The Lecture Title

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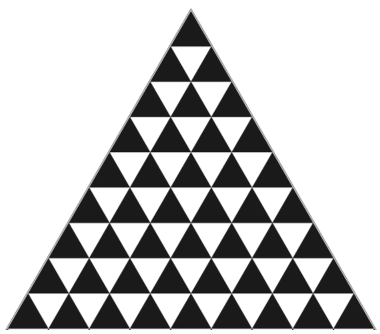
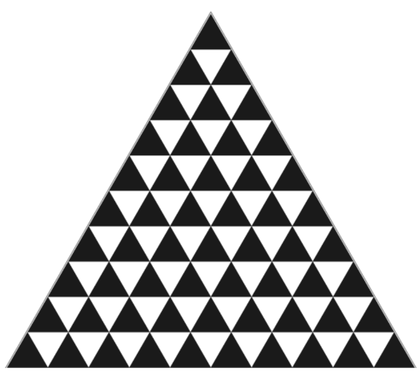
1 Number of Parallelograms in a Triangle filled with Congruent Equilateral Triangles

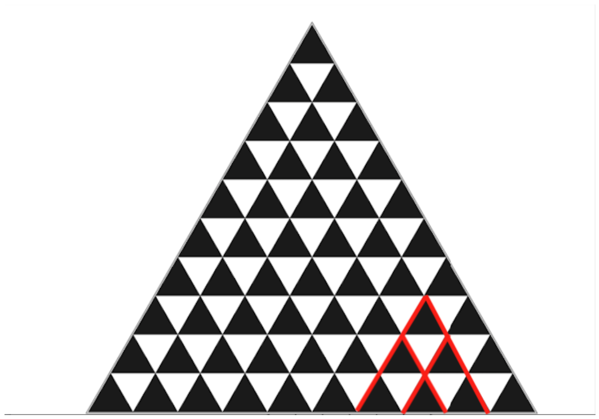


Problem 1. . How many parallelograms can be formed be formed using the vertices of the inner triangles in the triangle above?

Solution There are n=9 triangles, and n+1 points on the edge of the outer triangle. If you extend the triangle another layer, so that there 10 triangles on the edge of the triangle, there are n+2 points.



If we take any four points on the baseline of the n=10 triangle, and pick the left two to go inward to the right and the right two to go inward to the left, we will always form a parallelogram.



There are 11 points on the base of the extended triangle and you have to choose 4. However, you can extend the triangle in any direction of the sides of the triangle. So, multiply by 3.

Total Number of Parallelograms = $\binom{n+2}{4}$ * 3, where n = number of triangles on the baseline of the original outer triangle.

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2 Two Counting Problems

Problem 1. . 50 mathematicians walk into a bar. They each remove their hat and toss it in a pile as they arrive. Several hours later, they leave one by one, grabbing a hat at random to face the brutal March win. What is the probability that no mathematician received their own hat?

Solution Using principle of Exclusion and Inclusion: Probability[nobody gets own hat] = (Every possible permutation - At least one person gets their hat back + At least two people getting their hat back - At least 3 people getting their hat back ...)/(Every possible permutation)
= ((n!)-$\binom{n}{2}$*(n-1)!+$\binom{n}{2}(n-2)$!-$\binom{n}{3}$(n-3)!...)/(n!)
Which can simplify to (1-1/1!+1/2!-1/3!+1/4!...)
We know the e^x = $\sum_{n=0}^{+\infty}$x^n/n!
Therefore (1-1/1!+1/2!-1/3!+1/4!...) = 1/e ■

Problem 2. . You have a bag containing 100 numbered marbles. You draw 30 marbles out at random, with replacement. What is the probability you saw exactly 20 distinct marbles

Solution We need to draw n marbles with replacement, where n is less than or equal to x, the total marbles. We are seeking k distinct marbles.
1) The total number of events = x^n
2) Choose k marbles = $\binom{x}{k}$
3) Partition sequence of marbles if n draws of k subjects = S(n,k)
4) Assign the marbles to subsets = k!
Pr[seeing k] = ($\binom{x}{k}$*k!*S(n,k))$/x^n$
= ($\binom{100}{20}$*20!*S(30,20))$/100^{30}$

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