$$\frac{dV}{dt} + \frac{V}{RC} = \frac{V_{in}}{RC} \quad , \quad \text{where} \quad V_{in} = A_o \cos \omega t$$

$$\frac{dV}{dt} + \frac{V}{RC} = \frac{A_o \cos \omega t}{RC}$$

Solution has the form

$$V = V_{tr} + V_{ss}$$

$$= V_H + V_P$$

Homogeneous (Transient)

Set $f(t) = 0$ so

$$\frac{dV_H}{dt} + \frac{V_H}{RC} = 0$$

Solve using separation of variables

Show all steps!

You should get $V_H = K e^{-t/RC}$

↑
arbitrary constant

Don't solve for K yet!

Particular (Steady-State)

$$\frac{dV_P}{dt} + \frac{V_P}{RC} = \frac{A_o \cos \omega t}{RC}$$

$$\overline{dt} \quad \overline{RC} \quad \overline{RC}$$

Assume V_p has a solution of the form

$$V_p = A\sin\omega t + B\cos\omega t$$

Need to find A & B.

Find $\frac{dV_p}{dt} = ?$

Plug V_p and $\frac{dV_p}{dt}$ into the ODE

$$\frac{dV_p}{dt} + \frac{V_p}{RC} = \frac{A_o\cos\omega t}{RC}$$

Multiply both sides by RC

Now do some algebra

Treat the left and right hand sides separately

Get into this form:

$$\left(\text{sin coeffs}\right)\sin\omega t + \left(\text{cos coeffs}\right)\cos\omega t$$

$$= 0\sin\omega t + A_o\cos\omega t$$

$$\left.\begin{array}{l}\left(\text{sin coeffs}\right) = 0 \\ \left(\text{cos coeffs}\right) = A_o\end{array}\right\}$$

Solve for A and B using these equations

After solving for A and B, plug A and B into V_p

After solving for A and B, plug A and B into V_P

Lastly solve for K using the full solution and I.C.

$$V = V_H + V_P$$

$$V = K e^{-t/RC} + A \sin \omega t + B \cos \omega t$$

Plug in for A and B

Apply I.C., solve for K

Write full solution using the K you found.