What's Next?

Bryan Lu

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Question. What does math/doing math mean to you?
More specific prompts:

- Why do you want to study math?
- Do you like math? Why?
- How could you learn about math outside of school/VMT?
- What does being "good" at math mean?
- How diverse is the math community?
- Does doing math make you "smarter?"
- Why should people study math?

1 The Landscape

Competition math categories:

- algebra (inequalities, FEs)
- geometry (mostly Euclidean)
- combinatorics
- number theory (elementary)

TJ Math (engineering requirements):

- calculus sequence (BC + multi)
- linear algebra
- differential equations
- statistics

This is **VERY FAR** from everything that's out there!
Classical undergraduate categories:

Main categories of study:

- (abstract) algebra
- analysis/measure
- topology/geometry

Less central categories:

- combinatorics
- logic/set theory
- probability/statistics

These are not clean divisions – lots of overlap between these areas of study!
Categorization is even trickier beyond curriculums, but some broad fields that incorporate many of the above fields and carry significant historical importance/development (there are many more!):

- algebraic number theory
- analytic number theory
- dynamical systems
- Lie theory
- harmonic analysis
- operator theory
- computer science
- foundations
- game theory/economics
- algebraic geometry
- algebraic topology
- mathematical physics

2 Ideas

Myths/Misconceptions:

(a) Proving really **hard** theorems/solving really **hard** problems (misplaced emphasis)
(b) Formalism vs. pedantry
(c) Working on your own vs. community of ideas
(d) Difficulty as a barrier to growth
(e) (there are probably tons more . . .)

Core ideas to keep in mind:

(1) Connecting seemingly unrelated objects/procedures.
(2) Extending/generalizing ideas we already know.
(3) Analyzing when/why patterns break and seeking complexity.
(4) Staying grounded with "standard" examples/calculations.

Goal: I want to show you these ideas in action with examples and stories!

3 Examples/Stories

The rest of this lecture will proceed in a "choose your own adventure" format: For each of the first three **Core Ideas**, you will get to pick one of the two stories to talk about via democracy. Some topics are, by my estimation, a little more difficult/inaccessible, so these have been marked with a ★.

3.1 Connections

Both of the following stories/examples revolve around the use of graphs combined with algebra in the pursuit of various avenues of study:

- *How to Tell Spaces Apart (Simplicial Homology)* – We will use graphs as a model for describing various topological spaces (spheres, toruses, Klein bottles) and use an extension of the Euler characteristic with linear algebra to distinguish spaces from each other.
- (★) *Word Problems (Geometric Group Theory)* – We will take an alternate view of groups and study graphs induced by a group's structure, and use these graphs to explain why deciding whether an element in a group is the identity element is unsolvable.

(Enter break here, informal time for questions)

3.2 Generalizations

The following two stories describe generalizations of ideas and concepts you may have seen before via the formalisms of analysis and geometry:

- (★) *Doing Calculus Anywhere (Manifolds, Differential Forms)* – One of the staples of calculus is the Fundamental Theorem of Calculus, which has many further generalizations further afield in multivariable calculus. Let's see why these are actually all the same thing!
- *A Case Study in Volume (Measure Theory)* – So you think you know what volume is? Here we test your intuitive understanding of volume and use edge cases as a case study for generalizing what you might think of as "volume."

3.3 Breaking and Repairing

These final two stories describe ways in which natural constructions might fail to behave as expected, and when possible, how we can make sense of the complexity that arises as a result.

- *I Forgot How to Factor (Algebraic Number Theory)* – We will try solving some Diophantine equations in some creative ways, and realize that I don't really know how to prime-factorize things anymore. Can we fix that?
- *The Answer Is Not True (Analysis, Foundations)* – Because everyone loves the true/false section of a math exam, we're going to have a true/false party!
  Wait, I forgot to take the spoiler out of the section header... fine, to make it more interesting, be prepared to explain your answers!

Questions?