ECE 83

Spring 2014 [EXAM #2]

INDUCTANCE AND CAPACITANCE → Energy-storage Elements

CAPACITORS → C

Store energy through electric fields

• current trough capacitor:

$$i_c = C\frac{dV_c}{dt}$$

• capacitors act as open circuits for steady state dc voltages
→ No current

• C in series ⇒ R parallel

$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$$

• capacitor problem ⇒ solve for voltage

• Units: farads [F]

• voltage does not change instantaneosly

INDUCTORS → L

Store energy through magnetic fields

• voltage in inductor:

$$V_L = L\frac{di_L}{dt}$$

0 in steady state

• Inductors act as short circuits for steady state dc voltages
→ no voltage

• L in series ⇒ R series

$$L_{eq} = L_1 + L_2 + L_3$$

• Inductor problem ⇒ solve for current

• Units: henries [H]

• current cannot change instantaneosly

TRANSIENTS/SWITCHING

$$\boxed{x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}}$$

[SS condition after switch flips] [condition just after switch flips]

time constant
$\tau = R_{th}C$ (C)
$\tau = \frac{L}{R_{th}}$ (L)

→ R_{th} → resistance as seen through the capacitor as a lens

capacitor:
$x(t) \Rightarrow v(t)$

inductor:
$x(t) \Rightarrow i(t)$

⇒ $v(0^-) = \underline{v(0^+)}$ = SS before switch flips

• voltage cannot change instantaneosly across a capacitor

• current cannot change instantaneosly across an inductor

• To solve switching problems:

1. make steady-state assumptions
2. solve
3. flip the switch
4. solve

V_0 = constant (DC)

$V(t) = A\sin(\omega t + \theta)$ (AC)

Time constant → measure of time to reach steady state

(AC) → surges

STEADY-STATE SINUSODIAL ANALYSIS → AC

$\boxed{v(t) = V_m \cos(\omega t + \theta)}$ sinusodial voltage

- V_m → peak value of the voltage (amplitude)
- ω → angular frequency (rad/s)
- θ → phase angle (degrees)

ω → natural freq.
f → driving frequen
$\omega = 2\pi f$ $f = \frac{1}{T}$ → [Hz]

*to evaluate at a fixed time, convert θ to radians so you can evaluate $(\omega t + \theta)$

$\boxed{\sin(z) = \cos(z - 90°)}$
→ sinusodial function is normally expressed using cosine

Ex: $V_x(t) = 10\sin(200t + 30°)$
$V_x(t) = 10\cos(200t + 30° - 90°)$
$= 10\cos(200t - 70°)$

$\boxed{p(t) = \frac{V^2(t)}{R}}$ power

$\boxed{E(t) = \int_0^T p(t)\,dt}$ energy of one period of oscillation

$\boxed{P_{avg} = \frac{V_{rms}^2}{R} \qquad P_{avg} = I_{rms}^2 R}$ Average Power

$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$

• **PHASORS:** vector in the complex-number plane → transform function that represent sinusoidal voltages or currents

*phasors make equations not in the time domain

→ $v(t) = V_m \cos(\omega t + \theta)$
→ $\bar{V} = V_m \angle \theta$

$\boxed{V\cos(\omega t + \theta) = \bar{V}\angle\theta = V\cos\theta + jV\sin\theta}$ Transform function (phasor)

• **Impedence:**

$Z_L = j\omega L$ [inductor] lags
$Z_C = -\frac{j}{\omega C}$ [capacitor] leads
$Z_R = R$ [resistor]

• Ohm's Law in the complex domain:
$V = IR \Rightarrow \boxed{V = (Z)I}$

•NOTES: $(\angle\theta_1)(\angle\theta_2) = \angle(\theta_1 + \theta_2)$
$\frac{\angle\theta_1}{\angle\theta_2} = \angle(\theta_1 - \theta_2)$ $j = \angle 90°$

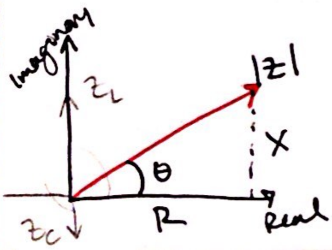
CONVERTING FORMS
$C\angle\theta = A + jB$
Polar → $C\angle\theta$; Rectangular → $A + jB$
Mult/Div → Polar Form
ADD/Sub → Rectangular Form

Impedence = Resistance in phasor domain

$\boxed{\text{Norton-Thevinin, Mesh Analysis, Nodal Analysis, KVL, KCL, Ohm's Law} \Rightarrow \text{STILL APPLY!!!}}$

$C = \sqrt{A^2 + B^2}$
$\theta = \tan^{-1}\left(\frac{B}{A}\right)$

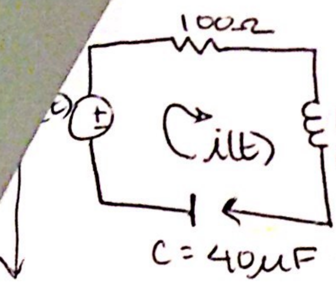
• capacitance → current leads voltage by 90°
• resistance → voltage is in phase with current
• inductance → current lags voltage by 90°



$Z = R + jX$ (R: Real, jX: Imaginary)

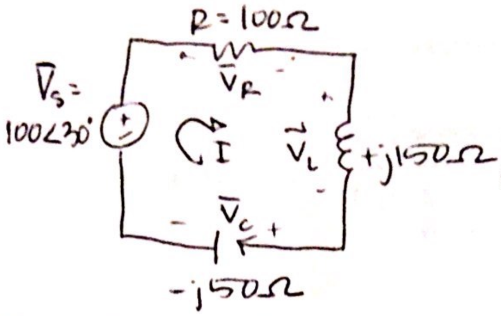
R = real part of impedence
X = imaginary part of impedence
$\left[-\frac{j}{\omega C} \text{ or } j\omega L\right]$

Steady-State AC analysis



$V_s(t) = 100\cos(500t + 30°)$

$1\,\mu F = 10^{-6}\,F$



$Z_{eq} = Z_L + Z_C + R$
$= 100 + j150 - j150 = 100 + j100$
$= 141.4\angle 45°$

① Replace time descriptions of sources with phasors (sources must have the same frequency)

② Replace inductances + capacitances with their complex impedences

$Z_L = j\omega L = \omega L\angle 90°$

$Z_C = -\frac{j}{\omega C}$

$\bar{I} = \frac{\bar{V}_s}{Z} = \frac{100\angle 30°}{141.4\angle 45°} = 0.707\angle -15° \rightarrow i(t) = 0.707\cos(500t - 15°)$

You can find voltage across each element too $\rightarrow V_R = R \times I = 100 \times 0.707\angle -15°$
$= 70.7\angle -15°$

• **POWER** $P = VI$

• Power for general loads ⇒ resistor, inductor, capacitor

$v(t) = V_m\cos(\omega t)$
$i(t) = I_m\cos(\omega t - \theta)$
$P(t) = V_m I_m \cos(\omega(t))\cos(\omega t - \theta)$

$$\boxed{\begin{aligned} P &= \frac{V_m I_m}{2}\cos(\theta) \\ P &= V_{rms} I_{rms}\cos(\theta) \end{aligned}}$$

Average Power

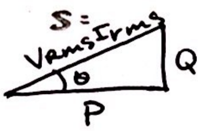
units [W] watts

$\boxed{Q = V_{rms} I_{rms} \sin(\theta)}$ Reactive Power

$\cos(\theta) \rightarrow$ power factor
$= \cos(\theta_v - \theta_i)$
$\theta \rightarrow$ power angle $(\theta_v - \theta_i) = \theta$

$\boxed{\text{apparent power} = V_{rms} I_{rms} = S}$ Apparent Power

• Power Triangle:



(θ positive) ↳ inductance

$V_{rms} = \frac{V_m}{\sqrt{2}}$

$I_{rms} = \frac{I_m}{\sqrt{2}}$

$[Q] = [VARs]$
$[\text{appt. power}] = [\text{volt-amperes (VA)}]$
units

$\boxed{P^2 + Q^2 = (V_{rms} I_{rms})^2}$

$\boxed{S = P + jQ}$ complex power | apparent power $S = VI^*$

Real power, Avg. Power, P-power, P → resistor

Reactive Power, Imaginary power, P → inductor, capacitor

$Z = R + jX$
$S = P + jQ$

$Q(+) \Rightarrow$ Inductors
$Q(-) \Rightarrow$ capacitors

$P_{app} = \sqrt{P^2 + Q^2} = |S|$

$$\begin{bmatrix} P = \frac{V_{rms}^2}{R} = I_{rms}^2 R \\ Q = \frac{V_{rms}^2}{X} = I_{rms}^2 X \end{bmatrix}$$

$\mu \rightarrow 10^{-6}$

$P = \left(\frac{I}{\sqrt{2}}\right)^2 R = \frac{I^2}{2} R$

ECE 83: Intro. To Electrical Engineering

Spring 2014
[EXAM #1]

3 MAIN CONCEPTS

① current = flow of electrons
[A]

② voltage = energy transferred; "potential energy"
[V]

③ resistance = "friction"
[Ω]

- fluid-flow analogy: current is a measure of the flow of charge through the cross section of a circuit element; voltage is measured across the ends of a circuit element or between any two points in a circuit.

SOURCES

12V ⊕ Independent voltage source

2A ⊕ Independent current source

$4V_x$ ◇ Dependent voltage source
- voltage-controlled voltage source
- current-controlled voltage source

$2i_y$ ◇ Dependent current source
- voltage-controlled current source
- current-controlled current source

these sources depend on something else in the circuit

KIRCHHOFF'S CURRENT LAW

- KCL: the net current (i) entering a node is zero.
  → to compute net current: add currents leaving a node and subtract currents entering
  → assume all unknown currents leave the node (+)
- Use Nodal Analysis to add currents and solve for voltages at nodes
  (a) choose reference node ($V_g = 0$), or ground node
  (b) Assume all unknown currents leave the node (If you get a negative result, you know the current actually enters the node)
  (c) write KCL equations

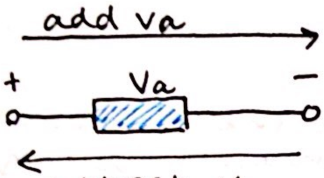
  V_1: $\frac{V_1 - V_2}{R_{12}} + \frac{V_1 - V_3}{R_{13}} = 0$

  (d) note: node voltages do not exist - they are mathematical constructs

* often it's useful to put ground node on (−) side of ⊕

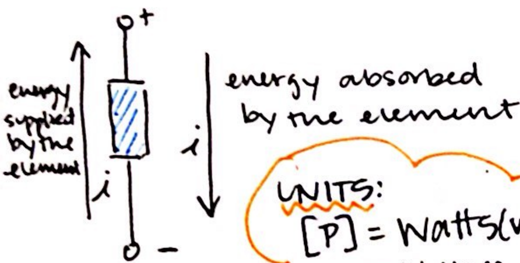
KIRCHHOFF'S VOLTAGE LAW

- **KVL**: The sum of the voltages equals zero for any closed path (loop) in an electrical circuit
- Use Mesh Analysis of currents to solve for voltages
  - (a) identify loops in the circuit
  - (b) write and solve KVL equations for each constructed I-loop
  - * draw each loop in the same direction (clockwise)
- NOTE:



$$R_1(I_1 - I_2) + R_2(I_1) + V = 0$$

POWER



UNITS:
$[P] = \text{Watts}(W)$
$= \text{Volts} \times \text{Amperes}$

Power = Voltage × Current

$$\boxed{P = VI}$$

$$\boxed{P = Vi = i^2 R = \frac{V^2}{R}}$$

- resistors always dissapate (absorb) power
- sources may dissapate or supply power
  - → if current enters (−) side → supply power!
  - → current enters (+) side → dissapate!

CIRCUITS

SERIES CIRCUIT vs. **PARALLEL CIRCUIT**

- current is the same through resistors in series
- voltage changes

↓

* Voltage divider

$$\boxed{V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_s}$$

*$(R_{eq})_{series}$

$$\boxed{(R_{eq})_{series} = \sum_{i=1}^{N} R_i}$$

↳ add up resistances for resistors in series

- current splits
- voltage is the same for elements in parallel

↓

* current divider

$$\boxed{i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s}$$

*$(R_{eq})_{||}$

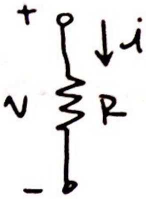
$$\boxed{(R_{eq})_{||} = \frac{R_1 R_2}{R_1 + R_2}}$$

'S LAW

$\boxed{V = iR}$

- **Ohm's Law** relates voltage and current
- usually applied after KCL or KVL for solving problems

- **Resistors:**



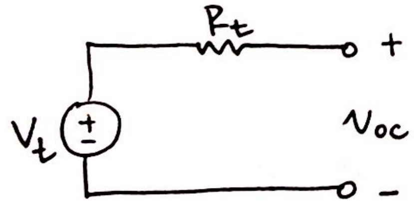
- current moves from higher to lower potential through resistors - ALWAYS! (i.e. 10V to 5V)
- current travels the path of least resistance

SUBCIRCUITS

- goal → simplify circuits
- lens: what are we looking through? (usually a resistor)
  - neglect the lens element in analysis of circuit
- open circuit: no current flow
- short circuit: current flows, but no voltage (connected by an ideal conductor)

THEVENIN SUBCIRCUIT

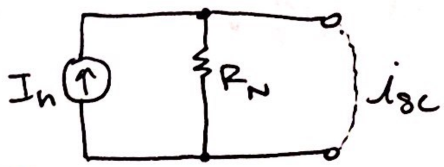
- consists of an independent VS in series with a resistance



$\boxed{V_t = V_{oc}}$ → original circuit

NORTON SUBCIRCUIT

- consists of an independent current source in parallel with the Thevenin resistance



$\boxed{i_{sc} = I_n}$ → original circuit

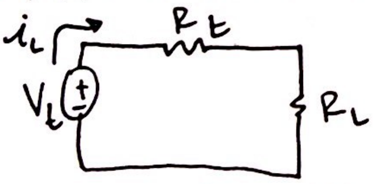
$\boxed{R_t = R_N}$

- use short circuit to get rid of parallel resistor

- If and only if, you have no dependent sources ◇, you can solve for R_t directly by zeroing out sources

  (±) ⇒ SC   (↑) ⇒ OC

- we can replace any circuit with a thevenin or norton equivalent ← Source Transformation!
- What is the maximum power across the resistor?



if $\underbrace{R_L = R_t}_{\text{requirement for max. power!}}$ ...

$$\boxed{P_{max} = \frac{V_t^2}{4R_t}}$$

- to find a (T) or (N) equivalent, you need to find 2 of the following 3:

  V_t, I_N, R_t OR i_{sc}, V_{oc}, R_t

- Steps to Solve:
  1) Zero out sources to solve for $R_{eq} = R_t = R_N$
  2) go back to original circuit w/ sources to solve for one other variable
- you can replace norton w/ thevenin (or opposite) to solve problems!

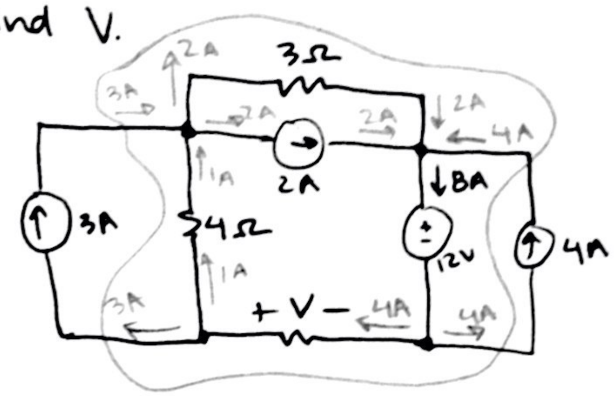
AMPLE PROBLEMS

$V = IR$

$[V] = [A][\Omega]$

-VL

Find V.



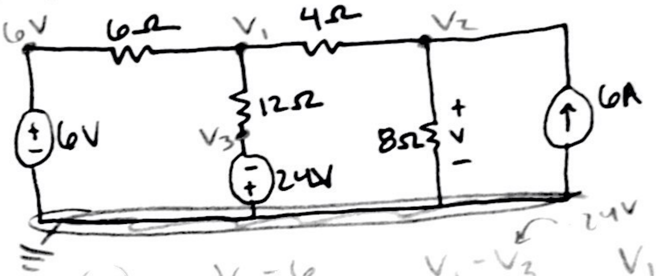
- observe circuit to add currents
- KVL of loop

$(4\Omega)(1A) + (3\Omega)(2A) + 12V - V = 0$

$4 + 6 + 12 = V = \boxed{22\,V}$

NODAL ANALYSIS (KCL)

Find V.



- choose ground node on (−) side of ⊕ to give voltage at a node
- (+) voltages leaving node

$(V_1)\quad \frac{V_1 - 6}{6} + \frac{V_1 - V_3}{12} + \frac{V_1 - V_2}{4} = 0$

$V_1 = 9V$

$V_2 = 22V$

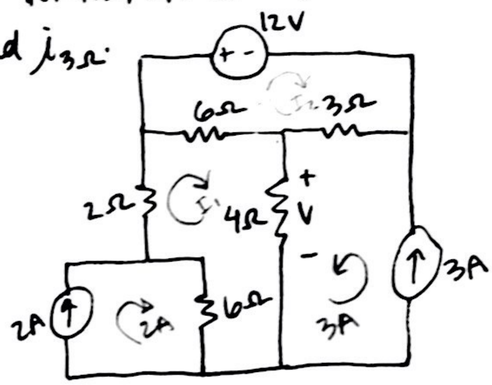
$(V_2)\quad \frac{V_2 - V_1}{4} + \frac{V_2}{8} - 6 = 0$

$V = \boxed{V_2 = 22V}$

$(V_2)\quad V_3 = -24V$

MESH ANALYSIS (KVL)

Find $i_{3\Omega}$.



Mesh I_1: $2I_1 + 6(I_1 - I_2) + 4(I_1 + 3) + 6(I_1 - 2) = 0$

Mesh I_2: $3(I_2 + 3) + 6(I_2 - I_1) + 12V = 0$

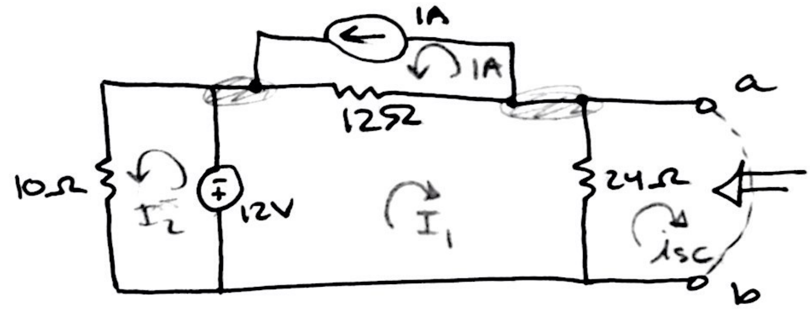
↳ $9I_2 - 6I_1 = 21$ → $I_1 = 1A$

$-6I_2 + 18I_1 = 0$ $\quad I_2 = -3A$

$i_{3\Omega} = I_2 + 3 = \boxed{0}$

NORTON / THEVENIN

Find the Norton & Thevenin equivalent subcircuits

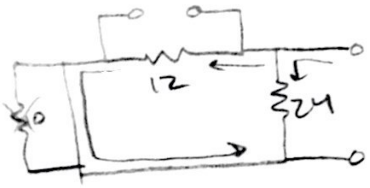


• no ◇ → zero out sources

⊕ → SC

⊕ → OC

• go back to original circuit to find i_{sc} or V_{oc}



$$R_{eq} = R_t = 12 // 24 = \frac{(12)(24)}{12+24} = 8\,\Omega$$

• Mesh Analysis

Mesh i_{sc}: $24(i_{sc} - I_1) = 0$

Mesh I_1: $24(I_1 - i_{sc}) + 12 + 12(I_1 + 1) = 0$

$\hookrightarrow 24I_1 - 24i_{sc} + 24 + 12I_1 = 0$

$i_{sc} = -2A \longrightarrow I_N = i_{sc}$ $\quad V_t = i_{sc} R_t$