1. **(4 points)** Prove by induction for every natural number n, $\sum_1^n 2^i = 2^{n+1} - 2$.

Base Case: $n=1$. $\sum_{i=1}^{1} 2^i = 2 = 4-2 = 2^{1+1}-2$ ✓

Inductive Step: assuming $\sum_{i=1}^{n-1} 2^i = 2^n - 2$ want to show that $\sum_{i=1}^{n} 2^i = 2^{n+1}-2$.

Algebraically, $\sum_{i=1}^{n} 2^i = 2^n + \underbrace{\sum_{i=1}^{n-1} 2^i}_{\text{by hypothesis}} = 2^n + (2^n - 2) = 2 \cdot 2^n - 2 = 2^{n+1} - 2$ □

(NB here I showed that $P(k-1) \rightarrow P(k)$ instead of the more typical $P(k) \rightarrow P(k+1)$. These are logically equivalent since the former is quantified over $\forall k > 1$ and the latter over $\forall k \geq 1$. The choice just simplifies the algebra a little, and is otherwise arbitrary.)

2. **(6 points)** Prove by induction that for all $n \geq 2$, $6^k > 5^k + 9$.

Base Case: $n=2$. $6^2 = 36 > 34 = 25 + 9 = 5^2 + 9$ ✓

Inductive Step: assuming $6^n > 5^n + 9$, want to show that $6^{n+1} > 5^{n+1} + 9$.

$$
\begin{aligned}
6^{n+1} = 6(6^n) &> 6(5^n + 9) && \text{by hypothesis} \\
&> 5(5^n + 9) && \text{since } 5^n + 9 > 0 \text{ and } 5 < 6 \\
&= 5^{n+1} + 5 \cdot 9 \\
&> 5^{n+1} + 9 && \square
\end{aligned}
$$