

# Modeling and Simulation of Vehicle Steer by Wire System

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**Abstract**— The steer by wire system offer many benefits compare with conventional steering system. By eliminating the mechanical linkage of column shaft between the steering wheel and the front wheel system, it gives more space efficiency, fuel efficiency in term of functionality and at the same time present challenges to the designer. Many researchers have done their control strategy on steer by wire system in past recent years. This paper presents the control strategy for the wheel synchronization and the variable steering ratio. Mathematical modeling was created for steering wheel and front wheel model. The steering wheel and the front wheel system is control using PID controller and introduce a new feedforward variable steering ratio based on under propensity equation method. A simulation was made and compared in order to analysis the system performance.

**Index Terms**; - Steer by wire system, variable steering ratio, front tire angle, modeling and simulation.

## I. INTRODUCTION

Steer-by-wire systems (SBW) are new technology in vehicle system application. In steer-by-wire system, there is no mechanical coupling between the steering wheel and front wheel system as shown in fig. 1. Even though the mechanical coupling between the steering wheel and the front wheel system are eliminated, a steer by wire system expected not only implement same function as conventional mechanical coupling steering system, but it expected to provide advanced steering function. There are several advantage offers by steer by wire system such as no oil leaking, freedom in car interior design, large space in cabin and less injury in car accidents [1].

There are main several requirements for steer by wire system [2]:

- (1) Directional control and wheel synchronization. It is required that front wheel follow the driver input command from the steering wheel.

- (2) Capability of steering wheel return or free control. The steering wheel should return automatically to the center if the hands of driver remove or release from the steering.
- (3) Variable steering ratio. The steering ratio between steering wheel angle and front wheel angle. For example, steering ratio 15:1. By means steering wheel it to 15 degree angle, the front tire wheel should turn to 1 degree angle.
- (4) Adjustable variable steering feel. The vehicle driver relies on steering feel to sense the force of road condition with tire to ground contact and maintain control of the vehicle.

Several works has been undertaken to study the modeling and control of steer by wire system. Reza Kazemi et al [4] presented the control strategies of steering wheel using PID controller and Active front steering (AFS) controller for front wheel system. The research focus on an integrated control system of AFS and direct yaw moment control (DYC) by actively controlling the front tire angle, this control system designed using model matching technique [8]. Oh S-W et al [6], introduced the steering ratio using feed forward controller. Gradient propensity equation is used to control the steering ratio. The author claim less feedback sensor is use based on vehicle state equation [6]. While in paper [2], the steering ratio is determine based on steering wheel angle and vehicle speed. The author introduced fuzzy logic to control the steering ratio.

These papers focus on directional control and variable steering ratio control strategies. The following section will describe the control methods for directional control strategy by using PID controller and variable steering ratio with the aims to satisfy the steer by wire requirements.

### A. Steer by Wire System

In conventional mechanical steering system, the column shafts are directly connected to rack pinion gear and tire system as shown in fig. 1a. Existing conventional steering system use hydraulic power steering (HPS) or Electric power steering (EPS) to assist the driver. The advantage of EPS compared to HPS is the system comparatively new technology with less complicated build mechanism, taking less space and more durable.

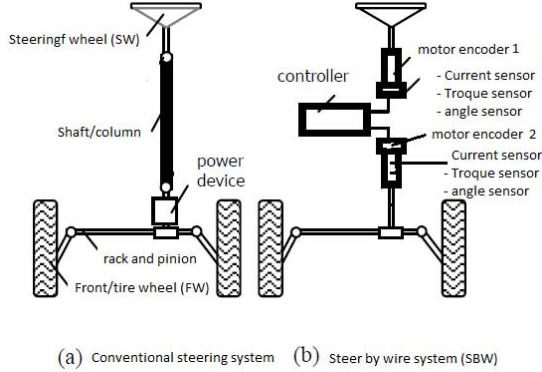


Fig.1 Conventional steering and Steer by wire (SBW)[1]

Fig. 1b show the steer by wire system where the power assist system (i.e. using hydraulic or electric) and column shaft are removed. Sensors and actuators were attached to the steering wheel and the front wheel system. The signal from steering wheel motor encoder is used to observe the rotation angle from driver input. This rotation angle is then converted to electrical signal and wired to an electronic control unit (ECU). The ECU controls the signal and sends it to front wheel actuators for rotating the front wheel parts in the same manner of the steering wheel behavior.

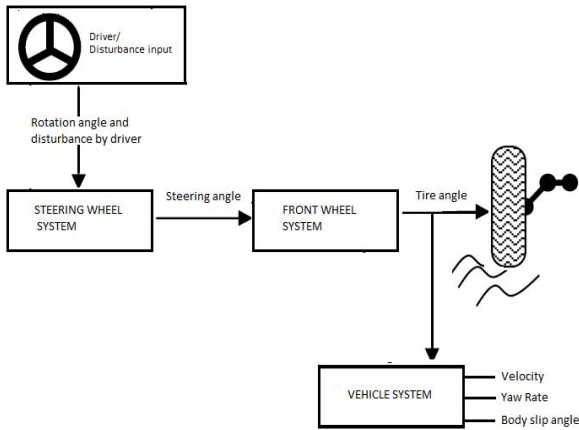


Fig.2 Steer by wire system (SBW) overview [1]

In general, a steer by wire system can be divided into three main subsystems [3]. There are steering wheel, front wheel and vehicle model system as shown in fig. 1b and fig. 2. The steering wheel system contains torque sensor, current sensor, steering angle sensor and motor encoder while front wheel system contains rack pinion gear, angle sensor, motor encoder and other mechanisms that are related. The vehicle model consists of 2 degree of freedom (D.O.F) which is lateral and yaw motion. The model of steering wheel and the front wheel system are model and simulate using parameters as shown in Appendix I

### B. Steering Wheel System Modelling

The steering wheel dynamic equation was modelled using mathematical equation based on Newton's law.

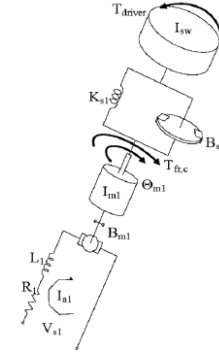


Fig.3 Steering wheel system diagram [4]

In steering wheel system, the input to the system are the steering angle ( $\theta_{sw}$ ) and driver torque ( $T_{driver}$ ). While motor torque ( $T_{m1}$ ) and torque friction ( $T_{frc}$ ) will react as a disturbance. The output of the system is the steering motor current ( $i_{a1}$ ) and steering motor angular displacement ( $\theta_{m1}$ ). ( $B_{sc}$ ) is steering column damping, ( $k_{s1}$ ) lumped torque stiffness, ( $I_{sw}$ ) steering lumped inertia, ( $R_1$ ) motor electrical resistance, ( $L_1$ ) motor electrical inductance, ( $I_{m1}$ ) lumped inertia motor and ( $k_{b1}$ ) steering motor emf. The mathematical equations of the steering wheel subsystem are given below:

Steering angle:

$$\ddot{\theta}_{sw} = \frac{1}{I_{sw}} [T_{driver} - T_{frc} - B_{sc} \dot{\theta}_{sw} - k_{s1} \theta_{sw} + B_{sc} \dot{\theta}_{m1} + k_{s1} \theta_{m1}] \quad (1)$$

Current of steering Motor:

$$\dot{i}_{a1} = \frac{1}{L_1} [-R_1 i_{a1} - k_{b1} \dot{\theta}_{m1} + \theta_{sw}] \quad (2)$$

Steering Motor angular displacement:

$$\ddot{\theta}_{m1} = \frac{1}{I_{m1}} [-k_{s1} \theta_{m1} - B_{m1} \dot{\theta}_{m1} - B_{sc} \dot{\theta}_{m1} + k_{s1} \theta_{sw} + B_{sc} \dot{\theta}_{sw} + T_{m1}] \quad (3)$$

Consequently, the state equation of the steering wheel system is given as:

$$\dot{x}(t) = A_{sw} x(t) + B_{sw} u(t) \quad (4)$$

$$y(t) = C_{sw} x(t) + D_{sw} u(t) \quad (5)$$

$$x(t) = [\theta_{sw} \quad \dot{\theta}_{sw} \quad \theta_{m1} \quad \dot{\theta}_{m1} \quad i_{a1}]^T \quad (6)$$

And  $u(t)$  is considered as the input of the steering wheel subsystem, and parameter states are:

$$u(t) = [T_{driver} \quad \theta_{sw}]^T \quad (7)$$

$$A_{sw} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -k_{s1} & -B_{sc} & K_{s1} & B_{sc} & 0 \\ I_{sw} & I_{sw} & I_{sw} & I_{sw} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ k_{s1} & -B_{sc} & -K_{s1} & -(B_{sc} + B_{m1}) & 0 \\ I_{m1} & I_{m1} & I_{m1} & I_{m1} & (-R_1/L_1) \\ 0 & 0 & 0 & -K_{b1}/L_1 & 0 \end{bmatrix}$$

$$B_{sw} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ (1/I_{sw}) & 0 & (-1/I_{sw}) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & (-1/I_{m1}) \\ 0 & (1/L_1) & 0 & 0 \end{bmatrix}$$

$$C_{sw} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D_{sw} = 0$$

### C. Front Wheel System Modelling

Fig. 4 shows the front wheel model.

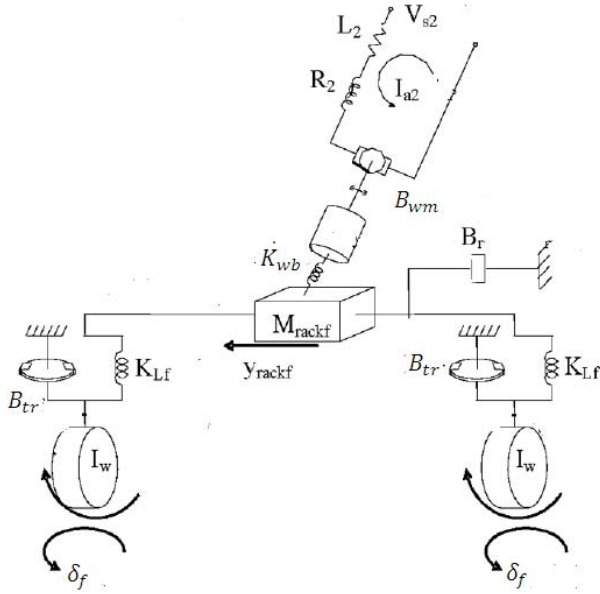


Fig.4 Front tire wheel system diagram [4]

The mathematical equations of the front tire wheel subsystem are written as follows:

Front motor current

$$\dot{I}_{a2} = -\frac{R_2}{L_2} I_{a2} - \frac{K_{wb}}{J_{wm}} T_{m2} + V_{s2} \quad (8)$$

The Torque of Front motor

$$\dot{T}_{m2} = \frac{K_{wb}}{L_2} I_{a2} - \frac{B_{wb}}{J_{wm}} T_{m2} - \frac{\theta_{m2}}{C_{wm}} \quad (9)$$

The Rack force;

$$\dot{y}_{rack} = -\frac{b_r}{m_r} y_{rack} - \frac{\theta_{m2}}{C_{wm} * g_{mr}} - \frac{g_r}{C_{tr}} v_{tr} \quad (10)$$

and Front tire angle;

$$\dot{\delta}_f = -\frac{B_{tr}}{j_t} \delta_f + \frac{v_{tr}}{C_{tr}} \quad (11)$$

Front Angular displacement motor

$$\dot{\theta}_{m2} = \frac{T_{m2}}{J_{wm}} + \frac{y_{rack}}{mr * g_{mr}} \quad (12)$$

Tierod velocity

$$\dot{v}_{tr} = \frac{y_{rack}}{m_r * g_r} - \frac{v_{tr}}{j_t} \quad (13)$$

Consequently, the state equation of the front tire wheel system is given as:

$$\dot{x}(t) = A_{ftw} x(t) + B_{ftw} u(t) \quad (14)$$

$$y(t) = C_{ftw} x(t) + D_{ftw} u(t) \quad (15)$$

where state model is given as:

$$x(t) = [i_{a2} \ T_{m2} \ y_{rack} \ \delta_f \ \theta_{m2} \ v_{tr}]^T \quad (16)$$

The steering motor angular displacement (\$\theta\_{m1}\$) is considered the input of the front wheel system. The mathematical model in states matrix forms are:

$$u(t) = [\theta_{m1}] \quad (17)$$

$$A_{ftw} = \begin{bmatrix} \frac{-R_2}{L_2} & \frac{-K_{wb}}{J_{wm}} & 0 & 0 & 0 & 0 \\ \frac{K_{wb}}{L_2} & \frac{-B_{wb}}{J_{wm}} & \frac{-b_r}{M_r} & 0 & \frac{C_{wm}}{-1} & \frac{-g_r}{C_{tr}} \\ 0 & 0 & \frac{-1}{j_t} & \frac{C_{wm} * g_{mr}}{-1} & 0 & \frac{1}{C_{tr}} \\ 0 & 0 & \frac{g_{mr} * M_r}{1} & 0 & 0 & 0 \\ 0 & \frac{1}{J_{wm}} & \frac{1}{g_r * M_r} & \frac{-1}{j_t} & 0 & 0 \\ 0 & 0 & \frac{1}{g_r * M_r} & \frac{1}{j_t} & 0 & 0 \end{bmatrix}$$

$$B_{ftw} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C_{ftw} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D_{ftw} = 0$$

### D. Controller Design

#### 1) Directional control and Wheel Synchronization

Fig. 5 shows the block diagram for the control structure of the steer by wire system (SBW). It includes the steering wheel system, the front wheel system and the controller. Two PID controllers were used for steering wheel and front wheel system. Based on the fig. 5, when the driver holds and turn the steering wheel, an adjustable angle is created by the motor of the steering wheel system, the PID controller will adjust the steering angle to a desired angle by calculate the error. This angle will substitute in to the front wheel system and feedback to the PID controller.

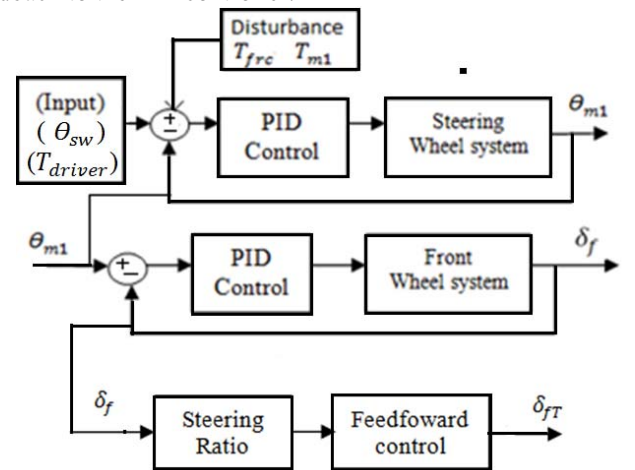


Fig.5 Control structure SBW system diagram

## II. VARIABLE STEERING RATIO

The steering ratio is one of the main advantages in advanced vehicle steering system. The ratio is referred to the amount of turn in steering wheel to the amount of degree of

the front tire angle. An advantage of steering ratio is that the driver applies a small force on steering wheel, which will result in large steering force at the front wheel. Thus will reduce the amount of steering wheel turn compared to the tire angle. In conventional steering system, the steering ratio is set between 12:1 and 20:1 depending to the manufacturer.

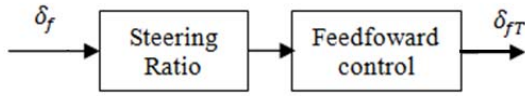


Fig.6 Variable steering ratio diagram

Feedback sensors from the front tire wheel angle and vehicle speed were used as a input to the steering ratio algorithm in order to control the tire angle. Fig. 6 shows the implementations of the variable steering ratio. The concept of the variable steering is to improve maneuverability by adjusting the tire angle based on vehicle speed. If the vehicle speed is increased, smaller steering ratio will use while in vehicle at stop condition (i.e in parking) a higher ratio will use. By using this method, the maneuverability and stability can be improved [6]. A feed forward control is used to control the steering ratio based on understeer gradient equation.. Equations (18) present the understeer gradient equation. An improvement was made based on paper [6], by adding the characteristics of the initial front tire angle and the steering ratio to the control strategies and it is present in equation (20). Equation below shows the improvement of the understeer gradient equation.

$$K = \frac{R}{V^2} (\delta_f - 57.3 \frac{(a+b)}{R}) \quad (18)$$

$$K = [(W_f/C_f) - (W_r/C_r)] \quad (19)$$

$$\delta_{fT} = \left( \frac{(a+b)\delta_{ratio}}{(V^2 K + (a+b))} \right) + \frac{\delta_f}{\delta_{ratio}} \quad (20)$$

Where;

$\delta_{ratio}$  = steering ratio

$\delta_{fT}$  = Front tire angle (with steering ratio)

$V$  = vehicle speed

$\delta_f$  = Front tire angle without steering ratio

$K$  = adjustable gain

$a$  = distance of front tire to vehicle COG

$b$  = distance of rear tire to vehicle COG

$R$  = radius of COG path

$W_f$  = weight of front wheel

$W_r$  = weight of rear wheel

$C_f$  = total front cornering stiffness

$C_r$  = total rear cornering stiffness

### III. SIMULATION AND RESULTS

To investigate the effectiveness of the proposed control algorithm, a computer simulation based on Matlab software were conducted and output responses are compared with passive system.

#### Wheel Synchronization Controller

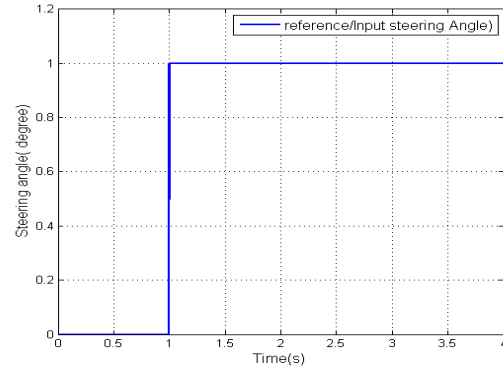
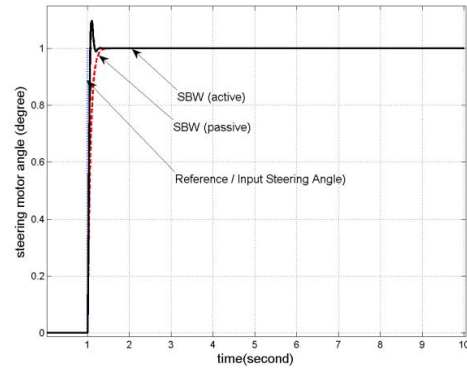
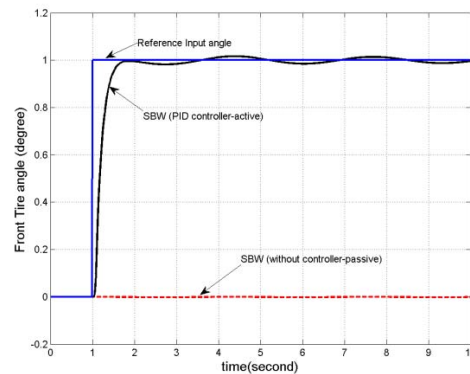


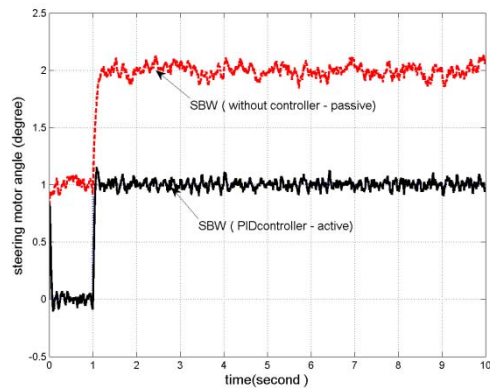
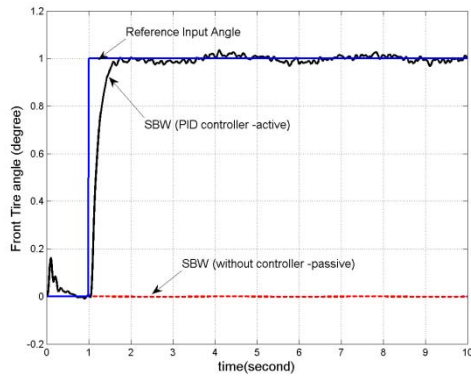
Fig 7: Input steering angle

To simulation the robustness of the system, two scenarios were conducted, case 1 without disturbance input and case 2 with a disturbance inputs. The input to the system is a steering angle ( $\theta_{sw}$ ) simulated as a step response, as shown in fig. 7 and driver torque ( $T_{driver}$ ) = 4Nm. The torque friction ( $T_{fric}$ ) and the torque motor ( $T_{m1}$ ) = 2Nm, react as disturbance input, where torque friction ( $T_{fric}$ ) is created using random noise. Fig. 8 and fig 9 shows the output response for the steering motor angular displacement ( $\theta_{m1}$ ) and the front tire angle ( $\delta_f$ ) without disturbance input, however fig.10 and fig. 11 with disturbance input to the system.

Fig 8: Steering motor angular displacement ( $\theta_{m1}$  case 1 )

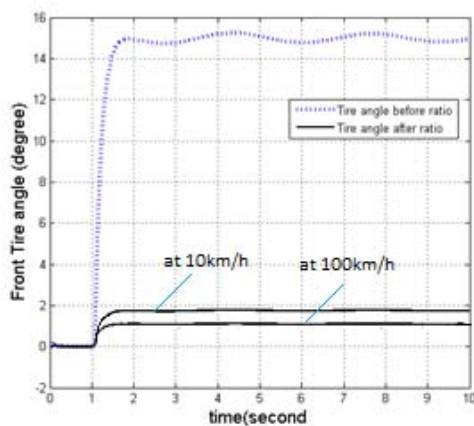
Based on fig. 11, the front tire angle ( $\delta_f$ ) response is unstable due to the used of open loop system (without using controller), however by using PID controller the system is stable even there are exist disturbances at the input to the system..

Fig 9: Front Tire angle ( $\delta_f$  case 1)

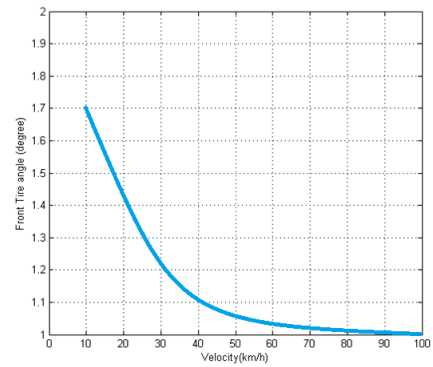
Fig 10: Steering motor angular displacement ( $\theta_{m1}$  case 2)Fig 11: Front Tire angle ( $\delta_f$  case 2)

#### A. Variable Steering Ratio

A simulation is conducted by applying steering ratio of 15:1, steering angle input as fig. 7 and a random noise disturbance input friction torque ( $T_{frc}$ ) was applied and torque of motor ( $T_{m1}$ ) = 2Nm. In variable steering ratio, at high speed the tire angle is stiff and at lower speed the tire angle is increased to provide easy maneuverability.

Fig 12: Front Tire angle ( $\delta_{FT}$ )

From fig 12, it can be seen, the front tire angle ( $\delta_{FT}$ ) response are varies at low and high speed and fig 13 shows the relationship between of the front tire angle ( $\delta_{FT}$ ) against the vehicle speed.

Fig 13: Front tire angle ( $\delta_{FT}$ ) against Vehicle Speed (V)

#### IV. CONCLUSION

In this paper, mathematical model and control strategy for a steer by wire system was proposed. From the simulation, it can be concluded that by using the PID controller, the front tire angle ( $\delta_{FT}$ ) is stable even disturbance and torque are applied. Based on the results, the controller is able to reduce the disturbance error. In variable steering ratio, the proposed feed forward control is suitable to use due to less sensor use and simple control strategy with constant steering ratio. From the simulation result, the variable steering ratio is achieved, where by using the proposed control strategy, the front tire angle ( $\delta_{FT}$ ) is increased and proportional to the vehicle speed. Thus the proposed control strategy improved maneuverability and stability of the system. For future work, a yaw rate and body slip angle will be investigated to include into the system for in order to improve the maneuverability and stability.

#### ACKNOWLEDGEMENT

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## APPENDIX I

## REFERENCES

BIL	Steer ByWire System Parameter		
	Items	Values	units
$K_{s1}$	Lumped torque stiffness(SM)	3500	Nm/rad
$I_{sw}$	Steering Lumped Inertia(SM)	0.0079	Kgm2
$B_{sc}$	Steering column damping(SM)	0.136	Nmsec/rad
$I_{m1}$	Inertia of steering motor(SM)	0.0021	Kgm2
$K_{b1}$	Steering motor emf constant(SM)	0.35	V-s/rad
$L_1$	Motor electrical inductance (SM)	0.002	H
$R_1$	Motor electrical resistance (SM)	4.6	Ohm
$R_2$	Motor electrical resistance (FM)	5.0	Ohm
$L_2$	Motor electrical inductance (FM)	0.002	H
$K_{wmb}$	Motor torque constant (FM)	2	Nm/rad
$J_{wm}$	Motor moment inertia (FM)	0.0079	Nm/rad
$B_{wm}$	Motor resistance coefficient (FM)	1.0	Kgm2
$C_{wm}$	Motor shaft compliance (FM)	0.4	Nm/s
$b_r$	Resistance rack (FM)	25	Nm/rad
$m_r$	Mass rack (FM)	2.0	Kg
$g_{mr}$	column pinion radius (FM)	0.015	m
$g_r$	length ratio steering arm (FM)	4.5	m
$C_{tr}$	Compliance of tie rod (FM)	0.2	rad/Nm
$B_{tr}$	resistance of tie rod (FM)	0.004	Nms/rad
$j_t$	Inertia of tire (FM)	1.36	Kgm2
$\delta_{FT}$	Front tire angle (with ratio)	-	degree
$T_{driver}$	torque of driver	[0,4]	Nm
$T_{fre}$	torque friction	Random	Nm
$T_{m1}$	steering motor torque	[0,2]	Nm
$V_s$	voltage source	-	V
$\theta_{m1}$	Angular displacement of steering motor	-	degree
$i_{a1}$	Current of steering motor	-	ampere
$i_{a2}$	Current of front motor	-	ampere
$T_{m2}$	Torque of front motor	-	Nm
$y_{rack}$	Rack force	-	Nm
$\theta_{m2}$	Angular displacement of front motor	-	degree
$V_{tr}$	Tie rod velocity	-	Km/h
V	Vehicle speed	[10,100]	Km/h

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