Introduction to Neural Networks

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Engineering for Professionals Program

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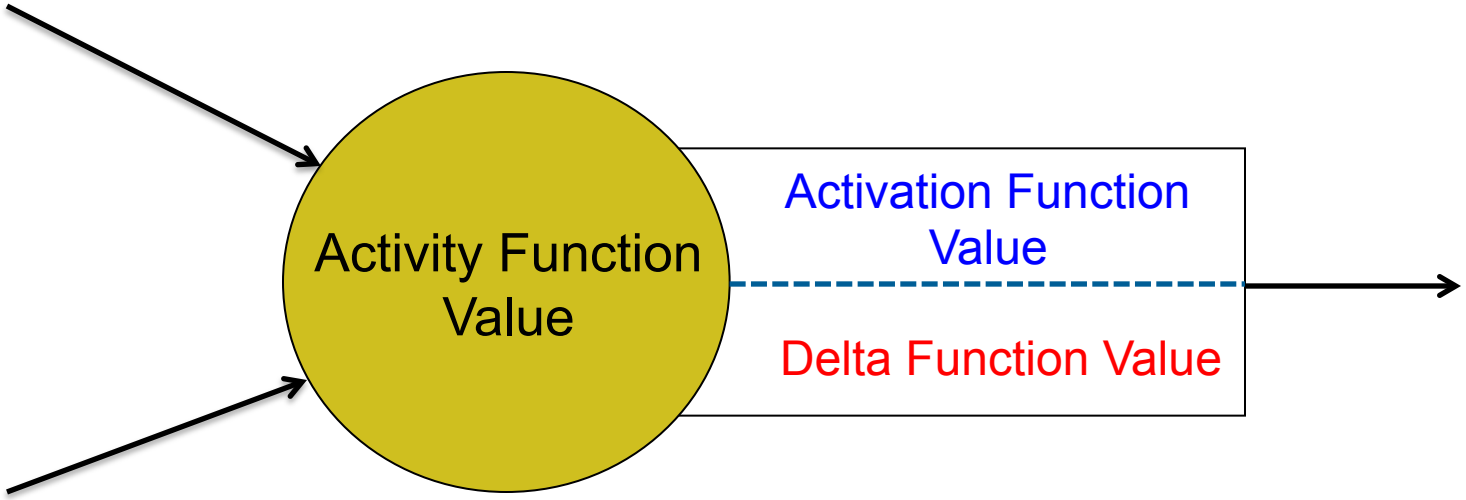
Module 5.2: An Example of the FFBP

This Sub-Module Covers …

- An example of applying the FFBP algorithm to a simple, multi-layer network.
- It will demonstrate the structure and behavior of the FFBP.

Conventions for Our Perceptron

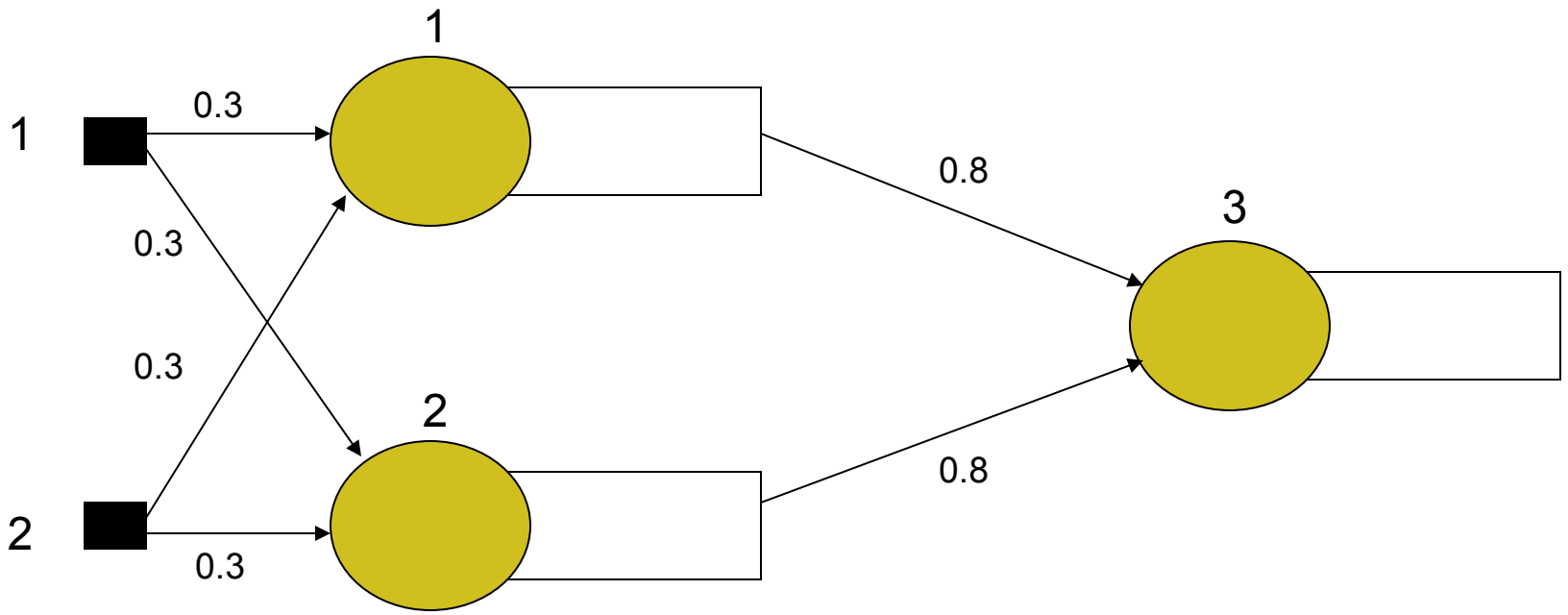


An FFBP Example

- A simple two input, single output, two layer network
- Inputs are 1 and 2: I = {1,2}
- Want to train the network so the inputs produce an output value of 0.7. Thus, {1,2} → {0.7}
- Arbitrarily set initial weights for all links.
- For convenience, we'll set the biases all to 0.
- Set all the weights of the hidden layer to 0.3.
- Set all the weights in the output layer to 0.8.
- Set the value of η = 1.0 (that's convenient!).

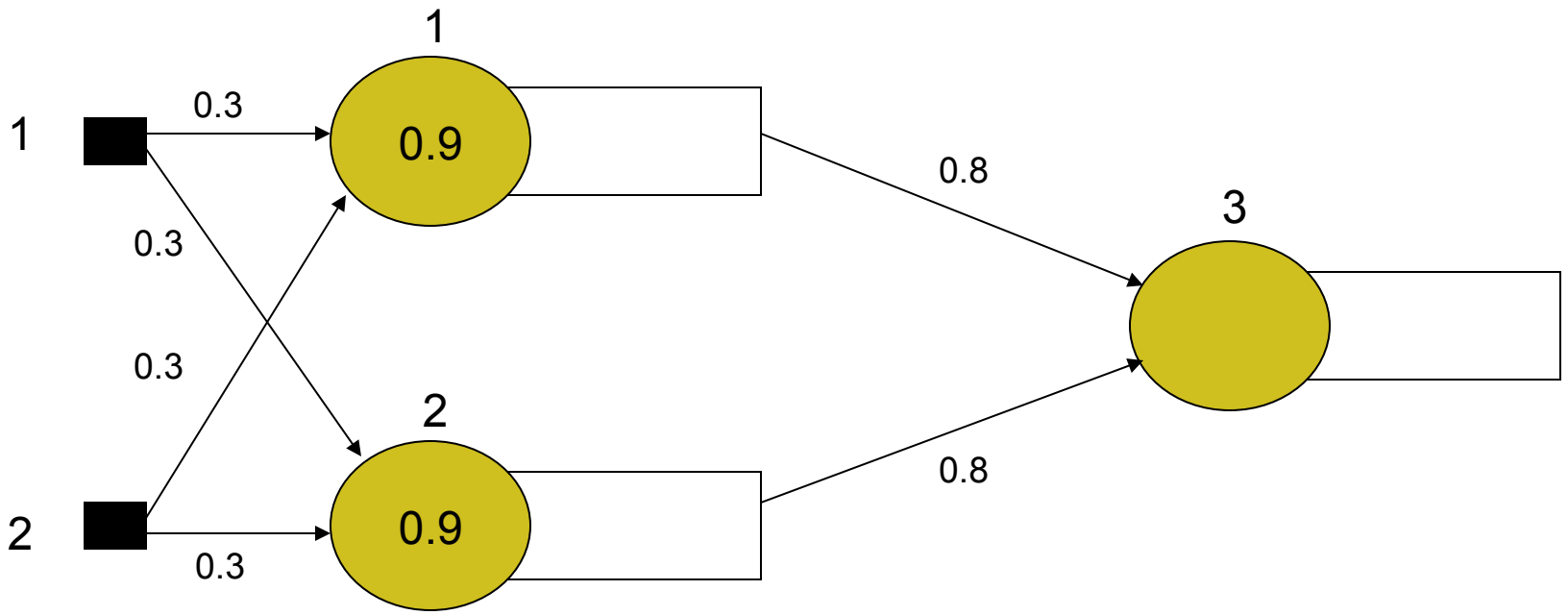
Topology of Net

After Initialization of Weights



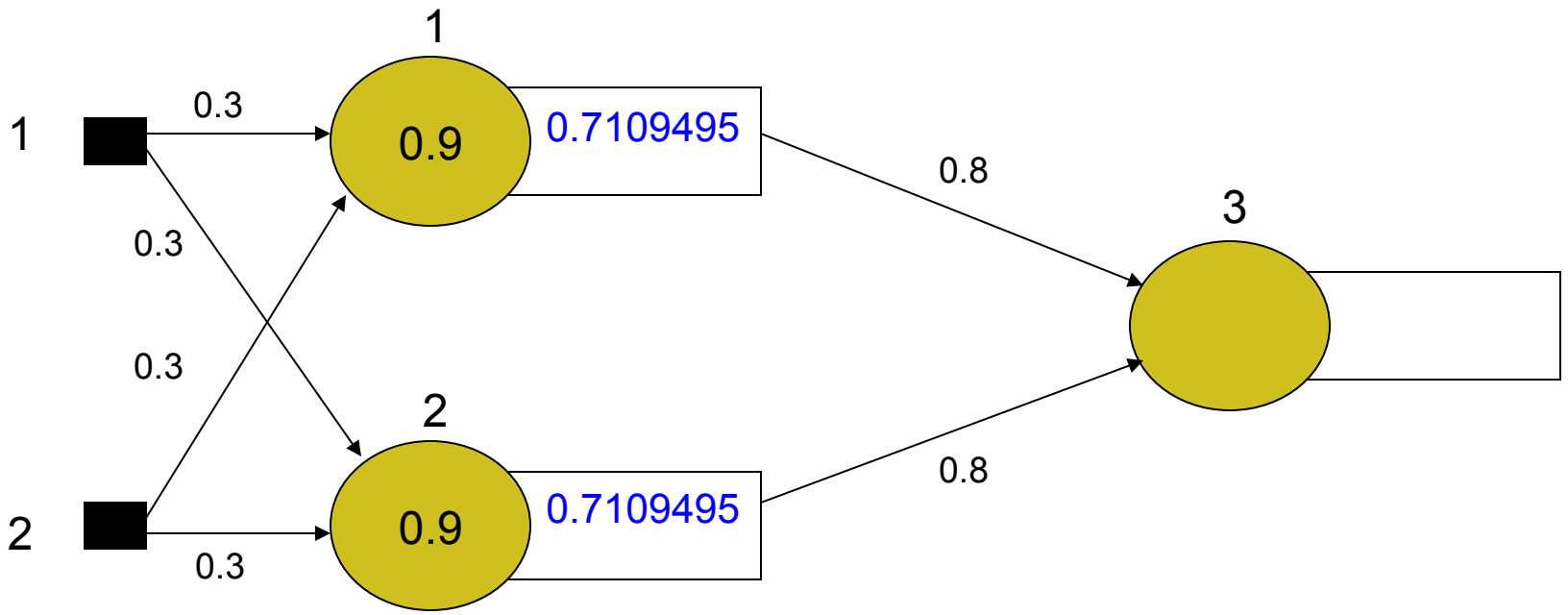
Topology of Net

Feed-forward Epoch



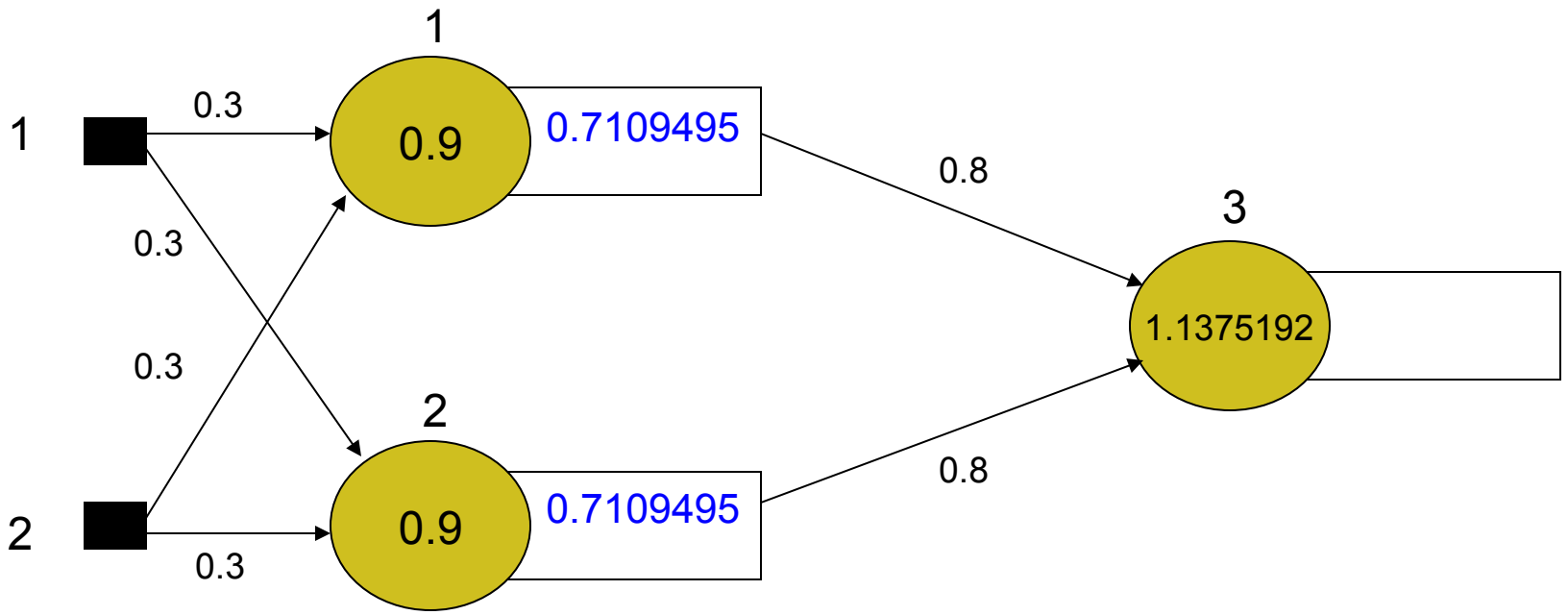
Topology of Net

Feed-forward Epoch



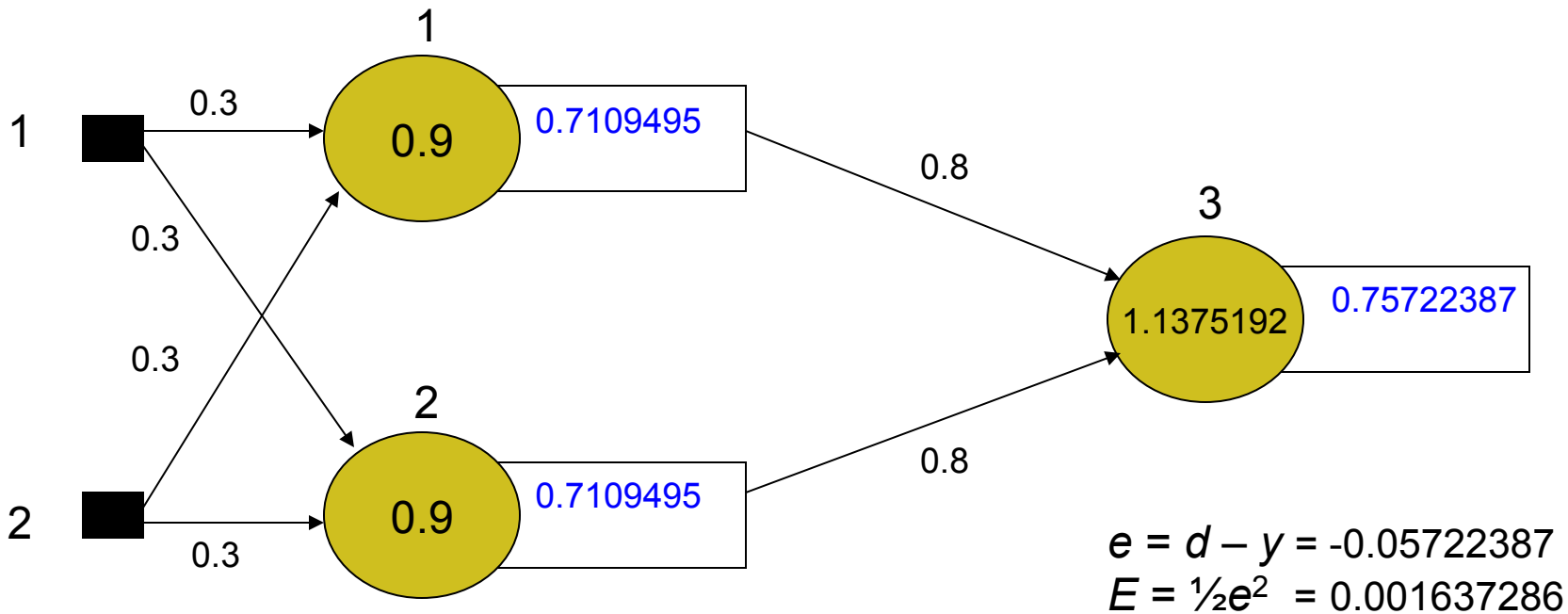
Topology of Net

Feed-forward Epoch



Topology of Net

Feed-forward Epoch



$e = d - y$ = -0.05722387

$E = \frac{1}{2}e^2$ = 0.001637286

Calculate the Deltas for the Output Layer

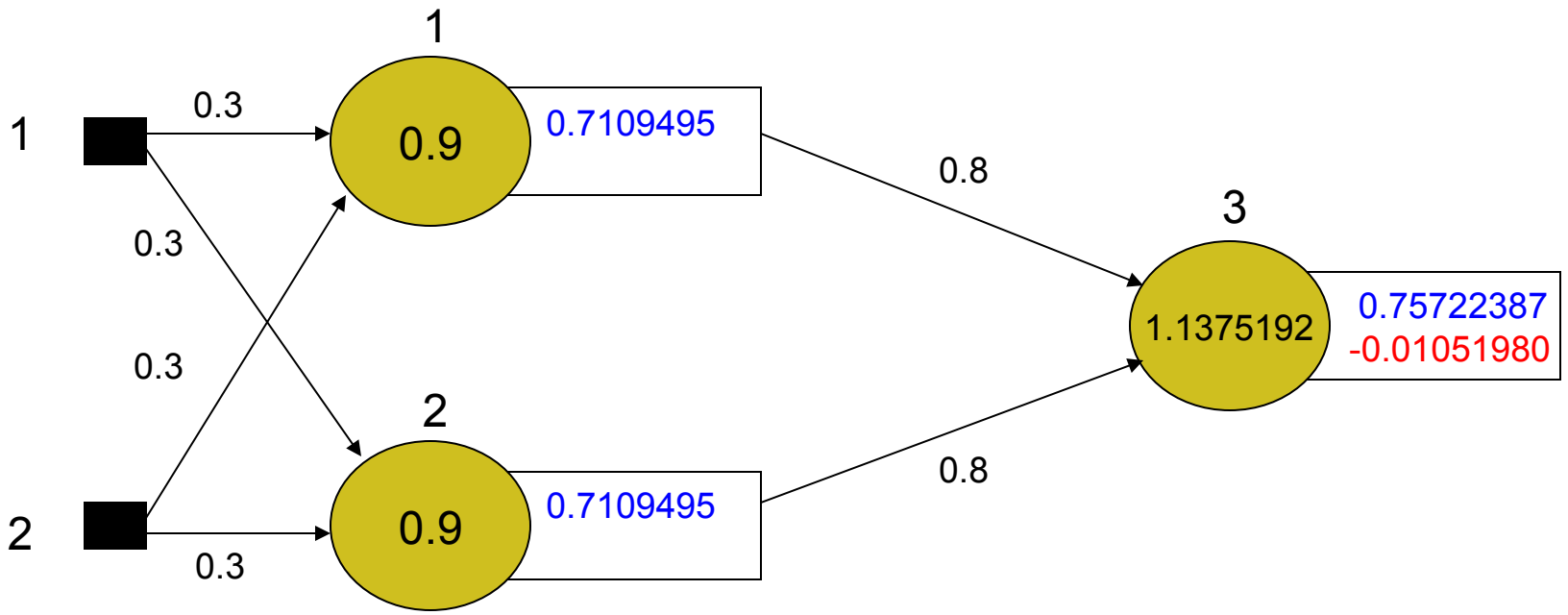
$$w_{jk}(t+1) = w_{jk}(t) - \eta\left(-e_k[1-y_k]y_k x_j\right)$$

$$w_{jk}(t+1) = w_{jk}(t) + \eta\left(\delta_k x_j\right)$$

$$\delta_k = e_k\left[1-y_k\right]y_k$$

Delta (node 3) = (0.7 - 0.757224) (1 - 0.757224) 0.757224
= -0.0105198

Back Propagation Epoch



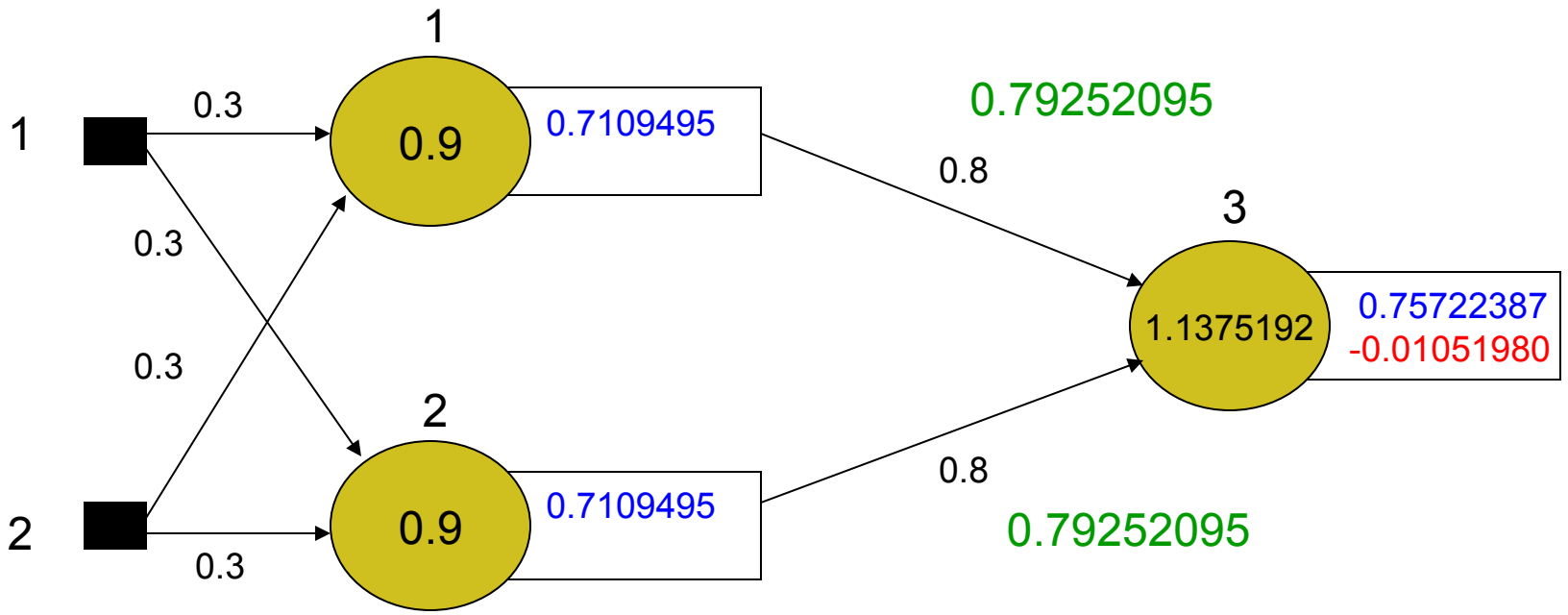
Calculate the New Weights for the Output Layer

$$w_{jk}(t+1) = w_{jk}(t) - \eta\left(-e_k[1-y_k]y_k x_j\right)$$

$$w_{jk}(t+1) = w_{jk}(t) + \eta\left(\delta_k x_j\right)$$

$w_{jk}(t+1)$ = (0.8) + 1*(-0.0105198)*0.7109495
= 0.79252095

Back Propagation Epoch



Calculate the Deltas for the Input Layer

Recall that for the input layer, the gradient vector element

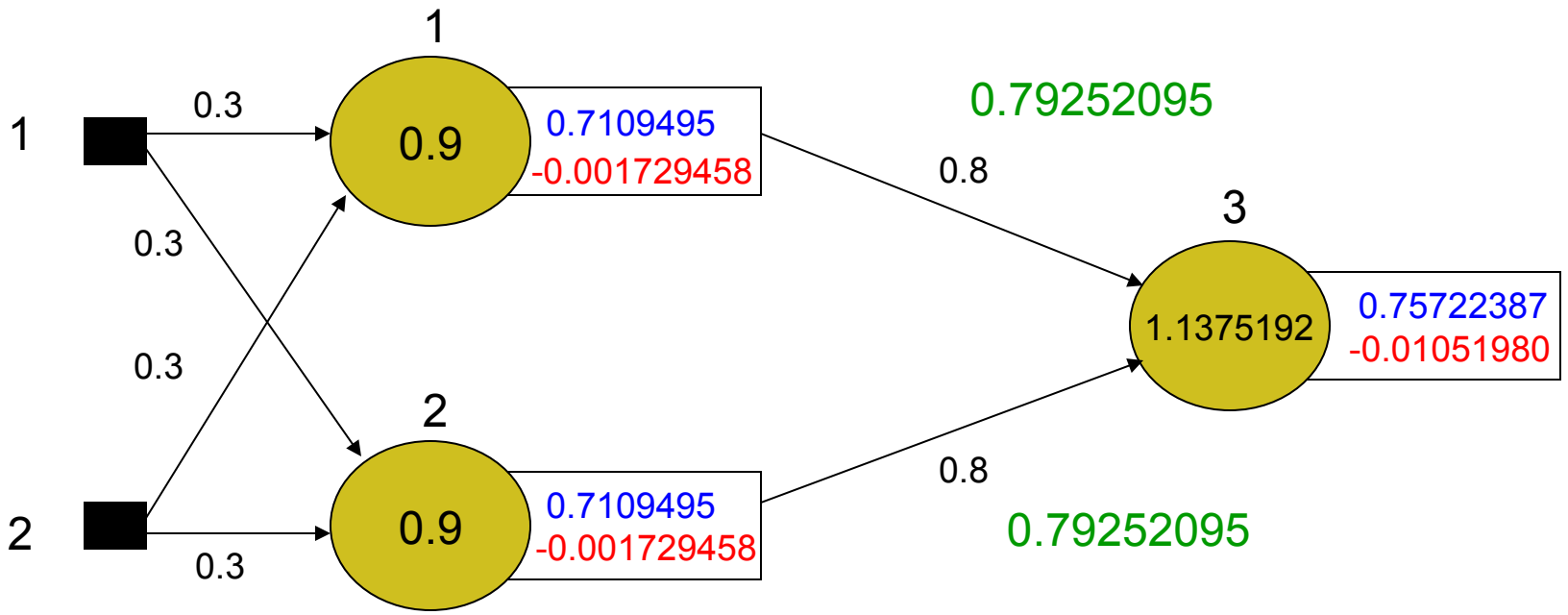
$$\frac{\partial E}{\partial w_{ij}} = -[1 - x_j]x_j \left(\sum_k \delta_k w_{jk} \right) x_i = \delta_j x_i$$

where

$$\delta_j = [1 - x_j]x_j \left(\sum_k \delta_k w_{jk} \right)$$

δ_j = (1 - 0.7109495) * 0.7109495 *(-0.01051980*0.8)
= -0.001729458

Back Propagation Epoch



Calculate the New Weights for the Hidden Layer

$$w_{ij}(t+1) = w_{ij}(t) - \eta\left(\frac{\partial E}{\partial w_{ij}}\right)$$

But now $$\frac{\partial E}{\partial w_{ij}} = -[1 - x_j]x_j\left(\sum_k \delta_k w_{jk}\right)x_i = -\delta_j x_i$$

Thus, the updated weights for the input layer are $w_{ij}(t+1) = w_{ij}(t) + \eta\left(\delta_j x_i\right)$

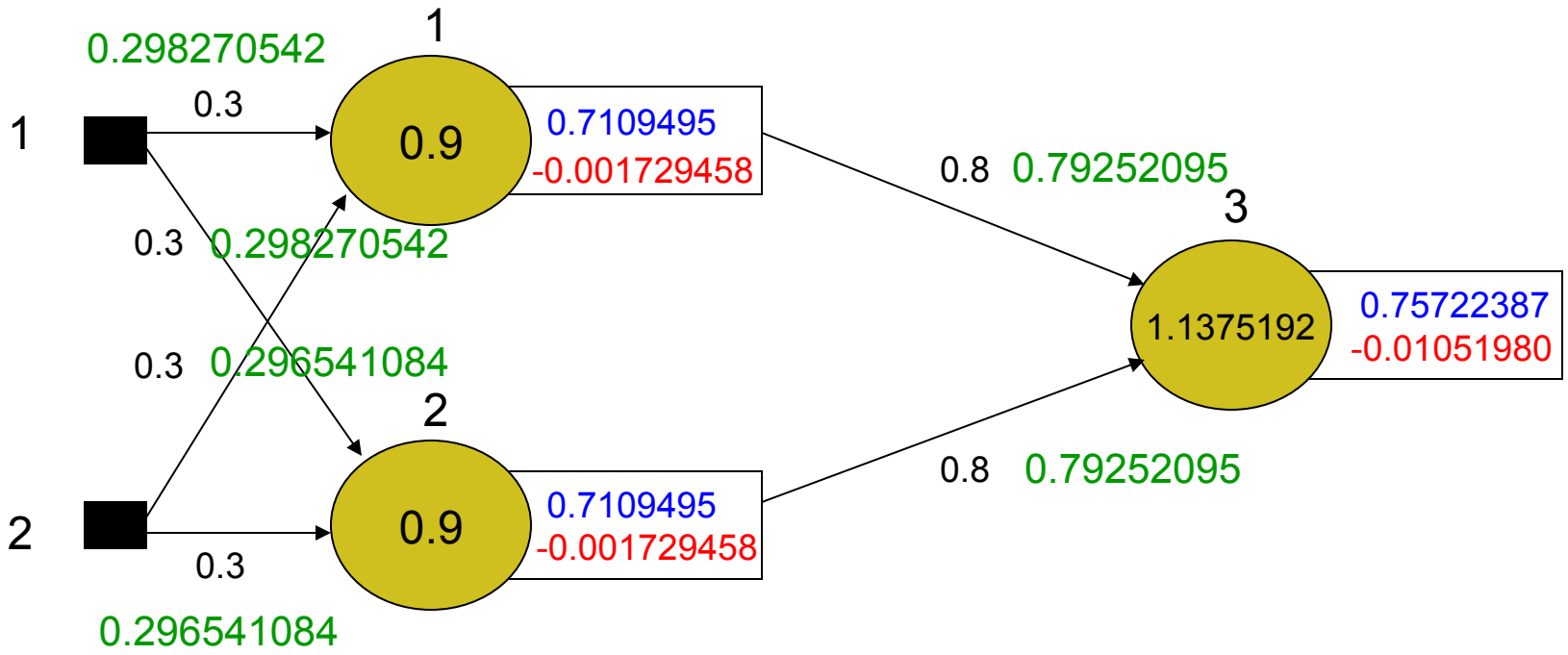
w_{11} (t+1) = (0.3) + 1*(-0.001729458)*1 = 0.298270542

w_{12} (t+1) = (0.3) + 1*(-0.001729458)*1 = 0.298270542

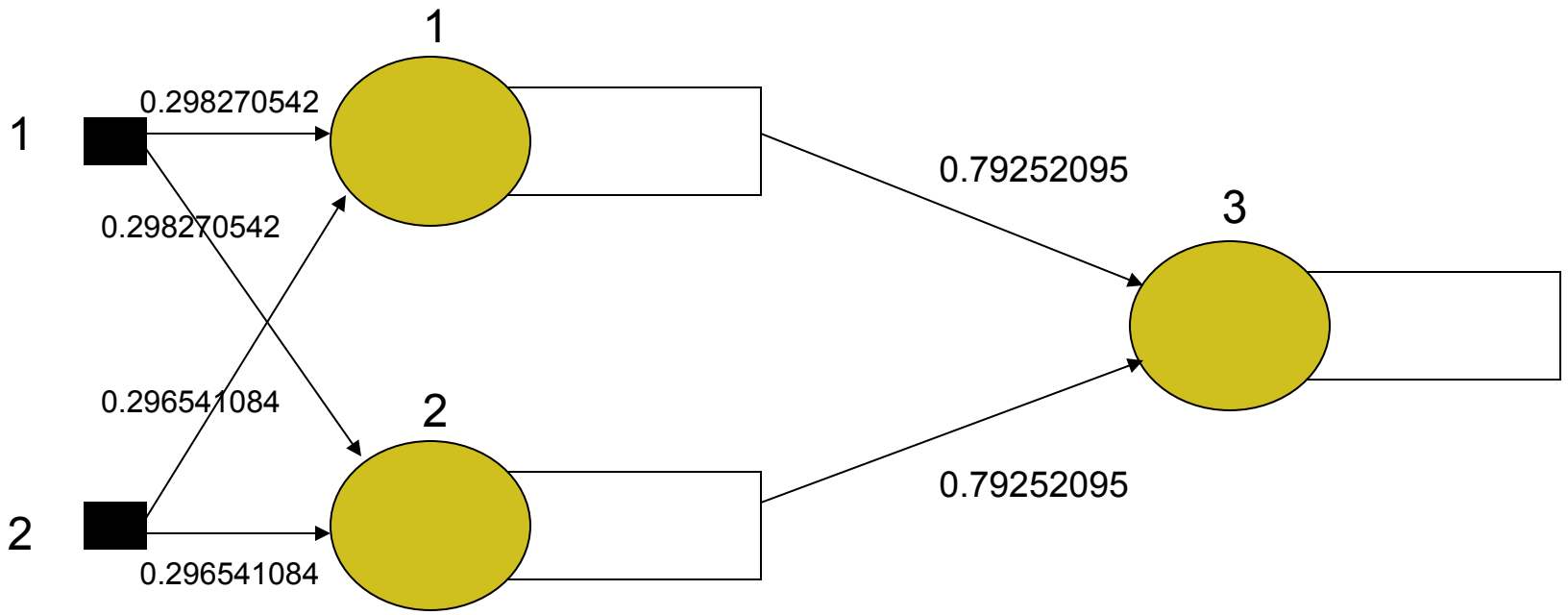
w_{21} (t+1) = (0.3) + 1*(-0.001729458)*2 = 0.296541084

w_{22} (t+1) = (0.3) + 1*(-0.001729458)*2 = 0.296541084

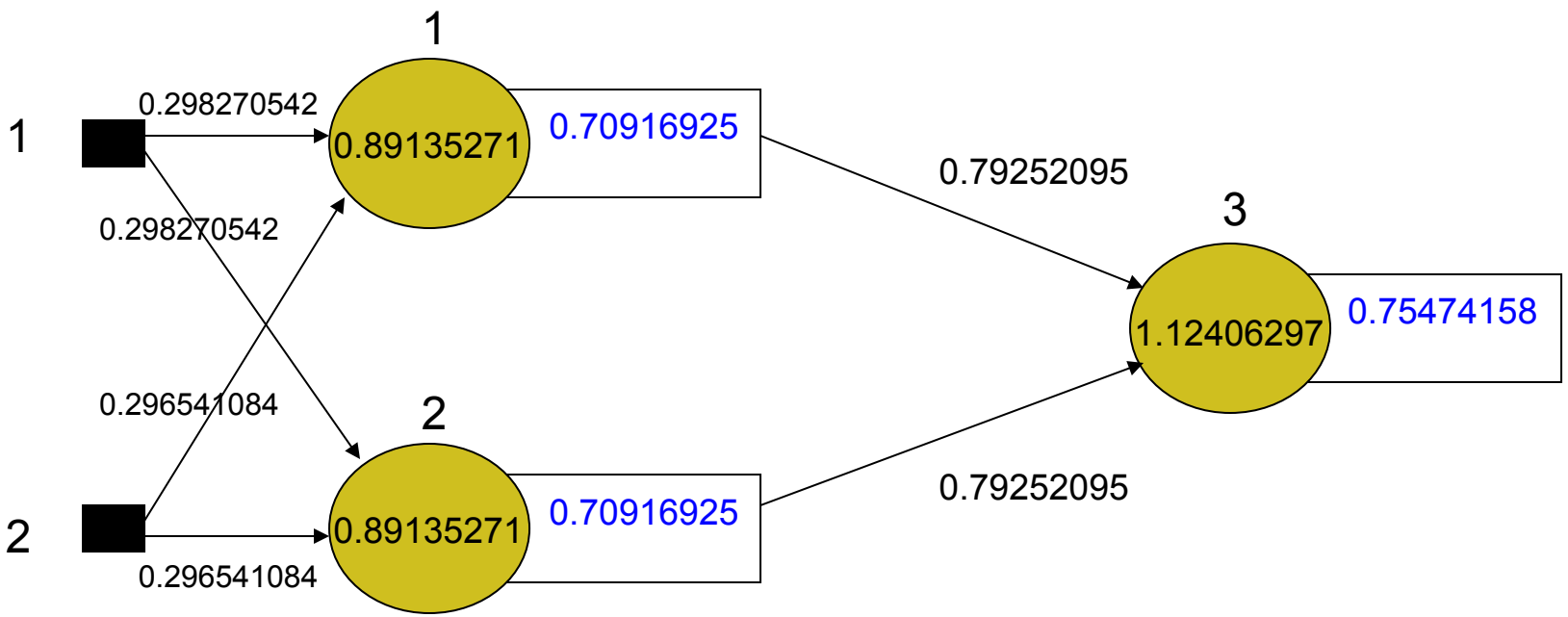
Back Propagation Epoch



Start of a New Day



2nd Feed-forward Epoch



First Epoch

$e = d - y$ = -0.05722387

$E = \frac{1}{2}e^2$ = 0.001637286

Second Epoch

$e = d - y$ = -0.05474158

$E = \frac{1}{2}e^2$ = 0.00149832

The FFBP Summary

Feed-forward Epoch:

- We presented input values at the input layer to the hidden layer nodes.
- Computed activity and activation values in the hidden layer.
- Used those activation function values as input values for the output layer node(s).
- Computed the output error values.

Back-Propagation Epoch:

- Used the output error values in conjunction with the inputs to and the outputs of the output layer node(s) to compute the delta value(s) (gradient value(s)) for the output layer node(s).
- Used these delta value(s) to compute updated weights for the output layer node(s).
- Used these delta values in conjunction with the output and input values of the hidden layer nodes along with the original weights in the output layer to compute delta values for the hidden layer nodes.
- Used the delta values associated with the hidden layer nodes to compute updated weights for the hidden layer.