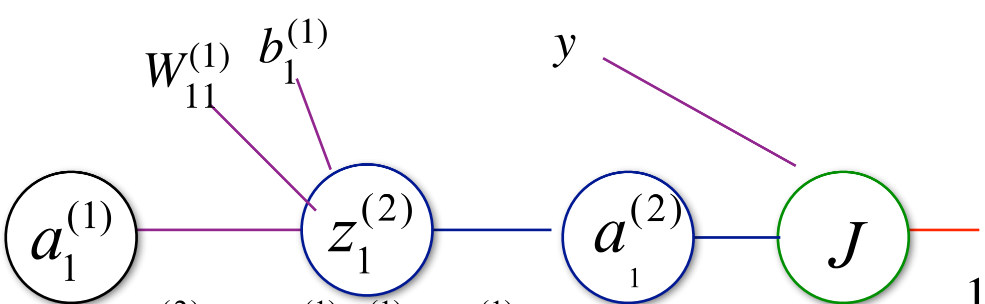
Notation and Math

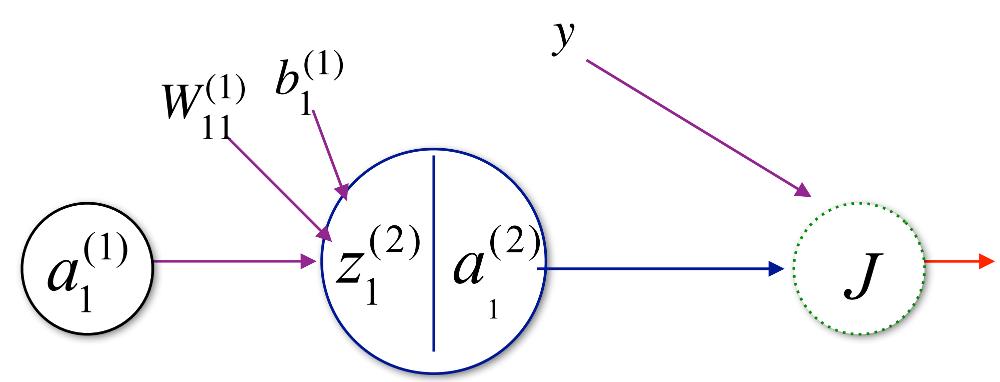
Partial derivatives

$$J = \frac{1}{2}(f(\underbrace{W_{11}^{(1)}a_1^{(1)} + b_1^{(1)}}_{z_1^{(2)}}) - y)^2$$

$$= \frac{1}{2}(f(\underbrace{z_1^{(2)})}_{a_1^{(2)}} - y)^2 \quad = \frac{1}{2}(a_1^{(2)} - y)^2$$



$z_1^{(2)} = W_{11}^{(1)}a_1^{(1)} + b_1^{(1)}$ $\quad a_1^{(2)} = f(z_1^{(2)})$ $\quad J = \frac{1}{2}(a_1^{(2)} - y)^2$



$$a_1^{(2)} = f(z_1^{(2)}) \qquad J = \frac{1}{2}(a_1^{(2)} - y)^2$$

$$z_1^{(2)} = W_{11}^{(1)}a_1^{(1)} + b_1^{(1)}$$

$$\frac{\partial J}{\partial b_1^{(1)}} = \frac{\partial J}{\partial a_1^{(2)}}\frac{\partial a_1^{(2)}}{\partial z_1^{(2)}}\frac{\partial z_1^{(2)}}{\partial b_1^{(1)}} \quad = (a_1^{(2)} - y)\frac{\partial a^{(2)}}{\partial z_1^{(2)}}\frac{\partial z_1^{(2)}}{\partial b_1^{(1)}}$$

values propagate this way

$$= (a_1^{(2)} - y)\frac{\partial f(z^{(2)})}{\partial z_1^{(2)}}\frac{\partial z_1^{(2)}}{\partial b_1^{(1)}}$$

If $f(\cdot)$ is the logistic function

$$\frac{\partial f(z)}{\partial z} = f(z)(1 - f(z))$$

$$= (a_1^{(2)} - y)f'(z^{(2)})\frac{\partial z_1^{(2)}}{\partial b_1^{(1)}}$$

$$= \underbrace{(a_1^{(2)} - y)f'(z^{(2)})}\cdot 1$$

$$\frac{\partial J}{\partial W_{11}^{(1)}} = \frac{\partial J}{\partial a_1^{(2)}}\frac{\partial a_1^{(2)}}{\partial z_1^{(2)}}\frac{\partial z_1^{(2)}}{\partial W_{11}^{(1)}} \quad = \frac{\partial J}{\partial z_1^{(2)}}\frac{\partial z_1^{(2)}}{\partial W_{11}^{(1)}} \quad = (a_1^{(2)} - y)f'(z^{(2)})\cdot a_1^{(1)}$$

$$\frac{dJ}{db_1^{(1)}} = \frac{dJ}{da_1^{(2)}}\frac{da_1^{(2)}}{dz_1^{(2)}}\frac{dz_1^{(2)}}{db_1^{(1)}}$$

values propagate this way

$$\frac{\partial J}{\partial b_1^{(1)}} = (a_1^{(2)} - y)\ \ f'(z_1^{(2)})\ \ 1$$