The Johns Hopkins University
JHU ENGINEERING FOR PROFESSIONALS PROGRAM
NEURAL NETWORKS: 625-438.71

Solutions to Problem Set #8

8.1 Can the following matrices represent the weight matrix for a Hopfield net? Please explain why or why not.

(a)

$$\begin{bmatrix} 0 & 2 & -2 \\ -2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix}$$

Answer: No because it is not symmetric.

(b)

$$\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

Answer: No because not all diagonal elements are 0.

8.2 Can the following matrix represent the weight matrix for a Hopfield net that has been trained to recognize 3 exemplars? Please explain why or why not.

$$\begin{bmatrix} 0 & 2 & -2 \\ 2 & 0 & 2 \\ -2 & 2 & 0 \end{bmatrix}$$

Answer: No because it is not possible for 3 exemplars to result in values of either 2 or -2. Consider the possible combinations of matrix elements that are summed from three exemplars (there are 8 possible 3-tuples):
1: (-1 -1 -1) = -3 **2:** (-1 -1 +1)= -1 **3:** (-1 +1 – 1) = -1 **4:** (-1 + 1 + 1) = 1
5: (1 – 1 -1) = -1 **6:** (1 – 1 + 1) = 1 **7:** (1 + 1 – 1) = 1 **8:** (1 + 1 + 1) = 3
Hence it is impossible for a matrix term to equal 2 or -2.

8.3 For the matrix in 8.2, can it represent the weight matrix for a Hopfield net that has been trained to recognize 2 exemplars? Please explain why or why not.

Answer: For two exemplars **p** = (p_1 p_2 p_3) **and q** = (q_1 q_2 q_3) we get the following three equations from their outer products and the resultant matrix of the Hopfield net: $p_1p_2 + q_1q_2 = 2$ $\quad$ $p_1p_3 + q_1q_3 = -2$ $\quad$ $p_2p_3 + q_2q_3 = 2$
Note there are 8 possible vectors **p.** If p1 and p2 are both -1, then clearly q1q2 = 1 and so q1 = q2. Likewise if p1 and p2 are both 1. Thus, we get the following implication: If p1 = p2, then q1 = q2 in order for the first equation to be satisfied. In fact, for the first equation to be satisfied, p1 **must equal** p2 and similarly for p2

and p3 (from the third equation). Thus, p1 = p2 = p3 and q1 = q2 = q3. If that is the case, then the second equation is impossible. Thus, this matrix **cannot** be the weight matrix for 2 exemplars. Surprise!

8.4 Given the Hopfield net weight matrix

$$W_f = \begin{bmatrix} 0 & -2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

answer the following questions.

(a) Starting with the input vector **e** = [-1 -1 1] and using synchronous updating, iterate using the scheme described in class to show the sequence of output vectors until you can show that it either converges or does not converge.

Answer:

$$W_f e = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \to \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \qquad W\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} \to \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

$$W\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \to \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

and we see that the system oscillates.

(b) **(extra)** In part (a), suppose that we had used asynchronous updating. Would the result have been the same? Why or why not? **Note:** you do <u>not</u> need to iterate to answer this part.

Answer: Using the energy function described in class, asynchronous updating reduces the energy function or keeps it the same, hence cannot return to a previous state. Thus, the result would not have been the same.