

LEHIGH UNIVERSITY  
Department of Mechanical Engineering and Mechanics

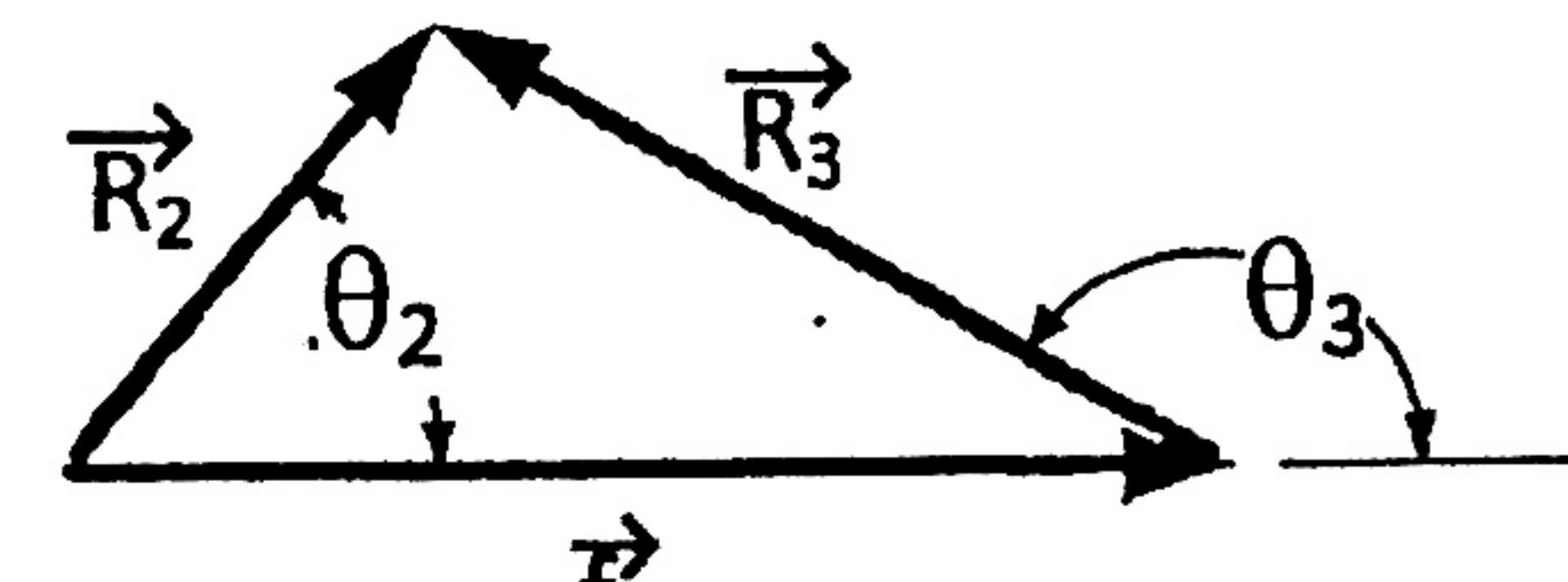
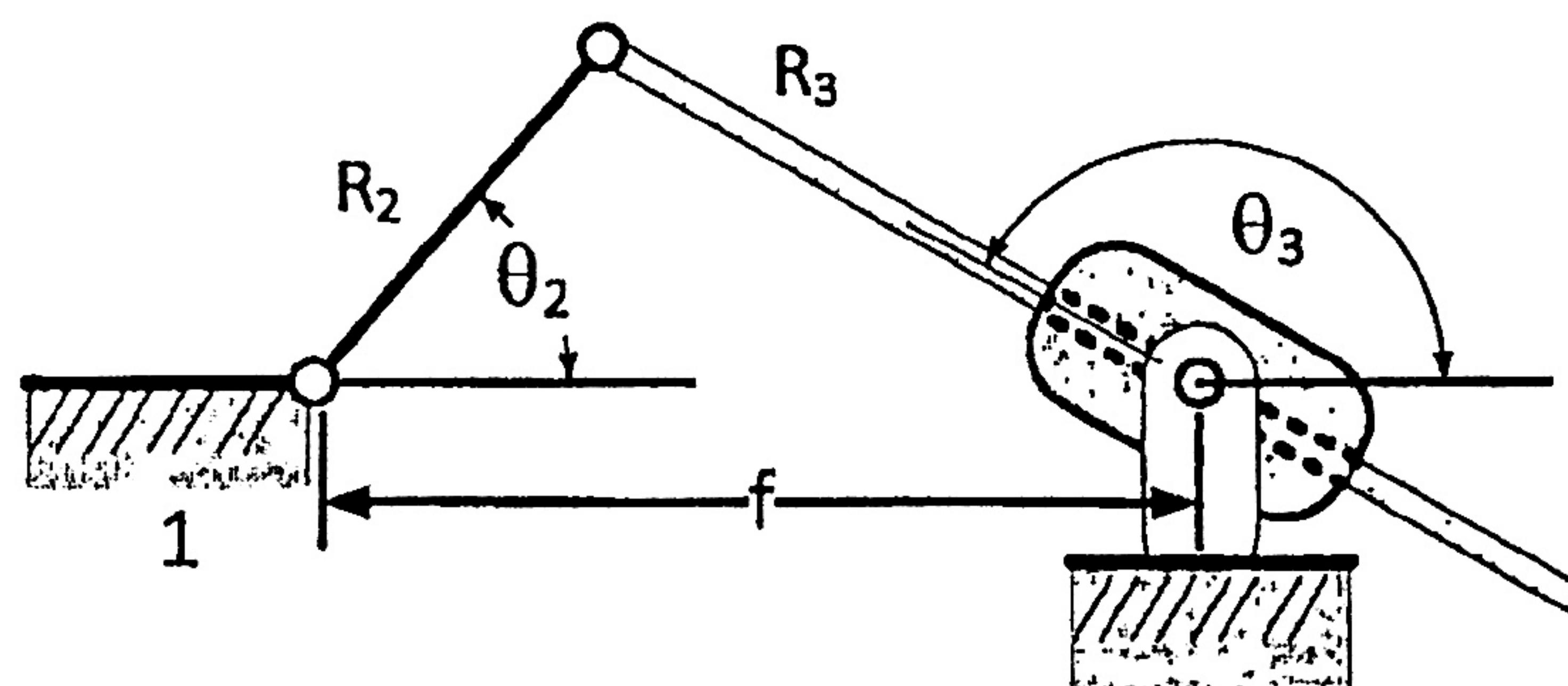
ME 252

Mechanical Elements

Spring 2015 Test 1

1. (25) For the oscillating cylinder engine shown and its corresponding vector loop representation with known inputs for the crank's angular displacement ( $\theta_2$ ), velocity ( $\dot{\theta}_2$ ) and acceleration ( $\ddot{\theta}_2$ ), determine the following:

- Determine the velocity loop equations in their real form. List or indicate the unknowns.
- Determine the Jacobian Matrix for the system and find its inverse.
- Set up the matrix equations to find the velocity coefficients. Do not solve.



Position loop equations

$$\vec{R}_2 - \vec{R}_3 - \vec{f} = \vec{0}$$

$$Real: R_2 \cos \theta_2 - f - R_3 \cos \theta_3 = 0$$

$$Imaginary: R_2 \sin \theta_2 - R_3 \sin \theta_3 = 0$$

where  $R_3$  &  $\theta_3$  unknowns and  $R_2, f$  &  $\theta_2$  known

a) 
$$\begin{aligned} -R_2 \sin \theta_2 \dot{\theta}_2 - R_3 \cos \theta_3 + R_3 \cos \theta_3 \dot{\theta}_3 &= 0 \\ R_2 \cos \theta_2 \dot{\theta}_2 - R_3 \sin \theta_3 - R_3 \cos \theta_3 \dot{\theta}_3 &= 0 \end{aligned}$$

Unknowns:  $R_3 \quad \theta_3$   
 $\dot{\theta}_3 \quad \ddot{\theta}_3$

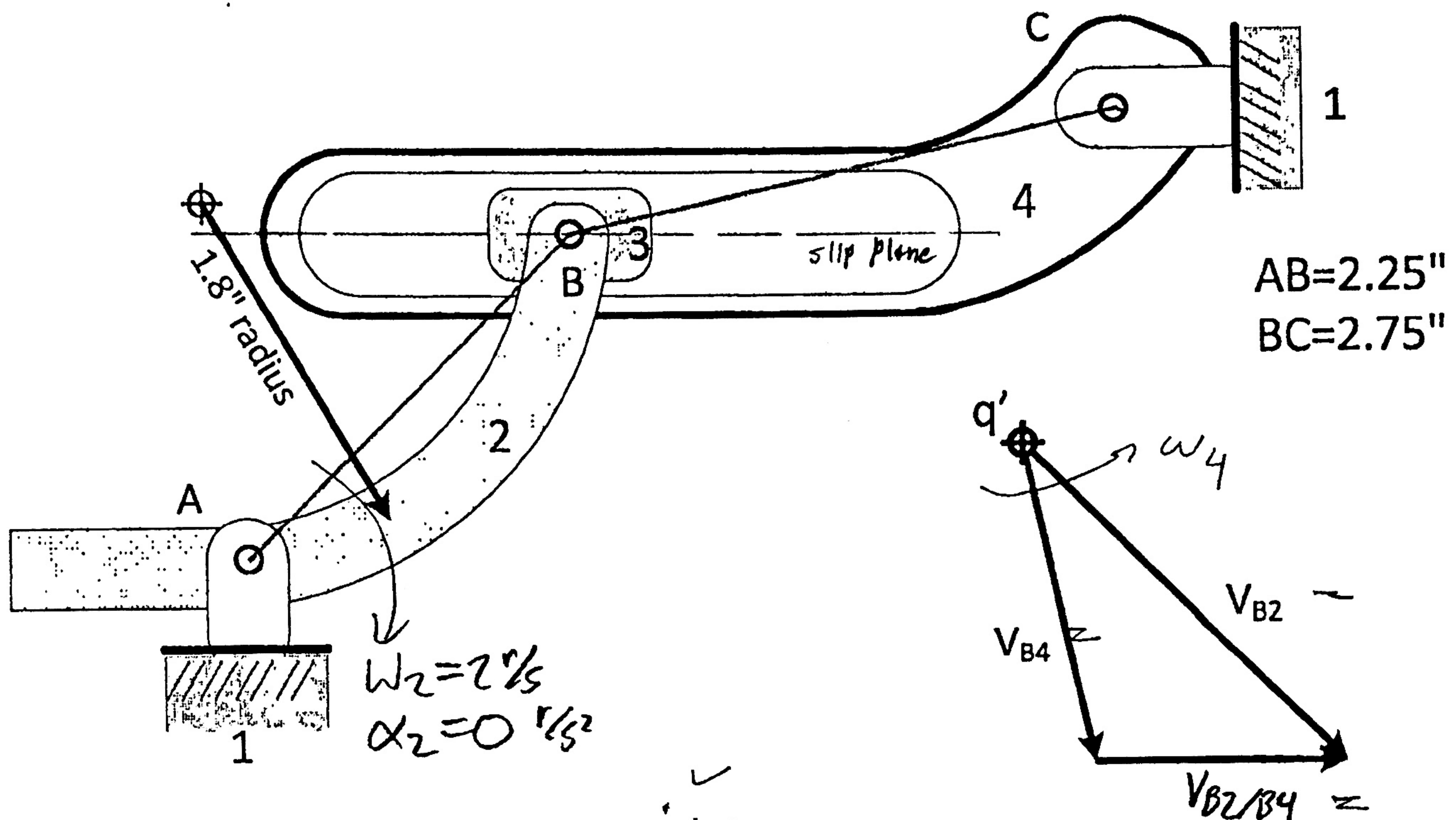
b) 
$$\begin{aligned} -R_3 \cos \theta_3 + R_3 \sin \theta_3 \dot{\theta}_3 &= R_2 \sin \theta_2 \dot{\theta}_2 \\ -R_3 \sin \theta_3 - R_3 \cos \theta_3 \dot{\theta}_3 &= -R_2 \cos \theta_2 \dot{\theta}_2 \end{aligned}$$

$$\boxed{\begin{bmatrix} -\cos \theta_3 & R_3 \sin \theta_3 \\ -\sin \theta_3 & R_3 \cos \theta_3 \end{bmatrix}} \begin{bmatrix} \dot{\theta}_3 \\ \ddot{\theta}_3 \end{bmatrix} = \dot{\theta}_2 \begin{bmatrix} R_2 \sin \theta_2 \\ -R_2 \cos \theta_2 \end{bmatrix}$$

Jacobian

c)

3. (25) If, in the mechanism shown, the angular velocity of body 2 is 2 rad/sec clockwise and constant, determine by a graphical technique, the angular acceleration of body 4  
 Linear scale 1" = 1", Velocity scale: 1" = 2 in/s, Acceleration scale: 1" = 3 in/s<sup>2</sup>.



$$\alpha_{B2}^n = \omega_2^2 \times BA = 2^2 \times 2.25 = 9 \text{ in/s}^2$$

$$\alpha_{B2}^T = \alpha_2 \times BA = 0 \text{ in/s}^2$$

$$\alpha_c \leq 2 V_{B4/B2} \times \omega_4 = 2(1.25)(0.59) = 1.475 \text{ in/s}^2$$

↑  
scalling factor.

$$\omega_4 = \frac{V_{B4}}{BC} = \frac{1.625 \times 2}{2.75} = 0.59 \text{ rad/s}$$

$$\alpha_{B4} = \alpha_{B2} + \alpha_{B4/B2} + \alpha_{coriolis}$$

$$\alpha_{B4} = \alpha_{B2}^n + \alpha_{B2}^T + \alpha_{B4/B2} + \alpha_{coriolis}$$

$$\alpha_{B4} = \alpha_{B2}^n + \alpha_{B2}^T + \alpha_{B4/B2} + \alpha_{coriolis}$$

18

$$\alpha_{B4} = 2.625 \text{ rad/s}^2 = \alpha_{B4}^n + \alpha_{B4}^T$$

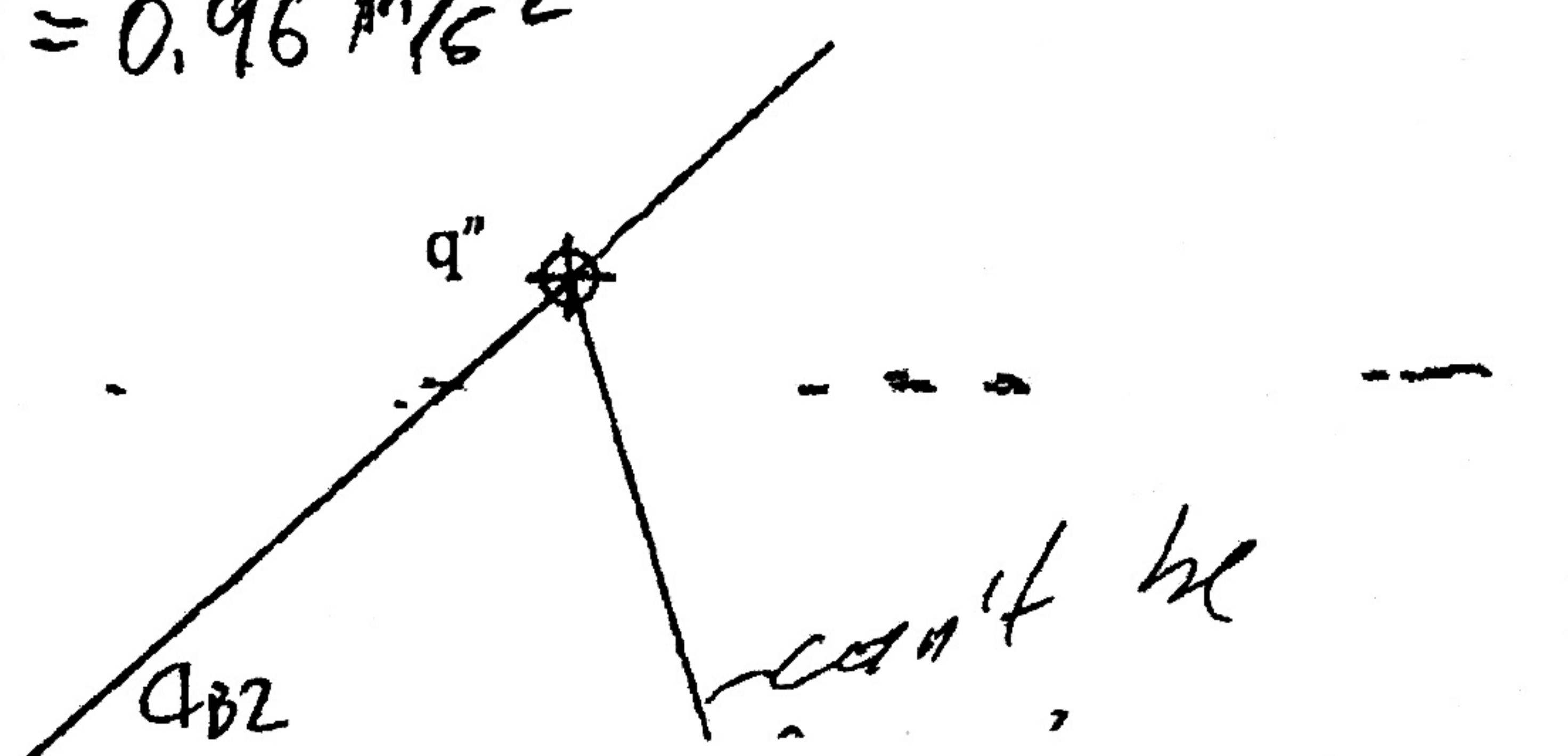
$$\alpha_{B4}^n = \omega_4^2 \times BC = 0.59^2 \times 2.75 = 0.96 \text{ in/s}^2$$

$$\alpha_{B4}^T = \alpha_4 \times BC$$

$$\alpha_4 = \frac{\alpha_{B4} - \alpha_{B4}^n}{BC} + \text{various!}$$

X 10 + various!

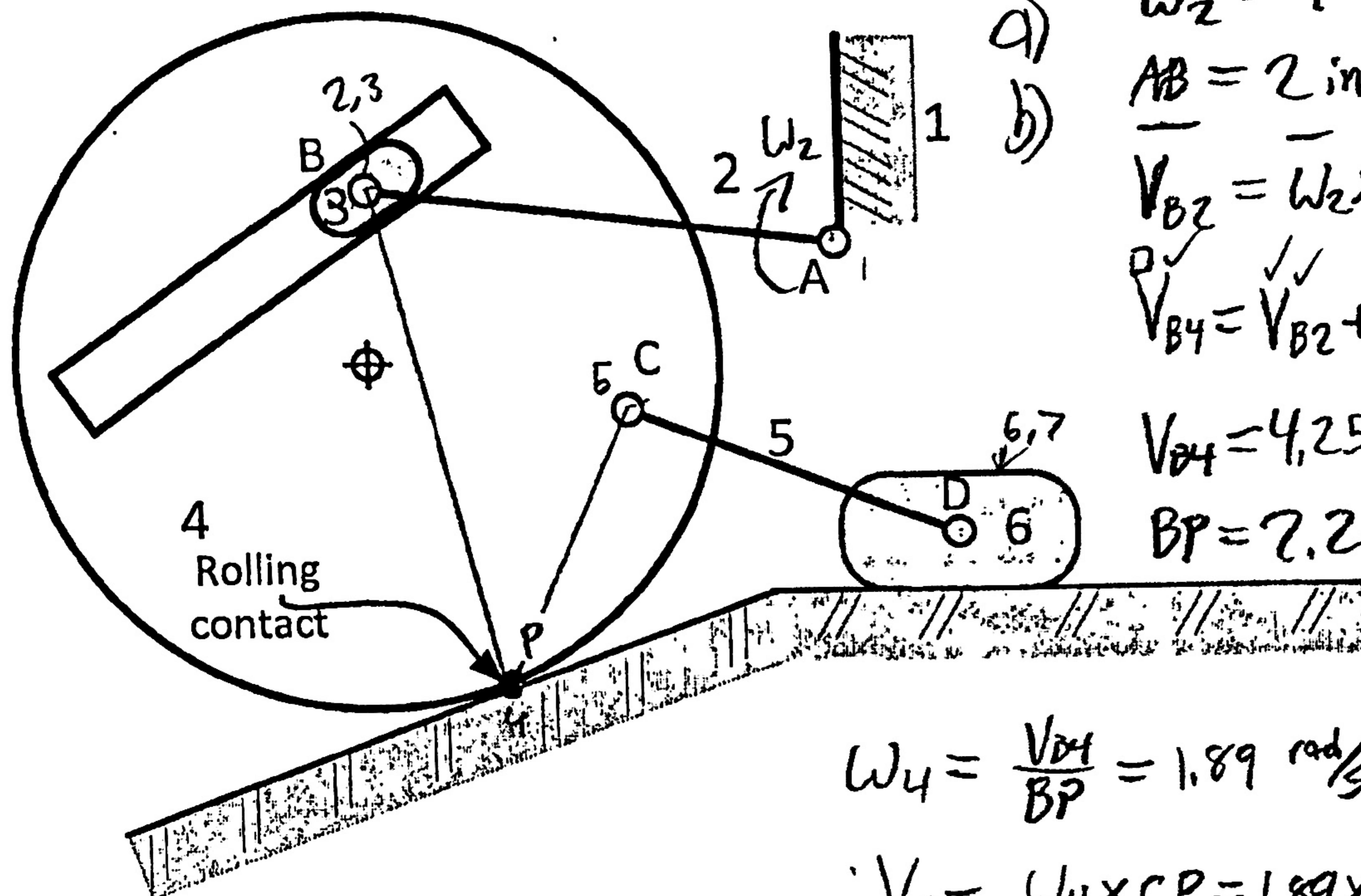
$$\sim 2.625 - 0.960$$



2. (25) Body 2 rotates clockwise at an angular velocity of 1 rad/sec. Using only the relative velocity method of solution, find  
 (10) a) the linear velocity of body 6, slider  
 (10) b) the angular velocity of bodies 3, 4 and 5.  
 (5) c) Using the Kutzbach mobility equation  $M = 3(L - 1) - 2J_1 - J_2$ , determine the degree(s) of freedom.

Note: Body 3 slides in the slot cut in body 4.

Linear scale: 1"=1", velocity scale: 1"= 1"/sec



$$\omega_2 = 1 \text{ rad/s}$$

$$AB = 2 \text{ in}$$

$$V_{B2} = \bar{\omega}_2 \times \bar{AB} = 1 \times 2 = 2 \text{ in/s}$$

$$V_{B4} = V_{B2} + V_{B4/B2}$$

$$V_{B4} = 4.25 \text{ in/s}$$

$$BP = 7.25 \text{ in}$$

$$\omega_4 = \frac{V_{D4}}{BP} = 1.89 \text{ rad/s}$$

$$\omega_4 = 1.89 \text{ rad/s}$$

$$V_C = \omega_4 \times CP = 1.89 \times 1.25 = 2.36 \text{ in/s}$$

$$V_D = V_C + V_{D/C}$$

$$V_{D/C} = 1.75 \text{ in/s}$$

$$V_D = 2.5 \text{ in/s}$$

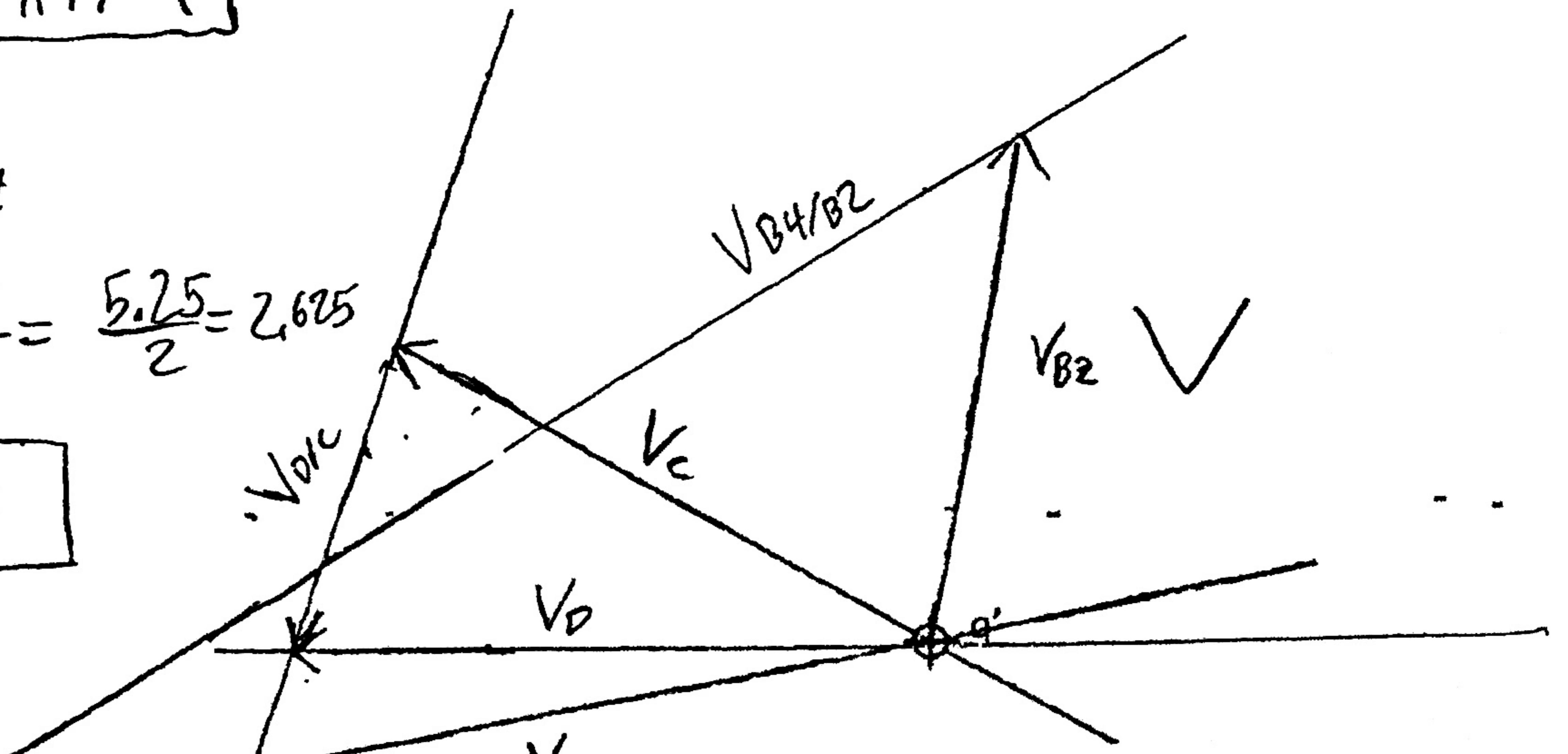
$$\omega_5 = \frac{V_{D/C}}{DC} = \frac{1.75}{1.5}$$

$$\omega_5 = 1.17 \text{ rad/s}$$

$$\omega_3 = \omega_4 + \omega_{3/4}$$

$$\omega_{3/4} = \frac{V_{B2/B4}}{AB} = \frac{5.25}{2} = 2.625$$

$$\omega_3 = 4.515 \text{ rad/s}$$



4. (25)

(5)

(10)

(10)

For the mechanism shown in the position given, find

a) the degree(s) of freedom (see prob. 2)

b) Locate all instant centers for the mechanism. Show all constructions lines used in locating these instant centers

c) When the angular velocity of the link 2 is  $\omega_2 = 0.5 \text{ rad/sec}$  clockwise, find the velocity of the slider (link 5) and the angular velocity of bodies 3 and 4. Linear scale 1" = 1"

$$a) M = 3(L-1) - 2J_1 - J_2 = 3(5-1) - 2(5) - 1$$

$$L = 5$$

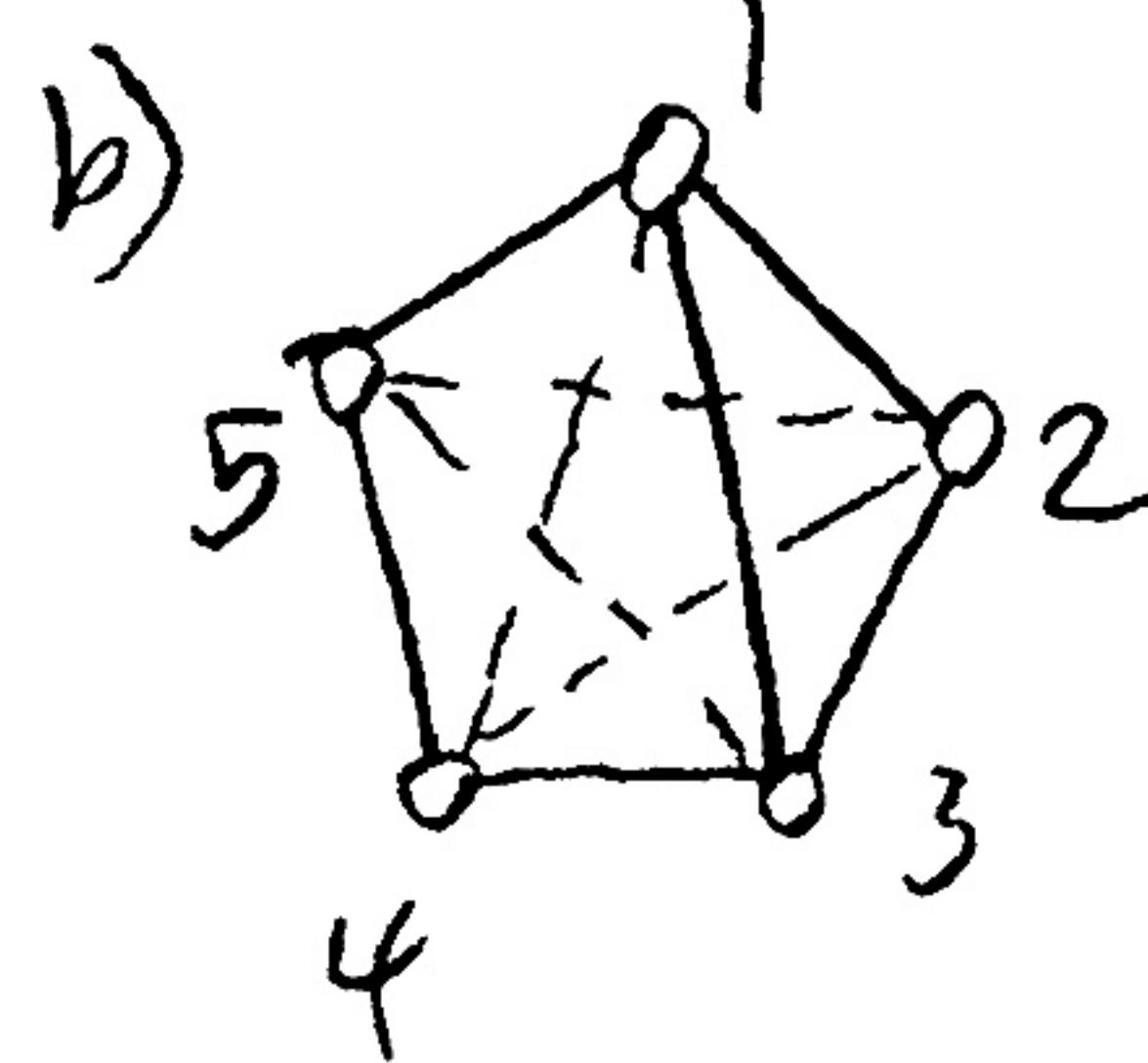
$$J_1 = 5$$

$$J_2 = 1$$

$$= 12 - 10 - 1$$

$$= 1 \quad \checkmark \quad \text{YES}$$

1 Degree of Freedom



$$n = \frac{m(m-1)}{2} \\ = \frac{5(4)}{2} \\ = 10$$

$$b) \quad \textcircled{a} \quad \omega_2 = 0.5 \text{ rad/s } \leftarrow$$

$$V_5 = \omega_2 (12 \text{ to } 52) \\ = 0.5 (0.875)$$

$$= 0.4375$$

$$\omega_2 (12 \text{ to } 23) = \omega_3 (13 \text{ to } 23)$$

$$\omega_3 = \frac{0.5(1.25)}{0.625}$$

$V_5 = 0.4375 \text{ in/s } \leftarrow$

$\omega_3 = 1 \text{ rad/s } \leftarrow$

$$\omega_3 (13 \text{ to } 34) = \omega_4 (14 \text{ to } 34)$$

$$\omega_4 = \frac{1(0.4)}{2.5}$$

$\omega_4 = 0.16 \text{ rad/s } \leftarrow$

$$43 + 13 \} 41$$

$$45 + 51 \} 51$$

$$41 + 12 \} 42$$

$$43 + 32 \} 32$$

$$13 + 15 \} 35$$

$$34 + 45 \} 45$$

$$53 + 32 \} 52$$

$$51 + 12 \} 51$$

