Introduction to Neural Networks

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Module 3.1: Basic Symbolic Logic

This Sub-Module Covers …

- Basic review of Symbolic Logic and Truth Tables.
- Rules of Inference.
- The Truth Value of Compound Statements
- Perceptrons and Logic.

What is …

Truth?

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msnbc

… information or statements on which we can act with confidence.

Meaningful … but vague!

Learning Truth

Young children and babies often learn how to assess things in their brand new world, but often in a very dualistic fashion!

“Is that bad?”

Let's keep things simple…

- Avoid all the vagaries of the human condition.
- Simplify issues … make them amenable to analysis.
- Provide for a rich set of possibilities … allow encoding of all the shades of gray.

Logic-Truth Tables

```
True  = 1
False = 0
```

AND

A	B	A∧B
0	0	0
0	1	0
1	0	0
1	1	1

OR

A	B	A∨B
0	0	0
0	1	1
1	0	1
1	1	1

Logic-Truth Tables

NAND

A	B	$\overline{A \wedge B}$
0	0	1
0	1	1
1	0	1
1	1	0

Logic-Truth Tables

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \vee \overline{B}$
0	0	1	1	
0	1	1	0	
1	0	0	1	
1	1	0	0	

Logic-Truth Tables

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \vee \overline{B}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

Logic-Truth Tables

NAND

A	B	$\overline{A \wedge B}$
0	0	1
0	1	1
1	0	1
1	1	0

$\equiv$

Not A OR Not B

A	B	$\overline{A} \vee \overline{B}$
0	0	1
0	1	1
1	0	1
1	1	0

Logically Equivalent

$$\overline{A \wedge B} \equiv \overline{A} \vee \overline{B}$$

Rules of Inference

What does $A \Rightarrow B$ mean?

Recited often as

A implies B… or

If A then B…

But what does this mean?

Answer:

If A is a true statement, then B is a true statement.

Sometimes stated as:

A is sufficient for B, or

B is a necessary consequence of A.

Truth Table of $A \Rightarrow B$

A	B	$A \Rightarrow B$
0	0	1
0	1	1
1	0	0
1	1	1

Logic-Truth Tables

NAND

A	B	$\overline{A \wedge B}$
0	0	1
0	1	1
1	0	1
1	1	0

XOR

A	B	$A \otimes B$
0	0	0
0	1	1
1	0	1
1	1	0

Evaluating Compound Statements

NAND

A	B	$\overline{A \wedge B}$
0	0	1
0	1	1
1	0	1
1	1	0

Not A OR Not B

A	B	$\overline{A} \vee \overline{B}$
0	0	1
0	1	1
1	0	1
1	1	0

How would we determine the truth value of this statement?

$$\overline{A \wedge B} \Rightarrow \overline{A} \vee \overline{B}$$

A' B'

Compound Statements

A	B	$A \Rightarrow B$
0	0	1
0	1	1
1	0	0
1	1	1

A' B'

A	B	$\overline{A \wedge B}$	$\overline{A} \vee \overline{B}$	$\overline{A \wedge B} \Rightarrow \overline{A} \vee \overline{B}$
0	0	1	1	1
0	1	1	1	1
1	0	1	1	1
1	1	0	0	1

Tautology

Really Compound Statements!

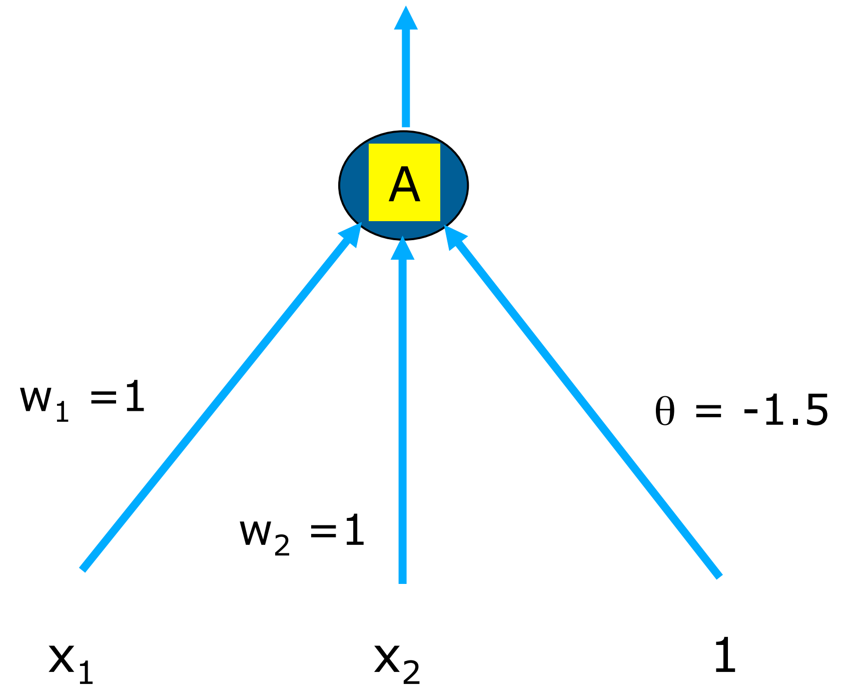
$$[(A \Rightarrow B) \wedge (B \Rightarrow C)] \Rightarrow (A \Rightarrow C)$$

Rule of Inference
Basis of Deductive Reasoning

A	B	C	$A \Rightarrow B$	$B \Rightarrow C$	$\overbrace{(A \Rightarrow B) \wedge (B \Rightarrow C)}^{A'}$	$\overbrace{A \Rightarrow C}^{B'}$	$[(A \Rightarrow B) \wedge (B \Rightarrow C)] \Rightarrow (A \Rightarrow C)$
0	0	0	1	1	1	1	1

Can Perceptrons Model AND?

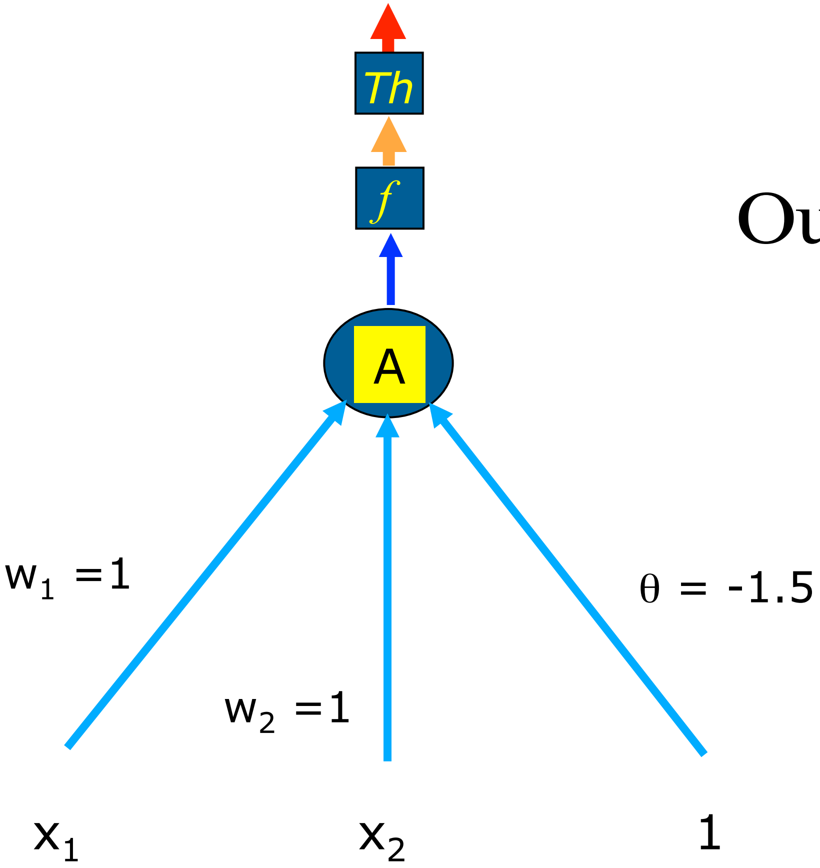
A	B	A∧B
0	0	0
0	1	0
1	0	0
1	1	1



$$A = w_1x_1 + w_2x_2 + \theta = 1 + 1 - 1.5 = 0.5$$

We can now input this value into the activation function.

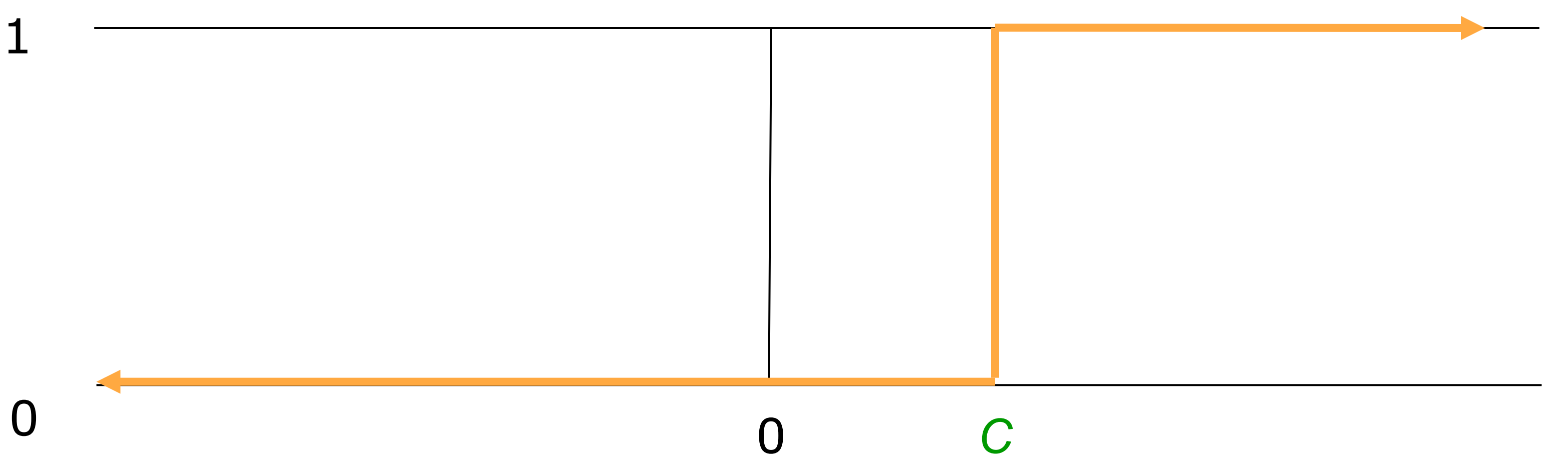
Threshold Logic



$$\text{Output} = \begin{cases} 1 & \text{if } f(A) \geq C \\ 0 & \text{otherwise} \end{cases}$$

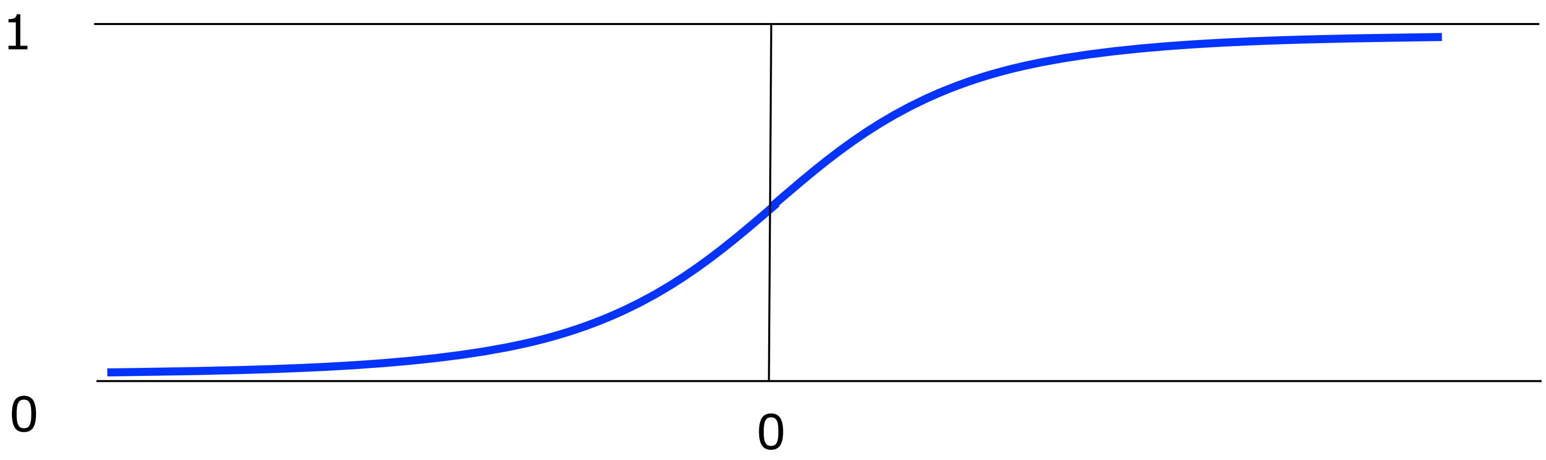
Threshold Logic

$$\text{Output} = \begin{cases} 1 & \text{if } f(A) \geq C \\ 0 & \text{otherwise} \end{cases}$$

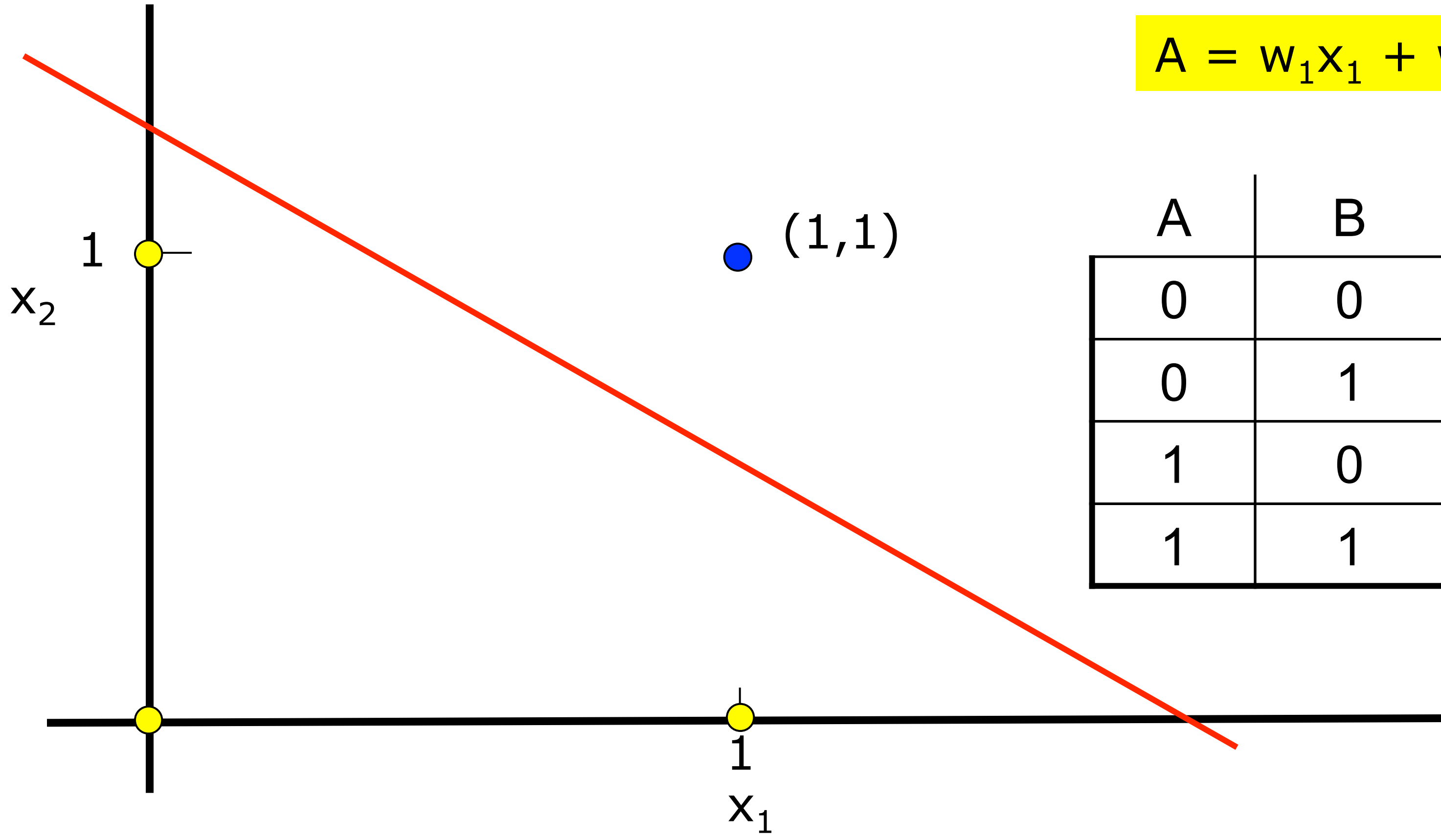


Activation Function

$$f(A) = \frac{1}{1+e^{-A}}$$



A New Angle to Perceptrons



$A = w_1x_1 + w_2x_2$

A	B	A∧B
0	0	0
0	1	0
1	0	0
1	1	1