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Game Theory and its Applications in Multi-Agent Systems

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Outline

- Introduction
- Matching Games
- Stackelberg Games
- Coalition Games
- Future works

Part1:
Introduction

Multi-agent Systems

- Agent capabilities:
  - Sensing the environment
  - Interaction with the environment and other agents
  - Taking autonomous decisions
  - They are designed to mimic human behavior
- Multi-agent system is a system composed of multiple interacting intelligent agents that can interact, collaborate, and act together in order to perform different tasks.

Goals of Designing Multi-Agent Systems

- Simulating the way humans act in their environment
- interact with one another,
- cooperatively solve problems
- Task-distribution
- Enhancing their problem solving performances by competition.

Communication in Heterogeneous Cooperative Networks

- Wireless communication is necessary in multi-agent systems:
  - Mobility
  - Sharing the information
  - Communicating with control-centers
  - Negotiating task-scheduling

Challenges of designing a wireless network

- Power constraints for nodes using batteries or energy harvesting
- Ability to cope with node failures (resilience)
- Mobility of nodes
- Heterogeneity of nodes
  - Different capabilities
  - Different resources
  - Different demands
- Scalability to large scale of deployment
- Ability to withstand harsh environmental conditions

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How to Model the Network?

- A proper network design and communication protocol, should capture the following elements in the model:
  - Heterogeneity of the nodes
  - Interaction among the nodes
  - Decision making algorithm
  - Context information

- How to model such different aspects of the protocol?
  - ***Game theory*** provides a suitable framework

Game Theory

- Definition:
  - Game theory consists of mathematical models and techniques developed in economics to analyze interactive decision processes, predict the outcomes of interactions, identify optimal strategies.
- Elements of Game:

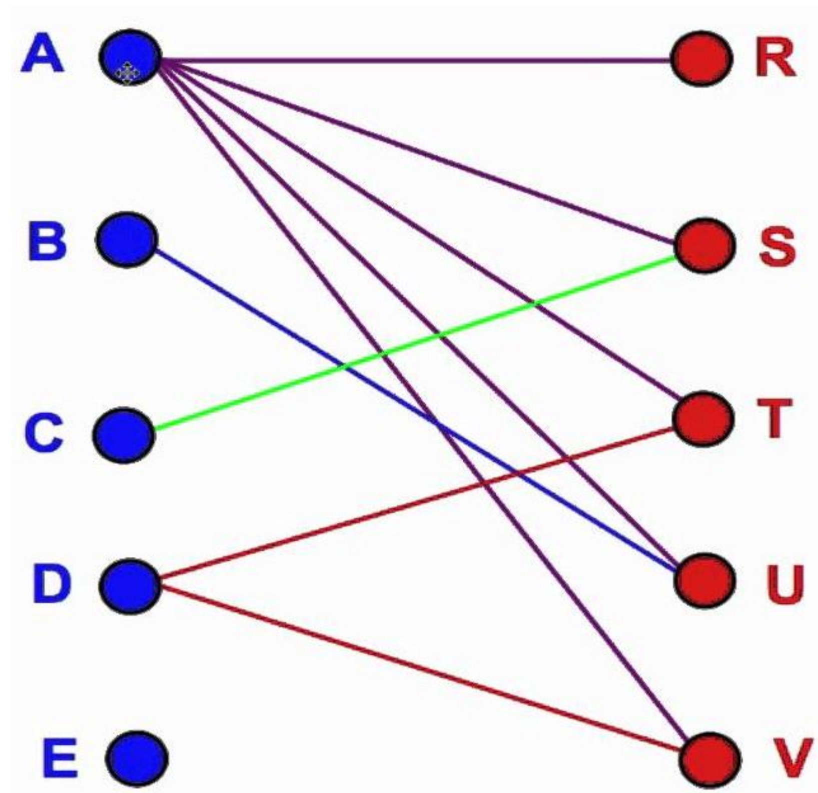
Elements of Game	Example in wireless network
Set of players	Nodes in the wireless network, agents, servers
Strategies	modulation scheme, Coding rate, transmit power level, forwarding packet or not.
Payoffs	Performance metrics (e.g. Throughput, Delay, SINR)

Game Theory

Three specially useful games in network design

- ***Matching games***
  - Resource allocation
  - Task scheduling
- ***Stackelberg games***
  - Optimization of agent's performance
- ***Coalition games***
  - Modeling the cooperation among the agents

Part II:
Matching Games



Introduction

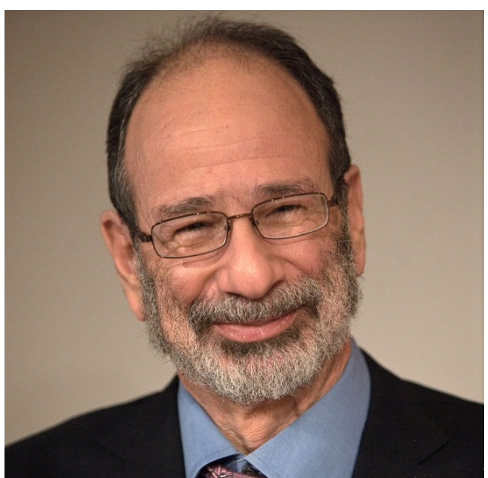
- Examples
  - Matching channels and agents
  - Matching resources and users
  - Matching tasks and groups of cooperating agents
- Existing Theories:
  - Matching Theory
  - Contract Theory

Matching Theory

- The Nobel Prize in Economics: 2012
  - Lloyd Shapley and Alvin Roth

Alvin Roth

Generated further analytical developments



Lloyd Shapley

Developed the theory in the 1960s

Matching Theory

- Definition
  - a mathematical framework attempting to describe the formation of mutually beneficial relationships over time.

➢ A matching μ is a function of $\mathbb{N}\times\mathbb{M}$ to itself such that $\forall n \in \mathbb{N}$ and $\forall m \in \mathbb{M}$: I) $\mu(n) = m$ if and only if $\mu(m) = n$; II) $\mu(n) \in \mathbb{M} \cup \emptyset$ and $\mu(m) \in \mathbb{N} \cup \emptyset$.

- Assumptions:
  - Players are rational and selfish
  - The preference function is transitive.

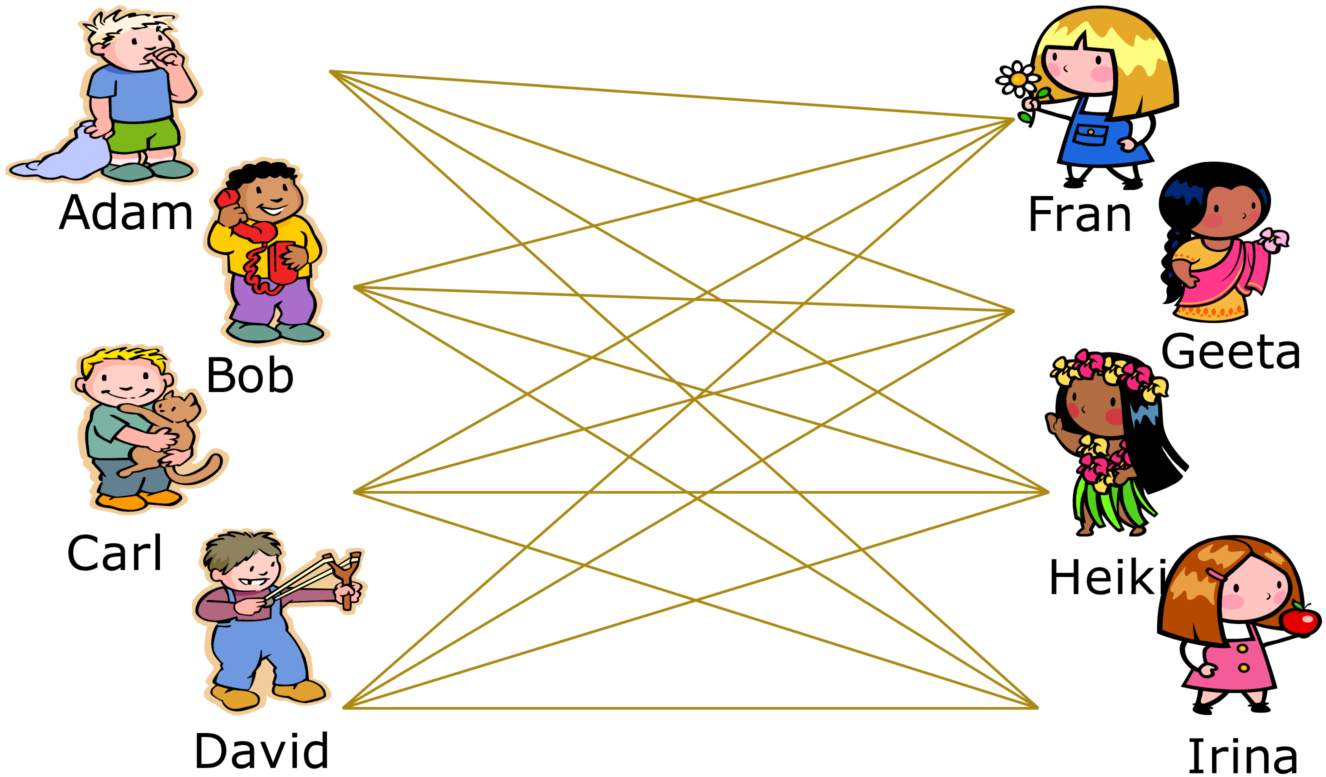
Matching Problem with Complete Preference List

- Three types of matching problems
  - ***One-to-one*** (stable marriage)
    - Assigning channels to agents
  - ***One-to-many*** (college admissions)
    - Assigning servers to agents
  - ***Many-to-many*** (complex scenarios)

Solution Concepts

- We seek to find a **stable matching** such that
  - There does not exist any pair of players, *i* and *j* matches, respectively to players, *a* and *b* but..
  - *j* prefers *a* to *b* and *a* prefers *j* to *i*
- How can we find a stable matching?
  - many approaches(minimizing sum/ max of ranks, minimizing diff of total ranks, Gale and Shapley algorithm)
  - **Most popular:** Deferred acceptance or GS algorithm
  - Illustrated via an example

Example 1: Matching partners



- women and men be matched
- respecting their individual preferences

Example 1: Matching partners

- The Gale-Shapley algorithm can be set up in two alternative ways:
  - men propose to women
  - women propose to men
- Each man proposing to the woman he likes the best
  - Each woman looks at the different proposals she has received (if any)
  - retains what she regards as the most attractive proposal (but defers from accepting it) and rejects the others
- The men who were rejected in the first round
  - Propose to their second-best choices
  - The women again keep their best offer and reject the rest
- Continues until no men want to make any further proposals
- Each of the women then accepts the proposal she holds
- The process comes to an end

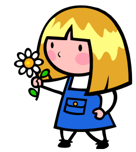
Here Comes the Story…

Adam: Geeta, Heiki, Irina, Fran

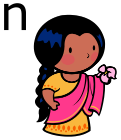
Bob: Irina, Fran, Heiki, Geeta

Carl: Geeta, Fran, Heiki, Irina

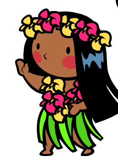
David: Irina, Heiki, Geeta, Fran



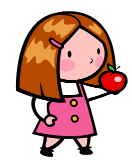
Fran: Adam, Bob, Carl, David



Geeta: Carl, David, Bob, Adam

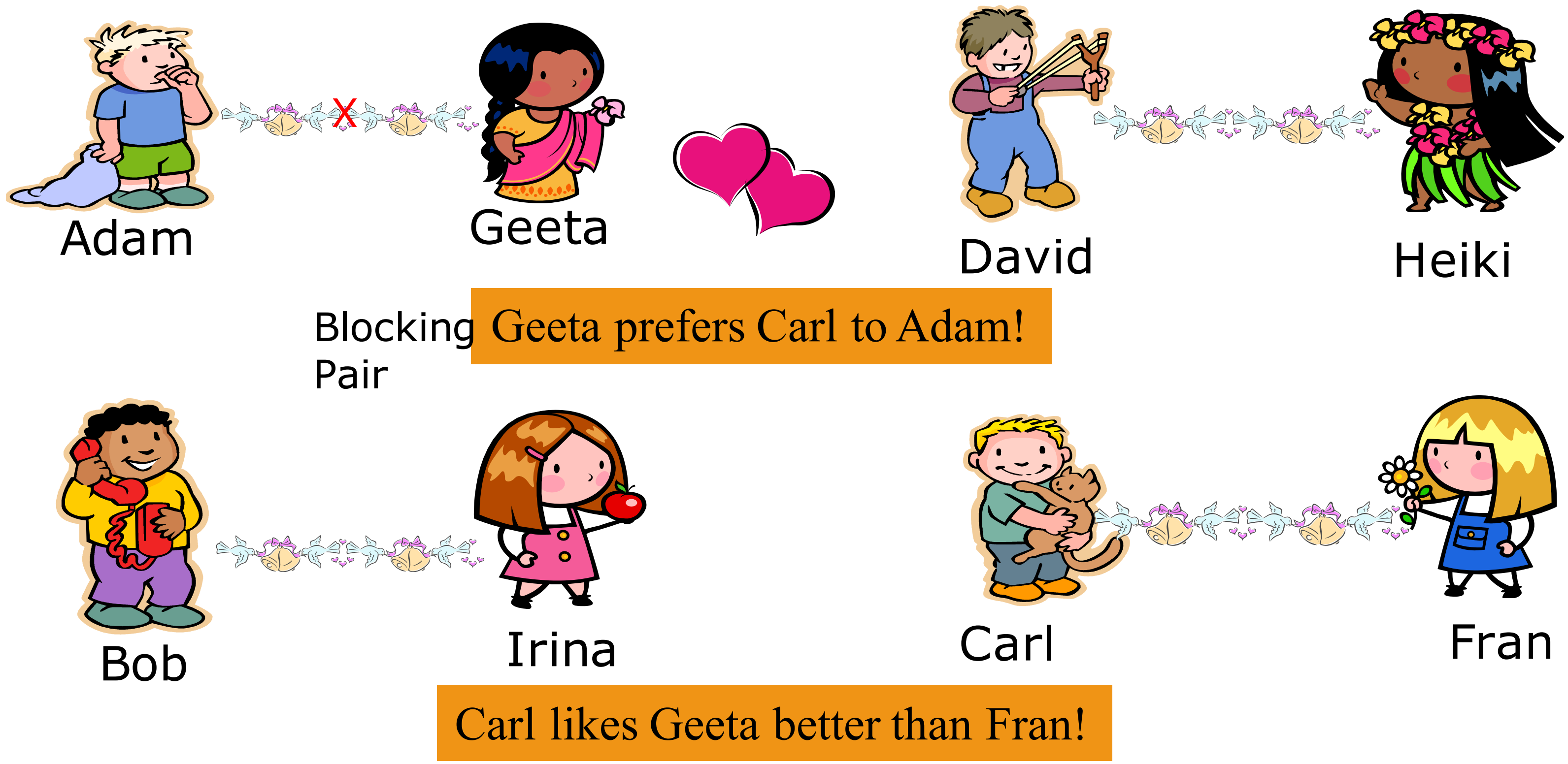


Heiki: Carl, Bob, David, Adam

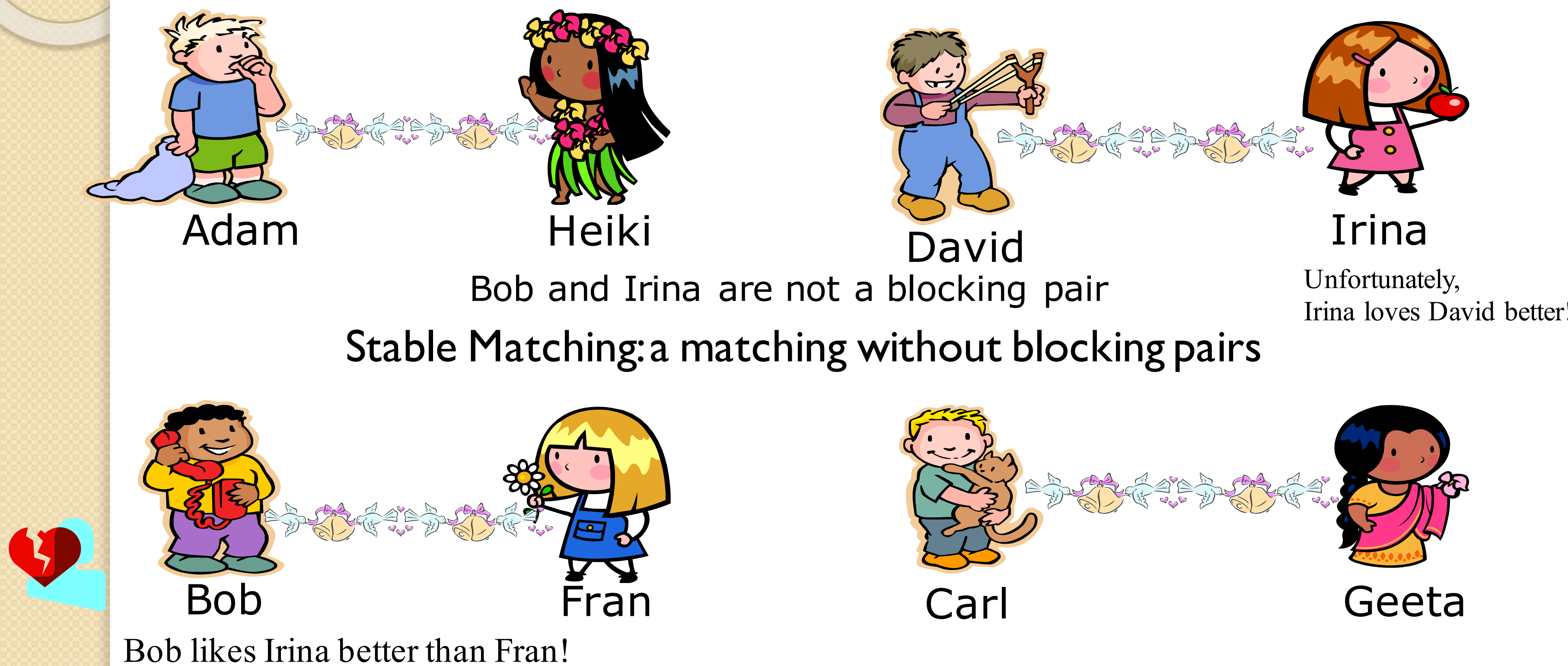


Irina: Adam, Carl, David, Bob

Search for possible matching



Stable Matching



GS Algorithm

1:	a	c	b	d	e	a:	2	1	3	4	5
2:	c	a	e	b	d	b:	2	1	4	5	3
3:	b	a	e	d	c	c:	1	2	3	5	4
4:	c	b	d	e	a	d:	3	1	4	2	5
5:	c	d	b	e	a	e:	4	3	1	2	5

1, 2, 3, 4 ,5 represent men
a, b, c, d,e represent women

no blocking pairs

Properties of Matching

- The setup of the algorithm have important distributional consequences
  - it matters a great deal whether
    - the right to propose is given to the women or to the men
  - If the women propose
    - the outcome is better for them than if the men propose
  - Conversely, the men propose
    - leads to the worst outcome from the women's perspective

- ***Optimality* is defined on each side, difficult *to guarantee* on both sides**
  - The matching may not be unique

Part III:
Stackelberg Games

Stackelberg Games

- Stackelberg models are a classic real-life example of bi-level programming.
- The upper level is called leader. It makes the first move and has all relevant information about the possible actions the follower might take in response to his own actions.
- On the other hand, the follower usually observes and reacts optimally to the actions of the leader.
- The leader solves a Stackelberg competition model in order to determine the optimal actions which he should take.

Definition: Stackelberg Equilibrium

- Equilibrium: A strategy $s_1{}^*$ is called a Stackelberg equilibrium strategy for the leader, if

$$\min_{s_2 \in \mathcal{R}_2(s_1^*} u_1(s_1^*, s_2) = \max_{s_1 \in \mathcal{S}_1} \min_{s_2 \mathcal{R}_2(s_1)} u_1(s_1, s_2) \triangleq u_1^*$$

The quantity $u_1{}^*$ is the Stackelberg utility of the leader.

Applications

- Whenever there is hierarchy in decision making process, Stackelberg game is suitable for modeling.
- Cognitive radio: resource allocation
- Relay networks: bandwidth allocation
- Optimal Control

Example I: simple firm production

- <u>Assumptions</u>:
  - Price of the market: $P = 30 - 0.25Q$
  - Quantity: $Q = q_1 + q_2$
- <u>Question</u>:
  - If player I first introduces its product into the market, what is the optimum quantity it should make?
- <u>Solution</u>:
  - The optimum quantity is the Stackelberg equilibrium

Example I continued…

- Response function of follower:

$$\pi_2 = (30 - 0.25q_1 - 0.25q_2)q_2 - q_2$$

$$\frac{\partial \pi_2}{\partial q_2} = 0 \Longrightarrow q_2 = 58 - 0.5q_1$$

- Plugging it into utility function of leader, we get:

$$\pi_1 = (30 - 0.25q_1 - 0.25(58 - 0.5q_1))q_1 - q_1$$

$$\frac{\partial \pi_1}{\partial q_1} = 0 \Longrightarrow \begin{array}{l} q_1 = 58 \\ q_2 = 29 \end{array}$$

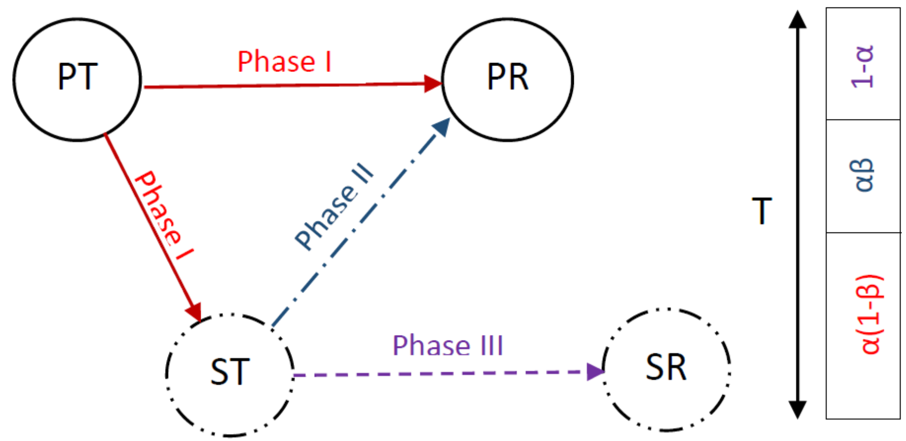
Example II: Time Allocation in Cognitive Radio

- Transmission rate:

$$R_{P_i}^{NC} = \log\left(1 + \frac{\left|h_{P_i}\right|^2 P_{P_i}}{N_0}\right)$$

$$r_1 = \log\left(1 + \frac{\left|h_{P_i S_j}\right|^2 P_{P_i}}{N_0}\right)$$

$$r_2 = \log\left(1 + \frac{\left|h_{S_j P_i}\right|^2 P_{S_j}}{N_0}\right)$$



(a) Network Model (b) Time Frame Model

- Primary user (leader) utility:

$$\boxed{R_{P_i}^{C}\left(\alpha_{ij}, \beta_{ij}, P_{S_j}\right) = \min\left(\left(1-\alpha_{ij}\right) r_1, \alpha_{ij}\beta_{ij} r_2\right)}$$

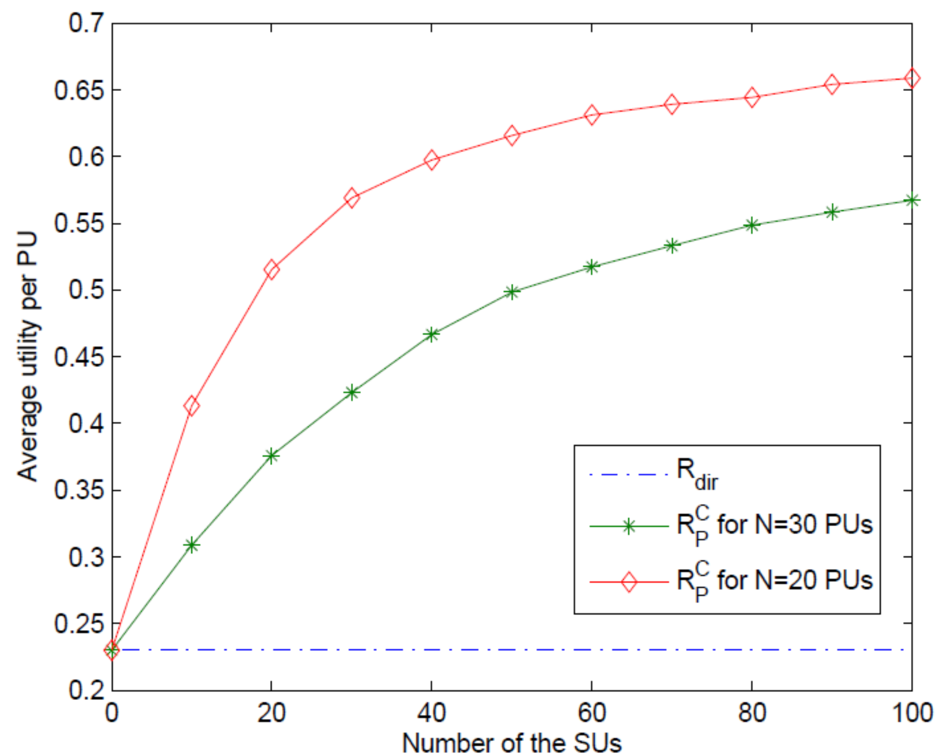
Optimum Time Sharing Parameters

- Using backward induction according to Stackelberg game; we can find the optimum values for $(\alpha_{ij}, \beta_{ij})$ as:

$$\beta_{ij}^* = \arg\max_{\beta_{ij}\in[0,1]} \log\left(1 + \frac{\left|h_{S_jP_i}\right|^2 P_{S_j}}{N_0}\right)$$

$$\alpha_{ij}^* = \frac{r_1}{r_1 + \beta_{ij}^* r_2}$$

$$R_{P_i}^C\left(\alpha_{ij}^*, \beta_{ij}^*, P_{S_j}\right) = \frac{\beta_{ij}^* r_1 r_2}{r_1 + \beta_{ij}^* r_2}$$



Part IV:
Coalition Games

Coalition Games

- Robot Networks witness a highly complex and dynamic environment whereby the nodes can interact and cooperate for improving their performance.
- In this context, cooperation has emerged as a novel communication paradigm that can yield tremendous performance gains from the physical layer all the way up to the application layer

- a significant amount of research efforts has been dedicated to studying cooperation in wireless networks.
- Examples of cooperation:
- MIMO: reduction of BER
- Relay forwarding: throughput increase, improving connectivity

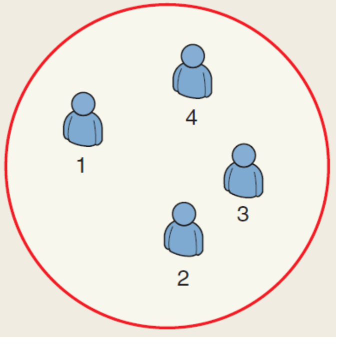
- But….What is the cost of cooperation? The impact on the network structure? How to model it?

- Challenges of modeling:
  - Cooperation entails cost: power
  - It is desirable to have a distributed algorithm

- Solution? Coalition game theory

Cooperative Coalitional Game Theory

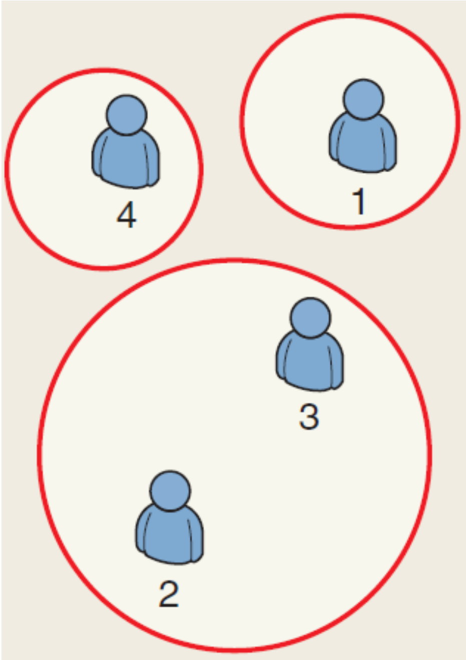
- Canonical Coalition Game
  - It is beneficial to all players to join the coalition. Grand coalition is the optimal solution



  - Main objective:
    - properties and stability of grand coalition
    - how to distribute gain from cooperation in fair manner between player (Core)
    - Pay-off allocation solution : Core, Shapley value

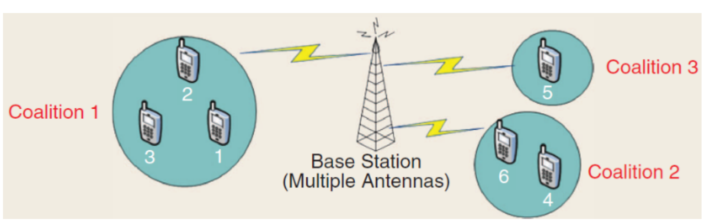
Cooperative Coalitional Game Theory

- Coalition Formation Game
  - Forming a coalition brings gains to its members, but the gains are limited by a cost for forming the coalition
  - Main objective
    - optimal coalition size
    - assess the structure's characteristics



Application in Wireless Network

- Virtual MIMO
  - A single transmitter sends data in a TDMA system to multiple receivers.
  - Non-cooperative approach:
    - Sending the data in an allotted slot.
  - Cooperative approach:
    - For improving their capacity, the receivers form coalitions, whereby each coalition S is seen as a single user MIMO that transmits in the slots that were previously held by the users of S.



Time Slot 1	Time Slot 2	Time Slot 3	Time Slot 4	Time Slot 5	Time Slot 6
User 1	User 2	User 3	User 4	User 5	User 6

Time Slot 1	Time Slot 2	Time Slot 3	Time Slot 4	Time Slot 5	Time Slot 6
Coal. 1	Coal. 1	Coal. 1	Coal. 2	Coal. 3	Coal. 2

- Applications:
- Distributed task allocation in multi-agent systems
- Network formation in multihop networks
- Distributed collaborating environment sensing

Conclusion

- Game theory is a suitable mathematical framework to model and analyze the interaction among the agents.
- Three types of game are of special interest in protocol designing for wireless networks:
    - Matching games
    - Stackelberg games
    - Coalition games

Future work

- Power control in heterogeneous networks, considering different scenarios for the users, QoS, capacity, etc.
- Designing distributed algorithms for coalition games which can be efficiently implemented within the network.
- Matching algorithm when the preferences of the agents are dynamic and change.

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Thank You!

- *Perhaps it is good to have a beautiful mind, but an even greater gift is to discover a beautiful heart.*

  -John Nash