The Johns Hopkins University

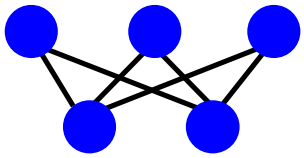
JHU ENGINEERING FOR PROFESSIONALS PROGRAM

NEURAL NETWORKS: 625-438.71

Solutions to Problem Set #11

Let the graph below depict a Restricted Boltzmann Machine with three hidden layer nodes labeled from left to right as 'a', 'b' and 'c' and two visible layer nodes numbered 1 and 2 (from left to right), none of which have any biases, with the following connecting weights:

$w_{1a} = 2$, $w_{1b} = -1$ $w_{1c} = 1$, $w_{2a} = -2$, $w_{2b} = 0$ $w_{2c} = -1$.



Answer the following questions:

a) If the initial visible vector **v** = (1, 1), what are the probabilities that the hidden layer nodes will be 1s?

Ans:

For the first hidden node *a*, the activity function is 1x(2) + 1x(-2) = 0. Hence, the probability for node a having a state of 1 is $1/(1+e^0) = ½$.

For the second hidden node *b*, the activity function is 1x(-1) + 1x0 = -1. Hence, the probability for node a having a state of 1 is $1/(1+e^1) = 0.2689$.

For the third hidden node *c*, the activity function is 1x(1) + 1x(-1) = 0. Hence, the probability for node a having a state of 1 is $1/(1+e^0) = ½$.

b) Use the following random numbers to determine the states of the hidden layer nodes given the probabilities you obtained in part a): 0.87, 0.14, 0.64.

Ans:

Is $0.87 \leq 0.5$: false →0; Is $0.14 \leq 0.2689$: true →1; Is $0.64 \leq 0.5$: false →0.
Therefore, the hidden vector is (0,1,0).

c) Given the states you obtained in part b) for the hidden layer nodes, what are the probabilities that the visible layer nodes will be 1s?

Ans:

For the first visible node 1, the activity function is 0x(2) + 1x(-1) + 0x(1) = -1. Hence, the probability for node a having a state of 1 is $1/(1+e^1) = 0.2689$.

For the second visible node 2, the activity function is 0x(-2) + 1x(0) + 0x(-1) = 0. Hence, the probability for node a having a state of 1 is $1/(1+e^0) = 0.5$.

d) Use the following random numbers to determine the reconstructed states of the visible layer nodes given the probabilities you obtained in part b): 0.25, 0.72.

Is $0.25 \leq 0.2689$: true $\rightarrow$1; Is $0.72 \leq 0.5$: false $\rightarrow$0;
Therefore, the visible vector is (1,0).