1) If we convert the affine geodesic equation to one with respect to $\lambda$, we get

$$\frac{D}{d\sigma}\frac{dx^\mu}{d\sigma} = \frac{d\lambda}{d\sigma}\frac{D}{d\lambda}\left(\frac{dx^\mu}{d\lambda}\frac{d\lambda}{d\sigma}\right) = \frac{d\lambda}{d\sigma}\left(\frac{D}{d\lambda}\left(\frac{dx^\mu}{d\lambda}\right)\frac{d\lambda}{d\sigma} + \frac{D}{d\lambda}\left(\frac{d\lambda}{d\sigma}\right)\frac{dx^\mu}{d\lambda}\right) = \left(\frac{d\sigma}{d\lambda}\right)^{-1}\left(\alpha(\lambda)\frac{dx^\mu}{d\lambda}\left(\frac{d\sigma}{d\lambda}\right)^{-1} + \frac{dx^\mu}{d\lambda}\left(\frac{d\sigma}{d\lambda}\right)^{-2}\frac{d^2\sigma}{d\lambda^2}\right) = 0$$

where the last step above uses the first equation and writes all terms with respect to $\lambda$.
Define $g(\lambda) \equiv \frac{d\sigma}{d\lambda}$ and rewrite the equation above as $0 = \frac{1}{g(\lambda)}\left(\frac{\alpha(\lambda)}{g(\lambda)} + \frac{1}{(g(\lambda))^2}\frac{dg}{d\lambda}\right)\frac{dx^\mu}{d\lambda}$.
Thus, $\frac{\alpha(\lambda)}{g(\lambda)} + \frac{1}{(g(\lambda))^2}\frac{dg}{d\lambda} = 0$ and $\alpha(\lambda)g(\lambda) = \frac{dg}{d\lambda}$. Solving for $g(\lambda)$, $\frac{dg}{g} = \alpha\, d\lambda \Rightarrow \int\frac{dg}{g} = \int \alpha\, d\lambda$
$\ln g = \int \alpha\, d\lambda + C \Rightarrow g = Ce^{\int \alpha\, d\lambda}$. Now remember that $g(\lambda) = \frac{d\sigma}{d\lambda}$, so integrating once again,
$\boxed{\sigma(\lambda) = C\int\left(e^{\int \alpha(\lambda)\, d\lambda}\right)d\lambda}$ where $C$ is an arbitrary constant. This works for any function $\alpha(\lambda)$, so this proves $\sigma(\lambda)$ can always exist.

2) a) For a point $p$ on the surface of a unit sphere with an orthonormal tangent vector $\hat{k}^\mu$, you can form a great curve using the parameterization of a circle. Here, $x^\mu(\lambda) = \cos\lambda\, p^\mu + \sin\lambda\, \hat{k}^\mu$. To check if this behavior is expected, consider $x^\mu(0) = p^\mu$. This motivates mapping $p^\mu$ as $x$ and $\hat{k}^\mu$ as $y$ for a unit circle.
Now to prove $x^\mu(\lambda)$ is a geodesic, evaluate $\frac{d^2x^\mu}{d\lambda^2} = -\cos\lambda\, p^\mu - \sin\lambda\, \hat{k}^\mu = -x^\mu(\lambda)$. This means $\frac{d^2x^\mu}{d\lambda^2}$ is proportional to $x^\mu(\lambda)$ itself, which is a curve of vectors normal to the surface of the sphere, meaning $\frac{d^2x^\mu}{d\lambda^2}$ has no tangential component and $x^\mu(\lambda)$ is a geodesic of the sphere.
To summarize, $\boxed{x^\mu(\lambda) = \cos\lambda\, p^\mu + \sin\lambda\, \hat{k}^\mu}$ describes a great curve on a sphere and is the geodesic for point $p$

b) The metric for the surface of a unit sphere is $ds^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$. Mapping this to Euclidian space, $x = \sin\theta\cos\varphi$ and $y = \sin\theta\cos\varphi$. Evaluating in a local space around the north pole ($\theta = 0$) allows us to use the first-order approximation $\sin\theta \approx \theta$ to define local RNC. Thus, $x \approx \theta\cos\varphi$ and $y \approx \theta\sin\varphi$, so $dx = d\theta\cos\varphi - d\varphi\,\theta\sin\varphi$ and $dy = d\theta\sin\varphi + d\varphi\,\theta\cos\varphi$. Therefore,

$$dx^2 + dy^2 = d\theta^2\cos^2\varphi + d\varphi^2\theta^2\sin^2\varphi - \cancel{d\theta\, d\varphi\, \theta\cos\varphi\sin\varphi} + d\theta^2\sin^2\varphi + d\varphi^2\theta^2\cos^2\varphi + \cancel{d\theta\, d\varphi\, \theta\cos\varphi\sin\varphi}$$

$= d\theta^2 + \theta^2 d\varphi^2$. Note that $x = \theta\cos\varphi$ and $y = \theta\sin\varphi$ form a polar mapping, so refering back to the spherical coordinates, $ds^2 = d\theta^2 + \sin^2\theta\, d\varphi^2 = dx^2 + dy^2 + (\sin^2\theta - \theta^2)\, d\varphi^2$
$\approx dx^2 + dy^2 + \left(\theta^2 - \frac{2\theta^4}{6} + \mathcal{O}(\theta^6) - \theta^2\right)d\varphi^2 = dx^2 + dy^2 - \frac{\theta^4}{3}d\varphi^2 + \cancel{\mathcal{O}(\theta^6)\, d\varphi^2}$. Note that $\theta^2 = x^2 + y^2$ and $(x\,dy - y\,dx) = \theta^2 d\varphi$, so $d\varphi^2 = \frac{(x\,dy - y\,dx)^2}{(x^2+y^2)^2}$ and $-\frac{\theta^4}{3}d\varphi^2 = -\frac{(x^2+y^2)^2}{3}\frac{(x\,dy - y\,dx)^2}{(x^2+y^2)^2} = -\frac{(x\,dy - y\,dx)^2}{3}$
Therefore, spherical $\boxed{ds^2 = dx^2 + dy^2 - \frac{1}{3}(x\,dy - y\,dx)^2}$, which is Euclidian space with quadratic terms $(dx^2, dx\,dy, dy^2)$

3) Using the diagonal Christoffel equations (3.213)-(3.216), and the metric $g = \begin{pmatrix} -r^2 & 0 \\ 0 & 1/r^2 \end{pmatrix}$, we have

$$\Gamma^r_{tt} = -\frac{1}{2}(g_{rr})^{-1}\partial_r g_{tt} = -\frac{1}{2}r^2\frac{\partial}{\partial r}(-r^2) = \boxed{r^3}$$

$$\Gamma^t_{rr} = -\frac{1}{2}(g_{tt})^{-1}\partial_t g_{rr} = -\frac{1}{2}\left(-\frac{1}{r^2}\right)\frac{\partial}{\partial t}\left(\frac{1}{r^2}\right) = \boxed{0}$$

$$\Gamma^r_{tr} = \Gamma^r_{rt} = \partial_t\left(\ln\sqrt{|g_{rr}|}\right) = \frac{\partial}{\partial t}\ln\frac{1}{r} = \boxed{0}$$

$$\Gamma^t_{rt} = \Gamma^t_{tr} = \partial_r\left(\ln\sqrt{|g_{tt}|}\right) = \frac{\partial}{\partial r}\ln r = \boxed{\frac{1}{r}}$$

$$\Gamma^r_{rr} = \partial_r\left(\ln\sqrt{|g_{rr}|}\right) = \frac{\partial}{\partial r}\ln\frac{1}{r} = -\frac{\partial}{\partial r}\ln r = \boxed{-\frac{1}{r}}$$

$$\Gamma^t_{tt} = \partial_t\left(\ln\sqrt{|g_{tt}|}\right) = \frac{\partial}{\partial t}\ln r = \boxed{0}$$

To find the Riemann tensor, we use (3.113).

$$R^r{}_{trt} = \partial_r \Gamma^r_{tt} - \partial_t \Gamma^r_{rt} + \Gamma^r_{r\lambda}\Gamma^\lambda_{tt} - \Gamma^r_{t\lambda}\Gamma^\lambda_{rt} = \partial_r\Gamma^r_{tt} - \cancelto{0}{\partial_t\Gamma^r_{rt}} + \Gamma^r_{rr}\Gamma^r_{tt} - \cancelto{0}{\Gamma^r_{tr}}\cancelto{0}{\Gamma^r_{rt}} + \cancelto{0}{\Gamma^r_{rt}}\cancelto{0}{\Gamma^t_{tt}} - \Gamma^r_{tt}\Gamma^t_{rt}$$

$$= \partial_r r^3 + \left(-\frac{1}{r}\right)(r^3) - (r^3)\left(\frac{1}{r}\right) = 3r^2 - r^2 - r^2 = r^2$$

If we then consider symmetries, it helps to lower the first index, so

$$R_{rtrt} = g_{r\lambda}R^\lambda{}_{trt} = g_{rr}R^r{}_{trt} + \cancelto{0}{g_{rt}}R^t{}_{trt} = \left(\frac{1}{r^2}\right)(r^2) = 1$$

Since $R_{\mu\nu\sigma\rho} = R_{\sigma\rho\mu\nu} = -R_{\rho\sigma\mu\nu}$, $R_{rttr} = R_{trrt} = -R_{rtrt} = -1$ $\quad (R_{\mu\nu\sigma\rho} = R_{\sigma\rho\mu\nu})$ 2

Also, $R_{\rho[\sigma\mu\nu]} = 0$, so $R_{rtrt} + R_{rrtt} + R_{rttr} = 1 + R_{rrtt} - 1 = 0$. Thus, $R_{rrtt} = R_{ttrr} = 0$

Finally, the same cyclic equation means $R_{trtr} = -R_{trrt} - R_{ttrr} = 1$

Therefore,

$$\boxed{\begin{aligned} R^t{}_{rtr} &= g^{tt}R_{trtr} = -\frac{1}{r^2} \\ R^r{}_{ttr} &= -R^r{}_{trt} = -r^2 \\ R^t{}_{rrt} &= -R^t{}_{rtr} = \frac{1}{r^2} \\ R^r{}_{rtt} &= R^t{}_{trr} = 0 \end{aligned}}$$

and all Riemann tensors with more than 2 r or t terms evaluates to zero due to the symmetry of the Christoffel symbols or the Riemann tensor definition.

Now, the Ricci tensor is given as $R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$, so $R_{rr} = R^\lambda{}_{r\lambda r} = \cancelto{0}{R^r{}_{rrr}} + R^t{}_{rtr} = \boxed{-\frac{1}{r^2}}$

$R_{rt} = R_{tr} = R^\lambda{}_{r\lambda t} = 0$ and $R_{tt} = R^\lambda{}_{t\lambda t} = R^r{}_{trt} + \cancelto{0}{R^t{}_{ttt}} = \boxed{r^2}$

so the Ricci tensor is $\boxed{R_{\mu\nu} = \begin{pmatrix} r^2 & 0 \\ 0 & -1/r^2 \end{pmatrix}}$

Finally, the Ricci Scalar is given as $R = R^\mu{}_\mu = g^{\mu\nu}R_{\mu\nu} = g^{rr}R_{rr} + \cancelto{0}{2g^{rt}R_{rt}} + g^{tt}R_{tt}$

$$= (r^2)\left(-\frac{1}{r^2}\right) + \left(-\frac{1}{r^2}\right)(r^2) = \boxed{-2}$$