Two-Way ANOVA vs. Mixed Model

A Tale of Two Thieves

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Traditional Two-Way ANOVA (Both Fixed)

Model Specification:

$$\text{ASSAY}_{ijk} = \mu + \beta_i \cdot \text{METHOD}_i + \alpha_j + \gamma_{ij} + \varepsilon_{ijk}$$

Where:

- μ = Grand mean
- β_i = Fixed METHOD effect (INTM or UNIT)
- α_j = Fixed LOCATION effect ($j = 1, \ldots, 6$)
- γ_{ij} = Fixed METHOD $\times$ LOCATION interaction
- ε_{ijk} = Random error, $\varepsilon_{ijk} \sim N(0, \sigma^2)$

Key Assumption:

- Inference applies only to these 6 specific locations
- Cannot generalize to other locations

Mixed Model (METHOD Fixed, LOCATION Random)

Model Specification:

$$\text{ASSAY}_{ijk} = \mu + \beta \cdot \text{METHOD} + b_i + \varepsilon_{ijk}$$

Where:

- μ = Grand mean
- β = Fixed METHOD effect (applies to any method)
- b_i = Random LOCATION intercept, $b_i \sim N(0, \sigma_b^2)$
- ε_{ijk} = Error term, $\varepsilon_{ijk} \sim N(0, \sigma_i^2)$

Key Feature: Heterogeneous Variance

$$\sigma^2_{\text{INTM}} = 0.7069 \neq \sigma^2_{\text{UNIT}} = 3.4001 \quad (4.81\times \text{ difference})$$

Generalizability:

- Inference generalizes to any location within the blender

Key Differences: Side-by-Side Comparison

Aspect	**Two-Way ANOVA**	**Mixed Model**
METHOD effect	β_i (fixed)	β (fixed)
LOCATION effect	α_j (fixed)	$b_i \sim N(0, \sigma_b^2)$
Interaction	Estimable	Absorbed in b_i
Variance	Homogeneous: σ^2	Heterogeneous: σ_i^2
Generalization	Limited to 6 LOCs	Beyond 6 LOCs
Variance decomp.	Not explicit	31% LOC + 2.3% METHOD

Why Mixed Model? (1/4)

1. Sampling Design

Key Question

Are these 6 locations *fixed* or a *random sample*?

Answer: The 6 locations represent a random sample from all possible locations within the V-blender.

- We didn't pre-select "important" locations
- They were chosen to represent the overall blender
- We want to infer about *any* future location

Conclusion: LOCATION should be random, not fixed

Why Mixed Model? (2/4)

2. Generalizability

Research Goal

Understand blender uniformity for future production batches

- Two-Way ANOVA: "Are these 6 locations different?"
- Mixed Model: "Does location systematically affect content?"

Example:

- ANOVA results apply only to Locations 1-6
- Mixed Model generalizes to any location in any future batch

Why Mixed Model? (3/4)

3. Precision Comparison

Question: Do INTM and UNIT have different measurement precisions?

Heterogeneous Variance Structure:

$$\sigma^2_{\text{INTM}} = 0.7069 \quad \text{vs} \quad \sigma^2_{\text{UNIT}} = 3.4001$$

- INTM is 4.81× more precise than UNIT
- This difference cannot be detected with homogeneous variance
- Precision matters for quality control!

Why Mixed Model? (4/4)

4. Explicit Variance Decomposition

Mixed model partitions total variation into three components:

Variance Decomposition Formula:

$$\text{Var}(\text{ASSAY}_{ijk}) = \text{Var}(b_i) + \text{Var}(\varepsilon_{ijk}) + \text{Var}(\beta \cdot \text{METHOD})$$

$$= \underbrace{0.9977}_{\text{Location}} + \underbrace{2.0535}_{\text{Within-Location}} + \underbrace{0.0731}_{\text{METHOD}}$$

Percentage Decomposition:

- Between-Location Variance: $\frac{0.9977}{3.1243} = 31.9\%$
  - Random intercepts: $b_i \sim N(0, 0.9977)$
  - Range: 35.01 to 37.98 mg/100mg (2.96 span)
- METHOD Effect: $\frac{0.0731}{3.1243} = 2.3\%$
  - Fixed effect: $\beta \cdot \text{METHOD}$
- Residual Error: $\frac{2.0535}{3.1243} = 65.7\%$
  - Heterogeneous: $\sigma^2_{\text{INTM}} = 0.7069$, $\sigma^2_{\text{UNIT}} = 3.4001$

Key Insight: Location effect (31.9%) dominates, not method choice

Summary: Mixed Model Choice

When to Use Mixed Model

- Some factors are fixed (METHOD) $\rightarrow$ conditional inference
- Other factors are random (LOCATION) $\rightarrow$ population inference
- Want to partition variance explicitly
- Need heterogeneous variance structure

Our Case

- METHOD: Fixed (only these 2 types matter)
- LOCATION: Random (represents any location in blender)
- Variance: Heterogeneous (INTM vs UNIT differ 4.81$\times$)
- Insight: Location effect (31%) dominates METHOD (2.3%)