For the questions below, assume $F_0 = 0, F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$.

1. **(3 points)** prove that $F_n < 2^n$ by induction on n.

Basis Cases: $n=0$: $F_0 = 0 < 2^0 = 1$ $\quad$ $n=1$: $F_1 = 1 < 2^1 = 2$

$0 < 1$ ✓ $\quad$ $1 < 2$ ✓

Inductive Step: $n > 0$, $f_k < 2^k$ for $0 \leq k \leq n+1$

$$F_{n+1} = F_n + F_{n-1} \underset{\text{by ind. hyp.}}{<} 2^n + 2^{n-1} < 2^n + 2^n = 2(2^n) = 2^{n+1}$$

Therefore, $\boxed{F_{n+1} < 2^{n+1}}$

2. **(7 points)** prove that $\sum_{i=1}^{n} F_i = F_{n+2} - 1$ by induction on n.

Basis Cases: $n=1$: $\sum_{i=1}^{1} F_i = 1$

$F_{1+2} - 1 = F_3 - 1 = 2 - 1 = 1$ $\quad$ $1 = 1$ ✓

$n=2$: $\sum_{i=1}^{2} F_i = F_1 + F_2 = 1 + 1 = 2$

$F_{2+2} - 1 = F_4 - 1 = 3 - 1 = 2$ $\quad$ $2 = 2$ ✓

Inductive Step: $\sum_{i=1}^{n+1} F_i = F_{(n+1)+2} - 1 \Rightarrow \sum_{i=1}^{n+1} F_i = F_{n+3} - 1$

$$\sum_{i=1}^{n+1} F_i = \sum_{i=1}^{n} F_i + F_{n+1}$$

$= F_{n+2} - 1 + F_{n+1}$ $\quad$ by ind. hyp.

$= (F_{n+2} + F_{n+1}) - 1$

$= F_{n+3} - 1$