Final Report

IEM 4013-24520

ROCH

By Aymen Charmi, Jacob Roach, and Brenna Rodgers

Alabama

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Executive Summary

The state being examined for redistricting is Alabama. There are seven congressional districts in Alabama that have not changed since 1973. Therefore, after 2010 Alabama had seven congressional districts and also seven after 2020. Alabama's district map has remained unchanged for over four decades. The criteria these maps meet are that the district lines are contiguous and that the district lines must be approved. In some states even a 1% population deviation will deem the redistricting unconstitutional.

Alabama's counties have been redistricted into these 7 districts. District 1is the smallest district with counties Cleburne, Tallapoosa, Crenshaw, Calhoun, Elmore, Randolph, Clay, Cherokee, Lee, Chambers, Lowndes, Talladega, Autauga, and Macon and has a total population of 714,963 people. District 2 is the largest district with the population of 720,310 it contains the following counties: Jefferson, Bibb, Hale, and Perry. District 3 contains counties Clarke, Washington, Mobile, Monroe, Baldwin, and Choctaw and has a population of 717,488 people. District 4 has a population of 718,247 people and contains counties Shelby, Dallas, Pickens, Walker, Chilton, Marengo, Greene, Fayette, Tuscaloosa, Wilcox, Coosa, Sumter, and Winston. District 5 contains counties Lawrence, Morgan, Etowah, DeKalb, Marshall, Blount, Jackson, St. Clair, and Cullman and has a population of 719,832 people. District 6 has a population of 717,940 people and contains counties Lauderdale, Madison, Lamar, Franklin, Colbert, Limestone, and Marion. District 7 contains counties Barbour, Geneva, Dale, Houston, Russell, Conecuh, Montgomery, Coffee, Covington, Henry, Butler, Escambia, Pike, Bullock and has a population of 715,499 people. This is the new district plan that was found from the minimization problem.

Introduction

"Redistricting is the way we change the districts that determine who represents us" (Levitt 2022).

Congressional redistricting occurs every 10 years after a new population census is produced,

which affects the deviations of populations in each congressional district. The new census also

provides changes in political affiliation, which will affect the current political parties in control

of districts. Alabama has 67 counties that are going to be divided into 7 districts. Figure 1 shows
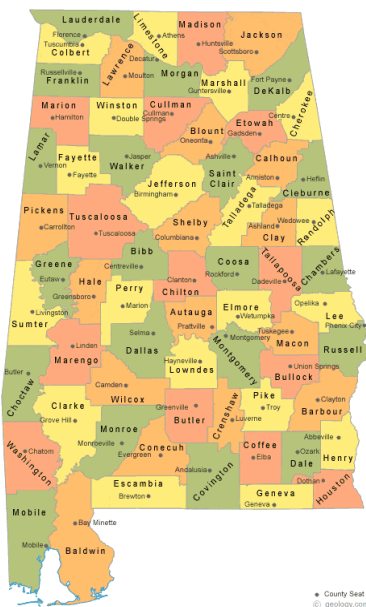
the counties of Alabama and the locations.

Figure 1: Alabama counties; https://geology.com/county-map/alabama.shtml

Problem Statement

The problem is deciding which counties go into which districts while satisfying federal and state
policies/criteria.

Criteria

Federal Criteria

The federal criteria is based on two categories: population equality, and racial & language minority protections.

Population Equality:

- The first criteria for a state is population equality. Each state needs to have around the same population in each of their own districts. "Across states, however, district population sizes can vary"(Eckman 2021). In other words, each states' district population can differ between states.

Racial and language minority protections:

- The second federal criteria is racial and language minority protections. "Under the VRA, states cannot draw district maps that have the effect of reducing, or diluting, minority voting strength" (Eckman 2021). Each state cannot divide the districts in a way that will make the minority population all in one district and not in the others.

These are the federal criteria for redistricting in states across the United States.

State Criteria

The state criteria is based on three things.

Compactness and Contiguity:

- First, the criteria is compactness and contiguity. A district is contiguous if travel is allowed between two points in a district without crossing into a different district. "For congressional districts, 34 states require contiguity" (Eckman 2021).

Political subdivisions and communities of interest:

- The second state criteria is the political subdivisions and communities of interest. Only 31 states require consideration of political divisions.

 Political competition or considering existing district/incumbent:

- The third and final state criteria is the political competition or considering existing district/incumbent. "Some states expressly allow the use or consideration of party identification information in the redistricting process, whereas others prohibit it" (Eckman 2021). Each state can choose to either use consideration of existing information or not.

This is the general state criteria for all states. Alabama's criteria follow these three criteria. Alabama must abide by the Voting Rights Act and constitutional rule on race. Alabama's districts should be reasonably compact, contiguous and follow state lines where possible. "In addition, the committee agreed to "attempt to avoid contests between incumbents" (Graves 2022). Alabama also considers the population in prison when redistricting.

Integer Programming Model

OR Model In Words

- Indices:
  - i is the county i $\in$ {1, 2, ..., n}
  - j is the district j $\in$ {1, 2, ..., k}

- Parameters:
  - n= # of counties in Alabama

- ○ k= # of districts desired

- ○ L= lowest district population

- ○ U= highest district population

- Variables:

  - ○ x_{ij}= 1 when county i ∈ {1, 2, …, n} is assigned to district j ∈ {1, 2, …, k} ; 0

    otherwise

  - ○ p_i= population of each district i

  - ○ f_{ij}^{v}= flow across arc [i,j,v] that is sent from source/ root v

- Objective Function:

  - ○ Objective is to minimize the moment of inertia: sum (d^2 * p * x over all i and j = w_{ij})
    - ■ This equation uses the districts as bodies with a centroid. The centroid can be at j or another location i, or some distance d_{ij} from the centroid j. "The mass of the parcel is its population" (Two MIPs for redistricting).

- Constraints:

  - ○ Add constraints saying that each county i is assigned to one district
  - ○ Add constraint saying there should be k district centers
  - ○ Add constraints that say: if j roots a district, then its population is between L and U
  - ○ Add coupling constraints saying that if i is assigned to j, then j is a center
  - ○ Add contiguity constraints
    - ■ Add constraints saying that if node i is assigned to node j then node i must consume one unit of node j's flow
    - ■ Add constraints saying that node i can receive flow of type j only if node i is assigned to node j
    - ■ Add constraints saying that j cannot receive flow of its own type

OR Model In Comments and Math

OR Model in Words	OR Model in Math
Minimize the moment of inertia for county i in district j	$\min \sum_{i \in V} \sum_{j \in V} w_{ij} x_{ij}$
Constraint- Vertex i is assigned to one district	$\sum_{j \in V} x_{ij} = 1 \qquad \forall j \in V$
Constraint-k districts are chosen	$\sum_{j \in V} x_{ij} = k \qquad \forall j \in V$
Constraint-population of each district i is between L and U	$Lx_{jj} \leq \sum_{i \in V} p_i x_{ij} \leq Ux_{jj} \qquad \forall j \in V$
Constraint- if i is assigned to j, then j is a center.	$x_{ij} \leq x_{jj} \qquad \forall i, j \in V$ $x_{ij} \in \{0, 1\} \qquad \forall i, j \in V$
Constraint- vertex i is assigned to center j, and i consumes one unit of flow of type j, otherwise 0	$\sum_{u \in N(i)} (f^j_{ui} - f^j_{iu}) = x_{ij} \qquad \forall i \in V \backslash \{j\}, \forall j \in V$
Constraint- vertex i can receive flow of only j and if i is assigned to center j	$\sum_{u \in N(i)} f^j_{ui} \leq (n - 1)x_{ij} \quad \forall i \in V \backslash \{j\}, \forall j \in V$
Constraint- prevent flow circulations	$\sum_{u \in N(i)} f^j_{uj} = 0 \qquad \forall j \in V$ $f^v_{ij}, f^v_{ji} \geq 0 \qquad \forall \{i, j\} \in E, \forall v \in V.$

Experiments

The computer we used for the coding was a microsoft surface pro that was running the code with around 75% CPU. The computer has 4 GB RAM. The gurobi version is 9.5.0, the objective value is 6,593,085,384.640669 given by minimizing the moment of inertia: sum (d^2 * p * x over all i and j). The required time to solve the optimization model was 614.72 seconds.

Solution

After optimization from the code we used we got the following results. The results found for each district were as follows:

- District 1 has population 714963 and contains counties ['Cleburne', 'Tallapoosa', 'Crenshaw', 'Calhoun', 'Elmore', 'Randolph', 'Clay', 'Cherokee', 'Lee', 'Chambers', 'Lowndes', 'Talladega', 'Autauga', 'Macon']

- District 2 has population 720310 and contains counties ['Jefferson', 'Bibb', 'Hale', 'Perry']

- District 3 has population 717488 and contains counties ['Clarke', 'Washington', 'Mobile', 'Monroe', 'Baldwin', 'Choctaw']

- District 4 has population 718247 and contains counties ['Shelby', 'Dallas', 'Pickens', 'Walker', 'Chilton', 'Marengo', 'Greene', 'Fayette', 'Tuscaloosa', 'Wilcox', 'Coosa', 'Sumter', 'Winston']

- District 5 has population 719832 and contains counties ['Lawrence', 'Morgan', 'Etowah', 'DeKalb', 'Marshall', 'Blount', 'Jackson', 'St. Clair', 'Cullman']

- District 6 has population 717940 and contains counties ['Lauderdale', 'Madison', 'Lamar', 'Franklin', 'Colbert', 'Limestone', 'Marion']

- District 7 has population 715499 and contains counties ['Barbour', 'Geneva', 'Dale', 'Houston', 'Russell', 'Conecuh', 'Montgomery', 'Coffee', 'Covington', 'Henry', 'Butler', 'Escambia', 'Pike', 'Bullock']
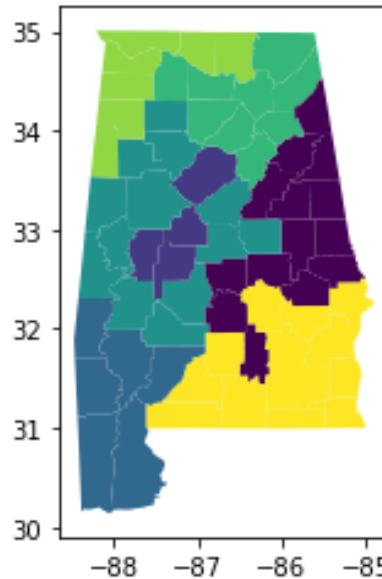

Figure 2: Alabama districting from code; Jupyter notebook code

In addition, figure 2 shows the redistrict map that was found after this optimal solution. This map and district size, with counties, is an optimal solution because it follows all of the constraints set and all of the criteria.

Districting Plan

The proposed district map has 7 districts, and they are all connected. Figure 3 shows the redistrict map proposed for our plan. We made this map from the district.org website. Each district is around 717,000 people for each population. As shown on figure 3, each population is shown on the right matching up with individual colors for each district. The deviation from this

proposed plan is 0.39%. Since it was found that 0.39% was the deviation, this plan can be considered as legal because it is less than 1%.
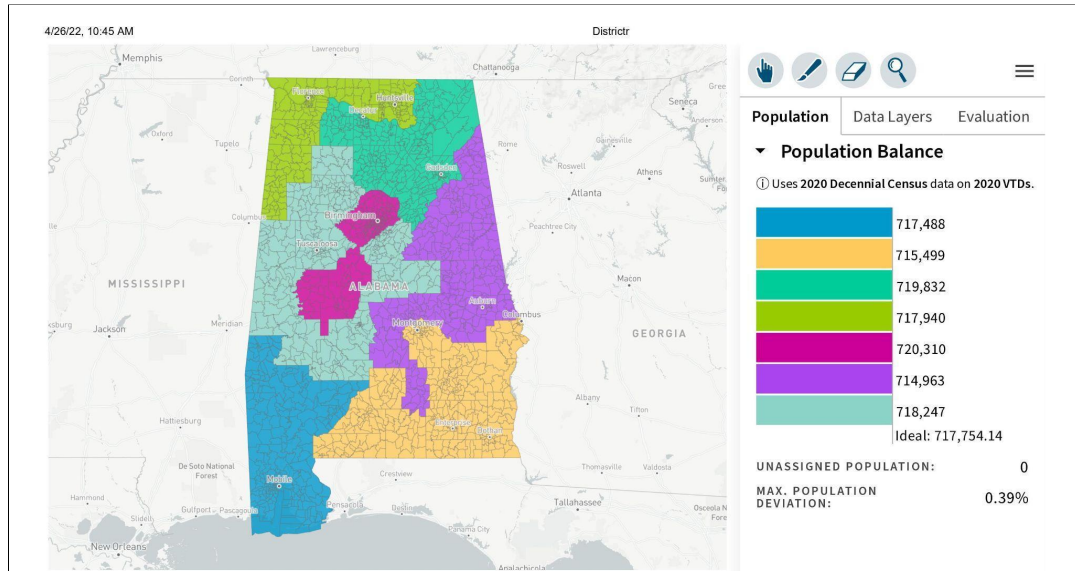

Figure 3: Redistrict Plan; https://districtr.org/plan/126331

Evaluation Of Plan

The proposed map, as seen in figure 2 meets most of the criteria, but the population deviation is 0.39% instead of 0.01%. The limitation of the plan is that it is more precise because we divided them by counties exactly, so it has no overlap. The map does meet all the required criteria and meets the contiguity constraints.

Conclusion

In conclusion, the new district map found has 7 districts, each with a population of around 717,000 people. The results found for each district were as follows:

- District 1 has population 714963 and contains counties ['Cleburne', 'Tallapoosa', 'Crenshaw', 'Calhoun', 'Elmore', 'Randolph', 'Clay', 'Cherokee', 'Lee', 'Chambers', 'Lowndes', 'Talladega', 'Autauga', 'Macon']

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The districts found follow contiguity guidelines and are close to equivalent in size. The map found from the code was similar to the map planned on the district website. The solution was optimal and could be used as a new district plan.

Appendices

Code

The code file is submitted separately.

GitHub Repository

Our OR code, report, and experimental results are on the github site. Here is a link to our post:

https://github.com/blrodgers26/IEM-4013-Final-Project

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