Magical Square

t.blokland.1998

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1 Issue

5	b	c
d	e	4
g	h	6

In the this square we want to find some values for the unknown variables to satisfy the restriction of a "Magical Square". This will be solved with row reduction.
The restriction of a Magical Square is:
There is a variable T for which the sums of each column, the sums of each row and the sums of each diagonal have the same value as T.

2 Approach

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \\ j \\ t \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ -6 \\ -5 \\ 0 \\ -10 \\ -11 \\ 0 \\ 0 \end{bmatrix}$$

The following operations have been applied to the matrix, in order to get as close as possible to the identity matrix:
R1 = R1-R6
R2 = R2-R7

R5 = R5-R3-R8+R6
R3 = (R3-R5)+R1+R7
R4 = R4-R3(New)
R6 = R6
R7 = R7
R8 = R8-R7-R3(New)
R9 = (R9-3R7)/2

All variables T are brought to zero with R9
R6 = R6+R9
R8 = R8-R9
R7 = R7+R9
R3 = R3+R9

The rows are ordered to finish the Echelon Form:
In order from top to bottom: R1,R6,R8,R4,R2,R7,R3,R5,R9

Now the matrix is capable of defining every unknown variable, without using other unknown variables:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \\ j \\ t \end{bmatrix} = \begin{bmatrix} 5 \\ 6.5 \\ 6.5 \\ 7 \\ 7 \\ 5.5 \\ 4.5 \\ 6 \\ 16.5 \end{bmatrix}$$

3 Result

5	5	6.5
7	5.5	4
4.5	6	6

The Magical Square can be solved with the matrix. The sum of the columns, rows and diagonals are 16.5, so the Magic Square is correct.