

Transpositions & Polya's Theorem

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1 Permutations & Transpositions

Any permutation can be written as the product of transpositions, or the permutation of two elements (ie. $(1\ 2)$, which can become $(2\ 1)$ by swapping the values). The order of the transpositions does not matter. If your cyclic permutation is made of more than 2 numbers, then permute the first number with every other number in the permutation.

For example, given $(1\ 2\ 3\ 4)$, the product of transpositions will be $(12)(13)(14)$. Given, (425) , the transpositions will be $(42)(45)$.

Any permutation can be considered odd or even based on the number of transpositions involved in the permutation. If there are an odd number of transpositions, then the permutation is odd. If there is an even number, then the permutation is even.

Below is a table filled in with examples using the information from above:

π	Product of Transpositions	Type/Sign/Parity
$(123)\ (45)$	$(12)\ (13)\ (45)$	odd
(1234)	$(12)\ (13)\ (14)$	odd
$(13)\ (425)\ (67)$	$(13)\ (67)\ (42)\ (45)$	even
(12345)	$(12)\ (13)\ (14)\ (15)$	even