Exam 2

MECH 326: Aerodynamics
Wednesday, November 16, 2016: 8:00–9:00 AM

Name: Solution

1. Thin airfoil design (25%)
2. Finite wing design (25%)
3. Compressible flow measurements (20%)
4. Concepts (30%)

Extra Credit

a) A10 Warthog
b) MiG-21
c) Bell X-1

Attempt 10
a) 10
b) 5

1. **Thin airfoil design** (25%) A supersonic aircraft has just taken off and it is still at subsonic speeds of $M < 0.3$. The wings can be modeled as thin airfoils with no camber at an angle of attack, $\alpha = 5^o$. However, the ailerons (the last 20% of the chord) are down at a trim angle of $\Delta\theta = 5^o$. We can model the wings and deflected ailerons with a piecewise camber function,

$$\frac{z}{c} = 0, \qquad 0 \le \frac{x}{c} \le 0.8$$
$$\frac{z}{c} = \beta_1 - \beta_2\left(\frac{x}{c}\right), \qquad 0.8 \le \frac{x}{c} \le 1$$

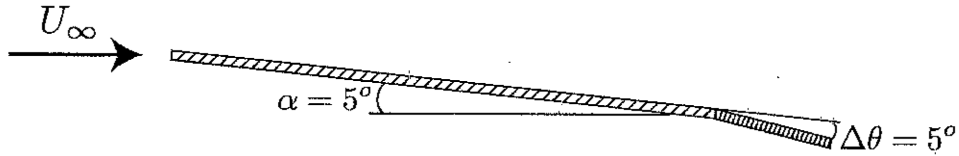
and

$$\frac{x}{c} = \frac{1}{2}(1 - \cos\theta_0)$$

where $\beta_1 = 0.07$ and $\beta_2 = 0.087$. Calculate the following,

(a) $\alpha_{L=0}$.

(b) Two-dimensional lift coefficient, c_l.



(a) $\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta_0 - 1)\,d\theta_0$ $\qquad \frac{dz}{dx} = \begin{cases} 0 & 0 \le x/c \le 0.8 \\ -\beta_2 & 0.8 \le x/c \le 1 \end{cases}$

$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{2.214} \cancel{(0)(\cos\theta_0 - 1)\,d\theta_0} - \frac{1}{\pi}\int_{2.214}^{\pi} -\beta_2(\cos\theta_0 - 1)\,d\theta_0$ $\qquad x/c = 0,\ 0.8,\ 1$; $\theta_0 = 0,\ 2.214,\ \pi$

$\alpha_{L=0} = +\frac{\beta_2}{\pi}\left[\sin\theta_0\Big|_{2.24}^{\pi} - \theta_0\Big|_{2.214}^{\pi}\right] = \frac{\beta_2}{\pi}\left[\sin\pi - \sin(2.214) - \pi + 2.214\right]$

$\alpha_{L=0} = -\beta_2(0.55)$

$$\boxed{\begin{aligned} \alpha_{L=0} &= -0.04785 \text{ rad} \\ \alpha_{L=0} &= -2.7 \text{ deg} \end{aligned}}$$

(b) $c_l = 2\pi\left[\alpha - \alpha_{L=0}\right]$

$= 2\pi\left[5\left(\frac{\pi}{180}\right) - (-0.04785)\right]$

~~$c_l = 0.448$~~ $\boxed{c_l = 0.849}$

10 2. **Finite wing design** (25%) A wing with an elliptical planform is flying through sea-level air at a speed of 50 m/s. The wing loading is $W/S = 1200$ N/m^2. The wing is untwisted and has the same section from root to tip. The lift curve slope of the section is $m_0 = 5.8$ rad^{-1}. The span of the wing is 8 m, and the aspect ratio is $\mathcal{R} = 6$. The density of air is $\rho = 1.23$ kg/m^3. Address the following:

5 (a) Determine the sectional-lift and induced-drag coefficients, that is, c_l and c_{di}, respectively.

5 (b) Determine the effective, induced and absolute angles of attack, that is, α_0, α_i and α_a, respectively.

5 (c) What is the power that is required to overcome the induced drag of the wing?

$u = 50$ m/s $\quad W/S = 1200$ N/m^2 $\quad m_0 = 5.8$ rad^{-1} $\quad b = 8$ m $\quad \mathcal{R} = 6$

$\rho = 1.23$ kg/m^3

$$\mathcal{R} = \frac{b^2}{S} \rightarrow S = \frac{b^2}{\mathcal{R}} = \frac{(8\text{ m})^2}{6} \rightarrow S = 10.67\text{ m}^2$$

(a) During steady-level flight $\rightarrow L = W \rightarrow L/S = W/S$
$= 1200$ N/m^2

$$C_L = \frac{L}{\frac{1}{2}\rho u^2 S} \quad \text{and} \quad C_L = c_\ell \text{ (elliptical wings)}$$

$$c_\ell = \frac{L/S}{\frac{1}{2}\rho u^2} = \frac{(1200\text{ N/m}^2)}{\frac{1}{2}(1.23\text{ kg/m}^3)(50\text{ m/s})^2}$$

$$\boxed{c_\ell = 0.78}$$

Also $\underbrace{c_{di} = C_{Di}}_{\text{elliptical wings}} = \frac{C_L^2}{\pi \mathcal{R}} = \frac{(0.78)^2}{\pi(6)} \rightarrow \boxed{c_{di} = 0.0323}$

(b) $\alpha_0 = c_\ell / m_0 = \frac{0.78}{5.8\text{ rad}^{-1}} \rightarrow \boxed{\begin{array}{l}\alpha_0 = 0.134\text{ rad}\\ \alpha_0 = 7.71\text{ deg}\end{array}}$

$\alpha_i = -\frac{c_{di}}{c_\ell} = -\frac{0.0323}{0.78} \rightarrow \boxed{\begin{array}{l}\alpha_i = -0.041\text{ rad}\\ \alpha_i = -2.37\text{ deg}\end{array}}$

$\alpha_a = \alpha_0 - \alpha_i \Rightarrow \boxed{\begin{array}{l}\alpha_a = 0.175\text{ rad}\\ \alpha_a = 10.08\text{ deg}\end{array}}$

(c) Power $= P = D_i u$

$$D_i = C_{Di}\left(\tfrac{1}{2}\rho u^2 S\right) = 0.0323\left(\tfrac{1}{2}\right)(1.23\text{ kg/m}^3)(50\text{ m/s})^2 (10.67\text{ m}^2)$$

$D_i = 529.9$ N

$P = (529.9\text{ N})(50\text{ m/s})$

$$\boxed{P = 26.5\text{ kW}}$$

5 5 5

3. **Compressible flow measurements** (20%) You have graduated Lehigh University and landed a job as a mechanical engineer working at Boeing's wind tunnel facilities outside of Philadelphia. You are in charge of setting up the measurement apparatus for their transonic wind tunnel, which is composed of a pitot-static probe connected to a differential pressure sensor. The pressure sensor measures the pressure difference between the stagnation pressure and the static pressure, that is, $\Delta P = P_0 - P$, at different locations in the wind tunnel. Assume that the **air** flow is **isentropic** throughout the wind tunnel and that the reservoir pressure and temperature have been measured at $P_0 = 100$ kPa and $T_0 = 300$ K, respectively. There is negligible flow speed in the reservoir. Determine the following:

   (a) What is the speed of sound, a_0, and density of air, ρ_0, in the reservoir?

   (b) At point 1 just inside the inlet of the wind tunnel the pitot-static probe measures a pressure difference of $\Delta P = 30$ Pa. What is the Mach number, M_1, at point 1?

   (c) Similarly, at the point 2 in the test section the pitot-static probe measures a Mach number of $M_2 = 0.74$ (this is the cruise Mach number of the Boeing 737!). What is the local speed of sound, a_2, and the flow speed, U_2 in the test section?

Bonus (5 pts, not to exceed 100%): Now the test section speed is reduced such that $M_1 = 0.01$ and $M_2 = 0.5$. Using a statement of conservation of mass calculate the contraction ratio of the tunnel, that is the ratio of the cross-sectional area at point 1 to that at point 2 or A_1/A_2.

(a) $a_0 = \sqrt{\gamma R T_0} \rightarrow \sqrt{(1.4)(287)(300)} \rightarrow \boxed{a_0 = 347.2 \text{ m/s}}$

$P_0 = \rho_0 R T_0 \rightarrow \rho_0 = \dfrac{P_0}{RT_0} = \dfrac{100{,}000 \text{ Pa}}{(287 \text{ J/kg K})(300 \text{ K})}$

$\boxed{\rho_0 = 1.16 \text{ kg/m}^3}$

(b) $\Delta P = 30$ Pa

$\Delta P = P_0 - P_1 = P_0 - \left(\dfrac{P_1}{P_0}\right) P_0 = P_0\left(1 - \dfrac{P_1}{P_0}\right)$

$\dfrac{\Delta P}{P_0} = 1 - \dfrac{P_1}{P_0} \rightarrow \dfrac{P_1}{P_0} = 1 - \dfrac{\Delta P}{P_0} = 1 - \dfrac{30 \text{ Pa}}{100{,}000 \text{ Pa}}$

$\dfrac{P_1}{P_0} = 0.9997$

Tables $\rightarrow \boxed{M_1 = 0.02}$

(c) $M_2 = 0.74$

$a_2 = \left(\dfrac{a_2}{a_0}\right) a_0$ Tables @ $M_2 = 0.74 \rightarrow \dfrac{a_2}{a_0} = 0.9494$

$a_2 = (0.9494)(347.2 \text{ m/s}) \rightarrow \boxed{a_2 = 329.6 \text{ m/s}}$

$U_2 = M_2 a_2 = (0.74)(329.6 \text{ m/s}) \rightarrow \boxed{U_2 = 243.9 \text{ m/s}}$

Bonus

$\rho_1 A_1 U_1 = \rho_2 A_2 U_2 \rightarrow \dfrac{A_1}{A_2} = \left(\dfrac{\rho_2}{\rho_1}\right)\left(\dfrac{U_2}{U_1}\right) = \left(\dfrac{\rho_2}{\rho_0}\right)\left(\dfrac{\rho_0}{\rho_1}\right)\left(\dfrac{M_2}{M_1}\right)\left(\dfrac{a_2}{a_0}\right)\left(\dfrac{a_0}{a_1}\right)$

From Tables: $\dfrac{\rho_2}{\rho_0} = 0.8852$ $\quad \dfrac{a_2}{a_0} = 0.9759$ $\quad \dfrac{\rho_1}{\rho_0} = 1$ $\quad \dfrac{a_1}{a_0} = 1$ $\quad \dfrac{M_2}{M_1} = \dfrac{0.5}{0.01} = 50$

$\dfrac{A_1}{A_2} = (0.8852)(1)(50)(0.9759)(1) \rightarrow \boxed{\dfrac{A_1}{A_2} = 43.2}$

4. **Concepts** (30%) How well do you conceptually understand aerodynamics:

3 each

(4.1) What is the theoretical lift slope of a two-dimensional airfoil?

$m_0 = 2\pi \text{ rad}^{-1} \cong 0.11 \text{ deg}^{-1}$

(4.2) What mechanism leads to induced drag and a smaller lift slope in finite wings?

Tip vortex System

(4.3) In thin airfoil theory, what condition must be applied at the trailing edge in order to model the effects of viscosity? Kutta Condition

(4.4) What assumption of the Bernoulli equation is invalidated when $M > 0.3$?

Incompressibility

(4.5) In a potential flow, does the strength of a vortex filament vary along it's length or stay the same?

Stays the same

(4.6) In a potential flow, does the strength of a vortex filament vary in time or stay the same?

Stays the Same

(4.7) In a potential flow, can a vortex filament end in the fluid?

No

(4.8) For an isentropic compressible flow of an ideal-gas, if the velocity increases does the temperature increase, decrease or stays the same? The temperature decreases

(4.9) For an irreversible adiabatic flow process, does the total enthalpy change or stay the same?

Stays the same

(4.10) For an irreversible flow process, does the total pressure change or stay the same?

Changes