1-(a). The size of vector w is $d+1$.

The size of vector y is n.

1-(b).

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 & \cdots & x_1^d \\ 1 & x_2 & x_2^2 & x_2^3 & \cdots & x_2^d \\ 1 & x_3 & x_3^2 & x_3^3 & \cdots & x_3^d \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & x_n^3 & & x_n^d \end{bmatrix}$$

$(d\times 1) \times n$.

1-c. If dim $= n$, $\det A = \prod_{1 \le i \le j \le n} (x_j - x_i)$

Let $n=2$. Then, $A = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix}$ $\det A = x_2 - x_1$.

이를 이용하여 A는 Vandermonde 행렬이므로, 행 연산을 하면

$$\det A = \det \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 0 & x_2 - x_1 & x_2^2 - x_1^2 & \cdots & x_2^{n-1} - x_1^{n-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & x_n - x_1 & x_n^2 - x_1^2 & \cdots & x_n^{n-1} - x_1^{n-1} \end{bmatrix}$$

$$= \det \begin{bmatrix} x_2 - x_1 & x_2^2 - x_1^2 & \cdots & x_2^{n-1} - x_1^{n-1} \\ x_3 - x_1 & x_3^2 - x_1^2 & \cdots & x_3^{n-1} - x_1^{n-1} \\ \vdots & \vdots & & \vdots \\ x_n - x_1 & x_n^2 - x_1^2 & \cdots & x_n^{n-1} - x_1^{n-1} \end{bmatrix}$$

$$= \det \left(\begin{bmatrix} x_2 - x_1 & 0 & \cdots & 0 \\ 0 & x_3 - x_1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & x_n - x_1 \end{bmatrix} \begin{bmatrix} 1 & x_2 + x_1 & \cdots & \sum_{i=0}^{n-2} x_2^{n-2-i} x_1^i \\ 1 & x_3 + x_1 & \cdots & \sum_{i=0}^{n-2} x_3^{n-2-i} x_1^i \\ \vdots & \vdots & & \vdots \\ 1 & x_n + x_1 & \cdots & \sum_{i=0}^{n-2} x_n^{n-2-i} x_1^i \end{bmatrix} \right)$$

$$= \prod_{j=2}^{n} (x_j - x_1) \det \begin{bmatrix} 1 & x_2 + x_1 & \cdots & \sum_{i=0}^{n-2} x_2^{n-2-i} x_1^i \\ 1 & x_3 + x_1 & \cdots & \sum_{i=0}^{n-2} x_3^{n-2-i} x_1^i \\ \vdots & \vdots & & \vdots \\ 1 & x_n + x_1 & \cdots & \sum_{i=0}^{n-2} x_n^{n-2-i} x_1^i \end{bmatrix}$$ ($\because$ by, Cauchy's Theorem)

$$= \prod_{j=2}^{n} (x_j - x_1) \det \left(\begin{bmatrix} 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-2} \end{bmatrix} \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-2} \\ 0 & 1 & x_1 & \cdots & x_1^{n-3} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \right)$$

$$= \prod_{j=2}^{n} (x_j - x_i) \det \begin{bmatrix} 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-2} \end{bmatrix}$$

$$= \prod_{j=2}^{n} (x_i - x_1) \prod_{2 \le i \le j \le n} (x_j - x_i)$$

$$= \prod_{1 \le i \le j \le n} (x_j - x_i)$$

1-(d) 행렬 A에서 $1 \le i \le j \le n$ 일때,

$x_i \ne x_j$를 만족하여야 한다.

1-(e)

$$Aw = y.$$

$$AA^{-1}w = A^{-1}y$$

$$w = A^{-1}y \qquad \therefore w = A^{-1}y$$

2

의사역 행렬의 정의로부터, $AA^{+}A = A$

$A^{+}AA^{+} = A^{+}$ 를 이끌어낼 수 있다

A는 선형 독립이므로, $A^{+} = (A^{T}A)^{-1}A^{T}$ 이다

이를 이용하여

$$Aw = y$$

$$A^{T}Aw = A^{T}y$$

$$w = (A^{T}A)^{-1}A^{T}y$$

$$w = A^{+}y$$

$$\therefore w = A^{+}y$$