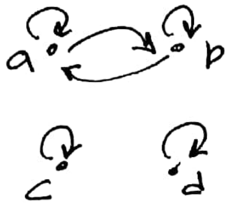
Key

1. **(4 points)** If $R_1 = \{(a,a),(b,b),(c,c),(d,d),(a,b),(b,a)\}$ on the set $A = \{a,b,c,d\}$ and $R_2 = \{(a,b),(a,c),(c,b),(b,c)\}$ on the set $B = \{a,b,c\}$, which of the relations is reflexive? Symmetric? Transitive? If the property does not hold for one of the relations, please say why.

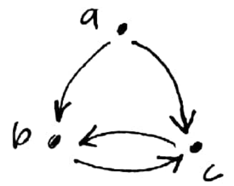
2 pts



R_1 { Reflexive ✓
Symmetric ✓
Transitive ✓

R_2 { Not reflexive (no self relations)
Not Symmetric (for every xRy we must have yRx, however we have (a,b) but not (b,a))
NOT TRANSITIVE (we have $(c,b) \to (b,c)$ but not (c,c).

2 pts



2. **(6 points)** if $a_n = 4a_{n-1} - 4a_{n-2}$, with $a_0 = 6$ and $a_1 = 8$, prove by induction that $a_n = 6 \cdot 2^n - 2n \cdot 2^n$ by induction on n,

Proof by Induction

2pts

B.S

$a_0 = 6 = 6 \cdot 2^0 - 2 \cdot 0 \cdot 2^0$
$6 = 6$ ✓
$a_1 = 8 = 6 \cdot 2^1 - 2 \cdot 1 \cdot 2^1$
$8 = 8$ ✓

2 pts

I.S.

we know $a_n = 4a_{n-1} - 4a_{n-2}$
assume (I.H.) $a_n = 6 \cdot 2^n - 2n \cdot 2^n$
we must show $a_{n+1} = 6 \cdot 2^{n+1} - 2(n+1) \cdot 2^{n+1}$

$a_{n+1} = 4a_n - 4a_{n-1}$
$= 4\left[6 \cdot 2^n - 2 \cdot n \cdot 2^n\right] - 4\left[6 \cdot 2^{n-1} - 2 \cdot (n-1) \cdot 2^{n-1}\right]$ by I.H.

Key step

2 pts (algebra)

$= 4 \cdot 6 \cdot 2^n - 4 \cdot 2 \cdot n \cdot 2^n - 4 \cdot 6 \cdot 2^{n-1} + 4 \cdot 2 \cdot (n-1) \cdot 2^{n-1}$
$= 2 \cdot 2 \cdot 6 \cdot 2^n - 2 \cdot 2 \cdot 2 \cdot n \cdot 2^n - 2 \cdot 2 \cdot 6 \cdot 2^{n-1} + 2 \cdot 2 \cdot 2 \cdot (n-1) \cdot 2^{n-1}$
$= 2 \cdot 6 \cdot 2^{n+1} - 6 \cdot 2^{n+1} - 2 \cdot 2 \cdot n \cdot 2^{n+1} + 2 \cdot (n-1) \cdot 2^{n+1}$
$= 2^{n+1}(2 \cdot 6 - 6) - 2 \cdot 2^{n+1}(2n - n + 1) = 2^{n+1}(6) - 2 \cdot 2^{n+1}(n+1)$ ✓

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