1. **(3 points)** Prove by contradiction that if n is an integer, and n is odd, then n^2 is odd.

Proof by Contradiction

Assume n^2 is even

we know n is odd $\rightarrow$ $n = 2b+1$, $b \in \mathbb{Z}$

Then, $n^2 = (2b+1)^2 = 4b^2 + 4b + 1 = 2(2b^2+2b)+1$

Let $a = 2b^2 + 2b$, $a \in \mathbb{Z}$

Then, $n^2 = 2a+1$ $\rightarrow$ odd

Therefore, we have n^2 is odd AND n^2 is even.

CONTRADICTION!

2. **(5 points)** Prove that the product of any five consecutive integers is divisible by 120. (For example, the product of 3,4,5,6 and 7 is 2520, which is equal to 120 times 21.

Proof

Given any 5 consecutive integers,
at least one must be a multiple of 2, and
at least one must be a multiple of 3, and
at least one must be a multiple of 4, and
at lest one must be a multiple of 5.

Hence, the product of these numbers is

$2 \cdot 3 \cdot 4 \cdot 5 = 120$ ✓

Another way to put it,

$2a \cdot 3b \cdot 4c \cdot 5d \cdot e = 2 \cdot 3 \cdot 4 \cdot 5 \underbrace{(a \cdot b \cdot c \cdot d \cdot e)}_{K} = 120 \cdot K$ divisible by 120

where $a, b, c, d, e, K \in \mathbb{Z}$

3. **(2 points)** Prove or disprove that if a and b are natural numbers, then $a + b < ab$.

Disproof by Counter example:

Let $a=1$, $b=1$ where $a, b \in \mathbb{N}$

Then, $a+b < a \cdot b$

$1+1 < 1 \cdot 1$

$2 < 1$

false!

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