The Lecture Title

Scribe: Your Name

Date: Day, Mon, Date Year

1 The Triangle Problem

Redo the triangle problem from before, but consider picking any two points at vertices between two unit triangles, and setting that to be the longest diagonal of the parallelogram. In this manner, we can see that the

2 Unit 2: Generating Functions

A **generating function** of a sequence $\{a_n\}_{n=1}^{\infty}$ is the function $g(x)$ s

Problem 1. Find the nth term in the sequence $1, 2, 4, 8, 16, \ldots$

Solution Clearly, we see that each term is ■

Problem 2. Find a generating function for the sequence 1, 2, 4, 8, 16, ...

Solution By definition, we have that

$$g(x) = 1x^0 + 2x^1 + 4x^2 \ldots = \frac{1}{1-2x}$$

by the geometric series. ■

With this analysis, we can clearly see that generating function g for the sequence $1, a, a^2, \ldots = \{a^k\}_{k=0}$ is $\frac{1}{1-ax}$.

Here is another example:

Problem F. ind an explicit form for the sequence $2, 5, 13, 35, 97, \ldots$ and a generating function for this sequence.

Solution By inspection, we see that this sequence is $2^n + 3^n$. Clearly, we see that ■ Notice that generating functions are linear, so we can

Problem C. onsider the *Fibonacci numbers*, defined by $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$. Find a generating function for the Fibonacci sequence, and then derive an explicit expression for the Fibonacci numbers, *Binet's Formula*

Solution Consider writing out this generating function $F(x)$:

$$F(x) = x + x^2 + 2x^3 +$$

■