The Johns Hopkins University

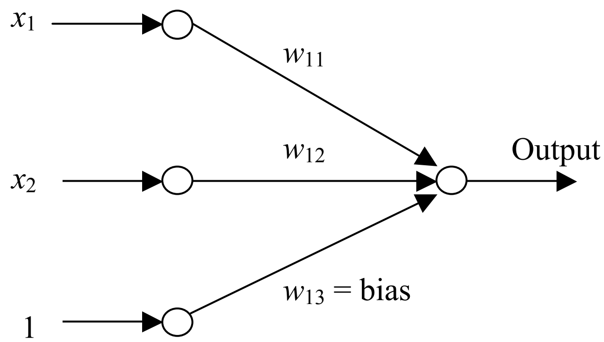
JHU ENGINEERING FOR PROFESSIONALS PROGRAM

NEURAL NETWORKS: 625-438/605-447

Solutions to Problem Set #3

3.1 Determine a standard perceptron solution (*i.e.,* a set of weights w_{11}, w_{12}, and w_{13} = bias) which represents the Logical OR function.

Input		Output
x_1	x_2	
0	0	0
0	1	1
1	0	1
1	1	1



Ans:

By inspection we can guess that

$$\boxed{w_{13} = \text{bias} = -0.5 \qquad w_{11} = 1 \qquad w_{12} = 1}$$

is a solution. This is easily verified

input x_1	x_2	(OR) Target output	Network output with above weights	Thresholded output
0	0	0	$f(0+0-0.5) = f(-0.5)$	0
0	1	1	$f(0+1-0.5) = f(+0.5)$	1
1	0	1	$f(1+0-0.5) = f(+0.5)$	1
1	1	1	$f(1+1-0.5) = f(1.5)$	1

OR solve the following equations

$$\left.\begin{array}{l} w_{11}\cdot 0 + w_{12}\cdot 0 + w_{13} < 0 \\ w_{11}\cdot 0 + w_{12}\cdot 1 + w_{13} \geq 0 \\ w_{11}\cdot 1 + w_{12}\cdot 0 + w_{13} \geq 0 \\ w_{11}\cdot 1 + w_{12}\cdot 1 + w_{13} \geq 0 \end{array}\right\} \quad \begin{array}{l} w_{13} < 0 \\ w_{12} + w_{13} \geq 0 \\ w_{11} + w_{13} \geq 0 \\ w_{11} + w_{12} + w_{13} \geq 0 \end{array}$$

$$\boxed{\begin{array}{l} w_{13} < 0 \\ w_{11} \geq -w_{13} \\ w_{12} \geq -w_{13} \end{array}} \leftarrow$$

Any w_{ij}'s satisfying the set of equations on the left is a solution.

3.2 Consider a standard perceptron as illustrated above except with a threshold value of 0.6, *i.e.,* a function such that

$$f(x) \geq 0.6 \rightarrow 1$$
$$f(x) < 0.6 \rightarrow 0$$

(a) Show that if we don't allow a bias term there doesn't exist a solution (*i.e.,* w_{11} and w_{12}) for the following truth table:

Input		Output
x_1	x_2	
0	0	1
0	1	0
1	0	1
1	1	0

Ans:

Here the "f" threshold is changed to 0.6 so the corresponding threshold on x is .405

x_1	x_2	output
0	0	1
0	1	0
1	0	1
1	1	0

1a/ No bias term

first pattern gives $w_{11} \cdot 0 + w_{12} \cdot 0 \geq .405$

so impossible.

No solution for w_{11}, w_{12}

(b) Now allow a bias term and show that the problem is now solvable by providing a solution for each of the weights

Ans:

(b) Allow bias term

Again A solution (by guessing) is

$$\boxed{w_{11} = 0 \quad w_{12} = -10 \quad w_{13} = 5}$$

which is easily verified
$$\begin{cases} 0\cdot 0 + 0(-10) + 5 = 5 \geq .4 \rightarrow 1 \\ 0\cdot 0 + 1(-10) + 5 = -5 \leq .4 \rightarrow 0 \\ 1\cdot 0 + 0(-10) + 5 = 5 \geq .4 \rightarrow 1 \\ 1\cdot 0 + 1(-10) + 5 = -5 \leq .4 \rightarrow 0 \end{cases}$$
Threshold

OR Again we can solve

$$\left.\begin{aligned} w_{11}\cdot 0 + w_{12}\cdot 0 + w_{13} &\geq .405 \\ w_{11}\cdot 0 + w_{12}\cdot 1 + w_{13} &< .405 \\ w_{11}\cdot 1 + w_{12}\cdot 0 + w_{13} &\geq .405 \\ w_{11}\cdot 1 + w_{12}\cdot 1 + w_{13} &< .405 \end{aligned}\right\} \quad \begin{aligned} w_{13} &\geq .405 \\ w_{12} + w_{13} &< .405 \\ w_{11} + w_{13} &\geq .405 \\ w_{11} + w_{12} + w_{13} &< .405 \end{aligned}$$

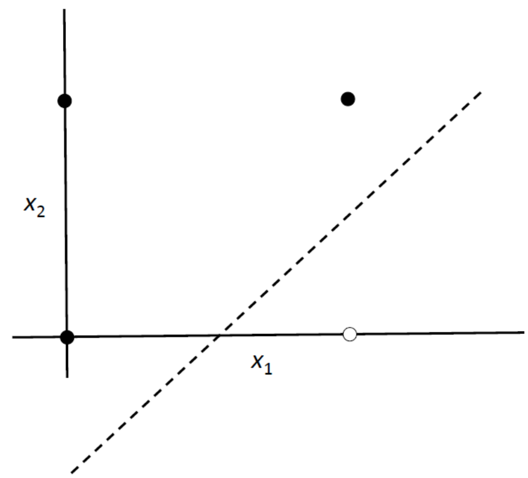
Any set of w_{ij}'s satisfying the above equation is a solution.

3.3 Design (determine weights and bias) of a first-order perceptron that models the implication statement: $x_1 \Rightarrow x_2$.

Ans: The truth table looks like this:

Input		Output
x_1	x_2	$x_1 \Rightarrow x_2$
0	0	1
0	1	1
1	0	0
1	1	1

We can plot the points on a graph as shown below where the equation of the dotted line is $x_1 - x_2 - \frac{1}{2} = A$ and the activation function is defined by $f(A) = \begin{cases} 1 & A > 0 \\ 0 & A \leq 0 \end{cases}$. Thus, the weights are $w_1 = 1$, $w_2 = -1$ and the bias $\theta = -1/2$.



3.4 Find fixed points (*i.e., x* = *f*(*x*)) for the equation $f(x) = \ln(1 + ax)$ where a = 3. Indicate the two fixed point solutions.

Ans: Starting with 5, thru the 30th iterate are:

x	LN(1+3*A1)
5	2.77258872
2.77258872	2.23192292
2.23192292	2.04067067
2.04067067	1.96319027
1.96319027	1.93000879
1.93000879	1.91545483
1.91545483	1.90900376
1.90900376	1.90613096
1.90613096	1.90484897
1.90484897	1.90427635
1.90427635	1.90402048
1.90402048	1.90390612
1.90390612	1.90385501
1.90385501	1.90383216
…	
1.90381369	1.90381369

Thus, the two fixed point solutions are x = 0 and 1.90381369.