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[Prescriptive Analytics with Mathematical Programming (CMSE11431)]

[Assignment 1 - Group Project]

Group 1

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Abstract

Allocating optimal proportion of drivers to passengers going to various destinations is crucial for ride-hailing platforms like Uber to maximize potential profit. We develop a novel LP approach to solve this issue which considers a much broader set of associated costs and constraints. The model is implemented to provide optimal proportion of active cab allocation around Boston. We find that, generally, cabs starting in a particular origin, most usually, are primarily allocated to a single destination. Furthermore, in most cases, the available supply is fully utilized at origins, and the respective demands are entirely fulfilled, resulting in minimal idle and opportunity costs.

1. Introduction

Uber was first introduced in Boston in 2011. Since then, ridership has been growing exponentially, with 45.3 million rides in 2019 alone. Through these ride-hailing platforms, customers can request for a ride through the mobile application, which matches ride requests with available drivers within the area, and fares may change, sometimes dynamically as demand fluctuates. Overall, the income of drivers is dependent on the working hours, total supply of drivers, and passenger demand (Chen et al., 2017).

As countries emerge from the COVID-19 pandemic, more individuals are heading out again through public and private transport, including via ride-hailing platforms such as Uber. In Boston, for example, the rate of increase in demand has outstripped that of supply of private hire cabs (Buell, 2021), resulting in constant surge pricing and hence frustration amongst citizens.

As the demand and supply of cabs are mostly dynamic in nature, it is essential to ensure that the supply of cabs is geographically optimized to always meet demand throughout the day. This solves the demand conundrum that individuals may face especially during peak periods, hence maximizing customer satisfaction, and ensures that cab drivers will not be driving around aimlessly, incurring unnecessary costs and pollution.

Through linear programming (LP), we can first identify the constraints limiting the factors that may impede optimal allocation of cabs around Boston. These may include costs of travelling between areas, drivers' working schedules, or even the total number of drivers available at any one time.

Through understanding these constraints, we can better allocate cabs to different geographical locations based on their relative demand at different times of the day, based on past data available. This minimizes the idle and opportunity costs incurred by cab companies from inefficient allocation of cabs in Boston, while maximizing customer satisfaction through sufficient supply of cabs, and hence improving profits for cab companies.

2. Literature Review

Prior to performing our analysis, preliminary research was done to explore the various methods used to address similar business problems. There have been similar studies that investigated optimum allocation of transportation vehicles within a certain area, albeit with bicycles. Nonetheless, the techniques and models used from related topics can provide some insight. Several research literatures deal with optimal allocation of public transport, including bicycles via Linear Programming and methods such as CPLEX. It is also interesting to note that not all models had the same problem statement, nor objective function, which may serve as future research inspiration.

One similar study conducted (Deng et al., 1992) lays the foundation of this report. The study used queuing models and integer programming to determine the optimal shifts of cab drivers to dictate the optimal supply of cabs that should be plying the roads based on the relative demand throughout the day. It also utilized a cost-analysis approach, which takes into account fixed, variable and opportunity costs, serving as the basis for mathematical formulations. Results were then fed back into scheduling management information system to minimize scheduling time and total distance travelled, thus maximizing profits. However, the model is inflexible as it presumes that cab drivers are to stick to their respective zones, limits the movement of cab drivers between zones, and insists that all customers are to be picked up regardless of time.

However, there are drawbacks of using LP to solve related business problems. One study (Pal and Zhang, 2017) attempted using mixed integer linear programming (MILP) to allow for optimal, complete rebalancing and reallocation of bicycles. The novel MILP proposed facilitated the complete rebalancing of bicycles, allowing a node to be visited multiple times by the same vehicle(s). However, many problems arose when using LP – the presence of big M whilst using the simplex algorithm limited its computational tractability, and symmetry in decision variables undermined the LP relaxation of the formulation. The paper also floated the presence of many decision variables as a factor for the formulation's computational intractability. This presented difficulties when running the model and may thus limit the efficiency in solving the business problem.

Another study conducted (Sun et al., 2019) utilized both MILP and clustering methods to optimize the location of virtual bike-sharing stations to meet the growing demands of such bicycles during morning and evening peak periods. The study introduced the concept of a service radius which was subsequently incorporated in our study. Moreover, MILP executed using CPLEX was able to generate optimal solutions in a short time, albeit with a smaller dataset. For larger problems, however, hardware used took excessive times to derive an optimal solution, CPLEX generated less accurate results vis-à-vis the clustering algorithm. Nonetheless, the solutions derived through the CPLEX method involving MILP remained feasible.

A similar study (Hosni et al., 2014) attempted linear programming using CPLEX and their alternative Lagrangian decomposition method in solving cab assignment and optimal routing problems. While linear programming using CPLEX was more efficient in finding optimal solutions for different scenarios, it consistently showed that CPLEX could generate optimal solutions for smaller datasets within a shorter period of time than the study's alternative algorithm. However, CPLEX was unable to derive optimal solutions within the set timeframe of 2 hours when the dataset increased in size and complexity. This shows that linear programming using CPLEX was more suitable for smaller and simpler datasets vis-à-vis more complex problems.

The aforementioned studies covering a variety of related business problems have provided us with insights and introduced us to the potential and limitations that utilizing IP/MILP can bring towards addressing our business problem, as further highlighted in Section 3.

3. Model and Problem Description

Boston has multiple ride-hailing services which all drive passengers simultaneously, it is reasonable to assume that they are in constant competition meaning that they would be constantly competing for clients. Hence, for each cab type, Uber is faced with an opportunity to allocate a different proportion of drivers to drive customers from the initial area to various destinations on any given day at any given hour. Simultaneously, it is crucial to allocate an appropriate proportion of drivers to passengers going to the most profitable destinations to yield the highest profit. Moreover, considering that Uber has been experiencing losses recently (Iqbal, 2022), such allocation would be particularly important.

The aim of this optimization model is to provide an optimal driver proportion allocation that would maximize potential profit. Opportunity costs of taking certain passengers instead of others must be considered as well as any associated idle cost of drivers when the cost of moving to a location is greater than the associated profit. Similarly, all drivers should never be assigned to a single area as this would mean that the company will lose market hold in corresponding locations which could be damaging to the overall reputation.

Hence, a reworked model vis-à-vis the model proposed by Tanizaki (2013) on a similar problem of allocating taxis to taxi stands is proposed. Apart from demand, supply and capacity of cabs, practical limitations such as budgeting, and further aforementioned costs to allocating cabs were included. The model also considers different classes of cabs, as demand differs for each type of cab at any given time. Nevertheless, as mentioned in the previous section, similar problems are usually tackled using IP or MIP; however, as was further noted by the literature review, such formulation on the scale of the presented problem takes an unreasonable amount of time to be solved. Thus, this report presents an alternative approach to determine the optimal allocation. Fundamentally, the presented mathematical model implements an LP relaxation as the decision variable will determine the proportion of drivers rather than the exact number of cabs to be allocated to driving passengers to different areas.

The assumptions made for this case are as follows:

- The number of available cabs in every timeframe is unrelated to the number of cabs in the subsequent or previous timeframes. This is because in ride-hailing platforms drivers can take breaks and leave work based on their preference.
- Data derived demand and supply for each area and hour are static and do not change within hours.
- One cab is only allowed to drop off passengers at one location during one timeframe. This suggests that cabs are not allowed to make multiple stops to drop off different passengers.
- Cabs can only drop off passengers within the areas covered by the nodes.

- Vehicles under the ‘Premium’ category are of a higher quality and have higher maintenance costs than that of the ‘Standard’ category, which in turn, are higher than that of the ‘Shared’ category.

4. Mathematical Formulations

The following list gives the breakdown of the model we have input in AIMMS:

Decision variable	$\boldsymbol{x_{zijt}}$	proportion of cabs to be allocated
Sets	$\boldsymbol{IL}$	Origin Locations
	$\boldsymbol{JL}$	Destination Locations
	$\boldsymbol{T}$	Time
	$\boldsymbol{Z}$	Type of taxis
Indices	$\boldsymbol{z}$	type of taxi
	$\boldsymbol{i}$	origin source
	$\boldsymbol{j}$	destination
	$\boldsymbol{t}$	time period
Parameters	$\boldsymbol{d_{ij}}$	Distance from point i to point j
	$\boldsymbol{D_{zjt}}$	Demand for taxis z at point j at time t
	$\boldsymbol{S_{zit}}$	Supply of taxis of type z at location i at given time t
	$\boldsymbol{M_{jt}}$	Minimum allocation requirement of taxis for location j at time t
	$\boldsymbol{B_z}$	Budget for each cab type z
	$\boldsymbol{\alpha_z}$	Variable cost for cab type z
	$\boldsymbol{f}$	Fixed costs
	$\boldsymbol{IC}$	Idle costs
	$\boldsymbol{OC}$	Opportunity costs
	$\boldsymbol{I}$	Constant used for idle costs
	$\boldsymbol{O}$	Constant used for opportunity costs

$\boldsymbol{\alpha_z}$	Variable costs
$\boldsymbol{r_{zijt}}$	Revenue
$\boldsymbol{c_{zij}}$	Allocation costs
$\boldsymbol{W_{zijt}}$	Number of allocated cabs of type z from origin i to destination j at time t

Objective function

$$Maximise \sum_z \sum_i \sum_j \sum_t (r_{zijt} - c_{zij}) * W_{zijt} - IC - OC \qquad - (1)$$

Where:

$$IC = I * \sum_z \sum_i \sum_t \left(S_{zit} - \sum_j W_{zijt} \right) \qquad - (2)$$

$$OC = O * \sum_z \sum_j \sum_t \left(D_{zjt} - \sum_i W_{zijt} \right) \qquad - (3)$$

$$c_{zij} = \alpha_z * d_{ij} + f \qquad - (4)$$

$$W_{zijt} = x_{zijt} * S_{zit} \qquad - (5)$$

(1) calculates profits by multiplying revenues less costs of cab operations by the number of cabs on the roads, barring idle costs and opportunity costs. (2) calculates idle costs by multiplying I by the difference between the total supply of cabs and the total number of available cabs on the road. (3) calculates opportunity costs by multiplying O by the difference between the demand of cabs and the total number of available cabs on the road. (4) calculates allocation costs by multiplying α by the distance travelled by the cabs, in addition to the fixed costs f. (5) calculates the number of cabs by multiplying the proportion of cabs to be allocated x_{zijt} by the total supply of cabs S_{zit}.

Constraints

$$\sum_{i}\sum_{j}\sum_{z} W_{zijt} * f \leq B_z, \qquad \forall z \qquad -(6)$$

$$\sum_{z}\sum_{i} W_{zijt} \geq M_{jt}, \qquad \forall j,t \qquad -(7)$$

$$\sum_{j} W_{zijt} \leq S_{zit}, \qquad \forall z,i,t \qquad -(8)$$

$$\sum_{i} W_{zijt} \leq D_{zjt}, \qquad \forall z,j,t \qquad -(9)$$

$$x_{zijt} \in [0,1], \qquad W_{zijt} \in \mathbb{Z}^{+} \qquad \forall z,i,j,t \qquad -(10)$$

(6) implies that the total fixed costs of operations for all types of cabs z, at all times t, will be less than the allocated budget. (7) suggests that for all cab types z, there should be more cabs allocated than the minimum allocation requirement for all locations i, at all times t. To preserve our market share and ensure that our presence will still be felt in the market, it is necessary to deploy a minimum number of cabs, even if it means that there will be some idle costs incurred. In this model, we have set it at an arbitrary value of 10%, since we are only shortlisting only 6 location nodes due to the nature of our dataset. (8) states that for all times t, origin locations i, and for all cab types z, the number of allocated cabs will be less than the total number of active cabs for all times. (9) determines that for all times t, destination locations j, and for all cab types z, the number of allocated cabs will always be less than the respective total demand for those cabs. (10) fixes x_{ijt} to between values 0 and 1 due to linear relaxation of our model, while the number of cabs W_{zijt} shall be a positive integer.

5. Computational Results and Analysis

5.1 Raw Dataset Description

The analysis presented in this report was based on a Kaggle dataset (Munde, 2017). Essentially, each entry in the dataset represents an individual trip between popular Boston areas using Uber or Lyft ride-hailing platforms and was collected through querying the firm's respective APIs. Table 5.1 explains the relevant columns used in the report and what they represent.

Table 5.1: A list of columns in raw dataset and their definition

Column	Definition
distance	Distance travelled between source and destination in miles
cab_type	Ride hailing platforms, in this case, Uber or Lyft
time_stamp	Time the ride was taken
destination	Destination node j
source	Source node i
price	Price of the ride in dollars ($)
name	Type of ride: Black, Black SUV, Taxi, UberPool, UberX, UberXL, WAV.

5.2 Dataset pre-processing

Unfortunately, the raw data presented in the original dataset could not be processed by the model. Instead, the parameters of the model were based on averages for the corresponding trips in this dataset. For instance, to determine the revenue r_{zijt} associated with making a trip using cab type z from origin i to destination j in time period t all the records that completed such trip were aggregated and divided by their count to produce a corresponding model input. Furthermore, the dataset did not contain enough trips from every origin to every destination for every working hour, hence only areas with enough sufficient trips at origins and destinations were selected for this project. The time periods between 0900 hours and 1700 hours were selected based on working hours of the common individual in Boston. Additionally, only data from Uber is used as the model should be customized for an individual company. The relevant data used for model input is shown in Tables 5.2, 5.3, 5.4 and 5.5 below.

Table 5.2: Distance in miles between origin and destination

		Destination (j)					
		Back Bay	Beacon Hill	Financial District	North Station	Theatre District	West End
Origin (i)	Boston University	1.50	2.51	4.86	3.25	3.05	2.95
	Fenway	1.44	2.40	4.39	3.13	2.77	2.84
	Haymarket Square	2.28	1.26	1.06	0.59	1.44	0.70
	North End	2.84	1.84	1.14	1.05	1.57	1.16
	Northeastern University	1.31	2.38	4.12	3.26	2.07	2.97
	South Station	2.81	2.37	0.51	2.01	0.91	1.95

Table 5.3: Price in dollars between origin and destination at different times of the day

		Back Bay								
		9	10	11	12	13	14	15	16	17
premium	Boston University	21.88	20.87	22.12	21.47	21.13	21.13	20.77	21.84	21.98
	Fenway	20.84	21.56	21.53	20.62	21.41	21.48	20.67	20.92	21.3
	Haymarket Square	26.42	25.21	26.02	25.4	26.49	25.2	26.15	24.69	26.17
	North End	29.9	29.82	30.45	30.49	29.57	29.88	29.45	29.92	29.71
	Northeastern University	20.19	20.72	20.47	20.75	21.21	20.79	21.47	20.49	20.85
	South Station	27.98	27.82	28.2	28.79	28.47	28.66	29.08	28.88	28.52
shared	Boston University	7.08	7.06	7.16	7.08	7.26	7.47	6.95	7.58	7.35
	Fenway	7.56	7.64	7	7.37	7.32	7.25	7.23	7.53	7.23
	Haymarket Square	9.5	9.54	9.23	10.09	9.26	9.13	9.55	9.32	9.27
	North End	10.71	10.65	10.78	10.5	10.83	10.63	10.71	11.37	10.74
	Northeastern University	7.7	7.61	8.03	7.32	7.8	7.52	7.54	7.88	7.66
	South Station	11.11	10.46	10.26	10.29	10.53	10.36	10.59	10.78	10
standard	Boston University	10.04	9.83	10.01	10.13	10.32	9.94	10.04	10.45	10.02
	Fenway	10.14	9.75	9.58	9.51	9.58	9.62	9.83	9.7	9.82
	Haymarket Square	12.31	12.31	13.02	12.28	12.15	11.83	11.65	13.01	11.73
	North End	14.25	13.59	14.88	14.14	14.58	14.45	13.75	14.33	13.71
	Northeastern University	10.38	9.93	10.07	10.02	10.38	9.69	10.13	10.59	10.42
	South Station	14.27	13.45	13.55	14.15	14.26	12.72	13.64	13.79	13.33

		Beacon Hill								
		9	10	11	12	13	14	15	16	17
premium	Boston University	26.34	27.65	27.24	27.97	26.25	27.04	27.25	27.42	26.78
	Fenway	26.17	25.03	24.89	25.23	25.29	25.71	25.68	25.49	24.89
	Haymarket Square	21.22	21.82	22.07	21.65	21.67	20.65	22.32	22.65	21.94
	North End	24.3	25.31	24.54	25	25.34	23.26	24.75	23.33	23.77
	Northeastern University	26.6	25.27	25.15	25.74	25.46	24.76	25.36	25.53	26.12
	South Station	25.58	25.33	25.27	25.44	24.97	25.65	25.24	25.17	27.3
shared	Boston University	8.59	8.71	8.1	8	8.57	8.32	8.36	8.34	7.89
	Fenway	8.54	8.76	8.81	9.07	9.02	8.61	8.9	8.79	8.71
	Haymarket Square	8.02	7.96	7.79	8.23	7.83	8.11	8.01	8.09	8.71
	North End	9.2	9.42	9.15	9.93	9.55	9.05	9.28	9.08	9.63
	Northeastern University	8.86	8.88	8.46	9.19	8.61	8.92	8.87	8.84	8.44
	South Station	9.48	8.95	9.39	9.1	8.76	9.06	9.5	8.53	8.88
standard	Boston University	13.08	12.46	13.05	12.17	12.26	12.32	12.71	12.01	11.94
	Fenway	11.89	12.09	12.14	12.37	11.86	11.88	12.34	11.76	12.16
	Haymarket Square	9.37	9.58	9.42	9.74	9.81	9.64	9.33	9.25	9.97
	North End	11.28	11.29	11.06	11.63	11.89	11.06	11.63	11.71	11.52
	Northeastern University	12.79	12.74	12.17	12.82	12.38	12.98	12.71	13.23	12.96
	South Station	12.34	12.54	13	13.04	12.33	12.45	12.7	12.78	12.58

		Financial District								
		9	10	11	12	13	14	15	16	17
premium	Boston University	36.21	36.07	36.34	35.34	37.73	35.65	35.92	35.51	36.34
	Fenway	34.69	33.65	34.14	34.56	33.84	34.54	34.2	33.94	33.56
	Haymarket Square	20.54	20.86	20.46	20.96	21.71	21.65	21.51	21.61	21.35
	North End	21.95	20.37	20.99	20.89	21.76	21.07	21.77	20.66	21.38
	Northeastern University	34.23	33.08	31.21	33.5	34.19	33.02	33.81	33.53	33.35
	South Station	20.8	20.45	20.48	21.18	21.58	22.05	21.44	21.12	20.79
shared	Boston University	10.36	10.17	10.67	10.26	11.37	10.48	11.44	10.42	11.94
	Fenway	11.45	10.87	10.71	10.85	11.21	10.67	11.28	11.58	11.26
	Haymarket Square	7.39	7.24	7.07	7.15	7.23	7.06	7.3	7.31	7.13
	North End	8.09	7.89	7.89	7.7	7.96	7.89	7.71	7.97	8.02
	Northeastern University	11.35	10.28	10.69	10.8	10.78	10.6	10.56	10.9	11.17
	South Station	5.35	5.66	5.45	5.62	5.22	5.43	5.36	5.25	5.46
standard	Boston University	17.06	17.83	17.23	17.78	18.47	18.61	17.75	17.27	18.07
	Fenway	16.56	17.15	16.24	16.58	16.14	16.31	16.64	16.86	17.25
	Haymarket Square	8.7	8.5	8.75	8.37	8.49	8.52	8.97	8.8	8.76
	North End	9.83	9.59	9.89	9.69	9.73	9.45	9.95	9.9	9.84
	Northeastern University	17.15	15.6	16.16	15.22	15.84	15.91	15.69	16.2	16.37
	South Station	7.98	8.19	8.12	8.79	8.34	8.17	8.26	8.19	8.45

		North Station								
		9	10	11	12	13	14	15	16	17
premium	Boston University	31.25	30.08	31.12	29.66	30.96	31.47	30.16	31.3	29.92
	Fenway	30.09	29.06	28.8	29.1	29.46	28.52	29.2	29.01	29.85
	Haymarket Square	20.8	22.47	22.17	21.72	20.24	21.03	21.09	21.41	20.8
	North End	20.34	21.86	21.57	21.06	21.65	20.28	21.16	21.3	23.29
	Northeastern University	29.28	30.1	28.11	29.79	29.65	29.15	29.46	28.42	30.34
	South Station	24.12	24.27	23.53	24.25	23.29	23.74	23.52	23.95	22.91
shared	Boston University	9.31	9.58	9.75	10.39	9.6	10.2	9.71	9.86	9.4
	Fenway	8.76	8.98	9.3	8.77	9.58	9.06	9.29	9.35	8.97
	Haymarket Square	6.7	6.86	6.84	6.67	6.79	6.57	6.81	6.79	6.56
	North End	6.48	6.56	6.45	6.52	6.31	6.29	6.4	6.71	6.73
	Northeastern University	10.07	9.29	9.06	10.29	9.8	10.23	10.15	9.92	9.91
	South Station	8.54	8.27	8.41	8.67	8.59	8.36	8.76	8.53	8.52
standard	Boston University	12.63	13.8	14.3	14.88	14.84	13.71	13.48	14.43	13.98
	Fenway	13.69	14.05	14.17	13.86	13.92	13.98	13.94	15.03	14.37
	Haymarket Square	8.23	7.92	8.08	7.96	8.03	8.09	8.1	8.13	8.1
	North End	8.65	8.6	8.64	8.75	8.57	8.82	8.94	8.83	9
	Northeastern University	14.72	14.35	14.33	15.65	13.54	14	14.14	14.22	14.49
	South Station	11.35	11.23	10.91	11.31	11.69	11.48	11.24	11.28	11.06

		Theatre District								
		9	10	11	12	13	14	15	16	17
premium	Boston University	28.7	27.71	27.98	28.39	28.16	28.36	28.1	28.65	28.03
	Fenway	27.14	26.11	26.09	26.48	27.15	27.58	26.83	25.8	27.21
	Haymarket Square	21.21	21.39	21.5	22.05	21.84	21.56	21.89	22.35	21.62
	North End	23.14	23.78	21.83	23.45	21.96	22.17	22.76	22.7	22.72
	Northeastern University	23.94	22.71	23.06	23.45	23.34	24.31	23.04	23.47	25.03
	South Station	22.94	22.37	22.51	21.49	21.31	21.35	21.9	22.41	22.43
shared	Boston University	8.68	8.96	8.88	8.79	9.14	9	8.57	8.89	8.52
	Fenway	9.36	9.85	9.09	9.72	9.46	9.73	9.28	9.55	9.56
	Haymarket Square	7.96	7.82	7.88	7.31	7.98	8.22	7.82	8.03	7.8
	North End	9.02	9.42	8.61	8.8	9.2	8.72	8.57	8.92	9.05
	Northeastern University	8.13	8.09	8.79	8.39	8.37	8.79	8.53	8.73	8.18
	South Station	7.88	7.63	7.6	7.96	8.34	7.67	7.56	7.79	8.29
standard	Boston University	13.12	14.24	13.06	13.38	13.03	13.25	13.24	13.1	13.1
	Fenway	13.8	12.94	13.25	13.32	13.92	13.2	12.89	13.69	12.69
	Haymarket Square	9.54	9.9	10	10.04	9.89	9.9	9.66	10.31	10.04
	North End	11.05	11.31	10.77	11.19	11.03	10.32	10.56	10.96	10.78
	Northeastern University	10.67	11.18	10.59	11.2	11.12	10.89	11.44	10.97	11.07
	South Station	10.43	9.93	9.85	10.65	10.43	10	10.42	9.99	10.57

		West End								
		9	10	11	12	13	14	15	16	17
premium	Boston University	28.64	29.04	28.28	29.41	28.78	28.99	27.72	29.13	28.18
	Fenway	27.52	27.04	26.9	26.5	26.26	26.9	27.24	27.29	26.71
	Haymarket Square	20.72	20.92	21.39	21.62	21.74	20.74	20.44	20.92	21.45
	North End	20.44	20.99	22.29	21.3	21.86	20.92	22.39	20.94	21.97
	Northeastern University	27.6	28.06	26.69	26.71	27.59	26.9	27.68	27.36	26.77
	South Station	25	23.49	23.94	22.68	24.91	24.33	24.74	23.77	24.87
shared	Boston University	8.93	9.05	8.83	9	8.84	9.01	9.05	8.95	9.43
	Fenway	9.51	9.48	9.4	9.31	9.38	9.62	9.45	9.29	9.14
	Haymarket Square	6.48	6.34	6.54	6.73	6.65	6.66	6.64	6.69	6.78
	North End	7.28	7.23	7.2	7.25	7.09	7.02	7.21	7.52	7.11
	Northeastern University	9.35	9	8.92	8.97	9.57	9.25	8.79	9.26	9.09
	South Station	8.49	8.71	8.73	8.44	8.12	8.69	9.17	8.88	8.37
standard	Boston University	12.65	12.88	12.58	12.21	12.35	12.1	13.06	12.43	12.61
	Fenway	12.82	13.05	12.39	13.1	12.83	13.27	13.36	13	12.39
	Haymarket Square	8.71	8.57	8.85	8.82	8.51	8.7	8.86	9.28	8.88
	North End	9.84	9.59	9.52	9.77	9.44	9.63	10.44	9.31	9.52
	Northeastern University	13.18	12.52	12.86	11.73	13.07	12.84	12.43	13.36	12.19
	South Station	11.08	11.74	11.5	11.53	11.41	11.71	11.19	11.46	11.46

Table 5.4: Demand at destination at different times of the day

Sum of Demand	Time									
Location	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**Grand Total**
⊟ premium	**2339**	**2196**	**1935**	**2317**	**2638**	**2360**	**2807**	**2476**	**2343**	**21411**
Back Bay	367	338	313	417	480	405	478	412	411	3621
Beacon Hill	381	363	339	389	452	376	492	385	388	3565
Financial District	363	389	315	406	461	390	465	416	393	3598
North Station	410	331	335	376	415	403	437	437	399	3543
Theatre District	414	385	288	371	423	395	460	440	388	3564
West End	404	390	345	358	407	391	475	386	364	3520
⊟ shared	**1180**	**1067**	**1001**	**1139**	**1366**	**1235**	**1421**	**1207**	**1146**	**10762**
Back Bay	189	173	151	200	249	200	247	191	185	1785
Beacon Hill	180	175	175	177	231	201	237	222	184	1782
Financial District	203	177	151	199	233	201	237	185	197	1783
North Station	184	169	155	190	223	221	226	202	202	1772
Theatre District	211	187	177	179	225	212	239	204	197	1831
West End	213	186	192	194	205	200	235	203	181	1809
⊟ standard	**4685**	**4290**	**4097**	**4692**	**5364**	**4718**	**5639**	**4907**	**4728**	**43120**
Back Bay	777	680	629	784	926	799	947	779	795	7116
Beacon Hill	762	699	719	774	973	761	1008	794	770	7260
Financial District	797	746	665	833	875	740	942	803	790	7191
North Station	765	657	720	796	886	801	878	885	794	7182
Theatre District	823	786	658	786	906	801	924	876	814	7374
West End	761	722	706	719	798	816	940	770	765	6997
Grand Total	**8204**	**7553**	**7033**	**8148**	**9368**	**8313**	**9867**	**8590**	**8217**	**75293**

Table 5.5: Supply at origin at different times of the day

Sum of Supply Location	Time 9	10	11	12	13	14	15	16	17	Grand Total
premium	**2410**	**2180**	**1966**	**2262**	**2713**	**2341**	**2805**	**2423**	**2374**	**21474**
Boston University	398	336	309	399	421	391	480	418	400	3552
Fenway	438	377	311	366	460	342	475	382	364	3515
Haymarket Square	383	390	338	372	481	422	418	421	396	3621
North End	424	350	347	387	436	405	462	393	419	3623
Northeastern University	370	364	309	366	465	394	480	405	415	3568
South Station	397	363	352	372	450	387	490	404	380	3595
shared	**1162**	**1073**	**1009**	**1162**	**1345**	**1233**	**1417**	**1206**	**1167**	**10774**
Boston University	195	173	162	189	217	223	232	198	198	1787
Fenway	202	164	168	189	220	188	215	205	202	1753
Haymarket Square	172	205	166	195	228	202	247	195	190	1800
North End	221	183	186	198	218	219	237	214	190	1866
Northeastern University	188	168	169	189	227	203	226	186	202	1758
South Station	184	180	158	202	235	198	260	208	185	1810
standard	**4757**	**4397**	**4093**	**4606**	**5469**	**4726**	**5639**	**4856**	**4722**	**43265**
Boston University	805	706	662	749	898	783	921	798	869	7191
Fenway	804	746	711	758	904	734	951	824	769	7201
Haymarket Square	748	776	642	734	937	877	901	816	745	7176
North End	852	724	732	794	912	826	877	790	789	7296
Northeastern University	789	717	668	774	873	725	985	817	803	7151
South Station	759	728	678	797	945	781	1004	811	747	7250
Grand Total	**8329**	**7650**	**7068**	**8030**	**9527**	**8300**	**9861**	**8485**	**8263**	**75513**

In addition, we grouped the following types of cab in the raw data into the categories as follows:

Table 5.6: Categories of cabs in raw data and in this report

Category (Raw Data)	New Category (in this report)
UberPool	Shared
Black SUV, Black	Premium
UberX, UberXL, WAV, Taxi	Standard

5.3 Determining parameters

We have also set out the following parameters for certain variables as follows, for the purposes of this report:

Table 5.7: Table of Parameters and their rationale of values

<table>
<tr><th>Parameter</th><th>Value</th><th>Explanation</th></tr>
<tr><td>$\boldsymbol{I}$</td><td>0.3</td><td>Reflects a slight mark-up of the amount that Uber absorbs (25%) from its drivers, accounting for any additional unseen miscellaneous idle costs</td></tr>
<tr><td>$\boldsymbol{O}$</td><td>1</td><td>An arbitrary value</td></tr>
<tr><td>$\boldsymbol{f}$</td><td>4.6</td><td>Reflects fixed costs (cost of revenue expenses) which may include cost of vehicle ownership, consumer discounts, promotional offers and consumer loyalty expenses (Uber, 2021).</td></tr>
<tr><td>$\boldsymbol{\alpha}$</td><td colspan="2">Consists of approximation of maintenance costs for each type of vehicle
• $\boldsymbol{\alpha}$ was set to reflect different values for different types of cab z.
• $\boldsymbol{\alpha}$ will be highest for premium cabs, followed by standard cabs and shared cabs, as seen in Table 5.7.1.
Table 5.7.1: Values of α for each type of cab z
<table><tr><th>Category ($\boldsymbol{z}$)</th><th>$\boldsymbol{\alpha_z}$</th></tr><tr><td>Standard</td><td>0.11</td></tr><tr><td>Premium</td><td>0.20</td></tr><tr><td>Shared</td><td>0.08</td></tr></table></td></tr>
<tr><td>$\boldsymbol{z}$</td><td colspan="2">The budget for each type of cab is different as well. As demand for ‘Standard’</td></tr>
</table>

<table>
<tr>
<td></td>
<td>

rides are generally expected to be highest amongst 'Premium' and 'Shared' rides the following arbitrary values are used, as shown in Table 5.7.2.

Table 5.7.2: Table showing budget for each type of cab z

Category (z)	B_z
Standard	18469
Premium	9287
Shared	4659

</td>
</tr>
</table>

Table 5.8: Decision variable results

		Back Bay 9	10	11	12	13	14	15	16	17
premium	Boston University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Fenway	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	0.00%	0.00%	0.27%	84.41%	0.00%	0.00%	0.00%	0.00%
	North End	86.56%	96.57%	90.20%	100.00%	16.97%	100.00%	100.00%	100.00%	98.09%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	South Station	0.00%	0.00%	0.00%	7.80%	0.00%	0.00%	3.27%	4.70%	0.00%
shared	Boston University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Fenway	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	84.39%	0.00%	88.21%	6.14%	0.00%	4.86%	0.00%	0.00%
	North End	2.26%	0.00%	81.18%	10.10%	0.00%	91.32%	99.16%	86.92%	92.11%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	South Station	100.00%	0.00%	0.00%	3.96%	100.00%	0.00%	0.00%	2.40%	5.41%
standard	Boston University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Fenway	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	86.21%	0.00%	0.00%	1.49%	0.00%	7.77%	0.00%	0.81%
	North End	91.20%	1.52%	85.93%	98.74%	100.00%	96.73%	100.00%	98.61%	100.00%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

		Beacon Hill 9	10	11	12	13	14	15	16	17
premium	Boston University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Fenway	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	4.31%	0.00%	0.00%
	North End	4.25%	3.43%	9.80%	0.00%	83.03%	0.00%	0.00%	0.00%	1.91%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	South Station	91.44%	96.69%	86.65%	92.20%	20.00%	97.16%	96.73%	95.30%	100.00%
shared	Boston University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Fenway	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	12.68%	0.00%	0.00%	0.00%	0.00%	0.00%	99.49%	88.95%
	North End	81.45%	81.42%	9.14%	89.39%	100.00%	8.68%	0.84%	13.08%	7.89%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	South Station	0.00%	0.00%	100.00%	0.00%	0.00%	91.92%	90.38%	0.00%	0.00%
standard	Boston University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Fenway	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	2.99%	7.75%	0.00%	0.00%	0.00%
	North End	0.35%	0.00%	5.60%	0.00%	0.00%	3.27%	0.00%	0.00%	0.00%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.41%	0.00%	2.86%
	South Station	100.00%	96.02%	100.00%	97.11%	100.00%	85.28%	100.00%	97.90%	100.00%

		Financial District 9	10	11	12	13	14	15	16	17
premium	Boston University	84.17%	100.00%	1.29%	12.53%	100.00%	12.28%	96.88%	0.00%	98.25%
	Fenway	6.39%	14.06%	100.00%	97.27%	0.00%	100.00%	0.00%	95.03%	0.00%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	8.60%	0.00%	0.00%	13.09%	0.00%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
shared	Boston University	0.00%	0.00%	3.70%	0.00%	100.00%	0.90%	100.00%	0.00%	99.49%
	Fenway	98.51%	6.71%	0.00%	5.82%	7.27%	0.00%	2.33%	90.24%	0.00%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	2.13%	98.81%	85.80%	99.47%	0.00%	98.03%	0.00%	0.00%	0.00%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
standard	Boston University	99.01%	89.66%	100.00%	100.00%	97.44%	94.51%	100.00%	100.00%	90.91%
	Fenway	0.00%	15.15%	0.42%	11.08%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	2.13%	0.61%	0.00%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

		North Station 9	10	11	12	13	14	15	16	17
premium	Boston University	0.00%	0.00%	98.71%	0.00%	0.00%	87.72%	0.00%	100.00%	0.00%
	Fenway	93.61%	0.00%	0.00%	2.73%	8.04%	0.00%	90.95%	4.97%	0.00%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	0.00%	90.93%	9.71%	100.00%	81.29%	15.23%	1.04%	0.00%	96.14%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
shared	Boston University	0.00%	97.69%	95.68%	100.00%	0.00%	99.10%	0.00%	8.08%	0.00%
	Fenway	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	97.87%	0.00%	0.00%	0.53%	98.24%	0.00%	100.00%	100.00%	100.00%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
standard	Boston University	0.00%	0.00%	0.00%	0.00%	2.56%	5.49%	0.00%	0.00%	0.00%
	Fenway	0.00%	0.00%	99.58%	2.90%	87.17%	4.50%	0.00%	100.00%	1.82%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	96.96%	91.63%	1.80%	100.00%	8.59%	100.00%	89.14%	7.47%	97.14%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

		Theatre District 9	10	11	12	13	14	15	16	17
premium	Boston University	15.83%	0.00%	0.00%	0.00%	0.00%	0.00%	3.13%	0.00%	0.00%
	Fenway	0.00%	0.00%	0.00%	0.00%	91.96%	0.00%	9.05%	0.00%	100.00%
	Haymarket Square	81.46%	98.72%	85.21%	99.73%	0.00%	93.60%	95.69%	100.00%	6.06%
	North End	9.20%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
shared	Boston University	0.00%	0.00%	0.62%	0.00%	0.00%	0.00%	0.00%	91.92%	0.00%
	Fenway	0.00%	93.29%	0.00%	94.18%	5.00%	5.32%	0.00%	9.76%	97.52%
	Haymarket Square	100.00%	0.00%	95.18%	0.00%	93.86%	100.00%	95.14%	0.51%	0.00%
	North End	16.29%	18.58%	9.68%	0.51%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
standard	Boston University	0.00%	10.34%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	8.63%
	Fenway	93.41%	0.00%	0.00%	5.54%	12.83%	0.00%	9.78%	0.00%	0.00%
	Haymarket Square	0.00%	0.00%	92.83%	100.00%	84.31%	91.33%	92.23%	100.00%	99.19%
	North End	8.45%	98.48%	8.47%	1.26%	0.00%	0.00%	0.00%	1.39%	0.00%
	Northeastern University	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	South Station	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

		West End 9	10	11	12	13	14	15	16	17
premium	Boston University	0.00%	0.00%	0.00%	87.47%	0.00%	0.00%	0.00%	0.00%	1.75%
	Fenway	0.00%	85.94%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Haymarket Square	0.00%	1.28%	5.62%	0.00%	0.00%	6.40%	0.00%	0.00%	86.11%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	100.00%	9.07%	90.29%	0.00%	10.11%	84.77%	98.96%	86.91%	3.86%
	South Station	8.56%	3.31%	13.35%	0.00%	80.00%	2.84%	0.00%	0.00%	0.00%
shared	Boston University	100.00%	2.31%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.51%
	Fenway	1.49%	0.00%	100.00%	0.00%	87.73%	94.68%	97.67%	0.00%	2.48%
	Haymarket Square	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	0.00%	1.19%	14.20%	0.00%	1.76%	1.97%	0.00%	0.00%	0.00%
	South Station	0.00%	100.00%	0.00%	96.04%	0.00%	8.08%	9.62%	97.60%	94.59%
standard	Boston University	0.99%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.46%
	Fenway	6.59%	84.85%	0.00%	80.47%	0.00%	95.50%	90.22%	0.00%	98.18%
	Haymarket Square	90.37%	0.00%	7.17%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	North End	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	Northeastern University	3.04%	8.37%	98.20%	0.00%	91.41%	0.00%	8.32%	91.92%	0.00%
	South Station	0.00%	3.98%	0.00%	2.89%	0.00%	14.72%	0.00%	2.10%	0.00%

5.4 Results

Table 5.8 illustrates the decision variable results denoting the optimal cab proportion allocation from different areas for every working hour. Exact numbers of each type of cab allocated throughout the day can be found in Appendix 9.1 and Appendix 9.2. Table 5.9 shows that the allocation proposed by our model should yield $1,261,063 in profit while keeping total idle and opportunity costs at around 0.0001% and 0.0002% of the total profit. From the observed results, it is clear that for the majority of origins, for all types, almost all cabs should be primarily allocated to a single destination. For instance, consider premium cabs starting at North End, where for almost all hours the majority of active drivers should be allocated to passengers going to Black Bay, especially at 1200, 1400, 1500 and 1600 hours when the entire supply should be allocated. Interestingly, allocation of shared cabs seems to be more time dependent, where all cabs starting at Northeastern University should be allocated to North Station at 0900, 1300, 1500, 1600 and 1700 hours. Simultaneously, almost no Northeaster University shared cabs should be allocated North Station at any other time. Thus, the model suggests that there may be a clear preferred destination for cabs depending on their origin.

Table 5.9: Table of Results

	Dollars ($)
Total Profit	1,261,063
Total Idle Cost	167
Total Opportunity Cost	338

Table 5.10: Supply utilization rate

		Supply								
		9	10	11	12	13	14	15	16	17
premium	Boston University	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Fenway	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Haymarket Square	81.5%	100.0%	90.8%	100.0%	84.4%	100.0%	100.0%	100.0%	92.2%
	North End	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Northeastern University	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	South Station	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
shared	Boston University	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Fenway	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Haymarket Square	100.0%	97.1%	95.2%	88.2%	100.0%	100.0%	100.0%	100.0%	88.9%
	North End	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Northeastern University	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	South Station	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
standard	Boston University	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Fenway	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Haymarket Square	90.4%	86.2%	100.0%	100.0%	88.8%	99.1%	100.0%	100.0%	100.0%
	North End	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Northeastern University	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	South Station	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Furthermore, Table 5.10 illustrates the supply utilization rate. Overall, almost the entirety of active drivers should be allocated to destinations irrespective of the hour. Nevertheless, the idle rate of Haymarket Square is high, especially for premium cabs at 0900. Essentially, the observed solution suggests that Uber could benefit from discouraging drivers to start working in Haymarket Square at specific hours highlighted by Table 5.10 as this would increase the overall profit by completely removing the associated idle cost.

Table 5.11: Demand satisfaction rate

		Demand								
		9	10	11	12	13	14	15	16	17
premium	Back Bay	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Beacon Hill	100.0%	100.0%	100.0%	88.2%	100.0%	100.0%	100.0%	100.0%	100.0%
	Financial District	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	North Station	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Theatre District	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	99.6%	95.7%	100.0%
	West End	100.0%	95.9%	100.0%	97.5%	100.0%	95.1%	100.0%	91.2%	100.0%
shared	Back Bay	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Beacon Hill	100.0%	100.0%	100.0%	100.0%	94.4%	100.0%	100.0%	100.0%	100.0%
	Financial District	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	North Station	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Theatre District	98.6%	100.0%	100.0%	100.0%	100.0%	100.0%	98.3%	99.5%	100.0%
	West End	93.0%	100.0%	100.0%	100.0%	96.1%	99.0%	100.0%	100.0%	100.0%
standard	Back Bay	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Beacon Hill	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Financial District	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	North Station	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Theatre District	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	94.4%	100.0%
	West End	100.0%	100.0%	99.4%	88.0%	100.0%	100.0%	100.0%	99.7%	99.2%

Additionally, Table 5.11 demonstrates demand satisfaction rate by the obtained solution which corresponds to proportion of destination demand that was met by the proposed cab allocation.

Similarly to supply utilization, most of demand for all cab types, hours and areas is satisfied with the lowest satisfaction rate dropping to 88.0% for standard cabs in the West End area at 12. This suggests that West End may be among the least profitable as a destination for Uber as higher opportunity costs are tolerated by the model in favour of higher profits elsewhere.

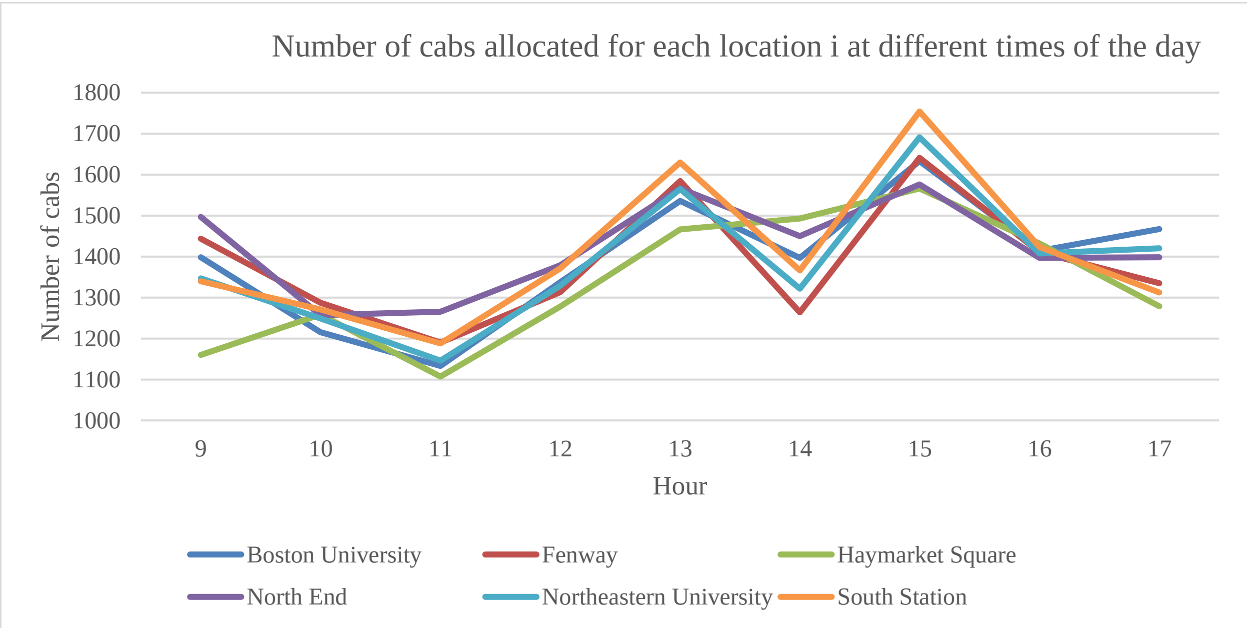

Figure 5.1: Number of cabs allocated for each location i at different times of the day

Figure 5.1 shows the trends for number of cabs allocated at each origin node throughout the day. We note that the allocation of cabs for North End and Haymarket Square does not seem to fluctuate as much as the different locations throughout the day. In addition, the pattern of allocation of cabs for Haymarket Square is inconsistent with other locations during some times of the day. This suggests that human traffic flow or types of activities conducted at Haymarket Square may be different as compared to other locations.

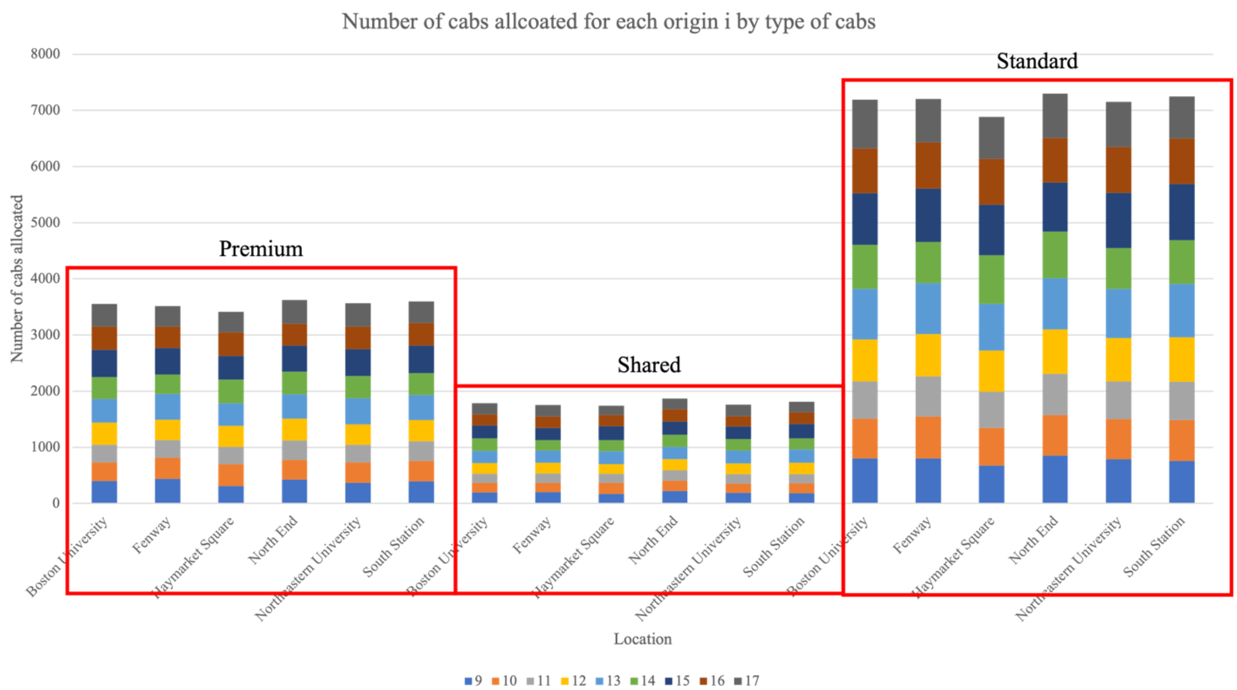

Figure 5.2: Number of cabs allocated for each origin i by type of cabs

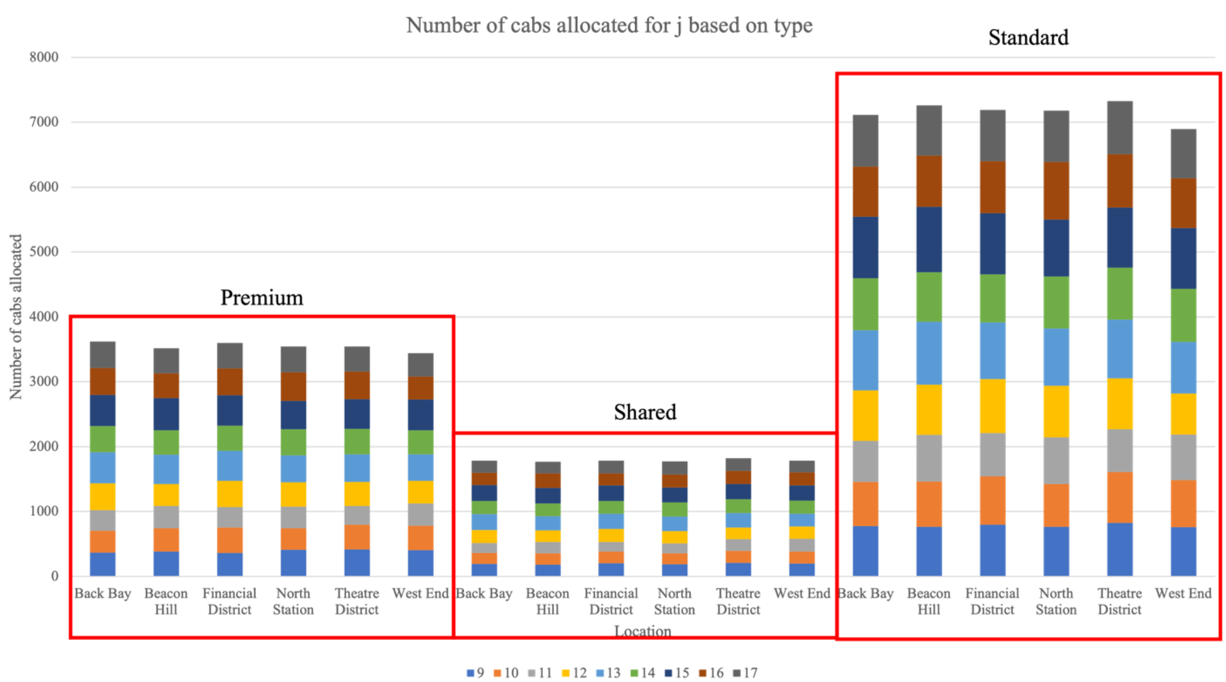

Figure 5.3: Number of cabs allocated for each destination j by type of cabs

From Figures 5.2 and 5.3, we deduce that for any single day, there is still an approximately equal number of cabs allocated to each origin and destination. Haymarket Square and West End has the lowest number of cabs allocated on average, further strengthening our hypothesis that the aforementioned locations may have reduced human activities.

5.5 Sensitivity Analyses

Table 5.12: Sensitivity Analysis on α_z

alpha (premium)	alpha (shared)	alpha (standard)	TotalProfit	Total Idle Cost	Total Opportunity Cost	Number of Cabs	Utilization Rate	Demand Satisfaction Rate
0.1	0.04	0.055	1275295	167	338	74955	99.26%	99.55%
0.15	0.06	0.0825	1268179	167	338	74955	99.26%	99.55%
0.2	0.08	0.11	1261063	167	338	74955	99.26%	99.55%
0.25	0.1	0.1375	1253947	167	338	74955	99.26%	99.55%
0.3	0.12	0.165	1246831	167	338	74955	99.26%	99.55%

Table 5.13: Sensitivity Analysis on I

I	TotalProfit	Total Idle Cost	Total Opportunity Cost	Number of Cabs	Utilization Rate	Demand Satisfaction Rate
0.15	1261147	84	338	74955	99.26%	99.55%
0.225	1261105	126	338	74955	99.26%	99.55%
0.3	1261063	167	338	74955	99.26%	99.55%
0.375	1261021	209	338	74955	99.26%	99.55%
0.45	1260979	251	338	74955	99.26%	99.55%

Table 5.14: Sensitivity Analysis on O

O	TotalProfit	Total Idle Cost	Total Opportunity Cost	Number of Cabs	Utilization Rate	Demand Satisfaction Rate
0.5	1261232	167	169	74955	99.26%	99.55%
0.75	1261147	167	254	74955	99.26%	99.55%
1	1261063	167	338	74955	99.26%	99.55%
1.25	1260978	167	423	74955	99.26%	99.55%
1.5	1260894	167	507	74955	99.26%	99.55%

As can be seen from Tables 5.12, 5.13 and 5.14, any changes in α, I and O resulted in changes in only the total profit. There were no changes to idle costs, opportunity costs, total number of cabs deployed, utilization rate nor demand satisfaction rate. Thus, this seems to suggest that any changes in these values may only impact Uber's financial earnings, and minimal impact on customer satisfaction.

Furthermore, shifts in demand and supply could similarly impact total profit as demonstrated by respective shadow prices in Table 5.15. For instance, an increase by a single premium cab at 1400 at Boston University would lead to a subsequent increase in total profit by \$29.83. Similarly, a rise in demand for a single premium cab at 1400 hours for Black Bay destination would only lead to a \$4.16 increase in profits. Note that all null values like demand for West End at 1700 hours mean that there would be no associated increase in total profit.

Table 5.15: Table depicting shadow prices

Supply (Constraint 8)

Premium

t	9	10	11	12	13	14	15	16	17
i									
Boston University	7.167105	29.6499	7.858098	30.0199	6.279726	29.82885	28.68966	30.58759	6.279153
Fenway	5.74187	27.67106	5.562589	29.33467	4.804218	27.55447	27.47628	28.32208	4.830312
Haymarket Square		21.98075		22.88154		21.80075	22.80256	23.26256	
North End	1.904373	24.38706	1.676305	25.83157	2.967934	22.85706	25.11666	23.82666	1.714102
Northeastern University	5.335843	28.66605	4.845304	29.99738	4.966933	27.50605	27.70899	27.96605	4.865304
South Station	3.078094	24.30078	2.300026	26.16529	2.491655	25.14078	25.50038	25.56038	5.137823

Shared

t	9	10	11	12	13	14	15	16	17
i									
Boston University	9.645132	1.23051	0.870842	0.637151	10.33354	10.03293	9.997335	9.845864	1.115618
Fenway	10.48243	1.747381	1.44034	1.188914	10.35243	10.59243	10.4058	10.52851	0.834082
Haymarket Square	8.961131				8.978935	8.873182	8.905022	9.115022	
North End	10.09477	1.413641	0.719749	0.365174	10.60263	9.766822	10.0202	10.05866	0.873641
Northeastern University	10.40401	1.17897	0.950337	1.160503	10.53242	10.21242	9.735795	9.904747	1.426113
South Station	10.33226	0.970859	0.917238	0.157501	9.770116	9.734311	10.19768	9.47099	0.135968

Standard

t	9	10	11	12	13	14	15	16	17
i									
Boston University	3.692034	3.984866	13.70487	14.99361	4.790078	3.865489	14.10431	14.44487	13.48545
Fenway	3.873672	3.356987	13.58834	13.98708	3.883548	4.14896	13.78545	15.05834	13.27708
Haymarket Square			9.973413	10.85354			10.70191	11.35191	10.60304
North End	1.256028	1.218364	10.72932	11.98944	2.016256	1.356256	11.82134	11.98781	12.5214
Northeastern University	4.219917	2.813233	13.73333	12.65677	3.488541	4.153953	12.8417	14.23333	13.38208
South Station	2.257575	2.14583	12.61086	12.51593	2.397803	2.687803	12.83289	12.44593	13.00327

Demand (Constraint 9)

Premium

t	9	10	11	12	13	14	15	16	17
j									
Back Bay	28.62712	6.064441	29.40519	3.262026	27.23356	4.156537	4.216936	3.956936	28.6274
Beacon Hill	23.22719	1.754512	23.69526		23.20363	1.234512	0.46491	0.33491	22.88747
Financial District	29.27084	6.301644	28.70984	5.548038	29.59975	6.049093	7.458281	5.940624	30.28879
North Station	24.9231	1.981631	23.81238	0.340302	25.23075	2.191631	2.298691	1.262889	26.02238
Theatre District	22.12256	0.321805	22.41256	0.081019	22.99206	0.671805			22.53256
West End	22.73268		22.45075		23.22912		0.57706		22.51075

Shared

t	9	10	11	12	13	14	15	16	17
j									
Back Bay	1.587827	10.55743	11.03285	11.10743	1.29849	1.835776	1.662403	2.283936	10.83896
Beacon Hill	0.157856	9.058986	9.482878	10.61745		0.335805	0.312431	0.073964	9.808986
Financial District	1.816657	9.971701	10.61033	10.51017	1.706657	1.25825	1.723284	1.900571	11.63556
North Station	0.60506	9.289682	9.81935	10.06857	0.206653	1.107265	0.652857	0.954328	9.422962
Theatre District		9.081131	8.965022	9.509598	0.086087	0.116087			9.70443
West End		8.783452	8.932085	9.32681			0.016627	0.453321	9.278343

Standard

t	9	10	11	12	13	14	15	16	17
j									
Back Bay	13.88129	13.25896	5.038005	3.037882	13.09896	13.98107	1.897054	3.229512	2.07592
Beacon Hill	11.02133	11.33308	1.328044	1.462981	10.87111	10.70111	0.806019	1.272981	0.515641
Financial District	14.03333	14.5105	3.369152	3.310404	14.34529	15.40988	3.595474	2.713843	5.249921
North Station	11.34131	12.37799	1.437897	0.72915	10.89269	10.68727	2.139528	0.827897	1.94915
Theatre District	10.82178	11.11945	1.068493	0.228369	10.93191	10.94191			0.478866
West End	9.833413	10.5801			10.45479	10.00812	0.461631		

6. Limitations

The limitations for our model are as follows:

1. LP relaxation could lead to a non-optimal solution; nevertheless, given that the data used in this project is an approximation, it is understood that the model would still achieve its aim of providing a suggested proportion of cabs to be allocated to each area from the source.
2. Given time and hardware constraints, a small dataset with limited number of variables and nodes can only be used for purposes of this report. As a result, any benefits to be gained by Uber from our optimization may be limited or even inaccurate as it fails to consider all nodes within the dataset.
3. Currently, α_z value for variable costs, f for fixed costs, as well as B_z, representing Uber's budget are arbitrary in nature and may not well reflect the extent of benefits that the model provided may bring. Hence, any profits to be gained may be inaccurate and/or understated.
4. Parameters are assumed to be constant, but this may not necessarily be true in real life as variables such as passenger demand may be dynamic in nature due to external circumstances, such as weather.

7. Conclusion

While integer programming may be influential in playing a part in optimizing the geographical allocation of cabs, one must consider the long computational times it takes as opposed to calculating the proportion of total fleet size. In addition, models should take into account other business factors such as idle costs and opportunity costs in the event of over-allocation or under-allocation of cabs, especially at different times of the day. Our model produced provides all of the aforementioned that other models may have missed, providing an all-round, customizable model that fits into Uber's hail-riding business prospective. Our study also showed the optimum proportion of cabs to be allocated at different times of the day at different locations and their resulting fleet utilization and demand fulfillment, such as allocating almost the entire fleet with their origin at North End to Back Bay almost throughout the day**.** Important assumptions such as the independence of the number of cabs from each interval to another, and fixed drop-off points needed to be made to ensure the overall feasibility of our solutions. Nonetheless, due to time and hardware constraints, only a limited section of data could be used, limiting the overall usefulness of our findings. The ceteris paribus assumption was also made, when in real-life, demand may be dynamic due to extraneous factors. More location nodes and more types of cabs can be considered in future research, barring any time nor hardware constraints. Similarly, more research can be done in future to depict a clearer picture of fixed and variable costs to ensure that the formulations and solutions derived can better fit Uber's business model. Future works can include considering the influence of extraneous factors in the model, through including additional constraints and parameters to better complement Uber's business strategy at a more holistic level.

8. References

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9. Appendix

Appendix 9.1

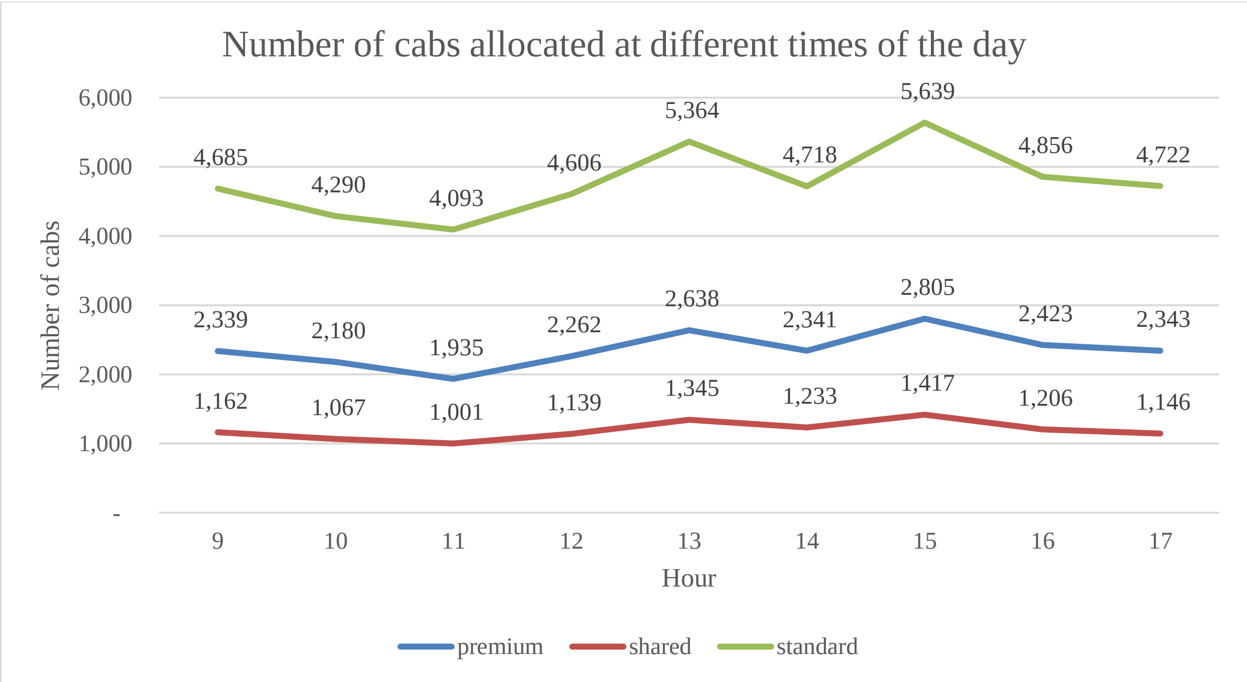

Figure 9.1: Total number of cabs allocated at different times of the day

Figure 9.1 shows that across all times, the number of standard cabs to be allocated is consistently higher than that of premium cabs and shared cabs. There is also a spike in number of cabs allocated across all types from 1300-1400 hours and 1500-1600 hours. This may be attributed to potential lunch times or increased activity during aforementioned hours.

Appendix 9.2

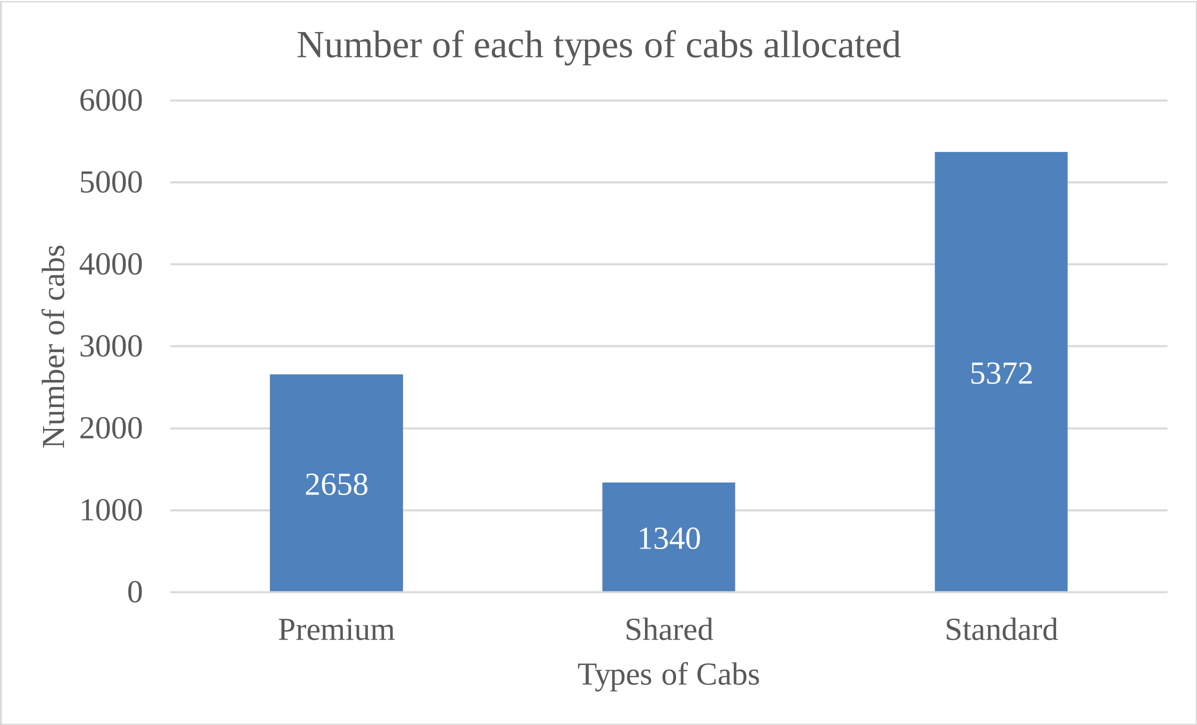

Figure 9.2: Number of each types of cabs allocated

The average number of standard cabs allocated for the day is the highest (5372), followed by that of premium cabs (2658) and shared cabs (1340).