The Lecture Title

Scribe: Your Name

Date: Day, Mon, Date Year

1 Groups

Given an element $s \in S$ and G a permutation group acting on S, we define the orbit of s and the stabilizers of s:

$$orbit(s) = \{\pi s | \pi \in G\}$$

$$stabilizer(s) = \{\pi \in G | \pi s = s\}$$

That is, the orbit of s is all the elements that s can turn into under a transformation, and the stabilizers are the set of permutations that fix s.

Theorem 1.1. *Orbit-Stabilizer Theorem. For all $s \in S$ and G acting on S,*

$$|orbit(s)||stabilizer(s)| = |G|$$

Notice that $stabilizer(s)$ is a subgroup of G so by Lagrange, $|stabilizer(s)|\big||G|$. Note also that if in the stabilizer, $\pi s = s$, then obviously $\pi^{-1} s = s$ by left multiplication.

Now, if we have the orbit of S forms cosets of $stab(s)$, there will be a 1 to 1 correspondence between the elements in the distinct cosets and the original group. There will be one coset for every element in the orbit, and all will be achieved.

Thus by Lagrange, $|orbit(s)||stabilizer(s)| = |G|$ and we are done.