Modeling and Analyzing the Effects of Delays - Consensus of Networks of Multi-Agent Systems

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Testing, Evaluation, and Control of Heterogeneous Large-scale Systems of Autonomous Vehicles

Overview

- Introduction
  - Consensus Problem
  - Applications
- Objective
- Time Delay
  - Methods
  - Modeling
  - Effects
- Network Topologies
  - Convergence Speed
  - Stability
  - Sensitivity
- Experiment
  - Results
- Summary
- Future Work

Introduction

[Crutchfield Team,2015]

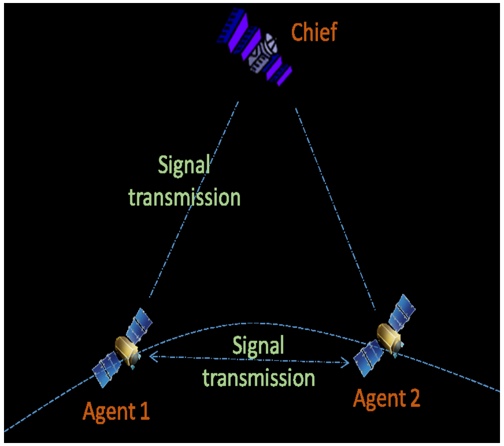
- Networks of Multi-agent systems
  - » Better performance than single agents
  - » Less setbacks or failures
  - » Improvement in outcomes of certain tasks

Consensus Problem

- Decision/ Task agreement
- Communication
  - » Issues
  - » Delayed information
  - » Time Delay
- Control Algorithm
  - » Coordinated and cooperative control

Applications

- Formation Control
  - » Leader and follower
- Attitude Alignment
  - » Satellite formation
- Rendezvous
  - » Sensor detection
- Flocking
  - » Aerial robots



[Sun Yi, 2015]

[Tobias Niebuhr, 2014]

Objective

- Analyze the stability of delayed systems
- Analyze the sensitivity and convergence speeds w.r.t delay vs topology
- Experiment in real-time

Methods for handling delays

- » Padè approximation
- » Psuedo-delay
- » Smith predictor
- » Bifurcation analysis
- » Runge-Kutta
- » Nyquist Theorem
- » Lyapunov method

Modeling Delay

- Analytically
  - Delay Differential Equation (1^{st} order)
    - Transcendental characteristic equation of DDE

$$S + A + A_d e^{-\tau S} = 0$$

- Delay $\approx e^{-\tau s}$
  - infinite number of roots
- No possible solution by standard methods
- Can be approximated

Lambert W Function

- Delay Differential Equation (1st order)

$$\dot{x} = Ax + A_d x(t-\tau)$$

- Lambert W Function

$$Z = W_k(Z)e^{W_k(Z)} = W(Ze^Z)$$

  - Transcendental characteristic equation of DDE

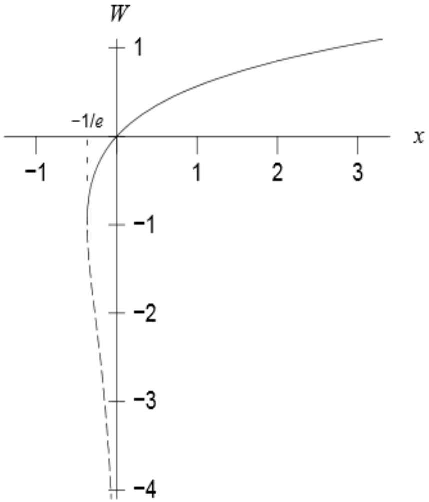
$$S + A + A_d e^{-\tau S} = 0$$

  - Multiply by $\tau e^{\tau(S+A)}$

$$\tau(S+A)e^{\tau(S+A)} = -A_d \tau e^{A\tau}$$

  - Use Lambert W definition $z = W(z)e^{W(z)}$

$$W\left(\tau(S+A)e^{\tau(S+A)}\right) = W(-A_d \tau e^{A\tau})$$
$$(S+A)\tau = W(-A_d \tau e^{A\tau})$$



[Sean Stewart, 2006]

Asl, F.M., and Ulsoy, A.G., "Analysis of a System of Linear Delay Differential Equations," *ASME J. Dynamic Systems, Measurement and Control*, Vol. 125, No.2, pp 215-223, 2003

Analyzing Stability

- characteristic spectrum

$$S = \frac{1}{\tau} W(-A_d \tau e^{A\tau}) - A$$

- $\lambda_i = -a \pm bi; a \rightarrow \ real\ and\ bi \rightarrow imaginary$
- System Stability- depends on real part of system roots
  Stable: if all $a < 0$
  Neutrally Stable: if $a = 0\ and\ bi < 0$
  Unstable: if any $a > 0$

Jordan Form

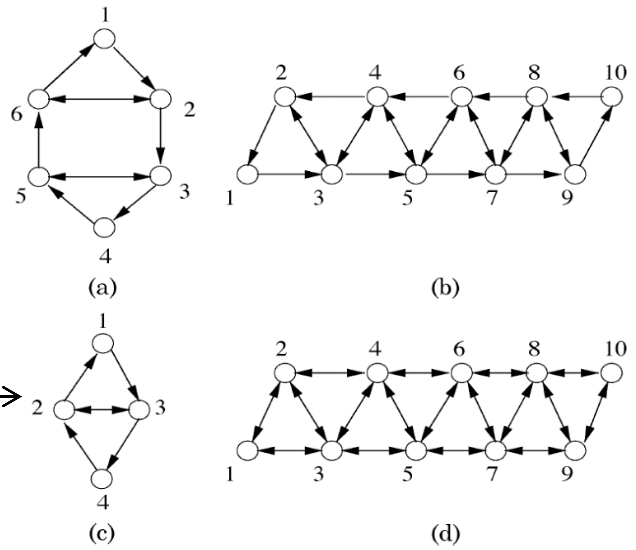
$$\mathbf{W}_k(J_{ki}(\hat{\lambda}_i)) = \begin{bmatrix} W_k(\hat{\lambda}_i) & W_k'(\hat{\lambda}_i) & \cdots & \frac{1}{(m-1)!} W_k^{(m-1)}(\hat{\lambda}_i) \\ 0 & W_k(\hat{\lambda}_i) & \cdots & \frac{1}{(m-2)!} W_k^{(m-2)}(\hat{\lambda}_i) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & W_k(\hat{\lambda}_i) \end{bmatrix}$$

Sun Yi, P. Nelson and A. Ulsoy. Analysis and Control Using the Lambert W Function Time Delay System

Approach

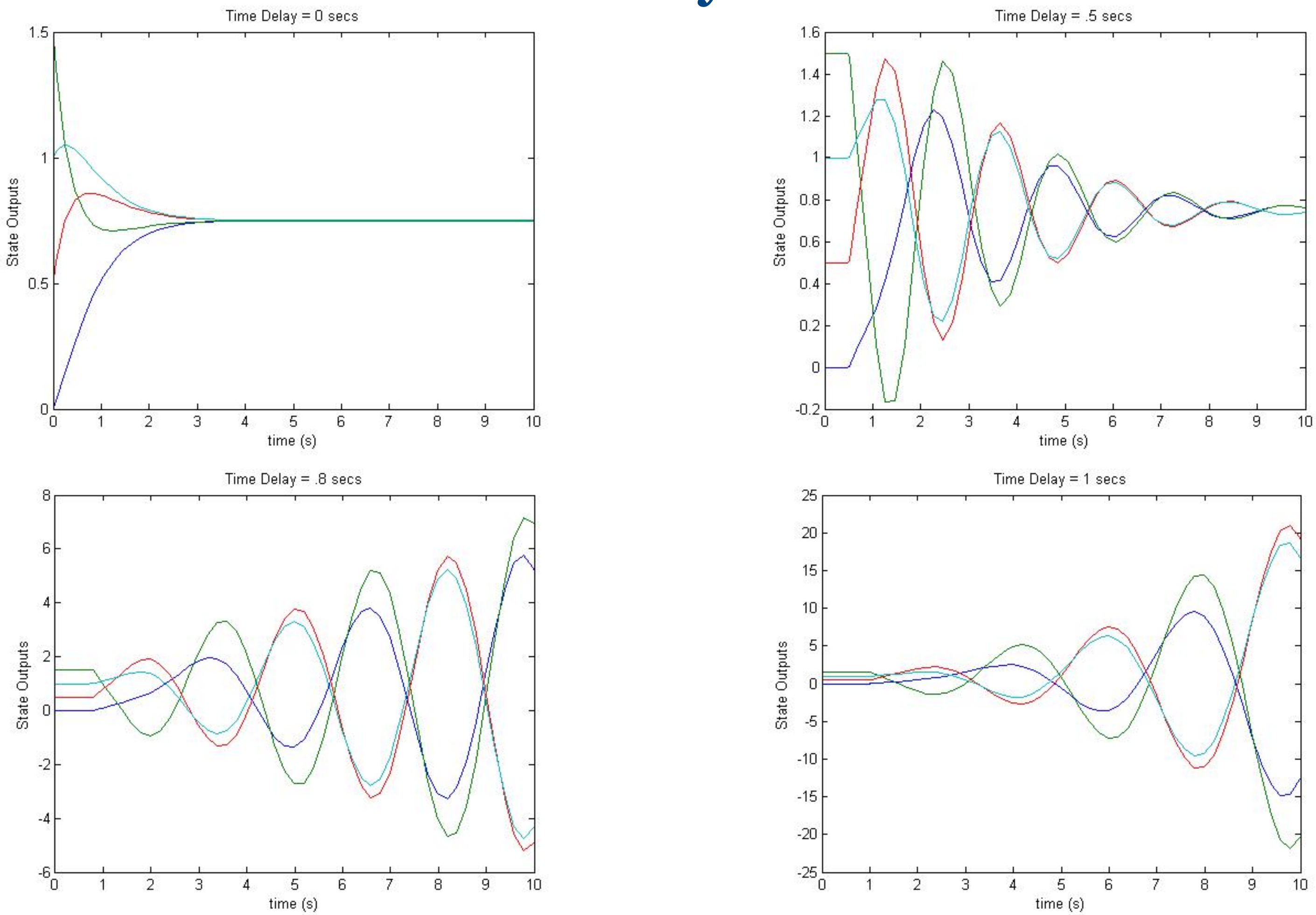
- Graph Theory
  - » Representation of how each agent communicates

- $L = \begin{bmatrix} 1 & 0 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & -1 & 0 & 1 \end{bmatrix}$



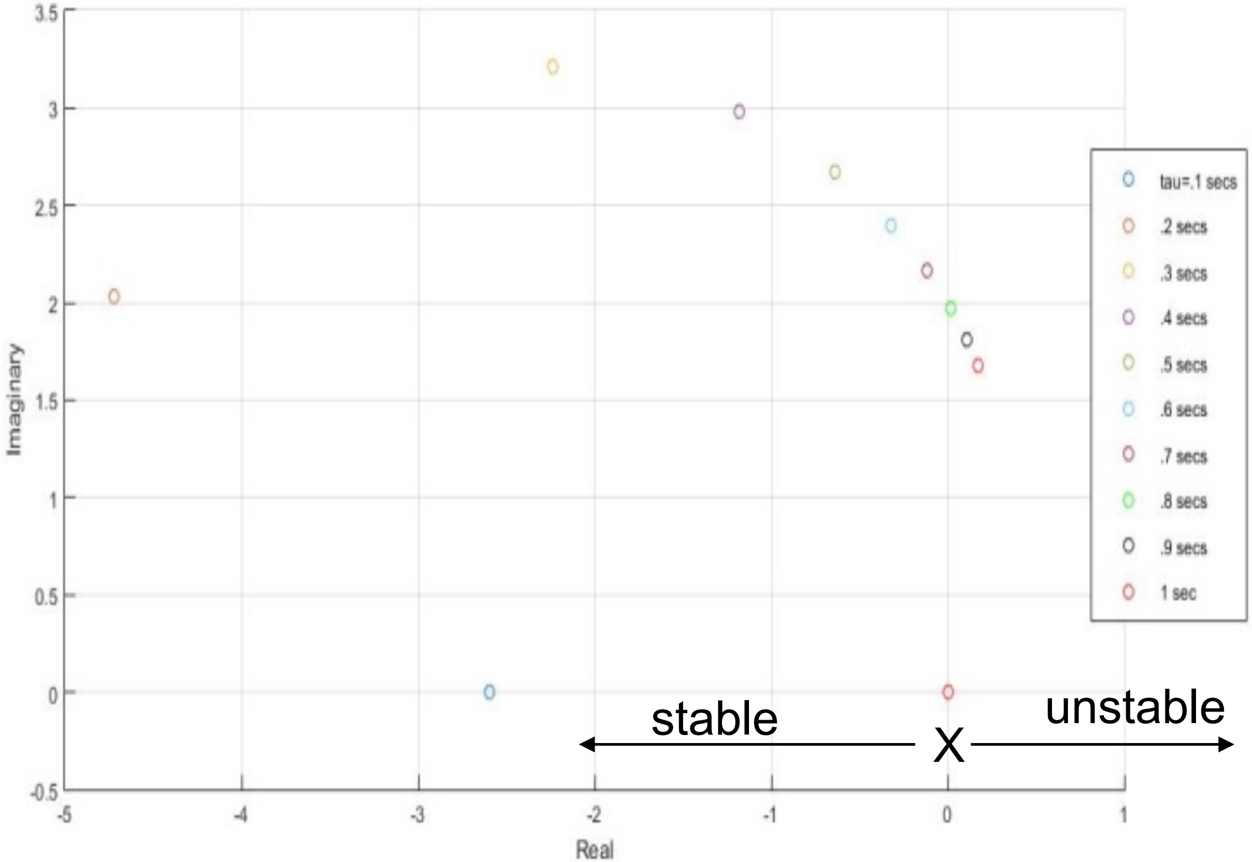
$\lambda_1 = 0;\ always$

Effects of Time Delay



- States become unstable as delay increase

$$S = \frac{1}{\tau} W(L\tau)$$

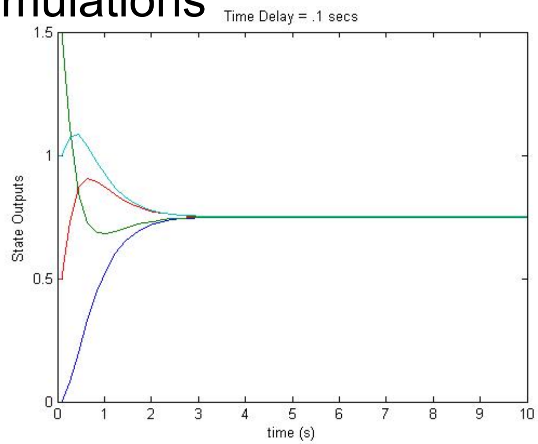


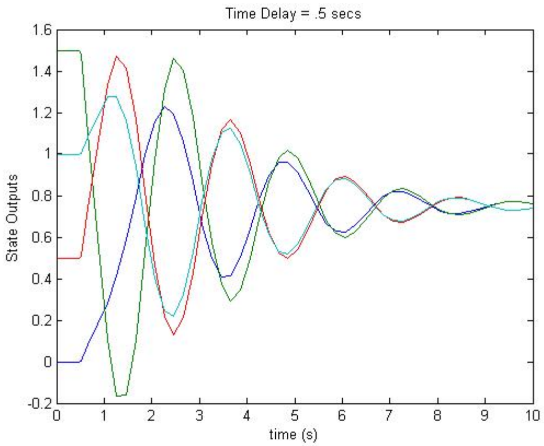
- Agrees with simulated results, adding quantity to the stability

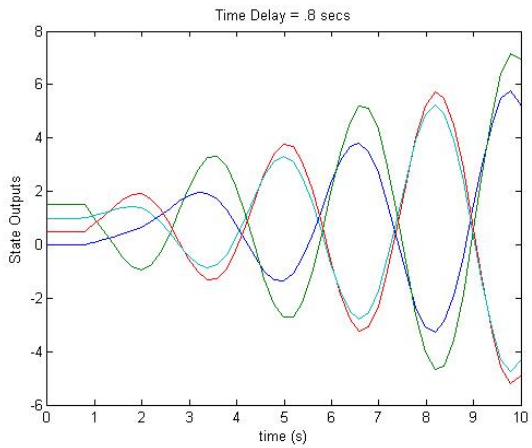
Lambert W

Delay	.1 secs	.5 secs	.8 secs
S_k	-0.0000	-0.0000 + 0.0000i	-0.0000 - 0.0000i
	-2.5917 + 0.0000i	-0.6363 + 2.6745i	0.0164 + 1.9739i
	-2.5917 - 0.0000i	-0.6363 + 2.6745i	0.0164 + 1.9739i
	-2.5917	-0.6363 + 2.6745i	0.0164 + 1.9739i

Simulations

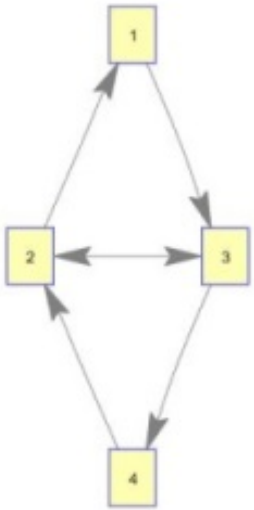


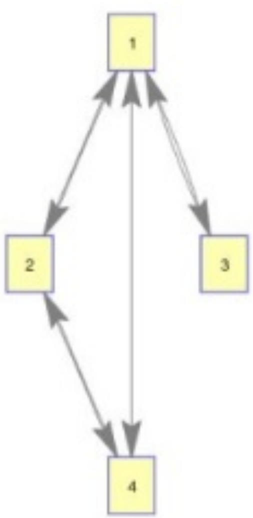




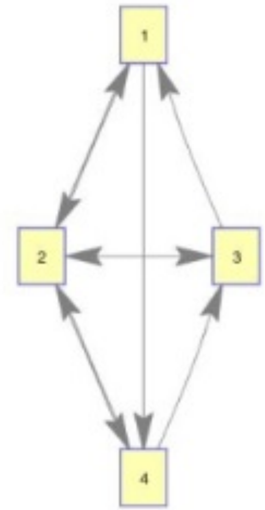
Accurately quantifies the stability

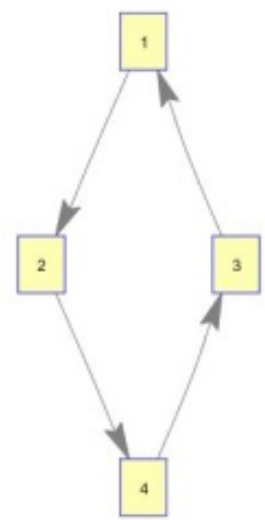
Different Topologies



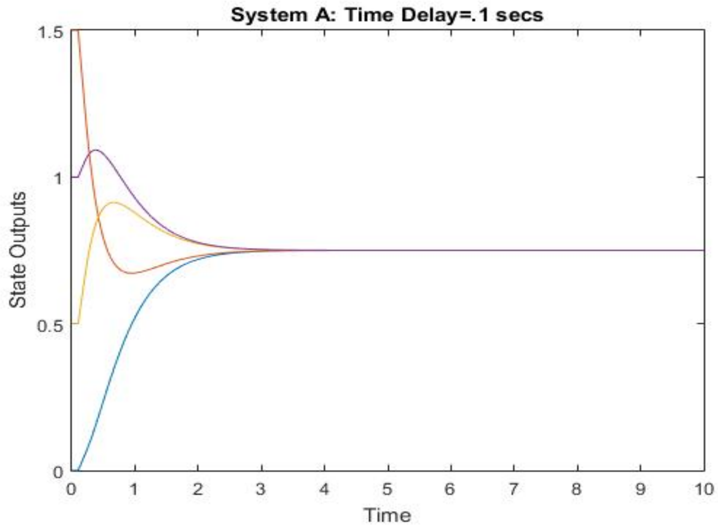


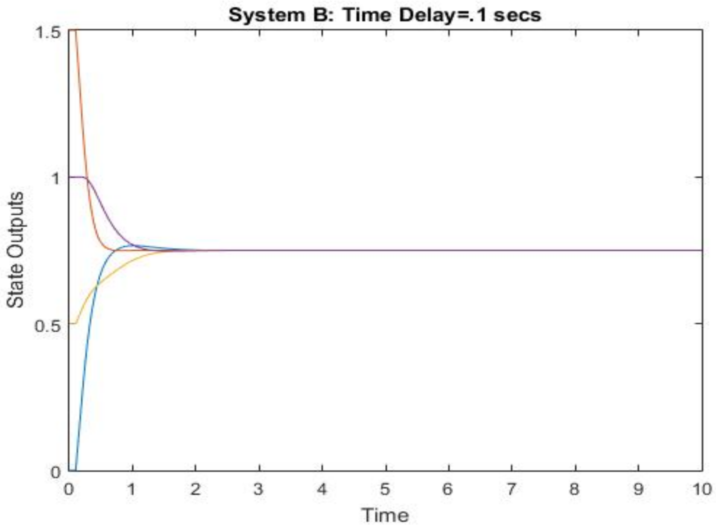
there are direct effects on the eigenvalues of the Laplacian matrix from changing the topology of a network[Saber, Fax, and Murray 2007]

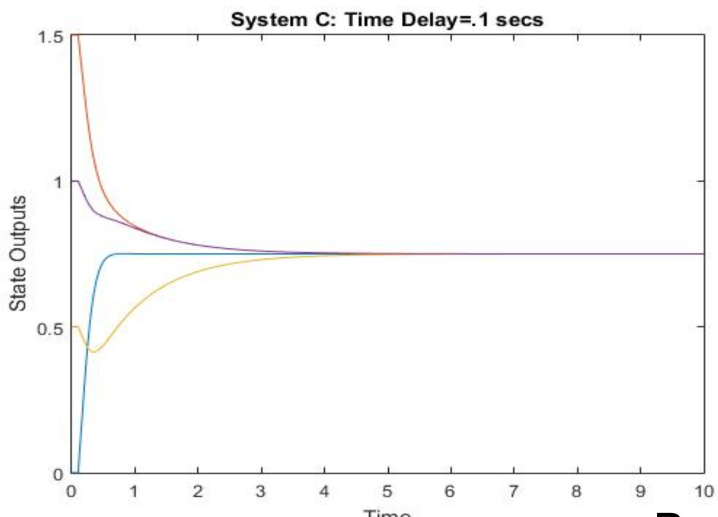


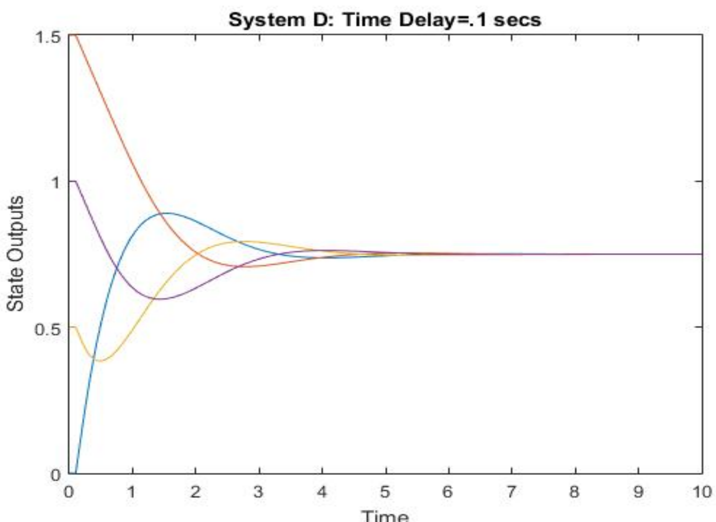


Convergence Speed



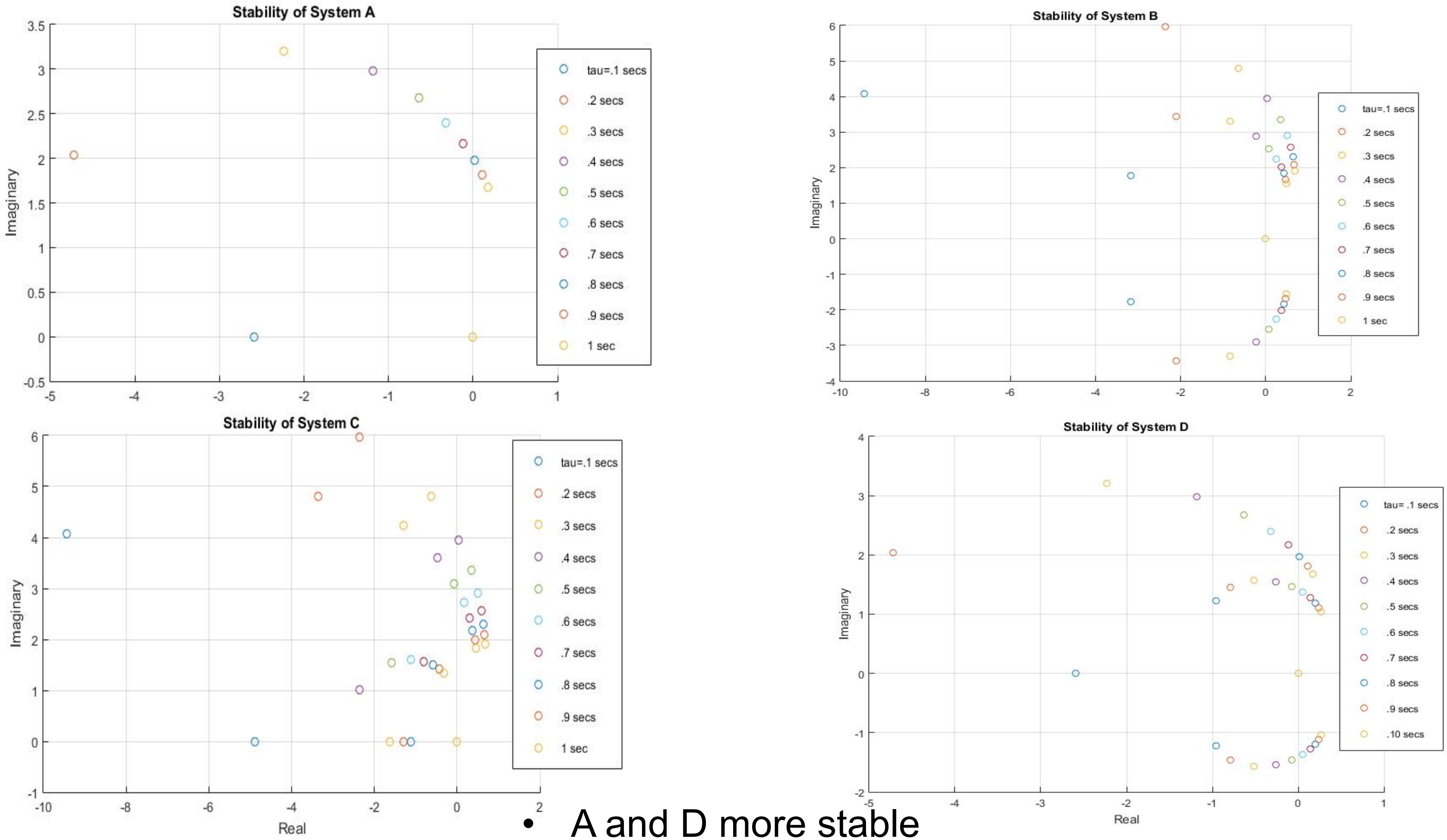






- B and C faster response

Stability Analysis



- A and D more stable

Sensitivity Analysis

$$\frac{ds}{d\tau} = -\frac{W(z)}{\tau^2} + \frac{\lambda_i}{\tau[e^{W(z)} + z]}; z = \lambda_i \tau$$

System A							
Delay	.1	.2	.3.	.4	.5	.6	.7 Critical pt.
λ_1	0.0000	0.0000+0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	-0.0000 + 0.0000i	-0.0000 + 0.0000i
λ_2	-9.0668	40.3312 +50.0675i	15.0194 - 0.3624i	7.2709 - 3.0772i	4.0620 - 2.9750i	2.4821 - 2.5317i	1.6116 - 2.1114i
λ_3	-9.0668	40.3312 +50.0675i	15.0194 - 0.3624i	7.2709 - 3.0772i	4.0620 - 2.9750i	2.4821 - 2.5317i	1.6116 - 2.1114i
λ_4	-9.0668	40.3312 +50.0675i	15.0194 - 0.3624i	7.2709 - 3.0772i	4.0620 - 2.9750i	2.4821 - 2.5317i	1.6116 - 2.1114i

System D					
Delay	.1	.2	.3	.4	.5 Critical pt.
λ_1	-9.0668 + 0.0000i	40.3312+ 50.0675i	15.0194 - 0.3624i	7.2709 - 3.0772i	4.0620 - 2.9750i
λ_2	0.9851 + 2.4647i	2.4328 + 1.8704i	2.7848 + 0.3456i	2.1780 - 0.6068i	1.5230 - 0.9292i
λ_3	0.9851 - 2.4647i	2.4328 - 1.8704i	2.7848 - 0.3456i	2.1780 + 0.6068i	1.5230 + 0.9292i
λ_4	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i	-0.0000 + 0.0000i	-0.0000 +0.0000i

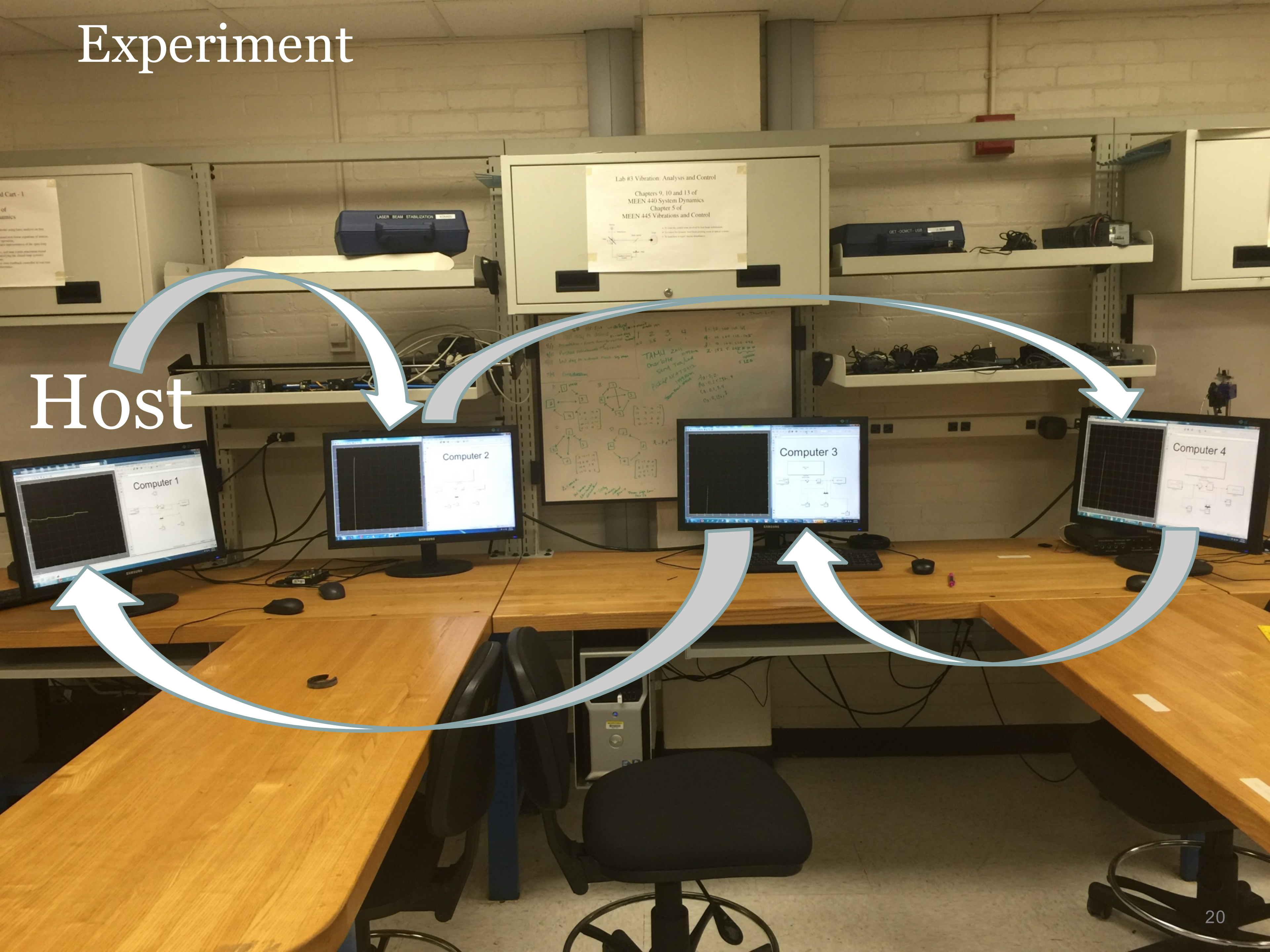
- Effects of changing topology

Sensitivity Analysis

System B				System C			
Delay	.1	.2	.3 Critical pt.	Delay	.1	.2	.3 Critical pt.
λ_1	0.0000 + 0.0000i	-0.0000 + 0.0000i	0.0000 + 0.0000i	λ_1	0.0000 + 0.0000i	0.0000 + 0.0000i	0.0000 + 0.0000i
λ_2	-5.42 + 17.88i	17.6204 + 4.0936i	8.4906 - 3.8467i	λ_2	-46.91 + 0.0000i	33.7937 - 0.8153i	12.0611 - 7.0302i
λ_3	-5.42 - 17.88i	17.6204 - 4.0936i	8.4906 + 3.8467i	λ_3	-1.41 + 0.0000i	-2.2667 + 0.0000i	-5.2121 +0.0000i
λ_4	161.32 + 200.27i	29.0836 - 12.3087i	9.9286 - 10.1267i	λ_4	161.32 +200.27i	29.0836 -12.3087i	9.9286 -10.1267i

- Sensitive to change in topology

Experiment


Host
Computer 1
Computer 2
Computer 3
Computer 4
Lab #3 Vibration: Analysis and Control
Chapters 9, 10 and 13 of
MEEN 440 System Dynamics
Chapter 5 of
MEEN 445 Vibrations and Control
LASER BEAM STABILIZATION

Computer Communication

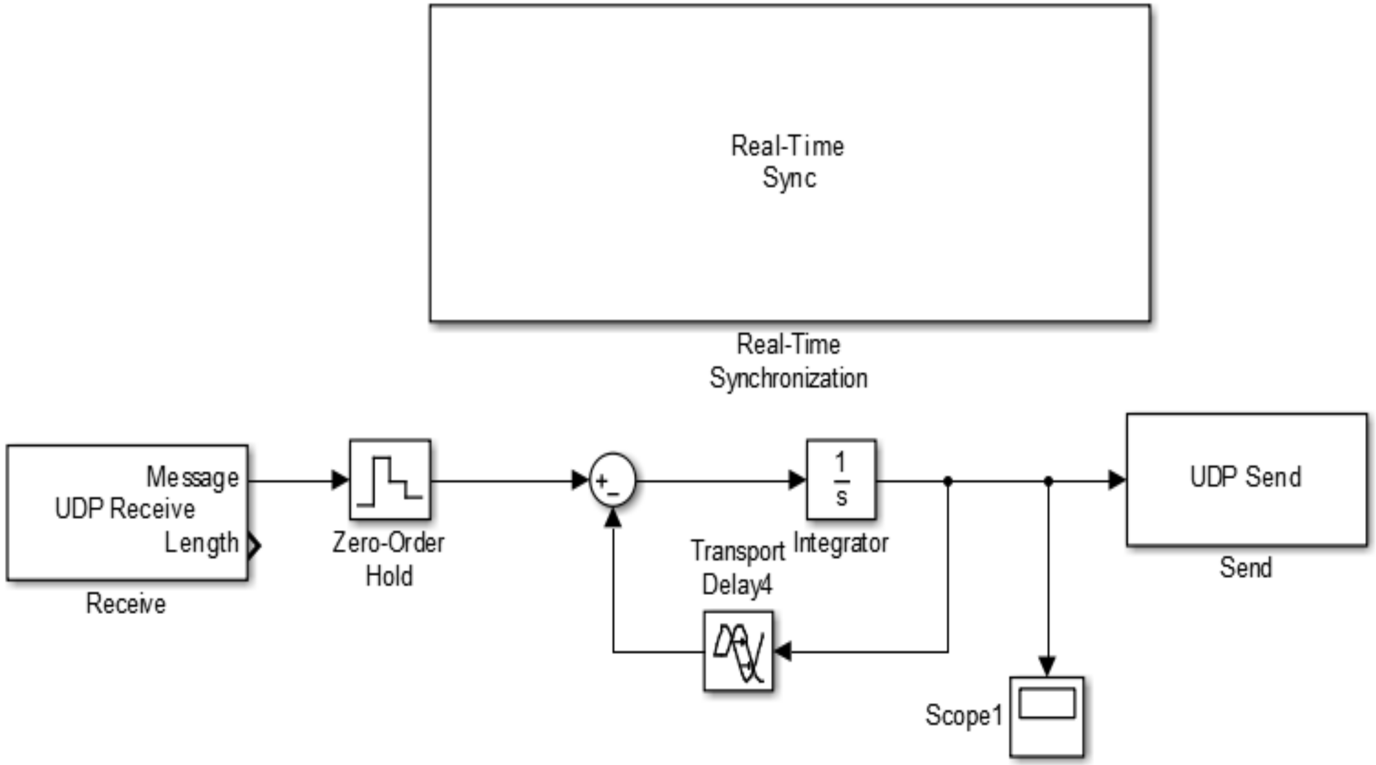
Transmission Control Protocol

- TCP/IP
- Connection required
  - » "Handshake"
- Flow control
- Emails, webpages,…

User Datagram Protocol

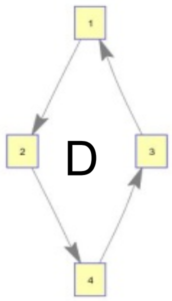
- UDP
- Connectionless
- Fast
  - » Ignores packet loss
- Real-time applications

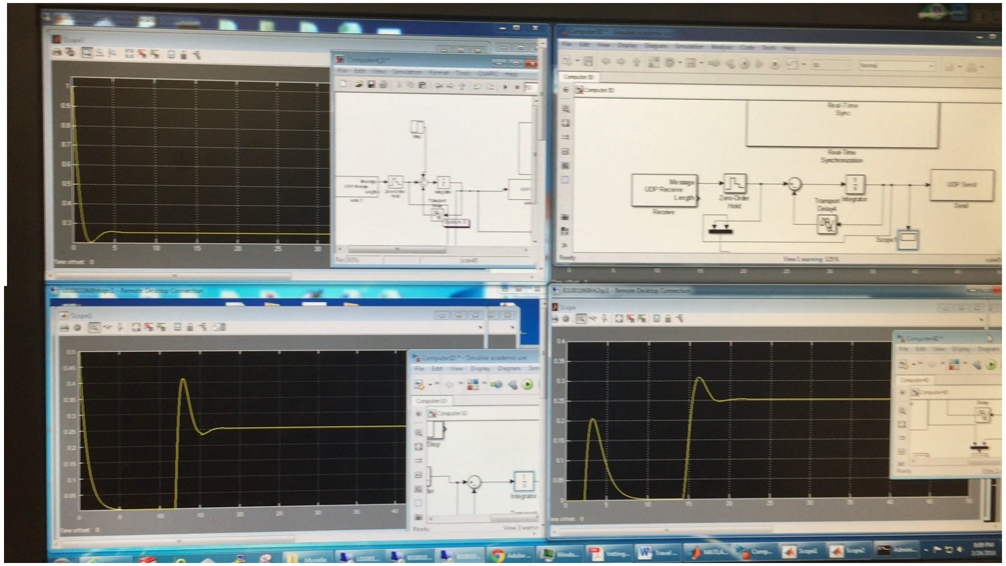
Simulink



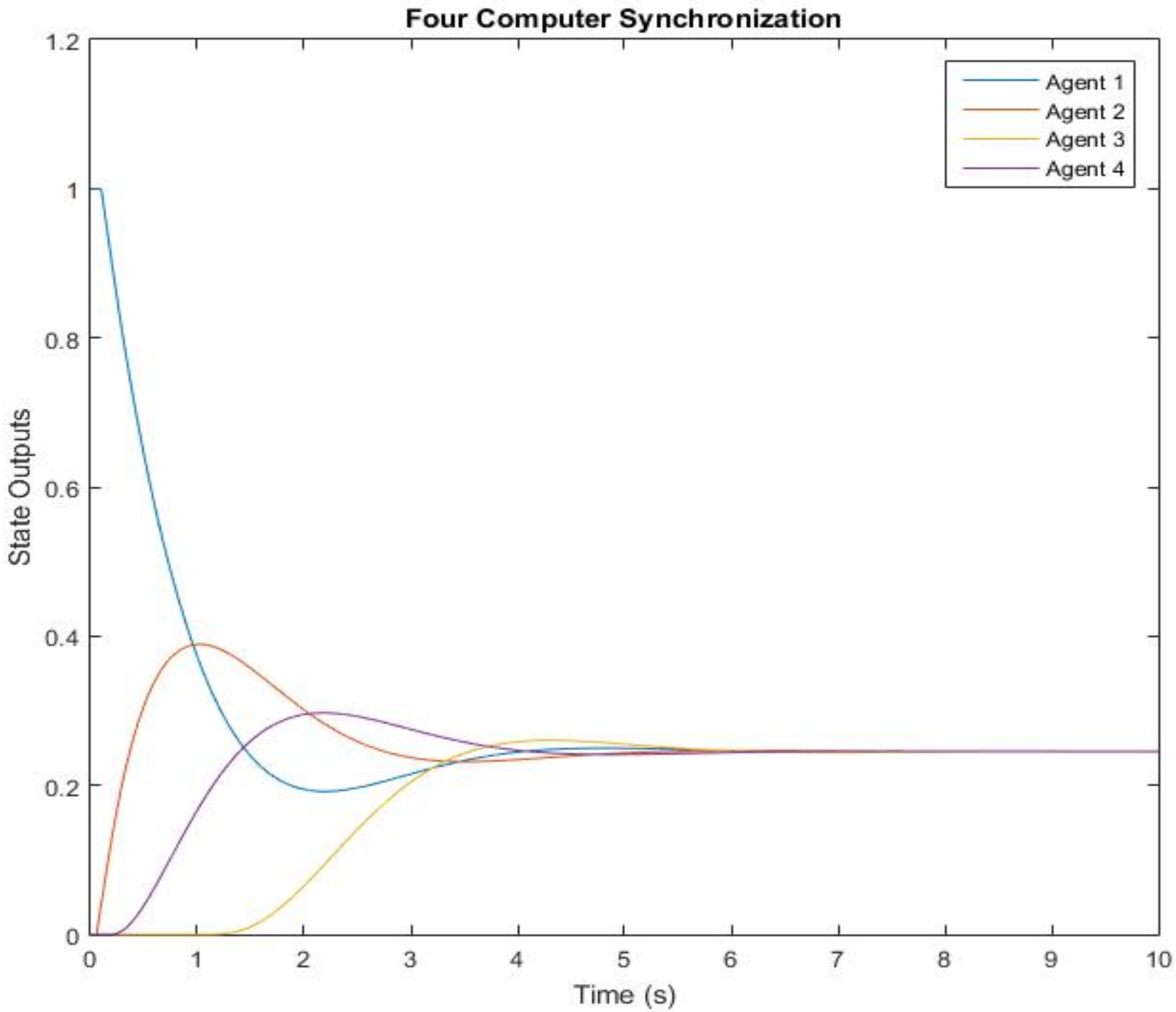
Real-Time
Sync
Real-Time
Synchronization
Message
UDP Receive
Length
Receive
Zero-Order
Hold
1
s
Integrator
Transport
Delay4
UDP Send
Send
Scope1

Experiment Run





Response



- Stable and synchronized

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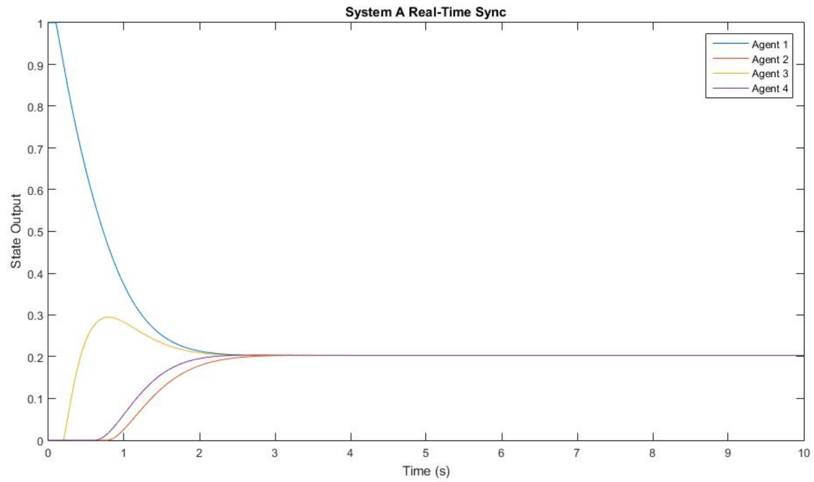
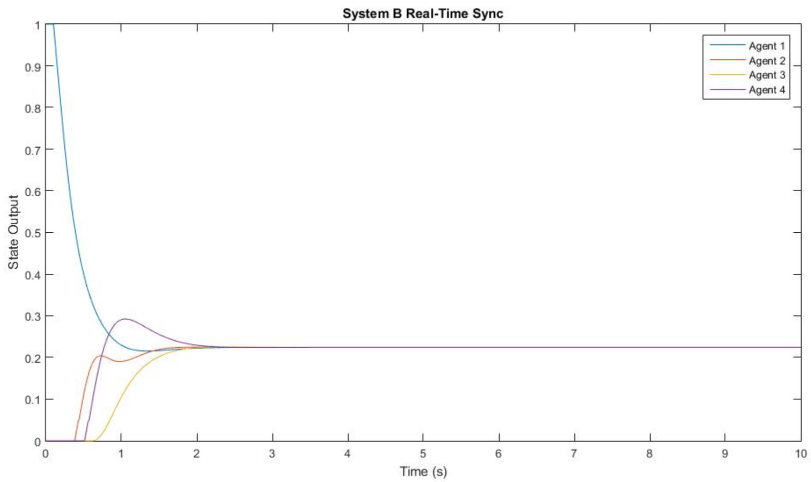
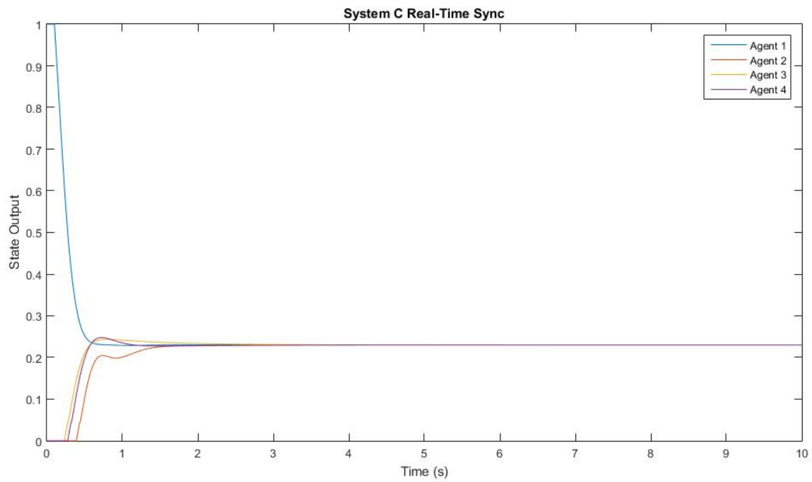
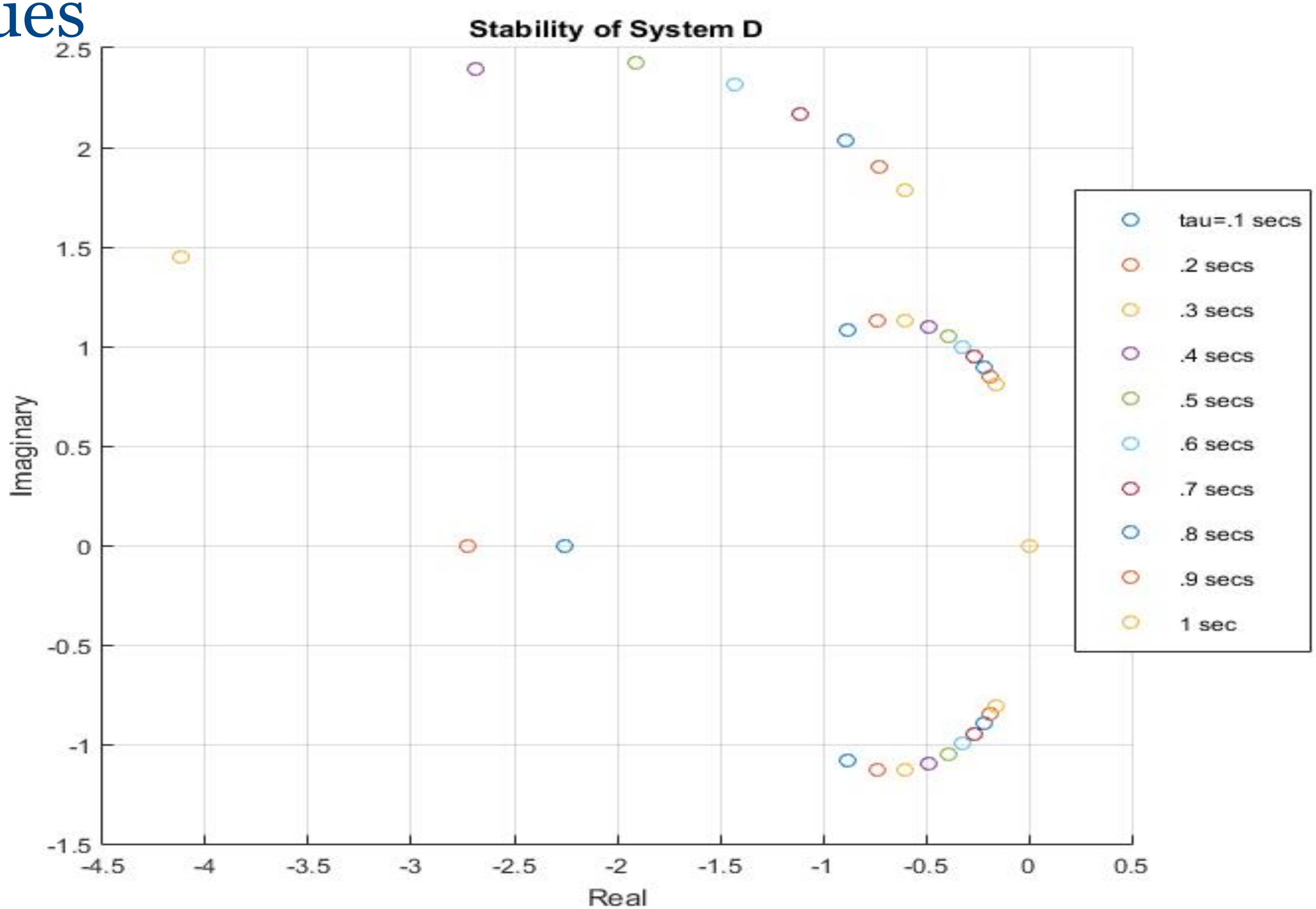
Responses of other topologies







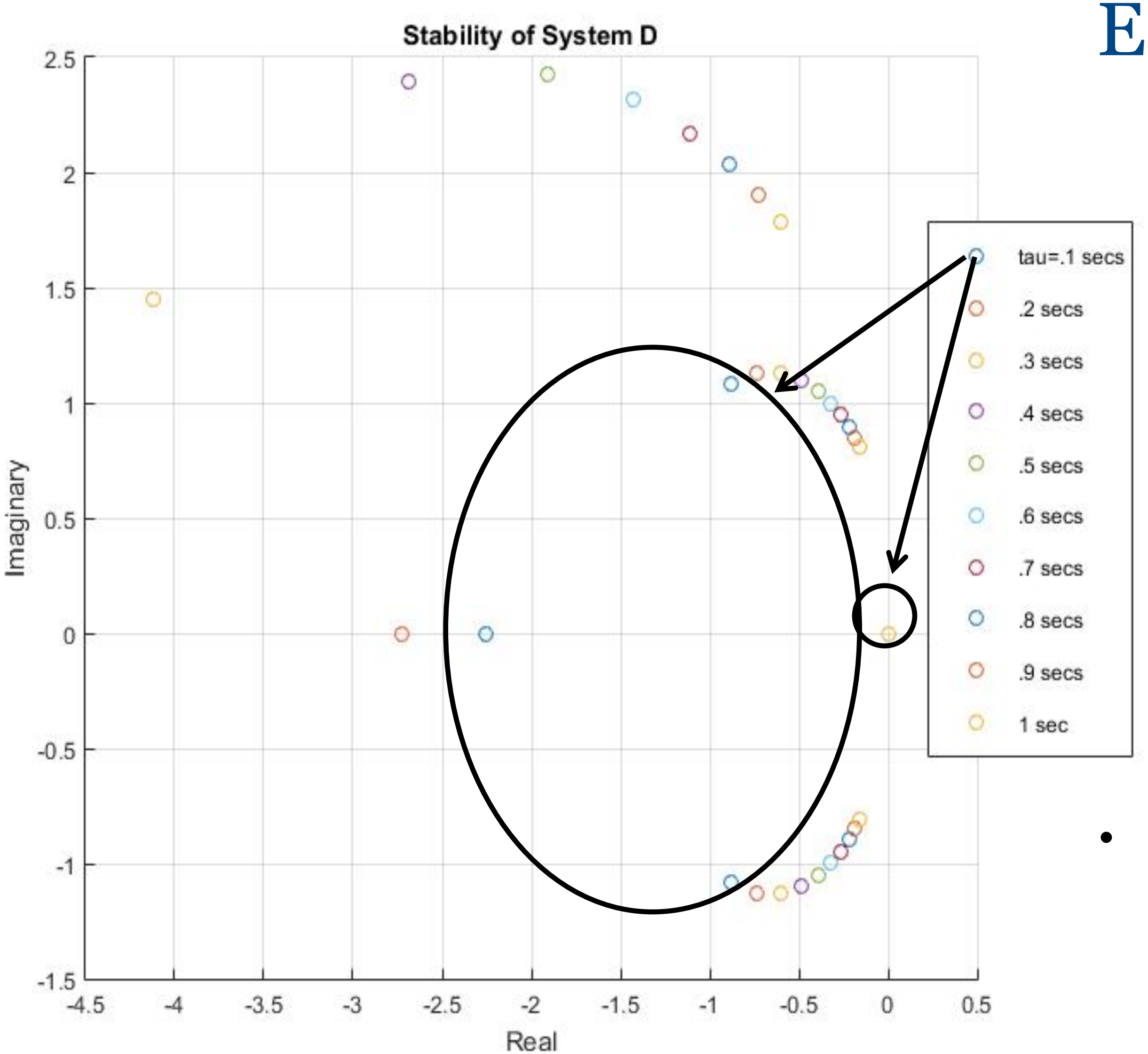
Eigenvalues



- Rightmost eigenvalues* for topology

[*Shinozaki,2007 & Yi et.al, 2007]

Eigenvalues



	Experimental Roots	Numerical Roots
Delay	.1 seconds	.1 seconds
λ_1	-0.0000 + 0.0000i	-0.0000 - 0.0000i
λ_2	-0.8817 - 1.0857i	-0.9575 - 1.2269i
λ_3	-0.8817 + 1.0857i	-0.9575 + 1.2269i
λ_4	-2.2527 + 0.0000i	-2.5917 + 0.0000i

- Rightmost eigenvalues for topology with .1 second delay

Summary

- Successfully analyzed the stability
- Performed sensitivity analysis of multiple topologies w.r.t delay
- Better approach than previous work
  - » Quantifies stability
  - » Examine sensitivity
  - » Examine and quantify convergence speed
- Built testbed for networks of real-systems

Future Work

- Use the rightmost eigenvalue as a key point for eigenvalue assignment or pole placement within a control loop to ensure the system remains stable while switching topology.
- Testbed can be implemented on systems such as drones, dc motors, and ground robots

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QUESTIONS