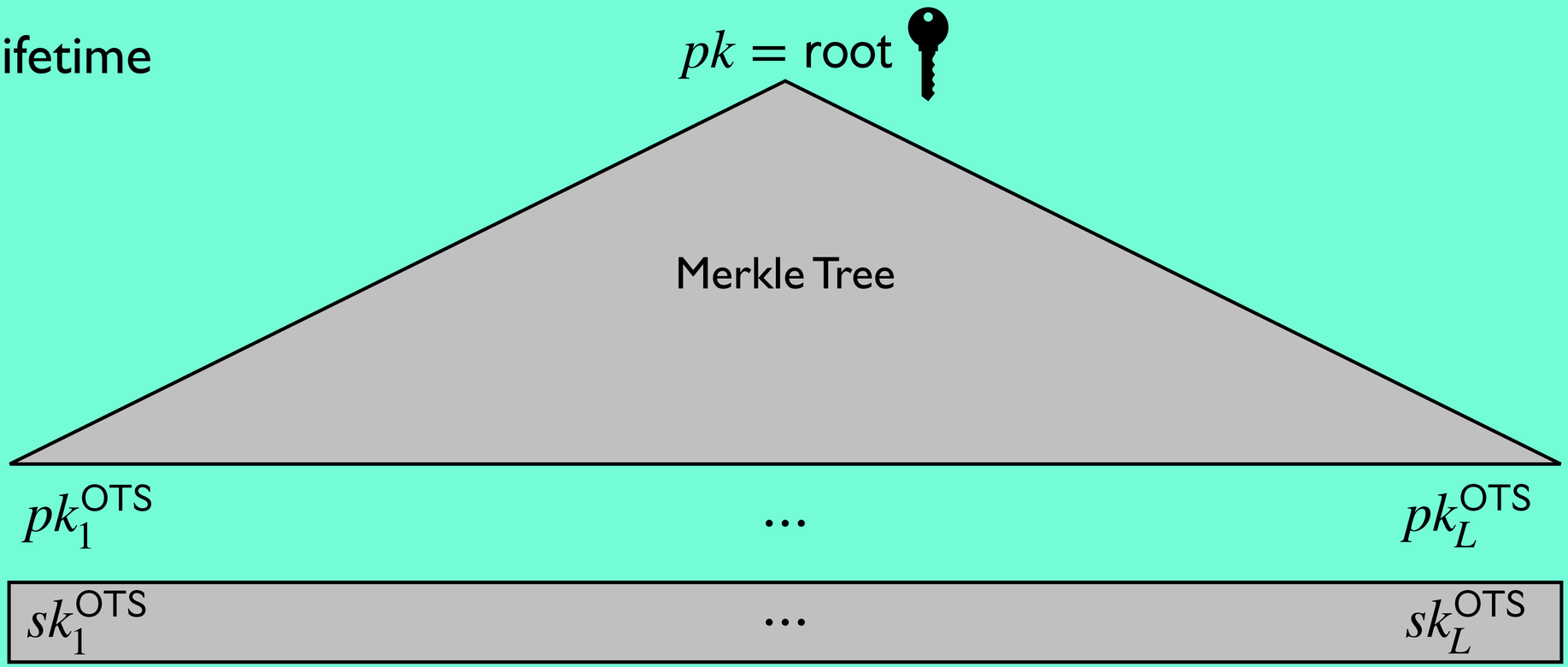
Verification

1. Check Merkle Path
2. Check OT Sig wrt to OT Public Key

From One-Time Signing to Many-Time Signing

$L =$ Key lifetime



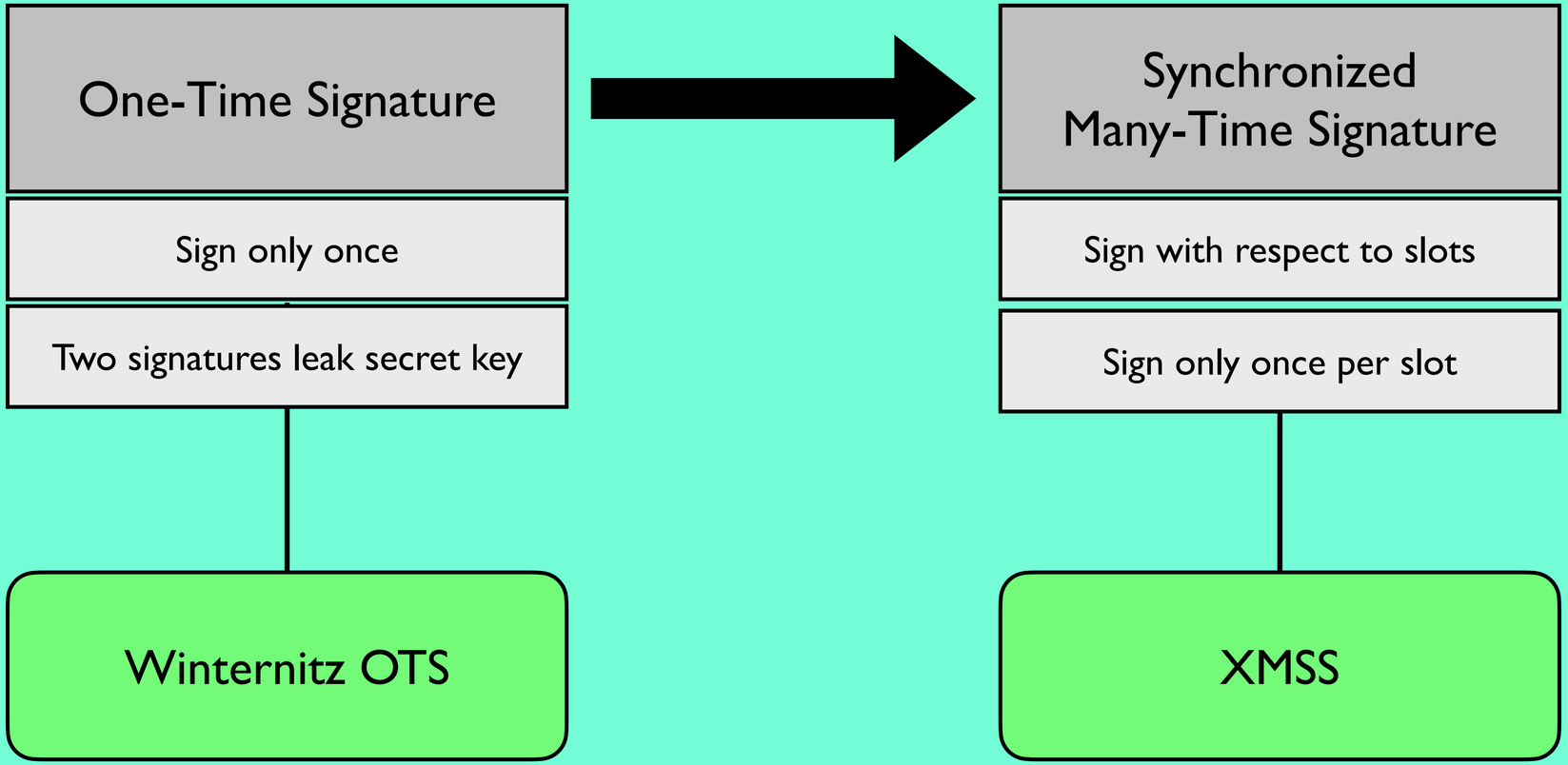
Signing message m wrt slot i

1. ~~OT Public Key pk_i^{OTS}~~
2. Merkle Path
3. OT Sig $Sig(sk_i^{\text{OTS}}, m)$

Verification

1. Check Merkle Path
2. Check OT Sig wrt to OT Public Key

The XMSS Approach



One-Time Signatures via Hash Chains

Example: $v = 4$ chains of length $2^w = 4$

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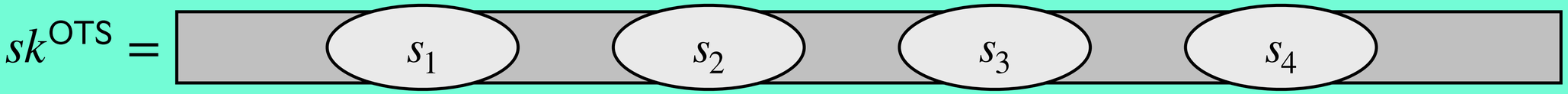

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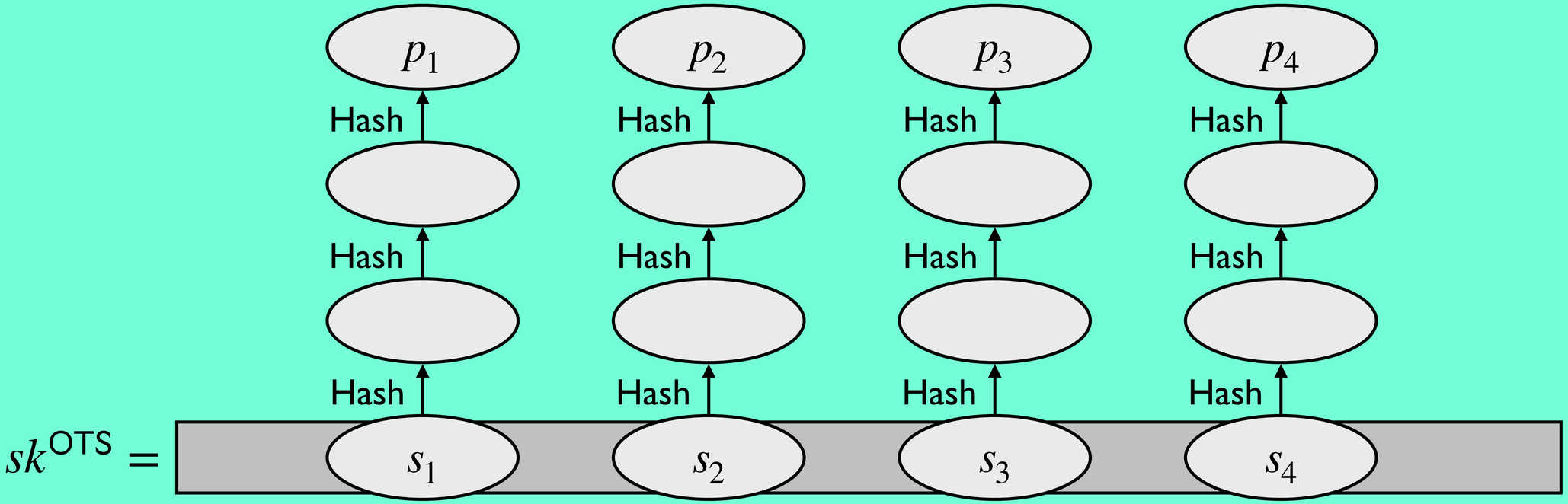

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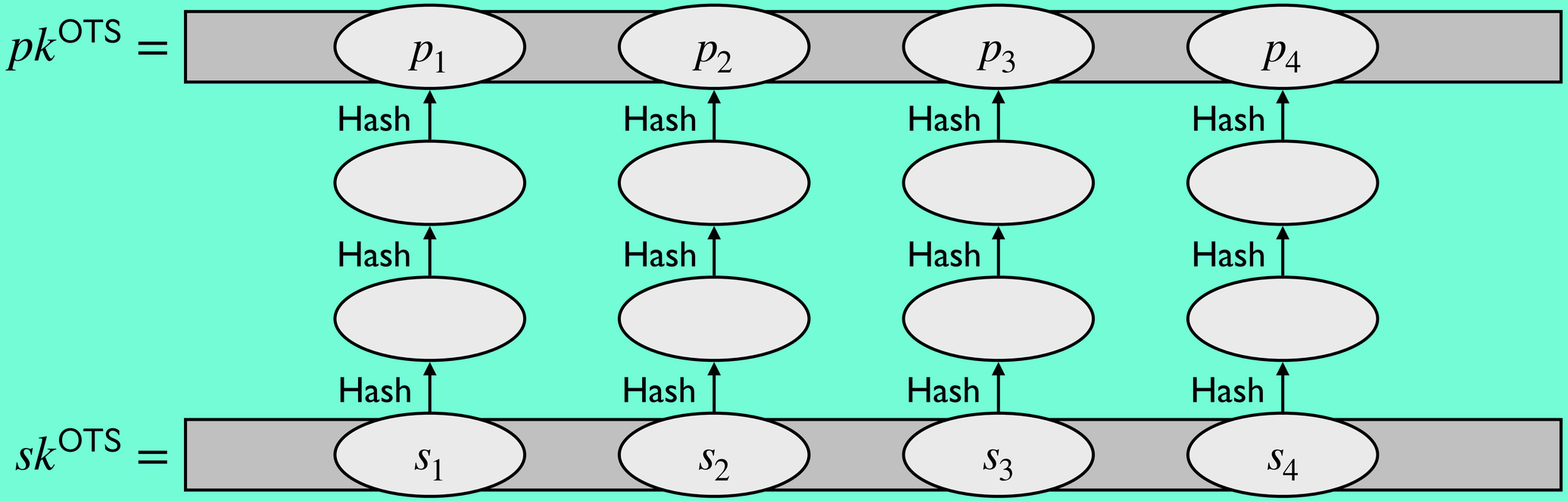

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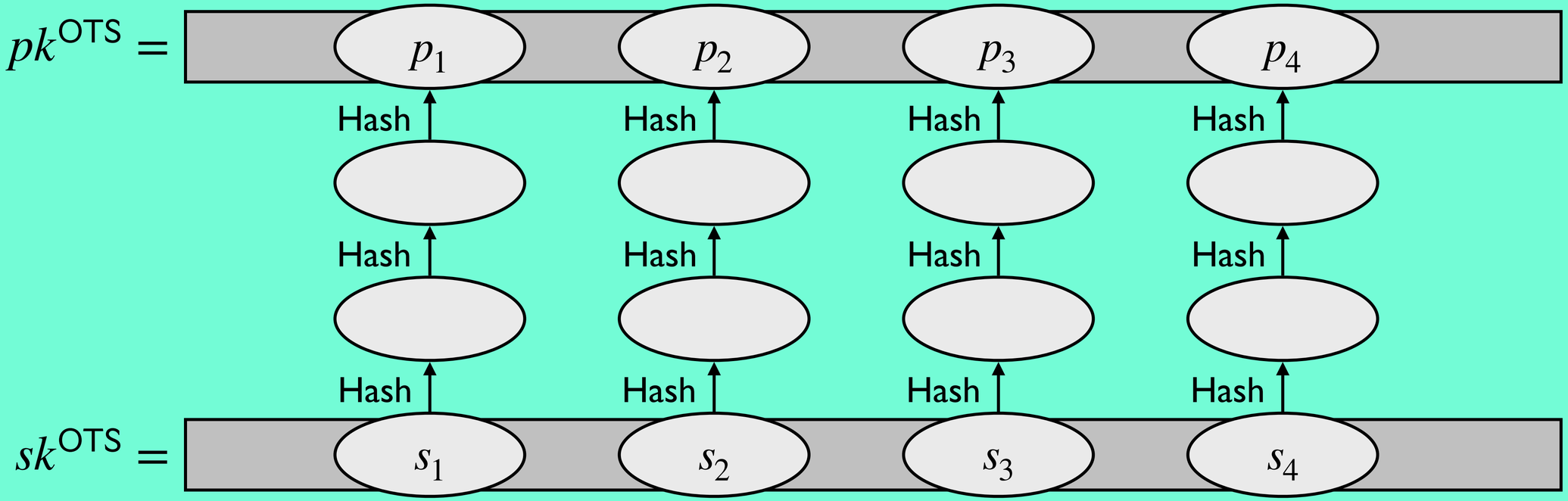

Message m →

Randomness ρ →

Encoding → $(x_1, \ldots, x_v)$ Positions in Chains

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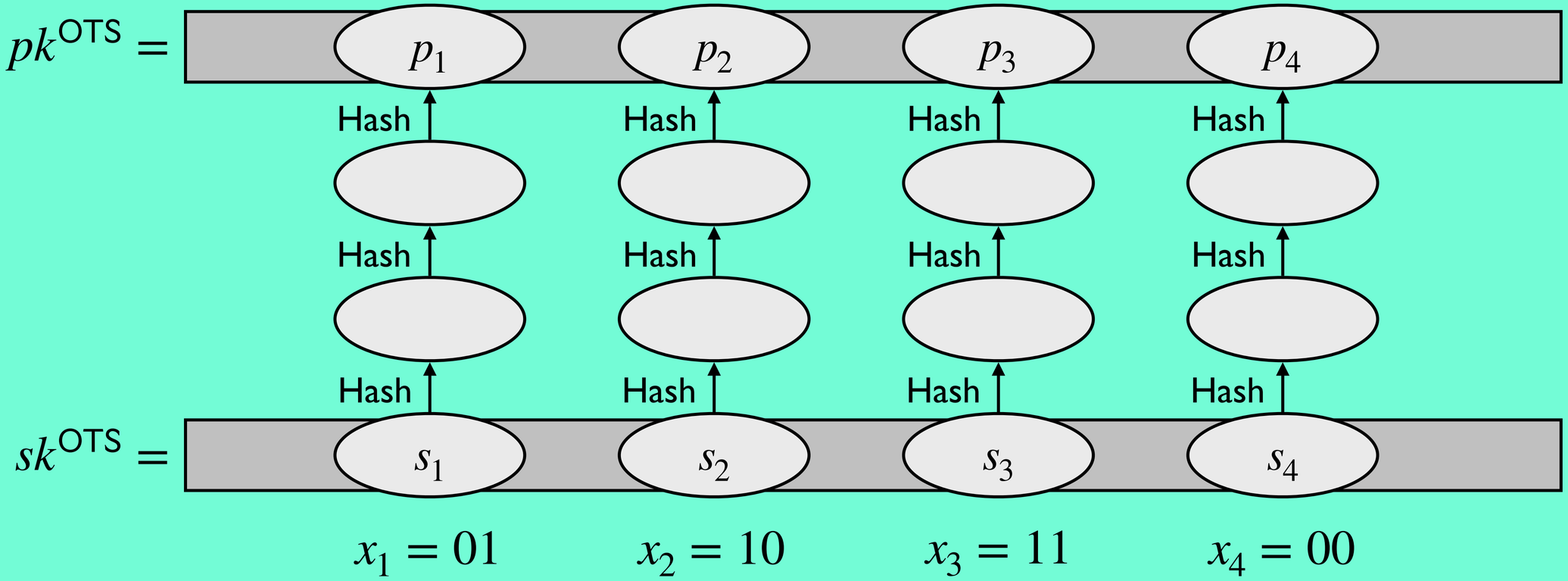

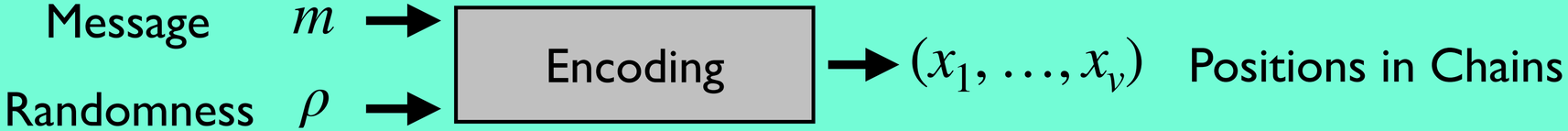

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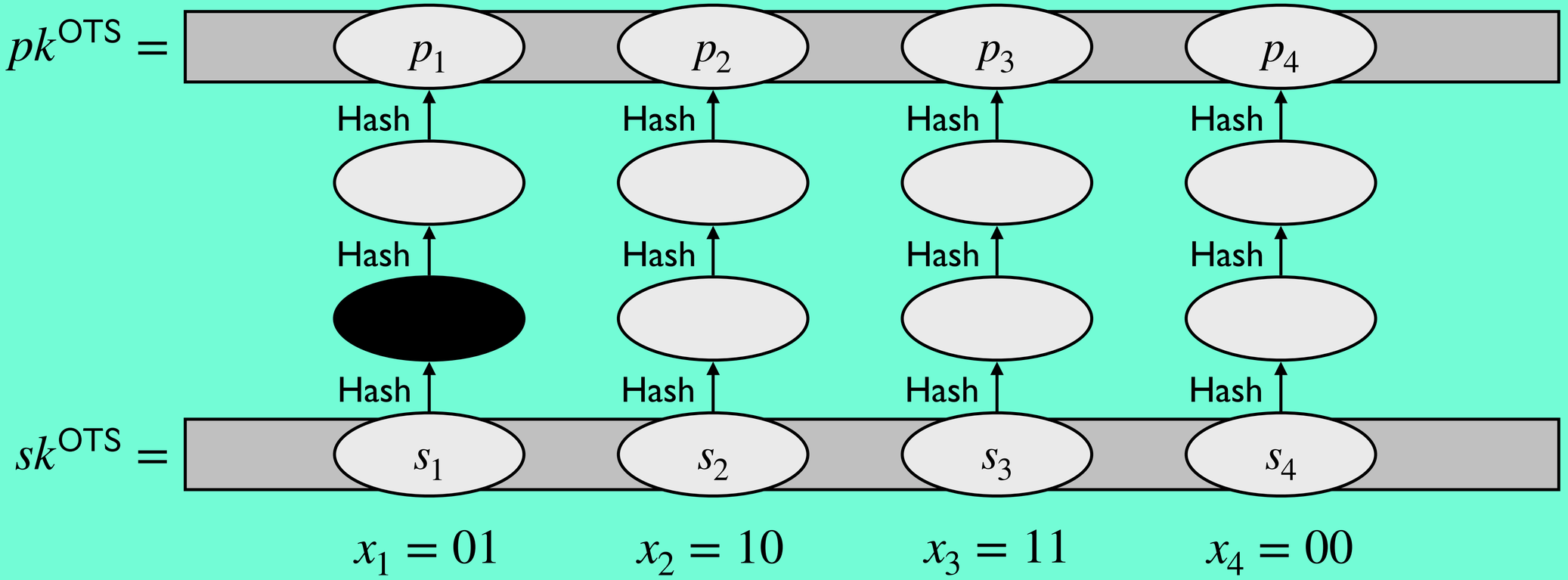

Part of Sig

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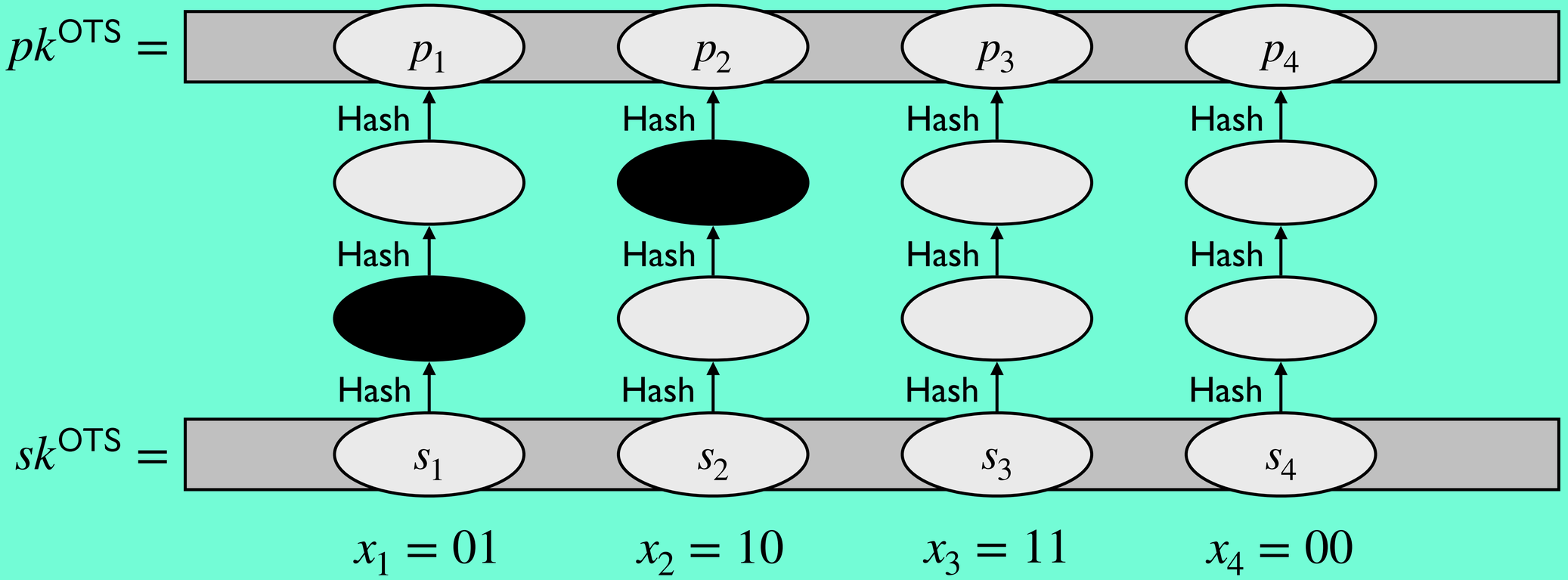

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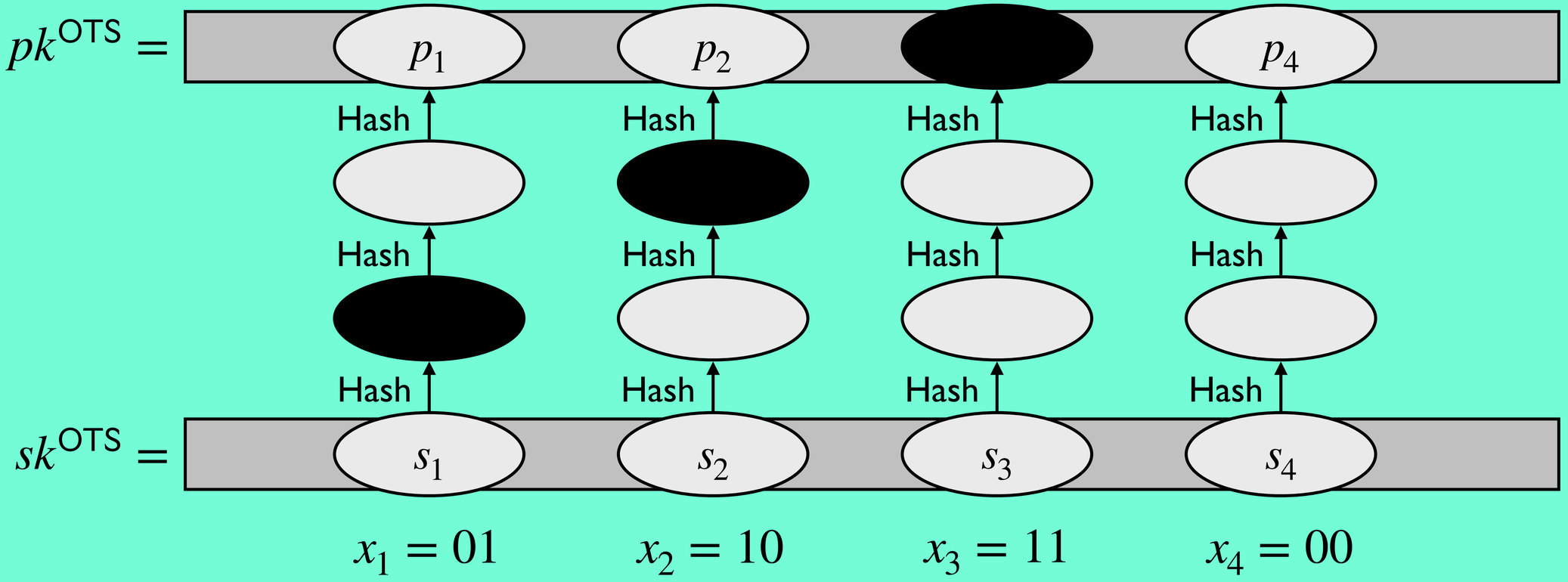

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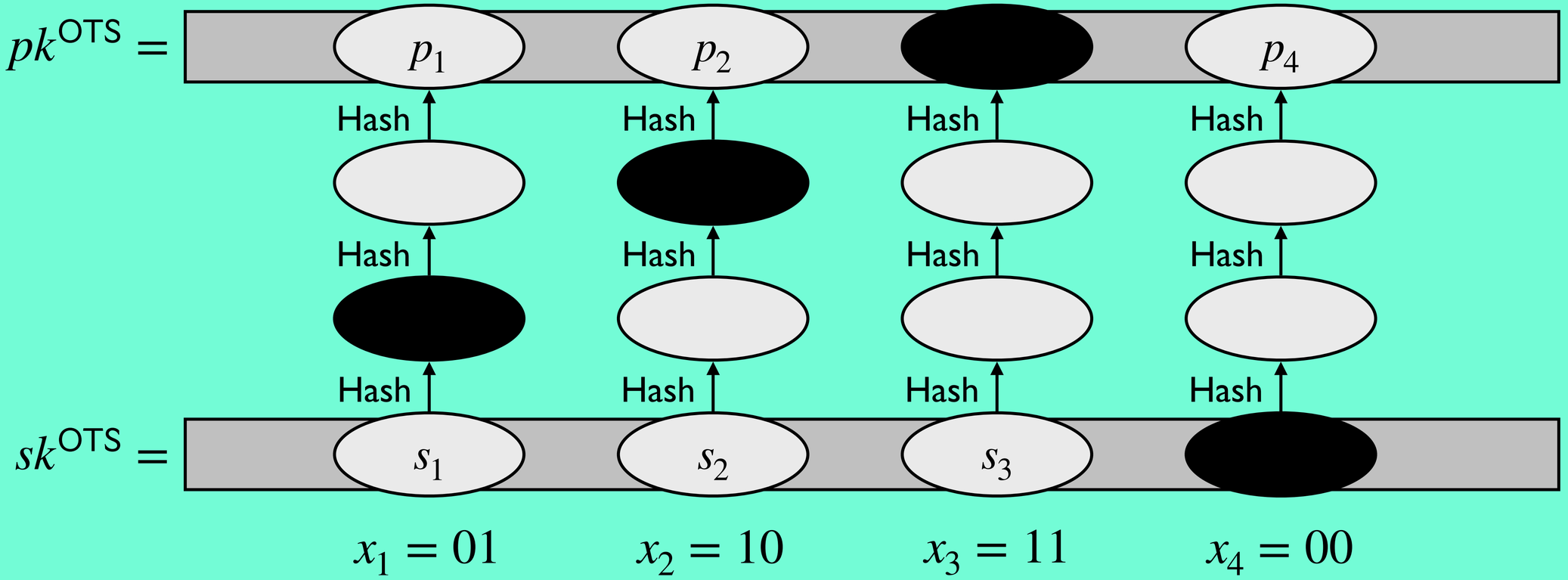

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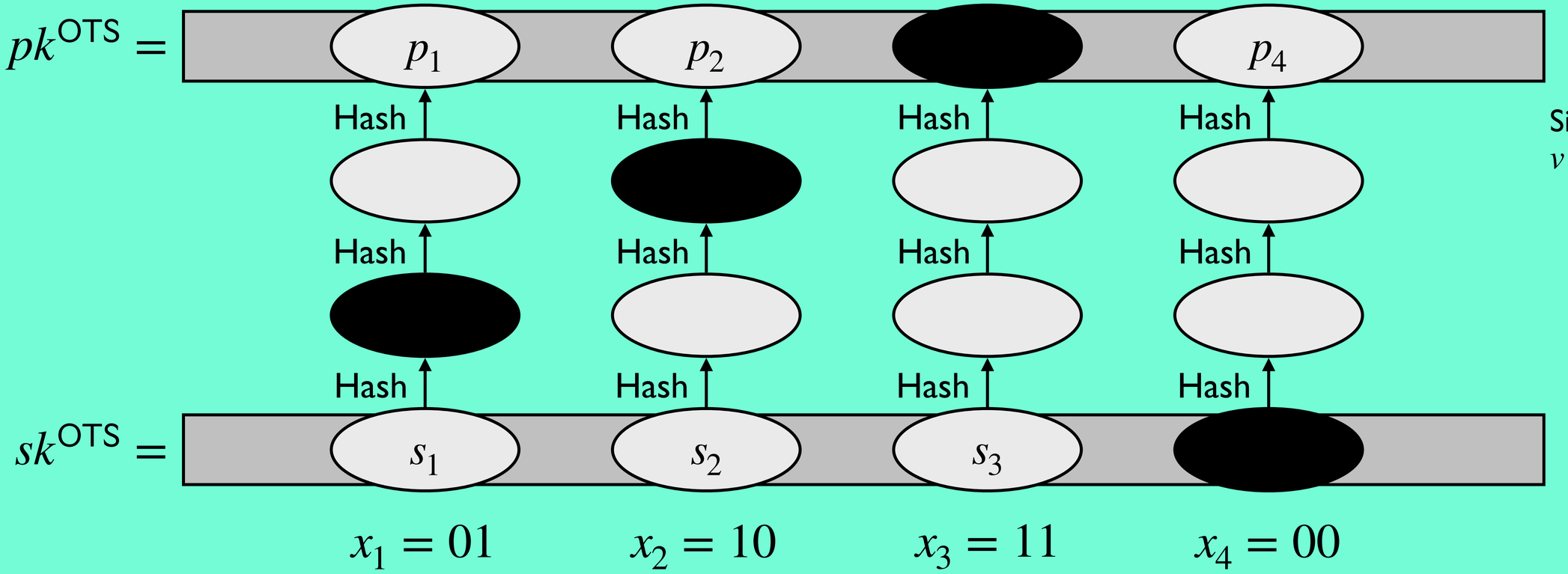

Part of Sig

Sig Size: $v \cdot \text{hash}$.
$v \cdot w \approx 128$

Message m →
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One-Time Signatures via Hash Chains

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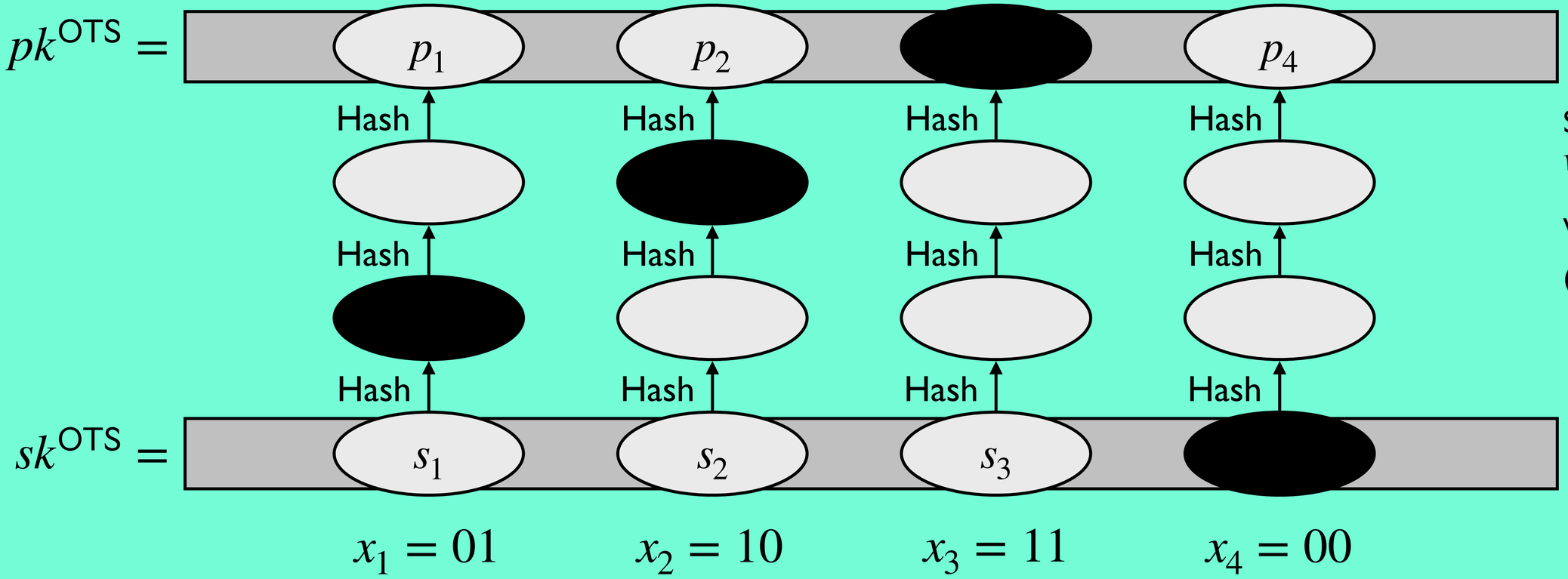
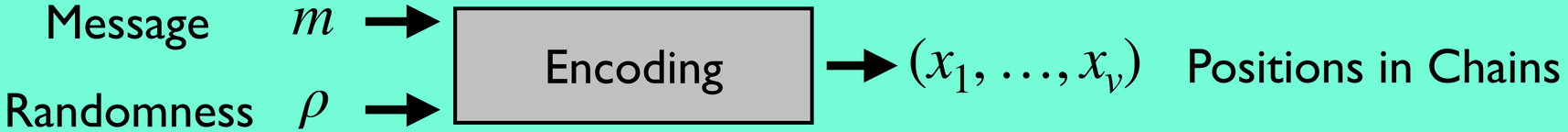


Part of Sig

Sig Size: $v \cdot$ hash.
$v \cdot w \approx 128$

Work: $v \cdot 2^w$
(Sign + Verify)



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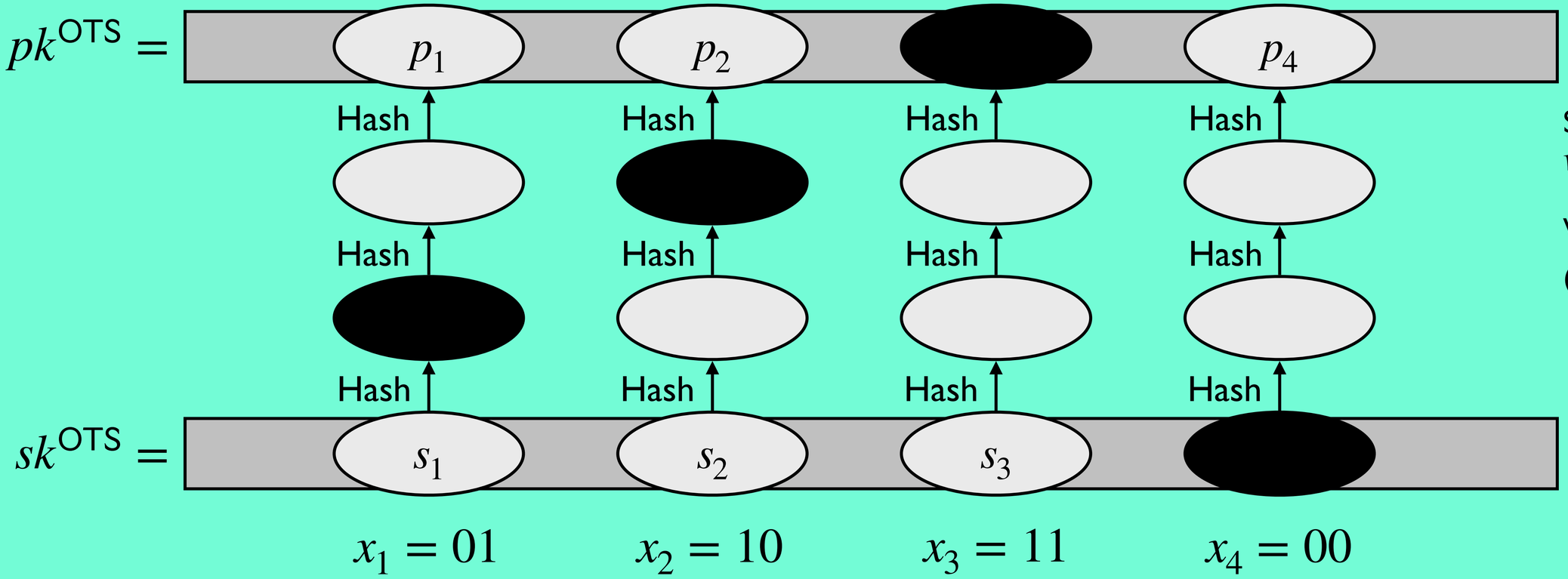

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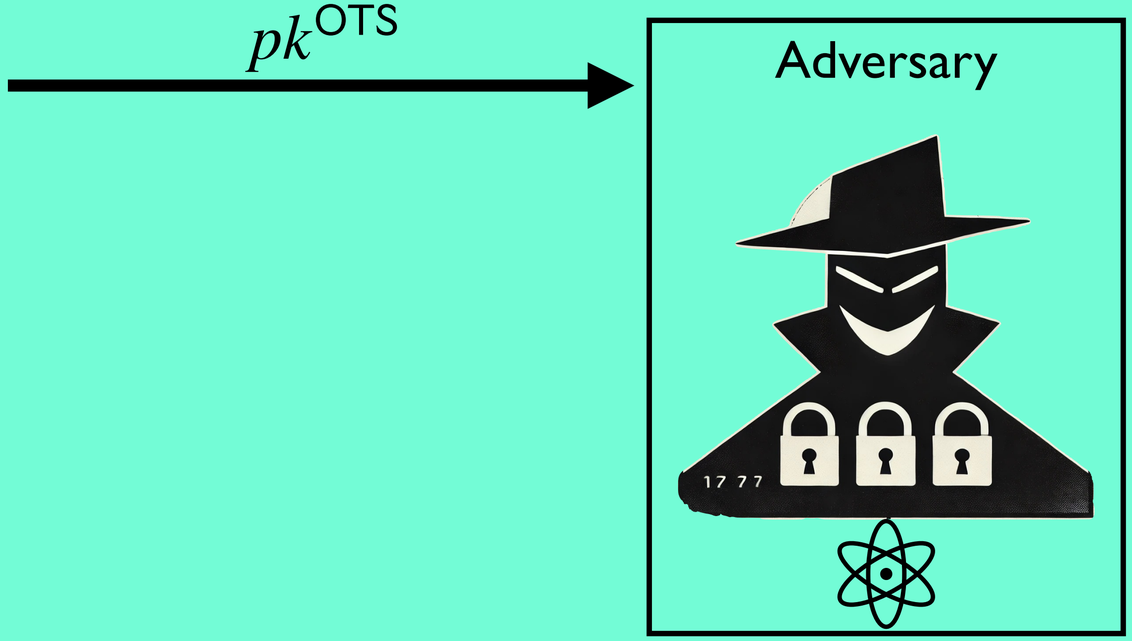
Need incomparability

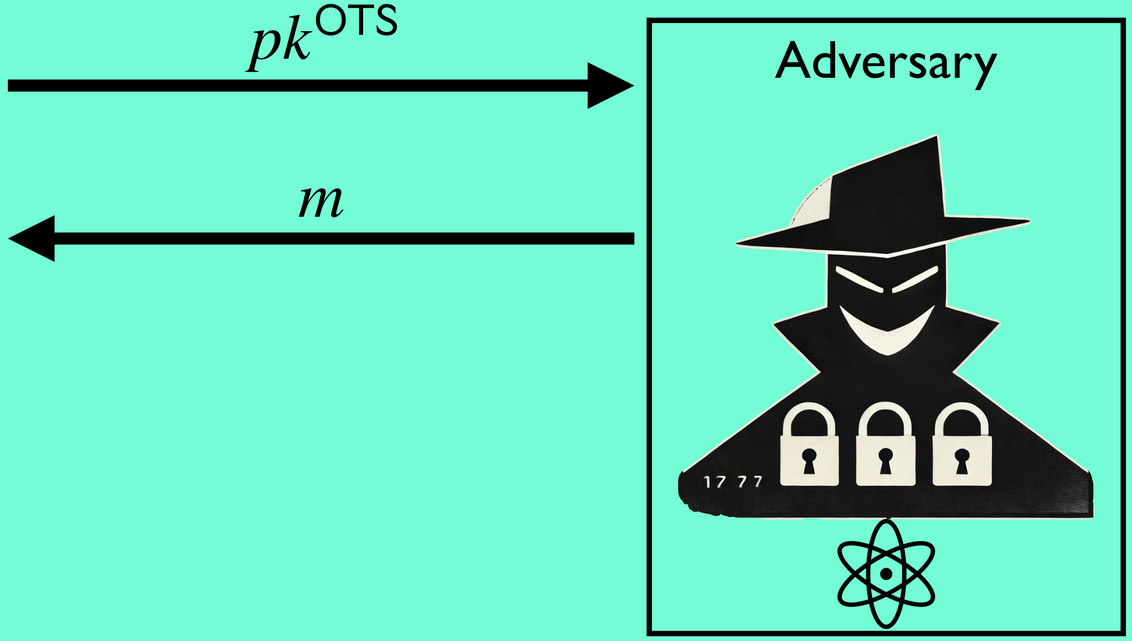
One-Time Signatures via Hash Chains

One-Time Signatures via Hash Chains

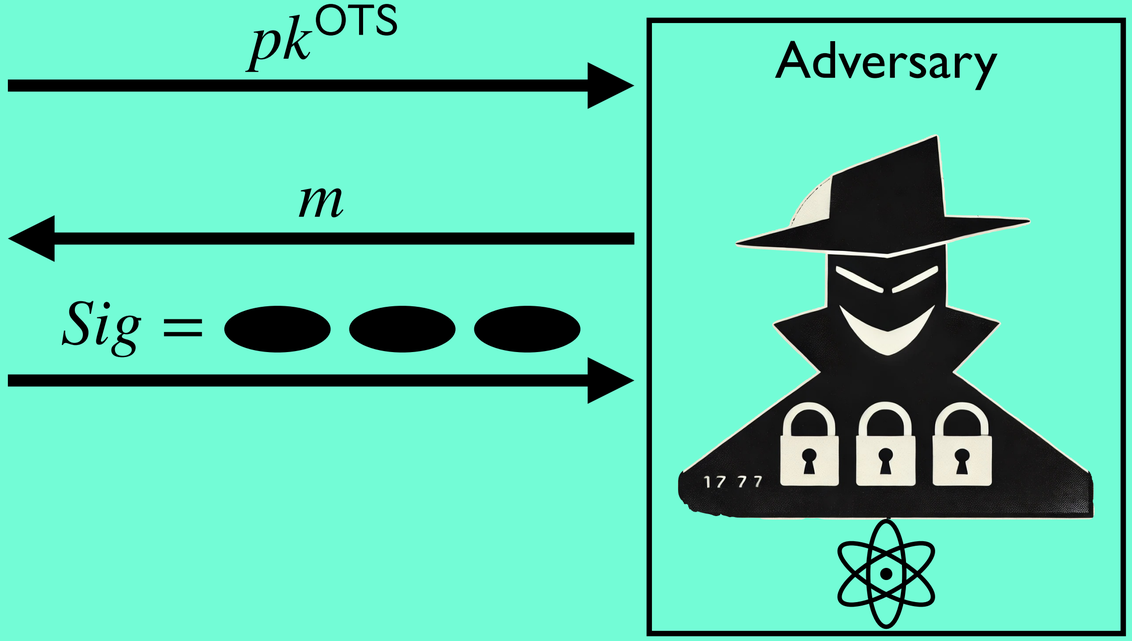
One-Time Signatures via Hash Chains



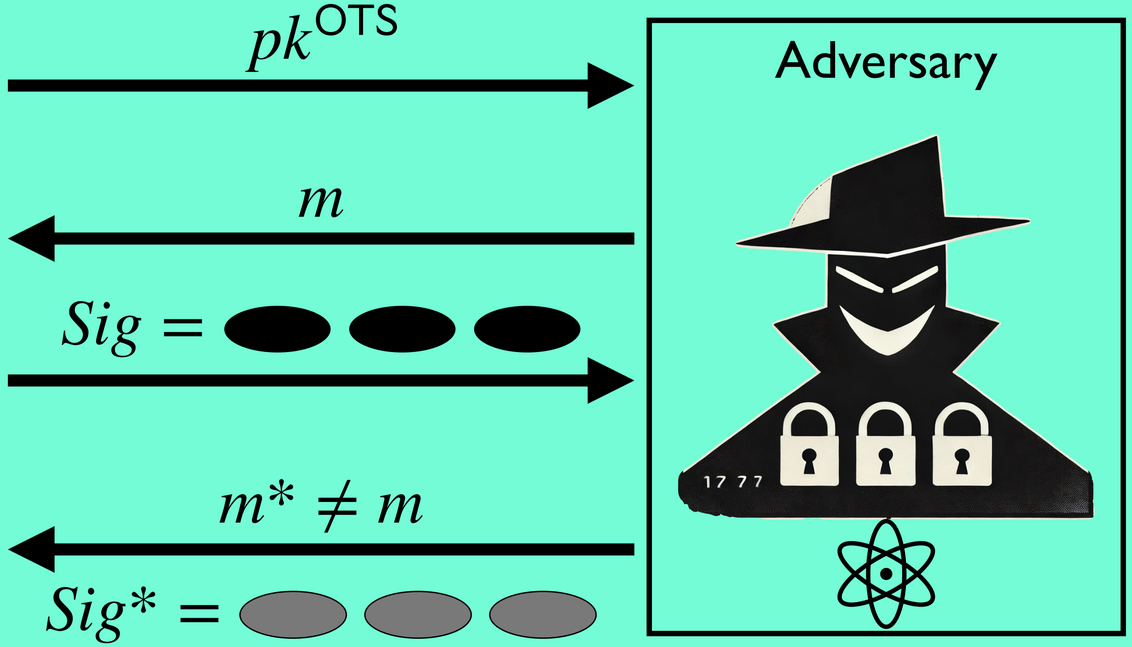
One-Time Signatures via Hash Chains



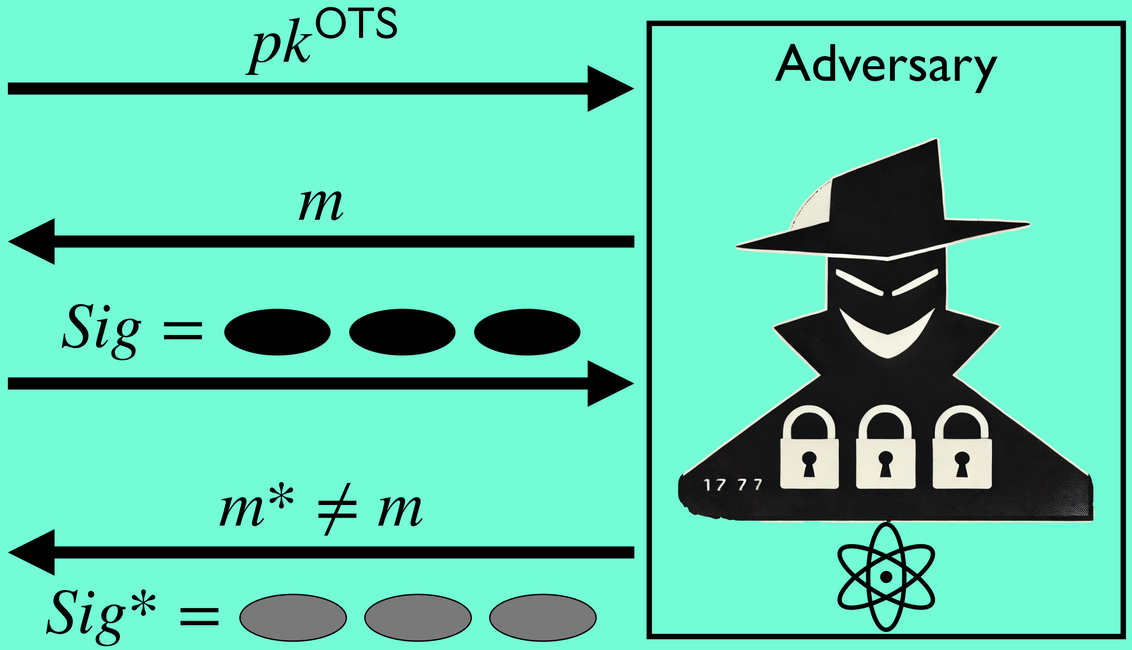
One-Time Signatures via Hash Chains

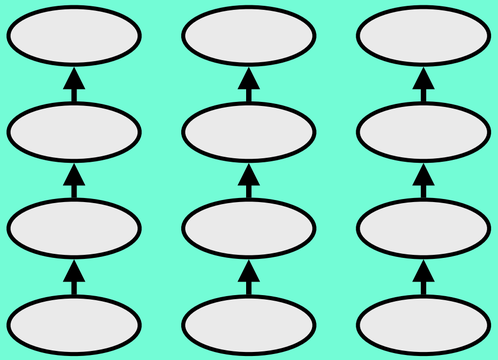


One-Time Signatures via Hash Chains



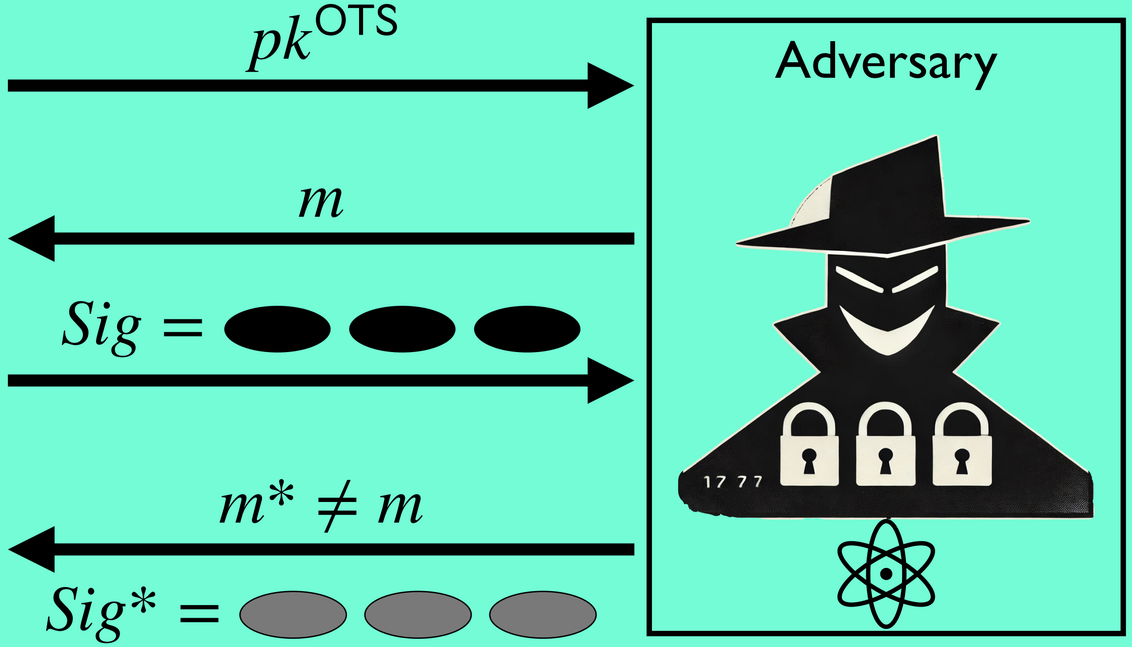
One-Time Signatures via Hash Chains

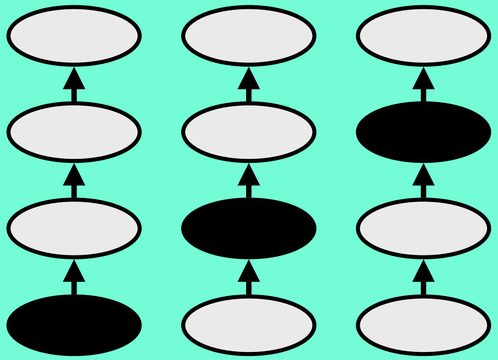




Bad Case

One-Time Signatures via Hash Chains





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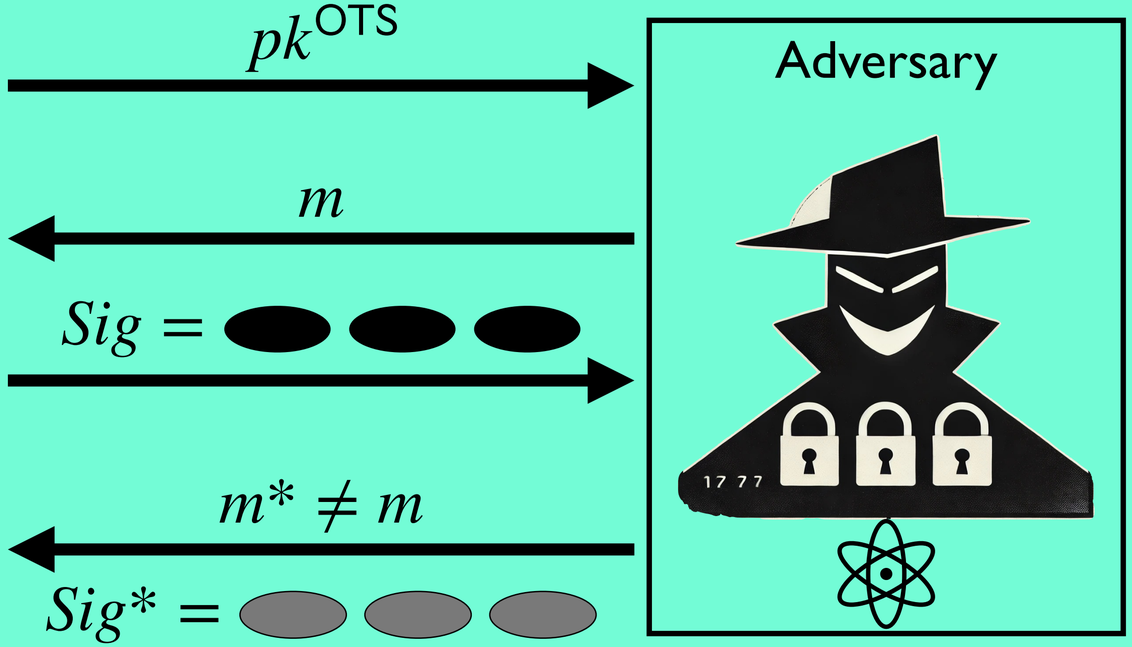

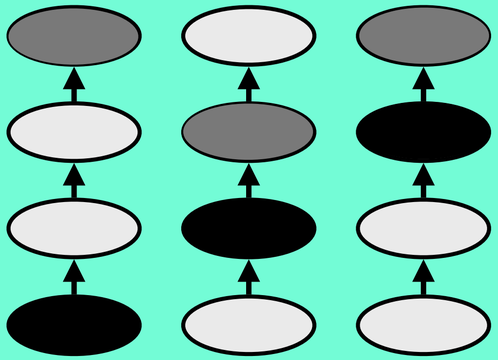

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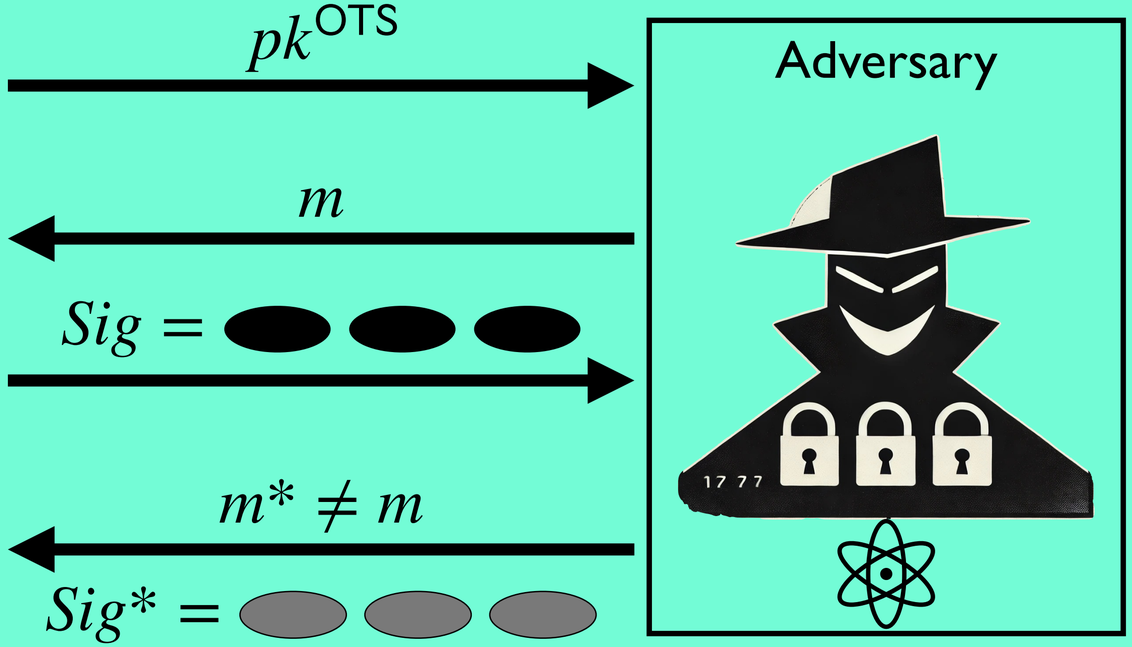

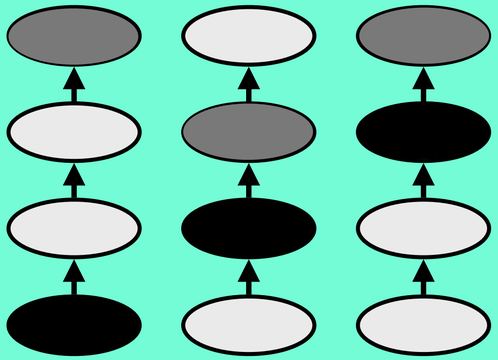

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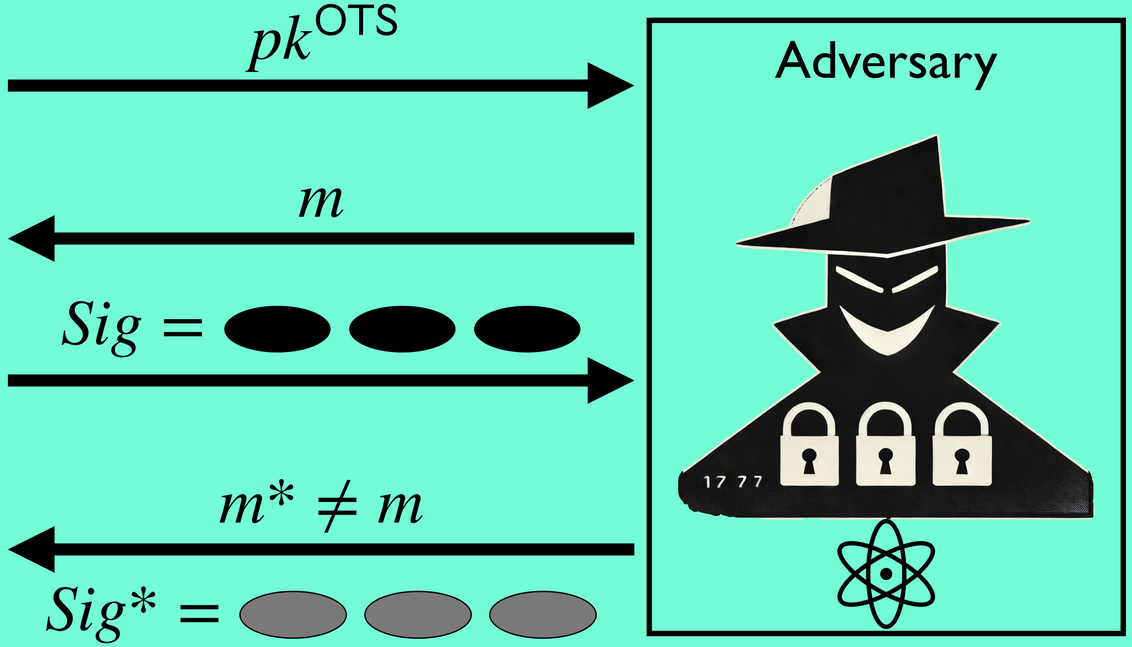
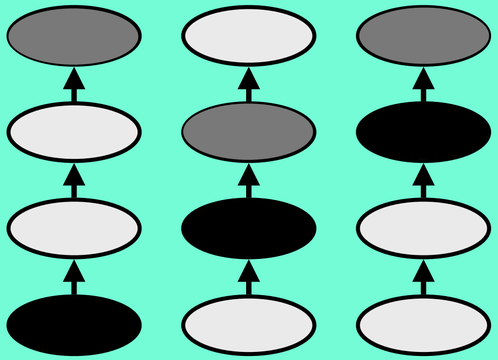
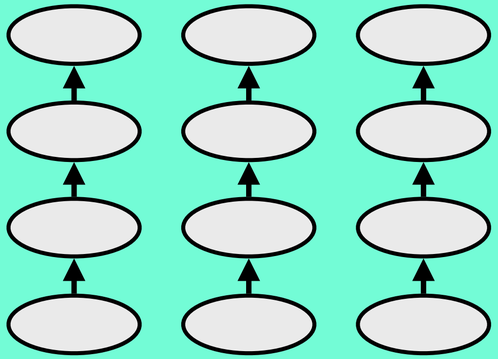




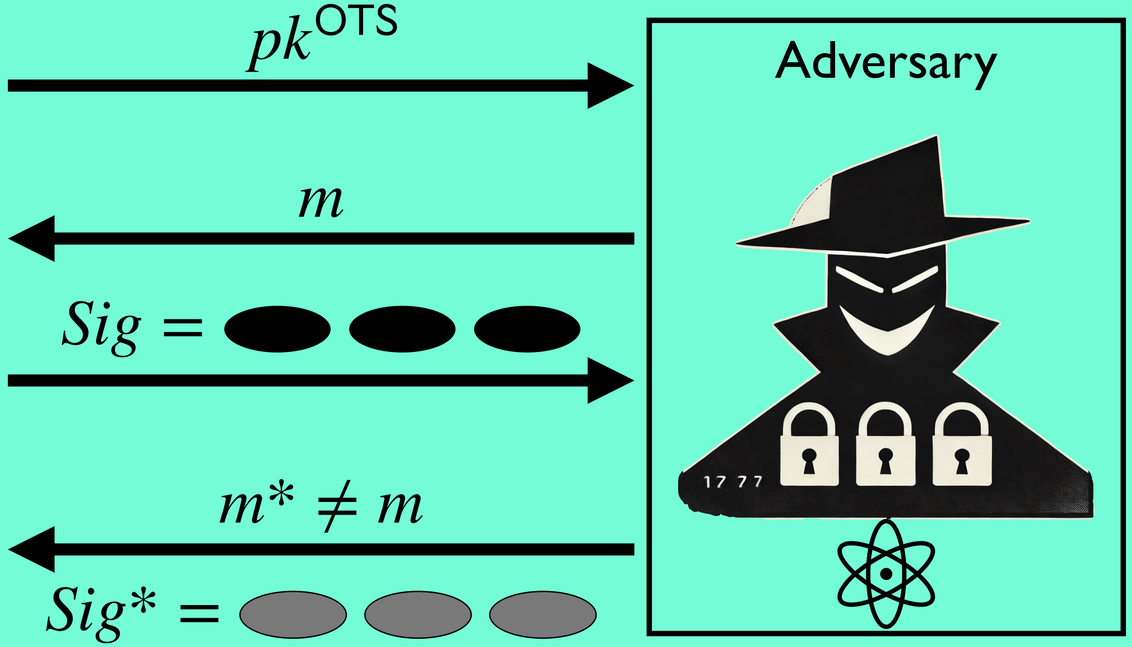
Bad Case

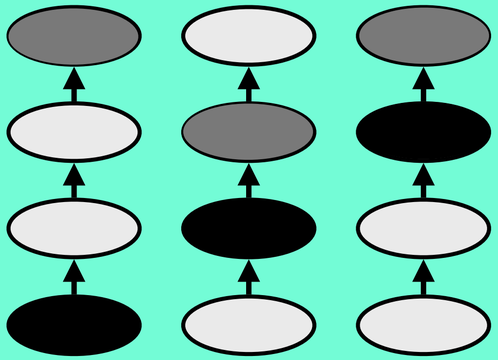
$\geq$



Good Case

One-Time Signatures via Hash Chains





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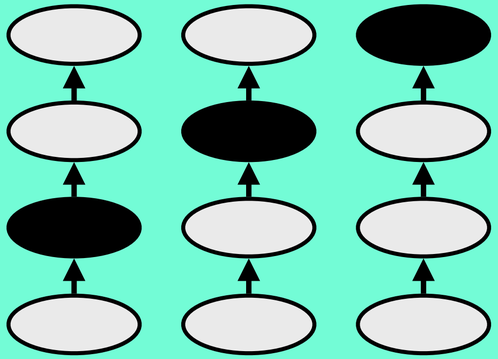

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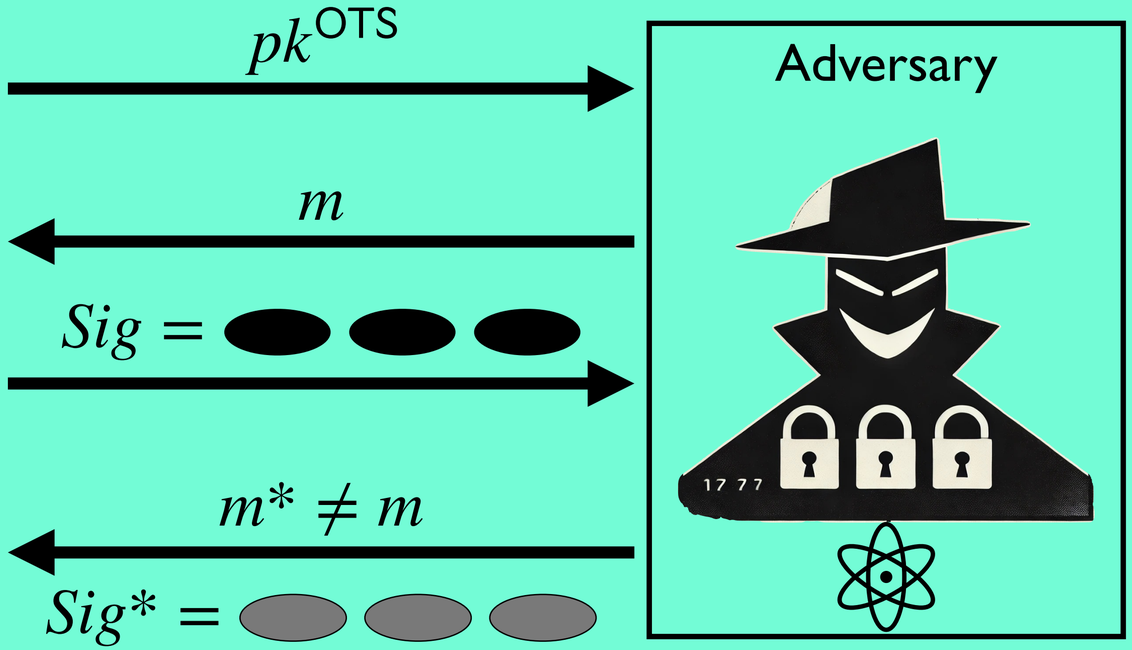
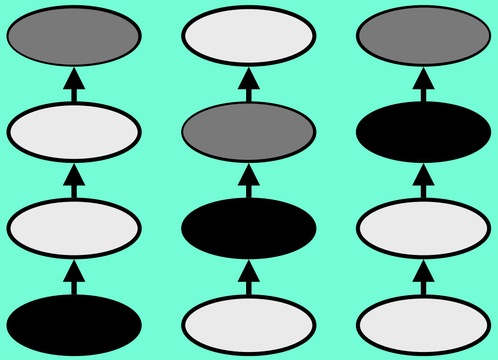
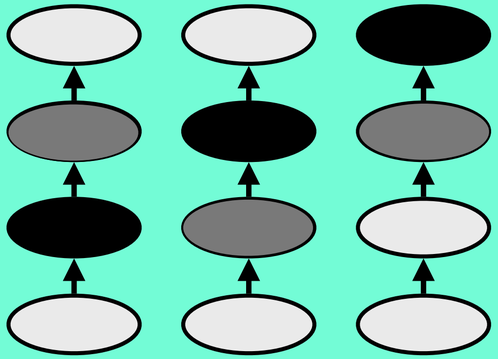


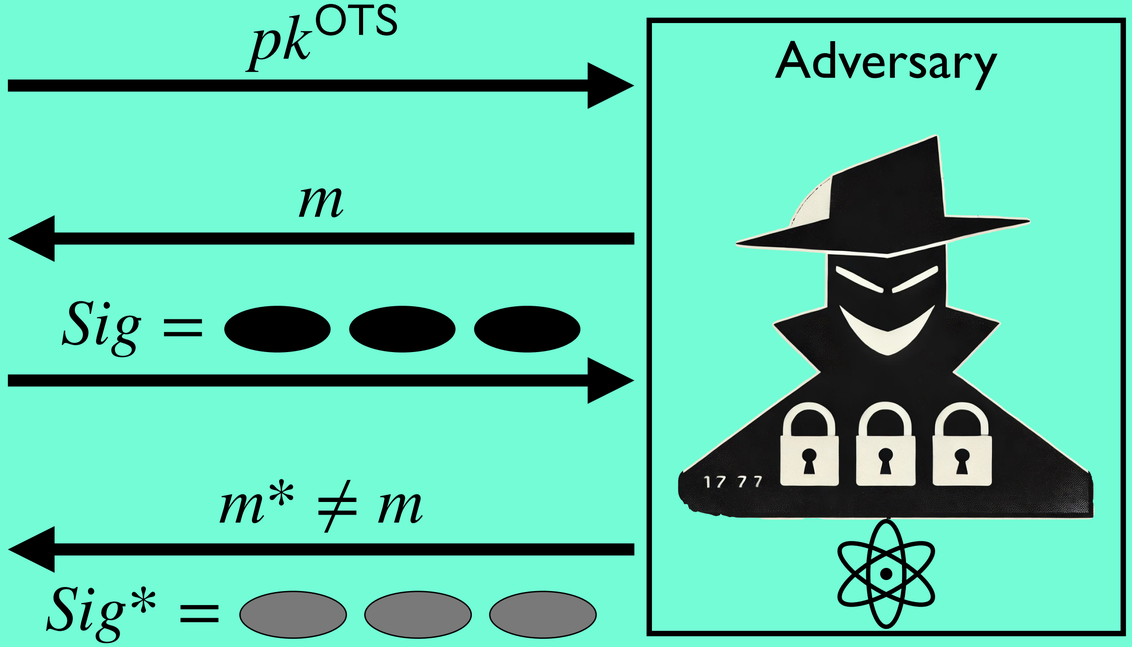


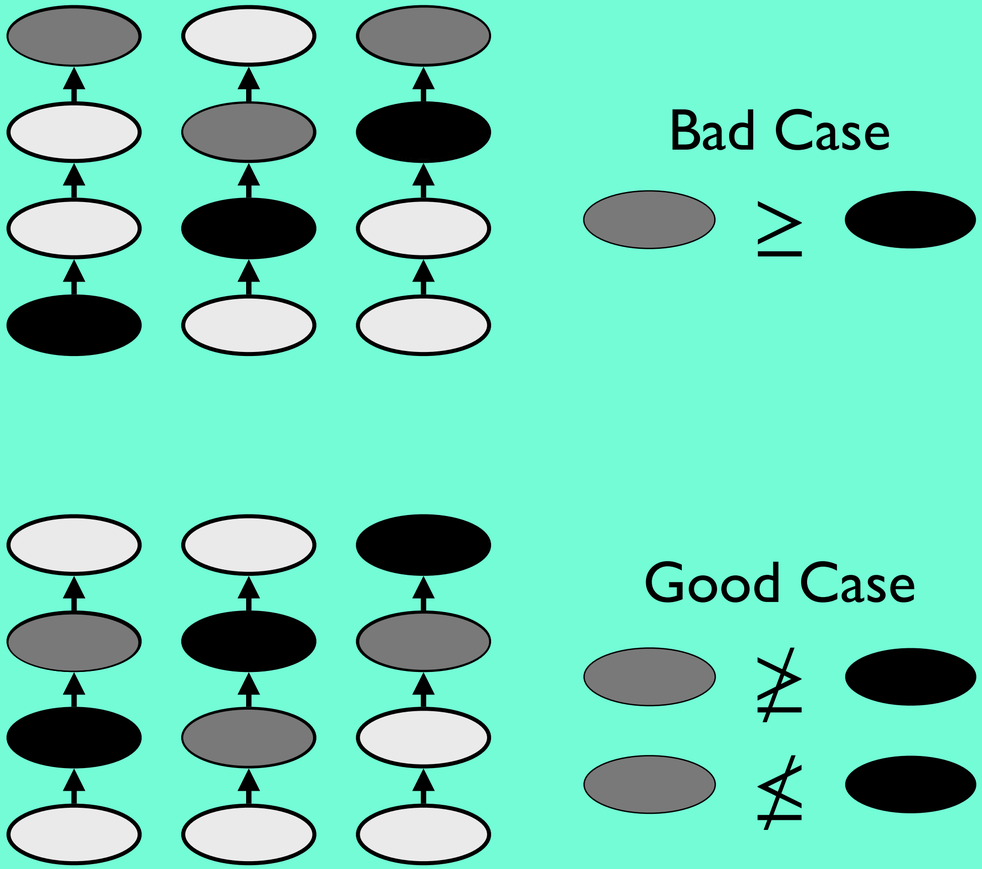
Bad Case



Good Case

One-Time Signatures via Hash Chains





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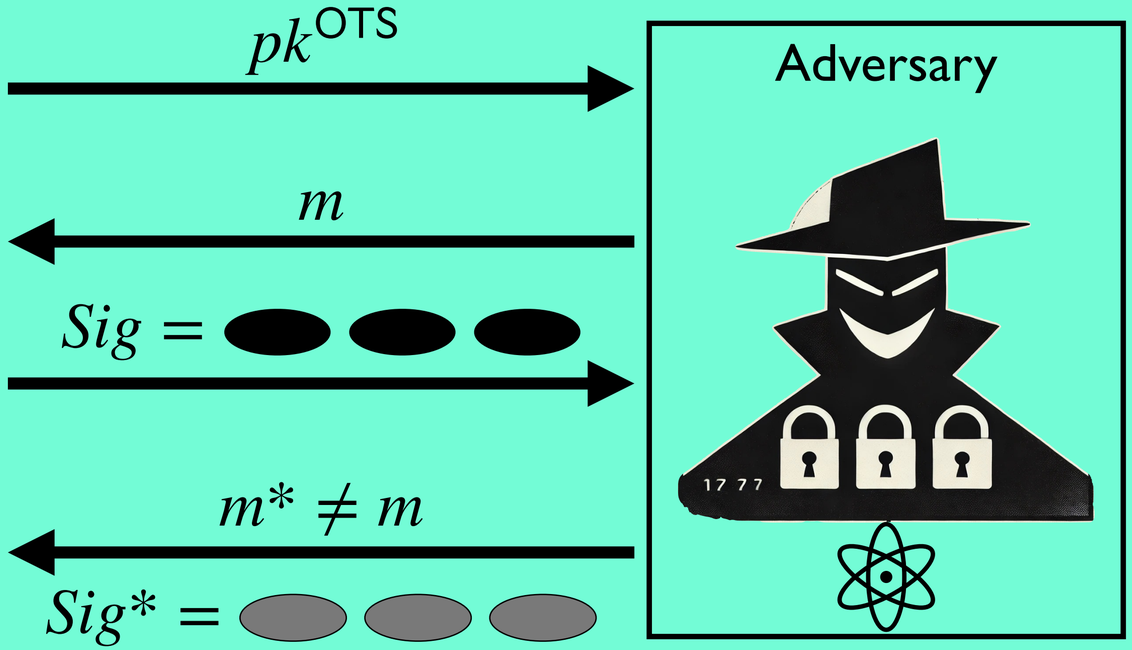
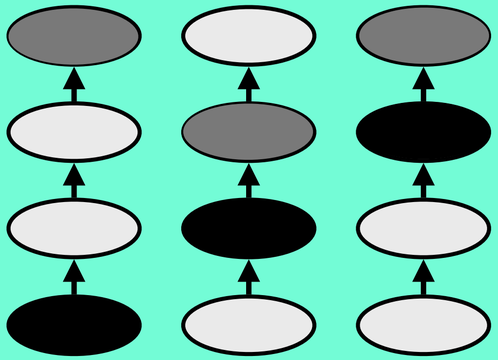
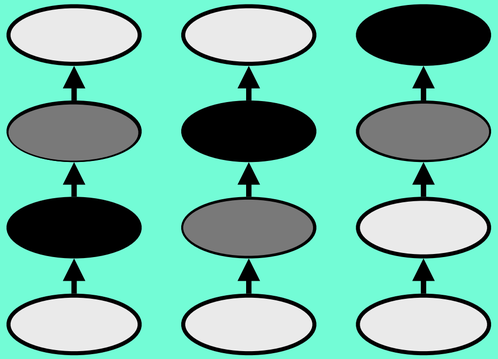




Bad Case

$\geq$



Good Case

$\not\geq$

$\not\leq$

Incomparable Encoding: Only Good Case

The XMSS Approach

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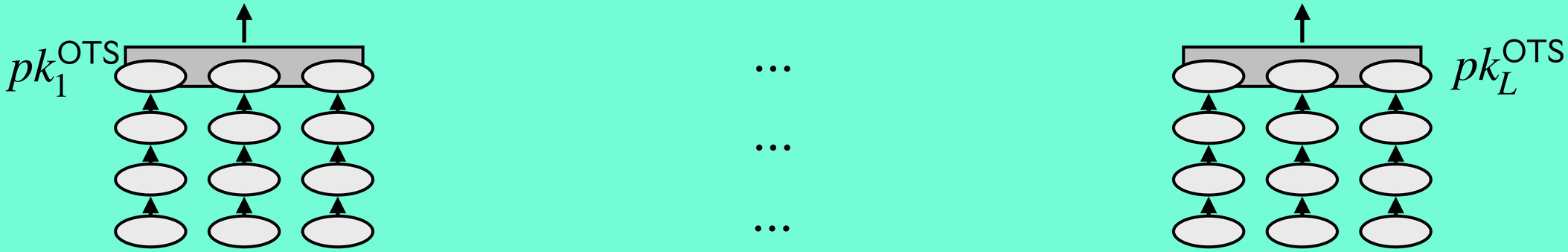
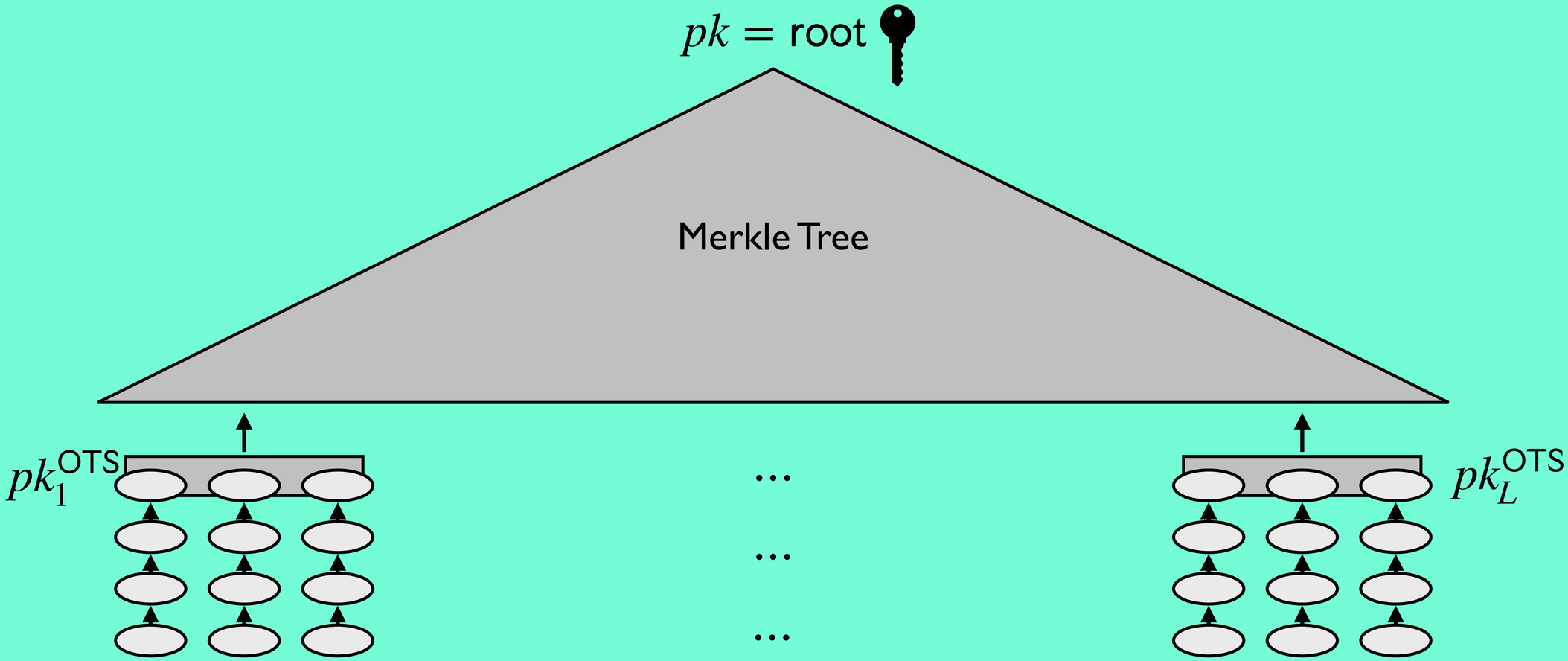
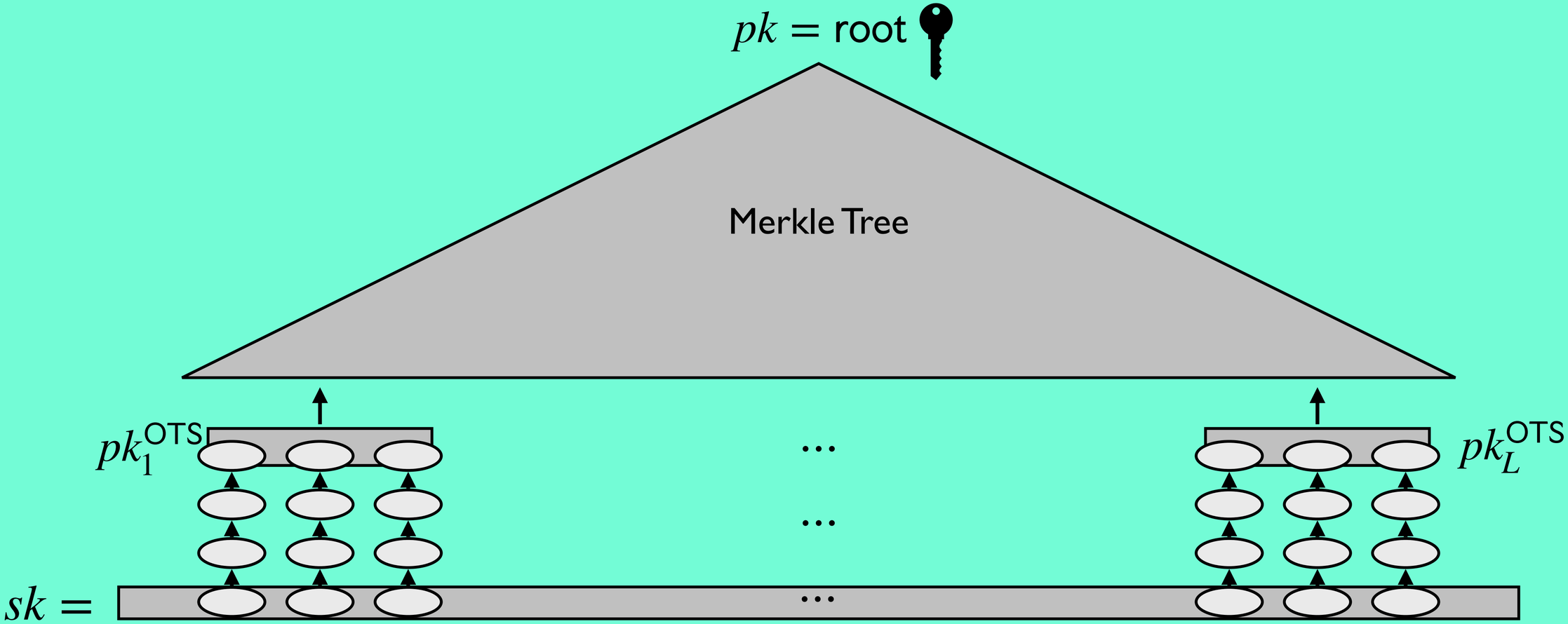


The XMSS Approach



The XMSS Approach



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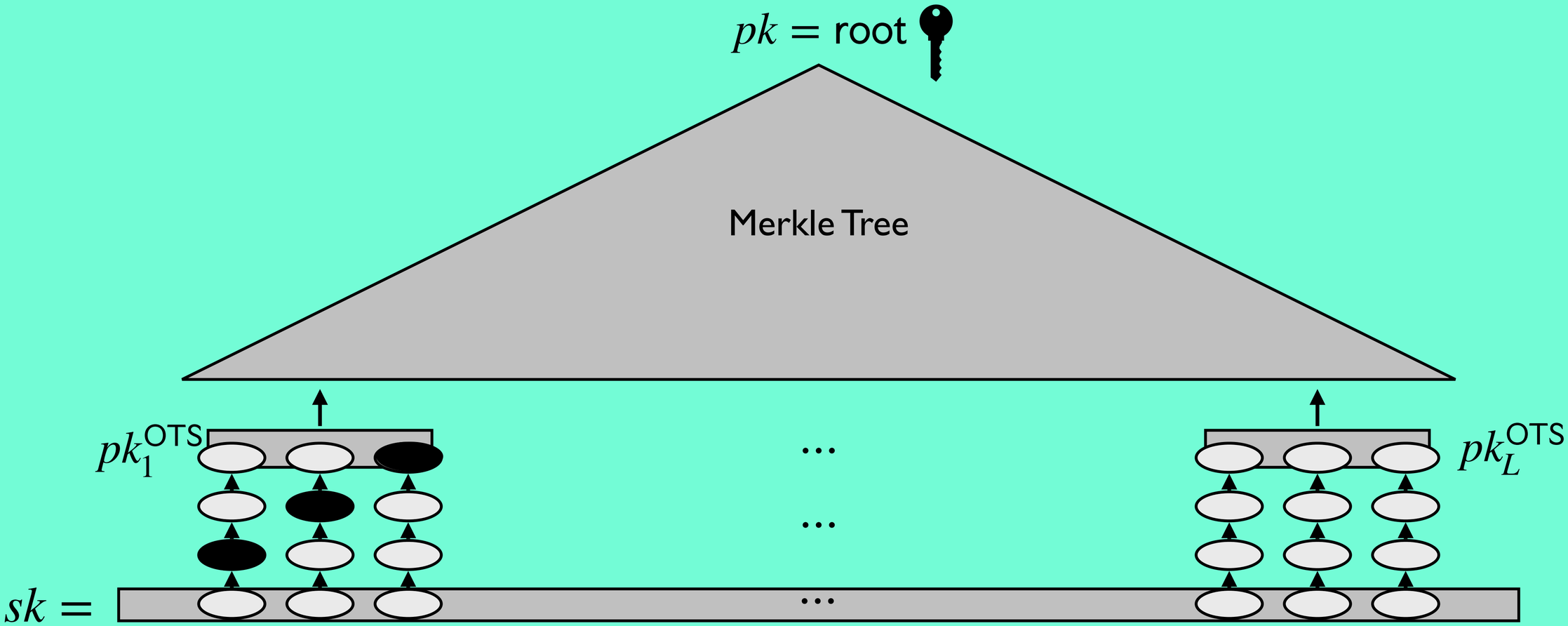

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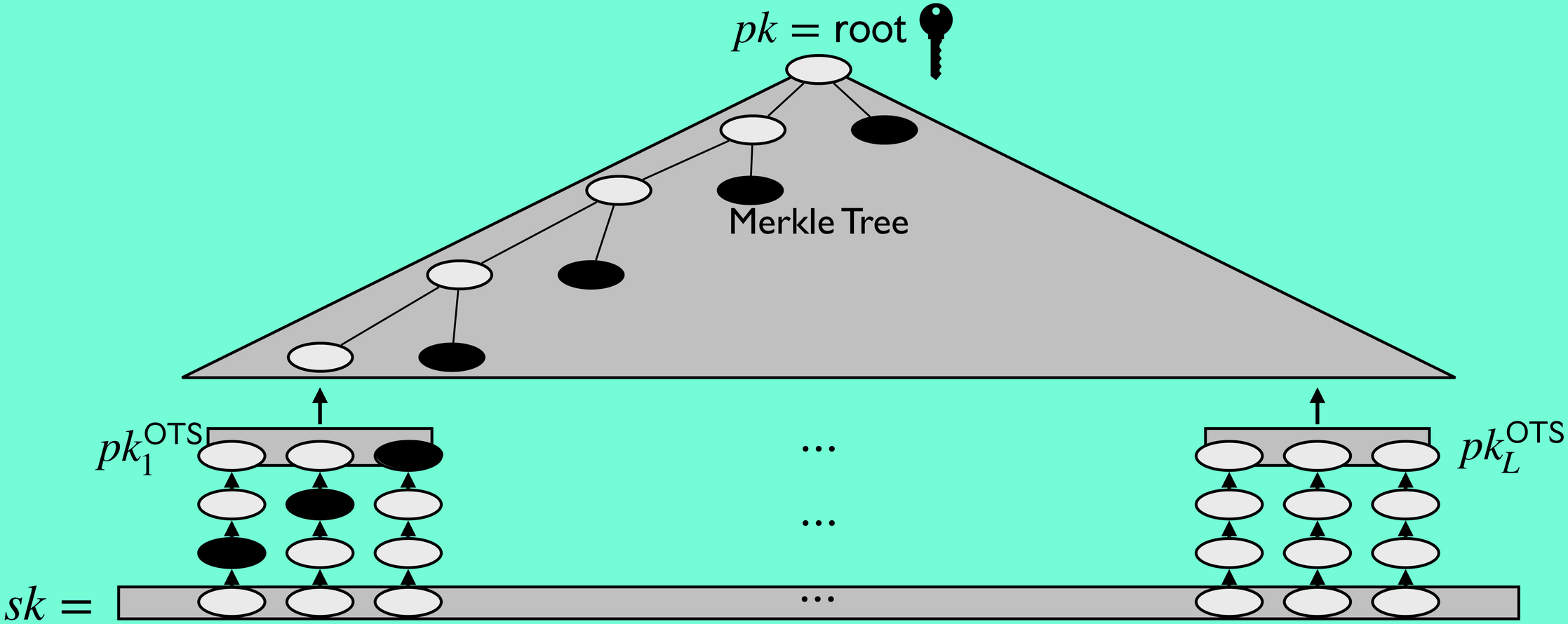
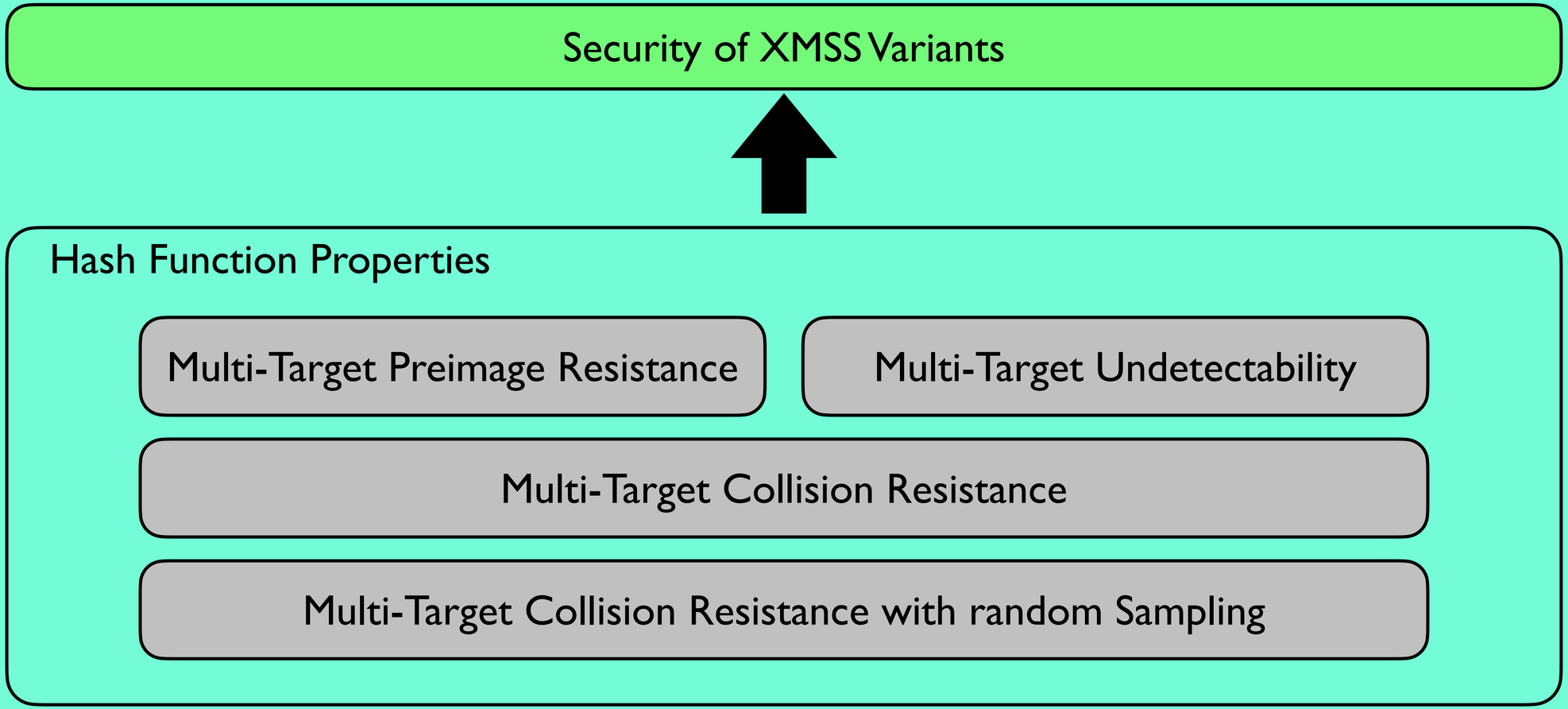


Security Properties for Hash-Functions

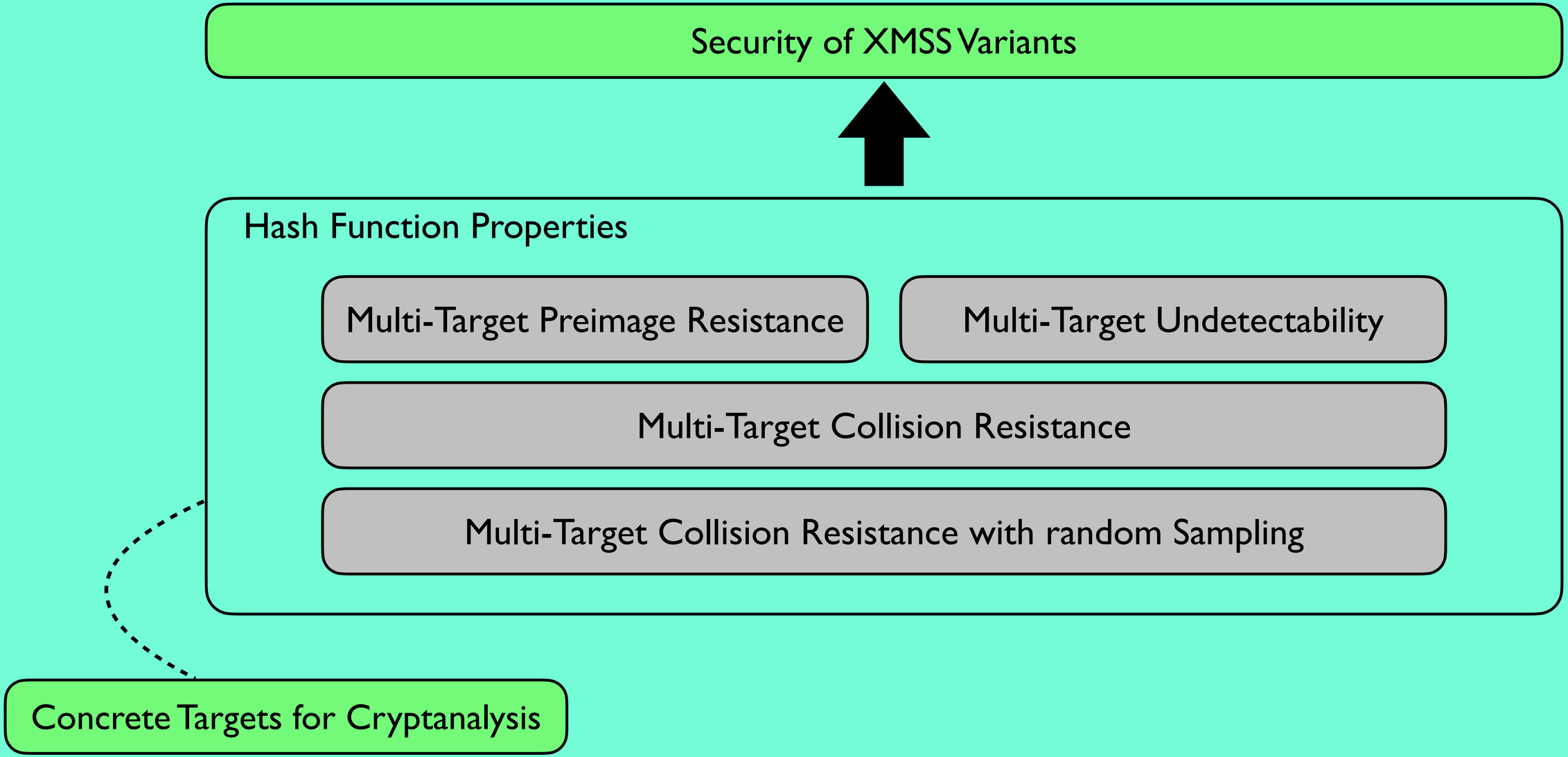
Security Properties for Hash-Functions

Security of XMSS Variants

Security Properties for Hash-Functions



Security Properties for Hash-Functions



Efficiency

	Encoding	Parameters	Gen [s]	Sign [μs]	Ver [μs]	Sig [KiB]	π_{16} AC	π_{24} AC	π_{16} WC	π_{24} WC
Lifetime $L = 2^{18}$	W	$w = 1$	179.01	362.59	416.54	4.97	81	97	158	97
	W	$w = 2$	168.19	350.04	408.67	2.75	122	59	237	59
	W	$w = 4$	330.52	638.08	769.41	1.66	325	41	615	41
	W	$w = 8$	2717.28	4820	5820	1.11	2917	31	5355	31
	TSW	$w = 1, \delta = 1$	172.67	541.45	396.56	4.75	77	93	77	93
	TSW	$w = 1, \delta = 1.1$	172.29	898.22	376.62	4.75	69	93	69	93
	TSW	$w = 2, \delta = 1$	166.51	530.83	372.93	2.65	117	57	117	57
	TSW	$w = 2, \delta = 1.1$	166.22	888.55	351.37	2.65	105	57	105	57
	TSW	$w = 4, \delta = 1$	312.49	1090.00	650.82	1.58	292	39	292	39
	TSW	$w = 4, \delta = 1.1$	312.64	1670.00	602.75	1.58	263	39	263	39
	TSW	$w = 8, \delta = 1$	2501.01	9760.00	4900.00	1.06	2550	30	2550	30
	TSW	$w = 8, \delta = 1.1$	2499.97	14570.00	4320.00	1.06	2295	30	2295	30
Lifetime $L = 2^{20}$	W	$w = 1$	780.89	362.44	418.31	5.03	82	99	158	99
	W	$w = 2$	705.42	336.30	400.60	2.81	122	61	237	61
	W	$w = 4$	1353.18	617.48	746.28	1.72	326	43	615	43
	W	$w = 8$	11122.95	4981.20	6039.40	1.34	2917	35	5355	35
	TSW	$w = 1, \delta = 1$	752.57	520.42	401.32	4.81	77	95	77	95
	TSW	$w = 1, \delta = 1.1$	731.79	844.01	381.23	4.81	69	95	69	95
	TSW	$w = 2, \delta = 1$	667.76	527.17	379.56	2.7	117	59	117	59
	TSW	$w = 2, \delta = 1.1$	668.14	853.66	354.09	2.7	105	59	105	59
	TSW	$w = 4, \delta = 1$	1249.52	1057.40	661.61	1.64	292	41	292	41
	TSW	$w = 4, \delta = 1.1$	1248.35	1600.00	603.65	1.64	263	41	263	41
	TSW	$w = 8, \delta = 1$	9972.32	9509.50	4870.60	1.27	2550	34	2550	34
	TSW	$w = 8, \delta = 1.1$	9927.97	14271.00	4358.60	1.27	2295	34	2295	34

* Assuming Poseidon2 is used, 128-bit classical security

Efficiency

	Encoding	Parameters	Gen [s]	Sign [μs]	Ver [μs]	Sig [KiB]	π_{16} AC	π_{24} AC	π_{16} WC	π_{24} WC
Lifetime $L = 2^{18}$	W	$w = 1$	179.01	362.59	416.54	4.97	81	97	158	97
	W	$w = 2$	168.19	350.04	408.67	2.75	122	59	237	59
	W	$w = 4$	330.52	638.08	769.41	1.66	325	41	615	41
	W	$w = 8$	2717.28	4820	5820	1.11	2917	31	5355	31
	TSW	$w = 1, \delta = 1$	172.67	541.45	396.56	4.75	77	93	77	93
	TSW	$w = 1, \delta = 1.1$	172.29	898.22	376.62	4.75	69	93	69	93
	TSW	$w = 2, \delta = 1$	166.51	530.83	372.93	2.65	117	57	117	57
	TSW	$w = 2, \delta = 1.1$	166.22	888.55	351.37	2.65	105	57	105	57
	TSW	$w = 4, \delta = 1$	312.49	1090.00	650.82	1.58	292	39	292	39
	TSW	$w = 4, \delta = 1.1$	312.64	1670.00	602.75	1.58	263	39	263	39
	TSW	$w = 8, \delta = 1$	2501.01	9760.00	4900.00	1.06	2550	30	2550	30
	TSW	$w = 8, \delta = 1.1$	2499.97	14570.00	4320.00	1.06	2295	30	2295	30
Lifetime $L = 2^{20}$	W	$w = 1$	780.89	362.44	418.31	5.03	82	99	158	99
	W	$w = 2$	705.42	336.30	400.60	2.81	122	61	237	61
	W	$w = 4$	1353.18	617.48	746.28	1.72	326	43	615	43
	W	$w = 8$	11122.95	4981.20	6039.40	1.34	2917	35	5355	35
	TSW	$w = 1, \delta = 1$	752.57	520.42	401.32	4.81	77	95	77	95
	TSW	$w = 1, \delta = 1.1$	731.79	844.01	381.23	4.81	69	95	69	95
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	TSW	$w = 8, \delta = 1.1$	9927.97	14271.00	4358.60	1.27	2295	34	2295	34

* Assuming Poseidon2 is used, 128-bit classical security

Outline

The Problem

Overall Paradigm

Signature Design

Next Steps

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The Problem

Overall Paradigm

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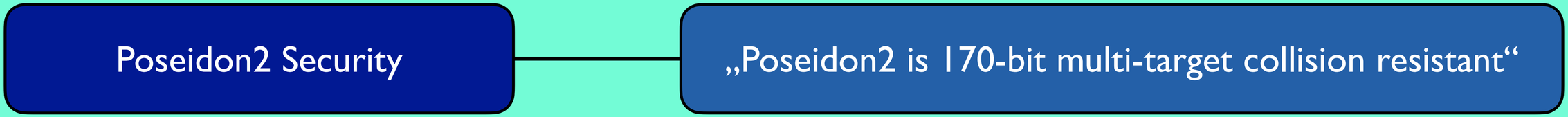
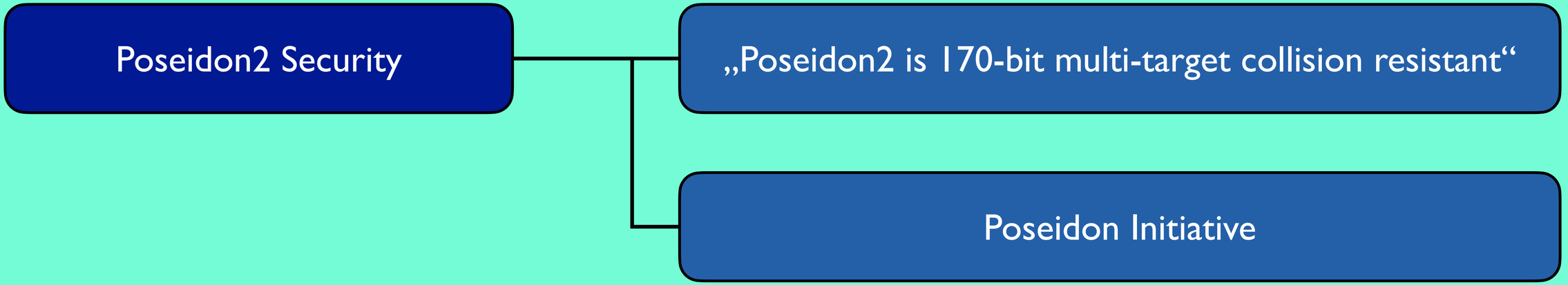
Next Steps

Poseidon2 Security

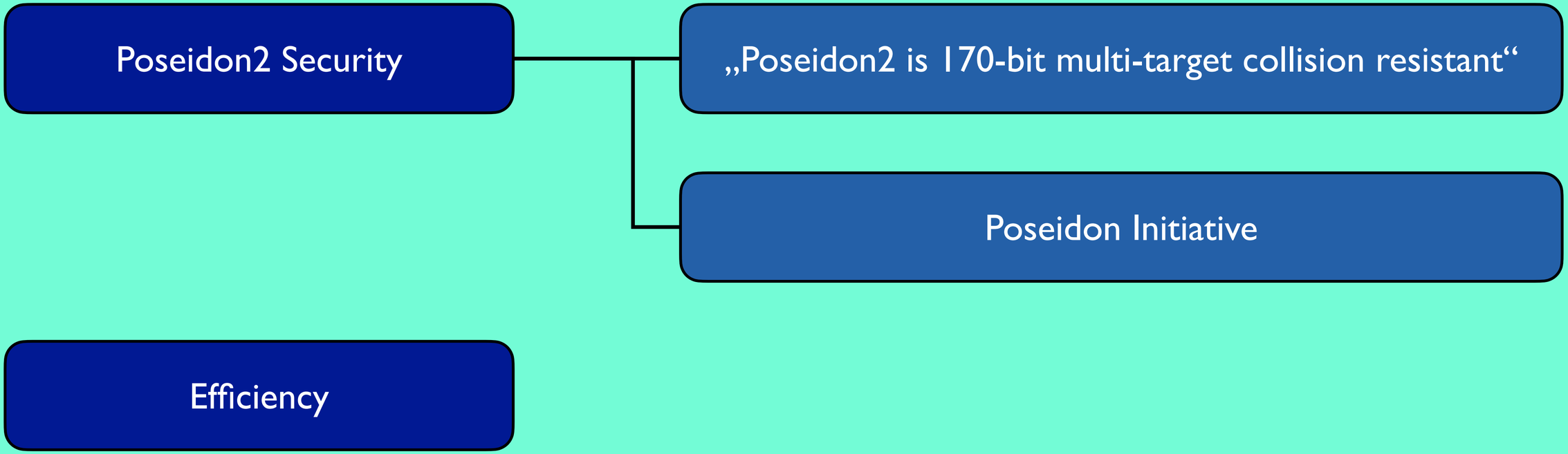
Next Steps


Poseidon2 Security
„Poseidon2 is 170-bit multi-target collision resistant“

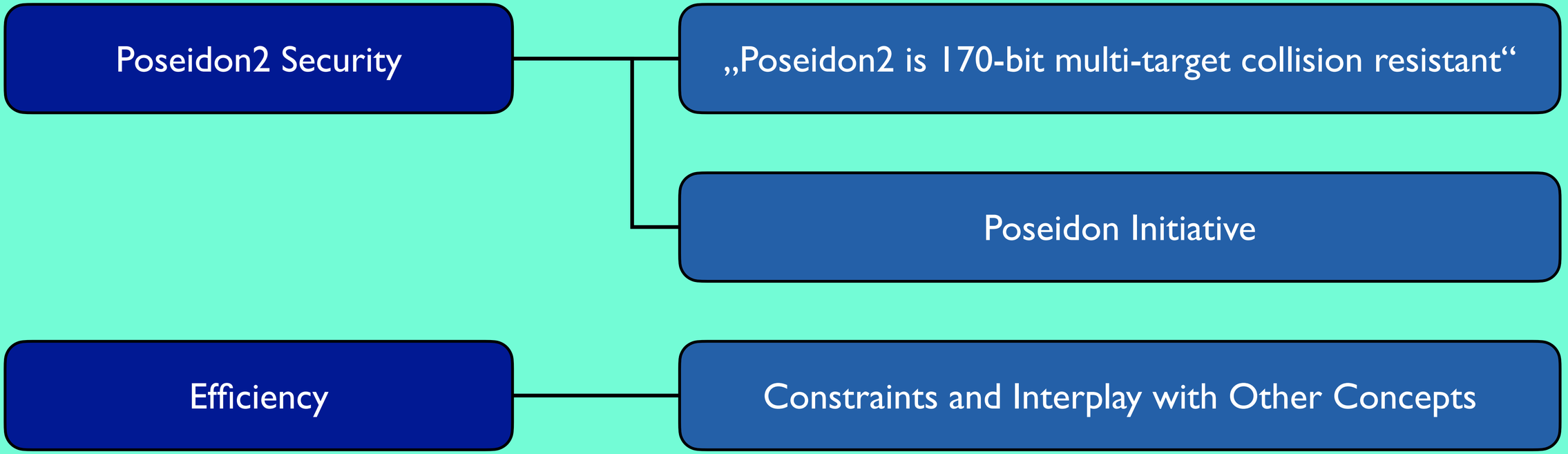
Next Steps


Poseidon2 Security
„Poseidon2 is 170-bit multi-target collision resistant“
Poseidon Initiative

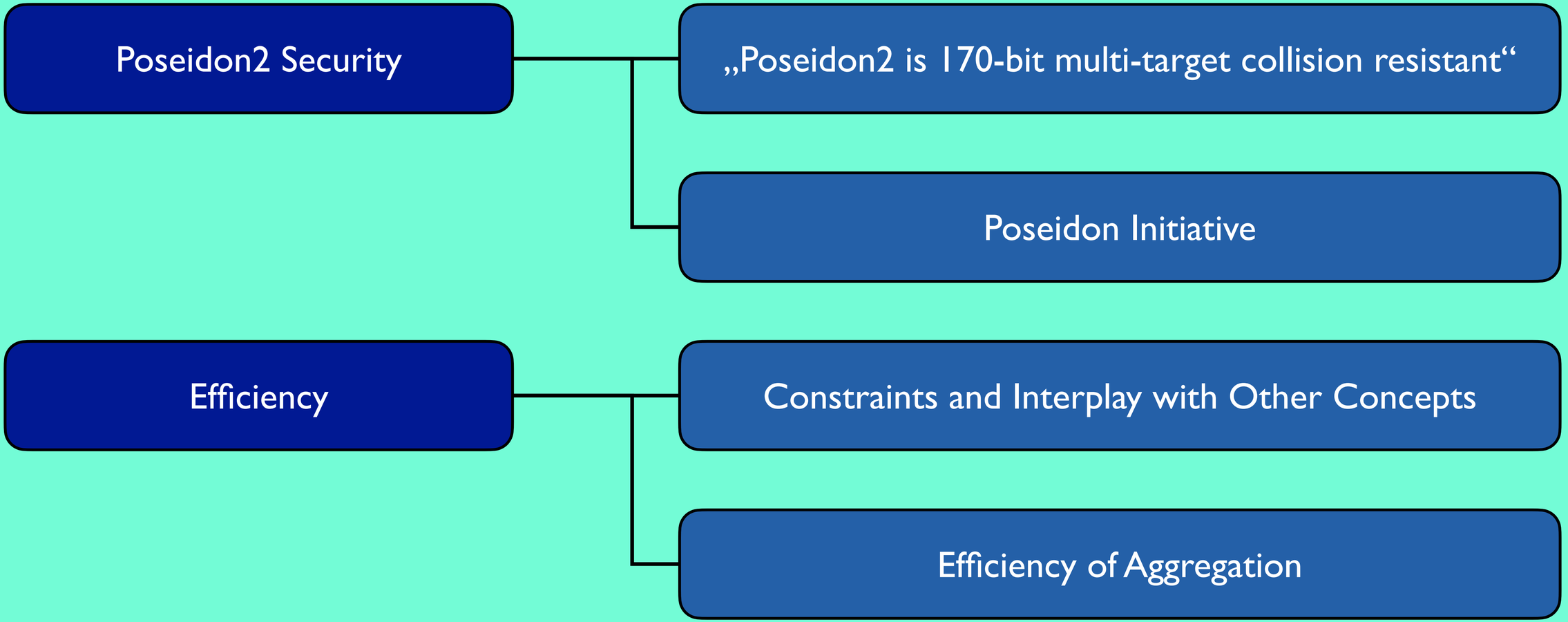
Next Steps


Poseidon2 Security
„Poseidon2 is 170-bit multi-target collision resistant“
Poseidon Initiative
Efficiency

Next Steps


Poseidon2 Security
„Poseidon2 is 170-bit multi-target collision resistant“
Poseidon Initiative
Efficiency
Constraints and Interplay with Other Concepts

Next Steps


Poseidon2 Security
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Efficiency of Aggregation

Summary

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Post-Quantum Multi-Signatures for Ethereum

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Our Work

XMSS Variants + Succinct Argument

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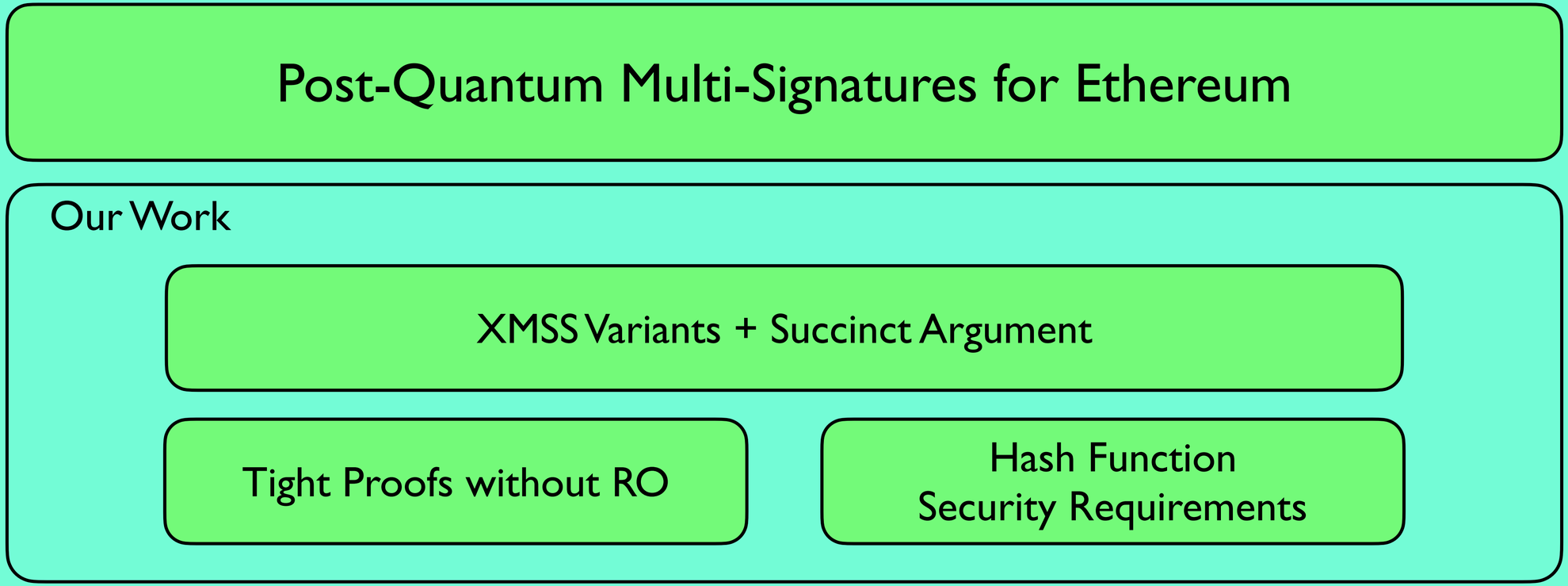
Tight Proofs without RO

Summary


Post-Quantum Multi-Signatures for Ethereum
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Is Poseidon2 secure enough?

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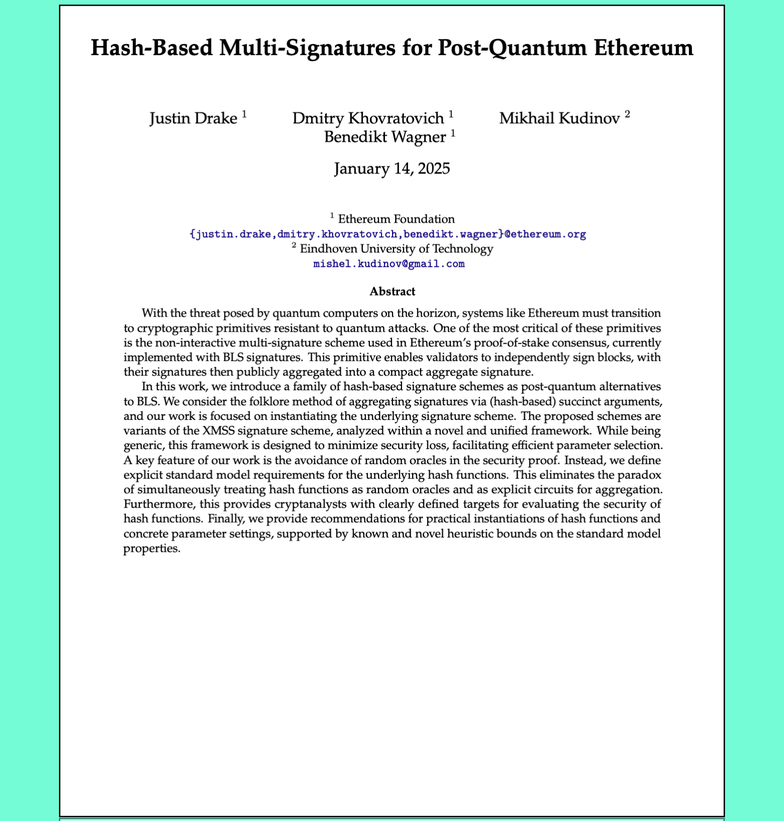
Is Poseidon2 secure enough?

Is this efficient enough?

Thank you!

Paper



Hash-Based Multi-Signatures for Post-Quantum Ethereum

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Abstract

With the threat posed by quantum computers on the horizon, systems like Ethereum must transition to cryptographic primitives resistant to quantum attacks. One of the most critical of these primitives is the non-interactive multi-signature scheme used in Ethereum's proof-of-stake consensus, currently implemented with BLS signatures. This primitive enables validators to independently sign blocks, with their signatures then publicly aggregated into a compact aggregate signature.

In this work, we introduce a family of hash-based signature schemes as post-quantum alternatives to BLS. We consider the folklore method of aggregating signatures via (hash-based) succinct arguments, and our work is focused on instantiating the underlying signature scheme. The proposed schemes are variants of the XMSS signature scheme, analyzed within a novel and unified framework. While being generic, this framework is designed to minimize security loss, facilitating efficient parameter selection. A key feature of our work is the avoidance of random oracles in the security proof. Instead, we define explicit standard model requirements for the underlying hash functions. This eliminates the paradox of simultaneously treating hash functions as random oracles and as explicit circuits for aggregation. Furthermore, this provides cryptanalysts with clearly defined targets for evaluating the security of hash functions. Finally, we provide recommendations for practical instantiations of hash functions and concrete parameter settings, supported by known and novel heuristic bounds on the standard model properties.

Code