Problem 4

NACA 2412 mean camber line:

$$z/c = 0.125\left[0.8\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right] \qquad 0 \le \frac{x}{c} \le 0.4$$

$$z/c = 0.0555\left[0.2 + 0.8\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right] \qquad 0.4 \le \frac{x}{c} \le 1$$

Compute: (a) C_ℓ vs. α
(b) $C_{m,c/4}$ (1)

(a) $C_\ell = 2\pi\left[A_0 + \frac{1}{2}A_1\right] = 2\pi\left[\alpha + \frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(\cos\theta_0 - 1)\,d\theta_0\right]$

Find $\frac{dz}{dx}$:

* $\frac{x}{c} = \frac{1}{2}(1 - \cos\theta_0)$ (Transformation) (1)

$$\frac{dz}{dx} = \cancel{dx}\, c\, \frac{d(z/c)}{dx} = \frac{c}{\cancel{c}}\, \frac{d(z/c)}{d(x/c)} = \frac{d(z/c)}{d(x/c)}$$

$$\frac{d}{d(x/c)}\left[0.125\left(0.8\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right)\right] = 0.125\left[0.8 - 2\left(\frac{x}{c}\right)\right]$$

and $$\frac{d}{d(x/c)}\left[0.0555\left(0.2 + 0.8\left(\frac{x}{c}\right) - \left(\frac{x}{c}\right)^2\right)\right] = 0.0555\left[0.8 - 2\left(\frac{x}{c}\right)\right]$$

$$\frac{dz}{dx} = \begin{cases} 0.125\left[0.8 - 2\left(\frac{x}{c}\right)\right] & 0 \le \frac{x}{c} \le 0.4 \\ 0.0555\left[0.8 - 2\left(\frac{x}{c}\right)\right] & 0.4 \le \frac{x}{c} \le 1 \end{cases}$$ (1)

We need to break the integrals into two parts!

1.) Transform $\frac{x}{c} \rightarrow f(\theta_0)$
2.) Determine the limits of integration in terms of θ_0.

$$\frac{dz}{dx} = \begin{cases} 0.125\left[0.8 - (1 - \cos\theta_0)\right] \\ 0.0555\left[0.8 - (1 - \cos\theta_0)\right] \end{cases} = \begin{cases} 0.125\left[\cos\theta_0 - 0.2\right] \\ 0.0555\left(\cos\theta_0 - 0.2\right) \end{cases}$$

$0 \le \frac{x}{c} \le 0.4$: $\frac{x}{c} = \frac{1}{2}(1 - \cos\theta_0)$

$\cos\theta_0 = 1 - 2\left(\frac{x}{c}\right)$

$\theta_0 = \cos^{-1}\left(1 - 2\left(\frac{x}{c}\right)\right)$

$\frac{x}{c} = 0 \rightarrow \theta_0 = \cos^{-1}(1)$

$\boxed{\theta_0 = 0}$

$\frac{x}{c} = 0.4 \rightarrow \theta_0 = \cos^{-1}\left(\frac{1}{5}\right)$

$\boxed{\theta_0 = 1.37 \text{ rad}}$

$\frac{x}{c} = 1 \rightarrow \theta_0 = \cos^{-1}(-1)$

$\boxed{\theta_0 = \pi \text{ rad}}$

(1)

$$\text{Integral} = \frac{1}{\pi}\int_0^{1.37} \underbrace{0.125(\cos\theta_0 - 0.2)(\cos\theta_0 - 1)}_{I1}\, d\theta_0 + \frac{1}{\pi}\int_{1.37}^{\pi} \underbrace{0.0555(\cos\theta_0 - 0.2)(\cos\theta_0 - 1)}_{I2}\, d\theta_0$$

$$I1: \frac{0.125}{\pi}\int_0^{1.37} \left[\cos^2\theta_0 - \cos\theta_0 - 0.2\cos\theta_0 + 0.2\right] d\theta_0$$

$$= \cos^2\theta_0 - 1.2\cos\theta_0 + 0.2$$

$$= \frac{1}{2}(\cos 2\theta_0 + 1) - 1.2\cos\theta_0 + 0.2$$

$$= \frac{1}{2}\cos(2\theta_0) - 1.2\cos\theta_0 + 0.7$$

$$I1 = \frac{0.125}{\pi}\left[\frac{1}{4}\sin 2\theta_0 - 1.2\sin\theta_0 + 0.7\theta_0\right]_0^{1.37}$$

$$I1 = \frac{0.125}{\pi}\left[-0.119 - 0\right]$$

$$\boxed{I1 = -0.0047 \text{ rad}}$$

$$I2 = \frac{0.0555}{\pi}\left[\frac{1}{4}\sin 2\theta_0 - 1.2\sin\theta_0 + 0.7\theta_0\right]_{1.37}^{\pi}$$

$I2 = 0.039 - (-0.002)$

$\boxed{I2 = 0.041}$ rad

$Integral = -0.0017 + 0.041 = 0.036$ rad

$2\pi\alpha + 0.226$

$\boxed{C_\ell = 2\pi(\alpha + 0.036)}$ (2) $\begin{cases} 1 \Rightarrow I_1 \\ 1 \Rightarrow I_2 \end{cases}$

(b) $C_{m,c/4} = \frac{\pi}{4}(A_2 - A_1)$ (2) $\qquad A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos(n\theta_0)\,d\theta_0$

$A_2 = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos(2\theta_0)\,d\theta_0$ → (1)

$A_1 = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos(\theta_0)\,d\theta_0$ →

$$C_{m,c/4} = \frac{\pi}{4}\left[\frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos(2\theta_0)\,d\theta_0 - \frac{2}{\pi}\int_0^\pi \frac{dz}{dx}\cos(\theta_0)\,d\theta_0\right]$$

$$= \frac{1}{2}\int_0^\pi \frac{dz}{dx}\left[\cos(2\theta_0) - \cos\theta_0\right]d\theta_0$$

$$= \frac{1}{2}\int_0^\pi B(\cos\theta_0 - 0.2)(\cos 2\theta_0 - \cos\theta_0)\,d\theta_0 \qquad B = \begin{cases} 0.125 \\ 0.0555 \end{cases}$$

$$= \frac{B}{2}\int_0^\pi \left[\cos\theta_0\cos 2\theta_0 - \cos^2\theta_0 - 0.2\cos 2\theta_0 + 0.2\cos\theta_0\right]d\theta_0$$

$$= \frac{B}{2}\int_0^\pi \left[2\cos^3\theta_0 - \cos\theta_0 - \cos^2\theta_0 - 0.2\cos 2\theta_0 + 0.2\cos\theta_0\right]d\theta_0$$

$$= \frac{B}{2}\int_0^\pi \left[2\cos^3\theta_0 - \cos^2\theta_0 - 0.2\cos 2\theta_0 - 0.8\cos\theta_0\right]d\theta_0$$

$$= \frac{B}{2}\left[\frac{1}{6}(9\sin\theta_0 + \sin 3\theta_0) - \frac{1}{2}(\theta_0 + \sin\theta_0\cos\theta_0) - \sin\theta_0\cos\theta_0 - 0.8\sin\theta_0\right]_0^\pi$$

$I_1 \Rightarrow B = 0.125 + \int_0^{1.37}$ $\qquad I_2 \Rightarrow B = 0.0555 + \int_{1.37}^{\pi}$

$I_1 = -0.017$ $I_2 = -0.036$

Integral = -0.053

$C_{m,c/4} = -0.053$

(1) $0,5 \Rightarrow I_1$

$0,5 \Rightarrow I_2$

Problem 5

NACA 2412 → Cessna 182

$b = 46\,ft \quad c = 4.8\,ft \quad U_\infty = 135\ knots \quad W = L = 2550\,lbs$

Lift per unit span: $L' = C_\ell \left(\frac{1}{2}\rho U_\infty^2 c\right)$ (3)

Total lift: $L = C_\ell \left(\frac{1}{2}\rho U_\infty^2 cb\right)$ (2)

NACA 2412: $C_\ell = 2\pi(\alpha + 0.036)$ (1)

$$\Downarrow$$

$$L = \pi(\alpha + 0.036)\rho U_\infty^2 cb$$

$$\boxed{\alpha = \frac{L}{\pi\rho U_\infty^2 cb} - 0.036} \quad (2)$$

$\rho(@8000ft) = 0.060\ \frac{lb}{ft^3} = 0.00189\ \frac{slug}{ft^3}$

$$\alpha = \frac{2550\ lbs}{\pi(0.00187\ slug/ft^3)(227.9\ ft/s)^2(4.8ft)(46ft)} - 0.036$$

$\alpha = 0.038 - 0.036$ rad

$\alpha = 0.002$ rad or $\alpha = 0.115°$

(2)