

Utilizziamo la formula fondamentale:

$$\int_a^b f(x) dx = [\varphi(x)]_a^b = \varphi(b) - \varphi(a),$$

dove  $\varphi(x)$  è una qualunque primitiva di  $f(x)$ .  
Determiniamo le primitive di  $3x^2 + x$ :

$$\int (3x^2 + x) dx = x^3 + \frac{1}{2}x^2 + c.$$

Una primitiva è dunque  $\varphi(x) = x^3 + \frac{1}{2}x^2$ .

Pertanto si ha:

$$\int_1^2 (3x^2 + x) dx = \left[ x^3 + \frac{1}{2}x^2 \right]_1^2.$$

Sostituiamo alla variabile  $x$  dentro la parentesi quadra prima 2 e poi 1 e calcoliamo la differenza:

$$\left[ x^3 + \frac{1}{2}x^2 \right]_1^2 = (8 + 2) - \left( 1 + \frac{1}{2} \right) = \frac{17}{2}.$$

$$\text{Quindi } \int_1^2 (3x^2 + x) dx = \frac{17}{2}.$$

Calcola i seguenti integrali definiti.

**51**  $\int_3^4 (5 - 6x) dx$

[-16]

**52**  $\int_{-2}^0 (3x^2 - x) dx$

[10]

**53**  $\int_0^1 (x^2 + x) dx$

$\left[ \frac{5}{6} \right]$

**54**  $\int_{-2}^1 \frac{3x^2 + 2x - 1}{3} dx$

[1]

**55**  $\int_1^2 \left( x^2 + \frac{1}{x^2} \right) dx$

$\left[ \frac{17}{6} \right]$

**56**  $\int_4^9 (3\sqrt{x} + 2x) dx$

[103]

**57**  $\int_0^1 (\sqrt[3]{x} - x) dx$

$\left[ \frac{1}{4} \right]$

**58**  $\int_2^3 \left( 2x + \frac{1}{x} + 1 \right) dx$

$\left[ 6 + \ln \frac{3}{2} \right]$

**59**  $\int_0^1 4(x + 1)^3 dx$

[15]

**60**  $\int_{-3}^0 (2x^2 + 5) dx$

[33]

**61**  $\int_{-1}^1 (x^3 - 3x^2 + 1) dx$

[0]

**62**  $\int_0^3 |x - 1| dx$

$\left[ \frac{5}{2} \right]$

**63**  $\int_{\frac{1}{4}}^{\frac{1}{2}} \left( \frac{1}{x^3} + \frac{1}{x^2} \right) dx$

[8]

**64**  $\int_1^4 \frac{x-1}{x} dx$

$[3 - 2\ln 2]$

**65**  $\int_2^7 \frac{1}{\sqrt{x+2}} dx$

[2]

**66**  $\int_1^{\pi+1} \sin(x-1) dx$

[2]

**67**  $\int_0^2 \frac{4x}{1+x^2} dx$

$[2\ln 5]$

**68**  $\int_3^8 \frac{3\sqrt{x+1}}{2} dx$

[19]

**69**  $\int_{-1}^{-\frac{1}{2}} \left( \frac{3}{x^4} - \frac{2}{x^2} \right) dx$

[5]

**70**  $\int_{-2}^{-1} \frac{x^2 + 1}{x} dx$

$\left[ -\frac{3}{2} - \ln 2 \right]$

**71**  $\int_1^2 \frac{3x^3 - 2}{x} dx$

$[7 - \ln 4]$

**72**  $\int_1^e \frac{1-x}{x^2} dx$

$\left[ -\frac{1}{e} \right]$

**73**  $\int_0^1 \frac{4x}{1+6x^2} dx$

$\left[ \frac{1}{3} \ln 7 \right]$

**74**  $\int_0^{\frac{1}{2}} \frac{4}{1+4x^2} dx$

$\left[ \frac{\pi}{2} \right]$

**75**  $\int_0^1 x^3(x^4 + 1)^5 dx$

$\left[ \frac{21}{8} \right]$

**76**  $\int_0^2 x(x^2 - 1)^3 dx$

[10]

**77**  $\int_{-1}^4 x|3-x| dx$

$\left[ \frac{9}{2} \right]$

**78**  $\int_0^{\sqrt{8}} 6x\sqrt{x^2+1} dx$

[52]

**79**  $\int_{-1}^0 \frac{x^3}{x^4+1} dx$

$\left[ -\frac{1}{4} \ln 2 \right]$

**80**  $\int_0^1 \frac{x}{(x^2-2)^4} dx$

$\left[ \frac{7}{48} \right]$

**81**  $\int_1^e \frac{6\ln^2 x}{x} dx$

[2]

**82**  $\int_0^{\frac{\pi}{3}} \tan x dx$

$[\ln 2]$

**83**  $\int_1^3 \frac{4x+3}{2x^2+3x} dx$

$[\ln 27 - \ln 5]$

**84**  $\int_2^{\sqrt{5}} 6x\sqrt{x^2-4} dx$

[2]

## ESERCIZI

- 85**  $\int_0^1 \frac{2}{x^2 + 6x + 9} dx$   $\left[\frac{1}{6}\right]$ 
**108**  $\int_0^2 \frac{e^x}{(e^x + 1)^2} dx$   $\left[\frac{e^2 - 1}{2(e^2 + 1)}\right]$
- 86**  $\int_2^4 \frac{2x^2 + x + 1}{2x - 1} dx$   $[8 + \ln 7 - \ln 3]$ 
**109**  $\int_{-\frac{\pi}{10}}^{\frac{\pi}{10}} \tan 2x dx$   $[0]$
- 87**  $\int_1^5 \left(3\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$   $[11\sqrt{5} - 3]$ 
**110**  $\int_{\frac{\pi}{8}}^{\pi} (\sin 2x + \cos 4x) dx$   $\left[\frac{\sqrt{2} - 3}{4}\right]$
- 88**  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos x dx$   $[0]$ 
**111**  $\int_1^3 \frac{1}{x(1+x)} dx$   $\left[\ln \frac{3}{2}\right]$
- 89**  $\int_{-1}^4 (x + \ln 2 \cdot 2^x) dx$   $[23]$ 
**112**  $\int_1^4 \left(5x\sqrt{x} - \frac{1}{x}\right) dx$   $[62 - \ln 4]$
- 90**  $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{\cos x}{\sin x + 1} dx$   $[\ln 4 - \ln 3]$ 
**113**  $\int_{\frac{2}{3}}^{\frac{e+1}{3}} \frac{3}{3x-1} dx$   $[1]$
- 91**  $\int_0^{\pi} (\sin x - \cos x) dx$   $[2]$ 
**114**  $\int_1^e \frac{1}{x(1 + \ln^2 x)} dx$   $\left[\frac{\pi}{4}\right]$
- 92**  $\int_0^{\pi} \sin 2x dx$   $[0]$ 
**115**  $\int_{\sqrt{2}}^{2\sqrt{2}} \frac{6x}{(x^2 + 1)^2} dx$   $\left[\frac{2}{3}\right]$
- 93**  $\int_2^4 x \ln\left(\frac{x}{2}\right) dx$   $[8 \ln 2 - 3]$ 
**116**  $\int_{\frac{1}{3}}^{\frac{1}{2}} \frac{e^x}{x^2} dx$   $[e^3 - e^2]$
- 94**  $\int_1^e (\ln x + x) dx$   $\left[\frac{e^2}{2} + \frac{1}{2}\right]$ 
**117**  $\int_0^{\ln \frac{1}{4}} \frac{e^x}{\cos^2(\pi e^x)} dx$   $\left[\frac{1}{\pi}\right]$
- 95**  $\int_0^1 \frac{e^x}{e^x + 1} dx$   $\left[\ln \frac{e+1}{2}\right]$ 
**118**  $\int_0^8 \frac{1}{\sqrt{64-x^2}} dx$   $\left[\frac{\pi}{2}\right]$
- 96**  $\int_0^1 2xe^{x^2} dx$   $[e - 1]$ 
**119**  $\int_0^{\frac{\sqrt{3}}{3}} \frac{4x}{1 + 9x^4} dx$   $\left[\frac{\pi}{6}\right]$
- 97**  $\int_1^{e^2} \ln x dx$   $[e^2 + 1]$ 
**120**  $\int_{\frac{\pi}{2}}^{\frac{3}{2}\pi} \frac{\sin 2x}{4 \sin^2 x} dx$   $\left[\frac{1}{2} \ln \frac{\sqrt{3}}{2}\right]$
- 98**  $\int_0^1 (2x - 1)5^{x^2-x} dx$   $[0]$ 
**121**  $\int_0^1 \arctan x dx$   $\left[\frac{\pi - \ln 4}{4}\right]$
- 99**  $\int_{-1}^2 xe^x dx$   $\left[e^2 + \frac{2}{e}\right]$ 
**122**  $\int_e^{e^2} \frac{2}{x \ln^2 x} dx$   $[1]$
- 100**  $\int_0^1 xe^{x-1} dx$   $\left[\frac{1}{e}\right]$ 
**123**  $\int_0^1 \arcsin x dx$   $\left[\frac{\pi - 2}{2}\right]$
- 101**  $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(\sin x + 1)^2} dx$   $\left[\frac{1}{2}\right]$ 
**124**  $\int_0^{\frac{\sqrt{3}}{2}} \frac{4x}{\sqrt{1-x^2}} dx$   $[2]$
- 102**  $\int_1^2 \frac{1}{x\sqrt{1-\ln^2 x}} dx$   $[\arcsin \ln 2]$ 
**125**  $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \sin x dx$   $[2]$
- 103**  $\int_1^2 2x \ln x dx$   $\left[4 \ln 2 - \frac{3}{2}\right]$ 
**126**  $\int_3^4 \frac{1}{(x-1)\ln(x-1)} dx$   $[\ln \ln 3 - \ln \ln 2]$
- 104**  $\int_4^9 \frac{e^{2\sqrt{x}-4}}{2\sqrt{x}} dx$   $\left[\frac{e^2}{2} - \frac{1}{2}\right]$ 
**127**  $\int_0^1 x \cdot 3^x dx$   $\left[\frac{3 \ln 3 - 2}{\ln^2 3}\right]$
- 105**  $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2 \tan x dx$   $\left[-2 \ln \frac{\sqrt{3}}{3}\right]$ 
**128**  $\int_0^{\frac{\pi}{2}} e^x \cos x dx$   $\left[\frac{1}{2}(e^{\frac{\pi}{2}} - 1)\right]$
- 106**  $\int_{\frac{\pi^2}{36}}^{\frac{\pi^2}{9}} \frac{\cos \sqrt{x}}{\sqrt{x}} dx$   $[\sqrt{3} - 1]$ 
**129**  $\int_0^2 \frac{x+2}{e^{x-3}} dx$   $[3e^3 - 5e]$
- 107**  $\int_0^{\pi} \frac{\sin x}{\sqrt{3 + \cos x}} dx$   $[4 - 2\sqrt{2}]$