MODULE 7

Estimation

PERCENTILES

DEFINING PERCENTILES

- **A percentile is the value at a particular rank.**
- Let p be a number between 0 and 100. The pth percentile of a collection is the smallest value in the collection that is **at least as large as p**% of all the values.
- Practically, suppose there are n elements in the collection. To find the pth percentile:
  - Sort the collection in increasing order.
  - Find p% of n: (p/100)×n. Call that k.
  - If k is an integer, take the kth element of the sorted collection.
  - If k is not an integer, round it up to the next integer, and take that element of the sorted collection.

COMPUTING PERCENTILES

The **p**th percentile is the first value on the sorted list that is **at least as large as p**% of the elements.

Example: **s = [1, 7, 3, 9, 5]**

s_sorted = [1, 3, 5, 7, 9] and

Percentile

Data set

percentile(80, s)is 7

The 80th percentile is the 4th ordered element: **(80/100) * 5**

- For a percentile that does not exactly correspond to an element, take the next greater element instead

THE PERCENTILE FUNCTION

- The pth percentile is the smallest value in a set that is at least as large as p% of the elements in the set
- Function in the ***datascience*** module:

  percentile(p, values)
- p is between 0 and 100

- Returns the *p*th percentile of the array

(Demo – notebook 7.1, Percentiles)

DISCUSSION QUESTION

Which are True, when s = [1, 7, 3, 9, 5]?

percentile(10, s) == 0

percentile(39, s) == percentile(40, s)

percentile(40, s) == percentile(41, s)

percentile(50, s) == 5

(Demo – notebook 7.1, Percentiles in class)

ESTIMATION

INFERENCE: ESTIMATION

- How do we calculate the value of an unknown parameter?
- If you have a census (that is, the whole population):
  - Just calculate the parameter and you're done
- If you don't have a census:
  - Take a random sample from the population
  - Use a statistic as an **estimate** of the parameter

(Demo – Notebook 7.1, Estimating Median - Sample Median)

VARIABILITY OF THE ESTIMATE

- One sample ➜ One estimate
- But the random sample could have come out differently
- And so the estimate could have been different
- Big question:
  - How different would it be if we did it again?

(Demo – Notebook 7.1, Variability of the Estimate)

QUANTIFYING UNCERTAINTY

- The estimate is usually not exactly right:

  Estimate = Parameter + Error

- How accurate is the estimate, usually?
- How big is a typical error?
- When we have a census, we can do this by simulation

(Demo – Notebook 7.1, Quantifying Uncertainty)

WHERE TO GET ANOTHER SAMPLE?

- We want to understand errors of our estimate
- Given the **population**, we could simulate
  - ...but we only have the **sample**!

- To get many values of the estimate, we needed many random samples
- Can't go back and sample again from the population:
  - No time, no money
- Stuck?

THE BOOTSTRAP

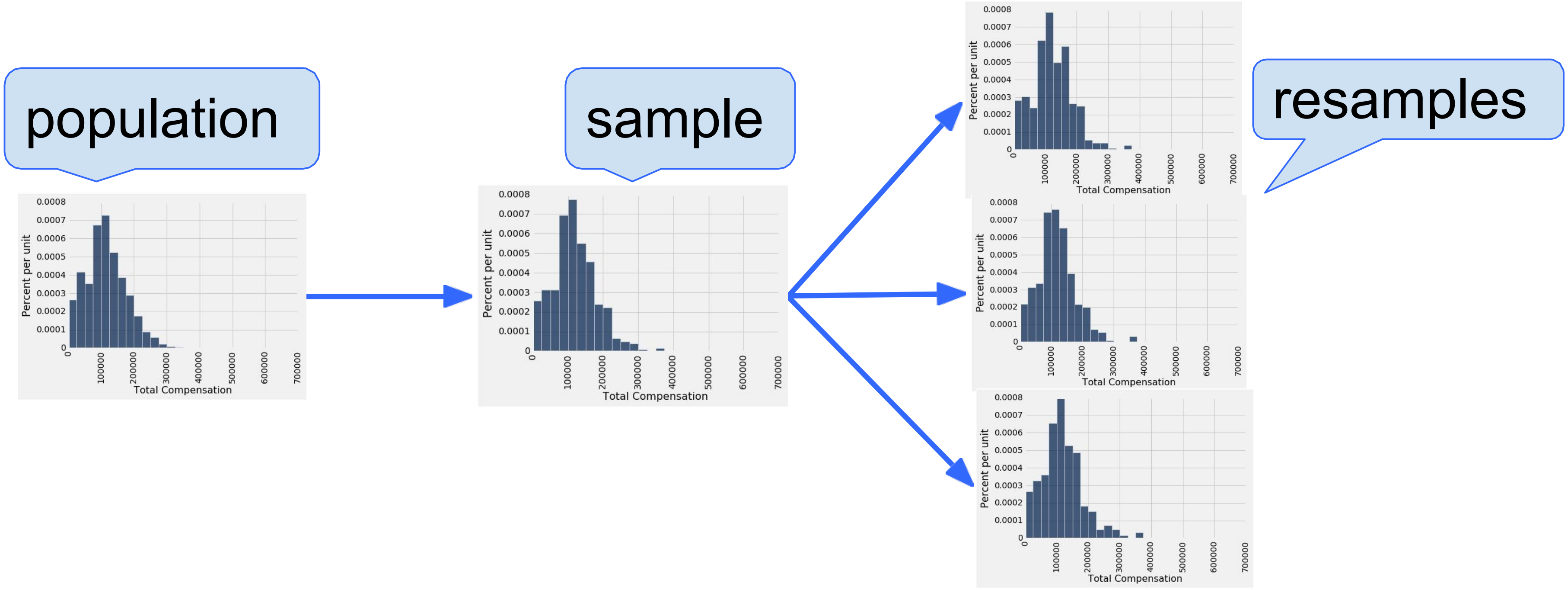
THE BOOTSTRAP

- A technique for simulating repeated random sampling

- All that we have is the original sample
  - … which is large and random
  - Therefore, it probably resembles the population

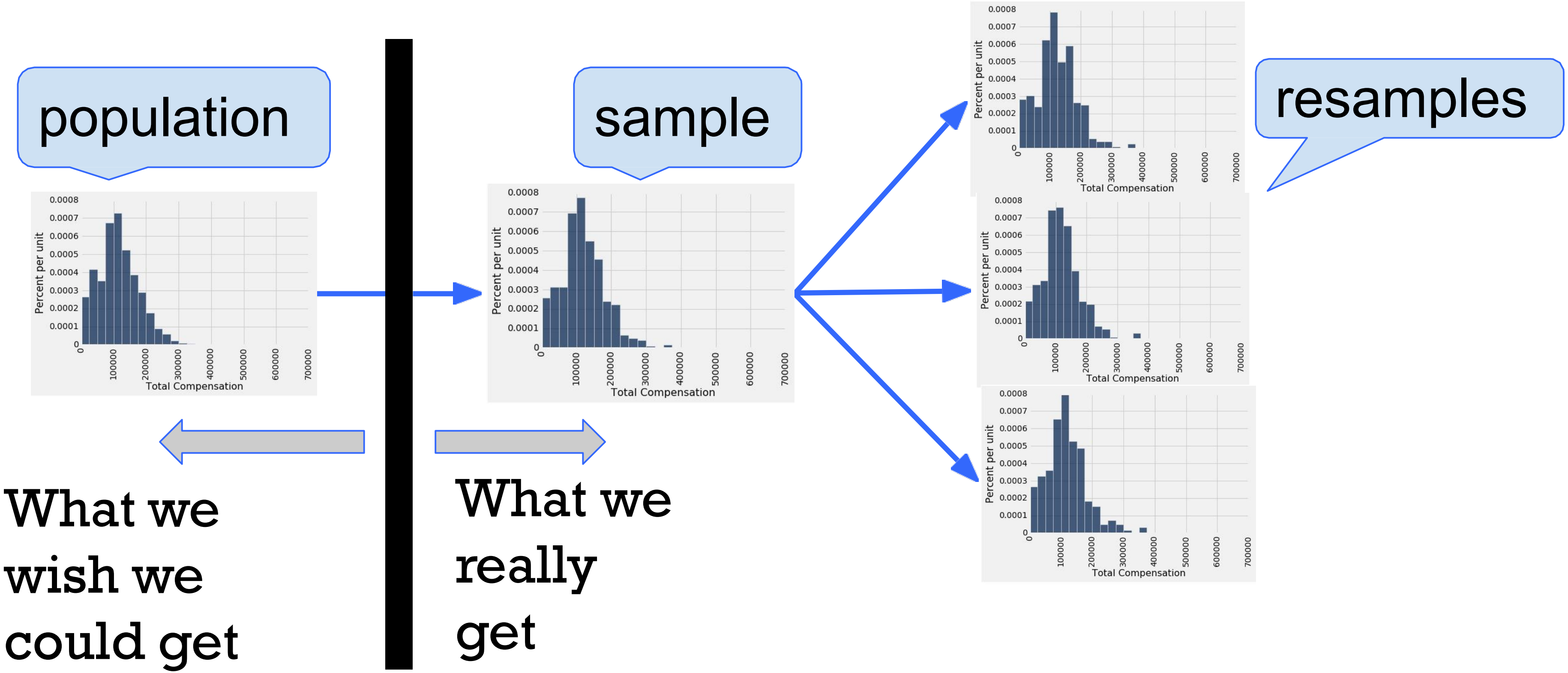
- So, we sample at random from the original sample!
  - AKA, *resampling*

WHY THE BOOTSTRAP WORKS



All of these look pretty similar, most likely.

WHY WE NEED THE BOOTSTRAP



REAL WORLD VS. BOOTSTRAP WORLD

Real world:

- True probability distribution (**population**)
  - → Random sample 1
    - → Estimate 1
  - → Random sample 2
    - → Estimate 2
  - …
  - → Random sample 10000
    - → Estimate 10000

Bootstrap world:

- Empirical distribution of original sample (**"population"**)
  - → Bootstrap sample 1
    - → Estimate 1
  - → Bootstrap sample 2
    - → Estimate 2
  - …
  - → Bootstrap sample 1000
    - → Estimate 1000

Hope: these two scenarios are analogous

Real vs. Bootstrap World

Real world (what we want):

- True probability distribution (**population**)
  - → Random sample 1
    - → Estimate 1
  - → Random sample 2
    - → Estimate 2
  - …
  - → Random sample 10000
    - → Estimate 10000

Can't get these :(

Bootstrap world:

- Empirical distribution of original sample (**"population"**)
  - → Bootstrap sample 1
    - → Estimate 1
  - → Bootstrap sample 2
    - → Estimate 2
  - ...
  - → Bootstrap sample 1000
    - → Estimate 1000

Hope: these two scenarios are analogous

THE BOOTSTRAP PRINCIPLE

- The bootstrap principle:
  - **Bootstrap-world** sampling ≈ **Real-world** sampling

- Not always true!
  - … but reasonable if sample is large enough

- We hope that:

  a. Variability of bootstrap estimate
  b. Distribution of bootstrap errors

  …are similar to what they are in the real world

KEY TO RESAMPLING

- From the original sample,
  - draw at random
  - **with replacement**
  - as many values as the original sample contained

- The size of the new sample has to be the same as the original one, so that the two estimates are comparable

(Demo – notebook 7.1, Bootstrap)

CONFIDENCE INTERVALS

DO THE ESTIMATES CAPTURE THE PARAMETER?

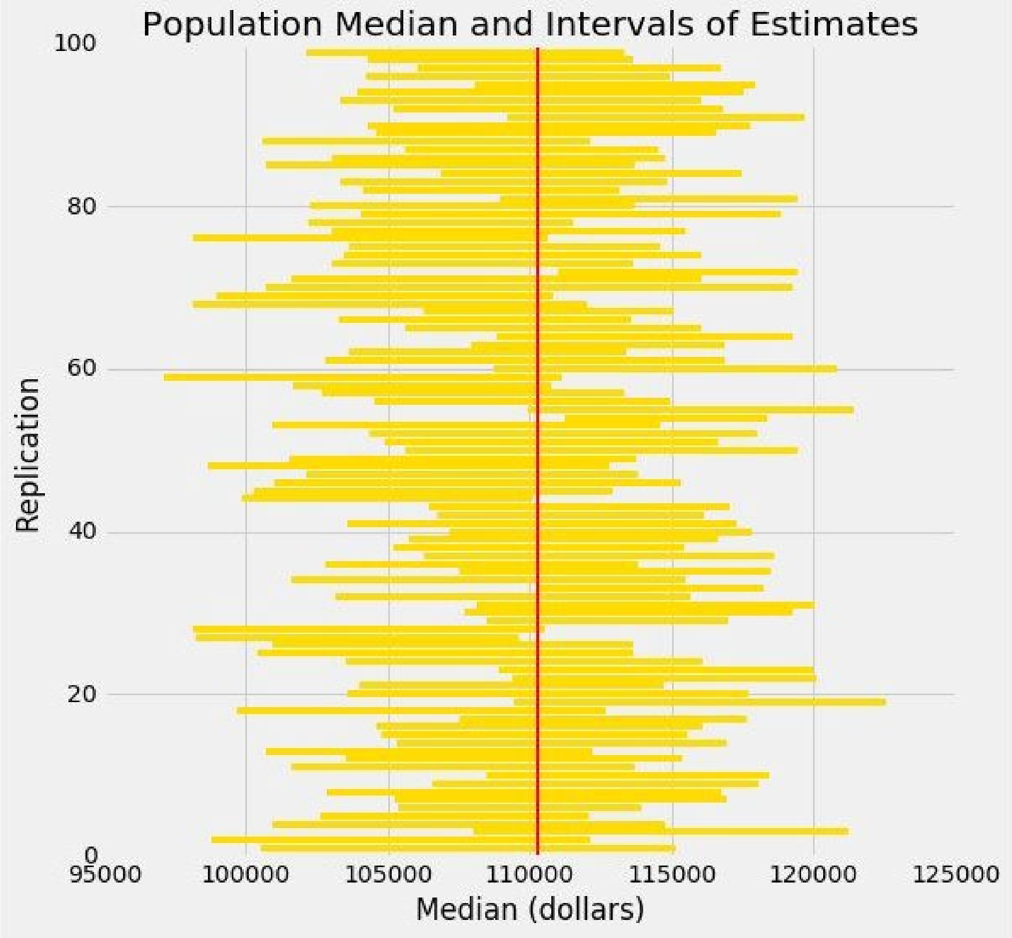
Q: How often does the empirical histogram of the resampled medians **contain our parameter?**

For instance, **how often** does the **middle 95% of the resampled medians contain our parameter?**

95% CONFIDENCE INTERVAL

- Interval of **a parameter**
- Based on random sampling
- 95% is called the ***confidence level***
  - Could be any percent between 0 and 100
  - Higher level means wider intervals
- The **confidence is in the process** that gives the interval:
  - It generates a "good" interval about 95% of the time.

(Demo – notebook 7.1, Confidence Intervals)



Each line here is a confidence interval from a fresh sample from the population

95% CI: Usage vs Interpretation

- **How to create it**
  - Middle 95% of the bootstrapped estimates

- **How to interpret it**
  - 95% of samples will give a 95% CI that contains the true parameter

USE METHODS APPROPRIATELY

CAN YOU USE A CI LIKE THIS?

By our calculation, an approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

- About 95% of the mothers in the population were between 26.9 years and 27.6 years old.

Answer: **False.** We're estimating that their **average age** is in this interval.

IS THIS WHAT A CI MEANS?

An approximate 95% confidence interval for the average age of the mothers in the population is (26.9, 27.6) years.

True or False:

- There is a 0.95 probability that the average age of mothers in the population is in the range 26.9 to 27.6 years.

Answer: **False.** The average age of the mothers in the population is unknown but it's a constant. It's not random. No chances involved.

WHEN *NOT* TO USE THE BOOTSTRAP

- If you’re trying to estimate very high or very low percentiles, or min and max
- If you’re trying to estimate any parameter that’s greatly affected by rare elements of the population
- If the probability distribution of your statistic is not roughly bell shaped (the shape of the empirical distribution will be a clue)
- If the original sample is very small

CONFIDENCE INTERVALS FOR TESTING

USING A CI FOR TESTING

- Null hypothesis: Population average = x
- Alternative hypothesis: Population average $\neq x$
- Cutoff for P-value: p%
- Method:
  - Construct a (100-p)% confidence interval for the population average
  - If x is not in the interval, reject the null
  - If x is in the interval, can't reject the null

QUESTIONS?