

Notation and Math

The derivative of $f(g(h(x)))$ is $f'(g(h(x)))g'(h(x))h'(x)$

$$\frac{df}{dx} = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dx}$$

Computation graphs

Represents a function as a graph

- Directed acyclic graphs that show the dependency structure of the computation to be performed
- Computation graph breaks the function into steps we can compute the derivative

Nodes

- input values
- functions for combining values

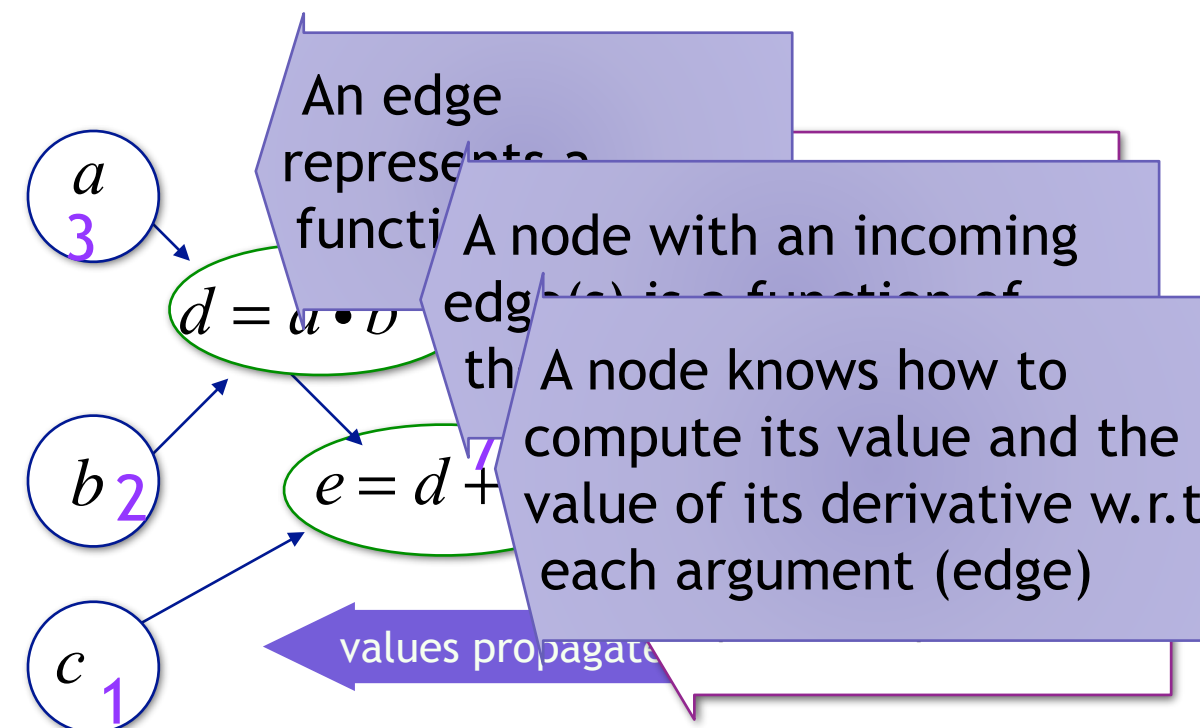
Example: $(ab+c)^2$

- input values: a, b, c
- functions for combining values:

$$d = a \cdot b$$

$$e = d + c$$

$$z = e^2$$

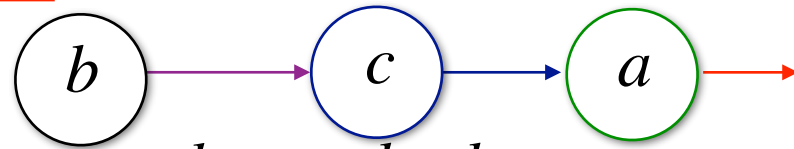


$$\begin{aligned} \frac{\partial z}{\partial a} &= \frac{\partial z}{\partial e} \cdot \frac{\partial e}{\partial d} \cdot \frac{\partial d}{\partial a} \\ &= 2e \cdot 1 \cdot b = 2(d + c) \cdot b \\ &= 2(a \cdot b + c) \cdot b \end{aligned}$$

$$(2 \cdot 7) (1) (2)$$

Basic Facts:

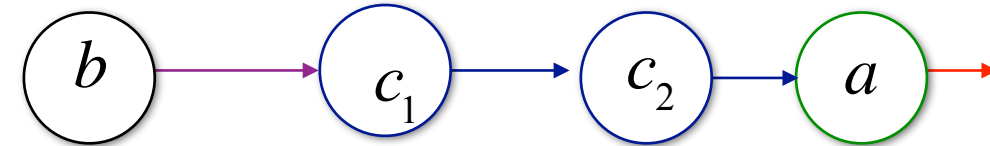
$$a = (3b - 1)^2$$



$$\frac{da}{db} = \frac{da}{dc} \frac{dc}{db} = 2(c - 1) \cdot 3$$

$$a = (c - 1)^2 \quad c = 3b$$

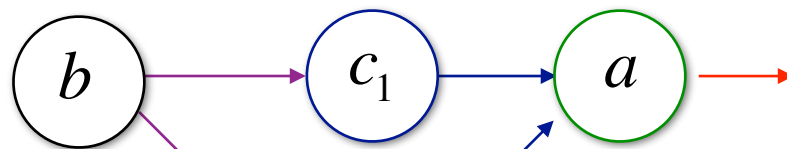
values propagate this way



$$\frac{da}{db} = \frac{da}{dc_2} \frac{dc_2}{dc_1} \frac{dc_1}{db} = \frac{da}{dc_1} \frac{dc_1}{db}$$

values propagate this way

$$a = ((4b + 1) + (5b + 1) - 1)^2$$



$$\frac{da}{db} = \frac{da}{dc_1} \frac{dc_1}{db} + \frac{da}{dc_2} \frac{dc_2}{db}$$

$$= 2(c_1 + c_2 - 1) \cdot 4 + 2(c_1 + c_2 - 1) \cdot 5$$

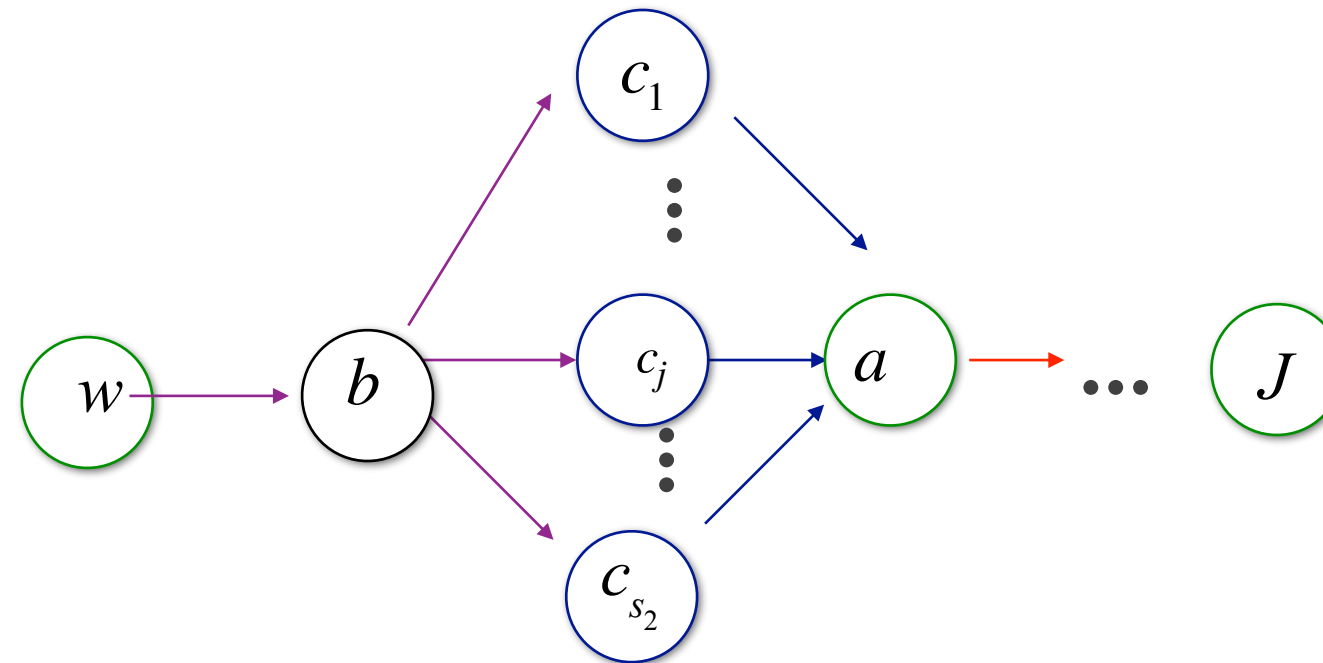
$$= 2((4b + 1) + (5b + 1) - 1) \cdot 4$$

$$+ 2((4b + 1) + (5b + 1) - 1) \cdot 5$$

$$a = (c_1 + c_2 - 1)^2$$

$$c_1 = (4b + 1)$$

$$c_2 = (5b + 1)$$



values propagate this way

$$\frac{da}{db} = \sum_{i=1}^{s_2} \frac{da}{dc_i} \frac{dc_i}{db}$$

$$\frac{dJ}{db} = \sum_{i=1}^{s_2} \frac{dJ}{dc_i} \frac{dc_i}{db}$$

$$\frac{dJ}{dw} = \frac{dJ}{db} \frac{db}{dw}$$