Introduction to Neural Networks

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Module 2.1: Mathematical Review-Linear Algebra

This Sub-Module Covers …

- Some mathematical review of Linear Algebra.
- Some of the essential elements of matrix/vector algebra.
- This sub-module is then followed by a short quiz.

Mathematical Review

- Preliminaries: Notational conventions

Summation $$\sum_{j=1}^{n} a_{ij} = a_{i1} + a_{i2} + \cdots + a_{in}$$

Product $$\prod_{j=1}^{n} a_{ij} = a_{i1} \cdot a_{i2} \cdot \ldots \cdot a_{in}$$

Vectors If a row vector $\vec{a} = (a_1, a_2, \ldots, a_n)$ then $\vec{a}^{\,T} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$

Vector Operations

Vector Addition:

Given two vectors **a** and **b** *of the same size,* **a + b =** ($a_1 + b_1$, $a_2 + b_2$, …, $a_n + b_n$)

For subtraction, the “+” is substituted with a “-”.

Vector Multiplication: (inner product, the dot product)

$$\vec{a} \cdot \vec{b} = (\vec{a}, \vec{b}) = \left\langle \vec{a}, \vec{b} \right\rangle = \sum_{i=1}^{n} a_i b_i = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

= some scalar quantity.

Matrix-Vector Operations

The inner product ---reprise:

$$\mathbf{ab}^T = (a_1, a_2, \ldots, a_n)\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix}$$

This operation results in a single scalar value. If $\mathbf{a}^T\mathbf{b} = 0$ when both vectors **a** and **b** are non-zero vectors, then the vectors **a** and **b** are said to be *orthogonal.*

A *non-zero vector* is a vector in which at least one element is not zero.

(0, 0, 0, 1.3, 0, 0)

The Outer Product

$$\mathbf{a}^T\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} (b_1, b_2, \ldots, b_n) = \begin{pmatrix} a_1b_1 & a_1b_2 & \cdots & a_1b_n \\ a_2b_1 & a_2b_2 & \cdots & a_2b_n \\ \vdots & \vdots & \ddots & \vdots \\ a_nb_1 & a_nb_2 & \cdots & a_nb_n \end{pmatrix}$$

Matrix Multiplication

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

2×2 2×2 2×2

$\cancel{[3 \times 2] \times [3 \times 2]}$

$[4 \times 10] \times [10 \times 2] = [4 \times 2]$

$$\begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix}\begin{bmatrix} \mathbf{b}_1 & \mathbf{b}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 \cdot \mathbf{b}_1 & \mathbf{a}_1 \cdot \mathbf{b}_2 \\ \mathbf{a}_2 \cdot \mathbf{b}_1 & \mathbf{a}_2 \cdot \mathbf{b}_2 \end{bmatrix}$$

Linear Independence

A set of non-zero vectors $\mathbf{v}_i$, $i = 1, \ldots, n$ is said to be *linearly independent* where $\sum_{i=1}^{n} a_i \mathbf{v}_i = \mathbf{0}$ if and only if $a_i = 0$ for all i.

$$a_1 \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} + a_1 \begin{bmatrix} v_{21} \\ v_{22} \\ v_{23} \end{bmatrix} + a_1 \begin{bmatrix} v_{31} \\ v_{32} \\ v_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$a_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Linear Independence