Engineering Vibrations & Systems

Module 7
Electromechanical Systems

ME 242
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Module 6

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1. Introduction

Electromechanical systems ---

1. Electrical/electromagnetic subsystem (generates force)
2. Mechanical subsystem (w/ mass, elasticity and damping)

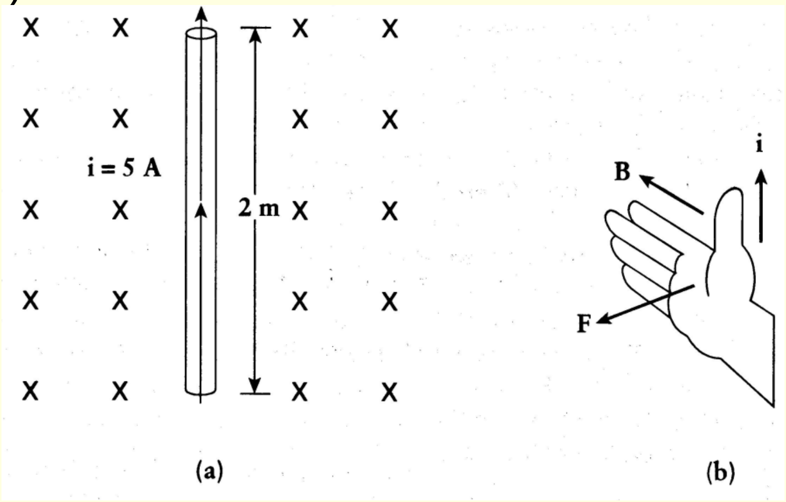
Both these systems are coupled.

Examples are:
electric motors, instrument meters, tachometers, accelerometers

2. Electromechanical Systems

2.1 Magnetic Coupling (Section 6.5 of SD)

Given:
1. length l of electric wire
2. magnetic field (of flux density B) ⊥ to the wire
3. current i in the wire

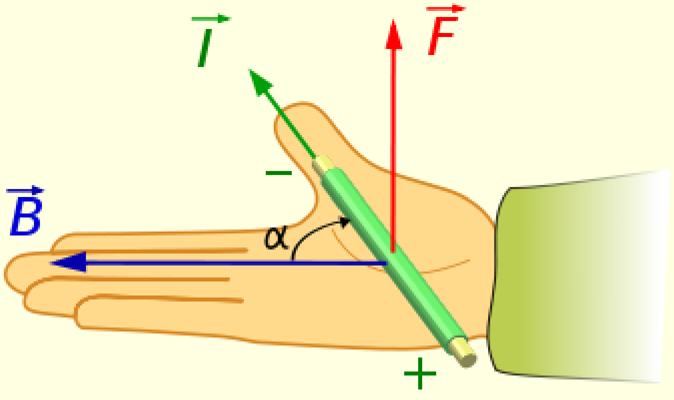


(i) Force f on stationary wire is:

$$f = Bil \quad \{ \vec{f} = \vec{i}l \times \vec{B} \} \qquad [1]$$

(ii) If the wire is moving with a velocity v there is an induced voltage on the wire given by e_b called the *back emf*:

$$e_b = Blv \qquad [2]$$



2. Electromechanical Systems

2.1 Magnetic Coupling (Continue)

Suppose:
1. magnetic force f acts on a mass m
2. there are no losses such as friction or damping
3. current I flows in the wire which moves with velocity v

(i) Power generated by the electrical circuit is:

$$e_b i = Bliv \qquad [3]$$

(ii) Motion due to Newton's Law:

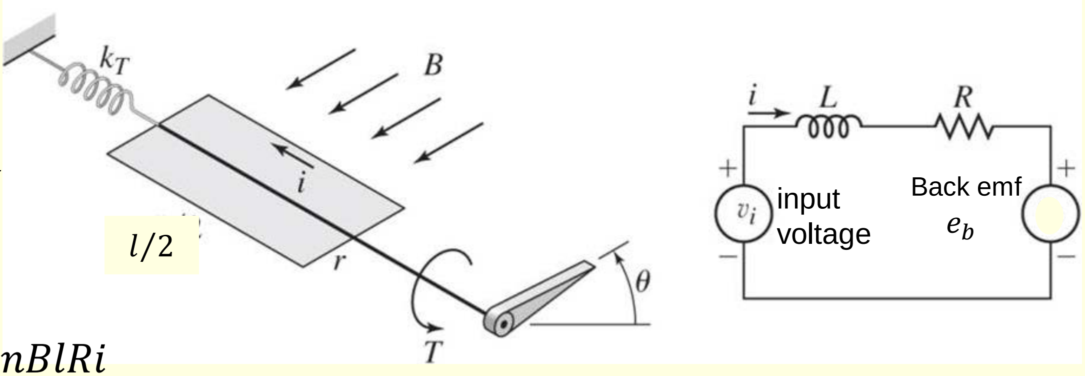
$$m\dot{v} = f = Bil \qquad [4]$$

2. Electromechanical Systems

2.2 D'Arsonval Meter (also called a voltmeter or ammeter)

D'Arsonval meter is used to measure the voltage across two points on a circuit or the current in a conductor. Current passes through a coil which is attached to a pointer in the meter. The coil is wrapped around an iron core and set within a magnetic field. Interaction between the current in the core with the magnetic field, rotates the coil and therefore the pointer. This rotation reacts against a torsion spring of stiffness k_T.

Length of one side of coil is $^{l}/_{2}$
Number of coils is n
Radius of coil is r
Mass moment of inertia of coils J
Damping constant on coils c
Torque T on the n coils is



$$T = fr = \left\{2nBi * {}^{l}/_{2}\right\} r = nBlRi$$

Therefore equation of motion of the coils:

$$J\frac{d^2\theta}{dt^2} + c\frac{d\theta}{dt} + k_T\theta = T = nBlRi$$

[5a]
Eq. for Mechanical subsystem

2. Electromechanical Systems

2.2 D'Arsonval Meter (Continue)

As the coil rotates, it induces a back emf that is proportional to the speed of rotation, or the speed $v = \dot{\theta} r$ in which the n coils cuts the magnetic field. Therefore the back emf

$$e_b = nBlv = nBlr\frac{d\theta}{dt} \qquad [5b]$$

The electrical circuit for the coil is modeled as a series connection of a resistor R and an inductor L. Applying KVL to this LR circuit,

$$v_i - L\frac{di}{dt} - Ri - e_b = 0 \qquad [5c]$$

or rewriting:

$$L\frac{di}{dt} + Ri + nBlr\frac{d\theta}{dt} = v_i \qquad [5d]$$

Eq. for Electromagnetic subsystem

Eqs [5a] and [5d] are the d.e. that govern the dynamic behavior of the meter. Together the two d.e. give a third order system. Suppose we suddenly apply a constant input voltage v_i, that would amount to a step input function. Depending on the mechanical parameters I, c, k_T the coil will rotate with critical damp, overdamp or underdamp.

At steady-state, $\dot{\theta} = 0$, [5d] → $i = \frac{v_i}{R}$

and [5a] → $\theta = \frac{nBlRi}{k_T} = \frac{nBlRv_i}{k_T R}$

2. Electromechanical Systems

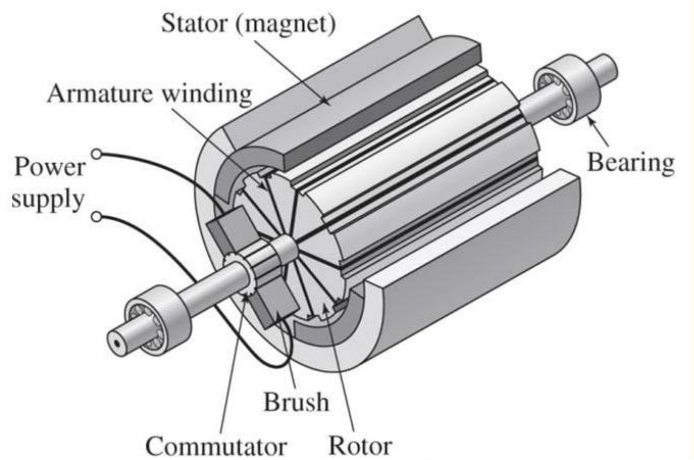
2.3 Armature Controlled DC Motor

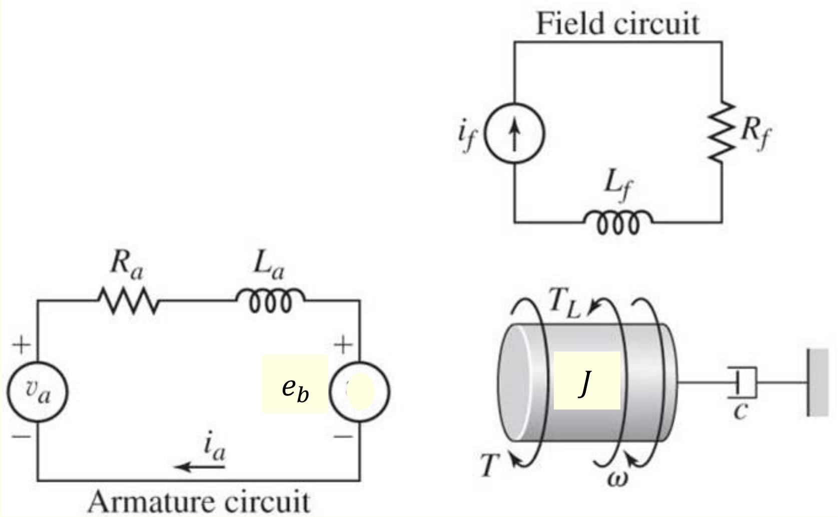
(Section 6.5.3 of SD)

Two main types of motors: direct current (*dc*) motors and alternating current (*ac*) motors. For *dc* motors, there are the *armature-controlled* motor and the *field-control* motor.

Basic elements of a motor:

1. Stator
2. Rotor
3. Armature
4. Commutator & Brushes





Electrical subsystem consists of the armature circuit and the field circuit. In a permanent magnet motor, the field circuit is replaced by a magnet. The *mechanical subsystem* consists Of a mass moment of inertia J and damping c.

2. Electromechanical Systems

2.3 Armature Controlled DC Motor (Continue)

Basic Relations of Motors

(i) Back emf e_b:

$$e_b = K_b\omega = K_b\dot{\theta} = (nBlr)\dot{\theta} \qquad [6a]$$

(ii) Motor torque T:

$$T = K_T i_a = (nBlr)i_a \qquad [6b]$$

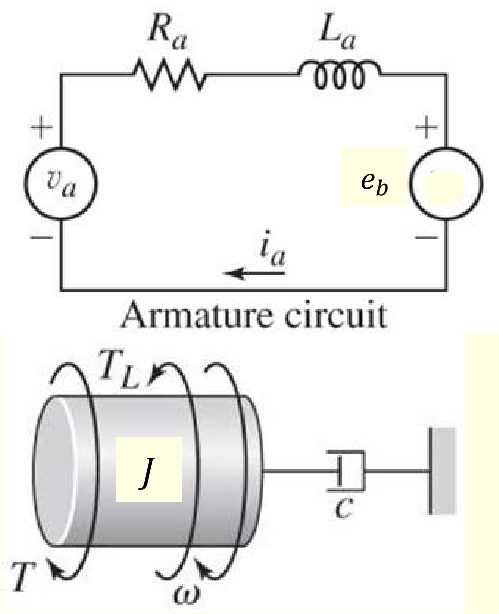
where:

e_b = back emf or voltage constant (*Volts)*
θ = angular rotation of rotor *(radians),* $\dot{\theta} = \omega$ (*radians/s*)
T = torque developed by rotor (*Nm*)
T_L = load torque; externally applied torque
K_b = emf constant (*V/Krpm*) or (*Nm/A*) *Krpm*=1000*rpm*; *A*=ampere
K_T = torque constant (*V/Krpm*) or (*Nm/A*)

Note that K_b and K_T have the same units **and** the same numerical value for a given dc motor. K_T is always given by the manufacturer because n, B, l, r are the parameters that the manufacturer chooses in designing the motor.

2. Electromechanical Systems

2.3 Armature Controlled DC Motor (Continue)



From the armature circuit, applying KVL:

$$v_a - R_a i_a - L_a \frac{di_a}{dt} - K_b \omega = 0 \qquad [6c]$$

For the mechanical subsystem:

$$J\frac{d\omega}{dt} = T - c\omega - T_L = K_T i_a - c\omega - T_L \qquad [6d]$$

A. Motor Block Diagram

Take Laplace Transform of [6c] with zero i.c.:

$$V_a(s) - R_a I_a(s) - L_a s I_a(s) - K_b \Omega(s) = 0$$

$$\Rightarrow \quad I_a(s) = \frac{1}{(L_a s + R_a)}[V_a(s) - K_b \Omega(s)] \qquad [6e]$$

write denominator as $\tau s + 1$ so that time constant $\tau = {}^{L_a}/_{R_a}$

2. Electromechanical Systems

Similarly take Laplace Transform of [6d] with zero i.c.:

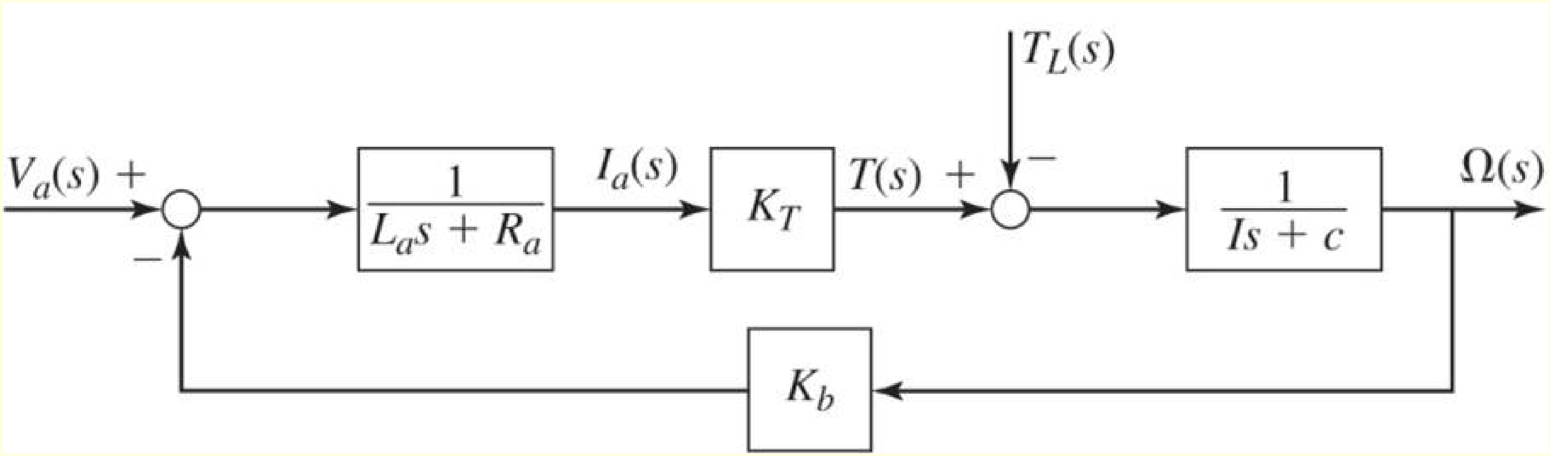
$$Js\Omega(s) = K_T I_a(s) - c\Omega(s) - T_L$$

$$\Omega(s) = \frac{1}{Js + c}[K_T I_a(s) - T_L] \qquad [6f]$$

write denominator as $\tau s + 1$ so that time constant $\tau = {}^{J}/_{c}$

$$I_a(s) = \frac{1}{(L_a s + R_a)}[V_a(s) - K_b \Omega(s)] \qquad [6e]$$

Block diagram of dc motor with motor speed $\Omega(s)$ as output:



2. Electromechanical Systems

B. Motor Transfer Function

Inputs: Applied voltage v_a; Load torque T_L
Outputs: Motor speed ω; Current i_a

➡ 4 transfer functions, one for each input-output pair. Solve Eqs. [6e] and [6f] for $I_a(s)$ and $\Omega(s)$. 2 eqs., 2 unknowns.

$$\left.\begin{array}{lll} \text{For output } I_a(s): & \dfrac{I_a(s)}{V_a(s)} = \dfrac{Js+c}{\Delta(s)}; & \dfrac{I_a(s)}{T_L(s)} = \dfrac{K_b}{\Delta(s)} \\[2ex] \text{For output } \Omega(s): & \dfrac{\Omega(s)}{V_a(s)} = \dfrac{K_T}{\Delta(s)}; & \dfrac{\Omega(s)}{T_L(s)} = \dfrac{L_a s+R_a}{\Delta(s)} \end{array}\right\} \quad [6g]$$

where $\Delta(s)$ is the characteristic polynomial that gives the characteristic eq.:

$$\Delta(s) = L_a J s^2 + (R_a J + cL_a) + cR_a + K_b K_T$$

Note that $\frac{I_a(s)}{V_a(s)}$ and $\frac{\Omega(s)}{T_L(s)}$ has numerator dynamics so that if v_a is a step function, output i_a can have a large overshoot. So would it be for ω if T_L is a step function.

2. Electromechanical Systems

C. State Variable Form of Motor Dynamics

Rewriting Eqs. [6c] and [6d]:

$$\frac{di_a}{dt} = \frac{1}{L_a}(v_a - R_a i_a - K_b \omega) \qquad [6h]$$

$$\frac{d\omega}{dt} = \frac{1}{J}(K_T i_a - c\omega - T_L) \qquad [6i]$$

The 2 variable are i_a and ω. So recasting the variables, let $x_1 = i_a$ and $x_2 = \omega$. Then Eqs. [6h] and [6i] become:

$$\frac{dx_1}{dt} = \frac{1}{L_a}(v_a - R_a x_1 - K_b x_2)$$

$$\frac{dx_2}{dt} = \frac{1}{J}(K_T x_1 - c x_2 - T_L)$$

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_{ab}}{L_a} \\ \frac{K_T}{J} & -\frac{c}{J} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} v_a \\ T_L \end{bmatrix} \qquad [6i]$$

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \qquad [6j]$$

2. Electromechanical Systems

2.4 Field-Controlled DC Motor (Section 6.5.5 of SD)

In a field-controlled dc motor, there are two power supplies, one for the armature circuit and one for the field circuit. The field circuit varies the intensity of the magnetic field surrounding the armature. There is also a control circuit to maintain a constant armature current in the presence of back emf which varies with motor speed and field strength.

Field strength B is a nonlinear function of field current i_f , i.e., $B(i_f)$. Therefore the torque on the armature is:

$$T = n\, B(i_f)\, L i_a r = (nLi_a r)\, B(i_f) = T(i_f)$$

which says that motor torque is a nonlinear function of the field current i_f. To simplify matters, let's assume the following *linear* approximation:

$$T - T_r = K_T(i_f - i_{fr})$$

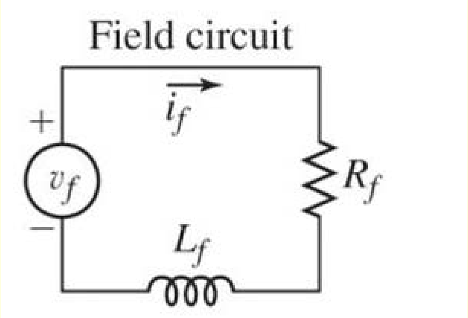
where T_r and i_{fr} are the torque and current values at the operating reference point and K_T is the slope of the $T(i_f)$ at that reference operating point. In this course, we will assume that T_r and i_{fr} are zero so that $T = K_T i_f$. [7a]

2. Electromechanical Systems

2.4 Field-Controlled DC Motor (Continue)

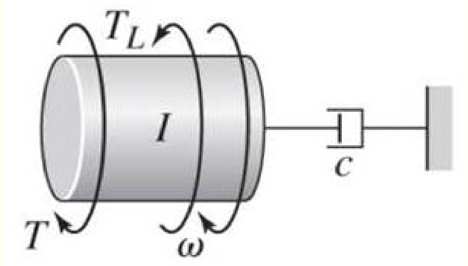
From the field circuit, applying KVL:

$$v_f = R_f i_f + L_f \frac{di_f}{dt} = 0 \qquad [7b]$$



For the mechanical subsystem:

$$J\frac{d\omega}{dt} = T - c\omega - T_L = K_T i_f - c\omega - T_L \qquad [7c]$$

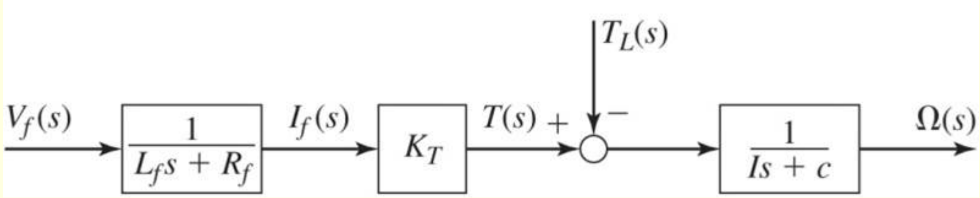


A. Motor Block Diagram

Take Laplace Transform of [7b], [7c] with zero i.c. and solving for $I_f(s)$ and $\Omega(s)$:

$$I_f(s) = \frac{1}{(L_f s + R_f)} V_f(s) \qquad [7d]$$

$$\Omega(s) = \frac{1}{Js + c}\left[K_T I_f(s) - T_L\right] \qquad [7e]$$



2. Electromechanical Systems

B. Motor Transfer Function

Inputs: Applied voltage v_f; Load torque T_L

Outputs: Motor speed ω; Current i_f

➡ Solve Eqs. [7d] and [7e] for $I_f(s)$ and $\Omega(s)$. 2 eqs., 2 unknowns.

For output $I_f(s)$:

$$\frac{I_f(s)}{V_f(s)} = \frac{1}{L_f s + R_f}; \qquad \frac{I_a(s)}{T_L(s)} = 0$$

For output $\Omega(s)$:

$$\frac{\Omega(s)}{V_f(s)} = \frac{K_T/R_f c}{[(L_f/R_f)s+1][(J/c)s+1]}; \qquad \frac{\Omega(s)}{T_L(s)} = -\frac{1}{Js+c}$$

[7f]

If the time constant for the electrical field system (L_f/R_f) is very small relative to that for the mechanical system (J/c), the dc motor system can be approximated by a first-order model:

$$\frac{\Omega(s)}{V_f(s)} = \frac{K_T/R_f c}{(J/c)s+1} = \frac{K_T/R_f}{Js+c} \qquad if \quad \frac{L_f}{R_f} \ll \frac{J}{c}$$

From [7a] the motor torque is:

$$T(s) = K_T i_f = \frac{K_T}{R_f} V_f(s)$$

[7g]

3. Analysis of Motor Performance

Repeated here are the 4 transfer functions given by Eq. [6g] for the armature-controlled dc motor.

Inputs: Applied voltage v_a; Load torque T_L
Outputs: Motor speed ω; Current i_a

$$\left.\begin{aligned} \frac{I_a(s)}{V_a(s)} &= \frac{Js+c}{\Delta(s)} \\ \frac{I_a(s)}{T_L(s)} &= \frac{K_b}{\Delta(s)} \\ \frac{\Omega(s)}{V_a(s)} &= \frac{K_T}{\Delta(s)} \\ \frac{\Omega(s)}{T_L(s)} &= \frac{L_a s+R_a}{\Delta(s)} \end{aligned}\right\} \qquad [8a]$$

where:

$$\Delta(s) = L_a J s^2 + (R_a J + cL_a) + cR_a + K_b K_T \qquad [8b]$$

3. Analysis of Motor Performance

A. Steady-State Motor Step-Response (Section 6.6 of SD)

Apply Final Value Theorem to the transfer functions. If v_a and T_L are step functions of magnitude $\widehat{V_a}$ and $\widehat{T_L}$ respectively,

Then, $\lim_{s\to 0} I_a(s)$:

$$i_a = \lim_{s\to 0}\left[\frac{(Js+c)}{\Delta(s)}V_a(s) + \frac{K_b}{\Delta(s)}T_L(s)\right]$$

$$\Rightarrow \boxed{i_a = \frac{c\widehat{V_a} + K_b\widehat{T_L}}{cR_a + K_bK_T}} \qquad [8c]$$

Similarly, $\lim_{s\to 0} \Omega(s)$:

$$\omega = \lim_{s\to 0}\left[\frac{K_T}{\Delta(s)}V_a(s) + \frac{L_a s + R_a}{\Delta(s)}T_L(s)\right]$$

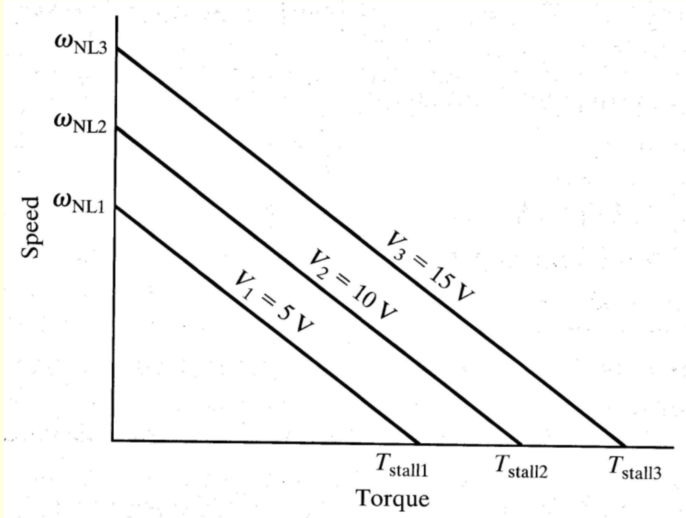
$$\Rightarrow \boxed{\omega = \frac{K_T\widehat{V_a} - R_a\widehat{T_L}}{cR_a + K_bK_T}} \qquad [8d]$$

3. Analysis of Motor Performance

A. Steady-State Motor Step-Response (Continue)

Observations:

1. From [8c] & [8d], an increase in load torque leads to an increase in current and a decrease in speed.
2. Steady-state speed is plotted against load torque T_L for various applied voltage V_a. This plot is called the load-speed curve of the motor. For any given applied voltage, it gives the maximum load torque that the motor can handle at that speed.



3. *No load speed* ω_{NL} is the motor speed when the load torque is zero. That is when: This is the highest motor speed for a given applied voltage.

$$\omega_{NL} = \frac{K_T \widehat{V}_a}{cR_a + K_b K_T}$$

4. The corresponding no load current occurs when:

$$i_a = \frac{c\widehat{V}_a}{cR_a + K_b K_T}$$

5. The *stall torque* T_{stall} occurs when the load torque causes the motor speed to be zero. That occurs when:

$$T_{stall} = \frac{K_T \widehat{V}_a}{R_a}$$

3. Analysis of Motor Performance

B. Motor Dynamic Response (Section 6.6 of SD)

Steady-state equations are often used because they are algebraic equations and therefore are easier to use. However, if we need to find out the dynamics of the electrical and/or the mechanical subsystems, we need to use the transfer functions and then take the inverse Laplace Transforms.

Example (p.361 of SD)
Motor parameters are:

$$K_T = K_b = 0.05\ \mathrm{N \cdot m/A}$$

$$c = 10^{-4}\ \mathrm{N \cdot m \cdot s/rad} \qquad R_a = 0.5\ \Omega$$

$$L_a = 2 \times 10^{-3}\ \mathrm{H} \qquad J = 9 \times 10^{-5}\ \mathrm{kg \cdot m^2}$$

From Eqs.[8a] & [8b]:

$$\frac{I_a(s)}{V_a(s)} = \frac{Js+c}{\Delta(s)}$$

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{\Delta(s)}$$

where:

$$\Delta(s) = L_a J s^2 + (R_a J + cL_a) + cR_a + K_b K_T$$

3. Analysis of Motor Performance

B. Motor Dynamic Response (Continue)

Example (Continue)

$$\frac{I_a(s)}{V_a(s)} = \frac{9 \times 10^{-5}s + 10^{-4}}{18 \times 10^{-8}s^2 + 4.52 \times 10^{-5}s + 2.55 \times 10^{-3}}$$

$$\frac{\Omega(s)}{V_a(s)} = \frac{0.05}{18 \times 10^{-8}s^2 + 4.52 \times 10^{-5}s + 2.55 \times 10^{-3}}$$

If v_a is a step function of magnitude 10 V,

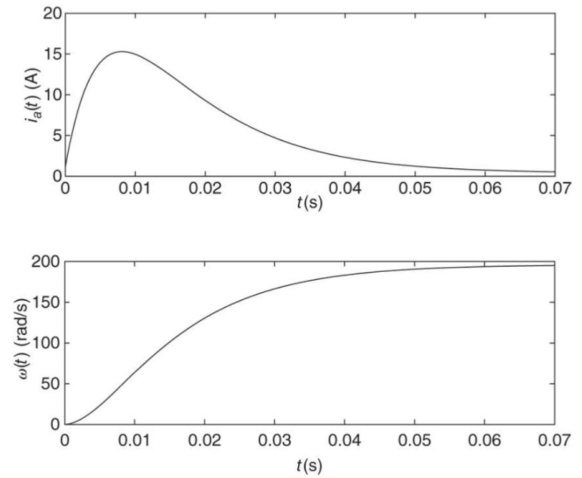
$$I_a(s) = \frac{5 \times 10^3 s + 5.555 \times 10^4}{s(s+165.52)(s+85.59)} = \frac{C_1}{s} + \frac{C_2}{s+165.52} + \frac{C_3}{s+85.59}$$

$$\Omega(s) = \frac{2.777 \times 10^6}{s(s+165.52)(s+85.59)} = \frac{D_1}{s} + \frac{D_2}{s+165.52} + \frac{D_3}{s+85.59}$$

Inverse Laplace Transform gives:

$$i_a(t) = 0.39 - 61e^{-165.52t} + 61.74e^{-85.59t}$$

$$\omega(t) = 196.1 + 210^{-165.52t} - 406e^{-85.59t}$$



4. Sensors & Electroacoustic Devices

4.1 Tachometer (Section 6.7.1 of SD)

A tachometer is used to measure velocity. One can think of a tachometer as similar in construction to a motor. Input is torque, and output is voltage. Equations are the same, just output and input has been changed.

From Eq. [6c]:

$$v_a - R_a i_a - L_a \frac{di_a}{dt} - K_b \omega = 0 \quad [9a]$$

At steady-state, $di_a/dt = 0$; $v_a = 0$ since there is no applied voltage. Therefore:

$$-R_a i_a - K_b \omega = 0 \quad [9b]$$

$$v_t = K_b \omega \quad [9c]$$

$R_a i_a$ is the voltage across the resistor which we call v_t. This means that by measuring the voltage across the resistor we can determine the velocity. If we want to determine the dynamics of the tachometer, we will need to include the mechanical subsystem which is given by Eq.[6d].

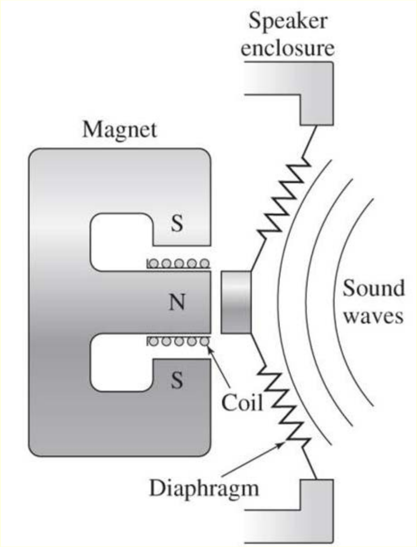
4. Sensors & Electroacoustic Devices

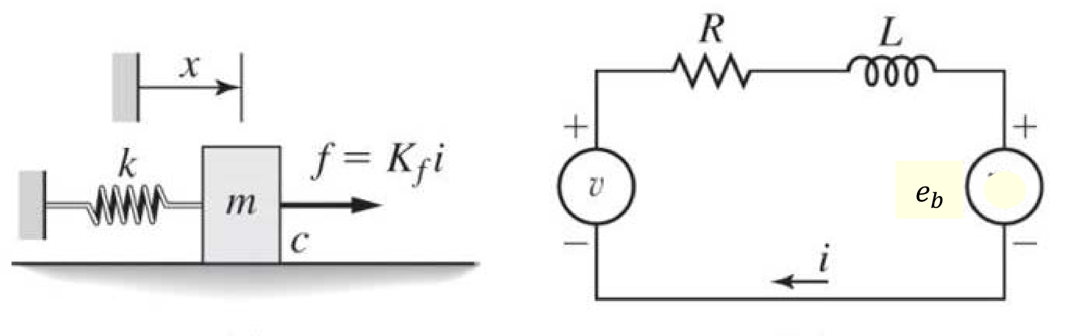
4.2 Accelerometer (Section 6.7.2 of SD)

An accelerometer is used to measure either acceleration, or displacement (not very good). We have covered this in Module 3 (Vibration Measurement Instruments)

4.3 Electroacoustic Devices (Section 6.7.5 of SD)

One example is the voice coil (cone speaker). A speaker converts electrical energy to mechanical energy – moving the coil and hence the cone. A microphone is another example. It converts vibrational sound energy into motion of a coil that then produces voltage and current in the coil.





(a) (b)

4. Sensors & Electroacoustic Devices

4.3 Electroacoustic Devices (Continue)

Using the mechanical subsystem model, Fig. (a), the magnetic force applied to the diaphragm (mass m) is due to the electrical subsystem. That force $f = nBli$ where n is the number of turns in the coil. Writing $K_f = nBl$, and from a FBD of the mass:

$$m\ddot{x} = -c\dot{x} - kx + K_f i \qquad [10a]$$

From the electrical subsystem, Fig. (b), the coil's inductance L and resistance R are in series. The coil sees back emf because it carries current and moves in a magnetic field. This back emf $e_b = K_b\dot{x}$. With an applied voltage v from the amplifier:

$$v = Ri + L\frac{di}{dt} + K_b\frac{dx}{dt} \qquad [10b]$$

The speaker is governed by Eqs. [10a] and [10b]. Taking Laplace Transform of [10a] and solving for $X(s)$:

$$X(s) = \frac{K_f}{ms^2 + cs + k} I(s) \qquad [10c]$$

Taking Laplace Transform of [10b] and solving for $I(s)$:

$$I(s) = \frac{1}{Ls + R}[V(s) - K_b s X(s)] \qquad [10d]$$

4. Sensors & Electroacoustic Devices

4.3 Electroacoustic Devices (Continue)

From Eqs. [10c] and [10d], eliminate $I(s)$, and then solve for the transfer function

$$X(s) = \frac{K_f}{ms^2 + cs + k} \frac{1}{Ls + R} [V(s) - K_b s X(s)]$$

$$X(s) \left\{ 1 + \frac{K_f K_b s}{D} \right\} = \frac{K_f}{D} V(s)$$

where:

$$D = [ms^2 + cs + k](Ls + R)$$

$$\frac{X(s)}{V(s)} = \frac{K_f}{D\left\{ 1 + \frac{K_f K_b s}{D} \right\}} = \frac{K_f}{D + K_f K_b s}$$

$$\Rightarrow \quad \frac{X(s)}{V(s)} = \frac{K_f}{mLs^3 + (cL + mR)s^2 + \left(kL + cR + K_f K_b\right)s + kR}$$

5. Matlab Applications

Matlab Functions for General Programming

1. global
2. plot
3. subplot

Matlab Functions for Analysis

1. lsim
2. step
3. tf
4. ss
5. ode45