The Johns Hopkins University

JHU ENGINEERING FOR PROFESSIONALS PROGRAM

NEURAL NETWORKS

Problem Set #9

9.1 Let $\mathbf{A_1}$ = (1, 1, -1, -1), $\mathbf{A_2}$ = (1, -1, 1, -1), $\mathbf{B_1}$ = (1, 1) and $\mathbf{B_2}$ = (-1, 1) in a Binary Associative Memory.

(a) Determine the weight matrix for this system.

(b) If the vector **X** = (-1, 1, -1, -1) is input into the system, what is the output vector **Y**?

(c) If the vector **Y** obtained in part b is input into the system, what is the output vector $\mathbf{X}^*$**?**

Ans: The weight matrix is given by the following equation:

(a)

$$\mathbf{A}_1^{\mathbf{T}}\mathbf{B}_1 + \mathbf{A}_2^{\mathbf{T}}\mathbf{B}_2 =$$

$$\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}\begin{pmatrix} -1 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ -1 & -1 \\ -1 & -1 \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 1 & -1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \\ -2 & 0 \\ 0 & -2 \end{pmatrix}$$

(b) **X** multiplied on the right of the matrix yields

$$\begin{pmatrix} -1 & 1 & -1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 2 \\ 2 & 0 \\ -2 & 0 \\ 0 & -2 \end{pmatrix} = \begin{pmatrix} 4 & 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 \end{pmatrix} \text{or} \begin{pmatrix} 1 & -1 \end{pmatrix}$$

depending on how you define the hard-limiting function insofar as the 0 is concerned.

(c) If (1, 1) or (1, -1) is multiplied by the weight matrix, we get:

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \\ -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 & -2 & -2 \end{pmatrix}^{\mathrm{T}} \to \begin{pmatrix} 1 & 1 & -1 & -1 \end{pmatrix}^{\mathrm{T}}$$

$$\begin{pmatrix} 0 & 2 \\ 2 & 0 \\ -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 & 2 & -2 & 2 \end{pmatrix}^{\mathrm{T}} \to \begin{pmatrix} -1 & 1 & -1 & 1 \end{pmatrix}^{\mathrm{T}}$$

9.2 Suppose we have a third layer of nodes and exemplar pairs associations between the **B** vectors and a set of **C** vectors where $\mathbf{C_1}$ = (1, 1, -1) and $\mathbf{C_2}$ = (1, -1, 1). How would you set up such a system? What would be its weight matrices? Note, there is no right answer here, but I am looking for creativity and exploration. Use your knowledge of BAMs to address this question.

Ans: One matrix can handle the A← →B transformations, and a second matrix can handle the B← →C transformations. Thus, the first transformation matrix is given above. The second transformation matrix is defined in similar fashion as the previous one:

$$\mathbf{B}_1^{\mathbf{T}}\mathbf{C}_1 + \mathbf{B}_2^{\mathbf{T}}\mathbf{C}_2$$