Chapter 3

Harmonically Excited Vibration

3.1 (a) $\delta = \frac{W}{k} = \frac{50}{4000} = 0.0125$ m

(b) $\delta_{st} = \frac{F_0}{k} = \frac{60}{4000} = 0.015$ m

(c) $\omega_n = \sqrt{\frac{k}{m}} = \left(\frac{4000 \times 9.81}{50}\right)^{1/2} = 28.0143$ rad/sec

$\omega = 6$ Hz $= 37.6992$ rad/sec

$$X = \delta_{st} \left| \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right| = 0.015 \left| \frac{1}{1 - \left(\frac{37.6992}{28.0143}\right)^2} \right| = 0.0185 \text{ m}$$

3.2 $$\tau_b = \frac{2\pi}{\omega_n - \omega} = \frac{2\pi}{2\pi\,(40.0 - 39.8)} = 5 \text{ sec}$$

3.3 $k = 4000$ N/m, $m = 10$ kg, $F(t) = 400 \cos 10t$ N

$F_0 = 400$ N, $\omega = 10$ rad/s

$\omega_n = \sqrt{\frac{k}{m}} = 20$ rad/s, $\frac{\omega}{\omega_n} = \frac{10}{20} = 0.5 < 1$

Response is given by Eq. (3.9):

$$x(t) = \left(x_0 - \frac{F_0}{k - m\omega^2}\right) \cos \omega_n t + \left(\frac{\dot{x}_0}{\omega_n}\right) \sin \omega_n t + \frac{F_0}{k - m\omega^2} \cos \omega t \tag{E.1}$$

(a) $x_0 = 0.1$, $\dot{x}_0 = 0$:

Eq. (E.1) becomes

$$x(t) = \left\{0.1 - \frac{400}{4000 - 10(100)}\right\} \cos 20t + \frac{400}{4000 - 10(100)} \cos 10t$$

$$= -0.033333 \cos 20t + 0.133333 \cos 10t \tag{E.2}$$

(b) $x_0 = 0$, $\dot{x}_0 = 10$:

Eq. (E.1) becomes

$$x(t) = \left\{0 - \frac{400}{4000 - 10(100)}\right\} \cos 20t + \frac{10}{20} \sin 20t$$

$$+ \left\{\frac{400}{4000 - 10(100)}\right\} \cos 10t$$

$$= -0.133333 \cos 20t + 0.5 \sin 20t + 0.133333 \cos 10t \tag{E.3}$$

(c) $x_0 = 0.1$, $\dot{x}_0 = 10$:

Eq. (E.1) becomes

$$x(t) = \left\{ 0.1 - \frac{400}{4000 - 10(100)} \right\} \cos 20t + \frac{10}{20} \sin 20t$$

$$+ \left\{ \frac{400}{4000 - 10(100)} \right\} \cos 10t$$

$$= -0.033333 \cos 20t + 0.5 \sin 20t + 0.133333 \cos 10t \qquad \text{(E.4)}$$

3.4 $k = 4000$ N/m, $m = 10$ kg, $F(t) = 400 \cos 20t$ N,

$F_0 = 400$ N, $\omega = 20$ rad/s

$\omega_n = \sqrt{\frac{k}{m}} = 20$ rad/s, $\frac{\omega}{\omega_n} = \frac{20}{20} = 1$

Response is given by Eq. (3.15):

$$x(t) = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t + \frac{\delta_{st} \, \omega_n t}{2} \sin \omega_n t \qquad \text{(E.1)}$$

where $\delta_{st} = F_0/k = 400/4000 = 0.1$

(a) $x_0 = 0.1$, $\dot{x}_0 = 0$:

Eq. (E.1) gives

$$x(t) = 0.1 \cos 20t + \frac{(0.1)(20)t}{2} \sin 20t$$

$$= 0.1 \cos 20t + t \sin 20t \qquad \text{(E.2)}$$

(b) $x_0 = 0$, $\dot{x}_0 = 10$:

Eq. (E.1) gives

$$x(t) = \frac{10}{20} \sin 20t + \frac{(0.1)(20)t}{2} \sin 20t$$

$$= 0.5 \sin 20t + t \sin 20t \qquad \text{(E.3)}$$

(c) $x_0 = 0.1$, $\dot{x}_0 = 10$:

Eq. (E.1) gives

$$x(t) = 0.1 \cos 20t + \frac{10}{20} \sin 20t + \frac{(0.1)(20)t}{2} \sin 20t$$

$$= 0.1 \cos 20t + 0.5 \sin 20t + t \sin 20t \qquad \text{(E.4)}$$

3.5 $k = 4000$ N/m, $m = 10$ kg, $F(t) = 400 \cos 20.1 t$ N

$F_0 = 400$ N, $\omega = 20.1$ rad/s, $\omega^2 = 404.01$ (rad/s)^2

$\omega_n = \sqrt{\frac{k}{m}} = 20$ rad/s

Solution is given by Eq. (3.9):

$$x(t) = \left(x_0 - \frac{F_0}{k - m\omega^2}\right) \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t + \left(\frac{F_0}{k - m\omega^2}\right) \cos \omega t \qquad (E.1)$$

(a) $x_0 = 0.1$, $\dot{x}_0 = 0$:

Eq. (E.1) reduces to

$$x(t) = \left\{0.1 - \frac{400}{4000 - 10(404.01)}\right\} \cos 20 t + \left\{\frac{400}{4000 - 10(404.01)}\right\} \cos 20.1 t$$

$$= 10.075062 \cos 20 t - 9.975062 \cos 20.1 t \qquad (E.2)$$

(b) $x_0 = 0$, $\dot{x}_0 = 10$:

Eq. (E.1) reduces to

$$x(t) = -\left\{\frac{400}{4000 - 10(404.01)}\right\} \cos 20 t + \frac{10}{20} \sin 20 t + \left\{\frac{400}{4000 - 10(404.01)}\right\} \cos 20.1 t$$

$$= 9.975062 \cos 20 t + 0.5 \sin 20 t - 9.975062 \cos 20.1 t \qquad (E.3)$$

(c) $x_0 = 0.1$, $\dot{x}_0 = 10$:

Eq. (E.1) gives

$$x(t) = \left\{0.1 - \frac{400}{4000 - 10(404.01)}\right\} \cos 20 t + \frac{10}{20} \sin 20 t + \left\{\frac{400}{4000 - 10(404.01)}\right\} \cos 20.1 t$$

$$= 10.075062 \cos 20t + 0.5 \sin 20t - 9.975062 \cos 20.1t \qquad (E.4)$$

3.6 $k = 4000$ N/m, $m = 10$ kg, $F(t) = 400 \cos 30t$ N

$F_0 = 400$ N, $\omega = 30$ rad/s, $\omega^2 = 900$ $(\text{rad/s})^2$

$\omega_n = \sqrt{\frac{k}{m}} = 20$ rad/s, $\frac{\omega}{\omega_n} = 1.5 > 1$

Solution is given by Eq. (3.9):

$$x(t) = \left(x_0 - \frac{F_0}{k - m\omega^2}\right) \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t + \left(\frac{F_0}{k - m\omega^2}\right) \cos \omega t \qquad (E.1)$$

(a) $x_0 = 0.1$, $\dot{x}_0 = 0$:

Eq. (E.1) yields

$$x(t) = \left\{0.1 - \frac{400}{4000 - 10(900)}\right\} \cos 20t + \frac{400}{4000 - 10(900)} \cos 30t$$

$$= 0.18 \cos 20t - 0.08 \cos 30t \qquad (E.2)$$

(b) $x_0 = 0$, $\dot{x}_0 = 10$:

Eq. (E.1) yields:

$$x(t) = -\left(\frac{400}{4000 - 10(900)}\right) \cos 20t + \frac{10}{20} \sin 20t + \left(\frac{400}{4000 - 10(900)}\right) \cos 30t$$

$$= 0.08 \cos 20t + 0.5 \sin 20t - 0.08 \cos 30t \qquad (E.3)$$

(c) $x_0 = 0.1$, $\dot{x}_0 = 10$:

Eq. (E.1) yields

$$x(t) = \left\{0.1 - \frac{400}{4000 - 10(900)}\right\} \cos 20t + \frac{10}{20} \sin 20t + \left\{\frac{400}{4000 - 10(900)}\right\} \cos 30t$$

$$= 0.18 \cos 20t + 0.5 \sin 20t - 0.08 \cos 30t \qquad (E.4)$$

3.7

$\delta_{st} = \frac{F_0}{k} = \frac{25}{2000} = 0.0125$ m

steady state solution at resonance $= x(t) = \frac{\delta_{st} \cdot \omega_n t}{2} \sin \omega_n t$

$= 0.00625\ \omega_n t \sin \omega_n t$ m

(a) At end of $\frac{1}{4}$ cycle, $\omega_n t = \frac{\pi}{2}$ and $x(t) = 0.00625\left(\frac{\pi}{2}\right) \sin\frac{\pi}{2} = 0.009817$ m

(b) At end of $2\frac{1}{2}$ cycles, $\omega_n t = 5\pi$ and $x(t) = 0.00625(5\pi) \sin 5\pi = 0$

(c) At end of $5\frac{3}{4}$ cycles, $\omega_n t = 11\frac{1}{2}\pi$ and

$x(t) = 0.00625\left(\frac{23}{2}\pi\right) \sin \frac{23}{2}\pi = -0.2258$ m

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3.8

$\delta_{st} = \frac{F_0}{k} = \frac{100}{4000} = 0.025$ m.

$X = \delta_{st} \cdot \frac{1}{\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|}$, $\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right| = \frac{\delta_{st}}{X} = \frac{0.025}{20 \times 10^{-3}} = 1.25$

$\frac{\omega}{\omega_n} = \sqrt{1.25 + 1} = 1.5$

$\omega_n = \omega/1.5 = 5(2\pi)/1.5 = 20.944$ rad/sec

$m = k/\omega_n^2 = 4000/(20.944)^2 = 9.1189$ kg

3.9

$\omega_n = \sqrt{k/m} = \sqrt{5000/10} = 22.3607$ rad/sec

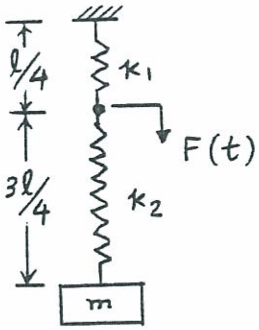
$\delta_{st} = F_0/k = 250/5000 = 0.05$ m

$X = \delta_{st} \left\{ \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right\}$

i.e., $\omega = \omega_n \left(1 - \frac{\delta_{st}}{X}\right)^{\frac{1}{2}} = 22.3607\left[1 - \frac{0.05}{0.10}\right]^{\frac{1}{2}}$

$= 15.8114$ rad/sec

3.10



$k_1 = 4k$; $\frac{1}{4k} + \frac{1}{k_2} = \frac{1}{k}$, $k_2 = \frac{4}{3}k$

Force transmitted to the mass through k_2:

$$\tilde{F}(t) = \frac{k_2}{k_1 + k_2} F(t) = \frac{k_1 k_2}{k_1 + k_2}\left(\frac{F_0}{k_1}\right)\cos\omega t$$

$$= k\,\delta_{st}\cos\omega t \quad \text{where} \quad \delta_{st} = \frac{F_0}{k_1}$$

Steady state response of m:

$$x(t) = \frac{\tilde{F}_0}{k\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}}\cos\omega t$$

$$= \left\{\frac{\delta_{st}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right\}\cos\omega t \quad \text{with } \tilde{F}_0 = k\cdot\delta_{st}.$$

3.16 Equivalent stiffness of wing (beam) at location of engine:

$$k = \frac{\text{force}}{\text{deflection}} = \frac{3\,E\,I}{\ell^3} = \frac{3\,E\,(\frac{1}{12}\,b\,a^3)}{\ell^3} = \frac{E\,b\,a^3}{4\,\ell^3}$$

Magnitude of unbalanced force: $= m\,r\,\omega^2 = m\,r\left(\frac{2\,\pi\,N}{60}\right)^2 = \frac{m\,r\,\pi^2\,N^2}{900}$

Equivalent mass of wing at location of engine: $M = \frac{33}{140}\,m_w = \frac{33}{140}\,(a\,b\,\ell\,\rho)$

Equation of motion: $M\,\ddot{x} + k\,x = m\,r\,\omega^2 \sin\omega\,t$

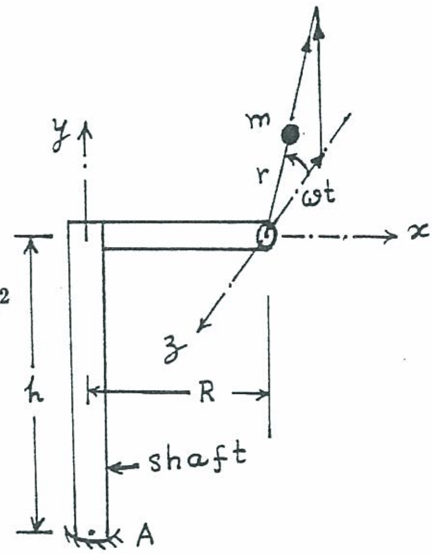
Maximum steady state displacement of wing at location of engine:

$$X = \left|\frac{m\,r\,\omega^2}{k - M\,\omega^2}\right| = \left|\frac{\left(\frac{m\,r\,\pi^2\,N^2}{900}\right)}{\left\{\frac{E\,b\,a^3}{4\,\ell^3} - \frac{33}{140}\,a\,b\,\ell\,\rho\left(\frac{2\,\pi\,N}{60}\right)^2\right\}}\right|$$

$$= \left|\frac{m\,r\,\ell^3\,N^2}{22.7973\,E\,b\,a^3 - 0.2357\,\rho\,a\,b\,\ell^4\,N^2}\right|$$

3.17 Rotating unbalanced force, $m\,r\,\omega^2$, can be resolved into two components as:

$$F_y = m\,r\,\omega^2 \sin\omega t \quad \text{(parallel y—axis)}$$
$$F_z = m\,r\,\omega^2 \cos\omega t \quad \text{(parallel z—axis)}$$



Maximum bending stress at A:

$$\sigma_b = \frac{1}{I_z}\,|M_z|\,\frac{d_0}{2} = \frac{m\,r\,\omega^2\,R\left(\frac{d_0}{2}\right)}{\frac{\pi}{64}\,(d_o^4 - d_i^4)}$$

$$= \frac{(0.1)\,(0.1)\,(31.416^2)\,(0.5)\,(\frac{0.1}{2})}{\frac{\pi}{64}\,(0.1^4 - 0.08^4)} = 8.5124\,(10^4)\ \mathrm{N/m^2}$$

Maximum torsional stress at A:

$$\sigma_t = \frac{1}{J_y}\,|M_y|\left(\frac{d_o}{2}\right) = \frac{m\,r\,\omega^2\,R\left(\frac{d_o}{2}\right)}{\frac{\pi}{32}\,(d_o^4 - d_i^4)}$$

$$= 4.2562\,(10^4)\ \mathrm{N/m^2}$$

3.18 Total stiffness with steel specimen:

$$k_{eq} = k_1 + k_2 = 10{,}217.0296 + 750{,}000.0 = 760{,}217.0296\ \mathrm{lb/in}$$

Force in specimen due to magnets (static) due to elongation $X = k_2\,X$.
Force in specimen due to a.c. current in magnets (dynamic) due to elongation $X = k_{eq}\,X - m\,\omega^2\,X$.

$$\text{Ratio of stresses} = \left|\frac{k_2\,X}{k_{eq}\,X - m\,\omega^2\,X}\right| = \frac{1}{2} \quad \text{i.e.,} \quad \left|\frac{k_2}{k_{eq} - m\,\omega^2}\right| = \frac{1}{2}.\text{sp}$$

$$\text{i.e.,} \quad \left|\frac{750{,}000.0}{760{,}217.0296 - \left(\frac{40}{386.4}\right)\omega^2}\right| = \frac{1}{2}$$

Squaring both sides of this equation and rearranging gives:

$$107.1225\,\omega^4 - 15.7365\,(10^8)\,\omega^2 - 167.207\,(10^{14}) = 0$$
$$\text{or} \quad \omega^2 = 0.218378\,(10^8) \quad \text{(positive value)}$$
$$\omega = 4673.0935\ \mathrm{rad/sec} = 743.7442\ \mathrm{Hz}$$

3.19 Equation of motion: $m_{eq}\,\ddot{x} + k_{eq}\,x = F(t)$
where m_{eq} = mass of valve and valve rod plus mass of spring at end = $(20 + (15/3))/386.4 = 0.0647\ \mathrm{lb{-}sec^2/in}$.
$k_{eq} = 400$ lb/in, $F(t) = A\ p(t) = A\ p_0 \sin\omega t = 100\,(10)\sin\omega t = 1000 \sin 8t$ lb.
Response of valve (steady state) $= x_p(t) = X \sin 8t$ in where

$$X = \frac{1000}{400 - 0.0647\,(8)^2} = 2.5261\ \mathrm{in}$$

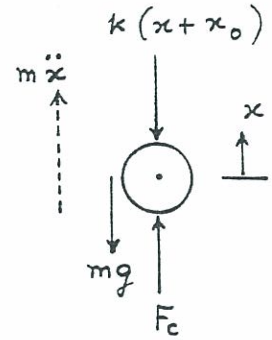
3.20 (a) Equation of motion:

$$m_0 \ddot{x} + k(x + x_0) + m_0 g = F_c; \quad \ddot{x} > 0 \tag{1}$$

where F_c = force exerted on the follower by the cam, m_0 = mass of follower plus one third the mass of the spring, and x_0 = initial displacement of the spring.

(b) Force exerted on the follower by the cam:

$$F_c = m_0 \ddot{x} + k(x + x_0) + m_0 g \tag{2}$$

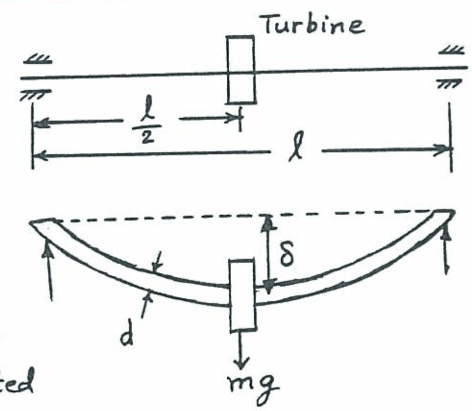


with $x = e \cos \omega t$.

(c) Condition under which follower loses contact with the cam is when F_c is zero and $\ddot{x}$ is negative. Equation (1) can be used to state this condition as:

$$k(x + x_0) + m_0 g \geq |m_0 \ddot{x}| \tag{3}$$

3.21 δ_{st} = static radial displacement of shaft under weight of turbine



δ = radial deflection of shaft during rotation

$k = \dfrac{48 E I}{l^3}$ = stiffness of centrally loaded simply supported beam

$$m \delta \omega^2 = k(\delta - \delta_{st}) \quad \text{or} \quad m\omega^2 = k - k \cdot \frac{\delta_{st}}{\delta}$$

or $$\frac{\delta}{\delta_{st}} = \frac{k}{k - m\omega^2} \tag{E$_1$}$$

Critical speed is $$\omega_{cri} = \sqrt{\frac{k}{m}} \tag{E$_2$}$$

If critical speed = $\frac{1}{5}$th of operating speed,

$$\sqrt{\frac{k}{m}} = \frac{1}{5}\omega \tag{E$_3$}$$

Here $m = 500/386.4 = 1.2940$ lb-s²/in

and $\omega = 3000 \times 2\pi/60 = 314.16$ rad/sec

For solid shaft (steel) of diameter d and length l, Eq. (E$_3$) gives

$$\frac{48\,E\,I}{m\,l^3} = \frac{\omega^2}{25} \quad \text{with } E = 30 \times 10^6 \text{ psi and } I = \frac{\pi d^4}{64}$$

i.e., $$\frac{l^3}{d^4} = 13836.8 \tag{E4}$$

Let $l = 30\,d$ in Eq. (E4):

$$d = \frac{27000}{13836.8} = 1.9513 \text{ inch} \quad \text{and hence } l = 58.5395 \text{ inch.}$$

3.22

$$I = \frac{\pi}{64}\left(d_o^4 - d_i^4\right) = \frac{\pi}{64}\left(4^4 - 3.5^4\right) = 5.2002 \text{ in}^4$$

$$k = \frac{48\,E\,I}{l^3} = \frac{48\,(30 \times 10^6)\,(5.2002)}{(100)^3} = 7488.288 \text{ lb/in}$$

$$m = 500/386.4 = 1.2940 \text{ lb-s}^2/\text{in}$$

$$\omega_n = \sqrt{k/m} = \sqrt{7488.288/1.2940} = 76.0719 \text{ rad/sec}$$

Eccentricity $= r = 2$ in, eccentric mass $= m_o = \frac{0.5}{386.4}\ \frac{\text{lb-s}^2}{\text{in}}$

Radial force due to eccentric mass at resonance

$$= F_0 = m_o\, r\, \omega^2 = \left(\frac{5}{386.4}\right)(2)(76.0719)^2 = 149.7654 \text{ lb}$$

Let $x(t)$ = radial displacement of turbine.

At resonance, Eq. (3.15) gives, for $x_o = \dot{x}_o = 0$,

$$x(t) = \frac{1}{2}\,\delta_{st}\,\omega_n t \sin \omega_n t$$

where $$\delta_{st} = \frac{F_0}{k} = \frac{149.7654}{7488.288} = 0.02 \text{ in}$$

To activate the limit switch, $x(t) = 0.5$ in. and hence

$$0.5 = \frac{1}{2}(0.02)(76.0719)\,t \sin 76.0719\,t$$

i.e., $$t \sin 76.0719\,t = 0.6573 \tag{E1}$$

Eq. (E1) is solved by trial and error (assuming values of $t = 1.0, 0.9, 0.8, 0.7$, etc.) as

$$t \simeq 0.6760 \text{ sec.}$$

3.23

Tip load = 0.1 lb, tip mass $= m_o = \frac{0.1}{386.4} = 2.588 \times 10^{-4}$ lb-s^2/in

$$I = \frac{1}{12}(0.2)(0.05)^3 = 2.0833 \times 10^{-6} \text{ in}^4$$

$$k = \frac{3EI}{l^3} = \frac{3(30 \times 10^6)(2.0833 \times 10^{-6})}{(10)^3} = 0.1875 \text{ lb/in}$$

$$m = \text{mass of beam} = \frac{0.283}{386.4}(10 \times 0.2 \times 0.05) = 7.324 \times 10^{-5} \text{ lb-s}^2/\text{in}$$

$$\omega_n = \left(\frac{k}{m_0 + 0.23\,m}\right)^{\frac{1}{2}} = \left[\frac{0.1875}{(2.588 + 0.7324)\,10^{-4}}\right]^{\frac{1}{2}} = 5.6469 \frac{\text{rad}}{\text{sec}}$$

Eq. (3.68) gives

$$\frac{X}{Y} = \left\{\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right\}^{\frac{1}{2}}$$

i.e.,

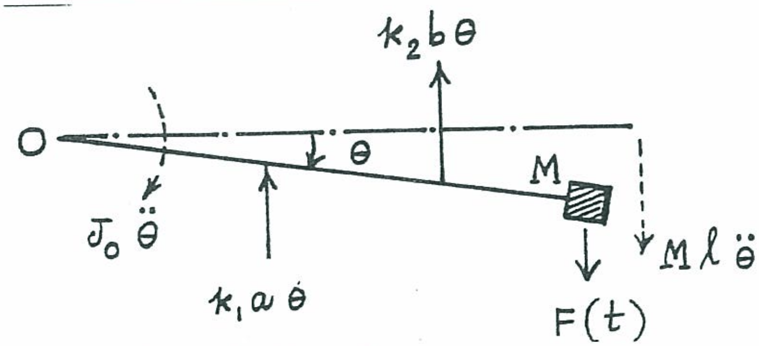
$$\frac{2.5}{0.05} = 50 = \left\{\frac{1 + (2 \times 0.01\, r)^2}{(1-r^2)^2 + (2 \times 0.01\, r)^2}\right\}^{\frac{1}{2}}$$

i.e. $r^4 - 1.9996\, r^2 + 0.9996 = 0$

i.e. $r = \frac{\omega}{\omega_n} = 0.9999$

$\therefore \quad \omega = 0.9999\, \omega_n = 5.6463$ rad/sec.

3.24



Equation of motion for rotational motion about the hinge O:

$$(J_0 + M\,\ell^2)\,\ddot{\theta} + (k_1\, a^2 + k_2\, b^2)\,\theta = F(t)\,\ell = F_0\,\ell \sin \omega t \tag{1}$$

Steady state response (using Eqs. (3.3) and (3.6)):

$$\theta_p(t) = \Theta \sin \omega t \tag{2}$$

$$\text{where} \quad \Theta = \frac{F_0\,\ell}{(k_1\, a^2 + k_2\, b^2) - (J_0 + M\,\ell^2)\,\omega^2} \tag{3}$$

$$\text{and} \quad J_0 = \frac{m\,\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 = \frac{1}{3}\, m\,\ell^2 \tag{4}$$

For given data, $J_0 = \frac{1}{3}(10)(1^2) = 3.3333$ kg–m^2, $\omega = \frac{1000\,(2\,\pi)}{60} = 104.72$ rad/sec, and

$$\Theta = \frac{500\ (1)}{5000\ (0.25^2\ + 0.5^2) - (3.3333 + 50\ (1^2))\ (104.72^2)} = -\ 8.5718\ (10^{-4})\ \text{rad}$$

3.25 Equation of motion for rotation about O:

$$J_0\ \ddot{\theta} = -\ k\ \frac{\theta\,\ell}{4}\ \frac{\ell}{4} - k\ \frac{\theta\,3\,\ell}{4}\ \frac{3\,\ell}{4} + M_0\ \cos\,\omega\,t$$

$$\text{i.e.,}\quad J_0\ \ddot{\theta} + \left(\frac{5}{8}\ k\ \ell^2\right)\ \theta = M_0\ \cos\,\omega\,t$$

$$\text{where}\quad J_0 = \frac{1}{12}\ m\ \ell^2 + m\ (\frac{\ell}{4})^2 = \frac{7}{48}\ m\ \ell^2 = \frac{7}{48}\ (10)\ (1^2) = 1.4583\ \text{kg--m}^2$$

and $\omega =$ 1000 rpm $=$ 104.72 rad/sec. Steady state solution is:

$$\theta_p(t) = \Theta\ \cos\,\omega\,t$$

where

$$\Theta = \frac{M_0}{\frac{5}{8}\ k\ \ell^2 - J_0\ \omega^2} = \frac{100}{5000\ (\frac{5}{8})\ (1^2) - 1.4583\ (104.72^2)} = -\ 0.007772\ \text{rad}$$

3.26 $k = 4000$ N/m, $m = 10$ kg, $c = 40$ N-s/m, $F(t) = 200 \cos 10t$,
$F_0 = 200$ N, $\omega = 10$ rad/s, $x_0 = 0.1$ m, $\dot{x}_0 = 0$

$\omega_n = \sqrt{\frac{k}{m}} = 20$ rad/s, $\delta_{st} = \frac{F_0}{k} = \frac{200}{4000} = 0.05$ m

$\zeta = c/c_c = \left(c/2\sqrt{km}\right) = \left(40/2\sqrt{4000(10)}\right) = 0.1$

$\omega_d = \sqrt{1-\zeta^2}\ \omega_n = \sqrt{1-(0.1)^2}\ (20) = 19.899749$ rad/s

$r = \frac{\omega}{\omega_n} = \frac{10}{20} = 0.5$

$$X = \delta_{st}\Big/\sqrt{(1-r^2)^2 + (2\zeta r)^2} = \frac{0.05}{\left\{(1-0.5^2)^2 + (2\,(0.1)(0.5))^2\right\}^{\frac{1}{2}}}$$

$= 0.066082$ m

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = \tan^{-1}\left(\frac{2 * 0.1 * 0.5}{1-0.5^2}\right) = 0.132552\ \text{rad}$$

steady state response, Eq. (3.25):

$$x_p(t) = X\ \cos(\omega t - \phi)$$
$$= 0.066082\ \cos(10t - 0.132552)\ \text{m}$$

Total response, Eq. (3.35):

$$x(t) = X_0 \, e^{-\zeta \omega_n t} \cos(\omega_d t - \phi_0) + X \cos(\omega t - \phi) \qquad (E.1)$$

Using the initial conditions x_0 and $\dot{x}_0$, Eq. (E.1) gives

$$x_0 = X_0 \cos \phi_0 + X \cos \phi \qquad (E.2)$$

or $$X_0 \cos \phi_0 = x_0 - X \cos \phi \qquad (E.3)$$

$$\dot{x}_0 = -\zeta \omega_n X_0 \cos \phi_0 + \omega_d X_0 \sin \phi_0 + \omega X \sin \phi \qquad (E.4)$$

or $$X_0 \sin \phi_0 = \frac{1}{\omega_d} \left\{ \dot{x}_0 + \zeta \omega_n X_0 \cos \phi_0 - \omega X \sin \phi \right\} \qquad (E.5)$$

For known values, Eqs. (E.3) and (E.5) yield

$$X_0 \cos \phi_0 = 0.034498 , \qquad X_0 \sin \phi_0 = -0.000922$$

Hence

$$X_0 = \left\{ (X_0 \cos \phi_0)^2 + (X_0 \sin \phi_0)^2 \right\}^{\frac{1}{2}} = 0.034510$$

$$\phi_0 = \tan^{-1} \left(\frac{X_0 \sin \phi_0}{X_0 \cos \phi_0} \right) = -0.026710$$

Thus the total response, Eq. (E.1), will be

$$x(t) = 0.034510 \, e^{-2t} \cos(19.899749\, t + 0.026710) + 0.066082 \cos(10t - 0.132552) \text{ m} \qquad (E.6)$$

3.27 $k = 4000$ N/m, $m = 10$ kg, $c = 40$ N-s/m, $F(t) = 200 \cos 10t$,
$F_0 = 200$ N, $\omega = 10$ rad/s, $x_0 = 0$, $\dot{x}_0 = 10$ m/s

From solution of Problem 3.26,

$\zeta = 0.1$, $\omega_d = 19.899749$ rad/s, $r = 0.5$, $X = 0.066082$ m,
$\phi = 0.132552$ rad

$$x_p(t) = 0.066082 \cos(10t - 0.132552) \text{ m}$$

$$X_0 \cos \phi_0 = x_0 - X \cos \phi = -0.065502$$

$$X_0 \sin \phi_0 = \frac{1}{\omega_d} \left\{ \dot{x}_0 + \zeta \omega_n X_0 \cos \phi_0 - \omega X \sin \phi \right\} = 0.491547$$

$$X_0 = \left\{ (X_0 \cos \phi_0)^2 + (X_0 \sin \phi_0)^2 \right\}^{\frac{1}{2}} = 0.495892$$

$$\phi_0 = \tan^{-1} \left(\frac{X_0 \sin \phi_0}{X_0 \cos \phi_0} \right) = -1.438320$$

Thus the total response, Eq. (3.35), is given by

$$x(t) = 0.495892\ e^{-2t} \cos(19.899749\,t + 1.438320)$$
$$+\ 0.066082 \cos(10t - 0.132552) \quad m$$

3.28 $k = 4000$ N/m, $m = 10$ kg, $c = 40$ N-s/m, $F(t) = 200 \cos 20t$

$F_0 = 200$ N, $\omega = 20$ rad/s, $x_0 = 0.1$ m, $\dot{x}_0 = 0$

$$\omega_n = \sqrt{\frac{k}{m}} = 20 \text{ rad/s}, \quad \delta_{st} = \frac{F_0}{k} = \frac{200}{4000} = 0.05$$

$$\zeta = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = \frac{40}{2\sqrt{4000\,(10)}} = 0.1$$

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n = \sqrt{1-0.1^2}\,(20) = 19.899749 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = 1$$

$$X = \frac{\delta_{st}}{\{(1-r^2)^2 + (2\zeta r)^2\}^{1/2}} = \frac{0.05}{\{(1-1^2)^2 + (2*0.1*1)^2\}^{1/2}} = 0.25$$

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$$

Steady state response, Eq. (3.25):

$$x_p(t) = X\cos(\omega t - \phi) = 0.25 \cos\left(20t - \frac{\pi}{2}\right) \quad m$$

Total response, Eq. (3.35):

$$x(t) = X_0\, e^{-\zeta\omega_n t} \cos(\omega_d t - \phi_0) + X\cos(\omega t - \phi) \qquad (E.1)$$

Using the initial conditions x_0 and $\dot{x}_0$, Eq. (E.1) gives

$$X_0 \cos\phi_0 = x_0 - X\cos\phi = 0.1 - 0.25 \cos\frac{\pi}{2} = 0.1$$

$$X_0 \sin\phi_0 = \frac{1}{\omega_d}\{\dot{x}_0 + \zeta\omega_n X_0 \cos\phi_0 - \omega X \sin\phi\}$$
$$= \left(0 + 0.1*20*0.1 - 20*0.25*\sin\frac{\pi}{2}\right)/19.899749$$
$$= -0.241209$$

Hence $X_0 = \{(X_0\cos\phi_0)^2 + (X_0\sin\phi_0)^2\}^{\frac{1}{2}} = 0.261117$

$$\phi_0 = \tan^{-1}\left(\frac{X_0 \sin\phi_0}{X_0 \cos\phi_0}\right) = \tan^{-1}\left(\frac{-0.241209}{0.1}\right) = -1.177783$$

Total response:

$$x(t) = 0.261117 \; e^{-2t} \cos(19.899749\, t + 1.777828)$$
$$+ 0.25 \cos\left(20t - \frac{\pi}{2}\right) \; m$$

3.29 $k = 4000$ N/m, $m = 10$ kg, $c = 40$ N-s/m,

$F(t) = 200 \cos 20t$ N, $F_0 = 200$ N, $\omega = 20$ rad/s

$x_0 = 0$, $\dot{x}_0 = 10$ m/s

From solution of Problem 3.28,

$\zeta = 0.1$, $\omega_n = 20$ rad/s, $\omega_d = 19.899749$ rad/s, $r = 1$

$X = 0.25$, $\phi = \frac{\pi}{2}$

$x_p(t) = 0.25 \cos\left(20t - \frac{\pi}{2}\right)$ m

$$X_0 \cos\phi_0 = x_0 - X\cos\phi = 0 - 0 = 0$$

$$X_0 \sin\phi_0 = \frac{1}{\omega_d}\left\{\dot{x}_0 + \zeta\omega_n X_0 \cos\phi_0 - \omega X \sin\phi\right\}$$

$$= \frac{1}{19.899749}\left\{10 + 0.1 * 20 * 0 - 20 * 0.25 * \sin\frac{\pi}{2}\right\}$$

$$= 0.251260$$

Hence $X_0 = \left\{(X_0\cos\phi_0)^2 + (X_0\sin\phi_0)^2\right\}^{\frac{1}{2}} = 0.251260$

$$\phi_0 = \tan^{-1}\left(\frac{X_0 \sin\phi_0}{X_0 \cos\phi_0}\right) = 1.570793$$

Total response:

$$x(t) = 0.251260 \; e^{-2t} \cos(19.899749\, t - 1.570793)$$
$$+ 0.25 \cos\left(20t - \frac{\pi}{2}\right) \; m$$

3.30 $m = \frac{500}{386.4}$ lb–sec²/in, $F(t) = 200 \sin 100 \pi t$ lb. Let $X_{max} = 0.05$ in < 0.1 in (maximum permissible value). From Eq. (3.33),

$$X_{max} = \delta_{st} \frac{1}{2 \varsigma \sqrt{1-\varsigma^2}} = 0.05 \qquad (1)$$

Let $\varsigma = 0.01$. Then $\delta_{st} = \frac{F_0}{k_{eq}} = \frac{200}{k_{eq}}$ and Eq. (1) gives

$$k_{eq} = \frac{200}{2\,(0.01)\sqrt{1-0.0001}\;(0.05)} = 20.0020\,(10^4)\ \text{lb/in}$$

Since shock mounts are in parallel, stiffness of each mount $= k = \frac{k_{eq}}{3} = 6.6673\,(10^4)$ lb/in.

$$\varsigma = \frac{c_{eq}}{c_c} = \frac{c_{eq}}{\sqrt{2\,k_{eq}\,m}}$$

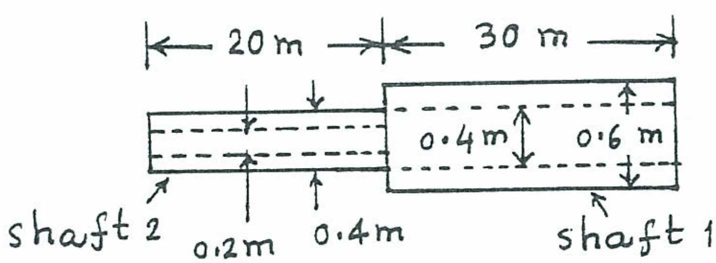
or $c_{eq} = \varsigma \sqrt{2\,k_{eq}\,m} = 0.01 \sqrt{2\,(20.0020\,(10^4))\,(\frac{500}{386.4})} = 7.1948$ lb–sec/in

and hence $c = \frac{c_{eq}}{3} = 2.3983$ lb-sec/in

3.31 Equation of motion for torsional system:

$$J_0\,\ddot{\theta} + c_t\,(\dot{\theta} - \dot{\alpha}) + k_t\,(\theta - \alpha) = 0 \qquad (1)$$

where $\theta =$ angular displacement of shaft and $\alpha =$ angular displacement of base of shaft $= \alpha_0 \sin \omega t$. Steady state response of propeller (Eq. (3.67)):



$$\theta_p(t) = \Theta \sin(\omega t - \phi) \tag{2}$$

$$\text{where } \Theta = \alpha_0 \left\{ \frac{k_t^2 + (c_t \omega)^2}{(k_t - J_0 \omega^2)^2 - (c_t \omega)^2} \right\}^{\frac{1}{2}} \tag{3}$$

$$\text{and } \phi = \tan^{-1} \left\{ \frac{J_0 c_t \omega^3}{k_t (k_t - J_0 \omega^2) + (c_t \omega)^2} \right\} \tag{4}$$

Here $J_0 = 10^4$ kg—m^2, $\varsigma_t = 0.1$, and $\omega = 314.16$ rad/sec. Torsional stiffnesses of shafts:

$$(k_t)_1 = \frac{G_1 J_1}{\ell_1} = \frac{(80 (10^9)) \left(\frac{\pi}{32} (0.6^4 - 0.4^4) \right)}{30} = 27.2272 (10^6) \text{ N—m/rad}$$

$$(k_t)_2 = \frac{G_2 J_2}{\ell_2} = \frac{(80 (10^9)) \left(\frac{\pi}{32} (0.4^4 - 0.2^4) \right)}{20} = 9.4248 (10^6) \text{ N—m/rad}$$

Series springs give:

$$k_t = \frac{(k_t)_1 (k_t)_2}{(k_t)_1 + (k_t)_2} = \frac{(27.2272 (10^6)) (9.4248 (10^6))}{27.2272 (10^6) + 9.4248 (10^6)} = 7.0013 (10^6) \text{ N—m/rad}$$

$$c_t = \varsigma (2 \sqrt{J_0 k_t}) = 0.1 (2) \sqrt{(10^4)(7.0013 (10^6))} = 52{,}919.8624 \text{ N—m—s/rad}$$

From Eq. (3),

$$\Theta = 0.05 \left[\frac{(7.0013 (10^6))^2 + \left\{ 5.2920 (10^4) (314.16^2) \right\}^2}{\left\{ 7.0013 (10^6) - (10^4)(314.16^2) \right\}^2 + \left\{ 5.2920 (10^4)(314.16) \right\}^2} \right]^{\frac{1}{2}}$$

$$= 9.2028 (10^{-4}) \text{ rad}$$

$$\phi = \tan^{-1} \left\{ \frac{(10^4)(5.2920 (10^4))(314.16^3)}{7.0013 (10^6) \left[7.0013 (10^6) - (10^4)(314.16^2) \right] + (5.2920 (10^4)(314.16))^2} \right\}$$

$$= \tan^{-1} (59.3664) = 89.0350^\circ = 1.5540 \text{ rad}$$

3.32

$$X = \frac{\delta_{st}}{\{(1-r^2)^2 + (2\zeta r)^2\}^{1/2}}$$

For maximum X, $\frac{dX}{dr} = -\delta_{st} \cdot \frac{1}{2} \frac{1}{\{(1-r^2)^2 + (2\zeta r)^2\}^{3/2}} \cdot \{2(1-r^2)(-2r) + 2(2\zeta r)(2\zeta)\}$
$= 0$

i.e., $-4r(1-r^2) + 8r\zeta^2 = 0$

i.e., $r = \sqrt{1-2\zeta^2}$

$$X \Big|_{at\ r=\sqrt{1-2\zeta^2}} = \frac{\delta_{st}}{\left[\{1-(1-2\zeta^2)\}^2 + \left(2\zeta\sqrt{1-2\zeta^2}\right)^2\right]^{1/2}} = \frac{\delta_{st}}{2\zeta\sqrt{1-\zeta^2}}$$

$$\therefore \quad \left(\frac{X}{\delta_{st}}\right)_{max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

3.33 Under a d.c. current (I) through the coil, core rotates by angle θ. Torque developed due to I balances the restoring torque of spring: $a\,I = k_t\,\theta$ where a is a constant and k_t is the torsional spring constant. Under an a.c. current I(t), torque developed is $T(t) = a\,I(t)$ and the equation of motion is:

$$J_0\,\ddot{\theta} + c_t\,\dot{\theta} + k_t\,\theta = T(t) = a\,I(t) = a\,I_0 \cos \omega t \tag{1}$$

Steady state angular displacement of core:

$$\theta_p(t) = \Theta \cos(\omega t - \phi).sp \tag{2}$$

$$\text{where} \quad \Theta = \frac{a\,I_0}{\left\{(k_t - J_0\,\omega^2)^2 + (c_t\,\omega)^2\right\}^{\frac{1}{2}}} = \frac{\left(\frac{a\,I_0}{k_t}\right)}{\left[\left\{1-(\frac{\omega}{\omega_n})^2\right\}^2 + \left\{2\,\zeta\,\frac{\omega}{\omega_n}\right\}^2\right]^{\frac{1}{2}}} \tag{3}$$

When $\omega = 0$ (d.c. current) and $I_0 = 1$ ampere, Eq. (1) gives

$$\Theta_{dc} = \left(\frac{a}{k_t}\right) = 1 \text{ (reading corresponding } \Theta_{dc})$$

and hence $a = k_t = 62.5$.

When $\omega = 50$ Hz $= 314.16$ rad/sec and $I_0 = 5$ amperes, Eq. (3) gives:

$$\Theta_{ac} = \frac{\left(\frac{a\,(5)}{k_t}\right)}{\left[\left\{1-\left(\frac{314.16}{250}\right)^2\right\}^2 + \left\{2\,(1)\left(\frac{314.16}{250}\right)\right\}^2\right]^{\frac{1}{2}}} = 1.9386 \text{ amperes}$$

where $J_0 = 0.001$ N–m^2, $k_t = 62.5$ N-m/rad, $c_t = 0.5$ N-m-s/rad, and $\omega_n = \sqrt{\dfrac{k_t}{J_0}} = \sqrt{\dfrac{62.5}{0.001}} = 250$ rad/s. The steady state value of current indicated by ammeter = 1.9386 amperes (this shows that the ammeter is not accurate).

3.34 Eq. (3.34):
$$\frac{X_{res}}{\delta_{st}} = \left.\frac{X}{\delta_{st}}\right|_{\omega=\omega_n} = \frac{1}{2\zeta}$$

i.e.,
$$\delta_{st} = 2\zeta\left(\frac{20}{1000}\right) = 0.04\,\zeta \qquad (E_1)$$

Eq. (3.30):
$$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\ ; \qquad r = 0.75 = \frac{\omega}{\omega_n}$$

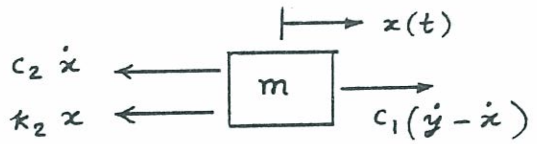
i.e.,
$$\frac{0.01}{\delta_{st}} = \frac{1}{\sqrt{(1-0.75^2)^2+(2\zeta\times 0.75)^2}} \qquad (E_2)$$

Eqs. (E_1) and (E_2) give
$$\frac{0.01}{0.04\,\zeta} = \frac{1}{\sqrt{0.1914+2.25\,\zeta^2}}$$

i.e., $0.1914 + 2.25\,\zeta^2 = 16\,\zeta^2$

i.e., $\zeta = 0.1180$

3.35 (a) Equation of motion of mass:



$$m\ddot{x} = c_1(\dot{y}-\dot{x}) - c_2\dot{x} - k_2 x$$

i.e.,
$$m\ddot{x} + (c_1+c_2)\dot{x} + k_2 x = c_1\dot{y} = -c_1\omega Y \sin\omega t$$

(b)
$$x_p(t) = \frac{-(c_1\omega Y/k_2)}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\sin(\omega t-\phi)$$

where $r = \omega/\omega_n$, $\zeta = (c_1+c_2)\,\omega/(2r\,k)$ and $\phi = \tan^{-1}\left(\dfrac{2\zeta r}{1-r^2}\right)$.

(c) steady-state force transmitted to point P:
$$= k_2 x_p + c_2\dot{x}_p$$
$$= \frac{-(c_1\omega Y)}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}\left\{\sin(\omega t-\phi) + \frac{c_2\omega}{k_2}\cos(\omega t-\phi)\right\}$$

3.40 Eq. (3.34) gives $\left(\frac{X}{\delta_{st}}\right)_{\omega=\omega_n} = \frac{1}{2\zeta}$

If $X = \frac{1}{\sqrt{2}} X_{max} = \frac{1}{\sqrt{2}} X\Big|_{\omega=\omega_n}$, Eq. (3.30) gives

$$\frac{1}{\sqrt{2}} \cdot \frac{1}{2\zeta} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Squaring and rearranging

$$8\zeta^2 = (1-r^2)^2 + 4\zeta^2 r^2 = 1 - 2r^2 + r^4 + 4r^2\zeta^2$$

$$r^4 + r^2(4\zeta^2 - 2) + (1 - 8\zeta^2) = 0$$

$$r^2 = 1 - 2\zeta^2 \pm 2\zeta\sqrt{1+\zeta^2}$$

Neglecting terms involving ζ^2,

$$r^2 = \frac{\omega^2}{\omega_n^2} = 1 \pm 2\zeta .$$

Let $\omega = \omega_1$ when $r^2 = 1 - 2\zeta$ and $\omega = \omega_2$ when $r^2 = 1 + 2\zeta$

$$\frac{\omega_2^2 - \omega_1^2}{\omega_n^2} = \frac{(\omega_2 + \omega_1)(\omega_2 - \omega_1)}{\left(\frac{\omega_2 + \omega_1}{2}\right)^2} = 4\zeta$$

$$\therefore \quad \frac{\omega_2 - \omega_1}{\omega_2 + \omega_1} = \zeta$$

3.41 $k_t = \frac{\pi G}{32\, l} d^4 = \frac{\pi\,(79.3 \times 10^9)}{32\,(1)} \left(\frac{4}{100}\right)^4 = 19930.31$ N-m/rad

$$\omega_n = \sqrt{\frac{k_t}{J_0}} = \sqrt{\frac{19930.31}{10}} = 44.6434 \text{ rad/sec}$$

$$\theta_{st} = M_{t0}/k_t = 1000/19930.31 = 0.0502 \text{ rad}$$

$$\zeta_t = \frac{c_t}{2 J_0 \omega_n} = \frac{300}{2(10)(44.6434)} = 0.336$$

(a) Eq. (3.30), when written for a torsional system, gives

$$\frac{\Theta}{\theta_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

i.e., $$\frac{(2/57.2956)}{0.0502} = \frac{1}{\sqrt{(1-r^2)^2 + (2 \times 0.336\, r)^2}}$$

i.e., $r^4 - 1.5484\, r^2 - 1.0679 = 0$

i.e., $r^2 = 2.0655,\ -0.5171$

$\therefore \quad \omega = r\,\omega_n = \sqrt{2.0655}\,(44.6434) = 64.16$ rad/sec

(b) Maximum torque transmitted to the support:

$$M_t(t) = k_t\,\theta(t) + c_t\,\dot{\theta}(t)$$
$$= k_t\,\Theta\cos(\omega t-\phi) - c_t\,\Theta\,\omega\sin(\omega t-\phi)$$
$$(M_t)_{max} = \sqrt{(k_t\,\Theta)^2 + (c_t\,\Theta\,\omega)^2}$$
$$= \sqrt{\left\{19930.31\left(\frac{2}{57.2956}\right)\right\}^2 + \left\{300\left(\frac{2}{57.2956}\right)(64.16)\right\}^2}$$
$$= 967.2 \text{ N-m}$$

3.42 Complete solution is $x(t) = X_0\,e^{-\zeta\omega_n t}\cos(\omega_d t+\phi_0) + X\cos(\omega t-\phi)$

$\omega = 2\pi(3.5) = 21.9912$ rad/sec, $\quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2500}{10}} = 15.8114$ rad/sec

$$\delta_{st} = \frac{F_0}{k} = \frac{180}{2500} = 0.072 \text{ m}$$

$$\zeta = \frac{c}{2m\omega_n} = \frac{45}{2(10)(15.8114)} = 0.1423, \quad r = \frac{\omega}{\omega_n} = \frac{21.9912}{15.8114} = 1.3908$$

$$\zeta\omega_n = 2.25, \quad \omega_d = \sqrt{1-\zeta^2}\,\omega_n = 15.6505$$

$$X = \frac{\delta_{st}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{0.072}{\left[(1-1.3908^2)^2 + (2\times 0.1423\times 1.3908)^2\right]^{1/2}}$$
$$= 0.07095 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) = \tan^{-1}\left(\frac{0.3958}{-0.9343}\right) = -22.9591^\circ$$

$$x(t) = X_0\,e^{-2.25t}\cos(15.6505t+\phi_0) + 0.07095\cos(21.9912t+22.9591^\circ)$$

$$\dot{x}(t) = -2.25X_0\,e^{-2.25t}\cos(15.6505t+\phi_0) - 15.6505X_0\,e^{-2.25t}\sin(15.6505t+\phi_0)$$
$$-21.9912(0.07095)\sin(21.9912t+22.9591^\circ)$$

$$x(0) = 0.015 = X\cos\phi_0 + 0.07095\cos 22.9591^\circ$$
$$X\cos\phi_0 = -0.05033 \quad ---- \quad (E_1)$$

$$\dot{x}(0) = 5 = -2.25X_0\cos\phi_0 - 15.6505X_0\sin\phi_0 - 1.5603\sin 22.9591^\circ$$
$$X\sin\phi_0 = \frac{-0.6086 - 2.25X_0\cos\phi_0 - 5}{15.6505} = -0.3511 \quad ---- \quad (E_2)$$

Eqs. (E_1) and (E_2) give

$$X_0 = \left\{(-0.05033)^2 + (-0.3511)^2\right\}^{1/2} = 0.3547$$
$$\phi_0 = \tan^{-1}\left(\frac{0.3511}{0.05033}\right) = \tan^{-1}(6.9760) = 81.8423^\circ$$

3.43 (a) Eq. (3.38) gives $\frac{1}{2\zeta} \approx \left(\frac{X}{\delta_{st}}\right)_{max} = \frac{0.2}{0.1} = 2$

$\therefore \; \zeta = 0.25$

(b) Eqs. (3.42) yield

$$\left(\frac{\omega_1}{\omega_n}\right)^2 \approx 1 - 2\zeta = 0.5 \;, \quad \omega_1 = \omega_n \sqrt{0.5} = (5 \times 2\pi)\sqrt{0.5} = 22.2145 \text{ rad/sec}$$

$$\left(\frac{\omega_2}{\omega_n}\right)^2 \approx 1 + 2\zeta = 1.5 \;, \quad \omega_2 = \omega_n \sqrt{1.5} = (5 \times 2\pi)\sqrt{1.5} = 38.4766 \text{ rad/sec}$$

3.44 Amplitude of vibration under base excitation:

$$X = Y \left\{ \frac{\sqrt{k^2 + (c\,\omega)^2}}{\left[\left(k - m\,\omega^2\right)^2 + (c\,\omega)^2\right]^{\frac{1}{2}}} \right\}$$

$$= \frac{(0.2)\sqrt{k^2 + c^2\,(157.08)^2}}{\left[\left\{k - 2000\,(157.08)^2\right\}^2 + c^2\,(157.08)^2\right]^{\frac{1}{2}}} = 0.1 \text{ m} \qquad (1)$$

Let $k = 5\,(10^6)$ N/m. Then Eq. (1) gives:

$$\frac{\sqrt{25\,(10^{12}) + 2.4674\,(10^4)\,c^2}}{\sqrt{1966.7717\,(10^{12}) + 2.4674\,(10^4)\,c^2}} = 0.5$$

i.e., $1.85055\,(10^4)\,c^2 = 466.6929\,(10^{12})$ i.e., $c = 158805.0$ N–s/m

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3.45

$$\frac{X}{Y} = \left[\frac{k^2 + c^2\,\omega^2}{\left(k - m\,\omega^2\right)^2 + c^2\,\omega^2}\right]^{\frac{1}{2}} .sp$$

$$\text{or} \quad \frac{10^{-6}}{Y} = \left[\frac{(10^6) + (10^3\,(200\,\pi))^2}{\left\{10^6 - \left(\frac{5000}{9.81}\right)(200\,\pi)^2\right\}^2 + \left\{(10^3)\,(200\,\pi)\right\}^2}\right]^{\frac{1}{2}}$$

or $Y = 169.5294\,(10^{-6})$ m

3.46 Equation of motion:

$$I_0\,\ddot{\theta} + \left(k\,\frac{\ell}{4}\,\theta\right)\frac{\ell}{4} + \left(c\,\frac{\ell}{4}\,\dot{\theta}\right)\frac{\ell}{4} + \left(k\,\frac{3\,\ell}{4}\,\theta\right)\frac{3\,\ell}{4} = M_0 \cos\omega t$$

$$\text{or} \quad I_0\,\ddot{\theta} + c\,\frac{\ell^2}{16}\,\dot{\theta} + \frac{5}{8}\,k\,\ell^2\,\theta = M_0 \cos\omega t$$

$$\text{where } I_0 = \frac{m\,\ell^2}{12} + m\left(\frac{\ell}{4}\right)^2 = \frac{7}{48}\, m\,\ell^2 = \frac{7}{48}\,(10)\,(1^2) = 1.4583 \text{ kg—m}^2$$

$$\frac{c\,\ell^2}{16} = \frac{(1000)\,(1^2)}{16} = 62.5 \text{ N—m—s/rad}$$

$$\frac{5}{8}\,k\,\ell^2 = \frac{5}{8}\,(5000)\,(1^2) = 3125.0 \text{ N—m/rad}$$

$$\omega = \frac{1000\,(2\,\pi)}{60} = 104.72 \text{ rad/sec}$$

Equation of motion becomes:

$$1.4583\,\ddot{\theta} + 62.5\,\dot{\theta} + 3125.0\,\theta = 100 \cos 104.72\,t$$

Steady state response is given by Eq. (3.28):

$$\theta_p(t) = \Theta \cos(\omega\,t - \phi) = \Theta \cos(104.72\,t - \phi).\text{sp}$$

$$\text{where } \Theta = \frac{100}{\left[\left\{3125.0 - 1.4583\,(104.72^2)\right\}^2 + \left\{62.5\,(104.72)\right\}^2\right]^{\frac{1}{2}}} = 0.006927 \text{ rad}$$

$$\text{and } \phi = \tan^{-1}\left(\frac{62.5\,(104.72)}{3125.0 - 1.4583\,(104.72^2)}\right) = -\,0.4705 \text{ rad} = -\,26.9606^\circ$$

3.47 $m = 100$ kg, $F_0 = 100$ N, $X_{max} = 0.005$ m at $\omega = 300$ rpm $= 31.416$ rad/sec. Equations (3.33) and (3.34) yield:

$$\omega = \omega_n \sqrt{1 - 2\,\zeta^2} = \sqrt{\frac{k}{m}}\,\sqrt{1 - 2\,\zeta^2} = 31.416$$

$$\text{or } k\,(1 - 2\,\zeta^2) = (100)\,(31.416^2) = 98{,}696.5056 \qquad (1)$$

$$\text{and } X_{max} = \delta_{st}\,\frac{1}{2\,\zeta\sqrt{1-\zeta^2}} = \frac{F_0}{k}\,\frac{1}{2\,\zeta\sqrt{1-\zeta^2}} = 0.005$$

$$\text{or } k\,\zeta\sqrt{1-\zeta^2} = \frac{F_0}{2\,(0.005)} = 10{,}000.0 \qquad (2)$$

Divide Eq. (1) by (2):

$$\frac{1 - 2\,\zeta^2}{\zeta\sqrt{1-\zeta^2}} = 9.8696 \qquad (3)$$

Squaring Eq. (3) and rearranging leads to:

$$101.4090\,\zeta^4 - 101.4090\,\zeta^2 + 1 = 0 \quad \text{or} \quad \zeta = 0.0998,\ 0.9950$$

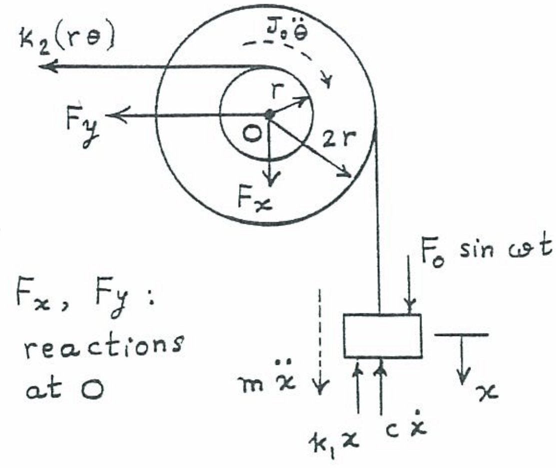
Using $\zeta = 0.0998$ in Eq. (1), we obtain

$$k = \frac{98696.5056}{1 - 2\,(0.0998^2)} = 100{,}702.4994 \text{ N/m}$$

Since $\zeta = \frac{c}{2\,m\,\omega_n}$, we find

$$c = 2\,m\,\omega_n\,\zeta = 2\,(100)\sqrt{\frac{100702.4944}{1000}}\,(0.0998) = 633.4038 \text{ N—s/m}$$

3.48



Equation of motion for rotation of pulley about O:

$$- k_2 (\theta r) r - J_0 \ddot{\theta} - k_1 x (2 r) - c \dot{x} (2 r) + F_0 \sin \omega t (2 r) - m \ddot{x} (2 r) = 0 \qquad (1)$$

where $\theta = x/(2r)$. Equation (1) can be rearranged as:

$$\left(\frac{J_0}{2 r} + 2 m r \right) \ddot{x} + 2 c r \dot{x} + \left(2 k_1 r + \frac{1}{2} k_2 r \right) x = 2 r F_0 \sin \omega t \qquad (2)$$

For given data, Eq. (2) becomes

$$11 \ddot{x} + 50 \dot{x} + 112.5 x = 5 \sin 20 t \qquad (3)$$

Steady state response is given by Eq. (3.25):

$$x_p(t) = X \cos (\omega t - \phi)$$

where $X = \dfrac{5}{\left[\left\{ 112.5 - 11 (20^2) \right\}^2 + \left\{ 50 (20) \right\}^2 \right]^{\frac{1}{2}}} = 0.001136$ m

and $\phi = \tan^{-1} \left(\dfrac{50 (20)}{112.5 - 11 (20^2)} \right) = - 0.2291$ rad $= - 13.1287°$

3.49 (a)

$$\sum M_0 = 0 \text{ (about hinge):}$$

$$I_0 \ddot{\theta} + \left(k \theta \frac{3 \ell}{4} \right) \frac{3 \ell}{4} + (c \ell \dot{\theta}) \ell = \frac{\ell}{2} F_0 \sin \omega t$$

$$\text{or} \quad I_0 \ddot{\theta} + c \ell^2 \dot{\theta} + \frac{9}{16} k \ell^2 \theta = \frac{F_0 \ell}{2} \sin \omega t$$

Magnitude of steady state response:

$$\Theta_a = \left(\frac{F_0 \ell}{2} \right) \Bigg/ \left[\left\{ \frac{9}{16} k \ell^2 - I_0 \omega^2 \right\}^2 + (c \ell^2 \omega)^2 \right]^{\frac{1}{2}} \qquad (1)$$

(b)

$$\sum M_0 = 0 \text{ (about hinge):}$$

$$I_0 \ddot{\theta} + (k \ell \theta) \ell + \left(c \frac{3\ell}{4} \dot{\theta}\right) \frac{3\ell}{4} = \frac{\ell}{2} F_0 \sin \omega t$$

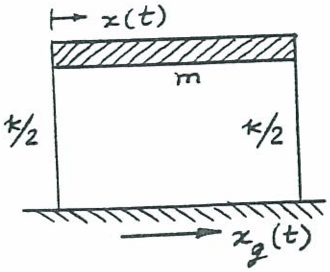
$$\text{or} \quad I_0 \ddot{\theta} + \frac{9}{16} c \ell^2 \dot{\theta} + k \ell^2 \theta = \frac{F_0 \ell}{2} \sin \omega t$$

Magnitude of steady state response:

$$\Theta_b = \left(\frac{F_0 \ell}{2}\right) \Bigg/ \left[\left\{k \ell^2 - I_0 \omega^2\right\}^2 + \left\{\frac{9}{16} c \ell^2 \omega\right\}^2\right]^{\frac{1}{2}} \qquad (2)$$

Usually, c is small compared to k. If the term containing c is negligible, Θ_a will be smaller than Θ_b. Hence arrangement (a) is desirable.

3.52 $\ddot{y}(t) = \ddot{x}_g(t) = A \cos \omega t \quad ; \qquad \dot{y}(t) = \frac{A}{\omega} \sin \omega t + B_1$



$$y(t) = -\frac{A}{\omega^2} \cos \omega t + B_1 t + B_2$$

Assuming $y(0) = \dot{y}(0) = 0$, we get

$$y(t) = -\frac{A}{\omega^2} \cos \omega t$$

Equation of motion:

$$m\ddot{x} + k(x - y) = 0$$

i.e., $m\ddot{z} + kz = -m\ddot{y} = -m\ddot{x}_g(t) = -mA \cos \omega t$

where $z = x - y$

Solution is :

$$z(t) = \frac{-mA \cos \omega t}{k - m\omega^2}$$

$$\therefore \quad x(t) = z(t) + y(t) = -\left(\frac{m}{k - m\omega^2} + \frac{1}{\omega^2}\right) A \cos \omega t$$

3.53 From solution of problem 3.52,

$$x(t) = \left|\frac{-mA}{k - m\omega^2}\right| \sin \omega t - \frac{A}{\omega^2} \sin \omega t$$

$$= \left|\frac{-2000\left(\frac{100}{1000}\right)}{0.1 * 10^6 - 2000(25)^2}\right| \sin 25t - \left(\frac{100}{1000}\right)\frac{1}{(25)^2} \sin 25t$$

For maximum $x(t)$,

$$x(t) = \left(\frac{-200}{1.15 \times 10^6} - \frac{1}{6250}\right) \sin 25t = -3.3391 \times 10^{-4} \sin 25t \text{ m}$$

$\therefore$ Maximum horizontal displacement of floor $= 0.3339$ mm

3.54

$$m(\ddot{x}-\ddot{y}) + k(x-y) = -m\ddot{y} = -m\ddot{x}_g \qquad (E_1)$$

Here $y(t) = x_g(t) = X_g \cos\omega t$, and Eq. (E_1) becomes

$$m\ddot{z} + kz = m\omega^2 X_g \cos\omega t \qquad \text{with } z = x-y$$

Solution is:

$$z(t) = \frac{m\omega^2 X_g \cos\omega t}{k - m\omega^2} = \frac{X_g r^2 \cos\omega t}{1-r^2}$$

with $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.5\times10^6}{2000}} = 15.8114$ rad/sec

and $r = \omega/\omega_n = 200/15.8114 = 12.6491$

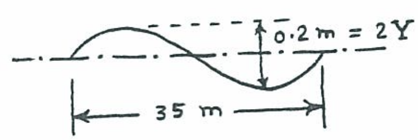
$$z(t) = \left(\frac{15}{1000}\right)\left\{\frac{12.6491^2}{1-12.6491^2}\right\}\cos 200t = -0.01509 \cos 200t \text{ m}$$

$$x(t) = y(t) + z(t) = \{0.015 \cos 200t - |0.01509| \cos 200t\} \text{ m}$$

$\therefore$ Amplitude of vibration of floor $= 0.03009$ m $= 30.09$ mm.

3.55

Time taken by car to travel one cycle (35 m) is



$$\tau = \frac{35\times3600}{60\times1000} = 2.1 \text{ sec}$$

Excitation frequency $= \omega = \frac{2\pi}{\tau} = 2.992$ rad/sec

$\omega_n = 2\pi(2) = 12.5664$ rad/sec, $\quad r = \frac{\omega}{\omega_n} = 0.2381$, $\quad \zeta = 0.15$

Amplitude of vibration of car is given by Eq. (3.68):

$$\frac{X}{Y} = \left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2} \qquad (E_1)$$

$$X = 0.1\left\{\frac{1+(2\times0.15\times0.2381)^2}{(1-0.2381^2)^2+(2\times0.15\times0.2381)^2}\right\}^{1/2}$$

$$= 0.105977 \text{ m}$$

The most unfavorable speed corresponds to the maximum of $\frac{X}{Y}$ in Eq. (E_1). For maximum of $\frac{X}{Y}$ with respect to r,

$$\frac{d}{dr}\left[\frac{1+4\zeta^2r^2}{1+r^4-2r^2+4\zeta^2r^2}\right] = 0$$

i.e., $$\frac{(1+r^4-2r^2+4\zeta^2r^2)(8\zeta^2r) - (1+4\zeta^2r^2)(4r^3-4r+8\zeta^2r)}{(1+r^4-2r^2+4\zeta^2r^2)^2} = 0$$

i.e., $$-4r(2\zeta^2r^4 + r^2 - 1) = 0$$

i.e., $$r = 0 \quad\text{or}\quad r^2 = \frac{-1\pm\sqrt{1+8\zeta^2}}{4\zeta^2}$$

Feasible value of $r^2 = \dfrac{-1 + \sqrt{1 + 8(0.15)^2}}{4(0.15)^2} = 0.9586$

$$r = \frac{\omega}{\omega_n} = 0.9791$$

$$\omega = 0.9791\ (12.5664) = 12.3035 \text{ rad/sec} = \frac{2\pi}{\tau}$$

where $\tau = \dfrac{35 \times 3600}{s \times 1000}$ and s = speed of car in km/hr.

$$\therefore \quad s = \frac{12.3035 \times 35 \times 3.6}{2\pi} = 246.7279 \text{ km/hr.}$$

3.56 Equations (3.73) and (3.68) give

$$F_T = m\omega^2 X = m\omega^2 Y \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

$$\frac{F_T}{kY} = \frac{m\omega^2}{k}\left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

$$= r^2 \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

3.57 Eq. (3.75): $\quad m\ddot{z} + c\dot{z} + kz = -m\ddot{y} = m\omega^2 Y \cos\omega t$

steady-state solution is:

$$z(t) = \frac{m\omega^2 Y \cos(\omega t - \phi_1)}{(k - m\omega^2)^2 + (c\omega)^2} = Z\cos(\omega t - \phi_1)$$

where $\phi_1 = \tan^{-1}\left(\dfrac{c\omega}{k - m\omega^2}\right)$

Damping force $= c\,\dfrac{dz}{dt} = -c\omega Z \sin(\omega t - \phi_1)$

Energy absorbed per cycle by the damper (E):

$$E = \int_{t=0}^{2\pi/\omega} c\,\frac{dz}{dt}\cdot dz = \int_0^{2\pi/\omega}\{-c\omega Z\sin(\omega t - \phi_1)\}\{-\omega Z \sin(\omega t - \phi_1)\}\,dt$$

$$= c\,\omega^2 Z^2 \int_0^{2\pi/\omega} \sin^2(\omega t - \phi_1)\,dt = \pi c\omega Z^2$$

Since $Z = m\omega^2 Y \Big/ \sqrt{(k - m\omega^2)^2 + (c\omega)^2}$,

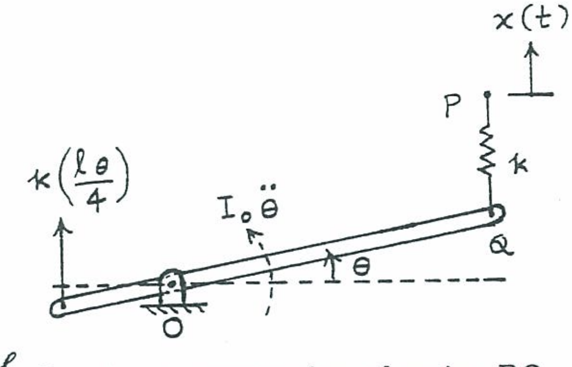
$$E = \left\{\frac{\pi c\omega\ (m^2\omega^4 Y)}{(k - m\omega^2)^2 + c^2\omega^2}\right\}$$

For maximum power, $\dfrac{dE}{dc} = 0$

i.e., $$\frac{\{(k-m\omega^2)^2+c^2\omega^2\}\,(\pi\omega^5 m^2 Y) - \pi c\,\omega^5 m^2 Y\,(2c\,\omega^2)}{\{(k-m\omega^2)^2+c^2\omega^2\}^2} = 0$$

i.e., $$c = \left(\frac{k-m\omega^2}{\omega}\right).$$

3.58



Linear displacement of point Q due to $\theta = \frac{3\,\ell}{4}\,\theta$ and net compression of spring PQ $= \frac{3}{4}\,\ell\,\theta - x(t)$. Equation of motion:

$$I_0\,\ddot{\theta} = -\,\frac{k\,\ell\,\theta}{4}\,\frac{\ell}{4} - k\left(\frac{3\,\ell\,\theta}{4} - x(t)\right)\frac{3\,\ell}{4} \qquad (1)$$

$$\text{where}\quad I_0 = \frac{1}{12}\,m\,\ell^2 + m\left(\frac{\ell}{4}\right)^2 = \frac{7}{48}\,m\,\ell^2 = \frac{7}{48}\,(10)\,(1^2) = 1.4583\ \text{kg}-\text{m}^2$$

Hence Eq. (1) can be rewritten as

$$I_0\,\ddot{\theta} + \left(\frac{5}{8}\,k\,\ell^2\right)\theta = \left(\frac{3}{4}\,k\,\ell\,x_0\right)\sin\omega\,t \qquad (2)$$

Steady state angular displacement of the bar is given by Eq. (3.6):

$$\Theta = \left(\frac{3}{4}\,k\,\ell\,x_0\right) \Big/ \left(\frac{5}{8}\,k\,\ell^2 - I_0\,\omega^2\right) \qquad (3)$$

$$= \left(\frac{3}{4}\,(1000)\,(1)\,(0.01)\right) \Big/ \left(\frac{5}{8}\,(1000)\,(1^2) - 1.4583\,(10^2)\right) = 0.01565\ \text{rad}$$

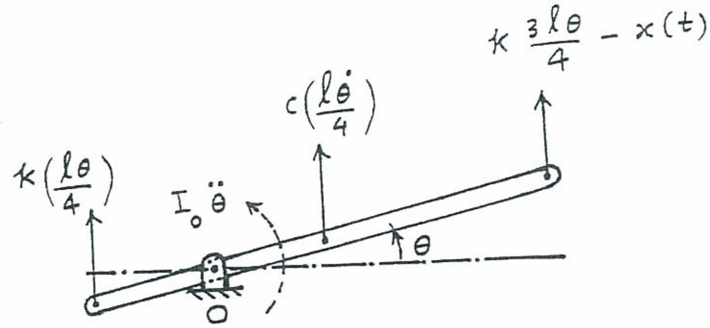
$$\text{and hence}\quad \theta(t) = \Theta\,\sin\omega\,t = 0.01565\,\sin 10\,t\ \text{rad}$$

3.59 Equation of motion:

$$I_0\,\ddot{\theta} = -\,k\,\frac{\ell\,\theta}{4}\left(\frac{\ell}{4}\right) - c\,\frac{\ell}{4}\,\dot{\theta}\left(\frac{\ell}{4}\right) - k\left(\frac{3\,\ell}{4}\,\theta - x(t)\right)\frac{3\,\ell}{4}$$

$$\text{i.e.,}\quad I_0\,\ddot{\theta} + \frac{1}{16}\,c\,\ell^2\,\dot{\theta} + \frac{5}{8}\,k\,\ell^2 = \frac{3}{4}\,k\,\ell\,x(t) = \frac{3}{4}\,k\,\ell\,x_0\,\sin\omega\,t \qquad (1)$$

$$\text{where}\quad I_0 = \frac{1}{12}\,m\,\ell^2 + m\left(\frac{\ell}{4}\right)^2 = \frac{7}{48}\,m\,\ell^2 = \frac{7}{48}\,(10)\,(1^2) = 1.4583\ \text{kg}-\text{m}^2 \qquad (2)$$



Using given data, Eq. (1) can be expressed as

$$1.4583\ \ddot{\theta} + \frac{1}{16}\ (500)\ (1^2)\ \dot{\theta} + \frac{5}{8}\ (1000)\ (1^2)\ \theta = \frac{3}{4}\ (1000)\ (1)\ (0.01)\ \sin 10\ t$$

$$\text{i.e.,}\quad 1.4583\ \ddot{\theta} + 31.25\ \dot{\theta} + 625.0\ \theta = 7.5\ \sin 10\ t \qquad (3)$$

Steady state angular displacement of the bar is given by Eq. (3.28) with:

$$\Theta = \frac{7.5}{\left\{\left[625.0 - 1.4583\ (10^2)\right]^2 + 31.25^2\ (10^2)\right\}^{\frac{1}{2}}} = 0.01311\ \text{rad}$$

$$\phi = \tan^{-1}\left(\frac{31.25\ (10)}{625.0 - 1.4583\ (10^2)}\right) = 0.5779\ \text{rad}$$

$$\therefore\ \theta(t) = \Theta \sin\ (\omega\ t - \phi) = 0.01311\ \sin\ (10\ t - 0.5779)\ \text{rad}$$

3.60 Displacement transmissibility (T):

$$T = \frac{X}{Y} = \left[\frac{1 + (2\ \zeta\ r)^2}{(1 - r^2)^2 + (2\ \zeta\ r)^2}\right]^{\frac{1}{2}}$$

For maximum of T,

$$\frac{dT}{dr} = \frac{1}{2}\left[\frac{1 + 4\ \zeta^2\ r^2}{(1 + r^4 - 2\ r^2) + 4\ \zeta^2\ r^2}\right]^{-\frac{1}{2}}$$

$$\frac{\left[(1 - r^2)^2 + (2\ \zeta\ r)^2\right](8\ \zeta^2\ r) - (1 + 4\ \zeta^2\ r^2)\left[4\ r^3 - 4\ r + 8\ \zeta^2\ r\right]}{\left[(1 - r^2)^2 + (2\ \zeta\ r)^2\right]^2} = 0$$

This equation can be simplified to obtain:

$$(2\ \zeta^2)\ r^4 + r^2 - 1 = 0$$

$$\text{Solution:}\quad r^2 = \frac{-1 \pm \sqrt{1 + 8\ \zeta^2}}{4\ \zeta^2}$$

$$\text{or}\quad r = r_m = \frac{1}{2\ \zeta}\sqrt{\sqrt{1 + 8\ \zeta^2} - 1}$$

3.61

Empty

$$m = \frac{1000}{32.2} = 31.0559 \ \frac{lb - sec^2}{ft}$$

At speed $v = 55$ mph, and wave length 12 ft,

$$\omega = 2\pi f = 2\pi \left(\frac{v\,(1760*3)}{3600} \right) \frac{1}{12}$$

$$= 42.2371 \text{ rad/sec}$$

$k = 30000$ lb/ft

$\zeta = 0.2$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{30000}{31.0559}}$$

$$= 31.0805 \text{ rad/sec}$$

$$r = \frac{\omega}{\omega_n} = \frac{42.2371}{31.0805}$$

$$= 1.3590$$

$$(2\zeta r)^2 = (2*0.2*1.3590)^2$$
$$= 0.2955$$

$$(1-r^2)^2 = (1-1.3590^2)^2$$
$$= 0.7170$$

$$\frac{X}{Y} = \left\{ \frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{1 + 0.2955}{0.7170 + 0.2955} \right\}^{\frac{1}{2}}$$

$$= 1.1311$$

Amplitude of vibration of automobile is magnified by a factor of 1.1311

Fully Loaded

$$m = \frac{3000}{32.2} = 93.1677 \ \frac{lb - sec^2}{ft}$$

$$\omega = 42.2371 \text{ rad/sec}$$

$k = 30000$ lb/ft

$\zeta = 0.2$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{30000}{93.1677}}$$

$$= 17.9444 \text{ rad/sec}$$

$$r = \frac{\omega}{\omega_n} = \frac{42.2371}{17.9444}$$

$$= 2.3538$$

$$(2\zeta r)^2 = (2*0.2*2.3538)^2$$
$$= 0.8864$$

$$(1-r^2)^2 = (1-2.3538^2)^2$$
$$= 20.6141$$

$$\frac{X}{Y} = \left\{ \frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right\}^{\frac{1}{2}}$$

$$= \left\{ \frac{1 + 0.8864}{20.6141 + 0.8864} \right\}^{\frac{1}{2}}$$

$$= 0.2962$$

Amplitude of vibration of automobile is diminished by a factor of 0.2962

3.63

Equation of motion: $M\ddot{x} + c\dot{x} + kx = m e \omega^2 \sin \omega t$

where $\omega = \frac{3000\,(2\pi)}{60} = 314.16$ rad/sec, $M = 100$ kg, $c = 2000$ N-s/m, $k = 10^6$ N/m, $m = 0.1$ kg and $e = r = 0.1$ m. Steady state response is:

$$x_p(t) = X \sin(\omega t - \phi)$$

where
$$X = \frac{m e \omega^2}{\left[(k - M\omega^2)^2 + (c\,\omega)^2\right]^{\frac{1}{2}}}$$

$$= \frac{0.1\,(0.1)\,(314.16^2)}{\left[\left\{10^6 - 100\,(314.16^2)\right\}^2 + (2000\,(314.16))^2\right]^{\frac{1}{2}}} = 110.9960\,(10^{-6})\text{ m}$$

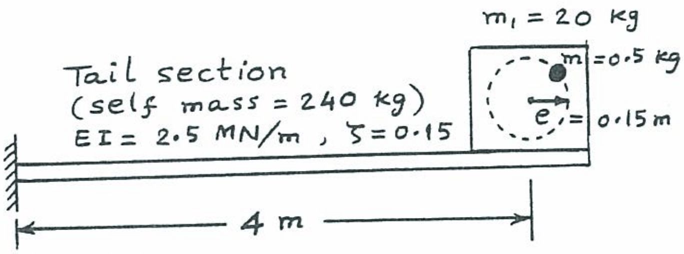
and
$$\phi = \tan^{-1}\left(\frac{c\,\omega}{k - M\omega^2}\right) = \tan^{-1}\left(\frac{2000\,(314.16)}{10^6 - 100\,(314.16^2)}\right)$$
$$= -0.07072\text{ rad} = -4.0520^\circ$$

3.64

k = spring constant of cantilever beam

$$= \frac{3EI}{l^3} = \frac{3\,(2.5\times10^6)}{4^3}$$

$$= 0.1172\times10^6 \text{ N/m}$$



$$\omega_n = \sqrt{\frac{k}{m_1 + 0.25\,m_b}} = \sqrt{\frac{0.1172\times10^6}{20 + 0.25\,(240)}} = 38.2753 \text{ rad/sec}$$

$$\omega = 2\pi\,(1500)/60 = 157.08 \text{ rad/sec}$$

$$r = \omega/\omega_n = 157.08/38.2753 = 4.1040\,, \qquad r^2 = 16.8428$$

Forced response is given by Eq. (3.79):

$$x_p(t) = X \sin(\omega t - \phi)$$

where
$$X = \frac{m e}{m_1}\cdot\frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

$$= \frac{(0.5)(0.15)}{20}\;\frac{16.8428}{\sqrt{(1 - 16.8428)^2 + (2\times0.15\times4.1040)^2}}$$

$$= 3.9747\times10^{-3}\text{ m} = 3.9747\text{ mm}$$

3.65 $\delta_{st} = \frac{45}{1000}\ m = \frac{Mg}{k} = \frac{380 \times 9.81}{k}$

i.e., $k = 82{,}840$ N/m

$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{82{,}840}{380}} = 14.7648$ rad/sec ; $\omega = \frac{2\pi(1750)}{60} = 183.26$ rad/sec

$r = \frac{\omega}{\omega_n} = \frac{183.26}{14.7648} = 12.412$; $r^2 = 154.0566$

(i) Amplitude of vibration

$$X = \frac{me}{M} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{0.15}{380} \frac{154.0566}{\sqrt{(153.0566)^2 + 0}}$$

$$= 3.9732 \times 10^{-4}\ m$$

(ii) Force transmitted to ground

$$= kX = (82840)(3.9732 \times 10^{-4}) = 32.9140\ N$$

3.66 $I = \frac{1}{12}(0.5)(0.1)^3 = 0.4167 \times 10^{-4}\ m^4$

$$k = \frac{192\,EI}{l^3} = \frac{192\,(2.07 \times 10^{11})(0.4167 \times 10^{-4})}{(5)^3} = 1.3248 \times 10^7\ N/m$$

(a) $\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{13.248 \times 10^6}{75}} = 420.2856$ rad/sec

$\omega = 2\pi(1200)/60 = 125.664$ rad/sec

$r = \omega/\omega_n = 125.664/420.2856 = 0.299$, $r^2 = 0.0894$

Amplitude of steady-state vibration is given by Eq. (3.30) with $\zeta = 0$:

$$X = \frac{\delta_{st}}{|r^2 - 1|} = \frac{F_0}{k|(r^2-1)|} = \frac{5000}{(1.3248 \times 10^7)(0.9106)}$$

$$= 0.4145 \times 10^{-3}\ m$$

(b) Using the effective mass due to self weight of beam (for a cantilever) to be valid here also,

$$\omega_n = \sqrt{\frac{k}{M + 0.2357\ m}}$$

where M = mass of motor = 75 kg, and

m = mass of beam = $(5 \times 0.5 \times 0.1)\left(\frac{76.5 \times 10^3}{9.81}\right) = 1949.5313$ kg

$$\omega_n = \sqrt{\frac{13.248 \times 10^6}{75 + (1949.5313)(0.2357)}} = 157.4339\ \text{rad/sec}$$

$r = \omega/\omega_n = 125.664/157.4339 = 0.7982$, $r^2 = 0.6371$

$$X = \frac{\delta_{st}}{|r^2 - 1|} = \frac{F_0}{k\,|r^2-1|} = \frac{5000}{(1.3248 \times 10^7)(0.3629)}$$

$$= 1.0400 \times 10^{-3} \text{ m}$$

3.67 Let width $= 0.5$ m and thickness $= t$ m.

$$I = \frac{1}{12}(0.5)\,t^3 = \frac{t^3}{24} \text{ m}^4$$

$$k = \frac{3EI}{l^3} = \frac{3(2.07 \times 10^{11})(t^3/24)}{(5)^3} = 2.07 \times 10^8\, t^3 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{M + 0.2357\, m}}$$

where m = mass of beam $= (5 \times 0.5 \times t)\left(\frac{76.5 \times 10^3}{9.81}\right) = 19495.41\,t$ kg

$$\omega_n = \sqrt{\frac{2.07 \times 10^8\, t^3}{75 + 0.2357\,(19495.41\,t)}}$$

$$r = \frac{\omega}{\omega_n} = 125.664 \sqrt{\frac{75 + 4595.0688\,t}{2.07 \times 10^8\, t^3}}$$

$$X = \frac{\delta_{st}}{|r^2-1|} = \frac{F_0}{k\,|r^2-1|}$$

i.e., $$0.5 = \frac{5000}{(2.07 \times 10^8\, t^3)\left\{(125.664)^2\left[\frac{75 + 4595.0688\,t}{2.07 \times 10^8\, t^3}\right] - 1\right\}}$$

i.e., $1.3108 \times 10^4\, t^3 - 4595.069\,t - 74.367 = 0$

By trial and error, the value of t is found as

$$t \approx 0.6 \text{ m}.$$

Since this is too large, we can start with a new width such as 1.0 m.

3.68 $m = (600/9.81)$ N, $\quad \omega = 2\pi(1000)/60 = 104.72$ rad/sec

$k = 6\,(6000) = 36{,}000$ N/m

$$\omega_n = \sqrt{k/m} = \sqrt{36000 \Big/ \left(\frac{600}{9.81}\right)} = 24.2611 \text{ rad/sec}$$

$r = \omega/\omega_n = 104.72/24.2611 = 4.3164$, $\quad r^2 = 18.6311$

$$X = \frac{F_0}{k\,|r^2-1|} = \frac{m_0\, e\, \omega^2}{k\,|r^2-1|}$$

where m_0 = unbalanced mass and e = eccentricity i.e., $$2.5 \times 10^{-3} = \frac{m_o e \, (104.72)^2}{36000 \, |17.6311|}$$

i.e., $m_o e = 0.1447$ kg-m

$\therefore$ Unbalance $= W_o e = m_o g e = 0.1447\,(9.81) = 1.4195$ N-m

---

3.69 $m = \frac{1000}{386.4} = 2.588 \ \frac{lb-s^2}{in}$, $\quad \omega = \frac{2\pi(1500)}{60} = 157.08 \ \frac{rad}{s}$

Possible isolators are:
(i) $k = 45\,000$ lb/in, $\zeta = 0$
(ii) $k = 90\,000$ lb/in, $\zeta = 0$
(iii) $k = 45\,000$ lb/in, $\zeta = 0.15$
(iv) $k = 90\,000$ lb/in, $\zeta = 0.15$

We will compare the force transmissibilities of these isolators.

Force transmissibility $= T_r = \sqrt{\dfrac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}}$

(i) $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{45000}{2.588}} = 131.8634$ rad/sec

$r = \omega/\omega_n = 157.08/131.8634 = 1.1912$, $\quad r^2 = 1.4190$

$$T_r = \frac{1}{|1-r^2|} = \frac{1}{0.419} = 2.3866$$

(ii) $\omega_n = \sqrt{\frac{90\,000}{2.588}} = 186.4829$ rad/sec

$r = \omega/\omega_n = 157.08/186.4829 = 0.8423$, $\quad r^2 = 0.7095$

$$T_r = \frac{1}{|1-r^2|} = \frac{1}{0.2905} = 3.4423$$

(iii) $\omega_n = \sqrt{\frac{45\,000}{2.588}} = 131.8634$ rad/sec

$r = 1.1912$, $\quad r^2 = 1.4190$, $\quad \zeta = 0.15$

$$T_r = \sqrt{\frac{1 + (2 \times 1.1912 \times 0.15)^2}{(1 - 1.4190)^2 + (2 \times 1.1912 \times 0.15)^2}} = 1.9282$$

(iv) $\omega_n = 186.4829$ rad/sec, $\quad r = 0.8423$, $\quad r^2 = 0.7095$, $\quad \zeta = 0.15$

$$T_r = \sqrt{\frac{1 + (2 \times 0.8423 \times 0.15)^2}{(1 - 0.7095)^2 + (2 \times 0.8423 \times 0.15)^2}} = 2.6789$$

$\therefore$ Isolation (iii) is best.

3.70 Eq. (3.82):
$$\frac{MX}{me} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

When $r = 1$,
$$\frac{MX}{me} = \frac{1}{2\zeta} \quad \text{or} \quad \frac{M}{me} = \frac{1}{2\zeta X} = \frac{1}{2\zeta(0.55)} = \frac{1}{1.1\zeta} \qquad (E_1)$$

When r = large,
$$\frac{MX}{me} \approx 1 \quad \text{or} \quad \frac{M}{me} \approx \frac{1}{X} = \frac{1}{0.15} \qquad (E_2)$$

Combining (E_1) and (E_2), we obtain
$$\frac{M}{me} = \frac{1}{0.15} = \frac{1}{1.1\zeta}$$
$$\therefore \quad \zeta = 0.1364$$

3.71 For each spring,
$$\underset{\sim}{k} = \frac{G d^4}{64 n R^3} = \frac{(11.5385 \times 10^6)(0.25)^4}{64(8)(1.5)^3} = 26.083 \text{ lb/in}$$

Total $k = 4(26.083) = 104.332$ lb/in

$$\omega = \frac{2\pi(1800)}{60} = 188.496 \text{ rad/sec}$$

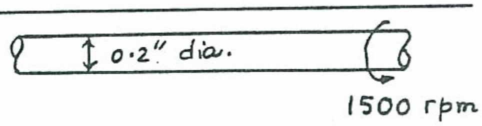
$m = 100/386.4$ lb-s²/in , $\quad M = 750/386.4$ lb-s²/in , $\quad \zeta = 0$

$$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{104.332}{(750/386.4)}} = 7.3316 \text{ rad/sec}$$

$r = 188.496/7.3316 = 25.7102$, $\qquad r^2 = 661.0144$

$$X = \frac{me}{M} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{100(0.01)}{750}\left(\frac{661.0144}{660.0144}\right)$$
$$= 1.3354 \times 10^{-3} \text{ in.}$$

3.72 $\omega = 2\pi(1500)/60 = 157.08$ rad/sec

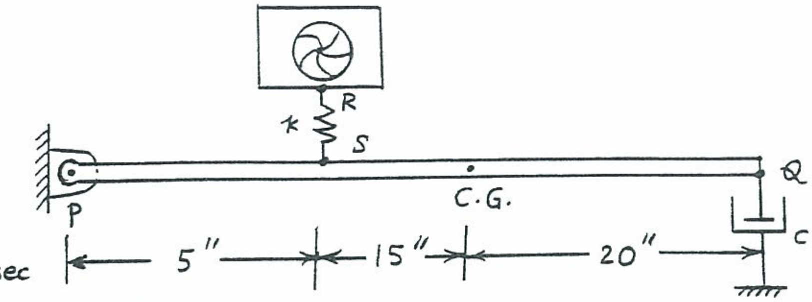


(a) Force due to eccentricity of rotor
$$= m e \omega^2 = \left(\frac{30}{386.4}\right)(0.01)(157.08)^2 = 19.1569 \text{ lb.}$$

(b) H.P. = (Force)(eccentricity)(angular velocity)
$$= (19.1569)\left(\frac{0.01}{12}\right)\left(\frac{157.08}{550}\right) = 0.004559 \text{ hp.}$$

3.74

$$\omega_{n,fan} = \sqrt{\frac{k}{m_{fan}}} = \sqrt{\frac{200}{50/386.4}} = 39.3141 \text{ rad/sec}$$



$$\omega = \frac{2\pi(750)}{60} = 78.54 \text{ rad/sec}$$

$$(J_P)_{plate+fan} = \frac{1}{3}\left(\frac{100}{386.4}\right)(40)^2 + \left(\frac{50}{386.4}\right)(5)^2 = 141.2612 \text{ lb-in-sec}^2$$

$$F_0 = m e \omega^2 = \left(\frac{50}{386.4}\right)(0.1)(78.54)^2 = 79.8205 \text{ lb}$$

Point R is subjected to the force, $F(t) = F_0 \cos \omega t = 79.8205 \cos 78.54\, t$ lb.

Assume that S is not moving.

Then R is displaced by:

$$x(t) = \frac{F_0 \cos \omega t}{|k - m\omega^2|} = \frac{F_0 \cos \omega t}{k\left|1 - \left(\frac{\omega}{\omega_n}\right)^2\right|} = \frac{79.8205 \cos \omega t}{200\left|1 - \left(\frac{78.54}{39.3141}\right)^2\right|}$$

$$= 0.1334 \cos 78.54\, t \text{ inch}$$

Let θ = angular displacement of plate PQ.

Displacement of $S = 5\,\theta$ inch

Extension of spring RS $= (5\theta - 0.1334 \cos 78.54\, t)$ inch

Restoring moment of spring force about P

$$= 200\,[5\theta - 0.1334 \cos 78.54\, t]\, 5 \text{ lb-in}$$

Velocity of $Q = 40\,\dot{\theta}$ inch/sec

Damping force at $Q = 40\,\dot{\theta}\,(1) = 40\,\dot{\theta}$ lb

Moment of damping force about $P = 40\,\dot{\theta}\,(40) = 1600\,\dot{\theta}$ lb-in

Equation of motion of plate PQ:

$$J_P\,\ddot{\theta} + 1600\,\dot{\theta} + 1000\,(5\theta - 0.1334 \cos 78.54\, t) = 0$$

i.e., $$141.2612\,\ddot{\theta} + 1600\,\dot{\theta} + 5000\,\theta = 133.4 \cos 78.54\, t \qquad (E_1)$$

Comparing (E_1) with Eq. (3.24), the solution of (E_1) can be expressed as $\theta_p(t) = \Theta \cos(\omega t - \phi)$

where, from Eqs. (3.30) and (3.31), we get

$$\Theta = \frac{(133.4/5000)}{\sqrt{(1 - 174.2751)^2 + (2 \times 0.9519 \times 13.2013)^2}} = 1.5238 \times 10^{-4} \text{ rad}$$

and $$\phi = \tan^{-1}(-25.1326/173.2751) = -8.2529^\circ$$

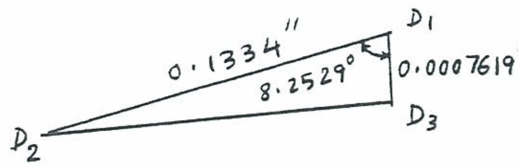
Steady state motion of $Q = \theta_p(40)$

$$= 0.006095 \cos(78.54\, t + 8.2529^\circ) \text{ inch}$$

Displacement of $S = \Theta(5)$ inch

$= (1.5238 \times 10^{-4})(5)$ inch

$= 0.0007619$ inch



$D_2 D_3$ = maximum deformation of spring $\approx 0.1334''$

Max. force transmitted to point $S = k(D_2 D_3)$

$= 200(0.1334) = 26.68$ lb

3.78

$$I = \int_0^{2\pi/\omega} \sin \omega t \cdot \cos(\omega t - \phi)\, dt$$

$$= \int_0^{2\pi/\omega} \sin \omega t \left[\cos \omega t \cdot \cos\phi + \sin \omega t \cdot \sin\phi \right] dt$$

$$= \int_0^{2\pi/\omega} \left\{ \cos\phi \left(\sin \omega t \cdot \cos \omega t \right) + \sin\phi \left(\sin^2 \omega t \right) \right\} dt$$

$$= \int_0^{2\pi/\omega} \left\{ \cos\phi \left(\frac{\sin 2\omega t}{2} \right) + \sin\phi \left(\frac{1 - \cos 2\omega t}{2} \right) \right\} dt$$

$$= \frac{\cos\phi}{2} \left(- \frac{\cos 2\omega t}{2\omega} \right) \Bigg|_0^{2\pi/\omega} + \frac{\sin\phi}{2} \left(t - \frac{\sin 2\omega t}{2} \right) \Bigg|_0^{2\pi/\omega}$$

$$= \frac{\pi}{\omega} \sin\phi$$

$$\Delta W' = \omega F_0 X \cdot I = \omega F_0 X \sin\phi$$

3.79

Let $x(t)$ = displacement of mass m

New length of each spring, $k_1 = (l^2 + x^2)^{1/2}$

New tension in each spring $k_1 = T = \left(\sqrt{l^2 + x^2} - l \right) k_1 + T_0$

Horizontal component of new tension in each spring k_1

$$= T x / \sqrt{l^2 + x^2}$$

Vertical component of new tension in each spring $k_1 = \dfrac{T \cdot l}{\sqrt{l^2 + x^2}}$

Total friction force $= \mu\, mg + \dfrac{2 T l}{\sqrt{l^2 + x^2}}$

when mass moves to right:

Equation of motion of mass m:

$$m\ddot{x} + k_2 x + \frac{2 T x}{\sqrt{l^2 + x^2}} - \mu \left[mg + \frac{2 T l}{\sqrt{l^2 + x^2}} \right] = p_0 A \sin \omega t$$

where A = area of piston.

i.e., $m\ddot{x} + x \left(k_2 + 2 \dfrac{T_0}{l} \right) = \mu mg + 2\mu T_0 + p_0 A \sin \omega t$

Similarly, when the mass moves to left:

$$m\ddot{x} + x \left(k_2 + 2 \frac{T_0}{l} \right) = -\mu mg - 2\mu T_0 + p_0 A \sin \omega t$$

3.82

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2100}{2}} = 32.403703 \text{ rad/sec}$$

N = vertical force = $mg = 2(9.81) = 19.62$ N

$$\frac{\omega}{\omega_n} = \frac{2.5173268 \times 2\pi}{32.403703} = 0.4881191$$

$$X = \frac{F_0}{k}\left[\frac{1 - \left(\frac{4\mu N}{\pi F_0}\right)^2}{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2}\right]^{1/2}$$

i.e.,
$$0.075 = \frac{120}{2100}\left[\frac{1 - \left\{\frac{4\mu(19.62)}{\pi(120)}\right\}^2}{(1 - 0.4881191^2)}\right]^{1/2}$$

i.e.,
$$1.3125 = \left(\frac{1 - 0.04334\,\mu^2}{0.5802473}\right)^{1/2}$$

i.e.,
$$0.9995666 = 1 - 0.04334\,\mu^2$$

i.e.,
$$\mu = 0.1$$

3.83

(a) $k = \frac{W}{\delta_{st}} = \frac{5000}{0.05} = 10^5$ N/m

When $\omega = \omega_n$, Eq. (3.102) gives

$$X = \frac{F_0}{k\beta} \Rightarrow 0.1 = \frac{1000}{(10^5)\beta} \Rightarrow \beta = 0.1$$

(b) $\Delta W = \pi\, c_{eq}\, \omega X^2 = \pi \beta k X^2$ where $c_{eq} = \frac{\beta k}{\omega}$ from Eq. (3.100)

$$\Delta W = \pi(0.1)(10^5)(0.1)^2 = 314.16 \text{ Joules/cycle}$$

(c) Steady state amplitude at one-quarter of resonant frequency:

$$\frac{\omega}{\omega_n} = 0.25$$

$$X = \frac{F_0}{k\left[\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \beta^2\right]^{1/2}} = \frac{1000}{10^5\left[\left\{1 - 0.25^2\right\}^2 + (0.1)^2\right]^{1/2}}$$

$$= 0.01061 \text{ m}$$

(d) Steady state amplitude at thrice the resonant frequency:

$$\frac{\omega}{\omega_n} = 3$$

$$X = \frac{1000}{10^5\left[(1 - 3^2)^2 + (0.1)^2\right]^{1/2}} = 0.00125 \text{ m}$$

3.84

$$\Delta W = \pi \beta k X^{\gamma}$$

$$\left.\begin{aligned} 3.8 &= \pi \beta (60\,000) (0.04)^{\gamma} \\ 9.5 &= \pi \beta (60000) (0.06)^{\gamma} \end{aligned}\right\} \Rightarrow \quad \begin{aligned} \beta (0.04)^{\gamma} &= 0.0000202 \\ \beta (0.06)^{\gamma} &= 0.0000504 \end{aligned}$$

Taking logarithms,

$$\ln \beta + \gamma \ln (0.04) = \ln (0.0000202)$$

$$\ln \beta + \gamma \ln (0.06) = \ln (0.0000504)$$

i.e.,

$$\ln \beta - 3.218876 \, \gamma = -10.809828 \quad ---- (E_1)$$

$$\ln \beta - 2.813411 \, \gamma = -9.895511 \quad ---- (E_2)$$

subtracting (E_1) from (E_2), $\quad 0.405465 \, \gamma = 0.914309$

$$\gamma = 2.254964$$

From (E_1), $\quad \ln \beta = -10.809828 + 3.218876 \, (2.254964) = -3.551378$

$$\beta = 0.028685$$

3.85

Work done $= W = \int F \, dx = \int F \, \dot{x} \, dt$

If $F(t) = F_0 \cos \omega t$ and $x(t) = X \cos(\omega t - \phi)$, work done in one cycle

$$= W = -\int_0^{2\pi/\omega} F_0 \cos \omega t . \; \omega X \sin (\omega t - \phi) \; dt$$

$$= -\frac{F_0 \omega X \cos \phi}{2} \left(-\frac{1}{2\omega} \cos 2\omega t \right)_0^{2\pi/\omega} + \frac{F_0 \omega X \sin \phi}{2} \left(t + \frac{1}{2\omega} \sin 2\omega t \right)_0^{\frac{2\pi}{\omega}}$$

$$= F_0 \pi X \sin \phi$$

Given data: $\quad F_0 = 5$ lb, $\omega = 3\pi \dfrac{\text{rad}}{\text{sec}}$, $\tau = \frac{2}{3}$ sec, $\phi = \frac{\pi}{3}$, $X = 0.5''$

$$W = F_0 \pi X \sin \phi = 5\pi (0.5) \sin \frac{\pi}{3} = 6.8018 \text{ lb-in}$$

(i) In one second, it will complete $1\frac{1}{2}$ cycles.

$$W\Big|_{1 \text{ second}} = 1.5 \, W = 10.2027 \text{ lb-in.}$$

(ii) In four seconds, it will complete 6 cycles.

$$W\Big|_{4 \text{ seconds}} = 6 \, W = 40.8108 \text{ lb-in.}$$

3.86

Damping force $= F = c\,(\dot{x})^n$

Energy dissipated per quarter cycle during harmonic motion $x(t) = X \sin \omega t$ is

$$\frac{\Delta W}{4} = \int_0^{\pi/2\omega} c\,(\dot{x})^n \, dx = \int_0^{\pi/2\omega} c \, (\omega X \cos \omega t)^n \, dx$$

But $\quad dx = \dot{x} \, dt = \omega X \cos \omega t \cdot dt$

$$\Delta W = 4c\,\omega^{n+1} X^{n+1} \int_0^{\pi/2\omega} \cos^{n+1}\omega t \cdot dt$$

$$= 4c\,\omega^{n+1} X^{n+1} \left\{ \frac{1}{(n+1)\omega} \cos^n \omega t \cdot \sin \omega t \Big|_0^{\pi/2\omega} + \frac{n}{n+1} \int_0^{\pi/2\omega} \cos^{n-1}\omega t \cdot dt \right\}$$

$$= 4c\,\omega^{n+1} X^{n+1} \left(\frac{n}{n+1}\right) \int_0^{\pi/2\omega} \cos^{n-1}\omega t \cdot dt$$

Equating this expression to $\pi\, c_{eq}\, \omega\, X^2$, we obtain

$$c_{eq} = \frac{4c\,\omega^n X^{n-1}}{\pi} \left(\frac{n}{n+1}\right) \int_0^{\pi/2\omega} \cos^{n-1}\omega t \cdot dt \equiv c\,\omega^n X^{n-1} \alpha_n$$

where $$\alpha_n = \frac{4}{\pi}\left(\frac{n}{n+1}\right) \int_0^{\pi/2\omega} \cos^{n-1}\omega t \cdot dt \quad \text{----} \; (E_1)$$

For example, for $n = 2$, (E_1) becomes

$$\alpha_n = \frac{4}{\pi}\left(\frac{2}{3}\right) \int_0^{\pi/2\omega} \cos\omega t \cdot dt = \frac{8}{3\pi}\left(\frac{\sin\omega t}{\omega}\right)_0^{\pi/2\omega} = \frac{8}{3\pi\omega}$$

and hence $$c_{eq} = \frac{8c\,\omega X}{3\pi}$$

which can be seen to be same as the expression found in Example 3.7.

For few other values of n, α_n can be found as follows:

n	1	2	3	4
α_n	$\frac{1}{\omega}$	$\frac{8}{3\pi\omega}$	$\frac{3}{4\omega}$	$\frac{32}{15\pi\omega}$

The amplitude can be found as

$$X = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + c_{eq}^2\,\omega^2}} = \frac{F_0}{\sqrt{k^2(1-r^2)^2 + c_{eq}^2\,\omega^2}}$$

$$= \frac{F_0}{\sqrt{k^2(1-r^2)^2 + c^2\,\omega^{2(n+1)}\, X^{2(n-1)}\, \alpha_n^2}}$$

3.87 Energy dissipated per cycle for viscous damping $= \pi c\, \omega X^2$

Energy dissipated per cycle for Coulomb damping $= 4\mu N X$

Equivalent viscous damping constant (c_{eq}) is given by

$$\pi\, c_{eq}\, \omega X^2 = \pi c\, \omega X^2 + 4\mu N X$$

$$c_{eq} = \left(c + \frac{4\mu N}{\pi\omega X}\right)$$

Amplitude X is given by

$$X = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + c_{eq}^2 \omega^2}} = \frac{F_0}{\sqrt{k^2(1-r^2)^2 + c_{eq}^2 \omega^2}}$$

substituting for c_{eq}, squaring and rearranging,

$$X^2 \left\{ k^2 (1-r^2)^2 + c^2 \omega^2 \right\} + X \left(\frac{8 \mu N c \omega}{\pi} \right) + \left(\frac{16 \mu^2 N^2}{\pi^2} - F_0^2 \right) = 0$$

3.88 (a) Equation of motion $m\ddot{x} \pm \mu N + c(\dot{x})^3 + kx = F_0 \cos \omega t$

Thus the system has combined Coulomb and velocity-cubed damping.

For Coulomb damping, $c_{eq\,1} = \dfrac{4 \mu N}{\pi \omega X}$ (E_1)

For velocity-cubed damping, the equivalent viscous damping coefficient can be obtained from the solution of problem 3.68:

$$c_{eq\,2} = c\, \omega^3 X^2 \alpha_3 \qquad (E_2)$$

where

$$\alpha_3 = \frac{4}{\pi} \left(\frac{3}{4} \right) \int_0^{\pi/2\omega} \cos^2 \omega t \; dt = \frac{3}{4\omega} \qquad (E_3)$$

$$\therefore \quad c_{eq\,2} = \frac{3}{4} c\, \omega^2 X^2 \qquad (E_4)$$

and

$$c_{eq} = c_{eq\,1} + c_{eq\,2} = \frac{4 \mu N}{\pi \omega X} + \frac{3}{4} c \omega^2 X^2 \qquad (E_5)$$

(b) Steady state amplitude under harmonic force:

$$X = \frac{F_0}{\sqrt{k^2 (1-r^2)^2 + c_{eq}^2 \omega^2}} = \frac{F_0}{\sqrt{k^2 (1-r^2)^2 + \left\{ \frac{4 \mu N}{\pi \omega X} + \frac{3}{4} c \omega^2 X^2 \right\}^2 \omega^2}} \qquad (E_6)$$

(c) Amplitude ratio:

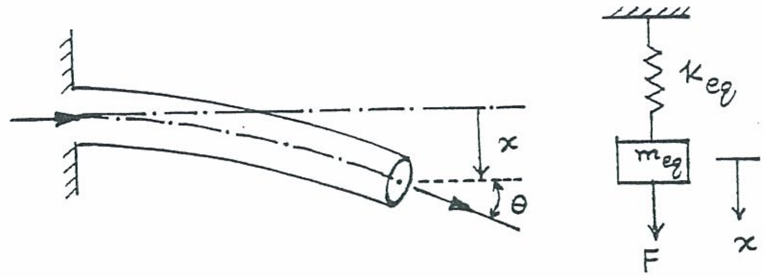
$$\frac{X}{\delta_{st}} = \frac{X}{(F_0/k)} = \frac{1}{\sqrt{(1-r^2)^2 + \left(\frac{c_{eq}^2 \omega^2}{k^2} \right)}}$$

$$= \frac{1}{\sqrt{(1-r^2)^2 + \left\{ \frac{4 \mu N}{\pi X k} + \frac{3}{4} \frac{c\, \omega^3 X^2}{k} \right\}^2}} \qquad (E_7)$$

At resonance, $r = 1$ and Eq. (E_7) reduces to

$$\left. \frac{X}{\delta_{st}} \right|_{\text{resonance}} = \frac{1}{\left\{ \frac{4 \mu N}{\pi X k} + \frac{3}{4k} c\, \omega^3 X^2 \right\}} \qquad (E_8)$$

3.89 Model the pipe as a single degree of freedom system with m_{eq} = equivalent mass at end $= \dfrac{33}{140}$ m (m = mass of pipe; see Problem 2.46) and $k_{eq} = \dfrac{3\,E\,I}{\ell^3}$. Slope of pipe at end:



$$\theta = \frac{F\,\ell^2}{2\,E\,I} = \frac{F\,\ell^3}{3\,E\,I}\left(\frac{3}{2\,\ell}\right) = \frac{3\,x}{2\,\ell}$$

where x = end deflection of the cantilever pipe under a transverse load F. Force induced due to fluid velocity v is $\rho\,A\,v^2$. Force acting on the single degree of freedom system (in vertical direction):

$$F = \rho\,A\,v^2 \sin\theta \approx \rho\,A\,v^2\,\theta = \rho\,A\,v^2\,\frac{3\,x}{2\,\ell}$$

Equation of motion: $m_{eq}\,\ddot{x} + k_{eq}\,x = F$

or $$\frac{33}{140}\,m\,\ddot{x} + \left(\frac{3\,E\,I}{\ell^3} - \frac{3\,\rho\,A\,v^2}{2\,\ell}\right)x = 0$$

Instability occurs when $\dfrac{3\,E\,I}{\ell^3} - \dfrac{3\,\rho\,A\,v^2}{2\,\ell} < 0$ or $v > \sqrt{\dfrac{2\,E\,I}{\rho\,A\,\ell^2}}$

3.90 Assume Reynolds number (R_e) greater than 1000. Strouhal number (St) for vortex shedding is taken as: $St = \frac{f\,d}{V} = 0.21$ where f = frequency of vortex shedding, d = diameter of cylinder and V = velocity of fluid (air). At 50 mph speed,

$$V = \frac{50\,(1760)\,(36)}{3600} = 880 \text{ in/sec} \quad \text{and} \quad f = \frac{0.21\,V}{d} = \frac{184.8}{d} \text{ Hz (d in inches)}$$

For the three sections of the antenna, the vortex frequencies are:

$$f_1 = \frac{184.8}{0.3} = 616.0 \text{ Hz}$$
$$f_2 = \frac{184.8}{0.2} = 924.0 \text{ Hz}$$
$$f_3 = \frac{184.8}{0.1} = 1848.0 \text{ Hz}$$

At 75 mph speed,

$$V = \frac{75\,(1760)\,(36)}{3600} = 1320 \text{ in/sec} \quad \text{and} \quad f = \frac{0.21\,V}{d} = \frac{277.2}{d} \text{ Hz (d in inches)}$$

For the three sections of the antenna, the frequencies are:

$$f_1 = \frac{277.2}{0.3} = 924.0 \text{ Hz}$$
$$f_2 = \frac{277.2}{0.2} = 1386.0 \text{ Hz}$$

$$f_3 = \frac{277.2}{0.1} = 2772.0 \text{ Hz}$$

Since the natural frequencies are much smaller, no instability occurs.

3.91 (a) Equivalent mass of single d.o.f. system $= m_{eq} = M + \frac{33}{140}\, m$ where m = mass of cylindrical part of the sign post:

$$m = \frac{\pi}{4}(D^2 - d^2)\, h\, \rho = \frac{\pi}{4}(0.25^2 - 0.2^2)(10)\left(\frac{76500}{9.81}\right) = 1378.0527 \text{ kg}$$

$$\therefore m_{eq} = 200 + \frac{33}{140}(1378.0527) = 524.8267 \text{ kg}$$

$$I = \frac{\pi}{64}(D^4 - d^4) = \frac{\pi}{4}(0.25^4 - 0.2^4) = 113.208\,(10^{-6})\text{ m}^4$$

Equivalent stiffness of the system:

$$k_{eq} = \frac{3\,E\,I}{h^3} = \frac{3\,(207\,(10^9))\,(113.208\,(10^{-6}))}{10^3} = 70{,}302.168 \text{ N/m}$$

Natural frequency of transverse vibration of sign post:

$$\omega_1 = \left(\frac{k_{eq}}{m_{eq}}\right)^{\frac{1}{2}} = \left(\frac{70302.168}{524.8267}\right)^{\frac{1}{2}} = 11.5738 \text{ rad/sec} = 1.8420 \text{ Hz}$$

(b) Wind velocity corresponding to maximum vibration of sign post (V) is given by:

$$St = 0.21 = \frac{f_1\, D}{V} \quad \text{or} \quad V = \frac{f_1\, D}{0.21} = \frac{(1.8420)\,(0.25)}{0.21} = 2.1929 \text{ m/s}$$

(c) Maximum force acting on the system due to wind velocity:

$$F(t) = F_0 \sin\omega t = \frac{1}{2}\, c\, \rho\, V^2\, A \sin\omega t = \frac{1}{2}(1)(1.2215)(2.19299^2)(8.0)\sin\omega t \text{ N}$$
$$= 23.4958 \sin\omega t \text{ N}$$

where $c = 1$ for a cylinder, ρ = density of air $= 1.2215 \text{ kg/m}^3$, A = projected area of cylindrical part $= (0.8)(10) = 8.0 \text{ m}^2$, and ω = frequency of wind force. Equation of motion:

$$m_{eq}\,\ddot{x} + c_{eq}\,\dot{x} + k_{eq}\,x = F(t)$$

and the maximum steady state displacement of the sign post occurs when $\omega = \omega_1$ and is given by Eq. (3.34):

$$X = \frac{\delta_{st}}{2\,\varsigma} = \frac{F_0}{k_{eq}\,(2)\,\varsigma} = \frac{23.4958}{2\,(0.1)\,(70302.168)} = 0.001671 \text{ m}$$

3.92 (a) Equation of motion $m\ddot{x} + c\dot{x} + kx = F_0 x$

or $m\ddot{x} + c\dot{x} + (k - F_0)x = 0$ (E_1)

Assuming the solution $x(t) = C e^{st}$ (E_2)

where C is a constant, Eq. (E_1) gives the auxiliary equation

$$s^2 + \frac{c}{m}s + \left(\frac{k-F_0}{m}\right) = 0 \quad (E_3)$$

Roots of (E_3) are

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \left(\frac{k-F_0}{m}\right)} \quad (E_4)$$

First consider the case of positive stiffness $(k > F_0)$. For this case, following possibilities exist.

1. If $\left(\frac{c}{2m}\right)^2 > \left(\frac{k-F_0}{m}\right)$:

   Both s_1 and s_2 will be real and negative and hence

   $$x(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t} \quad (E_5)$$

   will be stable.

2. If $\left(\frac{c}{2m}\right)^2 = \left(\frac{k-F_0}{m}\right)$:

   Both s_1 and s_2 will be identical, real and negative.

   Solution $$x(t) = (C_1 + C_2 t)\, e^{s_1 t} \quad (E_6)$$

   will be stable since $e^{s_1 t} \to 0$ as $t \to \infty$.

3. If $\left(\frac{c}{2m}\right)^2 < \left(\frac{k-F_0}{m}\right)$:

   Here s_1 and s_2 will be complex conjugates and solution will be

   $$x(t) = C\, e^{-\left(\frac{c}{2m}\right)t} \sin\left(\sqrt{\left\{-\left(\frac{c}{2m}\right)^2 + \left(\frac{k-F_0}{m}\right)\right\}}\, t + \phi\right) \quad (E_7)$$

   This represents a converging oscillatory motion and hence the system will be stable.

Next consider the case of negative stiffness $(k < F_0)$. Here

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 + \left(\frac{F_0 - k}{m}\right)} \quad (E_8)$$

Thus s_1 will be positive and s_2 will be negative, and the solution becomes

$$x(t) = C_1 e^{+s_1 t} + C_2 e^{-|s_2| t} \quad (E_9)$$

This solution can be seen to diverge as $t \to \infty$.

(b) Equation of motion $m\ddot{x} + c\dot{x} + kx = F_0 \dot{x}$

or $$\ddot{x} + \left(\frac{c - F_0}{m}\right)\dot{x} + \frac{k}{m}x = 0 \quad (E_{10})$$

Assuming $x(t) = C\, e^{st}$ the auxiliary equation becomes

$$s^2 + \left(\frac{c - F_0}{m}\right) s + \frac{k}{m} = 0 \qquad (E_{11})$$

and hence

$$s_{1,2} = -\left(\frac{c - F_0}{2m}\right) \pm \sqrt{\left(\frac{c - F_0}{2m}\right)^2 - \frac{k}{m}} \qquad (E_{12})$$

First consider the case of positive damping ($c > F_0$) in (E_{10}). For this case, it can be seen that the system will be stable for all possible values of $\left\{ \left(\frac{c - F_0}{2m}\right)^2 - \frac{k}{m} \right\}$.

Next, consider the case of negative damping ($c < F_0$). Depending on the sign of the quantity under the radical in Eq. (E_{12}), we will have three types of solution.

1. $\left(\frac{c - F_0}{2m}\right)^2 > \frac{k}{m}$. Here both s_1 and s_2 are real and positive and hence $x(t) = C_1\, e^{s_1 t} + C_2\, e^{s_2 t}$ $\quad (E_{13})$
   This denotes a diverging nonoscillatory motion; so the system is unstable.

2. $\left(\frac{c - F_0}{2m}\right)^2 = \frac{k}{m}$. Here s_1 and s_2 are identical and are real and positive. Hence $x(t) = (C_1 + C_2\, t)\, e^{s_1 t}$ $\quad (E_{14})$
   This represents a diverging nonoscillatory solution; so the system will be unstable.

3. $\left(\frac{c - F_0}{2m}\right)^2 < \frac{k}{m}$. Here s_1 and s_2 are complex conjugates and hence
   $$s_{1,2} = \left(\frac{F_0 - c}{2m}\right) \pm i \sqrt{\frac{k}{m} - \left(\frac{c - F_0}{2m}\right)^2} \qquad (E_{15})$$
   The solution becomes
   $$x(t) = X\, e^{\left(\frac{F_0 - c}{2m}\right) t} \sin\left(\sqrt{\frac{k}{m} - \left(\frac{c - F_0}{2m}\right)^2}\; t + \phi\right) \qquad (E_{16})$$
   Since the exponent is positive, Eq. (E_{16}) denotes a diverging oscillatory motion and hence the system is unstable.

Thus the condition for dynamic stability of the system can be stated as

$$F_0 \leq c \qquad (E_{17})$$

(c) Equation of motion $\quad m\ddot{x} + c\dot{x} + k x = F_0 \ddot{x}$

or
$$(m - F_0)\ddot{x} + c\dot{x} + k x = 0 \qquad (E_{18})$$

with the solution
$$x(t) = C\, e^{st} \qquad (E_{19})$$

the auxiliary equation will be
$$s^2 + \left(\frac{c}{m - F_0}\right) s + \left(\frac{k}{m - F_0}\right) = 0 \qquad (E_{20})$$

The roots are
$$s_{1,2} = -\frac{c}{2(m - F_0)} \pm \sqrt{\left\{\frac{c}{2(m - F_0)}\right\}^2 - \left(\frac{k}{m - F_0}\right)} \qquad (E_{21})$$

First consider the case of positive mass $(m > F_0)$ in (E_{18}). In this case, the system will be stable for all values of

$$\left[\left\{\frac{c}{2(m - F_0)}\right\}^2 - \left(\frac{k}{m - F_0}\right)\right].$$

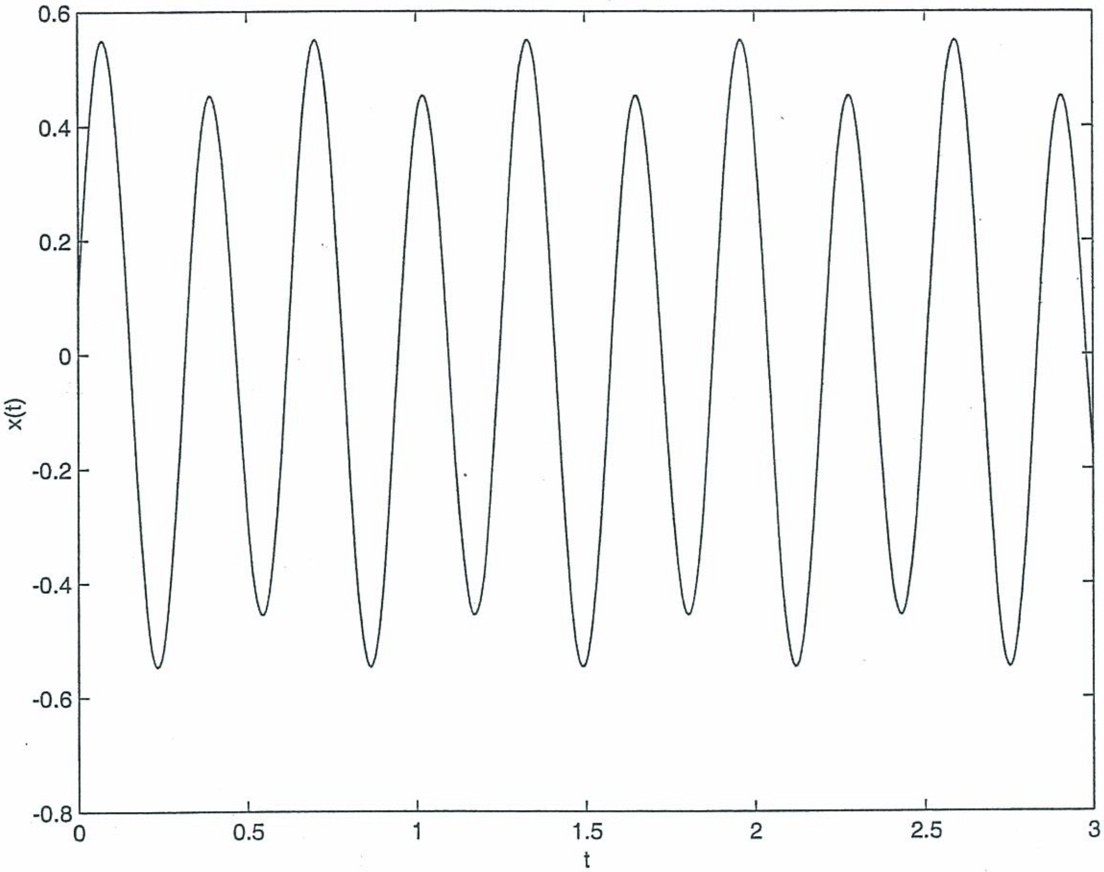
Next consider the case of negative mass $(m < F_0)$ in (E_{18}). For this case s_1 and s_2 can be expressed as

$$s_{1,2} = \frac{c}{2(F_0 - c)} \pm \sqrt{\left\{\frac{c}{2(F_0 - m)}\right\}^2 + \left(\frac{k}{F_0 - m}\right)} \qquad (E_{22})$$

This shows that s_1 will be positive and s_2 will be negative; thus the solution will be divergent.

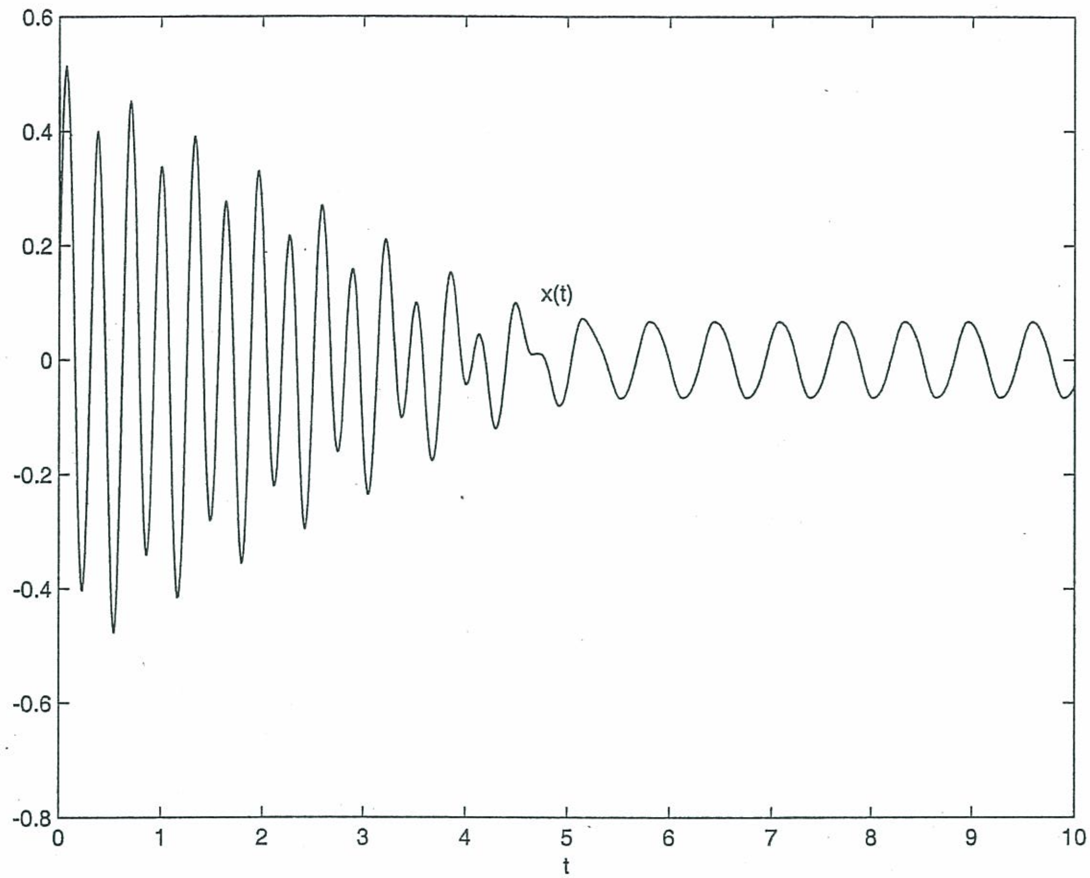
3.103

```
% Ex3_103.m
k = 4000;
m = 10;
w = 10;
F0 = 200;
wn = sqrt(k/m);
x0 = 0.1;
x0_dot = 10;
f_0 = F0/m;
for i = 1: 501
    t(i) = 3 * (i-1)/500;
    x(i) = x0_dot*sin(wn*t(i))/wn + (x0 - f_0/(wn^2-w^2))*cos(wn*t(i))...
        + f_0/(wn^2-w^2)*cos(w*t(i));
end
plot(t,x);
xlabel('t');
ylabel('x(t)');
```



3.104

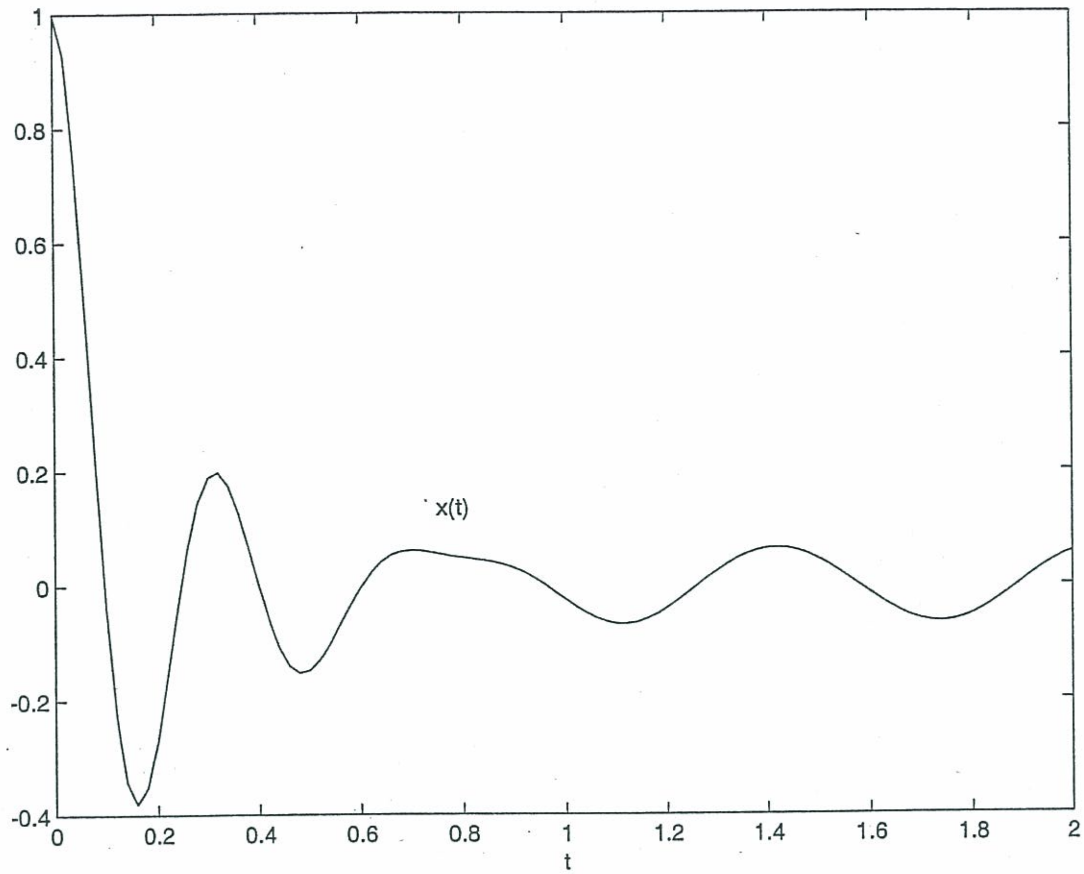
```
% Ex3_104.m
% This program will use the function dfunc3_104.m,they should
% be in the same folder
tspan = [0: 0.01: 10];
x0 = [0.1; 10];
[t,x] = ode23('dfunc3_104', tspan, x0);
disp('      t          x(t)      xd(t)');
disp([t x]);
plot(t,x(:,1));
xlabel('t');
gtext('x(t)');

% dfunc3_104.m
function f = dfunc3_104(t,x)
F0 = 200;
w = 10;
u = 0.3;
m = 10;
k = 4000;
f = zeros(2,1);
f(1) = x(2);
f(2) = (F0/m)*sin(w*t) - 9.81*u*sign(x(2)) - (k/m)*x(1);
```



3.105

```
% Ex3_105.m
% This program will use the function dfunc3_105.m, they should
% be in the same folder
tspan = [0: 0.02: 2];
x0 = [1; 0];
[t,x] = ode23('dfunc3_105', tspan, x0);
disp('      t         x(t)      xd(t)');
disp([t x]);
plot(t,x(:,1));
xlabel('t');
gtext('x(t)');
```

```
% dfunc3_105.m
function f = dfunc3_105(t,x)
m = 100;
k = 40000;
zeta = 0.25;
Y = 0.05;
w = 10;
c = 2 * zeta * sqrt(k*m);
f = zeros(2,1);
f(1) = x(2);
f(2) = k*Y*sin(w*t)/m + c*w*Y*cos(w*t)/m - c*x(2)/m - (k/m)*x(1);
```




```
>> Ex3_105
      t           x(t)       xd(t)
        0       1.0000           0
   0.0200       0.9272     -6.9328
   0.0400       0.7381    -11.5448
   0.0600       0.4823    -13.6094
   0.0800       0.2096    -13.3072
   0.1000      -0.0372    -11.1218
   0.1200      -0.2270     -7.7195
     .
     .
     .
   1.8000      -0.0523      0.3904
   1.8200      -0.0434      0.4869
   1.8400      -0.0329      0.5637
   1.8600      -0.0210      0.6179
   1.8800      -0.0083      0.6473
   1.9000       0.0047      0.6510
   1.9200       0.0176      0.6286
   1.9400       0.0297      0.5811
   1.9600       0.0406      0.5103
   1.9800       0.0499      0.4194
   2.0000       0.0572      0.3120
```

3.106

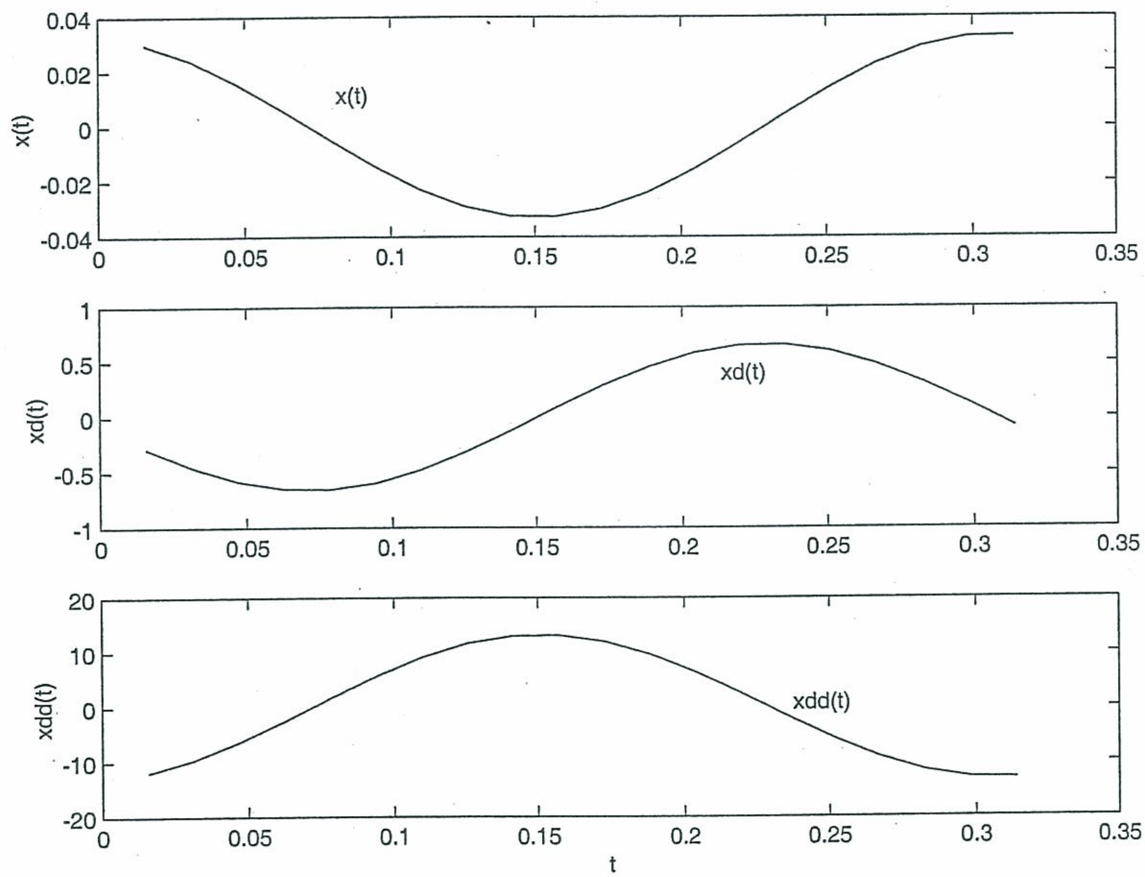
```
%=================================================================
%
% Ex3_106.m(Program3.m)
% Main program which calls HARESP
%
%=================================================================
% Run "Ex3_106.m" in MATLAB command window. Ex3_106.m and haresp.m should
% be in the same folder,and set the path to this folder
% following seven lines contain problem-dependent data
xm=10.0;
xk=1000;
zeta=0.1;
xc=2*zeta*sqrt(xk*xm);
f0=100.0;
om=20.0;
n=20;
ic=1;
% end of problem-dependent data
[t,x,xd,xdd,xamp,xphi]=haresp(xm,xc,xk,f0,om,ic,n);
% following lines output the results
fprintf('Steady state response of an undamped\n');
fprintf('Single degree of freedom system under harmonic force\n\n');
fprintf('Given data\n');
fprintf('xm = %10.8e\n',xm);
fprintf('xc = %10.8e\n',xc);
fprintf('xk = %10.8e\n',xk);
fprintf('f0 = %10.8e\n',f0);
fprintf('om = %10.8e\n',om);
fprintf('ic = %1.0f\n',ic);
fprintf('n  = %2.0f\n\n\n',n);
fprintf('Response: \n\n');
fprintf('   i          x(i)                    xd(i)              xdd(i)');
fprintf('\n\n');
for i=1:n
   fprintf('  %2.0f    %10.8e          %10.8e     %10.8e\n',i,x(i),...
        xd(i),xdd(i));
end
subplot(311);
plot(t,x);
ylabel('x(t)');
gtext('x(t)');
subplot(312);
plot(t,xd);
ylabel('xd(t)');
gtext('xd(t)');
subplot(313);
plot(t,xdd);
ylabel('xdd(t)');
gtext('xdd(t)');
xlabel('t');
```

```
%=====================================================================
%
%function haresp.m
%
%=====================================================================
function [t,x,xd,xdd,xamp,xphi]=haresp(xm,xc,xk,f0,om,ic,n);
omn=sqrt(xk/xm);
xai=xc/(2.0*xm*omn);
dst=f0/xk;
r=om/omn;
xamp=dst/sqrt((1.0-r^2)^2+(2.0*xai*r)^2);
xphi=atan(2.0*xai*r/(1.0-r^2));
delt=2.0*3.1416/(om*n);
time=0.0;
if ic~=0
   for i=1:n
      time=time+delt;
      t(i) = time;
      x(i)=xamp*cos(om*time-xphi);
      xd(i)=-xamp*om*sin(om*time-xphi);
      xdd(i)=-xamp*om^2*cos(om*time-xphi);
   end
else
   for i=1:n
      time=time+delt;
      t(i) = time;
      x(i)=xamp*sin(om*time-xphi);
      xd(i)=xamp*om*cos(om*time-xphi);
      xdd(i)=-xamp*om^2*sin(om*time-xphi);
   end
end
```

```
>> Ex3_106
Steady state response of an undamped
Single degree of freedom system under harmonic force

Given data
xm = 1.00000000e+001
xc = 2.00000000e+001
xk = 1.00000000e+003
f0 = 1.00000000e+002
om = 2.00000000e+001
ic = 1
n  = 20

Response:
```

i	x(i)	xd(i)	xdd(i)
1	2.97987095e-002	-2.85475021e-001	-1.19194838e+001
2	2.39294085e-002	-4.55669383e-001	-9.57176339e+000
3	1.57177193e-002	-5.81259445e-001	-6.28708774e+000
4	5.96746320e-003	-6.49951518e-001	-2.38698528e+000
5	-4.36693253e-003	-6.55021513e-001	1.74677301e+000
6	-1.42738605e-002	-5.95973141e-001	5.70954420e+000

7	-2.27835571e-002	-4.78586496e-001	9.11342282e+000
8	-2.90630300e-002	-3.14352252e-001	1.16252120e+001
9	-3.24975978e-002	-1.19346877e-001	1.29990391e+001
10	-3.27510596e-002	8.73410566e-002	1.31004238e+001
11	-2.97986046e-002	2.85479399e-001	1.19194419e+001
12	-2.39292411e-002	4.55672899e-001	9.57169644e+000
13	-1.57175058e-002	5.81261754e-001	6.28700233e+000
14	-5.96722446e-003	6.49952395e-001	2.38688978e+000
15	4.36717313e-003	6.55020872e-001	-1.74686925e+000
16	1.42740794e-002	5.95971044e-001	-5.70963176e+000
17	2.27837329e-002	4.78583148e-001	-9.11349314e+000
18	2.90631454e-002	3.14347982e-001	-1.16252582e+001
19	3.24976417e-002	1.19342102e-001	-1.29990567e+001
20	3.27510275e-002	-8.73458687e-002	-1.31004110e+001



3.107

```
%Ex3_107.m
Y= 0.05;
zeta = 0.1;
wn = 8.164966;
w = 2.90889;
r = w/wn;
x = Y * sqrt( (1 + (2 * zeta * r)^2)/((1 - r^2)^2 + (2 * zeta * r)^2) );
disp('    w                    wn                    x');
disp([w wn x]);
w = 14.54445;
r = w/wn;
x = Y * sqrt( (1 + (2 * zeta * r)^2)/((1 - r^2)^2 + (2 * zeta * r)^2) );
disp([w wn x]);
w = 29.08890;
r = w/wn;
x = Y * sqrt( (1 + (2 * zeta * r)^2)/((1 - r^2)^2 + (2 * zeta * r)^2) );
disp([w wn x]);
wn = 6.324555;
w = 2.90889;
r = w/wn;
x = Y * sqrt( (1 + (2 * zeta * r)^2)/((1 - r^2)^2 + (2 * zeta * r)^2) );
disp([w wn x]);
w = 14.54445;
r = w/wn;
x = Y * sqrt( (1 + (2 * zeta * r)^2)/((1 - r^2)^2 + (2 * zeta * r)^2) );
disp([w wn x]);
w = 29.08890;
r = w/wn;
x = Y * sqrt( (1 + (2 * zeta * r)^2)/((1 - r^2)^2 + (2 * zeta * r)^2) );
disp([w wn x]);
```

```
>> Ex3_107
    w                    wn                    x
  2.90889000000000   8.16496600000000   0.05722376420338

 14.54445000000000   8.16496600000000   0.02410324256879

 29.08890000000000   8.16496600000000   0.00524102723160

  2.90889000000000   6.32455500000000   0.06325355032007

 14.54445000000000   6.32455500000000   0.01275990975243

 29.08890000000000   6.32455500000000   0.00336736169683
```

3.108 Equation of motion is $m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

When $y(t) = Y \sin \omega t$, $\quad x_p(t) = X \cos(\omega t - \phi_1 - \phi_2)$

Complete solution can be expressed as

$$x(t) = X_0\, e^{-\zeta \omega_n t} \cos(\omega_d t + \phi_0) + X \cos(\omega t - \phi_1 - \phi_2) \qquad (E.1)$$

with $\quad X = Y\left[\dfrac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{1/2}$,

$$\phi_1 = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right), \quad \phi_2 = \tan^{-1}\left(\frac{1}{2\zeta r}\right), \quad r = \frac{\omega}{\omega_n}.$$

If the initial conditions are known $x(t=0) = x_0$ and $\dot{x}(t=0) = \dot{x}_0$,

$$x_0 = X_0 \cos\phi_0 + X \cos(\phi_1 + \phi_2)$$

and $\quad \dot{x}_0 = -\zeta\omega_n X_0 \cos\phi_0 - \omega_d X_0 \sin\phi_0 - \omega X \sin(-\phi_1 - \phi_2)$

Hence

$$X_0 = \left[\left\{x_0 - X\cos(\phi_1+\phi_2)\right\}^2 + \left\{\frac{-\dot{x}_0 - \zeta\omega_n x_0 + \zeta\omega_n X \cos(\phi_1+\phi_2) + \omega X \sin(\phi_1+\phi_2)}{\omega_d}\right\}^2\right]^{1/2}$$

$$\phi_0 = \tan^{-1}\left[\frac{-\dot{x}_0 - \zeta\omega_n x_0 + \zeta\omega_n X\cos(\phi_1+\phi_2) + \omega X \sin(\phi_1+\phi_2)}{\omega_d\left\{x_0 - X\cos(\phi_1+\phi_2)\right\}}\right]$$

If necessary, the velocity $\dot{x}(t)$ and acceleration $\ddot{x}(t)$ can be found from Eq. (E.1). The computer program and output are given below.

```
C ==================================================================
C
C SOLUTION OF PROBLEM 3.108
C MAIN PROGRAM WHICH CALLS BASEX
C RESPONSE OF A SINGLE D.O.F. SYSTEM SUBJECTED TO BASE EXCITATION,
C Y(T)=Y*SIN(OM*T)
C
C ==================================================================
```

```
C FOLLOWING 3 LINES CONTAIN PROBLEM-DEPENDENT DATA
      DIMENSION X(20),XD(20),XDD(20)
      DATA XM,XC,XK,Y,OM,N/2.0,10.0,100.0,0.1,25.0,20/
      DATA X0,XD0/0.01,5.0/
C END OF PROBLEM-DEPENDENT DATA
      CALL BASEX (XM,XC,XK,Y,OM,N,X,XD,XDD,X0,XD0)
      PRINT 100
 100  FORMAT (//,33H TOTAL RESPONSE OF AN UNDERDAMPED,/,
     2 52H SINGLE D.O.F. SYSTEM UNDER HARMONIC BASE EXCITATION)
      PRINT 200, XM,XC,XK,Y,OM,N
 200  FORMAT (//,12H GIVEN DATA:,/,5H XM =,E15.8,/,5H XC =,E15.8,/,
     2 5H XK =,E15.8,/,5H Y  =,E15.8,/,5H OM =,E15.8,/,5H N  =,I2)
      PRINT 300,X0,XD0
 300  FORMAT (/,20H INITIAL CONDITIONS:,/,6H X0  =,E15.8,/,6H XD0 =,
     2 E15.8)
      PRINT 400
 400  FORMAT (//,10H RESPONSE:,//,5H   I ,3X,5H X(I),12X,6H XD(I),
     2 11X,7H XDD(I),/)
      DO 500 I=1,N
 500  PRINT 600, I,X(I),XD(I),XDD(I)
 600  FORMAT (I4,2X,E15.8,2X,E15.8,2X,E15.8)
      STOP
      END
C ====================================================================
C
C SUBROUTINE BASEX
C
C ====================================================================
      SUBROUTINE BASEX (XM,XC,XK,Y,OM,N,X,XD,XDD,X0,XD0)
      DIMENSION X(N),XD(N),XDD(N)
      OMN=SQRT(XK/XM)
      XAI=XC/(2.0*XM*OMN)
      OMD=OMN*SQRT(1.0-XAI**2)
      R=OM/OMN
      DELT=2.0*3.1416/(OMD*REAL(N))
      XAMP=Y*SQRT(1.0+(2.0*XAI*R)**2/((1.0-R**2)**2+(2.0*XAI*R)**2))
      PHI1=ATAN(2.0*XAI*R/(1.0-R*R))
      PHI2=ATAN(1.0/(2.0*XAI*R))
      XCC=XC
      TIME=0.0
      DO 10 I=1,N
      TIME=TIME+DELT
      XC=X0-XAMP*COS(PHI1+PHI2)
      XS=(-XD0-XAI*OMN*XC+OM*XAMP*SIN(PHI1+PHI2))/OMD
      XZ=SQRT(XC**2+XS**2)
      PZ=ATAN(XS/XC)
      EX=EXP(-XAI*OMN*TIME)
      CS=COS(OMD*TIME+PZ)
      SI=SIN(OMD*TIME+PZ)
      CS12=COS(OM*TIME-PHI1-PHI2)
      SI12=SIN(OM*TIME-PHI1-PHI2)
      X(I)=XZ*EX*CS+XAMP*CS12
      XD(I)=-XAI*OMN*XZ*EX*CS-OMD*XZ*EX*SI-OM*XAMP*SI12
 10   XDD(I)=XZ*EX*CS*((XAI*OMN)**2-OMD**2)+2.0*XAI*OMN*OMD*XZ*EX*SI
     2 -(OM**2)*XAMP*CS12
```

```
      XC=XCC
      RETURN
      END
```

```
TOTAL RESPONSE OF AN UNDERDAMPED
SINGLE D.O.F. SYSTEM UNDER HARMONIC BASE EXCITATION


GIVEN DATA:
XM = 0.20000000E+01
XC = 0.10000000E+02
XK = 0.10000000E+03
Y  = 0.10000000E+00
OM = 0.25000000E+02
N  =20

 INITIAL CONDITIONS:
 X0   = 0.99999998E-02
 XD0  = 0.50000000E+01


 RESPONSE:
```

I	X(I)	XD(I)	XDD(I)
1	-0.50330855E-01	-0.57580528E+01	-0.10305269E+02
2	-0.30772516E+00	-0.44899216E+01	0.62558521E+02
3	-0.43412885E+00	-0.65008521E+00	0.85064461E+02
4	-0.38465756E+00	0.22720275E+01	0.28117821E+02
5	-0.27407524E+00	0.18295681E+01	-0.40405914E+02
6	-0.24066399E+00	-0.41657627E+00	-0.39765869E+02
7	-0.28432682E+00	-0.90879965E+00	0.23411983E+02
8	-0.27970907E+00	0.14345939E+01	0.64174423E+02
9	-0.14624044E+00	0.38781688E+01	0.26183165E+02
10	0.39102390E-01	0.33305802E+01	-0.47345394E+02
11	0.12840362E+00	0.26650119E+00	-0.67513680E+02
12	0.80883794E-01	-0.18088942E+01	-0.10970811E+02
13	0.96553117E-02	-0.68976790E+00	0.50778744E+02
14	0.38462169E-01	0.18209940E+01	0.40712753E+02
15	0.14735566E+00	0.22107751E+01	-0.27525837E+02
16	0.19996722E+00	-0.31552225E+00	-0.66972618E+02
17	0.11764607E+00	-0.28227291E+01	-0.26468327E+02
18	-0.18114097E-01	-0.23140993E+01	0.45068512E+02
19	-0.63696094E-01	0.51316518E+00	0.59701672E+02
20	0.10478826E-01	0.21295214E+01	0.43720150E+00

3.111 Unbalanced force in vertical direction $= m e \omega^2 \sin \omega t$ (E_1)

Unbalanced force in horizontal direction $= 0$

Let M = total mass of the shaker

Equation of motion is $M \ddot{x} + c \dot{x} + k x = m e \omega^2 \sin \omega t$ (E_2)

steady state solution of (E_2) is

$$x(t) = X \sin (\omega t - \phi) \qquad (E_3)$$

where

$$X = \frac{m\ e\ r^2}{M\left[(1-r^2)^2 + (2\zeta r)^2\right]^{1/2}} \qquad (E_4)$$

and

$$\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right) \qquad (E_5)$$

where

$$r = \frac{\omega}{\omega_n} = \omega\sqrt{\left(M/k\right)} \qquad (E_6)$$

Frequency range: 20 Hz to 30 Hz

i.e., 125.664 rad/sec to 188.496 rad/sec

$$\therefore \quad 125.664 \text{ rad/sec} \le \omega \le 188.496 \text{ rad/sec} \qquad (E_7)$$

$0.1'' \le X \le 0.2''$ where X is given by (E_4).

Mean power output over a time period τ is given by

$$P = \frac{1}{\tau}\int_0^{\tau} F(\tau)\ \frac{dx}{dt}(\tau)\ d\tau \qquad (E_9)$$

where $\tau = \frac{2\pi}{\omega}$,

$F(\tau) = m e \omega^2 \sin \omega t$, and

$\frac{dx}{dt} = \omega X \cos(\omega t - \phi)$

$$P \ge 1 \text{ hp} \qquad (E_{10})$$

$$\frac{M}{m} \ge 50 \qquad (E_{11})$$

Procedure:

Find ω, e, M, m, k and c so as to satisfy the requirements stated in (E_7), (E_8), (E_{10}) and (E_{11}).

3.112 $m = \frac{10^5}{386.4} = 258.7992$ lb-s²/in , $l = 600''$, $E = 30\times10^6$ psi

$$k = \frac{3EI}{l^3} = \frac{3\ (30\times10^6)}{(600)^3}\cdot\frac{\pi}{64}\left(D^4 - d^4\right) = 0.020453\left(D^4-d^4\right) \frac{lb}{in}$$

$\omega = 2\pi(15) = 94.248$ rad/sec ; $\zeta = 0.15$

Ground acceleration $\ddot{y}(t) = 193.2 \sin 94.248\, t$ in/s². (E_1)

Equation of motion:

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} = -50\,000 \sin 94.248\,t \qquad (E_2)$$

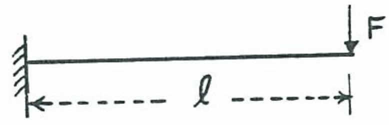
where $z = x - y$ = relative displacement.

Let $z(t) = Z \sin(\omega t - \phi) = Z \sin(94.248\,t - \phi) \qquad (E_3)$

with $$Z = \frac{-50\,000}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}} \qquad (E_4)$$

and $$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right) \qquad (E_5)$$

$$y_{max} = \frac{F\ell^3}{3EI} \Rightarrow \frac{3\,y_{max}}{\ell^2} = \frac{F\ell}{EI}$$



Max. bending moment $= M = F\ell$

Max. bending stress $= \dfrac{M\,(D/2)}{EI} = \dfrac{F\ell}{EI}\cdot\dfrac{D}{2} = \dfrac{3D}{2\ell^2}\,y_{max}$

If maximum relative displacement, $y_{max} = Z$, is known, max. bending stress (σ_1) is given by

$$\sigma_1 = \frac{3D}{2\ell^2}\cdot Z$$

Direct compressive stress (σ_2) due to weight of water tank is

$$\sigma_2 = \frac{mg}{\frac{\pi}{4}(D^2 - d^2)} = \frac{4\times 10^5}{\pi(D^2 - d^2)}$$

Total stress $= \sigma_1 + \sigma_2 \leq 30\,000$ psi

i.e., $$\frac{3D}{2\ell^2}Z + \frac{4\times 10^5}{\pi(D^2 - d^2)} \leq 30\,000 \qquad (E_6)$$

Natural frequency of water tank is

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.020453\,(D^4 - d^4)}{258.7992}} \geq 30\pi\ \frac{\text{rad}}{\text{sec}}$$

i.e., $$D^4 - d^4 \geq 1.1240\times 10^8 \qquad (E_7)$$

Weight of steel column is

$$W_s = \frac{\pi}{4}(D^2 - d^2)\,\ell\,\rho_{weight} = \frac{\pi}{4}(D^2 - d^2)(600)(0.283)$$

$$= 133.3609\,(D^2 - d^2)\ \text{lb.} \qquad (E_8)$$

Problem: Find $\begin{Bmatrix} D \\ d \end{Bmatrix}$ to minimize W_s subject to restrictions of (E_6) and (E_7). Also use the conditions : $D \geq d$, $D \geq 0$ and $d \geq 0$.

Procedure: Plot the inequalities (E_6) and (E_7) in the $D-d$ space and draw contours of W_s and identify the minimum weight design.