```
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(** HW4 **)
```
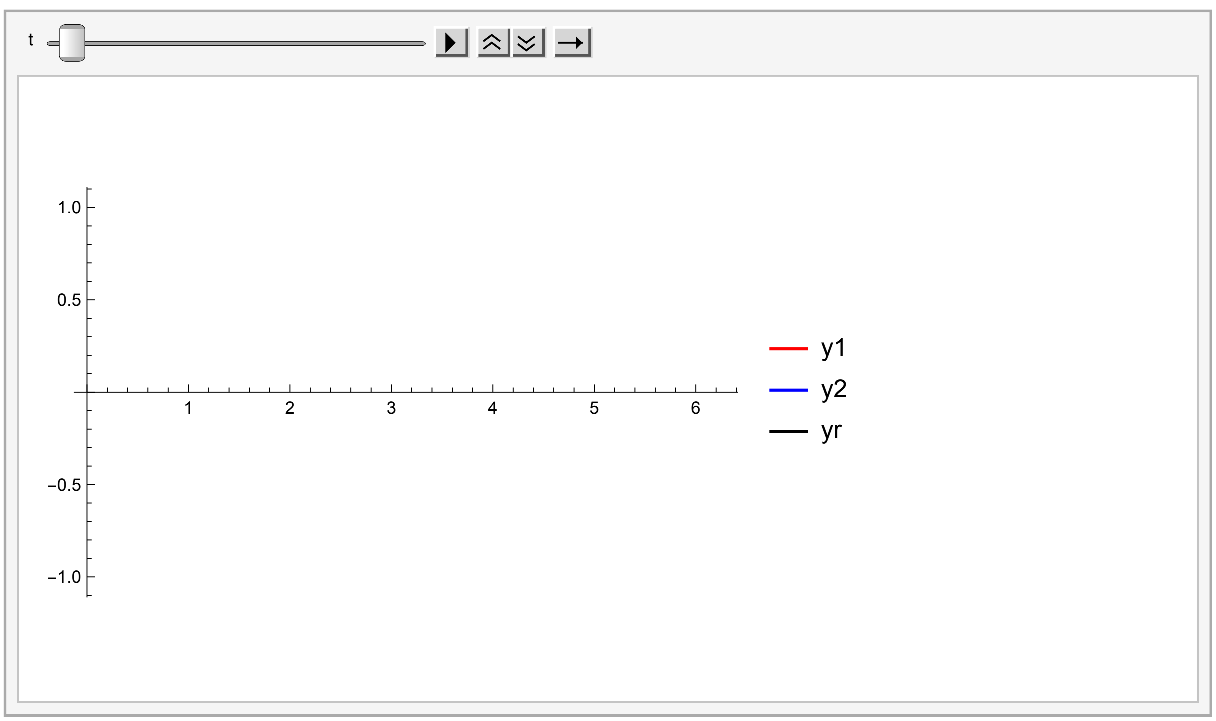
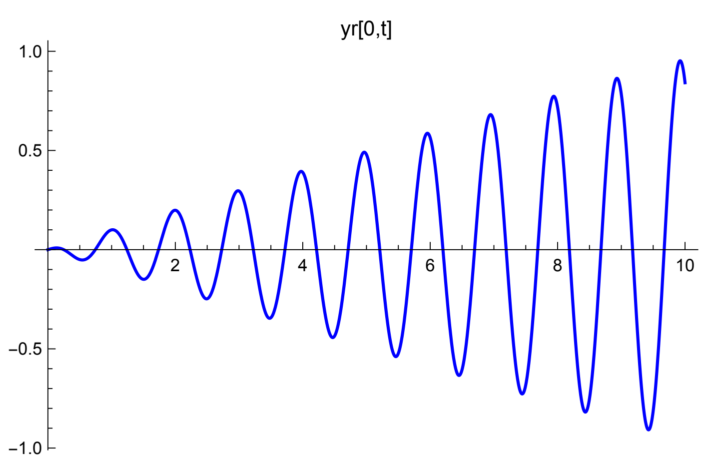

```
(* 1 *)
Remove["Global`*"]

y1[x_, t_] := A1 Sin[k1 (x - v t)]
y2[x_, t_] := A2 Sin[k2 (x + v t)](** The assignement had x ± vt,
but I think that's an error **)
yr[x_, t_] := y1[x, t] + y2[x, t]

k2 := k1 + δk;
v = 1;
k1 = 2 Pi;

A1 = 1;
A2 = 1;
δk = 0;

Animate[Plot[{y1[x, t], y2[x, t], yr[x, t]}, {x, 0, 2 Pi},
   PlotStyle → {Red, Blue, Black}, PlotLegends → {"y1", "y2", "yr"}],
  {t, 0, 10}, AnimationRate → 1, AnimationRunning -> False]

Print["I tried playing around with
    it but I was never able to produce standing waves..."]
```



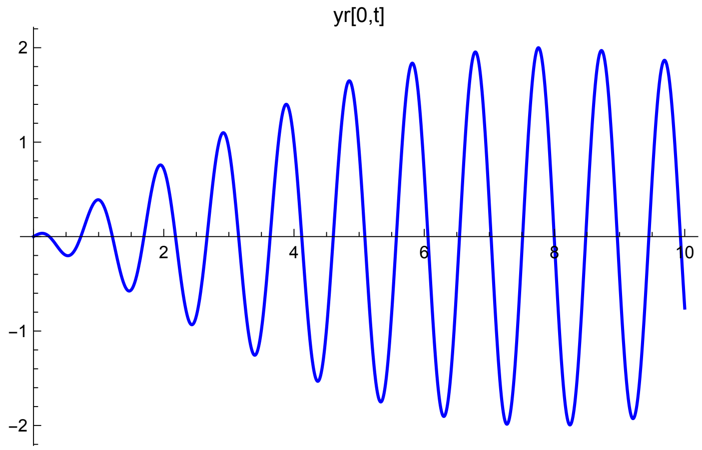

```
I tried playing around with it but I was never able to produce standing waves...
```

```
(*2*)

(* i *)

A1 = 1;
A2 = 1;
δklist = {.1, .4, .7, 1.5, 2.0, 3.0};


For[i = 1, i < Length[δklist] + 1, i++,
 k2 = k1 + δklist[[i]];
 Print["δk = ", δklist[[i]]];
 Print[Plot[yr[0, t], {t, 0, 10}, PlotStyle → Blue, PlotLabel → "yr[0,t]"]]
]

(* ii *)

δk = .6;
A2list = {1.2, 2.0, 5.0, 6.0};

For[i = 1, i < Length[A2list] + 1, i++,
 Print["A2 = ", A2list[[i]]];
 A2 = A2list[[i]];
 Print[Plot[yr[0, t], {t, 0, 10}, PlotStyle → Red, PlotLabel → "yr[0,t]"]]
]

(* iii *)

A2 = 2;
δk = .5;

Animate[Plot[yr[x, t], {x, 5, 15}], {t, 0, 10},
 AnimationRate → 1, AnimationRunning -> False]

Print["A person standing at x=10 would hear beats from the two interfering waves.
   That is, he would hear a pitch with a periodic change in volume."]
```

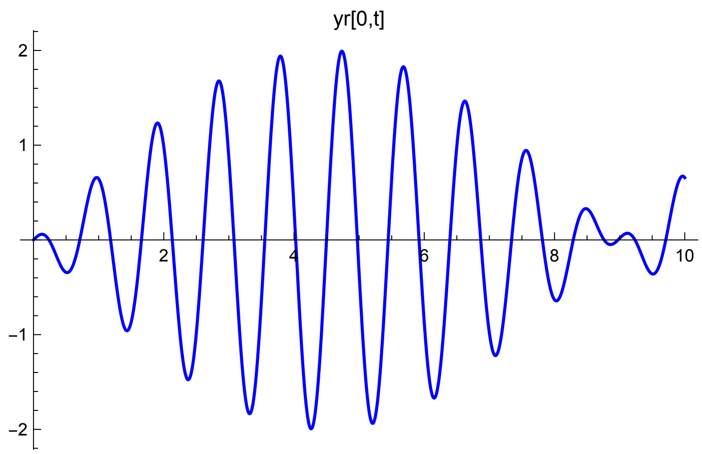
δk = 0.1



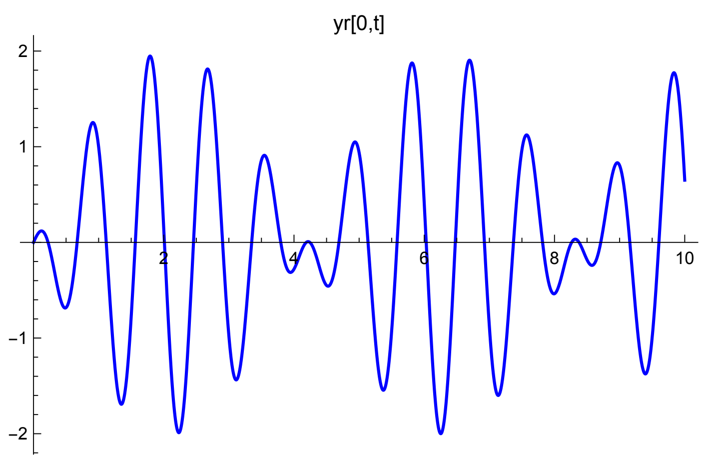
δk = 0.4



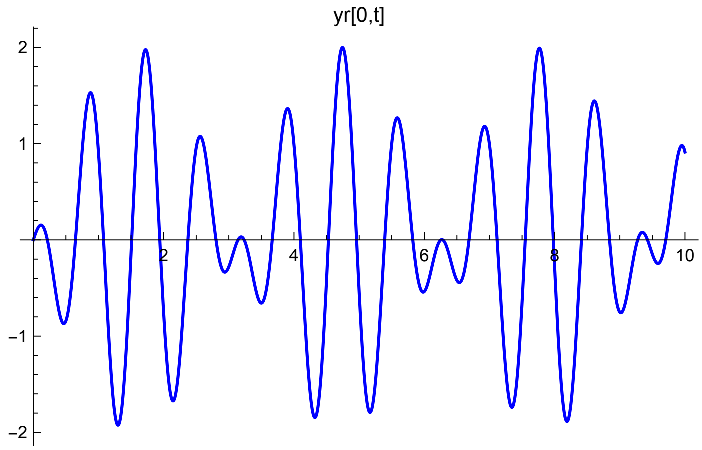
δk = 0.7



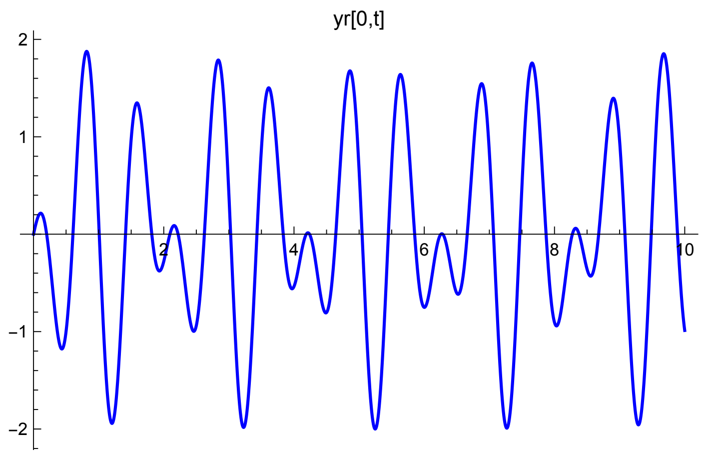
δk = 1.5



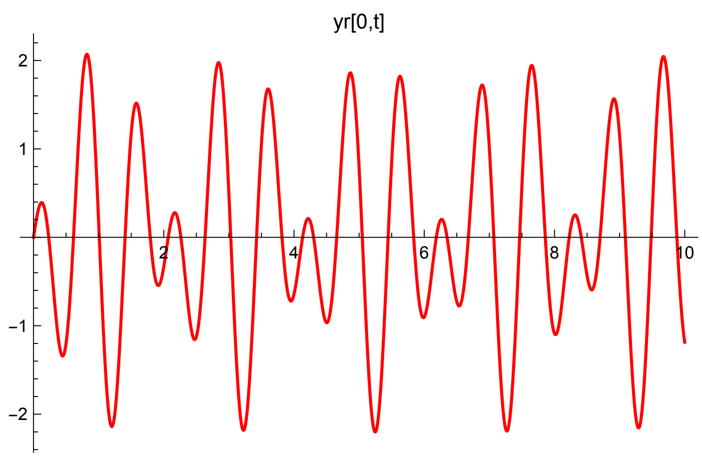
δk = 2.



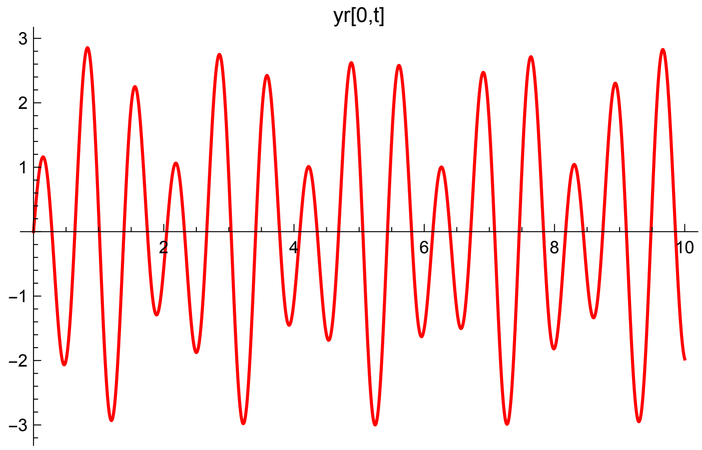
δk = 3.



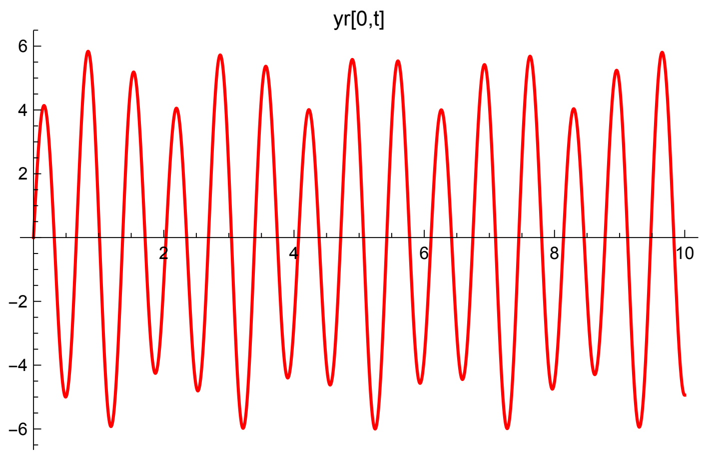
A2 = 1.2



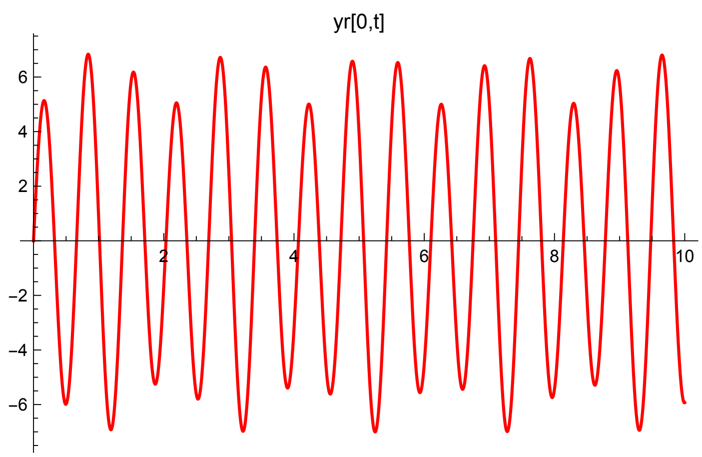
A2 = 2.

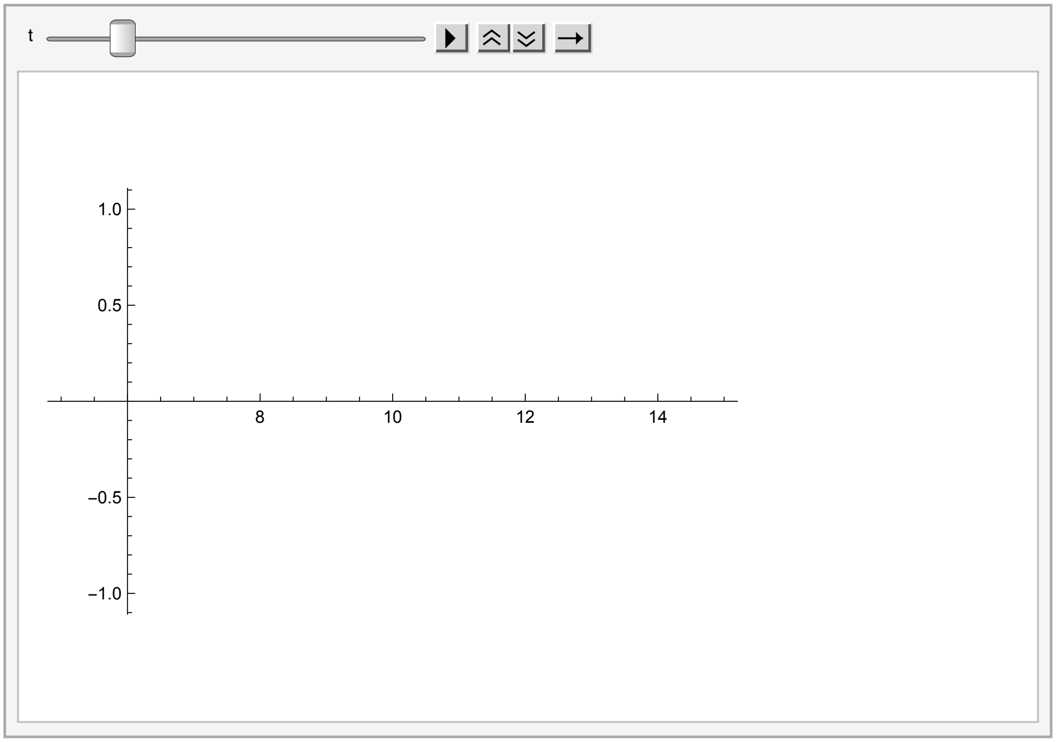



```
A2 = 5.
```




```
A2 = 6.
```





A person standing at x=10 would hear beats from the two interfering waves. That is, he would hear a pitch with a periodic change in volume.

```
(* 3 *)
Remove["Global`*"]

(** Marion 13-21 **)

SetAttributes[{k0, ω0, δk, ω0p}, Constant];
$Assumptions = {Element[{k0, ω0, δk, ω0p, t}, Reals] && k0 > 0 && ω0 > 0 && δk > 0};

A[k_] := Piecewise[{{1, Abs[k - k0] < Δk}}, 0]

Ψ[x_, t_] := Integrate[A[k] E^(I (ω t - k x)), {k, - Infinity, Infinity}] // Simplify

Print["Ψ[x,0]= ", Real[Ψ[x, 0]]]

(** k0=2Pi; **)
(** Δk=Pi; **)

(** Non-Marion Part **)

(* i *) A1[k_] := DiracDelta[k - k0];
(* ii *) A2[k_] := Piecewise[{{1, Abs[k - k0] < Δk}}, 0]
(** Isn't this the same thing as the Marion problem??? **)
(* iii *) A3[k_] := Piecewise[{{Cos[Pi/(2 Δk)] (k - k0), Abs[k - k0] < Δk}}, 0]

(* iv *) A4[k_] := Piecewise[{{(k - k0 + Δk)/(2 Δk), Abs[k - k0] < Δk}}, 0]

(* v *) A5[k_] := Piecewise[{{(k - k0)^2/Δk^2, Abs[k - k0] < Δk}}, 0]

Alist = {A1[k], A2[k], A3[k], A4[k], A5[k]};

For[i = 1, i < 6, i++,
 A[k] = Alist[[i]];
 Print["\nFor A[k]= ", A[k], "\n Ψ[x,0]= ", Real[Ψ[x, 0]]];
]

(** I'm not sure why but I can't figure out how to plot any of these. **)
```

$$\Psi[x,0] = \text{Real}\left[\begin{cases} \frac{2\, e^{-i\, k0\, x}\, \text{Sin}[x\, \Delta k]}{x} & \Delta k > 0 \\ 0 & \text{True} \end{cases}\right]$$

For $A[k] = \text{DiracDelta}[k - k0]$

$$\Psi[x,0] = \text{Real}\left[e^{-i\, k0\, x}\right]$$

For $A[k] = \begin{cases} 1 & \text{Abs}[k - k0] < \Delta k \\ 0 & \text{True} \end{cases}$

$$\Psi[x,0] = \text{Real}\left[\begin{cases} \frac{2\, e^{-i\, k0\, x}\, \text{Sin}[x\, \Delta k]}{x} & \Delta k > 0 \\ 0 & \text{True} \end{cases}\right]$$

For $A[k] = \begin{cases} (k - k0)\, \text{Cos}\left[\frac{\pi}{2\, \Delta k}\right] & \text{Abs}[k - k0] < \Delta k \\ 0 & \text{True} \end{cases}$

$$\Psi[x,0] = \text{Real}\left[\begin{cases} \frac{2\, i\, e^{-i\, k0\, x}\, \text{Cos}\left[\frac{\pi}{2\, \Delta k}\right]\, (x\, \Delta k\, \text{Cos}[x\, \Delta k] - \text{Sin}[x\, \Delta k])}{x^2} & \Delta k > 0 \\ 0 & \text{True} \end{cases}\right]$$

For $A[k] = \begin{cases} \frac{k - k0 + \Delta k}{2\, \Delta k} & \text{Abs}[k - k0] < \Delta k \\ 0 & \text{True} \end{cases}$

$$\Psi[x,0] = \text{Real}\left[\begin{cases} \frac{e^{-i\, x\, (k0 + \Delta k)}\, \left(1 - e^{2\, i\, x\, \Delta k} + 2\, i\, x\, \Delta k\right)}{2\, x^2\, \Delta k} & \Delta k > 0 \\ 0 & \text{True} \end{cases}\right]$$

For $A[k] = \begin{cases} \frac{(k - k0)^2}{\Delta k^2} & \text{Abs}[k - k0] < \Delta k \\ 0 & \text{True} \end{cases}$

$$\Psi[x,0] = \text{Real}\left[\begin{cases} -e^{-i\, k0\, x}\, \Delta k\, (\text{ExpIntegralE}[-2, -i\, x\, \Delta k] + \text{ExpIntegralE}[-2, i\, x\, \Delta k]) & \Delta k > 0 \\ 0 & \text{True} \end{cases}\right]$$

```
(* 4 *)

(** Marion 13-2 **)

$Assumptions = {Element[L, Reals] && τ > 0 && ρ > 0};

(** Inital conditions for the string **)
q[x, 0] = Piecewise[{{3 h/L x, 0 ≤ x ≤ L/3}, {3 h/(2 L) (L - x), L/3 ≤ x ≤ L}}];

q'[x, 0] = 0;

ω[n_] = π n/L Sqrt[τ/ρ];

A[n_] = 2/L Integrate[q[x, 0] Sin[n π x/L], {x, 0, L}] // Simplify;

A[n_] = A[n][[1, 1, 1]];(** Get's rid of the piecewise part **)

Print["\n The A[n] for the sum are: ", A[n]]

B[n_] = -2/(ω[n] L) Integrate[q'[x, 0] Sin[n π x/L], {x, 0, L}] // Simplify;

Print["\n The B[n] for the sum are: ", B[n]]

q[x, t] = Sum[A[i] Sin[i n π/L] Cos[ω[i] t], {i, 1, 5}];

Print["\n The first few terms of the wave function for the string are:\n
q[x,t]= ", q[x, t]]
```

The A[n] for the sum are: $\frac{12\, h\, \mathrm{Sin}\left[\frac{n \pi}{3}\right]^3}{n^2 \pi^2}$

The B[n] for the sum are: 0

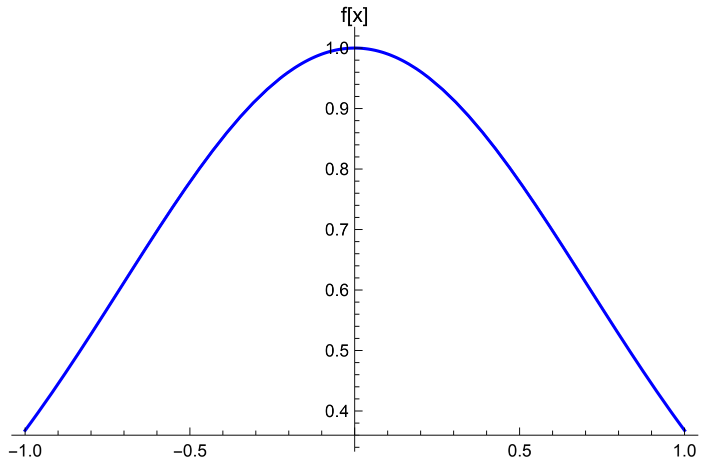
The first few terms of the wave function for the string are:

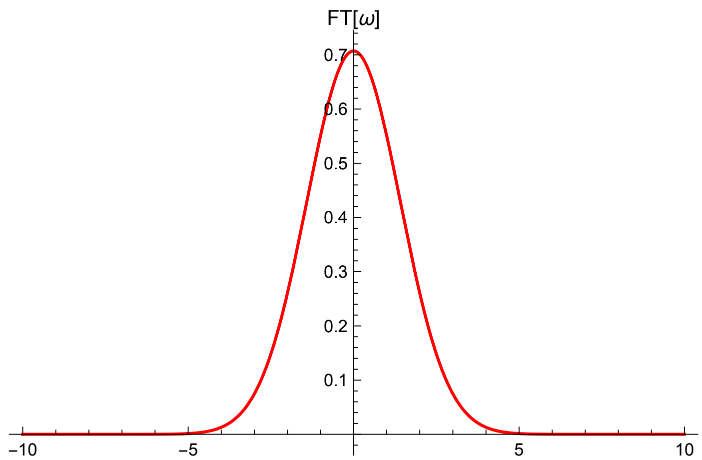
$$q[x,t]= \frac{9\sqrt{3}\, h\, \mathrm{Cos}\left[\frac{\pi t \sqrt{\frac{\tau}{\rho}}}{L}\right] \mathrm{Sin}\left[\frac{n \pi}{L}\right]}{2 \pi^2} + \frac{9\sqrt{3}\, h\, \mathrm{Cos}\left[\frac{2 \pi t \sqrt{\frac{\tau}{\rho}}}{L}\right] \mathrm{Sin}\left[\frac{2 n \pi}{L}\right]}{8 \pi^2} - \frac{9\sqrt{3}\, h\, \mathrm{Cos}\left[\frac{4 \pi t \sqrt{\frac{\tau}{\rho}}}{L}\right] \mathrm{Sin}\left[\frac{4 n \pi}{L}\right]}{32 \pi^2} - \frac{9\sqrt{3}\, h\, \mathrm{Cos}\left[\frac{5 \pi t \sqrt{\frac{\tau}{\rho}}}{L}\right] \mathrm{Sin}\left[\frac{5 n \pi}{L}\right]}{50 \pi^2}$$

```
(* 5 *)

(* a *)
f[x_] := M E^(-α x^2)
FT[ω_] := FourierTransform[f[x], x, ω]

M = 1;

Print["We will demonstrate that was the width of
   f[x] decreases (i.e. α increases) the width of FT[ω] increases
   graphically by plotting f[x],FT[ω] for various values of α.\n"]

αlist = {1, 5, 10, 15, 20, 35};

For[
 i = 1, i < Length[αlist] + 1, i++,
 α = αlist[[i]];
 Print["α = ", αlist[[i]]];
 Print[Plot[f[x], {x, -1, 1}, PlotLabel → "f[x]", PlotStyle → Blue]];
 Print[Plot[FT[ω], {ω, -10, 10}, PlotLabel → "FT[ω]", PlotStyle → Red]]
]

Print["From the plots it is clear that as α incrases
   (width of f[x] decreases) the width of FT[ω] increases!"]

Print["*******************************************************************"]

(* b *)

Clear[f, FT]

f[x_] := Piecewise[{{1, Abs[x] < a}, {0, Abs[x] > a}}]

FT[ω_] := FourierTransform[f[x], x, ω]

a = 1;

Plot[f[x], {x, -2, 2}, PlotLabel → "f[x]", PlotStyle → Blue]
Plot[FT[ω], {ω, -5, 5}, PlotLabel → "FT[ω]", PlotStyle → Red]

Print["The QM interpretation of these graphics is just the uncertainty
   principle. Since we definitely know the position f[x] of some
   particle, we cannot definitely know it's momentum, as Δx Δp ≤ ℏ/2.\n"]
```

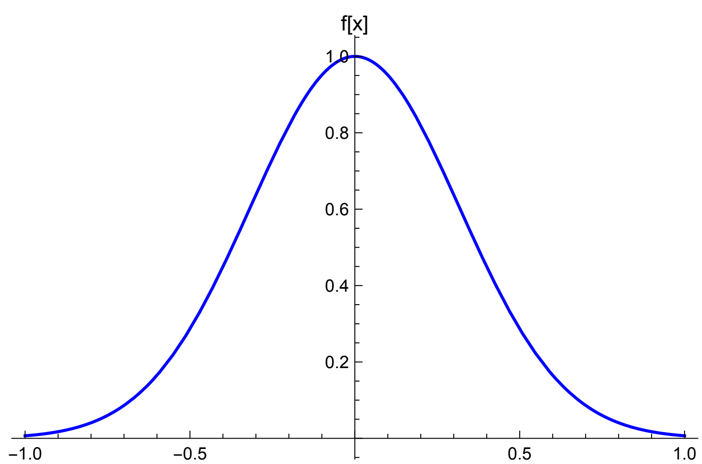
We will demonstrate that was the width of f[x] decreases (i.e. α increases) the width of FT[ω] increases graphically by plotting f[x],FT[ω] for various values of α.

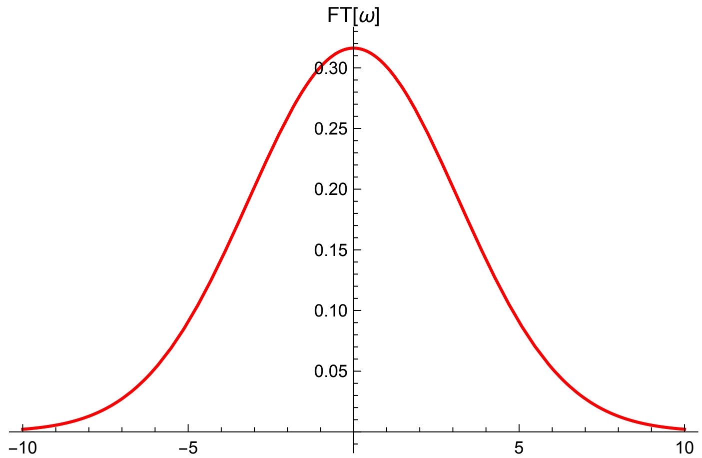
α = 1


f[x]
1.0
0.9
0.8
0.7
0.6
0.5
0.4
−1.0
−0.5
0.5
1.0

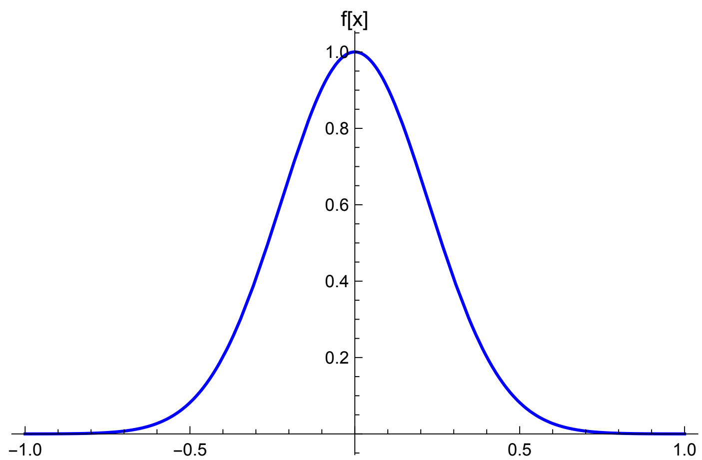

FT[ω]
0.7
0.6
0.5
0.4
0.3
0.2
0.1
−10
−5
5
10

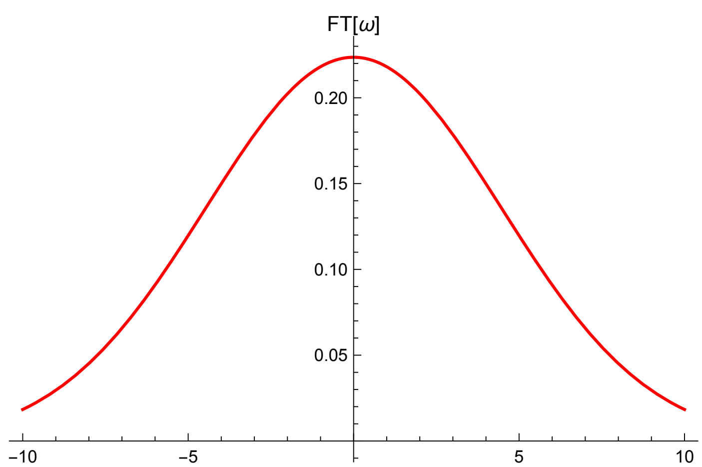
$\alpha = 5$


f[x]
1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5
1.0

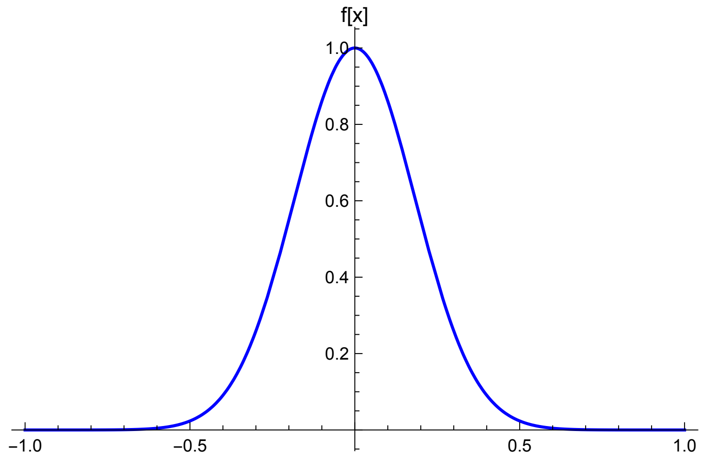

FT[ω]
0.30
0.25
0.20
0.15
0.10
0.05
−10
−5
5
10

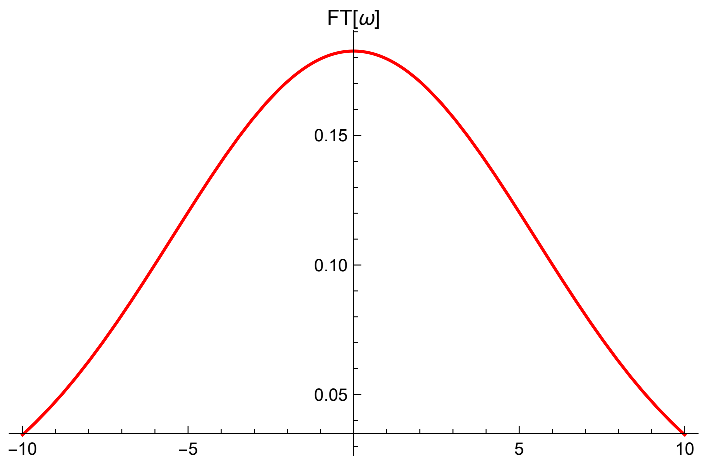
α = 10


f[x]
1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5
1.0

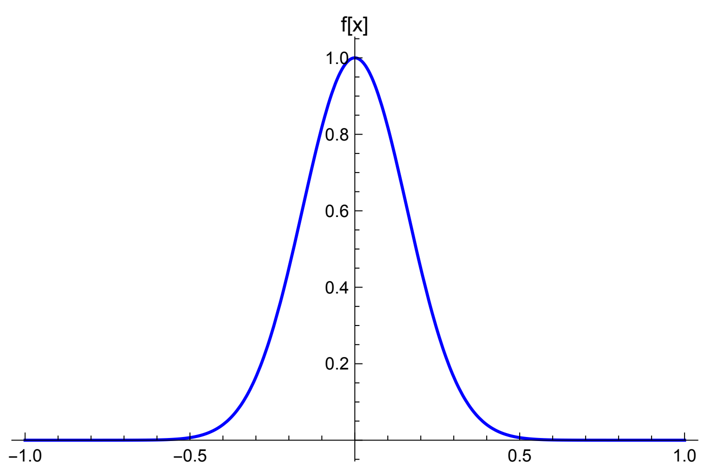

FT[ω]
0.20
0.15
0.10
0.05
−10
−5
5
10

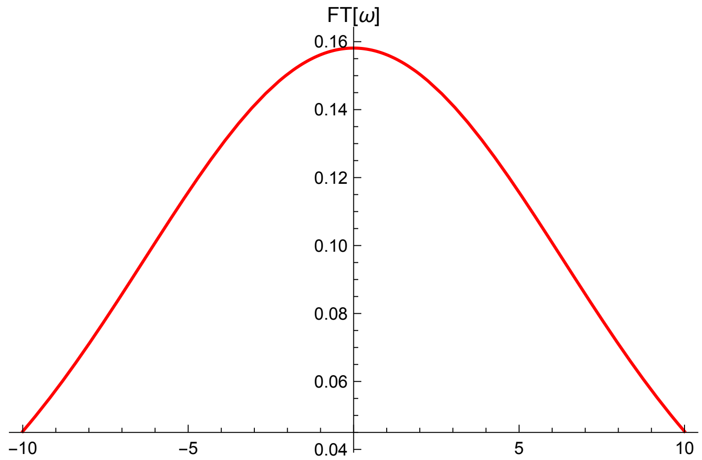
α = 15


f[x]
1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5
1.0

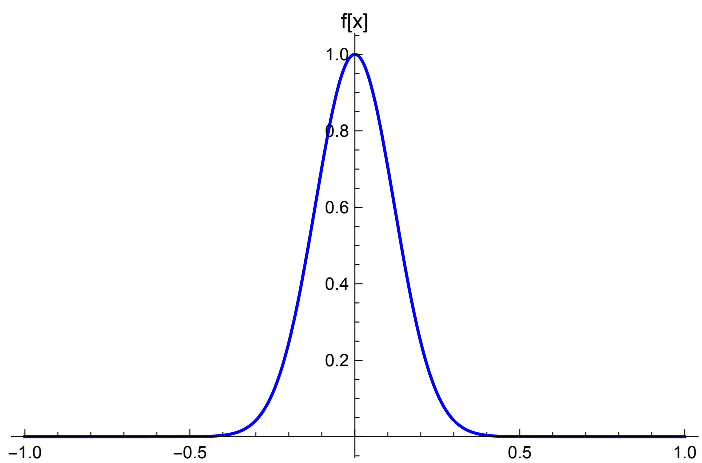

FT[ω]
0.15
0.10
0.05
−10
−5
5
10

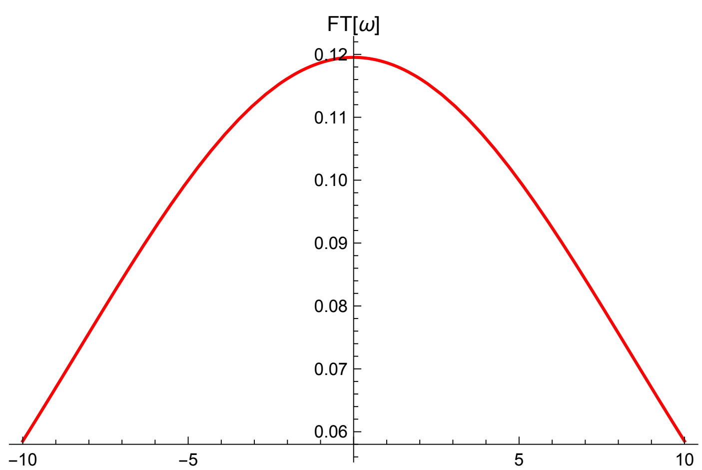
$\alpha = 20$


f[x]
1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5
1.0



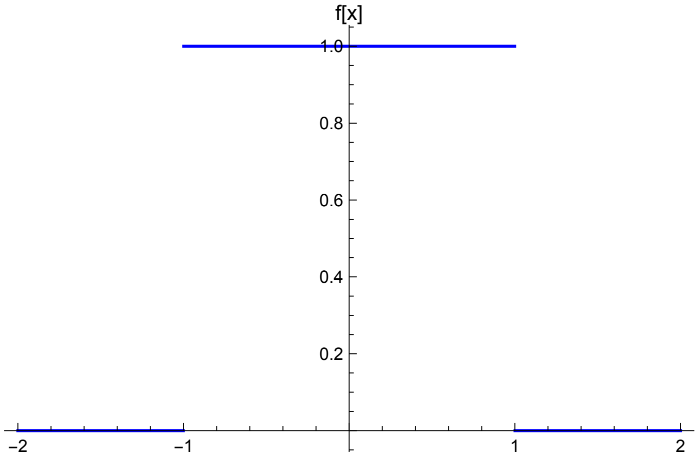
α = 35

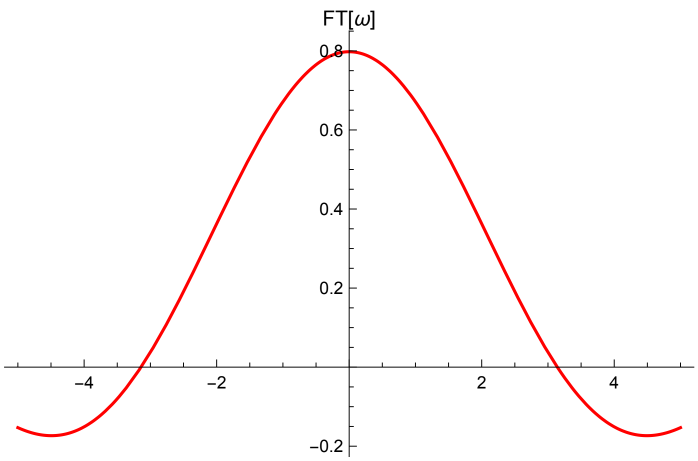




From the plots it is clear that as α incrases
(width of f[x] decreases) the width of FT[ω] increases!

****





The QM interpretation of these graphics is just the uncertainty principle. Since we definitely know the position f[x] of some particle, we cannot definitely know it's momentum, as $\triangle x \triangle p \leq \frac{\hbar}{2}$.

```
(* 6 *)
Clear[y1, y2, ω0, ω1, ω2]

y1[t_] := Sin[ω0 t]
y2[t_] := Sin[ω1 t] Sin[ω2 t]

A[k] = Alist[[5]];
Ψ5[x, 0] := Ψ[x, 0];

FTy1[ω_] = FourierTransform[y1[t], t, ω];
FTy2[ω_] = FourierTransform[y2[t], t, ω];
FTΨ5[ω_] = FourierTransform[Ψ5[x, 0], x, ω];

Print["\nThe Fourier Transform of y1[t]= ", y1[t], " is :\n"
 , "FTy1[ω]= ", FTy1[ω]]

Print["\nThe Fourier Transform of y2[t]= ", y2[t], " is :\n"
 , "FTy2[ω]= ", FTy2[ω]]
Print["\nThe Fourier Transform of Ψ5[x,0]= ", Ψ5[x, 0], " is :\n"
 , "FTΨ5[ω]= ", FTΨ5[ω]]
```

The Fourier Transform of y1[t]= Sin[t ω0] is :

FTy1[ω]= $i \sqrt{\frac{\pi}{2}}$ DiracDelta[ω − ω0] − $i \sqrt{\frac{\pi}{2}}$ DiracDelta[ω + ω0]

The Fourier Transform of y2[t]= Sin[t ω1] Sin[t ω2] is :

FTy2[ω]= $-\frac{1}{2}\sqrt{\frac{\pi}{2}}$ DiracDelta[ω − ω1 − ω2] + $\frac{1}{2}\sqrt{\frac{\pi}{2}}$ DiracDelta[ω + ω1 − ω2] + $\frac{1}{2}\sqrt{\frac{\pi}{2}}$ DiracDelta[ω − ω1 + ω2] − $\frac{1}{2}\sqrt{\frac{\pi}{2}}$ DiracDelta[ω + ω1 + ω2]

The Fourier Transform of Ψ5[x,0]=
$$\begin{cases} -e^{-i\, k0\, x}\, \triangle k\ (\text{ExpIntegralE}[-2, -i\, x\, \triangle k] + \text{ExpIntegralE}[-2, i\, x\, \triangle k]) & \triangle k > 0 \\ 0 & \text{True} \end{cases}$$
is :

FTΨ5[ω]= $-\frac{\sqrt{2\pi}\ (k0-\omega)^2\ (-1+\text{UnitStep}[-\triangle k])}{\triangle k^2}$