1. **(4 points total)** if $(a \mid b) \wedge (c \mid d)$, prove that $ac \mid bd$.

$a | b \Rightarrow b = a \cdot x \quad x \in \mathbb{Z}$

$c | d \Rightarrow d = c \cdot y \quad y \in \mathbb{Z}$

$bd = axcy$

$bd = ac(xy)$ let $z = xy$. since $x \in \mathbb{Z} \wedge y \in \mathbb{Z}$, $z \in \mathbb{Z}$

$bd = ac \cdot z$

$\therefore ac | bd$

2. **(4 points total)** Prove if $a, b, c, d, m \in \mathbb{Z}$ and $m \geq 2$ and $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then $(a+c) \equiv (b+d) \pmod{m}$.

$a \equiv b \pmod{m} \rightarrow a = b + xm \quad x \in \mathbb{Z}$

$c \equiv d \pmod{m} \rightarrow c = d + ym \quad y \in \mathbb{Z}$

$a + c = b + xm + d + ym$

$a + c = b + d + (x+y)m$ let $z = x+y$. since $x \in \mathbb{Z} \wedge y \in \mathbb{Z}$, $z \in \mathbb{Z}$

$a + c = b + d + zm \rightarrow \therefore (a+c) \equiv (b+d) \pmod{m}$

3. **(2 points total)** Rewrite $\neg \forall x P(x)$ so that the negation sign comes after the quantifier, and briefly explain why it is the correct negation.

$\neg \forall x P(x) \equiv \neg [P(x_1) \wedge P(x_2) \wedge \dots]$

$\equiv \neg P(x_1) \vee \neg P(x_2) \vee \dots$ (De Morgan's)

$\equiv \exists x \neg P(x)$

"not for all x $P(x)$" $\equiv$ "for some x, $\neg P(x)$"