Mathematica Homework #4

Email notebook to corbin@physics.ucla.edu
with a subject line: [Physics 105B]
Due on: Friday, 7 Dec

Please note: The absolute last day to submit Mathematica projects for a grade is Friday, 7 Dec. As my email filter is set to discard any and all nb attachments after 23:59 PST on 7 Dec, there will be no exceptions.

- In the first cell, enter all the usual stuff: your **name**, **student ID**, **email address** and the **assignment identifier** (eg. "HW 4").

  Let's have a little fun and develop some intuition for wave topics that are easy to take for granted.

- 1) **WaveLab:** Set up the following equations:

$$y_1(x,t) = A_1 \sin(\, k_1(x - vt)\,)$$
$$y_2(x,t) = A_2 \sin(\, k_2(x \pm vt)\,)$$
$$y_r(x,t) = y_1(x,t) + y_2(x,t)$$
$$k_2 = k_1 + \delta k$$
$$v = 1$$
$$k_1 = 2\pi$$

  Now, what I want you to do is to **Plot**[ ] $y1$ (in red), $y2$ (in blue) and y_r (in black) on a common x-axis and **Animate**[ ] that over time. Start off with $A_1 = A_2 = 1$ and try to reproduce standing waves. Vary the relative amplitudes, add a little difference to the wave numbers, don't be afraid to play around. Pay particular attention to how the relative phases of the contributing waves affect the amplitude of the resultant.

- 2) In lower-division physics, we derived the beat phenomenon using traveling waves of identical amplitude and similar frequency. Since we've got Mathematica to do the dirty-work for us (and it's just done most of it), let's generalize this a bit. Using the same equations in Problem 1:

  - *i*) Set $A_1 = A_2 = 1$ and do quick plots of $y_r(0,t)$ for $\delta k = 0.1, 0.4, 0.7, 1.5, 2.0, 3.0$ (**GraphicsArray**[ ] might help). Does what you see look reasonable?
  - *ii*) Now, set $A_1 = 1$, $\delta k = 0.6$ and do quick plots of $y_r(0,t)$ for $A_2 = 1.2, 2.0, 5.0, 6.0$. What happens when the amplitudes are different?
  - *iii*) Finally, take $A_1 = 1$, $A_2 = 2$, $\delta k = 0.5$, and Animate the passage of the wave through the medium. If this represented sound, and you were standing at $x = 10$, describe what you'd hear.

- 3) **Marion 13-21**. In addition to the book problem, I'd like you to play around a little more with the shape of the amplitude distribution. For each case below, plot $\Psi(x,0)$ on an appropriate scale, and see if you can understand what you're seeing in the context of the previous problem.

  - *i)* $A(k) = \text{DiracDelta}[k - k_0]$
  - *ii)* $A(k) = \begin{cases} 1 & |k - k_0| < \Delta k \\ 0 & \text{otherwise} \end{cases}$
  - *iii)* $A(k) = \begin{cases} \cos\left(\frac{\pi}{2\Delta k}(k - k_0)\right) & |k - k_0| < \Delta k \\ 0 & \text{otherwise} \end{cases}$
  - *iv)* $A(k) = \begin{cases} (k - k_0 + \Delta k)/2\Delta k & |k - k_0| < \Delta k \\ 0 & \text{otherwise} \end{cases}$
  - *v)* $A(k) = \begin{cases} (k - k_0)^2/(\Delta k)^2 & |k - k_0| < \Delta k \\ 0 & \text{otherwise} \end{cases}$

- 4) **Marion 13-2**

- 5) The Fourier transform is closely related to Fourier series, but it's different - it is a convolution of sorts. The Fourier **series** gives you a discrete set of amplitudes to go with each discrete harmonic component; the Fourier **transform** gives you a function that maps amplitude to frequency over a continuum of frequencies. If you feed a Fourier transform an arbitrary time-varying signal, it will generate a spectral decomposition of that signal - producing a function that tells you the strength with which every frequency is represented in the signal - this is what a 'Spectrum Analyzer' does in the lab. This is obviously used for signals analysis, but it also shows up in driven oscillations, in quantum mechanics (the position and momentum wavefunctions are related by Fourier transform), and so on.

  - a) Plot the Gaussian Probability function, $f(x) = Ne^{-\alpha x^2}$ and it's Fourier transform. Show that as the $f(x)$ decreases in width, it's transform increases in width. This is often used to explain the Heisenberg Uncertainty Principle. (For the engineers in the class, it's also a fundamental feature in spread-spectrum communications).
  - b) Plot $f(x)$ and its Fourier transform for the box function:

    $$\begin{array}{ll} f(x) = 1 & |x| < a \\ f(x) = 0 & |x| > a \end{array}$$

    If you've had some quantum mechanics, comment on the results in terms of momentum and position. You might verify your suspicions by playing with some different values of a.

- 6) This may be a challenge for those of us in the 'netbook' crowd, but let's give it a shot anyway. Use Fourier transformation to obtain the spectral decomposition of:
  - *i*) A "CW" signal: $y(t) = \sin(\omega_0 t)$
  - *ii*) Beats: $y(t) = \sin(\omega_1 t)\sin(\omega_2 t)$
  - *iii*) $\Psi(x, 0)$ produced in #3 part v above