Chapter Review

2.17 Exercises

2.1. Answer the following questions for the array shown.

$$\texttt{array1} = \begin{bmatrix} 0.0 & 0.5 & 2.1 & -3.5 & 5.0 \\ -0.1 & -1.2 & -6.6 & 1.1 & 3.4 \\ 1.2 & 0.1 & 0.5 & -0.4 & 1.3 \\ 1.1 & 5.1 & 0.0 & 1.4 & -2.1 \end{bmatrix}$$

(a) What is the size of `array1`?

(b) What is the value of `array1(1,4)`?

(c) What is the value of `array1(9)`?

(d) What is the size and value of `array1(:,1:2:4)`?

(e) What is the size and value of `array1([1 3],[end-1 end])`?

2.2. Are the following MATLAB variable names legal or illegal? Why?

(a) `dog1`

(b) `1dog`

(C) `dogs&cats`

(d) `Do_you_know_the_way_to_san_jose`

(e) `_help`

(f) `What's_up?`

2.3. Determine the size and contents of the following arrays. Note that the later arrays may depend on the definitions of arrays defined earlier in this exercise.

(a) `a = 2:3:12;`

(b) `b = [a' a' a'];`

(c) `c = b(1:2:3,1:2:3);`

(d) `d = a(2:4) + b(2,:);`

(e) `w = [zeros(1,3) ones(3,1)' 3:5'];`

(f) `b([1 3],2) = b([3 1],2);`

(g) `e = 1:-1:5;`

2.4. Assume that array `array1` is defined as shown, and determine the contents of the following subarrays:

$$\text{array1} = \begin{bmatrix} 2.2 & 0.0 & -2.1 & -3.5 & 6.0 \\ 0.0 & -3.0 & -5.6 & 2.8 & 2.3 \\ 2.1 & 0.5 & 0.1 & -0.4 & 5.3 \\ -1.4 & 7.2 & -2.6 & 1.1 & -3.0 \end{bmatrix}$$

(a) `array1(4,:)`

(b) `array1(:,4)`

(c) `array1(1:2:3,[3 3 4])`

(d) `array1([3 3],:)`

2.5. Assume that `value` has been initialized to 10 , and determine what is printed out by each of the following statements.

```
disp (['value = ' num2str(value)]);
disp (['value = ' int2str(value)]);
fprintf('value = %e\n',value);
fprintf('value = %f\n',value);
fprintf('value = %g\n',value);
fprintf('value = %12.4f\n',value);
```

2.6. Assume that `a`, `b`, `c`, and `d` are defined as follows, and calculate the results of the following operations if they are legal. If an operation is illegal, explain why.

$$\mathrm{a} = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \qquad \mathrm{b} = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$$

$$\mathrm{c} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \mathrm{d} = \mathrm{eye}(2)$$

(a) `result = a + b;`

(b) `result = a * d;`

(c) `result = a .* d;`

(d) `result = a * c;`

(e) `result = a .* c;`

(f) `result = a \ b;`

(g) `result = a .\ b;`

(h) `result = a .^ b;`

2.7. Evaluate each of the following expressions.

(a) `12 / 5 + 4`

(b) `(12 / 5) + 4`

(c) `12 / (5 + 4)`

(d) `3 ^ 2 ^ 3`

(e) `3 ^ (2 ^ 3)`

(f) `(3 ^ 2) ^ 3`

(g) `round(-12/5) + 4`

(h) `ceil(-12/5) + 4`

(i) `floor(-12/5) + 4`

2.8. Use MATLAB to evaluate each of the following expressions.

(a) $(3 - 4i)(-4 + 3i)$

(b) $\cos^{-1}(1.2)$

2.9. Evaluate the following expressions in MATLAB, where t = 2 s, $i = \sqrt{-1}$, and ω = 120 rad/s. How do the answers compare?

(a) $e^{-2t}\cos(\omega t)$

(b) $e^{-2t}[\cos(\omega t) + i\sin(\omega t)]$

(c) $e^{[-2t+i\omega t]}$

2.10. Solve the following system of simultaneous equations for x:

$$\begin{aligned}
-2.0\,x_1 + 5.0\,x_2 + 2.0\,x_3 + 3.0\,x_4 + 4.0\,x_5 - 1.0\,x_6 &= -3.0 \\
2.0\,x_1 - 1.0\,x_2 - 5.0\,x_3 - 2.0\,x_4 + 6.0\,x_5 + 4.0\,x_6 &= 1.0 \\
-1.0\,x_1 + 6.0\,x_2 - 4.0\,x_3 - 5.0\,x_4 + 3.0\,x_5 - 1.0\,x_6 &= -6.0 \\
4.0\,x_1 + 3.0\,x_2 - 6.0\,x_3 - 5.0\,x_4 - 2.0\,x_5 - 2.0\,x_6 &= 10.0 \\
-3.0\,x_1 + 6.0\,x_2 + 4.0\,x_3 + 2.0\,x_4 - 5.0\,x_5 + 4.0\,x_6 &= -6.0 \\
2.0\,x_1 + 4.0\,x_2 + 4.0\,x_3 + 4.0\,x_4 + 5.0\,x_5 - 4.0\,x_6 &= -2.0
\end{aligned}$$

2.11. **Position and Velocity of a Ball** If a stationary ball is released at a height h_0 above the surface of the Earth with a vertical velocity v_0, the position and velocity of the ball as a function of time will be given by the equations

$$h(t) = \frac{1}{2}gt^2 + v_0 t + h_0$$

(2.40)

$$v(t) = gt + v_0$$

(2.41)

where g is the acceleration due to gravity $(-9.81\ \mathrm{m/s^2})$, h is the height above the surface of the Earth (assuming no air friction), and v is the vertical component of velocity. Write a MATLAB program that prompts a user for the initial height of the ball in meters and the velocity of the ball in meters per second and plots the height and velocity as a function of time. Be sure to include proper labels in your plots.

2.12. The distance between two points (x_1, y_1) and (x_2, y_2) on a Cartesian coordinate plane is given by the equation

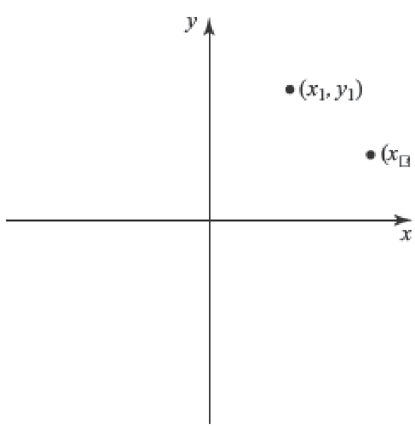
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(2.42)

(See Figure 2.21.) Write a program to calculate the distance between any two points (x_1, y_1) and (x_2, y_2) specified by the user. Use good programming practices in your program. Use the program to calculate the distance between the points (−3, 2) and (3, −6).

Figure 2.21

Distance between two points on a Cartesian plane.



2.13. A two-dimensional vector in a Cartesian plane can be represented in either rectangular coordinates (*x*,*y*) or the polar coordinates (*r*,*θ*), as shown in Figure 2.22. The relationships among these two sets of coordinates are given by the following equations:

$$x = r\cos\theta$$

(2.43)

$$y = r\sin\theta$$

(2.44)

$$r = \sqrt{x^2 + y^2}$$

(2.45)

$$\theta = \tan^{-1}\frac{y}{x}$$

(2.46)

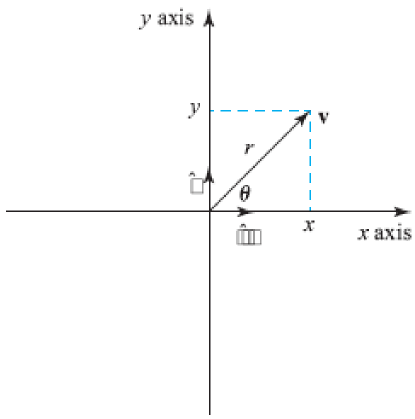
Use the MATLAB help system to look up function `atan2`, and use that function in answering the following questions.

(a) Write a program that accepts a two-dimensional vector in rectangular coordinates and calculates the vector in polar coordinates, with the angle θ expressed in degrees.

(b) Write a program that accepts a two-dimensional vector in polar coordinates (with the angle in degrees) and calculates the vector in rectangular coordinates.

Figure 2.22

A vector **v** can be represented in either rectangular coordinates (x, y) or polar coordinates (r, θ).



2.14. Write a version of the programs in Exercise 2.13 that uses functions `sind`, `cosd`, and `atan2d` instead of functions `sin`, `cos`, and `atan2`. What is the difference between these two sets of programs?

2.15. The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in a three-dimensional Cartesian coordinate system is given by the equation

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

(2.47)

Write a program to calculate the distance between any two points (x_1, y_1, z_1) and (x_2, y_2, z_2) specified by the user. Use good programming practices in your program. Use the program to calculate the distance between the points (−3, 2, 5) and (3, −6, −5).

2.16. A three-dimensional vector can be represented in either rectangular coordinates (x, y, z) or spherical coordinates (r, θ, ϕ), as shown in Figure 2.23. The relationships among these two sets of coordinates are given by the following equations:

$$x = r \cos\phi \cos\theta$$

(2.48)

$$y = r \cos\phi \sin\theta$$

(2.49)

$$z = r \sin \phi$$

(2.50)

$$r = \sqrt{x^2 + y^2 + z^2}$$

(2.51)

$$\theta = \tan^{-1} \frac{y}{x}$$

(2.52)

$$\phi = \tan^{-1} \frac{z}{\sqrt{x^2 + y^2}}$$

(2.53)

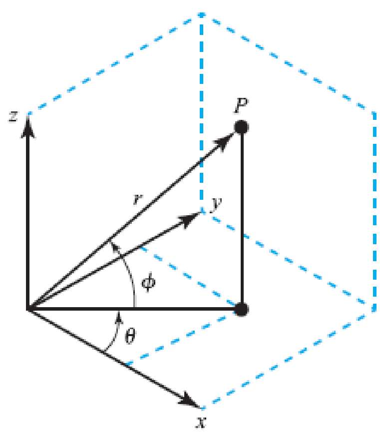
Use the MATLAB help system to look up function `atan2`, and use that function in answering the following questions.

(a) Write a program that accepts a three-dimensional vector in rectangular coordinates and calculates the vector in spherical coordinates, with the angles θ and ϕ expressed in degrees.

(b) Write a program that accepts a three-dimensional vector in spherical coordinates (with the angles θ and ϕ in degrees) and calculates the vector in rectangular coordinates.

Figure 2.23

A three-dimensional vector **v** can be represented in either rectangular coordinates (*x, y, z*) or spherical coordinates (*r, θ, ϕ*).



2.17. MATLAB includes two functions `cart2sph` and `sph2cart` to convert back and forth between Cartesian and spherical coordinates. Look these functions up in the MATLAB help system and rewrite the programs in Exercise 2.15 using these functions. How do the answers compare between the programs written using Equations (2.48), (2.49), (2.50), (2.51), (2.52), and (2.53) and the programs written using the built-in MATLAB functions?

2.18. **Unit Vectors** A unit vector is a vector whose magnitude is 1. Unit vectors are used in many areas of engineering and physics. A unit vector can be calculated from any vector by dividing the vector by the magnitude of the vector. A two-dimensional unit vector in the direction of vector $v = x\hat{\mathbf{i}} + y\hat{\mathbf{j}}$ can be calculated as

$$\mathbf{u} = \frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}}}{\sqrt{x^2 + y^2}}$$

(2.54)

A three-dimensional unit vector in the direction of vector $\mathbf{v} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\widehat{\mathbf{k}}$ can be calculated as

$$\mathbf{u} = \frac{x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\widehat{\mathbf{k}}}{\sqrt{x^2 + y^2 + z^2}}$$

(2.55)

(a) Write a program that accepts a two-dimensional vector in rectangular coordinates and calculates the unit vector pointing in that direction.

(b) Write a program that accepts a three-dimensional vector in rectangular coordinates and calculates the unit vector pointing in that direction.

2.19. **Calculating the Angle between Two Vectors** It can be shown that the dot product of two vectors is equal to the magnitude of each vector times the cosine of the angle between them:

$$\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}| \cos\theta$$

(2.56)

Note that this expression works for both two-dimensional and three-dimensional vectors. Use Equation (2.56) to write a program that calculates the angle between two user-supplied two-dimensional vectors.

2.20. Use Equation (2.56) to write a program that calculates the angle between two user-supplied three-dimensional vectors.

2.21. Plot the functions $f_1(x) = \sin x$ and $f_2(x) = \cos 2x$ for $-2 \leq x \leq 2$ on the same axes, using a solid blue line for $f_1(x)$ and a dashed red line for $f_2(x)$. Then calculate and plot the function $f_3(x) = f_1(x) - f_2(x)$ on the same axes using a dotted black line. Be sure to include a title, axis labels, a legend, and a grid on the plot.

2.22. Plot the function $f(x) = 2e^{-2x} + 0.5e^{-0.1x}$ for $0 \leq x \leq 20$ on a linear set of axes. Now plot the function $f(x) = 2e^{-2x} + 0.5e^{-0.1x}$ for $0 \leq x \leq 20$ with a logarithmic *y* axis. Include a grid, title, and axis labels on each plot. How do the two plots compare?

2.23. In the linear world, the relationship between the net force on an object and the acceleration of the object is given by Newton's law:

$$\mathbf{F} = m\mathbf{a}$$

(2.57)

where **F** is the net vector force on the object, *m* is the mass of the object, and **a** is the acceleration of the object. If acceleration is in meters per second squared and mass is in kilograms, then the force is in newtons.

In the rotational world, the relationship between the net torque on an object and the angular acceleration of the object is given by

$$\boldsymbol{\tau} = I\alpha$$

(2.58)

where **τ** is the net torque on the object, *I* is the moment of inertia of the object, and *α* is the angular acceleration of the object. If angular acceleration is in radians per second squared and the moment of inertia is in kilogram-meters squared, then the torque is in newton-meters.

Suppose that torque of 20 N-m is applied to the shaft of a motor having a moment of inertia of $15\ \mathbf{kg\text{-}m^2}$. What is the angular acceleration of the shaft?

2.24. **Decibels** Engineers often measure the ratio of two power measurements in *decibels*, or dB. The equation for the ratio of two power measurements in decibels is

$$\mathrm{dB} = 10\ \log_{10}\frac{P_2}{P_1}$$

(2.59)

where P_2 is the power level being measured and P_1 is some reference power level.

(a) Assume that the reference power level P_1 is 1 mW, and write a program that accepts an input power P_2 and converts it into decibels with respect to the 1 mW reference level. (Engineers have a special unit for decibel power levels with respect to a 1 mW reference: dBm.) Use good programming practices in your program.

(b) Write a program that creates a plot of power in watts versus power in dBm with respect to a 1 mW reference level. Create both a linear *xy* plot and a log-linear *xy* plot.

2.25. **Power in a Resistor** The voltage across a resistor is related to the current flowing through it by Ohm's law (see Figure 2.24):

$$V = IR$$

(2.60)

and the power consumed in the resistor is given by the equation

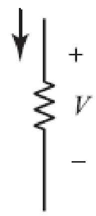
$$P = IV$$

(2.61)

Write a program that creates a plot of the power consumed by a 1000 Ω resistor as the voltage across it is varied from 1 V to 200 V. Create two plots, one showing power in watts, and one showing power in dBW (dB power levels with respect to a 1 W reference).

Figure 2.24

Voltage and current in a resistor.



2.26. **Hyperbolic Cosine** The hyperbolic cosine function is defined by the equation

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

(2.62)

Write a program to calculate the hyperbolic cosine of a user-supplied value `x`. Use the program to calculate the hyperbolic cosine of 3.0. Compare the answer that your program produces to the answer produced by the MATLAB intrinsic function `cosh`(x). Also, use MATLAB to plot the function `cosh(x)`. What is the smallest value that this function can have? At what value of `x` does it occur?

2.27. **Energy Stored in a Spring** The force required to compress a linear spring is given by the equation

$$F = kx$$

(2.63)

where *F* is the force in newtons and *k* is the spring constant in newtons per meter. The potential energy stored in the compressed spring is given by the equation

$$E = \frac{1}{2}kx^2$$

(2.64)

where *E* is the energy in joules. The following information is available for four springs:

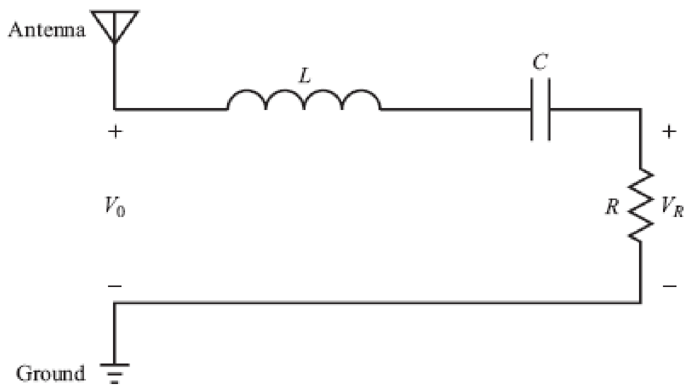
	Spring 1	**Spring 2**	**Spring 3**	**Spring 4**
Force (N)	20	30	25	20
Spring constant *k* (N/m)	150	200	250	300

Determine the compression of each spring and the potential energy stored in each spring. Which spring has the most energy stored in it?

2.28. **Radio Receiver** A simplified version of the front end of an AM radio receiver is shown in Figure 2.25. This receiver consists of an *RLC* tuned circuit containing a resistor, a capacitor, and an inductor connected in series. The *RLC* circuit is connected to an external antenna and the ground, as shown in Figure 2.25.

Figure 2.25

A simplified version of the front end of an AM radio receiver.



The tuned circuit allows the radio to select a specific station out of all the stations transmitting on the AM band. At the resonant frequency of the circuit, essentially all of the signal V_0 appearing at the antenna appears across the resistor, which represents the rest of the radio. In other words, the radio receives its strongest signal at the resonant frequency. The resonant frequency of the *LC* circuit is given by the equation

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(2.65)

where *L* is inductance in henrys (H) and *C* is capacitance in farads (F). Write a program that calculates the resonant frequency of this radio set given specific values of *L* and *C*. Test your program by calculating the frequency of the radio when *L* = 0.125 mH and *C* = 0.20 nF.

2.29. **Radio Receiver** The average (rms) voltage across the resistive load in Figure 2.25 varies as a function of frequency according to Equation (2.66):

$$V_R = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} V_0$$

(2.66)

where ω = 2 *f* and *f* is the frequency in hertz. Assume that *L* = 0.125 mH, *C* = 0.20 nF, *R* = 50 Ω, and $V_0 = 10\ \mathrm{mV}$.

(a) Plot the rms voltage on the resistive load as a function of frequency. At what frequency does the voltage on the resistive load peak? What is the voltage on the load at this frequency? This frequency is called the resonant frequency f_0 of the circuit.

(b) If the frequency is changed to 10 percent greater than the resonant frequency, what is the voltage on the load? How selective is this radio receiver?

(c) At what frequencies will the voltage on the load drop to half of the voltage at the resonant frequency?

2.30. Suppose two signals were received at the antenna of the radio receiver described in Exercise 2.29. One signal has a strength of 1 V at a frequency of 1000 kHz, and the other signal has a strength of 1 V at 950 kHz. Calculate the voltage V_R that will be received for each of these signals. How much power will the first signal supply to the resistive load *R*? How much power will the second signal supply to the resistive load *R*? Express the ratio of the power supplied by signal 1 to the power supplied by signal 2 in decibels (see Problem 2.24 for the definition of a decibel). How much is the second signal enhanced or suppressed compared to the first signal? (*Note:* The power supplied to the resistive load can be calculated from the equation $P = V_R^2/R$.)

2.31. **Equivalent Resistance** The equivalent resistance R_{EQ} of three resistors in parallel is given by Equation 2.67.

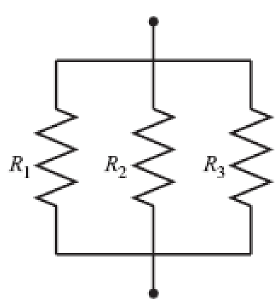
$$R_{EQ} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

(2.67)

Calculate the equivalent resistance R_{EQ} of the circuit shown in Figure 2.26 assuming that $R_1 = 100\ \Omega$, $R_2 = 50\ \Omega$, and $R_3 = 40\ \Omega$.

Figure 2.26

Three resistors in parallel.



2.32. **Aircraft Turning Radius** An object moving in a circular path at a constant tangential velocity *v* is shown in Figure 2.27. The radial acceleration required for the object to move in the circular path is given by the Equation (2.68):

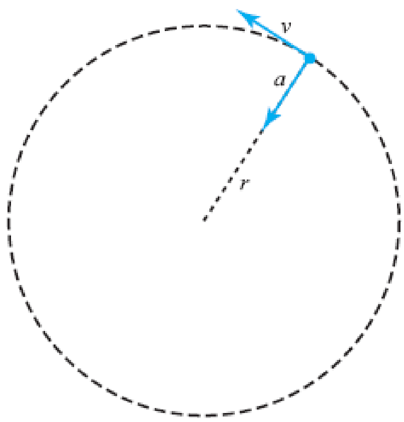
$$a = \frac{v^2}{r}$$

(2.68)

where *a* is the centripetal acceleration of the object in $\mathrm{m/s^2}$, *v* is the tangential velocity of the object in m/s, and *r* is the turning radius in meters. Suppose that the object is an aircraft, and answer the following questions about it:

Figure 2.27

An object moving in uniform circular motion due to the centripetal acceleration *a*.



(a) Suppose that the aircraft is moving at Mach 0.8, or 80 percent of the speed of sound. If the centripetal acceleration is 2 g, what is the turning radius of the aircraft? (*Note*: For this problem, you may assume that Mach 1 is equal to 340 m/s and that $1\ \mathrm{g} = 9.81\ \mathrm{m/s^2}$.)

(b) Suppose that the speed of the aircraft increases to Mach 1.5. What is the turning radius of the aircraft now?

(c) Plot the turning radius as a function of aircraft speed for speeds between Mach 0.5 and Mach 2.0, assuming that the acceleration remains 2 g.

(d) Suppose that the maximum acceleration that the pilot can stand is 7 g. What is the minimum possible turning radius of the aircraft at Mach 1.3?

(e) Plot the turning radius as a function of centripetal acceleration for accelerations between 2 g and 8 g, assuming a constant speed of Mach 0.8.