Exercise 1.14

Draw the tree illustrating the process generated by the `count-change` procedure of 1.2.2 in making change for 11 cents. What are the orders of growth of the space and number of steps used by this process as the amount to be changed increases?

Let's skip to `(cc 11 3)` - a higher value for `kinds-of-coins` will result in degenerate leaves.

```
(cc 11 3)
|-- (cc 11 2)
    |-- (cc 11 1)
        |-- (cc 11 0) -> 0
        |-- (cc 10 1)
            |-- (cc 10 0) -> 0
            |-- (cc 9 1)
                [...]
                |-- (cc 1 0) -> 0
                |-- (cc 0 1) -> 1
    |-- (cc 6 2)
        |-- (cc 6 1)
            [...]
            |-- (cc 1 0) -> 0
            |-- (cc 0 1) -> 1
        |-- (cc 1 2)
            |-- (cc 1 1)
                |-- (cc 1 0) -> 0
                |-- (cc 0 1) -> 1
            |-- (cc -4 2) -> 0
|-- (cc 1 3)
    |-- (cc 1 2)
        |-- (cc 1 1)
            |-- (cc 1 0) -> 0
            |-- (cc 0 1) -> 1
        |-- (cc -4 2) -> 0
    |-- (cc -9 2) -> 0
```

Space. The space required by tree recursion is simply the maximum depth of the tree, which in this case is $\Theta(n)$.

Time. Analyzing the recursion tree,

```
(cc n 1) := O(n)
(cc n 2) := (cc n 1) + (cc (- n k1) 2)
         := (cc n 1) + (cc (- n k1) 1) + (cc (- n (* 2 k1) 2)
         ...
         := (cc n 1) + (cc (- n k1) 1) + ... + (cc 0 1)
         := n * O(n)
         := O(n^2)
(cc n 3) := (cc n 2) + (cc (- n k2) 2)
         ...
         := (cc n 2) + (cc (- n k2) 2) + ... + (cc 0 2)
         := n * O(n^2)
         := O(n^3)
```

In general,

```
(cc n k) := (cc n (dec k)) + (cc (- n kn) k)
         ...
         := (cc n (dec k)) + (cc (- n kn) (dec k)) + ... + (cc 0 k)
         := n * O(n^(k-1))
         := O(n^k)
```

Therefore, `(cc n k)` requires $\Theta(n^k)$ steps.