UNIT-1
IMAGE FUNDAMENTALS

→ Resolution

1] Rows in digital image → vertical resolution
columns in digital image → horizontal resolution

2] Spatial resolution depends on:
- no. of pixels
-

3] No. of bits necessary to represent an image:
no. of rows × no. of columns × bit depth

4]

UNIT-2

IMAGE ENHANCEMENT

→ Point Processing

$$s = T(r)$$

where

r = pixel value of the original image at (x,y)

s = pixel value of the processed image at (x,y)

T = a grey level transformation function for a point.

$$f(x,y) \rightarrow g(x,y)$$

→ Neighbourhood Processing

$$g(x,y) = T[f(x,y)]$$

$f(x,y)$ is input image

$g(x,y)$ is processed image

1) Find the digital negative of the following image

20	0	100	15
10	25	255	30
0	10	55	10
15	0	200	100

235	255	155	240
245	230	0	225
255	245	200	245
240	255	55	155

2] Apply a threshold of 30% of maximum to given image.
Image matrix is given by $A = \begin{bmatrix} 2 & 3 & 0 & 7 \\ 0 & 3 & 7 & 2 \\ 5 & 3 & 2 & 0 \\ 4 & 2 & 2 & 0 \\ 1 & 7 & 6 & 5 \end{bmatrix}$

Applying threshold to the above image by assuming that threshold is 3.

$$s = \begin{cases} 1 ; & r > 3 \\ 0 ; & r \leq 3 \end{cases}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 7 \\ 0 & 0 & 7 & 0 \\ 7 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 \\ 0 & 7 & 7 & 7 \end{bmatrix}$$

$$T = 0.3 \times 7 = 2.1 = 2$$

$$s = \begin{cases} 1 ; & r > 2 \\ 0 ; & r \leq 2 \end{cases}$$

$$A = \begin{bmatrix} 0 & 7 & 0 & 7 \\ 0 & 7 & 7 & 0 \\ 7 & 7 & 0 & 0 \\ 7 & 0 & 0 & 0 \\ 0 & 7 & 7 & 7 \end{bmatrix}$$

→ Gray / Intensity Level Slicing

$$s = L-1, \quad r1 \le r \le r2$$
$$= 0, \quad \text{otherwise}$$
} without background.

with background.

$$s = L-1, \quad r1 \le r \le r2$$
$$= r, \quad \text{otherwise.}$$

1] Perform grey level slicing / intensity slicing such that Highlight pixel with intensity in the range 40-70% of max possible intensity and Keep other pixels unchanged.

3-bit image matrix, $f(x,y) = A = \begin{bmatrix} 0 & 3 & 2 & 6 & 4 \\ 6 & 3 & 4 & 5 & 2 \\ 5 & 3 & 2 & 1 & 2 \\ 4 & 2 & 3 & 6 & 5 \\ 5 & 3 & 6 & 4 & 5 \end{bmatrix}$

$(2^3-1$ (3-bits))

$r1 = 0.4 \times 7 \approx 3$

$r2 =$ to $0.7 \times 7 \approx 5$

$$s = L-1, \quad r1 \le r \le r2$$
$$= r, \quad \text{otherwise.}$$

$s = 7 \quad (2^3-1)$ (max possible value), $3 \le r \le 5$

$= r \quad r < 3$ and $r > 5$.

with background:

$$\begin{bmatrix} 0 & 7 & 2 & 6 & 7 \\ 6 & 7 & 7 & 7 & 2 \\ 7 & 7 & 2 & 1 & 2 \\ 7 & 7 & 7 & 6 & 7 \\ 7 & 7 & 6 & 7 & 7 \end{bmatrix}$$

without background:

$$\begin{bmatrix} 0 & 7 & 0 & 0 & 7 \\ 0 & 7 & 7 & 7 & 0 \\ 7 & 7 & 0 & 0 & 0 \\ 7 & 7 & 7 & 0 & 7 \\ 7 & 7 & 0 & 7 & 7 \end{bmatrix}$$

→ Log Transformations

1] Log operator is used to reduce something.

2] Reverse of log operation is applied to increase something.

3] When the difference between the intensities of pixels of an image is very large. i.e. some are very high intensity and some are very low intensity, so the low intensity ones will be ignored. To reduce the difference between these pixels, log transformations are applied.

4] $s = c * \log(1+r)$; c = constant

5] Eg: $r=0, s=0$

$r = 10^6$

$s = \log_{10}(1+10^6) = 6.$

6] $c = \dfrac{L}{\log_{10}(1+L)}$

1] Perform log transformations to the given image using constant $c = 8$.

Image matrix is given by:

$$A = \begin{bmatrix} 2 & 3 & 0 & 6 & 7 \\ 0 & 3 & 7 & 5 & 2 \\ 5 & 3 & 2 & 4 & 0 \\ 4 & 2 & 2 & 1 & 0 \\ 1 & 7 & 6 & 4 & 5 \end{bmatrix}$$

$$A2 \sim \begin{bmatrix} 4 & 5 & 0 & 7 & 7 \\ 0 & 5 & 7 & 6 & 4 \\ 6 & 5 & 4 & 6 & 0 \\ 6 & 4 & 4 & 2 & 0 \\ 2 & 7 & 7 & 6 & 6 \end{bmatrix}$$

$A_{11} = 8 \times \log(1+2) = 3.81$

2] Find constant c and perform log transformation for the given 7-bit image.

$$A = \begin{bmatrix} 110 & 120 & 90 \\ 91 & 94 & 98 \\ 90 & 91 & 99 \end{bmatrix}$$

$$c = \frac{L}{\log_{10}(1+L)}$$

$$c = \frac{2^b}{\log_{10}(1+2^b)}$$

$$= \frac{2^7}{\log_{10}(1+2^7)} = 60.64 = 61$$

$$A2 = 61 \times \log_{10}(1+r)$$

$$= \begin{bmatrix} 125 & 127 & 120 \\ 120 & 121 & 122 \\ 120 & 120 & 122 \end{bmatrix}$$

→ Power Law Transformation

1] $s = c \times r^{\gamma}$ $\gamma \rightarrow$ gamma
↳ alphabet 'r'

c and γ are positive constant.

2] For $\gamma < 1$, map a narrow range of dark input values into a wider range of output values.

3] For $\gamma > 1$, map a narrow range of light input values into a wider range of output values
i.e. you can see more details

1] Perform power law transformation on input image A given below with C = 3 and 2nd root.
Image matrix is given by $A = \begin{bmatrix} 2 & 3 & 0 & 6 & 7 \\ 0 & 3 & 7 & 5 & 2 \\ 5 & 3 & 2 & 4 & 0 \\ 4 & 2 & 2 & 1 & 0 \\ 1 & 7 & 6 & 4 & 5 \end{bmatrix}$

$S = 3 \times r^{1/2}$ [r = 2nd root = square root of r].

$A = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$

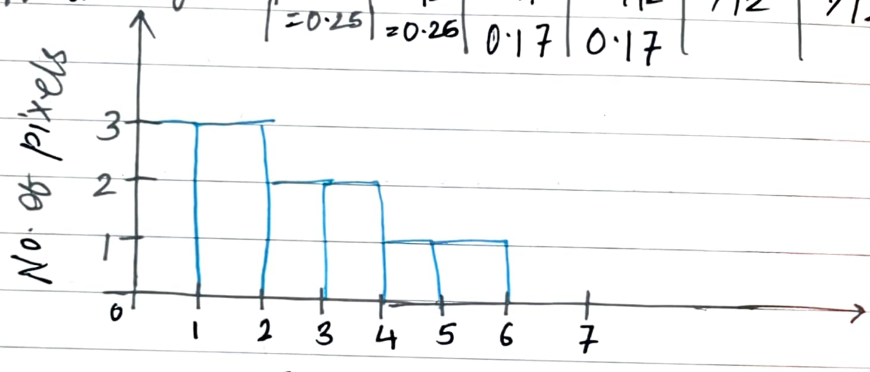
→ Image Enhancement by Histogram Processing

→

1] Plot of number of the grey level of following image

$$A = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & 2 \\ 1 & 2 & 4 \\ 3 & 5 & 0 \end{bmatrix}$$

Pixel intensity	0	1	2	3	4	5
No. of pixels	3	3	2	2	1	1
Probability	3/12 =0.25	3/12 =0.26	2/12 0.17	2/12 0.17	1/12	1/12



$$\text{Mean} = \sum_i r_i \, p(r_i)$$
$$= (0 \times 0.25) + (1 \times 0.25) + \cdots$$
$$= 1.82$$

→ Information shown by Histogram

1] Dark image → Right skewed histogram.
2] Bright image → left skewed.
3] High contrast → Distributed histogram.
4] Types of Processing :
(i) Histogram Equalization.
(ii) Histogram Stretching

→ Histogram Equalization

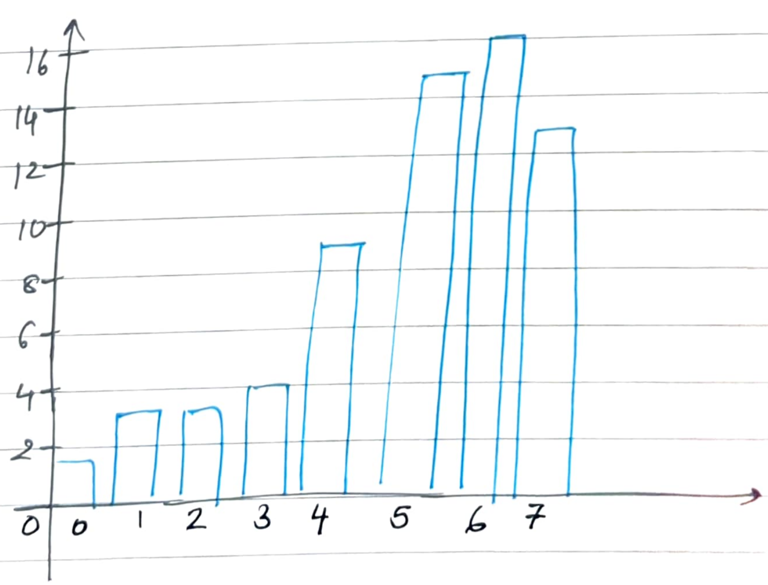
1] Also known as histogram flattering.

2] It is a preprocessing technique that removes noise, etc. from images to generate clean images.

3]

1] Histogram of Original Image. Apply Histogram Equalization.

Gray Level (i.e. intensity)	Count
0	1
1	3
2	3
3	4
4	9
5	15
6	16
7	13
	Total: 64



Equalization:

Gray level (K=8 total levels)	Count	Probability Density Func. (Count/Total Count)	Cumulative Distribution Freq. CDF	(K-1)*CDF	FLOOR ((K-1)*CDF)
0	1	0.0156	0.0156	0.1092	0
1	3	0.0468	0.0624	0.4368	0
2	3	0.0468	0.1092	0.7644	0
3	4	0.0625	0.1717	1.2019	1
4	9	0.1406	0.3123	2.1861	2
5	15	0.2343	0.5466	3.8262	3
6	16	0.25	0.7966	5.5762	5
7	13	0.2031	0.9997	6.9979	7
	64				

in this column round off the fourth digit according to the fifth one otherwise the ans. will vary

Gray level	Count
0	7
1	4
2	
3	
4	
5	
6	
7	

2] Equalize the given histogram:

Gray levels (r)	0	1	2	3	4	5	6	7
No. of pixels	790	1023	850	656	329	245	122	81

Gray Level	Count	PDF	CDF
0	790	0.1929	0.1929
1	1023	0.2498	0.4427
2	850	0.2075	0.6502
3	656	0.1602	0.8104
4	329	0.0803	0.8907
5	245	0.0598	0.9505
6	122	0.0298	0.9803
7	81	0.0198	1.000
	4096		

→ Spatial Filtering

1] It is used to blur or sharpen images.

2]

- Image Processing
  - Spatial Domain
    - Intensity Transformation (operate on single pixel by pixel)
    - Spatial Filtering
  - Frequency Domain

3] It is never going to be of the size 1×1. It will always be of 3×3 size of more than 3×3 but odd number only like 5×5 or 7×7, etc.

$\xrightarrow{\quad}$ x

$\downarrow$ y

$f(x-1, y-1)$	$f(x, y-1)$	$f(x+1, y-1)$
$f(x-1, y)$	$f(x, y)$	$f(x+1, y)$
$f(x-1, y+1)$	$f(x, y+1)$	$f(x+1, y+1)$

1) Original image.

7	9	11
10	50	8
9	5	6

3x3 Avg. Mask.

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

→ * if the filter is not given in the question, use this one by default.

Input image after zero padding:

0	0	0	0	0
0	7	9	11	0
0	10	50	8	0
0	9	5	6	0
0	0	0	0	0

Output:

8.4	10.56	8.67
10	12.78	9.89
8.2	9.78	7.67

$\xrightarrow{\text{Round}}$

8	11	9
10	13	10
8	10	8

1] Bigger the filter, the more blurry the image is.
2] Sharpness decreases when mask size increases.

→ Spatial Domain Low Pass / Averaging / Smoothing Filter

1] Replace every pixel with average of intensity levels of neighbourhood defined by mask.
2] The filter reduces the sharp transitions in intensity and also called a smoothening filter for this reason.
3] Reduces noise and highlights gross details in images.
~~4] Advantages:~~
4] Mask size determines degrees of smoothening (i.e. loss of detail).

→ Weighted Smoothing Filters.

1] The maximum weight must be at the centre.
2] (i) Min: The centre pixel will be the replaced by the least value ~~o~~ present in the neighbourhood.
(ii) Max: The centre pixel will be replaced by the highest value present in the neighbourhood.
(iii) Median: The centre pixel will be replace by the median of the neighbouring + centre pixel.
3] Median filter works better than averaging filter for salt & pepper noise. With averaging filter the noise also gets blurred, it gets reduced but not eliminated entirely.

→ Salt and Pepper Noise

1] Black and white noise
2] Introduced when some pixels become dead i.e. black 0 or white 255.

3] Median filter preserves the edges whereas average filter blurs the edges.

→ Filter Categories

1] Sharpening (high pass) : (i) sum of the filter is 0.
   (ii) used for medical images
   (iii) The output has no bg.
   (iv) Enhance the edges, ~~blur~~ sharpen blur images.
   (v) The positive (or highest) value is kept at the center.

→ Sharpening spatial Filters

1] Low pass filter → allows low frequency components to pass through it and blocks high frequency components
2] High pass filter → allow high freq. components to pass through it and blocks low freq. component
3] Highlights edges.

4] 1st derivative:

$$\frac{\partial f(x)}{\partial x} = \cancel{f(x) =} \ \cancel{f(x+1)} - f(x) - f(x-1)$$

5] 2nd derivative:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x).$$

6] The Laplacian is defined as follows:

$$\nabla^2 f = \frac{\partial^2 f}{\partial^2 x} + \frac{\partial^2 f}{\partial^2 y}$$

x - direction:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y).$$

y - direction:

$$\frac{\partial^2 f}{\partial^2 y} = f(x, y+1) + f(x, y-1) - 2f(x, y).$$

$$g(x, y) = f(x, y) - \nabla^2 f$$

↳ simplified Image Enhancement Formula

1] 3x3 High pass masks.

0	-1	0
-1	4	-1
0	-1	0

Image:

	0	1	2	3	4	5	6	7
0	10	10	10	10	10	10	10	100
1	10	10	10	10	10	10	10	10
2	10	10	10	10	10	10	10	10
3	10	10	10	10	10	10	10	10
4	100	100	100	100	100	100	100	100
5	100	100	100	100	100	100	100	100
6	100	100	100	100	100	100	100	100
7	100	100	100	100	100	100	100	100

I_{11}:

$$\frac{\partial^2 f}{\partial x^2} = 10 + 10 - 20 = 0$$

$$\frac{\partial^2 f}{\partial^2 y} = 10 + 10 - 20 = 0.$$

$$I_{11} : \nabla^2 f = 0 + 0 = 0.$$

I_{12}:

$$\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2} = 0.$$

$$\nabla^2 f = 0 + 0 = 0.$$