

$$\begin{aligned}\text{CPU Time} &= \text{CPU Clock Cycles} \times \text{Clock Cycle Time} \\ &= \frac{\text{CPU Clock Cycles}}{\text{Clock Rate}}\end{aligned}$$

- Performance can be improved by
 - Reducing number of clock cycles
 - Increasing clock rate
 - Hardware designer must often trade off clock rate against cycle count

- Computer A: 2GHz clock, program takes 10s CPU time
- Designing Computer B
 - Desire is for 6s CPU time
 - Suppose using a faster clock requires $1.2 \times$ clock cycles
- How fast must Computer B clock be?

$$\text{Clock Rate}_B = \frac{\text{Clock Cycles}_B}{\text{CPU Time}_B} = \frac{1.2 \times \text{Clock Cycles}_A}{6\text{s}}$$

$$\begin{aligned}\text{Clock Cycles}_A &= \text{CPU Time}_A \times \text{Clock Rate}_A \\ &= 10\text{s} \times 2\text{GHz} = 20 \times 10^9\end{aligned}$$

$$\text{Clock Rate}_B = \frac{1.2 \times 20 \times 10^9}{6\text{s}} = \frac{24 \times 10^9}{6\text{s}} = 4\text{GHz}$$

Clock Cycles = Instruction Count \times Cycles per Instruction

CPU Time = Instruction Count \times CPI \times Clock Cycle Time

$$= \frac{\text{Instruction Count} \times \text{CPI}}{\text{Clock Rate}}$$

- Instruction Count for a program
 - Determined by program, ISA and compiler
- Average cycles per instruction
 - Determined by CPU hardware
 - If different instructions have different CPI
 - Average CPI is affected by instruction mix

- Computer A: Cycle Time = 250ps, CPI = 2.0
- Computer B: Cycle Time = 500ps, CPI = 1.2
- Same ISA
- Which is faster, and by how much?

$$\begin{aligned}\text{CPU Time}_A &= \text{Instruction Count} \times \text{CPI}_A \times \text{Cycle Time}_A \\ &= 1 \times 2.0 \times 250\text{ps} = 1 \times 500\text{ps} \end{aligned}$$

A is faster...

$$\begin{aligned}\text{CPU Time}_B &= \text{Instruction Count} \times \text{CPI}_B \times \text{Cycle Time}_B \\ &= 1 \times 1.2 \times 500\text{ps} = 1 \times 600\text{ps} \end{aligned}$$

$$\frac{\text{CPU Time}_B}{\text{CPU Time}_A} = \frac{1 \times 600\text{ps}}{1 \times 500\text{ps}} = 1.2$$

...by this much

- With different instruction classes:

$$\text{Clock Cycles} = \sum_{i=1}^n (\text{CPI}_i \times \text{Instruction Count}_i)$$

- Weighted average CPI

$$\text{CPI} = \frac{\text{Clock Cycles}}{\text{Instruction Count}} = \sum_{i=1}^n \left(\text{CPI}_i \times \frac{\text{Instruction Count}_i}{\text{Instruction Count}} \right)$$

Relative frequency

- Alternative compiled code sequences using instructions in classes A, B, C

Class	A	B	C
CPI for class	1	2	3
IC in sequence 1	2	1	2
IC in sequence 2	4	1	1

- Sequence 1: IC = 5

- Clock Cycles
 $= 2 \times 1 + 1 \times 2 + 2 \times 3$
 $= 10$
- Avg. CPI = $10/5 = 2.0$

- Sequence 2: IC = 6

- Clock Cycles
 $= 4 \times 1 + 1 \times 2 + 1 \times 3$
 $= 9$
- Avg. CPI = $9/6 = 1.5$

$$\text{CPU Time} = \frac{\text{Instructions}}{\text{Program}} \times \frac{\text{Clock cycles}}{\text{Instruction}} \times \frac{\text{Seconds}}{\text{Clock cycle}}$$

- Performance depends on
 - Algorithm: affects IC, possibly CPI
 - Programming language: affects IC, CPI
 - Compiler: affects IC, CPI
 - Instruction set architecture: affects IC, CPI, T_c