A Cyclic Permutation Problem

1. How many ways may 5 boys and 3 girls be seated around a table, if each individual is distinct and no girl sits next to another girl?

   Solution 1 Arrange the 5 boys cyclically in $(5-1)!$ ways. Now select 3 of the 5 locations between the boys to seat a girl. Order the girls in 3! ways to sit in these 3 seats. Total count is $4!3!\binom{5}{3}$

   Solution 2/Wrong Arrange 5 boys in a line. Select 3 of the boys and seat a girl to the immediate left of each. There are $5!3!\binom{5}{3}$ ways to do this. Now to acount for rotations, divide by 8. Total count is $\frac{5!3!\binom{5}{3}}{8}$

   Solution 3/Right Permute boys and girls as in Solution 2. Let S be the set of permutations resulting from the process listed above. $|S| = 5!3!\binom{5}{3}$. Select any member $s \in S$. It is *not* the case that s and its 7 rotations $s = s_1, s_2, \ldots, s_8$. are all in S. In fact, 3 of the rotations s_x, s_y, s_z will result in a permutation with a girl at the far right end, to the left of no one. There are only 4 rotations that are valid. This means each equivalence group has size 5. So the final count is $\frac{5!3!\binom{5}{3}}{5} = 4!3!\binom{5}{3}$.

   Example: Let $s =$ `1a2b3cde` where 1,2,3 are girls and a,b,c,d,e are boys. Then the 8 rotations are

s_1	`1a2b3cde`
s_2	`a2b3cde1`
s_3	`2b3cde1a`
s_4	`b3cde1a2`
s_5	`3cde1a2b`
s_6	`cde1a2b3`
s_7	`de1a2b3c`
s_8	`e1a2b3cd`

   And we see s_2, s_4, s_6 are each not in S.