MCV4U Course Notes

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Abstract

This document is a review of the Calculus and Vectors 12 course. I had some time on my hands and I really needed to study for exams so here goes nothing. If you find any mistakes, please let me know so I can fix them. I hope this helps you!

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Credits **6**

1 Calculus

1.1 Limits

The **Limit** of a function is the value of a function as its input approaches some value.

$$\lim_{x \to a} f(x) \tag{1}$$

The **Right-hand Limit** is the value of a function as its input approaches some value from the positive side.

$$\lim_{x \to a^+} f(x) \tag{2}$$

The **Left-hand Limit** is the value of a function as its input approaches some value from the negative side.

$$\lim_{x \to a^-} f(x) \tag{3}$$

The left and right hand limits only exist if there is no vertical asymptotes at x and $f(x)$ exists at the x-values approaching x.
The limit of a function at a point only exists if both the left and right limits exist and are equal to each other.
A function is said to have **Continuity** at a point if:

$$\lim_{x \to a} f(x) = f(x) \tag{4}$$

1.1.1 Limit Properties

Suppose that $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.

1. $$\lim_{x \to a} c = c \ , \ c \in \mathbb{R} \tag{5}$$

2. $$\lim_{x \to a} x = a \tag{6}$$

3. $$\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x) \ , \ c \in \mathbb{R} \tag{7}$$

4. $$\lim_{x \to a} [f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) \tag{8}$$

5. $$\lim_{x \to a} [f(x) \times g(x)] = \lim_{x \to a} f(x) \times \lim_{x \to a} g(x) \tag{9}$$

6. $$\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}, \text{ if } \lim_{x \to a} g(x) \neq 0 \tag{10}$$

7. $$\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n \tag{11}$$

If $\lim_{x \to a} \left(\frac{f(x)}{g(x)} \right)$ and $\lim_{x \to a} g(x) = 0$ you must remove the factor of x from $f(x)$ and $g(x)$ by simplifying to evaluate the limit. In most cases this requires forming a difference of squares on the numerator. If you cannot remove the factor of x, the limit does not exist.

1.2 Rate of Change

The **Rate of Change** of a function is how much the y-values of the function change relative to the x-values.

1. The **average rate of change** of a function over a given interval $[a, b]$ is given by $\frac{f(b)-f(a)}{b-a}$. In other words, it is the slope of the secant that joins the points $(a, f(a))$ and $(b, f(b))$.

2. The tangent to a function, f, at $x = a$, is the line that touches f at $(a, f(a))$, and best approximates the function near a.

The rate of change of a function at $x = a$ can be determined by evaluating the following limit:

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \tag{12}$$

This is often used to find and prove derivatives.

1.3 Derivatives

The derivative of a function $f(x)$ (denoted as $f'(x)$) is a function such that $f'(x)$ at x is equal to the slope of $f(x)$ at x.
Use these rules to determine the derivative of a function.

Basic Rules	
Function	**Derivative**
$f(x) = c$	$f'(x) = 0$
$g(x) = cf(x)$	$g'(x) = cf'(x)$
$f(x) = x^n$	$f'(x) = nx^{n-1}$
Composite Rules	
$h(x) = f(x) \pm g(x)$	$h'(x) = f'(x) \pm g'(x)$
$h(x) = f(x)g(x)$	$h'(x) = f'(x)g(x) + f(x)g'(x)$
$h(x) = \frac{f(x)}{g(x)}$	$h'(x) = \frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}$
$h(x) = f(g(x))$	$h'(x) = f'(g(x))g'(x)$
Trigonometric Derivatives	
$f(x) = \sin(x)$	$f'(x) = \cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$
$f(x) = \tan(x)$	$f'(x) = \sec^2(x)$
$f(x) = \sec(x)$	$f'(x) = \sec(x)\tan(x)$
$f(x) = \cot(x)$	$f'(x) = -\csc^2(x)$
$f(x) = \csc(x)$	$f'(x) = -\csc(x)\cot(x)$
Exponential Derivatives	
$f(x) = a^x$	$f'(x) = \ln(a)a^x$
Logarithmic Derivatives	
$f(x) = \log_a(x)$	$f'(x) = \frac{1}{\ln(a)x}$
$f(x) = \ln(x)$	$f'(x) = \frac{1}{x}$

1.4 Implicit Differentiation

2 Vectors

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