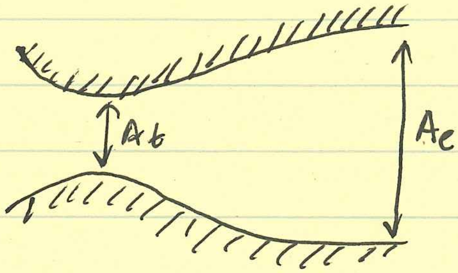
Homework 11

Problem 1



$\frac{A_e}{A_t} = 1.616$

$P_e = 0.947\,atm$ $P_0 = 1\,atm$

Isentropic flow

$M_t = ?$ $P_t = ?$

Is the flow at the throat sonic or not? If $\frac{A_e}{A^*} = \frac{A_e}{A_t}$ → then $A_t = A^*$ and flow is sonic at the throat.

~~(App A)~~ $\frac{P_0}{P_e} = \left(\frac{1}{0.947}\right) = 1.056$

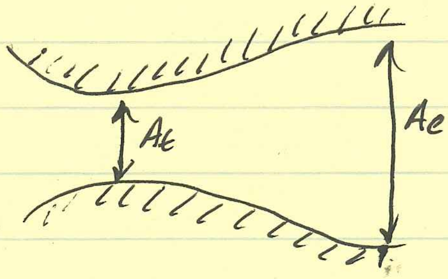
(App. A) → $M_e = 0.28$ $\frac{A_e}{A^*} = 2.166$ (not sonic at the throat)

$$\frac{A_t}{A^*} = \left(\frac{A_t}{A_e}\right)\left(\frac{A_e}{A^*}\right) = \left(\frac{1}{1.616}\right)\left(2.166\right) = 1.34$$

$\frac{A_t}{A^*} = 1.34$ → (App. A) → $\boxed{M_t = 0.5}$ $\frac{P_0}{P_t} = 1.186$

$P_t = \left(\frac{P_t}{P_0}\right) P_0$ → $P_t = \left(\frac{1}{1.186}\right) 1\,atm$ $\boxed{P_t = 0.843\,atm}$

Problem 2



$T_0 = 288\ K$ $M_t = 0.5$
$A_t = 0.3\,m^2$ $P_t = 0.843\ atm$

Calculate $\dot{m}$.

$\dot{m} = \rho A u = \rho_t A_t u_t$

$u_t = a_t M_t$ $a_t = \sqrt{\gamma R T_t} \longrightarrow$ find $\frac{T_0}{T_t} \rightarrow$ (App. A)

$\rho_t = \frac{P_t}{R T_t}$

For $M_t = 0.5 \rightarrow \frac{T_0}{T_t} = 1.05$

$T_t = \left(\frac{T_t}{T_0}\right) T_0 = \left(\frac{1}{1.05}\right) 288\ K \rightarrow T_t = 274.3\ K$

$\rho_t = \frac{(0.843\ atm)(1.01 \times 10^5\ N/m^2/atm)}{(287)(274.3)}$

$\rho_t = 1.08\ \frac{kg}{m^3}$

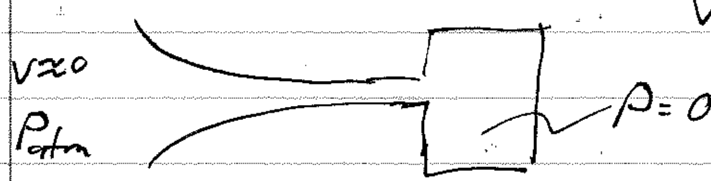
$a_t = \sqrt{\gamma R T_t} = \sqrt{(1.4)(287)(274.3)} \rightarrow a_t = 332\ m/s$

$u_t = a_t M_t = (332\ m/s)(0.5) \rightarrow u_t = 166\ m/s$

$\dot{m} = \rho_t A_t u_t = \left(1.08\ \frac{kg}{m^3}\right)(0.3\ m^2)(166\ m/s)$

$\boxed{\dot{m} = 53.8\ kg/s}$

Problem 9.2.2



What is the maximum air speed that can be obtained?

Energy Equation: $$\underbrace{\frac{V^2}{2} + \frac{a^2}{\gamma - 1}}_{\text{flow through duct}} = \underbrace{\frac{a_0^2}{\gamma - 1}}_{\text{duct inlet}}$$

V_{max} when $T \to 0$ and $a \to 0$:

$$\frac{V_{max}^2}{2} = \frac{a_0^2}{\gamma - 1}$$

$$V_{max} = \sqrt{\frac{2}{\gamma - 1}}\, a_0$$

At sea-level $\to a_0 = \sqrt{\gamma R T_0}$ $\quad T_0 = 288\ K$

$$a_0 = 340\ m/s$$

$$\boxed{V_{max} = 760.3\ m/s}$$

If $v_{max} = 1000\ m/s \longrightarrow$ what must T_0 be?

$$V_{max} = \left(\sqrt{\frac{2}{\gamma - 1}}\right)\sqrt{\gamma R T_0} \longrightarrow \left(\frac{2\gamma R}{\gamma - 1}\right) T_0 = V_{max}^2$$

$$T_0 = \frac{V_{max}^2 (\gamma - 1)}{2\gamma R}$$

$$\boxed{T_0 = 497.8\ K}$$

Problem 9.3.3

Test section $\rightarrow M = 4$

Isentropic flow

$\rho_\infty = 1.044\ kg/m^3$

$h = 1650 m$

What is the density at the test section?

$$\rho_0 = \rho_\infty \quad \rightarrow \quad \frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

$$\text{Tables} \rightarrow \frac{\rho}{\rho_0} = 0.02766$$

$$\rho = 0.02766\left(1.044\ kg/m^3\right)$$

$$\boxed{\rho = 0.0289\ kg/m^3}$$

Problem 9.7.1

$h = 2\,km \rightarrow P_\infty = 7.948 \times 10^{+4} \rightarrow 79.48\,kPa$

(a) $V_{inlet} = 23.24\,m/s$

Mach number $\rightarrow$ $M = \dfrac{V_{inlet}}{a_{inlet}}$ $\quad a_{inlet} = \sqrt{\gamma R T_{inlet}}$

$T_{inlet} = T_{atm} = 2\,^\circ C = 275\,K \quad a_{inlet} = 332\,m/s$

$$\boxed{M_1 = 0.07}$$

$P_0 = P_\infty + \frac{1}{2}\rho_\infty V^2 \qquad \rho_\infty = 1.007\,kg/m^3$

$P_0 = 79.48\,kPa + \frac{1}{2}(1.007\,kg/m^3)(23.24\,m/s)^2$

$= 79.48\,kPa + 271.93\,\dfrac{kg}{m\,s^2}$

$1\,Pa = 1\,\dfrac{N}{m^2} = 1\,\dfrac{kg\,m}{s^2\,m^2} = 1\,\dfrac{kg}{m\,s^2}$

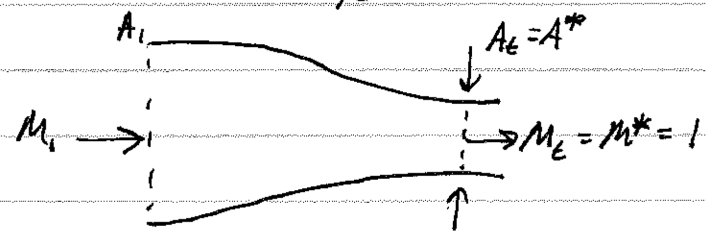
$P_0 = 79.48\,kPa + 0.272\,kPa$

$$\boxed{P_0 = 79.752\,kPa}$$

(b) $A_1 = 1\,m^2$ Isentropic flow $A_t = A^*$

Relation: $$\left(\frac{A_1}{A^*}\right)^2 = \frac{1}{M_1^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M_1^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

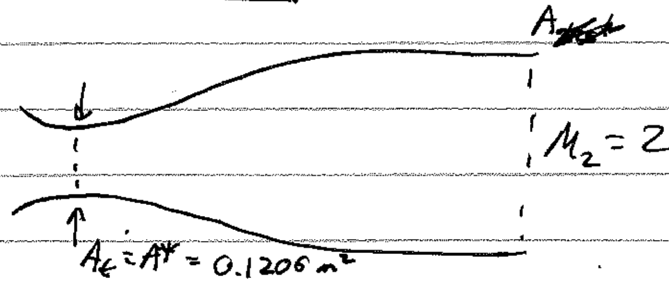
Or from Tables



$@ M_1 = 0.07 \rightarrow \dfrac{A^*}{A_1} = 0.1206$

$$\boxed{A^* = 0.1206\ m^2}$$

(c)

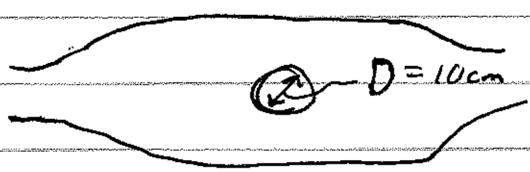


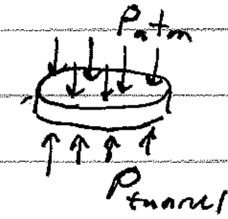
Use tables $\rightarrow$ $@ M_2 = 2 \rightarrow \dfrac{A^*}{A_2} = 0.5926$

$$A_2 = \left(\frac{1}{0.5926}\right)(0.1206\ m^2)$$

$$\boxed{A_2 = 0.2035\ m^2}$$

(d)





$F_{net} = (P_{atm} - P_{tunnel}) A_{window}$ $\quad A_{window} = \dfrac{\pi D^2}{4} = \dfrac{\pi (0.1m)^2}{4}$

$$A_{window} = 0.00785\ m^2$$

$P_{atm} = \cancel{\ } \ 79.48\ kPa$

$P_{tunnel} = \left(\dfrac{P_2}{P_0}\right) P_0$ $\qquad @ M_2 = 2 \rightarrow \dfrac{P_2}{P_0} = 0.1278$

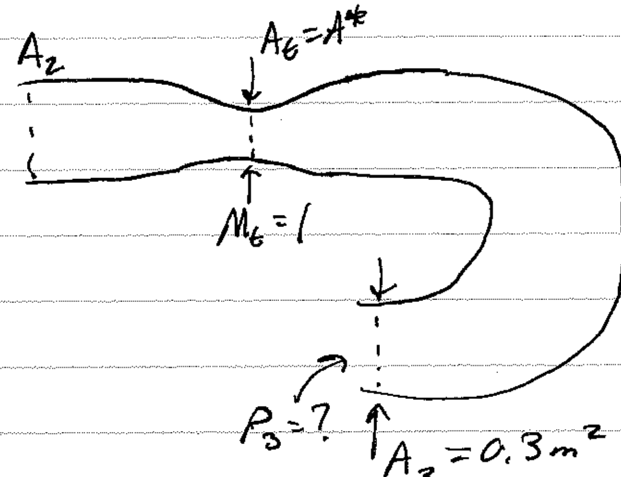
$P_{tunnel} = (0.1278)(79.752\ kPa)$

$P_{tunnel} = 10.1923\ kPa$

$$F_{net} = (79.48 - 10.1923\ kPa)(0.00785\ m^2)$$

$$\boxed{F_{net} = 544N}$$

(e)



$$\frac{A^*}{A_3} = \left(\frac{0.1206\,m^2}{0.3\,m^2}\right) \rightarrow \frac{A^*}{A_3} = 0.402 \rightarrow M_3 \approx 0.24$$

subsonic isentropic flow table

$$\text{and} \rightarrow \frac{P_3}{P_0} = 0.9607$$

$$P_3 = \left(\frac{P_3}{P_0}\right) P_0 \rightarrow \boxed{P_3 = 76.618\ kPa}$$

If there is an entropy increase in the test section, the total pressure will decrease and $\frac{P_{o,1}}{P_{o,2}} > 1$

$$\text{then} \quad P_3 = \left(\frac{P_3}{P_{o,2}}\right)\overbrace{\left(\frac{P_{o,2}}{P_{o,1}}\right)}^{<1}\left(P_{o,1}\right)$$

$$\text{so} \quad \boxed{P_{3,\text{non-isentropic}} < P_{3,\text{isentropic}}}$$

(f) At sea level, $h = 0\ km \rightarrow a = 340\ m/s$

$T_\infty = 288\ K$

$P_\infty = 101.32\ kPa$

$\rho_\infty = 1.226\ kg/m^3$

$$M_1 = \frac{V_{inlet}}{a} \Rightarrow M_1 = 0.0684$$

$A_t = 0.1206\ m^3$

$A_1 = 1\ m^2$

given $M_1 \rightarrow$ find $\rightarrow \frac{A^*}{A_1} = 0.1172$

$M_1 = 0.06 \rightarrow \frac{A^*}{A_1} = 0.1035$ } interpolate
$M_1 = 0.07 \rightarrow \frac{A^*}{A_1} = 0.1206$

Find $A^* = 0.1172\ m^2 \longrightarrow \frac{A^*}{A_t} = \left(\frac{0.1172\ m^2}{0.1206\ m^2}\right)$

$$\frac{A^*}{A_t} = 0.9716$$

Given $\frac{A^*}{A_t} = 0.9716 \rightarrow$ find $\boxed{M_t \approx 0.825}$

For choked flow at sea level $\rightarrow \frac{A^*}{A_1} = 0.1206$

$$M_1 = 0.07$$

Since $a = 340\ m/s \longrightarrow V_1 = M_1 a$

$$\boxed{\begin{aligned} V_1 &= 23.8\ m/s \\ &= 53.2\ mph \end{aligned}}$$