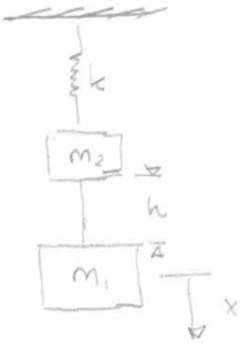
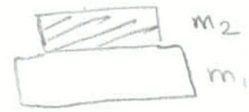


find governing eqn of motion: $x(t)$.

after impact



initial conditions $x(t=0) = 0\,m$

need 2nd I.C. → find velocity by relating PE_{spring} to KE
(no gravity due to static equilibrium)

$$\dot{x}(t=0) = V_0$$

$$W_2 h = m_2 g h$$

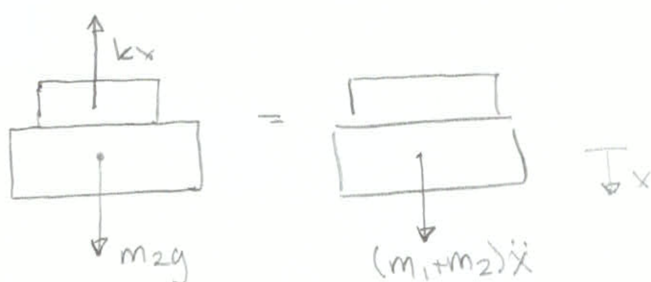
KE: $T = m_2 g h$ for $t \geq 0$ need to take into acct 2 masses

$$T = \frac{1}{2}(m_1+m_2)V_0^2 = m_2 g h$$

$$V_0 = \sqrt{\frac{2m_2 g h}{m_1+m_2}}$$

$$\therefore \dot{x}(t=0) = \sqrt{\frac{2m_2 g h}{m_1+m_2}}$$

FBD Analysis



$$\Sigma F = ma$$

$$-kx + m_2 g = (m_1+m_2)\ddot{x}$$

$$(m_1+m_2)\ddot{x} + kx = m_2 g$$

$$\ddot{x} + \frac{k}{m_1+m_2}x = \frac{m_2 g}{m_1+m_2} \quad (1)$$

solving eqn (1) using diff eqns.

$$x(t) = x_h(t) + x_p(t)$$

finding homogeneous soln $[x_h(t)]$

1) set RHS = 0

$$\ddot{x} + \frac{k}{m_1+m_2}x = 0$$

let $\omega_n^2 = \frac{k}{m_1+m_2}$

$$\therefore \ddot{x} + \omega_n^2 x = 0$$

2. solve for roots of eqn

$\ddot{x} \rightarrow r^2$
$\dot{x} \rightarrow r$
$x \rightarrow 1$

$r^2 + \omega_n = 0$

$r = \pm i\omega_n$

$\therefore$ we have complex roots (form: $r = a \pm ib$)
So $x_n(t)$ has form:

$$X_n(t) = e^{-at}\left[A\sin(bt) + B\cos(bt)\right]$$

$a = 0$
$b = \omega_n$

so: $X_h(t) = A\sin(\omega_n t) + B\cos(\omega_n t)$

ASIDE: other soln forms

2 distinct, real roots: $r_{1,2} = \lambda_1, \lambda_2$

$$X_n(t) = Ae^{-\lambda_1 t} + Be^{-\lambda_1 t}$$

repeated, real roots: $r_{1,2} = \lambda$

$$X_n(t) = Ae^{-\lambda t} + Bte^{\lambda t}$$

---

finding particular soln $[x_p(t)]$

1. examine RHS of eqn, $x_p(t)$, must be in form of RHS.

$$RHS = \frac{m_2 g}{m_1 + m_2} = Const$$

$\therefore x_p(t) = C_1$

2. plug $x_p(t)$ into original diff. eqn.

$$\cancel{\ddot{x}_p}^{\,0} + \frac{k}{m_1 + m_2} x_p = \frac{m_2 g}{m_1 + m_2}$$

$$\frac{k}{\cancel{m_1 + m_2}} C_1 = \frac{m_2 g}{\cancel{m_1 + m_2}}$$

$$C_1 = \frac{m_2 g}{k}$$

$$\therefore x_p(t) = \frac{m_2 g}{k}$$

ASIDE x2: other $x_p(t)$ soln forms.

$$\left[\begin{array}{l} RHS = e^{-at} \\ x_p(t) = C_1 e^{-at} \end{array}\right.$$

if real, repeated roots w/ $r = \lambda_1 = a$
multiply x_p by t^2 to get
$x_p(t) = C_1 t^2 e^{-at}$

if real, distinct roots w/ $r = \lambda_1, \lambda_2 = a, \lambda_2$
multiply x_p by t to get
$x_p(t) = C_1 t e^{-at}$

$$\left[\begin{array}{l} RHS = \cos(bt) \\ x_p(t) = C_1 \sin(bt) + C_2 \cos(bt) \end{array}\right.$$

similarly if RHS has sin.

$$x(t) = x_p(t) + x_n(t)$$

$$x(t) = \frac{m_2 g}{k} + A\sin(\omega_n t) + B\cos(\omega_n t)$$

use I.C.

$$x(t=0) = 0 = \frac{m_2 g}{k} + \cancel{A\sin(0)}^{0} + B\cos(0)$$

$$B = -\frac{m_2 g}{k}$$

$$\dot{x}(t) = A\omega_n \cos(\omega_n t) - B\omega_n \sin(\omega_n t)$$

$$\dot{x}(t=0) = \sqrt{\frac{2m_2 gh}{m_1 + m_2}} = A\omega_n \cos(0) - \cancel{B\omega_n \sin(0)}$$

Note: $\omega_n = \sqrt{\frac{k}{m_1 + m_2}}$

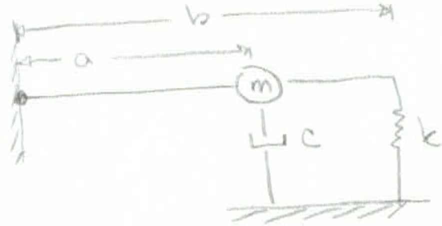
$$A = \frac{1}{\omega_n}\sqrt{\frac{2m_2 gh}{m_1 + m_2}}$$

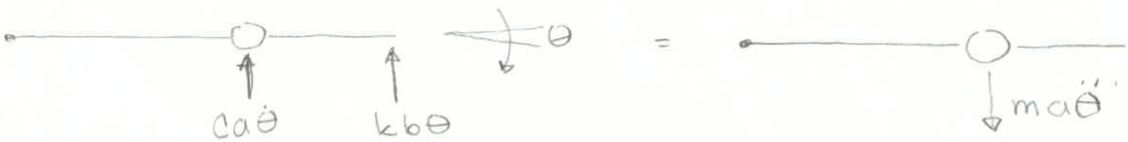
$$A = \frac{1}{\sqrt{k/m_1 + m_2}}\left[\sqrt{\frac{2m_2 gh}{m_1 + m_2}}\right] = \sqrt{\frac{\cancel{m_1 + m_2}}{k}}\sqrt{\frac{2m_2 gh}{\cancel{m_1 + m_2}}}$$

$$A = \sqrt{\frac{2m_2 gh}{k}}$$

$$\boxed{x(t) = \frac{m_2 g}{k} + \sqrt{\frac{2m_2 gh}{k}}\sin(\omega_n t) - \frac{m_2 g}{k}\cos(\omega_n t)}$$

2.42





$$\circlearrowleft{+})\ (ma\ddot{\theta})a = (-ca\dot{\theta})a + (-kb\theta)b$$

$$ma^2\ddot{\theta} + ca^2\dot{\theta} + kb^2\theta = 0$$

$$\ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{k}{m}\frac{b^2}{a^2}\theta = 0$$

$$\boxed{\ddot{\theta} + \frac{c}{m}\dot{\theta} + \frac{k}{m}\left(\frac{b}{a}\right)^2\theta = 0}$$

$$\omega_n^2 = \frac{k}{m}\left(\frac{b}{a}\right)^2$$

$$\boxed{\omega_n = \frac{b}{a}\sqrt{\frac{k}{m}}}$$ ← natural frequency

$$2\zeta\omega_n = \frac{c}{m}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{c}{2m\left(\frac{b}{a}\right)\sqrt{\frac{k}{m}}}$$

$$\zeta = \frac{ac}{2b\sqrt{mk}}$$

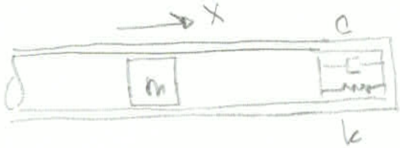
$$\zeta = \frac{c}{C_c}$$

$$C_c = \frac{c}{\zeta} = \frac{\cancel{c}}{\frac{a\cancel{c}}{2b\sqrt{mk}}}$$

$$\boxed{C_c = \frac{2b\sqrt{mk}}{a}}$$ ← critical damping coefficient

Damped Natural Frequency → $\omega_d = \frac{2\pi}{\tau_d} = \sqrt{1-\zeta^2}\,\omega_n = \sqrt{1 - \frac{ac^2}{4b^2mk}}\left(\frac{b}{a}\sqrt{\frac{k}{m}}\right) = \boxed{\sqrt{\frac{k}{m}\left(\frac{b}{a}\right)^2 - \left(\frac{c}{2m}\right)^2}}$

2.47



$m = 4.53\,kg$

$\dot{x} = 15.24\,\frac{m}{s}$

$c = 1.75\,\frac{N\text{-}s}{cm}$

$k = 350\,\frac{N}{cm}$

email out

$$\omega_n = \sqrt{\frac{35000}{4.53}} = 87.89\,\frac{rad}{s}$$

$$\tau = \frac{2\pi}{\omega_n} = \frac{2\pi}{87.89}\,s = 0.0715\,s$$

$$\zeta = \frac{c}{2\sqrt{km}} = \frac{c}{c_c}$$

$$c_c = 2\sqrt{km} = 2\sqrt{(35000)(4.53)} = 797.04\,\frac{N\text{-}s}{m}$$

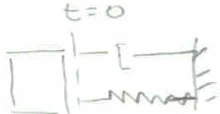
$$\zeta = \frac{c}{c_c} = \frac{1.75\times10^2}{797.04} = 0.2197 \rightsquigarrow \text{underdamped}$$

$$\tau_D = \sqrt{1-\zeta^2}\,\tau = \sqrt{1-(0.2197)^2}\,(0.0715\,s) = 0.0697\,s$$

$\zeta < 1$, underdamped

$$x(t) = e^{-\zeta\omega_n t}\left[\frac{\dot{x}_0 + \cancel{\zeta\omega_n x_0}}{\omega_n\sqrt{1-\zeta^2}}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right) + \cancel{x_0\cos\left(\sqrt{1-\zeta^2}\,\omega_n t\right)} \to 0\right]$$

$t=0$



$x_0 = x(0) = 0\,m$

$\dot{x}_0 = \dot{x}_0(0) = 20\,m/s$

at x_{max} $\sin(\quad) \to 1$, $\omega_n t = \pi/2$ as $\sqrt{1-\zeta^2} \rightsquigarrow 1$

b/c ζ^2 is v. small.

$$x_{max} = e^{-0.2197(\pi/2)}\left[\frac{15.24}{87.89\sqrt{1-0.2197^2}}\right] = \boxed{0.1259\,m}$$

$$\omega_n t = \frac{\pi}{2}$$

$$t = \frac{\pi}{2\omega_n} = \frac{\pi}{2(87.89)} = \boxed{0.0174\,s}$$