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(** HW1 **)

(* 1 *)
Remove["Global`*"]

ρ1 = k1 z^n;
ρ2 = k2 Sin[ϕ/2];
x = r Cos[ϕ];
y = r Sin[ϕ];
z = z;
m = Integrate[ρ1 r, {ϕ, 0, 2 Pi}, {z, 0, h}, {r, 0, a z/h}];
k = Solve[m == M, k1];

ρ1 = ρ1 /. k[[1]];

cm1 = 1/M {Integrate[ρ1 x r , {ϕ, 0, 2 Pi}, {z, 0, h}, {r, 0, a z/h}],
     Integrate[ρ1 y r , {ϕ, 0, 2 Pi}, {z, 0, h}, {r, 0, a z/h}],
     Integrate[ρ1 z r , {ϕ, 0, 2 Pi}, {z, 0, h}, {r, 0, a z/h}]};

m = Integrate[ρ2 r, {ϕ, 0, 2 Pi}, {z, 0, h}, {r, 0, a z/h}];
k = Solve[m == M, k2];

ρ2 = ρ2 /. k[[1]];

cm2 = 1/M {Integrate[ρ2 x r , {ϕ, 0, 2 Pi}, {z, 0, h}, {r, 0, a z/h}],
     Integrate[ρ2 y r , {ϕ, 0, 2 Pi}, {z, 0, h}, {r, 0, a z/h}],
     Integrate[ρ2 z r , {ϕ, 0, 2 Pi}, {z, 0, h}, {r, 0, a z/h}]};

Print["The center of mass for a cone with ρ1 is: ", cm1]
Print["The center of mass for a cone with ρ2 is: ", cm2]
```

The center of mass for a cone with $\rho 1$ is: $\left\{0, 0, \frac{h\,(3+n)}{4+n}\right\}$

The center of mass for a cone with $\rho 2$ is: $\left\{-\frac{a}{6}, 0, \frac{3\,h}{4}\right\}$

In[203]:=

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(* 2 *)
Remove["Global`*"]

$Assumptions = { Element[{a, r, θ, TCM0, b, μ}, Reals] && a > 0 &&
    r > 0 && θ ≥ 0 && θ ≤ π && TCM0 > 0 && b < a && μ > 0 && rmin > 0 };

V[r_] = k/r^2;

ECM = 1 / 2 μ (r')^2 + 1 / 2 μ r^2 (θ')^2 + V[r];

θ' = θ' /. Solve[μ r^2 θ' == μ b v0 , θ'] [[1]] // Simplify;
v0 = v0 /. Solve[TCM0 == 1 / 2 μ v0^2, v0] [[2]] // Simplify;

drdt = r' /. Solve[ECM == TCM0, r'][[2]];

dθdr = dθdr /. Solve[ θ' == dθdr drdt, dθdr][[1]] // FullSimplify;

drdθ = 1 / dθdr;

rmin = r /. Solve[drdθ == 0, r][[3]];

rmin = rmin // Simplify;

Δθ = 2 Integrate[dθdr, {r, rmin, Infinity}];

b = b /. Solve[θ == π - Δθ, b][[1]] // Quiet // FullSimplify;

σ := - b / Sin[θ] D[b, θ] // Simplify

σtot = Integrate[σ 2 π Sin[θ], {θ, 0, π}];

Print["\nThe differential cross-section: σ[θ] = ", σ]
Print["\nThe total cross-section: σtot = ", σtot]
```

The differential cross-section: $\sigma[\theta] = \frac{k\,\pi^2\,(\pi-\theta)\,\mathrm{Csc}[\theta]}{\mathrm{TCM0}\,\theta^2\,(-2\,\pi+\theta)^2}$

The total cross-section: $\sigma_{tot} = \infty$

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(* 3 *)
Remove["Global`*"]

r[t_] := {x[t], y[t], z[t]};

ω = {0, 0, Ω};

ic = { x[0] == x0,         y[0] == y0,   z[0] == 0 ,
      x'[0] == v0x, y'[0] == v0y, z'[0] == 0 };

Fcen[t_] := -m Cross[ω, Cross[ω, r[t]]];
Fcor[t_] := -2 m Cross[ω, r'[t]];
Fext[t_] = {0, 0, 0};

F [t_] = Fcen[t] + Fcor[t] + Fext[t];

N2 = { F[t][[1]] == m x''[t], F[t][[2]] == m y''[t], F[t][[3]] == m z''[t]};

eqnlist = Join[N2, ic];
soln = DSolve[eqnlist, r[t], t][[1]] // ExpToTrig // Simplify;

r[t] = r[t] /. soln;

v[t] = D[r[t], t] // Simplify;

Print["The equations of motion for the puck are: \n
x[t]= ", r[t][[1]], "\n
y[t] = ", r[t][[2]], "\n
z[t] = ", r[t][[3]], "\n"]

Print["The velocity components of the puck are: \n
vx[t]= ", v[t][[1]], "\n
vy[t] = ", v[t][[2]], "\n
vz[t] = ", v[t][[3]]]
```

```
The equations of motion for the puck are:

x[t]= (x0 + t (v0x - y0 Ω)) Cos[t Ω] + (y0 + t (v0y + x0 Ω)) Sin[t Ω]

y[t] = (y0 + t (v0y + x0 Ω)) Cos[t Ω] - (x0 + t (v0x - y0 Ω)) Sin[t Ω]

z[t] = 0

The velocity components of the puck are:

vx[t]= (v0x + t Ω (v0y + x0 Ω)) Cos[t Ω] + (v0y + t Ω (-v0x + y0 Ω)) Sin[t Ω]

vy[t] = (v0y + t Ω (-v0x + y0 Ω)) Cos[t Ω] - (v0x + t Ω (v0y + x0 Ω)) Sin[t Ω]

vz[t] = 0
```

```
(* 4 *)
Remove["Global`*"]

r[t_] := {x[t], y[t], z[t]};

ω = {0, 0, Ω};

ic = { x[0] == x0,          y[0] == y0,   z[0] == 0 ,
      x'[0] == v0x, y'[0] == v0y, z'[0] == v0z};

Fcen[t_] := -m Cross[ω, Cross[ω, r[t]]];
Fcor[t_] := -2 m Cross[ω, r'[t]];
Fext[t_] = {0, 0, -m g};

F [t_] = Fcen[t] + Fcor[t] + Fext[t];

N2 = { F[t][[1]] == m x''[t], F[t][[2]] == m y''[t], F[t][[3]] == m z''[t]};

eqnlist = Join[N2, ic];
soln = DSolve[eqnlist, r[t], t][[1]] // ExpToTrig // Simplify;

r[t] = r[t] /. soln;

T = t /. Solve[r[t][[3]] == 0, t][[2]];

r[t] /. t → T;

Print["The puck will land at the following coordinates on the platform: \n
x = ", r[t][[1]] /. t → T, "\n
y = ", r[t][[2]] /. t → T, "\n
z = ", r[t][[3]] /. t → T]
```

The puck will land at the following coordinates on the platform:

$$x = \left(x0 + \frac{2\,v0z\,(v0x - y0\,\Omega)}{g}\right) \mathrm{Cos}\left[\frac{2\,v0z\,\Omega}{g}\right] + \left(y0 + \frac{2\,v0z\,(v0y + x0\,\Omega)}{g}\right) \mathrm{Sin}\left[\frac{2\,v0z\,\Omega}{g}\right]$$

$$y = \left(y0 + \frac{2\,v0z\,(v0y + x0\,\Omega)}{g}\right) \mathrm{Cos}\left[\frac{2\,v0z\,\Omega}{g}\right] - \left(x0 + \frac{2\,v0z\,(v0x - y0\,\Omega)}{g}\right) \mathrm{Sin}\left[\frac{2\,v0z\,\Omega}{g}\right]$$

$$z = 0$$