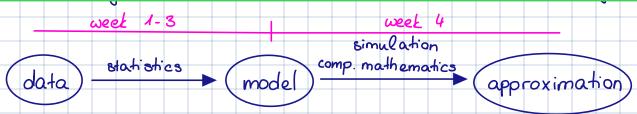
## Lecture Week 4

Random number generation and Monte Carlo methods with Python



Recall on probability theory

- (Ω, A, P) probability space

  Lo probability measure

  b 6- algebra

  o set
- · X: Ω → R random variable

• 
$$P_{\kappa}(B) = P(E\omega \in \Omega, X(\omega) \in BJ)$$
 image measure  $B \in B(R)$ 

$$P_{x}(B) = \int_{B} g(x) dx$$
,  $G \in B(R)$ 

$$E[X] = \int_{\Omega} x dP = \int_{-\infty} x g(x) dx$$

Monte Carla methods

Strong law of large numbers

• (U(m), me N) sequence of independent 21([0,1])-distributed random variables

strong law of large numbers

· therefore for M sufficiently large

$$\frac{1}{H} \stackrel{\mathcal{E}}{=} f(u^{(i)}) \approx \mathcal{E}[f(u)]$$

Algorithm I

• Simulate 4 5 f(U'i)

· given: MEN P: R- R

 $\theta \leftarrow 0$ for k = 0, ..., M - 1 do generate  $U \sim \mathcal{U}([0,1])$ 

Strong law of large numbers

• /E[ /1/4 (U1, U2)] = & A = E(x,y) ∈ [0,1]2, y ≤ f(x)}

o ((U, U, U, w)), me N) sequence of independent  $2((c, 1)) \times 2((c, 1))$  distributed random variables

strong law of large numbers

$$\frac{1}{11} \sum_{m=1}^{M} I_{A} \left( U_{1}^{(m)}, U_{2}^{(m)} \right) \longrightarrow \mathbb{E} \left[ I_{A} \left( U_{1}, U_{2} \right) \right] = \Im$$

```
· therefore for M sufficiently large
              ± Σ (u, (u, m)) ≈ E[f(u)]
 Algorithm II (Hit-or- Miss)
· Simulate # 5 1 (U1m) U2m)
 o given: McN, P:R->R
         count & O
         for m=0, ... M-1 do
               generate U, Uz ~ U(EO,1)
               if Uz = f(Uz) then
                     count 4 count + 1
               end if
         end for
         return count/M
Speed of convergence
    IE[ [ [ (u)] - 1 Σ f(u'm)] 2] 1/2
        = I \in \left[ \left( \frac{1}{M} \sum_{m=1}^{M} \left( I \in \left[ f(u) \right] - f(u^{(m)}) \right) \right]^{2} \right]^{4/2}
        =\frac{1}{H} \mathbb{E} \left[ \left( \sum_{m=1}^{H} \left( f\left( U^{(m)} \right) - \mathbb{E} \left[ f\left( U^{(m)} \right) \right] \right) \right)^{2} \right]^{1/2}
        = # Var [ = f (U'm)] 1/2
       = \( \frac{1}{\Sigma} \left( \Gamma^{(m)} \) \] \) \( \frac{1}{2} \)
        = 1 (M Var [f(u)])1/2
        = Im (Var [f(u)]) 1/2
```