Introduction to Neural Networks

Johns Hopkins University
Engineering for Professionals Program

605-447/625-438

Dr. Mark Fleischer

Module 7.4: Receiver Operating Characteristics

This Sub-Module Covers …

- Performance evaluation.
- Proper training does not merely involve minimizing the error function using the training data.
- Must have some appropriate way of gauging how well the network performs using data it hasn't seen during the training phase.

Let's Examine Classification Problems

- How well does the network correctly classify an input it has not previously seen?
- Simplest approach is to consider two classes
  - Corresponds nicely to mathematical logic,
  - Corresponds nicely to computer logic,
  - Some action is good/bad, or true/false
- How do we come up with reasonable, mathematically sound ways of evaluating performance in such a context?
- Consider the early days of radar development in WWII.
- Receiver Operating Characteristics.
- Basis is that everyone has their own particular vexations that set them 'off'!

Mandrake, those missiles are coming. Red Alert!

or are they?.........

Our friend Gen. Ripper …

- has an apparent ‘sensitivity’ to communist infiltration, indoctrination, subversion and conspiracies.
- This sensitivity led him to trigger and order …

Receiver Operating Characteristics

- Partition a set into two mutually exclusive subsets:
  - one subset contains entities with a characteristic *C* and the other subset contains contains entities that do not have characteristic *C*.
  - Sets could be a "real target" and "not a target"; or "signal" and "noise"; "Wow" and "no extraterrestrial signal"; "heart disease" and "absence of heart disease", etc. …
- A detector to determine the "detected set" and to indicate which set a detected "signal" belongs to—define set *D*.
  - The detector is not perfect---won't give the correct information all the time.
  - i.e., sometimes it'll 'detect' things that you don't want it to 'detect'.
  - Other times, it'll detect something (send a signal) when it isn't truly warranted.
- Partition D into two mutually exclusive subsets: Entities that are detected and entities that are not detected.

Assessing the Most Basic Classification

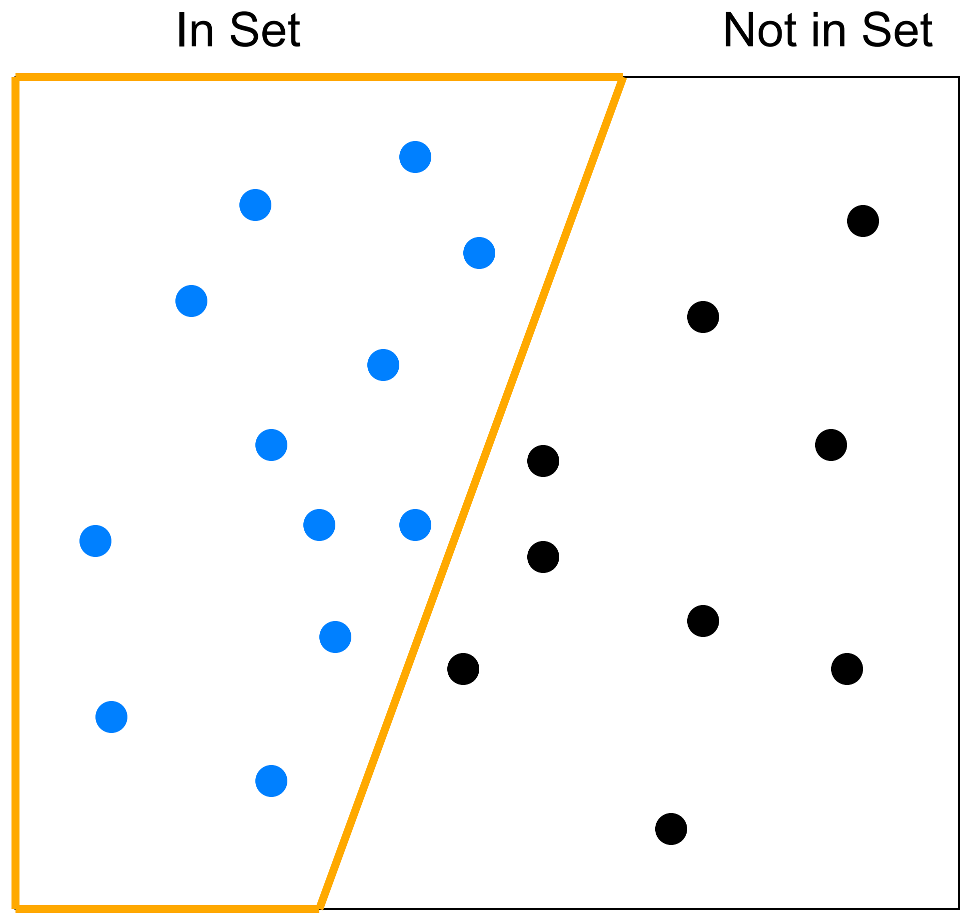
- Either an input produces the desired response, or it does not.

Duh!

Say we want to “detect” only certain types of ‘signals’.

Don’t want to detect signals that are not of this ‘type’.

Receiver Operating Characteristics



The Purpose of Detection

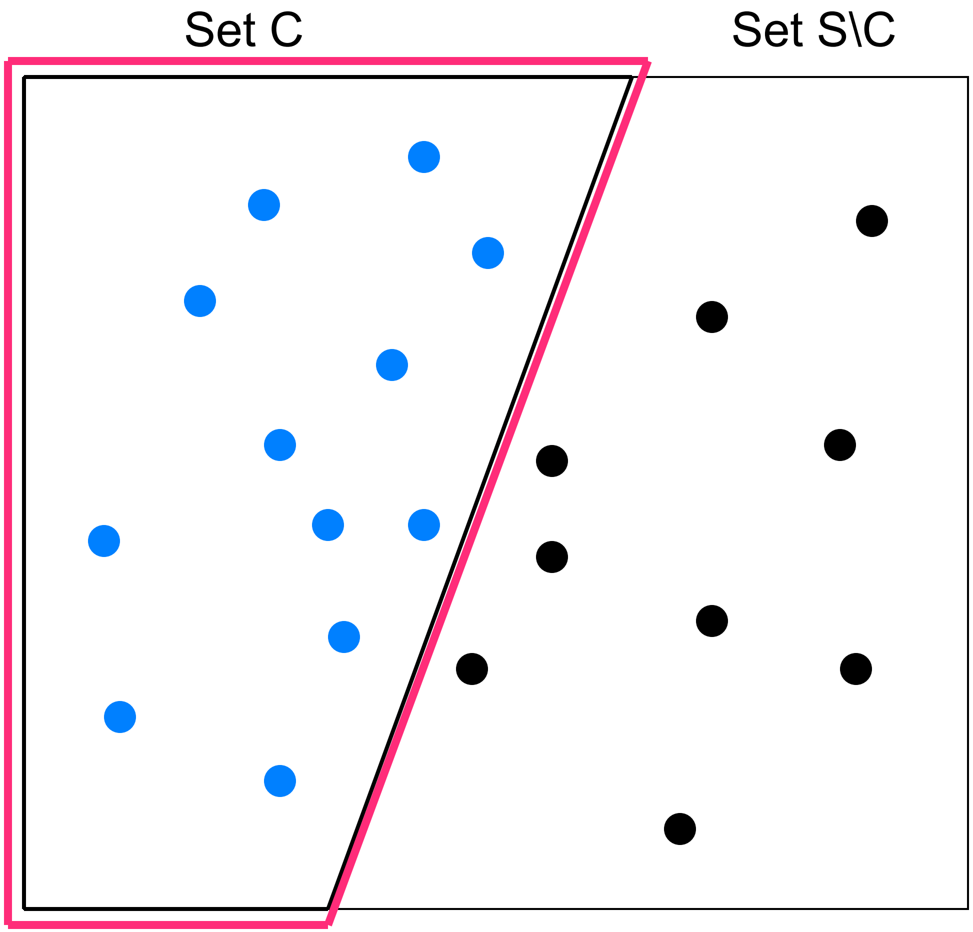
- We want to design our detector system (or medical test or neural network) to function such that a 'detected' signal means that which caused it to be detected also has the 'characteristic'.
- We want detection to be strongly associated with a specified characteristic.

Set Definitions

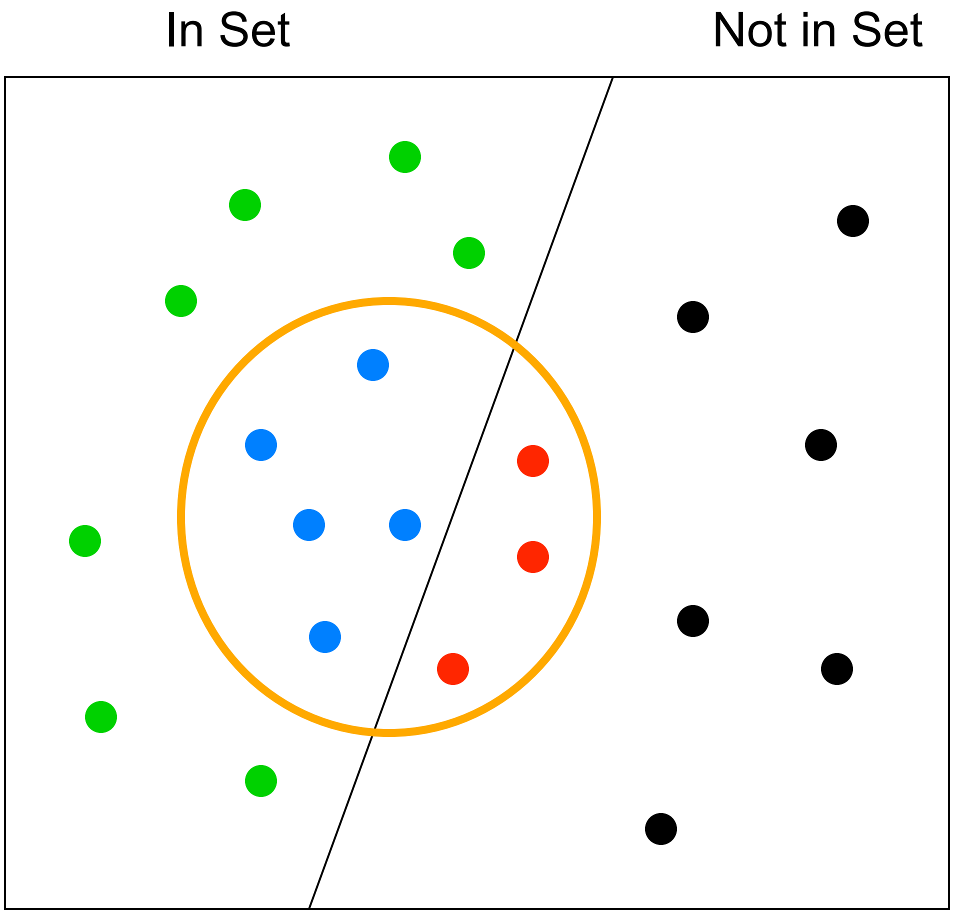
We want to detect signals from set C, i.e., want to detect whether an entity has the characteristic.

- Define the following sets.
  - $S_1 = \{x_i \mid x \in D \wedge x \in C\}$
  - $S_2 = \{x_i \mid x \in C\}$
  - $S_3 = \{x_i \mid x \notin D \wedge x \notin C\}$
  - $S_4 = \{x_i \mid x \notin C\}$

Ideally, we want this



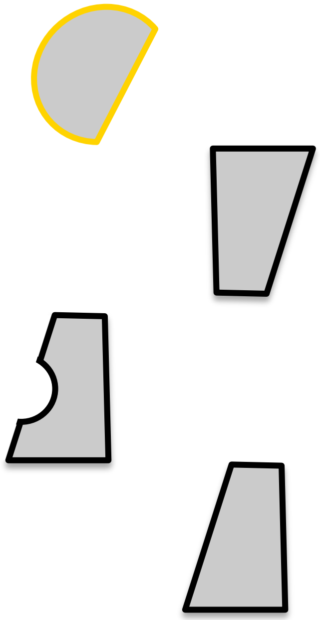
In Reality, we usually have this



Everything in the yellow circle is ‘detected’; i.e., produces a positive test result

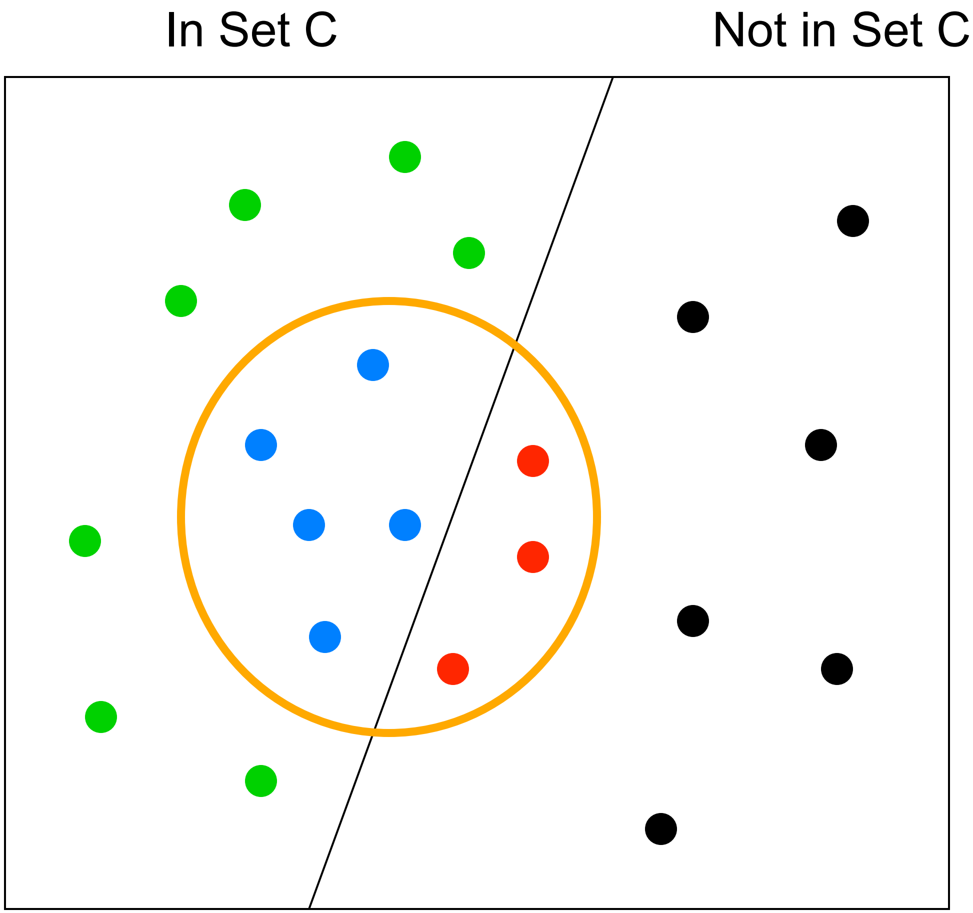
Receiver Operating Characteristics

- Define the following sets.
  - $S_{TP} = \{x_i \mid x \in D \wedge x \in C\}$
  - $S_{TP+FN} = \{x_i \mid x \in C\}$
  - $S_{TN} = \{x_i \mid x \notin D \wedge x \notin C\}$
  - $S_{TN+FP} = \{x_i \mid x \notin C\}$



Note, any element $x_i \notin D$ is an element which tests negative.
Any element $x_i \in D$ is an element which tests positive.

Receiver Operating Characteristics

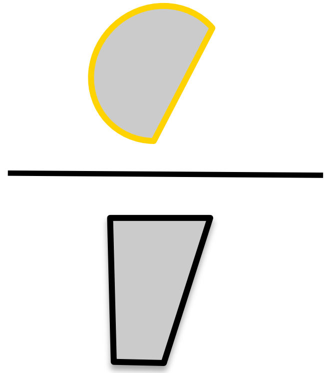


Blue is TP and Red is FP. Green is FN. Black is TN

Some Quantities we are interested in:

$$\Pr\left\{x \in D \middle| x \in C\right\} = \frac{\Pr\left\{x \in D \cap C\right\}}{\Pr\left\{x \in C\right\}}$$

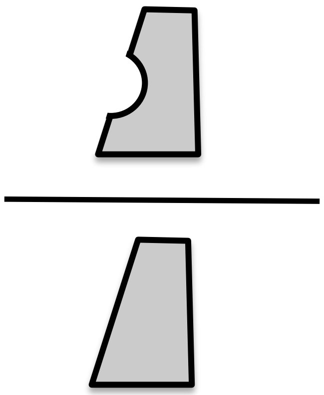
$$\Pr\left\{D \middle| C\right\} = \frac{\Pr\left\{D \cap C\right\}}{\Pr\left\{C\right\}}$$



Sensitivity

Some Quantities we are interested in:

$$\Pr\left\{x \notin D \middle| x \notin C\right\} = \Pr\left\{\bar{D} \middle| \bar{C}\right\} = \frac{\Pr\left\{\bar{D} \cap \bar{C}\right\}}{\Pr\left\{\bar{C}\right\}}$$



Specificity

Receiver Operating Characteristics

Recall the following sets.

- $S_{TP} = \{x_i \mid x \in D \wedge x \in C\}$
- $S_{TP+FN} = \{x_i \mid x \in C\}$
- $S_{TN} = \{x_i \mid x \notin D \wedge x \notin C\}$
- $S_{TN+FP} = \{x_i \mid x \notin C\}$

$$\text{Sensitivity} = \frac{\text{TP}}{\text{TP + FN}}$$

$$\text{Specificity} = \frac{\text{TN}}{\text{TN + FP}}$$

The Terms

Sensitivity: The higher this is, the greater the likelihood that if an object is in the characteristic set, it will result in a positive test result---*i.e.*, it will be detected hence the term.

The Terms

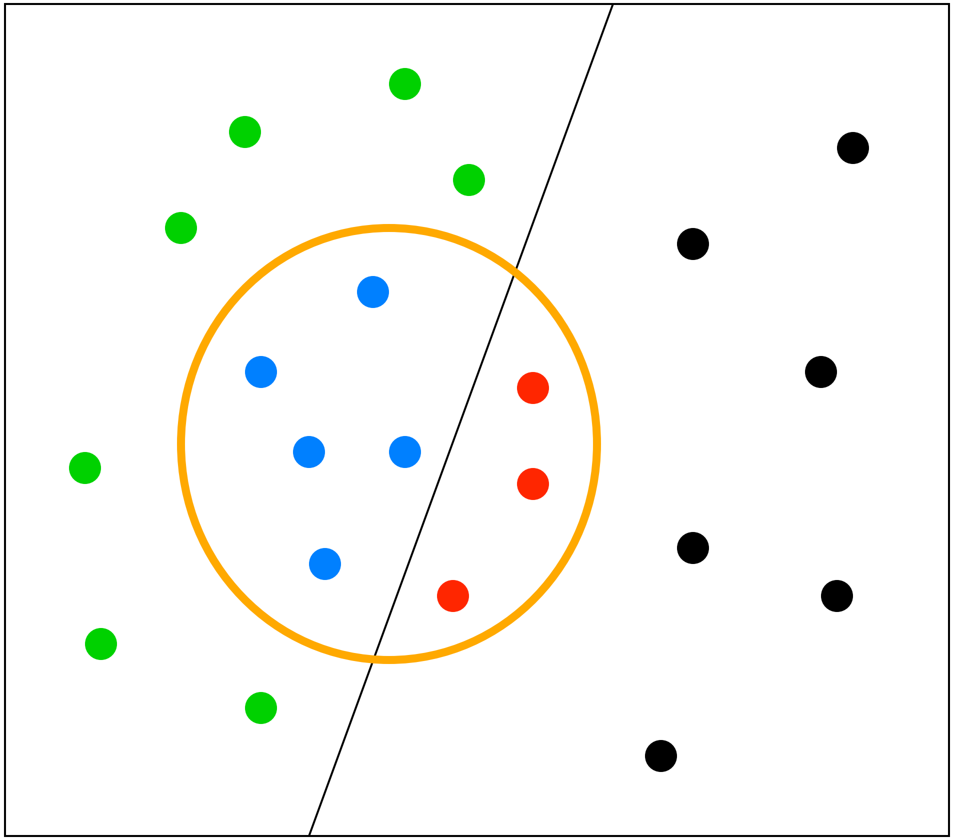
Specificity: A measure of the probability that if an object is *not* in the characteristic set then it will *not* be detected. Thus, the detection is specific to the object having the characteristic.

Matrix Form

	In Set	Not in Set
Test Positive	##	##
Test Negative	##	##

Confusion Matrix

Receiver Operating Characteristics



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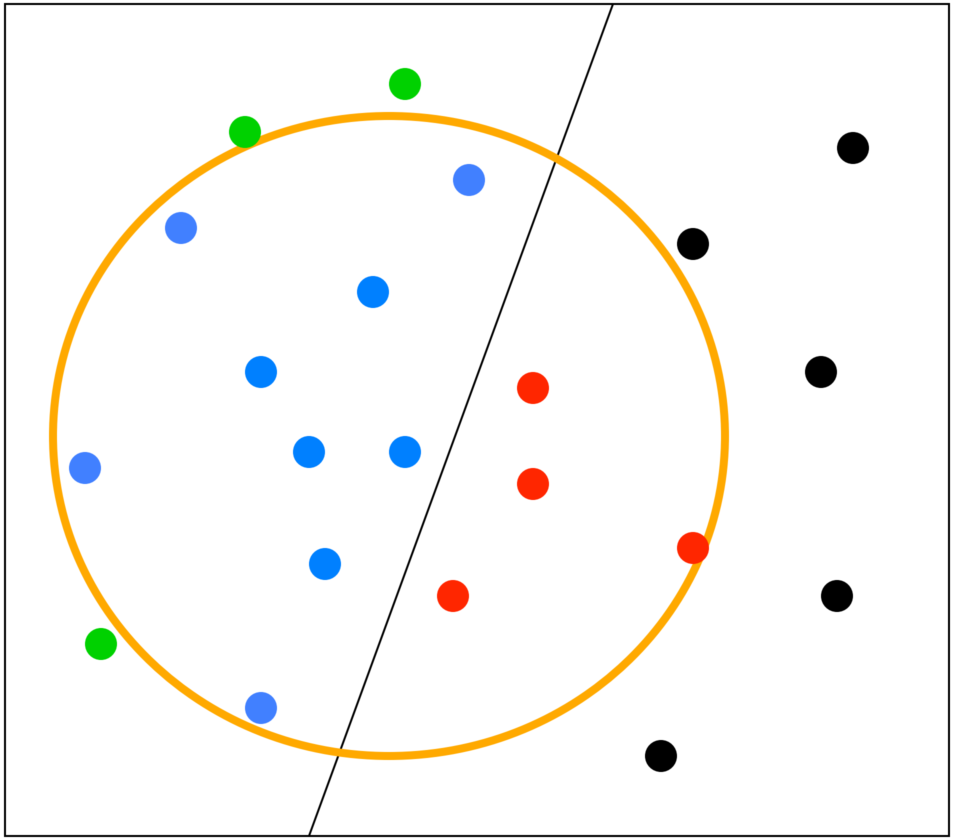
Receiver Operating Characteristics

In Set *C* Not in Set *C*



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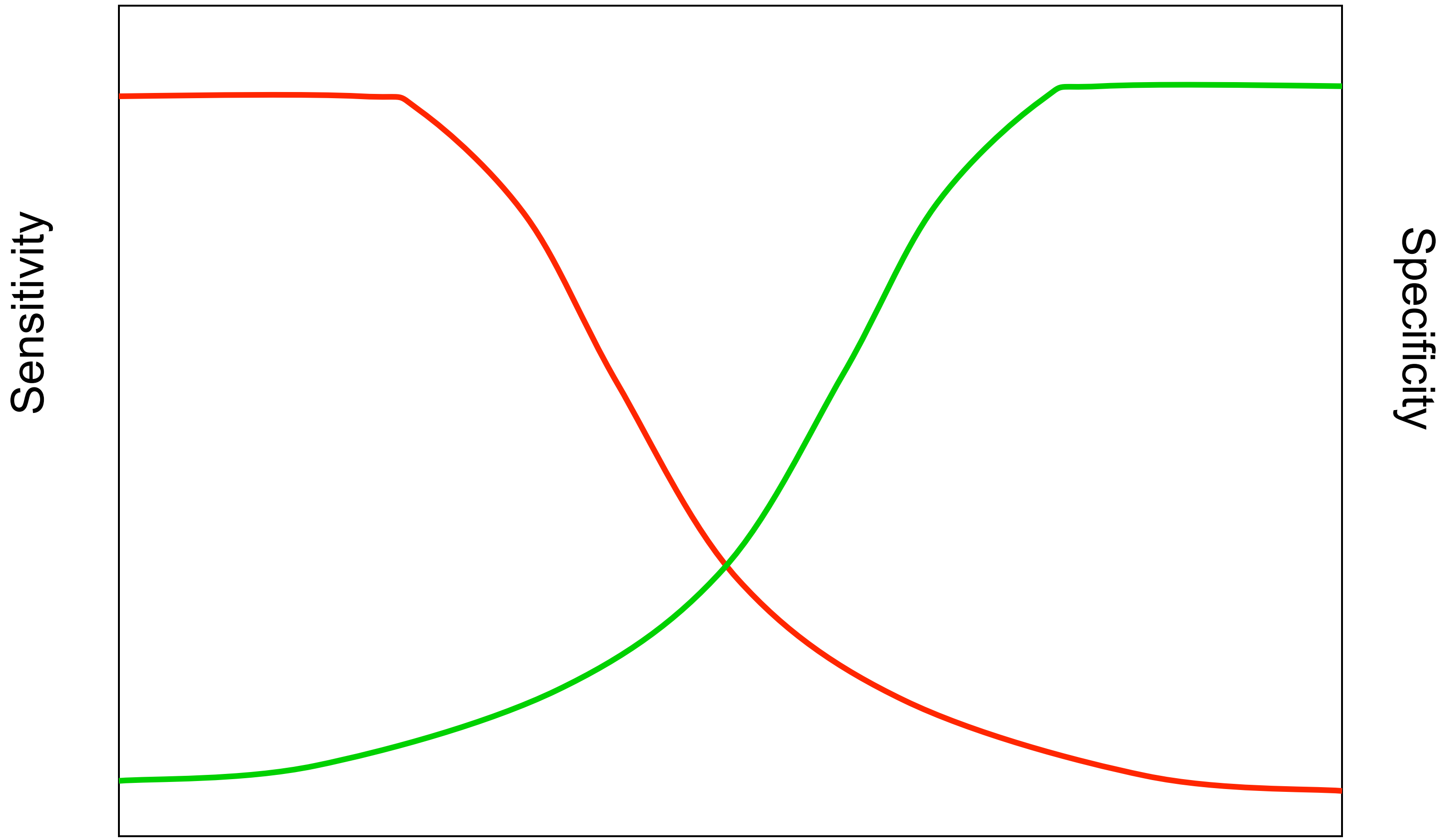
Characteristic Curves



Other Useful Quantities

$$\Pr\left\{x \in C \middle| x \in D\right\} = \Pr\left\{C \middle| D\right\} = \frac{\Pr\left\{C \cap D\right\}}{\Pr\left\{D\right\}}$$

Positive Predictive Value---PPV

$$\Pr\left\{x \notin C \middle| x \notin D\right\} = \Pr\left\{\bar{C} \middle| \bar{D}\right\} = \frac{\Pr\left\{\bar{C} \cap \bar{D}\right\}}{\Pr\left\{\bar{D}\right\}}$$

Negative Predictive Value---NPV

Summary

- ROCs are useful ways of assessing performance of neural networks especially in the context of the accuracy of classification-type applications.
- Can be used with classic MSEs to assess neural network training and performance.