The Johns Hopkins University
JHU Engineering for Professionals Program
NEURAL NETWORKS
Problem Set #6

1. Recall that in the simulated annealing program, the acceptance probability for a candidate solution becoming the current solution is given by

$$\Pr\left\{\text{Accept } X_j \text{ as current solution}\right\} = e^{-\Delta f_{ji}^{+}/t}$$

$$\text{where } \Delta f_{ji}^{+} = \begin{cases} f_j - f_i & \text{if } f_j - f_i > 0 \\ 0 & \text{if } f_j - f_i \le 0 \end{cases}$$

and where $f_i \equiv f(X_i)$

a) If the current objective function value corresponding to current solution i is f_i = 100 and the objective function value of the candidate solution is f_j = 120 and the temperature parameter t is 100, what is an estimate of the probability that the current solution in the next iteration is 120? Is this the maximum probability? Why or why not?

Answer: $e^{-0.20} = 0.81873...$ or 81.8% chance.

b) Given the data in part a, what is the limit of the acceptance probability as $t \to 0$?, as $t \to \infty$?

Answer: As $t \to 0$ the limit of the probability = 0. As $t \to \infty$ the limit of the probability = 1.

2. Recall that the stationary probability in simulated annealing for some energy/objective function state i is given by the Boltzmann Distribution:

$$\pi_i(t) = \frac{e^{-f_i/t}}{\sum_{j=1}^{n} e^{-f_j/t}}$$

If the temperature $t = 1$ was held fixed and only two solutions have the values of f_i = 100 and f_j = 120, what is the stationary probability of visiting states i and j?

Answer: for solution i the Boltzmann numerator is exp(-100/1) = 3.72×10^{-44}
For solution j the Boltzmann numerator is exp(-120/1) = 7.66×10^{-53} thus the Partition function value (the sum of all the numerator values) is approximately

3.72×10^{-44} because the j is sooo much smaller than it is for i. Thus, dividing the number for i with the Partition function value yields a probability of over 0.9999… and thus the probability of solution j is about 1×10^{-9} or on the order of 1 billionth!

3. In a genetic algorithm, a fitness function maps two chromosomes to a *fitness value* based on the following: for odd numbered positions in a string of 8 values indexed from 1 to 8, the values in odd numbered positions get multiplied by 2 and then summed to obtain f_{odd}. The numbers of values in even indexed positions get multiplied by 3 then summed to obtain f_{even}. The fitness value of each chromosome $f = f_{odd} + f_{even}$. Given the following two chromosomes, answer the following questions and clearly indicate your answers.

   {1, 3, 2, 0, 2, 1, 1, 2} and {0, 0, 2, 2, 3, 1, 1, 3}

   a. Calculate the fitness values f of both chromosomes.
   b. Use a cross-over operation like the one in the video to splice the chromosomes between indices 5 and 6 to produce two child chromosomes.
   c. Calculate the fitness value f of each child chromosome.
   d. Do any of the child chromosomes have higher or lower fitness values? If so, indicate which ones have higher/lower fitness values.
   e. If the rule for natural selection requires the next generation to have fitness values at least as good as the fitness values of the current generation of chromosomes, what is the population size in the next generation?
   f. What is a necessary operation to perform before the next generation's fitness values can be initiated? Discuss possible mechanisms for performing this operation.

Answer:

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a)

F1odd = 2*(1,2,2,1) = 12 | F1even = 3*(3,0,1,2) = 18 | F1 = 30

F2odd = 2*(0,2,3,1) = 12 | F2even = 3*(0,2,1,3) = 18 | F2 = 30

b)

Child1 = 13202113 | Child2 = 00223112

c)

Fc1odd = 2*(1,2,2,1) = 12 | Fc1even = 3*(3,0,1,3) = 21 | Fc1 = 33

Fc2odd = 2*(0,2,3,1) = 12 | Fc2even = 3*(0,2,1,2) = 15 | Fc2 = 27

d)

Child 1 has a higher fitness value than both parents.

e) There are three chromosomes in the next generation.

f) Child2 must be eliminated. The remaining 3 chromosomes could be pair as in (1,2), (1,3) and (1,3) and the above process utilized to create child chromosomes. Alternatively, only the 2 chromosomes with the highest fitness could be used. Thus, Child1 with a fitness of 33 could be paired with either 'parent' with a fitness of 30. Kinda creepy though!