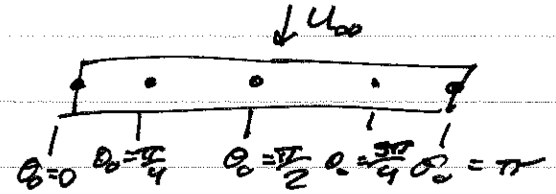
Problem #1

Untwisted rectangular wing $\rightarrow AR = 8 \quad \alpha = 4°$

$\theta_{0,k} = \frac{k\pi}{2N}$ Even terms $\rightarrow 0$ (symmetric Γ)

(a) Let $N = 2 \rightarrow$ Solve for A_1 and A_3, C_L, C_{D_i}.

$\theta_{0,1} = \frac{\pi}{4} \qquad \theta_{0,2} = \frac{\pi}{2}$



Equation (6.27) in book:

$$\alpha_a(\theta_0) = \frac{(m_0 c)_s}{(m_0 c)_{\theta_0}} \sum_{n=1}^{\infty} A_n \sin n\theta_0 + \frac{m_{0,s} c_s}{4\pi b} \int_0^{\pi} \frac{\frac{d}{d\theta}\left(\sum_{n=1}^{\infty} A_n \sin n\theta\right) d\theta}{\cos\theta - \cos\theta_0}$$

$\downarrow$

$$a_a(\theta_0) = \frac{m_{os} c_s}{m_0 c} \sum_{n=1}^{\infty} A_n \sin(n\theta) + \frac{m_{os} c_s}{4b} \sum_{n=1}^{\infty} n A_n \frac{\sin n\theta}{\sin\theta}$$

$c = c_s \quad m_0 = m_{0s} = 2\pi$ and $\alpha_a(\theta_0) = \alpha \quad AR = \frac{b}{c}$ (rectangular)

So:

$$\boxed{\alpha = \sum_{n=1}^{\infty} A_n \sin(n\theta) + \frac{\pi}{2AR} \sum_{n=1}^{\infty} n A_n \frac{\sin(n\theta)}{\sin\theta}}$$

$N = 2 \rightarrow n = 1, 3$ (skip even terms)

$$\alpha = A_1 \sin\theta + \frac{\pi}{2AR} A_1 \frac{\cancel{\sin\theta}}{\cancel{\sin\theta}}^{1} + A_3 \sin 3\theta + \frac{\pi}{2AR} 3A_3 \frac{\sin 3\theta}{\sin\theta}$$

$$\rightarrow \alpha = \left[\left(\sin\theta + \frac{\pi}{2AR}\right) \quad \sin 3\theta\left(1 + \frac{3\pi}{2AR \sin\theta}\right)\right]\begin{bmatrix} A_1 \\ A_3 \end{bmatrix}$$

Write this equation for each station:

$$\begin{bmatrix} \left(\sin\frac{\pi}{4} + \frac{\pi}{2AR}\right) & \sin\frac{3\pi}{4}\left(1 + \frac{3\pi}{2AR\sin\frac{\pi}{4}}\right) \\ \left(\sin\frac{\pi}{2} + \frac{\pi}{2AR}\right) & \sin\frac{3\pi}{2}\left(1 + \frac{3\pi}{2AR\sin\frac{\pi}{2}}\right) \end{bmatrix} \begin{bmatrix} A_1 \\ A_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \alpha \end{bmatrix}$$

$$B = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow B^{1} = \frac{1}{(ad-bc)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

S_c: $$B = \begin{bmatrix} 0.9035 & 1.296 \\ 1.196 & -1.589 \end{bmatrix} \longrightarrow B^{-1} = \begin{bmatrix} 0.5321 & 0.434 \\ 0.4006 & -0.3025 \end{bmatrix}$$

$$BA = \alpha \rightarrow A = B^{-1}\alpha$$

$$\begin{bmatrix} A_1 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0.5321 & 0.434 \\ 0.4006 & -0.3025 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix} \left(\frac{\pi}{180}\right)$$

$$\boxed{A_1 = 0.0674 \quad A_3 = 0.0068}$$

$$C_L = \frac{\pi^2 A_1}{2} \rightarrow \boxed{C_L = 0.333}$$

$$C_{Di} = \frac{C_L^2}{\pi e AR} \qquad e = \frac{1}{1+\sigma} \qquad \sigma = \frac{3A_3^2}{A_1^2} = 0.0309$$

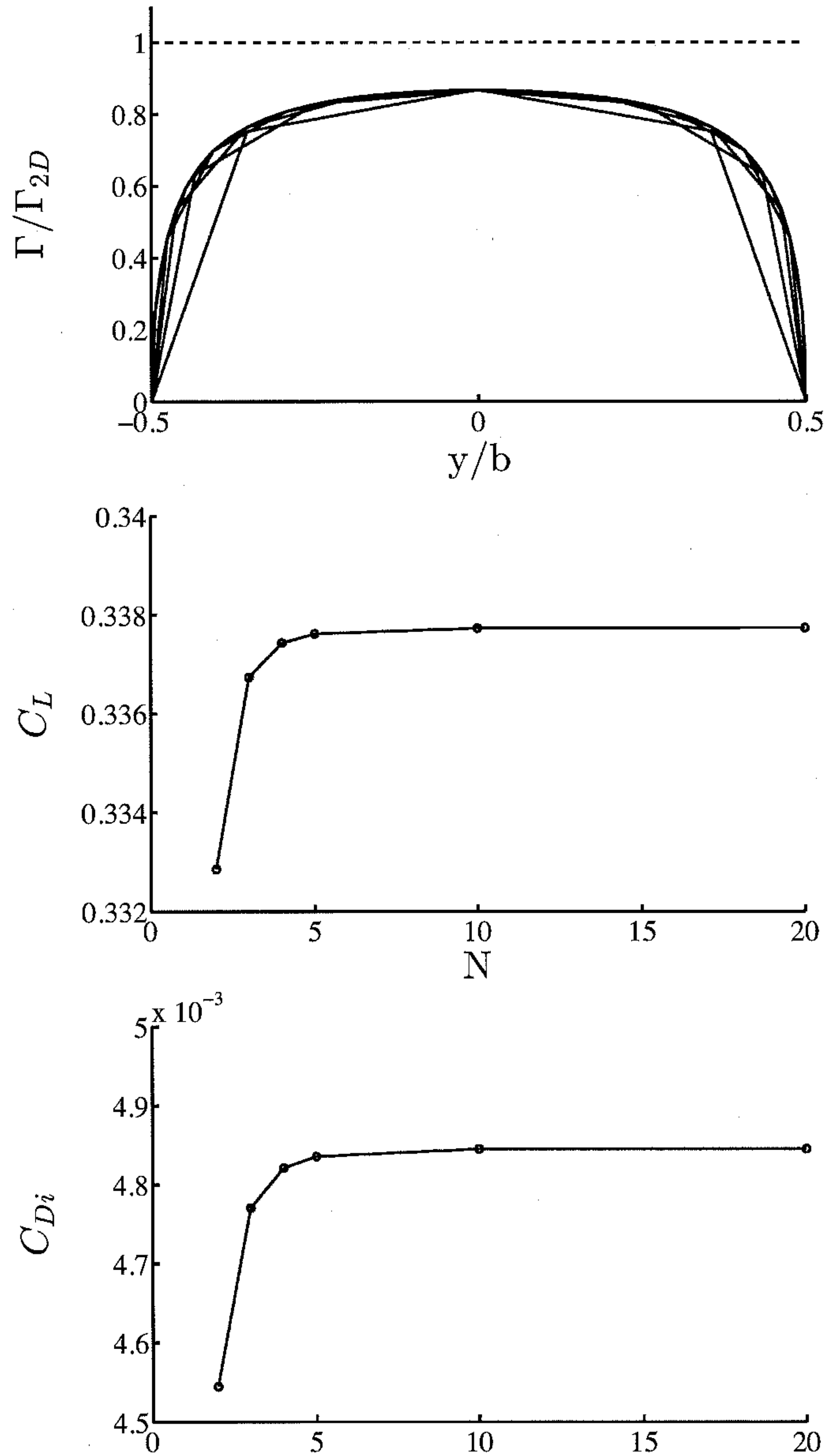
$$e = 0.97$$

$$\boxed{C_{Di} = 0.0045}$$

```
clear
clc
close all

% This program is to solve P1 of homework #8 for Aerodynamics MECH 326 Fall
% 2014.

% Problem Parameters
N = {2 3 4 5 10 20};            % Number of stations for half of the wing (symmetric
loading only)
alpha = 4*(pi/180);      % Geometric angle of attack convert from degrees to radians.
AR = 8;                  % Aspect ratio, b^2/S.  For this rectangular wing problem: AR =
b/c.

% Initialize Data structure.  The many of the fields within the structure
% are cell data types.  This allows the columns to be different lengths
% unlike an array.
Data.N = N;
Data.C_L = {};
Data.C_Di = {};
Data.Gamma_star = {};
Data.y_b = {};

% Running the main program and collecting the data.
for i = 1:numel(N)
    [C_L,C_Di,Gamma_star,y_b] = lifting_line_theory(N{i},alpha,AR);

    Data.C_L{i} = C_L;
    Data.C_Di{i} = C_Di;
    Data.Gamma_star{i} = Gamma_star;
    Data.y_b{i} = y_b;
end


%% Plotting
FontSizeAx = 24;
FontSizeLb = 32;
afFigurePosition = [0 0 25 15];
axespos = [0.175 0.15 0.76 0.76];
ylabelpos = [-0.15 0.5];
xlabelpos = [0.5 -0.1];

% Coefficient of lift
figure;
set(gcf, 'Units', 'centimeters','PaperPositionMode', 'auto','Position',
afFigurePosition);
set(gcf,'DefaultAxesFontSize',
FontSizeAx,'DefaultAxesFontName','TimesNewRoman','DefaultAxesGridLineStyle','-
.','DefaultAxesLineWidth',2,'DefaultAxesFontWeight','Normal')

hold on

    for i = 1:numel(N)-1
        plot([Data.N{i}; Data.N{i+1}],[Data.C_L{i}; Data.C_L{i+1}],'-ob','linewidth',2)
    end

hold off

set(gca, 'FontName', 'TimesNewRoman', 'FontSize', FontSizeAx)
set(gca, 'Units', 'normalized', 'Position', axespos);
xlabel('N','FontName', 'TimesNewRoman','FontUnit', 'points','FontSize',
FontSizeLb,'FontWeight', 'normal','Rotation', 0,'Units', 'Normalize','Position',
xlabelpos,'Interpreter', 'LaTeX');
ylabel('$$C_L$$','FontName', 'TimesNewRoman','FontUnit', 'points','FontSize',
FontSizeLb,'FontWeight', 'normal','Rotation', 90,'Units', 'Normalize','Position',
ylabelpos,'Interpreter', 'LaTeX');
```

```
print('-depsc','-r300','lift_coeff');


% Coefficient of induced drag
figure;
set(gcf, 'Units', 'centimeters','PaperPositionMode', 'auto','Position',
afFigurePosition);
set(gcf,'DefaultAxesFontSize',
FontSizeAx,'DefaultAxesFontName','TimesNewRoman','DefaultAxesGridLineStyle','-
.','DefaultAxesLineWidth',2,'DefaultAxesFontWeight','Normal')

hold on

    for i = 1:numel(N)-1
        plot([Data.N{i}; Data.N{i+1}],[Data.C_Di{i}; Data.C_Di{i+1}],'-ob','linewidth',
2)
    end

hold off

set(gca, 'FontName', 'TimesNewRoman', 'FontSize', FontSizeAx)
set(gca, 'Units', 'normalized', 'Position', axespos);
xlabel('N','FontName', 'TimesNewRoman','FontUnit', 'points','FontSize',
FontSizeLb,'FontWeight', 'normal','Rotation', 0,'Units', 'Normalize','Position',
xlabelpos,'Interpreter', 'LaTeX');
ylabel('$$C_{Di}$$','FontName', 'TimesNewRoman','FontUnit', 'points','FontSize',
FontSizeLb,'FontWeight', 'normal','Rotation', 90,'Units', 'Normalize','Position',
ylabelpos,'Interpreter', 'LaTeX');

print('-depsc','-r300','induced_drag_coeff');


% Circulation distribution
figure;
set(gcf, 'Units', 'centimeters','PaperPositionMode', 'auto','Position',
afFigurePosition);
set(gcf,'DefaultAxesFontSize',
FontSizeAx,'DefaultAxesFontName','TimesNewRoman','DefaultAxesGridLineStyle','-
.','DefaultAxesLineWidth',2,'DefaultAxesFontWeight','Normal')

hold on

    for i = 1:numel(N)
        plot(Data.y_b{i},Data.Gamma_star{i},'-b','linewidth',2)
    end

    plot(Data.y_b{end},Data.Gamma_star{end},'-k','linewidth',2)
    plot(Data.y_b{end},ones(numel(Data.y_b{end}),1),'--k','linewidth',2)

hold off

axis([-0.5 0.5 0 1.1])

set(gca, 'FontName', 'TimesNewRoman', 'FontSize', FontSizeAx)
set(gca, 'Units', 'normalized', 'Position', axespos);
xlabel('y/b','FontName', 'TimesNewRoman','FontUnit', 'points','FontSize',
FontSizeLb,'FontWeight', 'normal','Rotation', 0,'Units', 'Normalize','Position',
xlabelpos,'Interpreter', 'LaTeX');
ylabel('$$\Gamma/\Gamma_{2D}$$','FontName', 'TimesNewRoman','FontUnit',
'points','FontSize', FontSizeLb,'FontWeight', 'normal','Rotation', 90,'Units',
'Normalize','Position',ylabelpos,'Interpreter', 'LaTeX');

print('-depsc','-r300','circ_distribution');
```

```
function [C_L,C_Di,Gamma_star,y_b] = lifting_line_theory(N,alpha,AR)
    % Equation (6.27) in Kuethe and Chow, Foundations of
    % Aerodynamics, Fifth edition is used.  This is the fundamental equation
    % for lifting line theory where there is an arbitrary circulation
    % distribution.  For this problem, c = c_s,  m_0 = m_0s = 2*pi,
    % alpha_a(theta_0) = alpha.  The simplified fundamental equation is then:
    % Sum_1^inf (A_n sin(n*theta_0)) +  (pi/(2*AR))*Sum_1^inf (n*A_n sin(n*theta_0)/sin
(theta_0)) = alpha

    % Calculating the RHS vector
    alpha_vec = alpha*ones(N,1);

    % Calculating the theta_0 vector
    theta_0 = (1:N)'*(pi)/(2*N);

    % Calculating n-vector
    n = 1:2:2*N-1;

    % Building matrix B: B*A = alpha, where A's are the coefficients vector and
    % alpha is the vector of angles of attack.

    B = sin(repmat(n,N,1).*repmat(theta_0,1,N)) + pi/(2*AR)*repmat(n,N,1).*sin(repmat(n,
N,1).*repmat(theta_0,1,N))./sin(repmat(theta_0,1,N));

    % Invert the B matrix and solve for the coefficients, A = B^-1*alpha
    A = B\alpha_vec;

    % Calculate the lift coefficient
    C_L = pi^2*A(1)/2;

    % Calculate the correction factor, sigma, and the span efficiency factor, e = 1/(1 +
sigma)
    sigma = sum(n(2:end)'.*A((2:end)',1).^2,1)/A(1).^2;
    e = 1/(1 + sigma);
    C_Di = C_L^2/(pi*e*AR);

    % Calculating the circulation distribution. Gamma_star is the
    % non-dimensional circulation distribution.  It is non-dimensionalized
    % by the 2D circulation around a symmetric thin airfoil at an angle of
    % attack, alpha.  That is, gamma_star = gamma/(pi*U*c*alpha) and
    % gamma = (pi*U*c) * sum_1^inf (A_n sin(n theta_0)).

    Gamma_star = (1/alpha)*sum(repmat(A',N,1).*sin(repmat(n,N,1).*repmat(theta_0,1,N)),
2);

    % Add end-point conditions: theta_0 = [0, pi], Gamma = [0 0].  This comes
    % from the sin(n*theta_0) = [0, 0] when theta_0 = [0, pi].
    theta_0 = [0; theta_0];
    Gamma_star = [0; Gamma_star];

    % Transform theta_0 back to y/b
    y_b = -1/2*cos(theta_0);

    % Apply symmetry condition
    y_b = [y_b;-y_b(end-1:-1:1)];
    Gamma_star = [Gamma_star;Gamma_star(end-1:-1:1)];
end
```



1
0.8
0.6
0.4
0.2
0
Γ/Γ_{2D}
−0.5
0
0.5
y/b
0.34
0.338
0.336
0.334
0.332
C_L
0
5
10
15
20
N
5
x 10^{-3}
4.9
4.8
4.7
4.6
4.5
C_{Di}
0
5
10
15
20
N

Problem 2

Upstream

$T_1 = 288\,K$

$P_1 = 1\,atm$

Downstream

$T_2 = 690\,K$

$P_2 = 8.656\,atm$

Calculate Δh, Δe and Δs:

$c_p = 1004\ J/kg\,K \qquad c_v = 717.5\ J/kg\,K$

$$\Delta h = c_p(T_2 - T_1) = 1004(690-288) = \boxed{4.038 \times 10^5\ J/kg}$$

$$\Delta e = c_v(T_2 - T_1) = 717.5(690-288) = \boxed{2.884 \times 10^5\ J/kg}$$

$$\Delta s = c_p \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = 1004\ln\left(\frac{690}{288}\right) - 287\ln(8.656)$$

$$\boxed{\Delta s = 258.2\ J/kg \cdot K}$$