The Lecture Title

Scribe: Your Name

Date: Day, Mon, Date Year

1 Triangulation

Problem 1. How many triangulations are there of a labeled hexagon?

Note for a square, there are two (one that is just rotated): And for a pentagon, there are five - just one possible configuration that can be rotated:

Solution. This gives a total of $\boxed{14}$ total triangulations. ■

Proof. We can solve the more general case first. Let T_n be the number of ways to triangulate an n-gon. Notice that if we draw a diagonal of the n-gon ■

2 Laser Grid

Problem C. onsider walking on the coordinate plane, but you are constrained to either walk over by one

Solution

■

Problem 2. Consider walking on the coordinate plane, but you are constrained to either walk

Solution We will employ complementary counting by counting how many paths cross the line. The total number of paths (from the first problem) is known to be $\binom{2n}{n}$.

Now, define a *bad path* to be one that crosses the axis. Note that any bad path must touch the line $y = -1$. We will reflect the part of the path from $(0, 0)$ to the first intersection with $y = -1$ over the line $y = -1$. Doing so yields a path from $(0, -2)$ to $(2n, 0)$ with no restrictions.

We now claim that all bad paths from $(0, -2)$, when flipped in this manner, will yield a bad path in the original problem. This path must cross the line $y = -1$ at least once. Notice that when we undo this reflection, the path will now start at $(0, 0)$, and is forced to touch the line $y = -1$, and this uniquely determines exactly one bad path. Therefore, it suffices to count the number of paths from $(0, -2)$ to

Finally, the number of good paths is $\binom{2n}{n} - \binom{2n}{n-1}$. This can be simplified to yield $\frac{1}{n+1}\binom{2n}{n}$, which is the nth Catalan number. ■

Remember also that we encountered these before in the binomial expansion of $(1-x)^{\frac{1}{2}}$!