Exercise 2
4.-order Runge-Kutta

Solve computationally the time-dependent Schrödinger equation for the interacting two-level system using 4.-order Runge-Kutta method. The Hamiltonian is

$$H = \begin{pmatrix} E_1 & V \\ V & E_2 \end{pmatrix}. \tag{1}$$

Make plots with the time dependence of the occupation of the $|1\rangle$ and the $|2\rangle$ level for different energy splittings $E_1 - E_2$ and interaction strengths V. Check that your results agrees with the analytical solution for $E_1 - E_2 = 0$ and the approximate solution for $|V| \ll |E_1 - E_2|$. After you made sure that your numerical solution agrees with the analytical, make the interaction time-dependent: $V(t) = V_0 \cos(\omega t)$ and investigate the behavior of the occupations.

It makes sense to choose parameters that are reasonable for physical systems. You can see from your output, how fast the processes happen depending on energies. The energy splitting $|E_1 - E_2|$ depend on a particular quantum system. The splitting can be from $\sim$10 meV to $\sim$1 eV. The interaction V could be much smaller than the energy splitting $|E_1 - E_2|$ or could be of the same order as $|E_1 - E_2|$. It is usually not much larger than $|E_1 - E_2|$.

All parameters that can be varied must be defined and read from an input file. The time units must be attoseconds, femtoseconds or picoseconds. The report of the results must be in a pdf file/presentation.