Final Exam

Problem 1

Given three nodes in a competitive network running the MAXNET algorithm with *M* nodes, compute the activity function value associated with each node for the next iteration given their current values x_k indicated above the nodes. Use the following formulas:

$$A_j(t) = \sum_{k=1}^{M} w_{jk} x_k(t)$$

$$x_j(t+1) = f_t(A_j(t)) \ \ where$$

$$f_t(z) = \begin{cases} z & \text{for } z \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

and

$$w_{jk} = \begin{cases} 1 & \text{if } j = k \\ -\varepsilon & \text{otherwise} \end{cases} \quad \text{with } \varepsilon = 1/4$$

The current values of the nodes for nodes 1, 2 and 3 respectively are:

7.6, 8.8 and 4.7.

Problems 2 and 3

Consider the 1-D Self-Organizing Map neural network with nodes numbered from 0 to 255 (not all are shown) where the nodes on the two ends are also neighbors (thus the network has a ring-like topology).

RingTopology.pdf

The network has three inputs, each input value varying from 0 to 255, which represent color values for red, blue and green. The nodes' weights are each initialized with uniformly distributed random values between 0 and 255. The black node (see the pdf file) with index m = 2, has weights (95, 2, 255) that are 'closest' to the input vector **x**(k) = [100, 5, 250] presented to the network at time index k. The gray node with index n = 255, has weights [50, 100, 10].

Define the neighborhood function for the SOM where m and n refer to the nodes' indices thusly:

$$N(m,n) = \begin{cases} 0 & \text{if } m = n \\ \min(|m-n|, \, ||m-n| - 256|) & \text{if } m \neq n \end{cases}$$

which basically returns the 'distance' between two nodes --- i.e., the minimum number of nodes from one node m to another n.

Define the functions

$$f(z) = \begin{cases} \frac{1}{z} & for\; z > 0 \\ 0 & for\; z \le 0 \end{cases}$$

and learning parameter $\eta(k) = \dfrac{2}{\ln(10+k)}$ where k corresponds to a time index $\{ 1 \le k < \infty \}$; and the following update formula for the weights of a node n in which a node m corresponds to the node with weights that most closely matches the inputs (i.e., node m is the node that MAXNET indicates 'wins' the competition among all other nodes):

$$\mathbf{w}_n(k+1) = \mathbf{w}_n(k) + \eta(k)\, f(N(m,n)) \left[\mathbf{x}(k) - \mathbf{w}_n(k)\right]$$

Problems 4 - 7

The next question deals with Restricted Boltzmann Machines and asks a question that is easier than at first it might appear. Carefully consider the meaning of the expressions that follow.