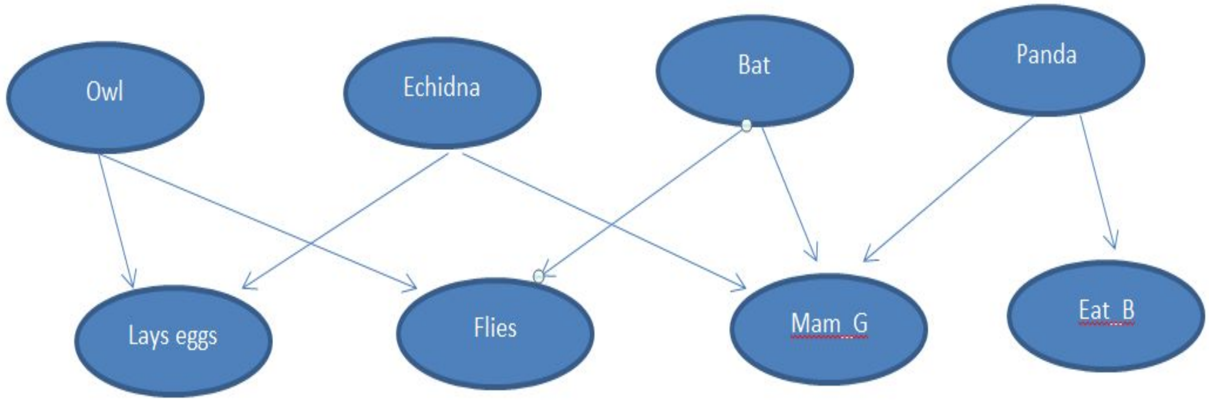
Q1.

1). Using the knowledge base, show the corresponding Bayes Net.



2). Using the ideas of conditional independence discussed in class and in Ch. 14 of R&N, identify which variables in this network are conditionally independent of which others given what conditions.

Using markov blanket rule:

1. Lays_eggs is conditionally independent of the network given owl and Ech
2. Flies is conditionally independent of the network given owl, Bat
3. Mam_G is conditionally independent of the given network given Ech, Bat and Panda
4. Eat_B is conditionally independent of the given network given Panda
5. Owl is conditionally independent of the network given Lays_eggs, Flies, Echidna, Bat
6. Ech is conditionally independent of the network given Lays_eggs ,Mam_G , Owl, BAt and Panda
7. Bat is conditionally independent of the network given Mam_G,Flies, Echidna, Panda, Owl
8. Panda is conditionally independent of the network given Eat_Bamboo, Mam_G, Echidna, Bat.

Using d-separation:X and Y are d-separated if all the paths that connect them are inactive.

Using common cause rule: (a<-c->b)

1. Lays_eggs and Flies are conditionally independent given Owl
2. Lays_eggs and Mam_G are conditionally independent given Echidna
3. Flies and Mam_G are conditionally independent given Bat
4. Mam_G and Eat_B are conditionally independent given Panda

3) Use the prior and conditional probabilities provided to construct probability tables for each node of this network.

For each node:

P(O):

P(owl) =	Owl	P(owl)
	T	0.05
	F	0.95

P(E):

e	P(e)
T	0.01
F	0.99

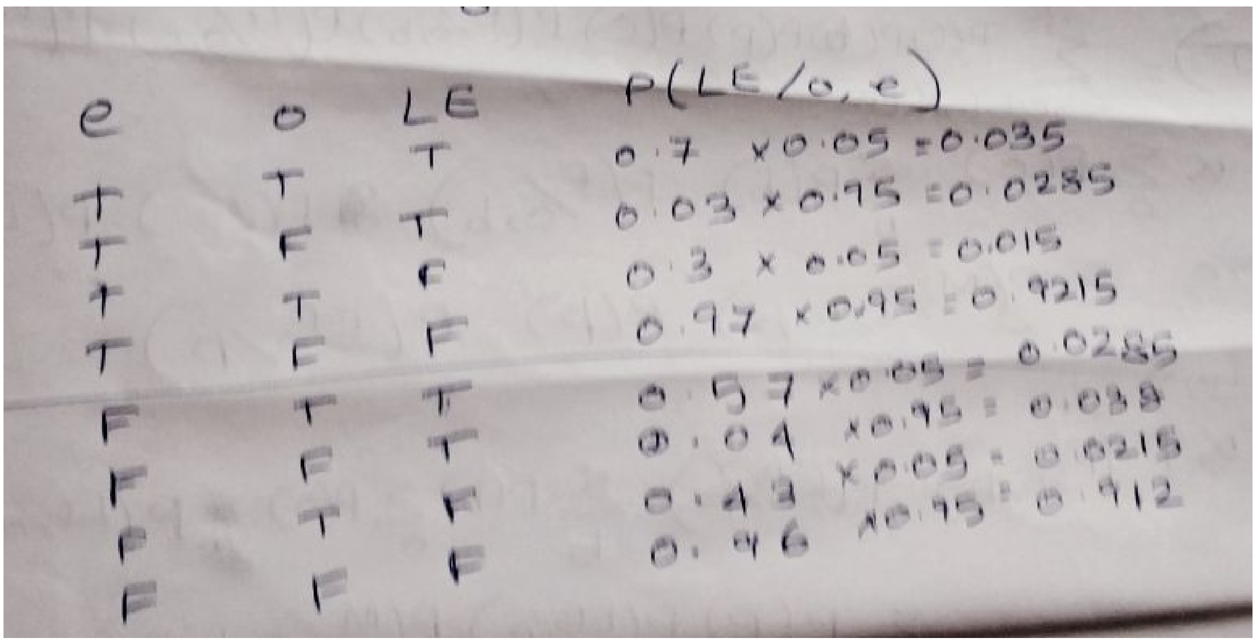
P(B):

P(Bat) :	Bat	P(Bat)
	T	0.03
	F	0.97

P(P):

e	P(e)
T	0.01
F	0.99

P(L_E):



e	o	LE	P(LE/o, e)
T	T	T	0.7 × 0.05 = 0.035
T	F	T	0.03 × 0.95 = 0.0285
T	T	F	0.3 × 0.05 = 0.015
T	F	F	0.97 × 0.95 = 0.9215
F	T	T	0.57 × 0.05 = 0.0285
F	F	T	0.04 × 0.95 = 0.038
F	T	F	0.43 × 0.05 = 0.0215
F	F	F	0.96 × 0.95 = 0.912

P(F):

F	O	B	P(F/O,B)
T	T	T	0.9 × 0.03 = 0.027
T	T	F	0.95 × 0.97 = 0.9215
T	F	T	0.86 × 0.03 = 0.0258
T	F	F	0.03 × 0.97 = 0.0291
F	T	T	0.1 × 0.03 = 0.003
F	T	F	0.05 × 0.97 = 0.0485
F	F	T	0.14 × 0.03 = 0.0042
F	F	F	0.97 × 0.97 = 0.9409

P(M_G):

e	b	p	M	P(M/e,b,p)
T	T	T	T	0.9 × 0.02 = 0.018 × 0.03 = 0.00054
T	T	F	T	0.75 × 0.98 = 0.735 × 0.03 = 0.0220
T	F	T	T	0.4 × 0.02 = 0.008 × 0.97 = 0.0776
T	F	F	T	0.02 × 0.98 = 0.0196 × 0.97 = 0.01901
T	T	T	F	0.1 × 0.02 = 0.02 × 0.03 = 0.0006
T	T	F	F	0.25 × 0.98 = 0.245 × 0.03 = 0.00735
T	F	T	F	0.6 × 0.02 = 0.012 × 0.97 = 0.01164
T	F	F	F	0.98 × 0.98 = 0.9604 × 0.97 = 0.93158
F	T	T	T	0.5 × 0.02 = 0.01 × 0.03 = 0.0003
F	T	F	T	0.08 × 0.98 = 0.0784 × 0.03 = 0.0235
F	F	T	T	0.5 × 0.02 = 0.01 × 0.97 = 0.0097
F	F	F	T	0.01 × 0.98 = 0.0098 × 0.97 = 0.0095
F	T	T	F	0.5 × 0.02 = 0.01 × 0.03 = 0.0003
F	T	F	F	0.92 × 0.98 = 0.9016 × 0.03 = 0.027
F	F	T	F	0.5 × 0.02 = 0.01 × 0.97 = [illegible]
F	F	F	F	0.99 × 0.98 = 0.9702 × 0.97 = [illegible]

P(E_B):

P	E_B	P(E-B)
T	T	0.7 x 0.02 = 0.014
F	T	0.03 x 0.98 = 0.0294
T	F	0.3 x 0.02 = 0.006
F	F	0.97 x 0.98 = 0.9506

Convention adopted:
OWL:O
BAT:B
Panda: P
Echidna:e
Flies: F
Lays_eggs:L_E
Eats Bamboo: E_B
Mammary glands: M_G

Then use exact inference to answer the following questions. What is the probability that --

a) The animal can fly.

b) The animal is an echidna.

c) The patient is a bat, given that it eats bamboo.

a) P(Flies=T)=T

Initial calculation and reduction:

$$P(F=T)=T.$$

$$P(F) = \alpha \sum_{o,b,e,p,Le,e\text{-}b,MG} P(o)P(b)P(e)P(p).P(F/o,b)P(E\text{-}b/p)\,P(Le/o,e).P(M\text{-}G/e,b,p)$$

$$= \alpha \sum_{o,Le,eB,MG} P(o) \sum_{b} P(b).P(F/o,b) \sum_{e} P(e).P(Le/o,e) \sum_{P} P(P).P(E\text{-}b/P).$$

we can see that

$$\sum_{P} P(P).P(E\text{-}b/P)=1, \quad \sum_{o}\sum_{e} P(e).P(o).P(Le/o,e)=1$$

$$\text{Hence } P(F) \,\alpha \sum_{o} P(o).\sum P(b).P(F/o,b)$$

With the given priors of o and b

$P(F=T)=T$

$$P(F=T) = \alpha \sum_{O} P(O) \cdot \sum_{B} P(B) \cdot P(F=T/O,B)$$

$P(O)$ =

O	P(O)
T	0.05
F	0.95

$P(B)$:

B	P(B)
T	0.03
F	0.97

Calculating P(F=T/b,o)*P(b):

1) Calc. $P(F=T \wedge O/B) = \sum_{B} P(F=T/O,b) \cdot P(B)$

F	O	B	$P(F \wedge O/B)$
T	T	T	$0.9 \times 0.03 = 0.027$
T	T	F	$0.95 \times 0.97 = 0.9215$
T	~~F~~	~~F~~T	$0.86 \times 0.03 = 0.0258$
T	F	F	$0.03 \times 0.97 = 0.0291$
F	T	T	$0.1 \times 0.03 = 0.003$
F	T	F	$0.05 \times 0.97 = 0.0485$
F	F	T	$0.14 \times 0.03 = 0.0042$
F	F	F	$0.97 \times 0.97 = 0.9409$

Combining common values of b: calculating P(F=T/o):

2) combing ~~of~~ over B : P(F=T/o)

F	O	P(F/o)
T	T	0.9485
F	T	0.0515
T	F	0.0549
F	F	0.9451

Calculating P(F=T/o)*P(o)

Calc : $P(F=T) = \sum_{o} P(o) \cdot P(F=T/o)$

F	O	P(F=T)
T	T	0.9485 × 0.05 = 0.047425
F	T	0.0515 × 0.05 = 0.002575
T	F	0.0549 × 0.95 = 0.052155
F	F	0.9451 × 0.95 = 0.897845

Combining over all values of o : P(F=T)

4) combining

~~P(F=)~~ $P(F=T)$

T $= 0.09958.$

F $= 0.90042.$

$P(F=T) = T\ (0.09958)$

Calculating P(F=T)=T

P(F=T) = 0.09958/0.90042 = 0.09958

b) P(Echidna)= P(E=T)=T

Given :

$$P(E=T) = T$$

$$P(E) \propto \sum_{o,b,p,Le,F,MG,EB} P(O)P(E)P(B)P(P)P(LE)P(F)P(MG)P(E.B)$$

Moving summation inside:

$$P(E=T) \propto \sum_{O} P(O).P(E).\sum_{B,P} P(B).P(P).P(LE/O,E=T)\,P(F/O,B)\,P(M/E,B,P)\,P(EB/P)$$

Rewriting, P(E) to P(E=T):

$$\alpha \sum_{O} P(O)\, P(E=T) . P(LE/O, E=T) \sum_{B} P(B) . P(F/O, B) .$$

$$\sum_{P} P(P) . P(E\_B/P) . P(M/E=T, B, P) .$$

Using variable elimination:

knowing that (i) $\sum_{O} P(O) . P(E=T) . P(LE/O, E=T) = 1$

(ii) $\sum_{O} \sum_{B} P(B) . P(F/O, B) = 1$

(iii) $\sum_{P} P(P) . P(E\_B/P) = 1 .$

we have

$$P(E=T) \propto \sum_{B,P} P(P) . P(B) . P(M/E=T, B, P) . P(E=T)$$

by var elimination $\sum_{B,P} P(P) . P(B) . P(E=T) . P(M/E=T, B, P) = P(E=T)$

$P(E=T) \propto P(E=T)$ = prior probability = 0.01

P(E=T) is left after variable elimination of all other variables: which gives prior probability of 0.01

c) **P(B=T/E_B=T)**

With the following equation:

Calculating P(B=T, E_B=T)

$$P(B=T / E\text{-}B=T) \propto P(B / EB=T)$$

$$P(B/EB=T) \propto \sum_{O,E,P,Le,F,MG} P(B)P(O)P(e)P(P)P(Le/o,e)P(F/o,B)\,P(M\text{-}G/e,b,p)\cdot P(e\text{-}B=T/P)$$

Moving summation inside:

$$\propto \sum_{o} P(O)\cdot P(B)\cdot \sum_{e} P(e)\cdot P(Le/o,e)\cdot P(F/o,B)\cdot \sum_{P} P(P)\cdot P(E\text{-}B=T/P)\cdot P(M\text{-}G/e,b,P)$$

$$\propto P(B) \sum_{o} P(O)\cdot P(F/o,B) \sum_{e,Le} P(e)\cdot P(Le/o,e)\cdot \sum_{P,M\text{-}g} P(P)\cdot P(E\text{-}B=T/P)\cdot P(M\text{-}G/e,b,P)$$

Starting with variable elimination:

$$\sum_{e} P(e)\cdot P(Le/o,e) = 1$$

$$\sum_{P} P(P)\cdot P(E\text{-}B=T/P) = P(E\text{-}B=T)$$

P	E-B	$P(P)\cdot P(E\text{-}B=T/P)$
T	T	0.7×0.02
F	T	0.03×0.02

Summing over all values of p to get P(E_B=T):

E_B	$P(E\_B=T)$
T	$0.73 \times 0.02 = 0.0146$.

Substituting the value:

$$P(B/E\_B=T) \propto P(B) \cdot \sum_{o} P(o) \cdot P(F/o,B) \cdot \sum_{e} P(e) \cdot \sum_{p} P(p) \cdot P(M\text{-}G/e,b,p)$$

$$\times 0.0146$$

For P(B=T):

$$P(B=T/E\_B=T) \propto P(B=T) \times 0.0146 \cdot \sum_{o,F} P(o) \cdot P(F/o,B=T) \cdot \sum_{P,Mg} P(P) \cdot P(M\text{-}G/e,b=T,P)$$

$$\sum_{M\text{-}G} P(M\text{-}G/e,b=T,P) = 1, \quad \sum_{P} P(P) = 1, \quad \sum_{o} P(o) = 1, \quad \sum_{F} P(F/o,B=T) = 1$$

Doing variable elimination:

$$\sum_{M\text{-}G} P(M\text{-}G/e,b=T,P) = 1, \quad \sum_{P} P(P) = 1, \quad \sum_{o} P(o) = 1, \quad \sum_{F} P(F/o,B=T) = 1$$

Finally:

Hence.

$$P(B=T/E\_B=T) \propto P(B=T) \times 0.0146$$
$$= 0.03 \times 0.0146$$
$$= 0.000438$$

P(B=T, E_B=t)= 0.000438

P(B=T/E_B=T)= P(B=T, E_B=t) /P(E_B= T)
= 0.000438 / 0.0146 as calculated above:

Hence
P(B=T/E_B=T)= 0.03