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Probability Approach to Casing Design Using Monte Carlo Simulation

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Abstract

Casing design plays an important role in the successful drilling of a well and accounts for a substantial percentage of well costs. The goal of casing design is to get an optimal design that will withstand the stress and other factors that affect the casing throughout the lifetime of the well. The conventional approach to casing design uses a deterministic working stress design (WSD), where minimum strength requirements of the casing are determined by comparing casing strength to the magnitude of severe accidental loads that may occur during the lifetime of the well. Uncertainties in the load and strength of the casing are accounted for by multipliers called safety factors (SFs) that are mostly based on experience and do not reflect the probability or consequence of the different casing failure modes. This approach may result in overly conservative casing designs, or design requirements for severe conditions that are expensive, leading to higher well costs.

In this paper, the Monte Carlo Simulation (MCS) method is applied to casing design, where uncertainties in casing loads are considered explicitly by assigning probability distributions to safety factors that affect design variables and parameters. As the MSC method predicts the casing safety factor probability, it gives a better view of the real uncertainties involved in the design. Acceptable probabilities of casing safety factor can be selected based on the cost and operating conditions the casing will undergo; thus, a probability approach to casing design is more flexible as it allows an explicit uncertainty-consistent designs compared to traditional working stress designs.

Keywords – Casing Design, Probability Density Function, Monte Carlo Simulation, Cumulative Distribution Function, Statistical Analysis.

Introduction

Casing is the major structural component of a well. Casing is needed to maintain borehole stability, prevent contamination of water sands, isolate water from producing formations, and control well pressures during drilling, production, and workover operations.¹ Designing of casing string requires knowledge of the operating conditions the casing will undergo as well as the concepts related to pipe properties.² While underdesigned casings are prone to failure at their early stage of operation, the overdesigned casing system adds a higher cost to the total project expense.3 Therefore, optimization of the casing design is essential to keep the casing cost minimal and the casing functioning properly.

The conventional approach to casing design uses a deterministic working stress design (WSD), which is based on multipliers called safety factors (SFs).4 The primary role of a safety factor is to account for uncertainties in the design variables and parameters. The magnitude of the SF is usually based on experience. Different companies use different acceptable SFs for their casing design presented in the relevant guidelines and recommendations, such as American Petroleum Institutes (API). Experience as shown that, SFs give little indication of the probability of failure of a given casing, as they do not explicitly consider the randomness of the design variables and parameters. Some other limitations of this approach are listed in 4.

Real-world problems involve uncertainties.5 To face engineering problems under uncertainty, it is necessary to prearrange proper probabilistic models capable of providing the quantification of such uncertainty, so that it can be taken into account in the process of decision making for engineering planning and design.6

Several studies on the probability approach to casing design have been published. 7 found that material yield limit is the most inﬂuential random variable in failure probability, followed by the wall thickness. The casing grade was also found to provide signiﬁcantly impact in the serviceability limit state. Some high failure probability values are noticed in K55 steel grade. 8 examined the probabilistic forecast workflow for WSD key output "Minimum Absolute Safety Factor". Using Monte-Carlo random sampling method. 9 obtained a probability of casing failure with different pressure and a relationship between safety factor and the probability of casing failure. It was shown that casings of different types and under the effect of different external loads have similar safety coefficient and different probabilities of failure.

The objective of this paper is to apply different probability distributions to determine the design load statistical values, with a Monte Carlo Simulation methodology applied for the selection of the most appropriate safety factor. Two probability distributions models are simulated using a C# programming language. Distributions used in the analysis are Uniform and Triangular probability distribution models.

Monte Carlo Simulation

Monte Carlo Simulation (also known as the Monte Carlo Method) is a way to account for uncertainties in analysis. The method finds all possible outcomes of your decisions and assesses the impact of risk. The method consists of generating random possible events and the number of failure events . The failure probability can be estimated mathematically as

1

The random working stress loads are defined by generating random statistical values for each random safety factor assumed in the analysis, by following the respective distribution model. By its nature, Monte Carlo provides very accurate results, since an adequate number of simulation is performed.

Random Number Generation

The C# algorithm in 10 is based on a pseudorandom number generator that produces a sequence of numbers that are uniformly distributed in the half-open interval [0,1]. These random variates are then transformed through some algorithm to create a new random variate having the required probability distribution. With this source of uniform pseudo-randomness, the realization of any random variable can be generated.11

In this study, the casing design safety factor, SF, is taken as a continuous random variable. The Cumulative Distribution Function (CDF), of the random variable provides the probability that asumes values lower than or equal to a specific value while the Probability Density Function (PDF), of the random variable, provides the density of probability of i.e. provides the probability that assumes values in the interval .

Selection of Adequate Probability Density Functions

To obtain the more adequate representation of the actual probability distribution of the casing design safety factor, the two following distributions are selected and considered in the statistical analysis reported in this research work:

1. the Continous Uniform distribution
2. the Triangular distribution.

The Uniform distribution has been selected because it best characterizes the casing design safety factor random variable provided by API. Where all values have an equal chance of occurrence.12 While Triangular distribution is based on experience, where values around the most likely (mode) safety factor, can be defined together with the minimum and maximum values calculated from API BULL 5C3.

Probability Distributions of Casing Design

Casing design safety factors data can be obtained through experience or from relevant guidelines and recommendations, such as American Petroleum Institutes (API). The tables in API 5C2 use formulae defined in API 5C3 (Superseded by ISO 10900). Understanding the relevant safety factors to apply requires an understanding of these formulae, as well as the manufacturing tolerances defined in API 5CT.13 In this study, two statistical probability distributions were applied to the casing design safety factor data, with the distributions fitted using the Monte Carlo simulation (MCS) method. The formulations of various distributions presented in this section follow 14 and 15.

Continuous Uniform Distribution

The continuous uniform distribution also referred to as the rectangular distribution, is commonly used to represent an equal chance of occurrence. The distribution is a family of symmetric probability distributions and its PDF and CDF functions are given in equations 3 and 5 below.

The uniform density function on the interval [a, b] is the constant function defined by

2.

Its graph is a horizontal line:

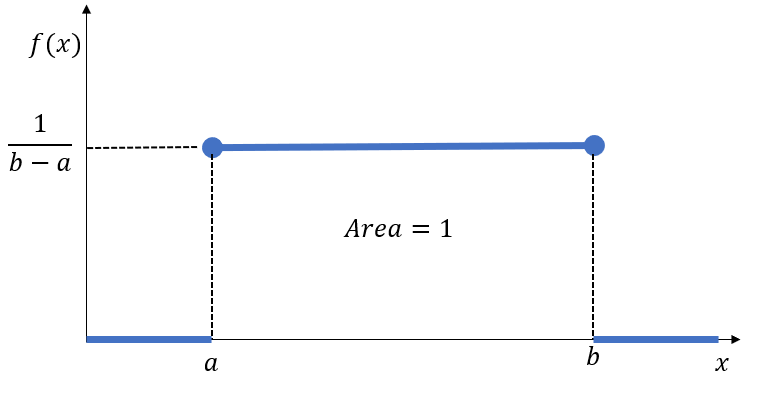


Figure 1. Uniform probability density function.

So,

3

The cumulative distribution function on the support is defined by

4

So,

5

If a random variable X admits a uniform density function, we say that X is uniformly distributed, or that X has the uniform distribution.

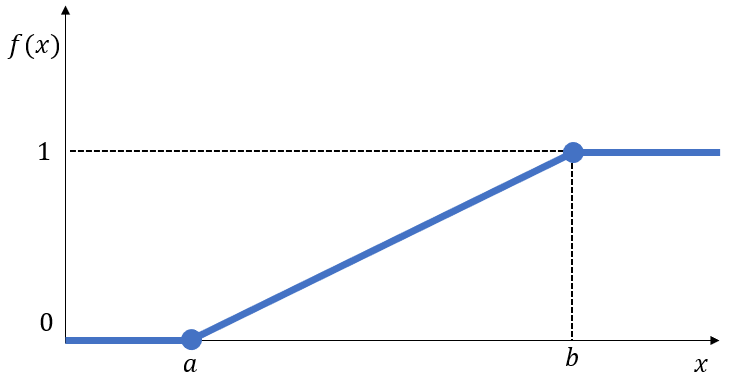


Figure 2. Uniform cumulative distribution function.

Triangular Distribution

The Triangular distribution is a continuous probability distribution whose parameters are defined within the limits of the interval [0,1]. This distribution is suitable for modelling processes that have a less conservative estimate of uncertainty. The application of the triangular distribution is based on the assumption that maximum and minimum values of the distribution are known and the mode of the triangular distribution occurs at zero. Its PDF and CDF functions are given in equation 6 and 9.

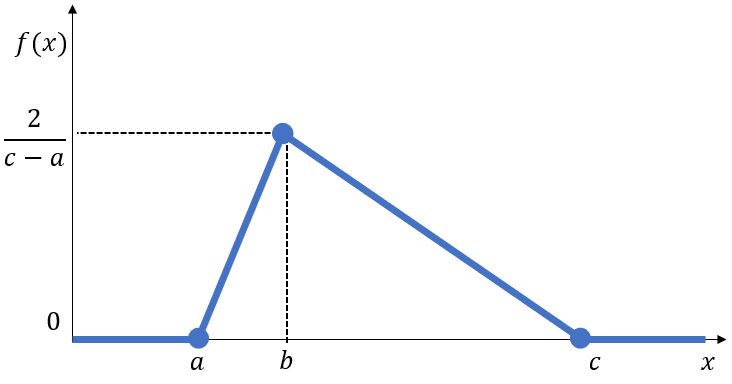


Figure 3. Triangular probability density function.

6

The CDF of triangular (a, b, c) on is

7

The CDF on is

8

So,

9

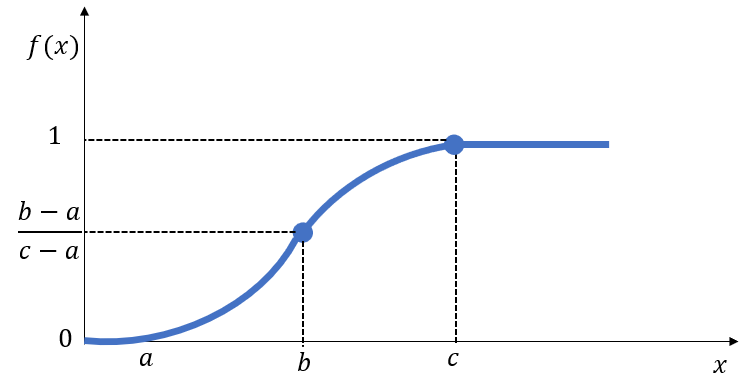


Figure 4. Triangular cumulative distribution function.

Goodness of Fit (GOF) Test

Several formal tests can be used to compare the actual empirical cumulative distribution function (CDF) of the data Fe(x) with the cumulative distribution used to characterise the data Ft(x). The tests used for cumulative distributions are the Anderson-darling and Cramer von-Mises tests.

Anderson Darling Statistic

Anderson Darling goodness of fit test is a modification of the Kolmogorov-Smirnov (K-S) test by providing more emphasis on the tails of the distribution. The Anderson Darling test is used to compare the fit of an observed cumulative distribution function to an expected cumulative distribution function. This test gives more weight to the tails than the Kolmogorov-Smirnov test.

The Anderson-Darling statistic (A2) is defined as:

10

11

Where:

is the weighting function and is the cumulative distribution function of the specified distribution.

When the Anderson Darling test’s weighting function 𝜔(𝑥) = 1 the test statistic is equal to the Cramer von-Mises Statistic. The addition of this weighting function allows the test to place more emphasis on the tails of the distributions tested.

Cramer von-Mises Statistic

The Cramer von-Mises statistic integrates across the square of CDF differences. This allows for a comparison of the fit along the entire distribution, unlike the Kolmogorov Smirnov statistic which is influenced by a single absolute maximum.

12

Additionally, an approximation for the p-value of the Cramer vonMises test can be computed directly using Monte-Carlo simulations.16 This approach was employed for the purpose of this study.

Application of Goodness of Fit Tests for Triangular Distributions

The results of these method do not conclusively state that the probability distribution is representative of the API provided safety factor. However, the method can be used to conclude whether the safety factor is over or underestimated by a specific distribution at a required level of confidence (significance) based on the properties of the relevant statistic. The uniform and triangular distribution were applied to the safety factor of the casing design using C# programming language and Oxyplot [<https://oxyplot.readthedocs.io/en/latest/>]. An iterative process was employed by which the significance level was altered within the range: 0.01, 0.02, 0.05, and 0.10. The probability (p-value) obtained through these method indicates the probability of failure of a casing, as it explicitly consider the randomness of the design variables and parameters.. The p-value is crucial in the determination of the real uncertainty involved in the design, with a low value (below the significance level) suggesting that the theoretical distribution is not representative of the dataset. Selection of the best distribution can be made upon by a combination of the p-value, the confidence interval and visual inspection of the PDF and CDF fit in the simulation plots. This method is largely dependent on the number of iterations of the random variable. Large simulation data affects the performance of the tests as there are a large number of data points, especially at the tails of the distribution, which greatly affect the CDF.

Abbreviations

**WSD** Working Stress Design

**MCS** Monte Carlo Simulation

**SFs** Satey Factors

**SF** Safety Factor

**API** American Petroleum Institutes

Nomenclature

C# C Sharp