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SLAM estimation method for uncertain model noise parameters

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Abstract

It is difficult to accurately obtain the statistical parameters of mobile robots motion parameters for its mobility. And it is also the same in observation system because of the environmental variability, which would cause the SLAM system noise statistical parameters uncertainty, and decrease the filtering performance. A SLAM adaptive filtering algorithm with noise statistical characteristics estimation is proposed, when the noise statistical parameters are constant but unknown, three additional maximum posteriori estimators are constructed, and adaptive CKF SLAM algorithm are designed. Compared with the standard CKF SLAM algorithm, the results show that under the uncertainty noise parameters, if the noise variance is Gaussian white noise distribution, the designed adaptive CKF SLAM with three maximum posterior can well estimate the state, track noise variance, and handle the problem of nonlinear systems. The simulation results verify the effectiveness of the proposed SLAM estimation method with the uncertainty of the model noise parameters.

Keywords SLAM · Uncertain model · Adaptive filter · Noise statistics characteristic estimation

1 Introduction

In the process of localization and mapping of mobile robots based on standard kalman filtering, the mathematical model of dynamic system and the statistical characteristics of noise need to be priori determined, and remain unchanged during the filtering process. The accurate estimation of the state can be obtained by correcting the prediction estimation from the measured values. However, in the practical localization and mapping of mobile robots, it is difficult to get the accurate mathematical model of system and statistical characteristics of noise. Moreover, in a dynamic environment, the motility of the target will disturb the motion state of the mobile robot, as well as the visual jitter caused by the unevenness of the road surface and the observed data error caused by the variation of the ambient light, which will cause the uncertainty of the noise statistical parameters of the actual system. In this case, the fixed noise model of the standard kalman filtering algorithm can not adapt to these changes, resulting in slower convergence and lower accuracy, and even divergence of the filtering. Therefore, the study on the SLAM estimation method with uncertain statistical characteristics of noise has important theoretical and practical significance for revealing the influence regular of noise statistical inaccuracy on SLAM estimation, and promoting the engineering application of SLAM estimation algorithm.

The adaptive filtering method is generally used to solve the problem of the filtering performance degradation caused by the uncertainty of the system model noise statistical parameters. It is mainly divided into three methods: one is the adaptive filtering algorithm of parameter estimation, which estimates the noise statistical parameters of the system model [1, 2], including Bayesian method, maximum likelihood method, variance matching method and correlation method.These methods are all for noise statistical parameters with time invariant characteristics. Even if they can be used for slowly varying noise statistical parameters, it is generally assumed that the convergence speed of the filter is faster than the noise statistical parameters changing. If the system noise statistical parameters is changed, the filtering effect of localization and mapping is poor. The other is a covariance correction algorithm based on innovation and residuals [3, 4]. The

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covariance matrix or gain matrix of noise is adjusted online according to the change of innovation sequence, so that the filter has self-adaptive ability [5, 6]. The third is interacting multiple models method [7, 8], which can estimate the time-varying noise statistical characteristics to a certain extent, and overcome the estimation error larger of single model. The above work provides a useful reference for the study of the filtering with uncertain model noise statistical parameters.

At present, the research on visual SLAM estimation of mobile robots with uncertain models is still lacking, especially the influence regular of inaccurate model noise on SLAM estimation is not clear. In view of this, This paper does a in-depth study on the filter performance declining caused by the uncertainty of the noise statistical parameters of mobile robot SLAM system. And the work mainly focuses on the following aspects:If the noise statistical parameters are constant and in line with the Gauss distribution, based on the principle of maximum a posteriori estimation, three kinds of additional maximum posteriori estimators [9–11] are constructed to estimate the inaccurate noise statistical parameters. The CKF SLAM adaptive filtering algorithm is designed in order to suppress SLAM filtering performance degradation.

2 Description of SLAM estimation problem with uncertain model noise statistical parameters

In dynamic environment, the motion of mobile robot will be disturbed by the moving target. The unevenness of the pavement will affect the observation value. so in practical application, there is a big error in the system model, the error is all as the statistical noise, then motion noise and observation noise statistical parameters Q and R are unknown or imprecisely known in the SLAM system, and the unknown or uncertain noise parameters are represented by θ, then mobile robot SLAM model of system noise parameter uncertainty is expressed as:

$$x_k = f(x_{k-1}, \theta) + w_{k-1} \tag{1}$$

$$z_k = h(x_k, \theta) + v_k \tag{2}$$

Where, $x_k \in \mathbb{R}^{\mathrm{n}}$ is state vector of the system, $x_k = [x_{r,k}, M_k]$, $x_{r,k}$ is the pose vector of the mobile robot, M_k is the state vector of the map. $z_k \in \mathbb{R}^{\mathrm{m}}$ is observation vector of the system, $f(\cdot)$ and $h(\cdot)$ are the known state transfer function and observation function, $w_k \in \mathbb{R}^{\mathrm{n}}$ is the motion noise, $v_k \in \mathbb{R}^{\mathrm{m}}$ is the observation noise.

Therefore, for the SLAM problem with uncertain model noise parameters, the main purpose of this paper is to design a noise statistical characteristic estimation algorithm in the practical probabilistic filtering SLAM process, under the noise statistical characteristic of the system model unknown and changing. Which can adaptively filter for the Gaussian distribution noise variance of the system model, to suppress the filtering divergence and estimate SLAM concurrently. To avoid filtering accuracy decreasing caused by the uncertain statistical parameters of the model noise or even filtering divergence, and we can achieve more accurate localization estimation and mapping for mobile robots.

3 SLAM estimation method with unknown constant noise statistical parameter

In the case of inaccurate statistical characteristics of the system model noise, any initialization of the system noise statistical parameters will lead to the problem of filter localization divergence and inconsistent mapping. Based on the principle of maximum a posteriori estimation, a noise statistical parameters estimation model is established. Combined with covariance matching, a model noise parameter estimation strategy is designed. Using CKF filtering algorithm, an adaptive filtering model of joint estimation 联合估计 of noise statistical parameters and system state parameters is established to suppress filter localization divergence and inconsistent mapping.

3.1 SLAM estimation modeling with constant unknown noise parameters

Since the noise statistic characteristics of mobile robot model are unknown during the implementation of SLAM filtering, statistical variance estimation involves the nonlinear transformation determination of formula (1) and formula (2), $f(\cdot)$ and $h(\cdot)$ are nonlinear transformation determined by parameter vector θ_k. The estimation of parameter θ_k needs to be determined by the data set $\{x_k, z_k\}$ containing the known inputs and the expected output. Therefore, the parameter estimation model is expressed as:

$$\theta_k = \theta_{k-1} + e_{k-1} \tag{3}$$

Where, $\theta_k \in \mathbb{R}^{n_\theta}$ is modeled as a stationary process driven by parameter process noise e_k (Which is represented by a Gaussian random vector with mean 0 and covariance matrix E_k), and its state transition matrix is unit matrix. It constitutes a SLAM estimation model of constant unknown noise with Eq. (1) and Eq. (2).

Here, the state of the SLAM system is taken as the state vector in the state estimation, and the statistical value θ of the noise parameter random variable is not added to the state vector. Instead, it is used as a new parameter vector to realize the alternation estimation of state and parameter.

The estimation of SLAM state x_k can be optimized with the non-linear Kalman filter family, the purpose is to minimize the following loss function:

$$J(x) = \sum_{i=1}^{k} \left\{ [z_i - h(x_i, \theta)]^{\mathrm{T}} R^{-1} [z_i - h(x_i, \theta)] + (x_i - \hat{x}_{i|i-1})^{\mathrm{T}} Q^{-1} (x_i - \hat{x}_{i|i-1}) \right\} \tag{4}$$

Due to the non-linearity of motion model and observation model of mobile robot, the EKF linearization will lead to the estimation precision reducing of localization and mapping of mobile robot SLAM, and even the divergence of filtering. In order to adapt to the strong nonlinearity of system model, the CKF filter is used to estimate the localization and mapping of the mobile robot.

In the Bayesian filtering theory, the posterior probability density also provides a complete statistical description of the parameter vectors at this time. Therefore, the estimation of the parameter vector θ_k is based on the estimation of the posterior probability density $p(\theta_k|z_{1:k})$, where $z_{1:k}$ represents k observation vectors, and the posterior probability density function is recursively calculated based on the observation sequence. Similar to the state estimation of SLAM, the parametric vector filter at this time can also be considered as an optimization algorithm, the purpose of which is to minimize the following loss function:

$$J(\theta) = \sum_{i=1}^{k} \left\{ [z_i - h(x_i, \theta)]^{\mathrm{T}} R^{-1} [z_i - h(x_i, \theta)] \right\} \tag{5}$$

The estimation of parameter vector θ_k by Bayesian filtering is equivalent to the calculation of multi-dimension integration. The nonlinear Kalman filter can realize the parameter estimation based on the state space model constructed by Eqs. (1)–(3). Therefore, for formula (5) using the maximum a posteriori estimation and it can also be solved by the Kalman filter, which is realized by the second-order string-interpolation filtering, Unscented Kalman filtering and spherical diameter volume filtering.

Therefore, CKF filter and maximum posteriori estimator constitute CKF SLAM estimation model with unknown noise parameter, which is given in Fig. 1, in order to accurately estimate the statistical properties of unknown system noise and the state of SLAM in motion model and observation model of mobile robot.

Where, the input of the maximum posteriori estimator is given by the state estimation of the CKF SLAM filter; the maximum posteriori estimator obtains the noise parameter estimation of the system model, and then passes it to the CKF SLAM filter to form a closed loop, which respectively estimates the SLAM state of the mobile robot and the noise variance parameter.

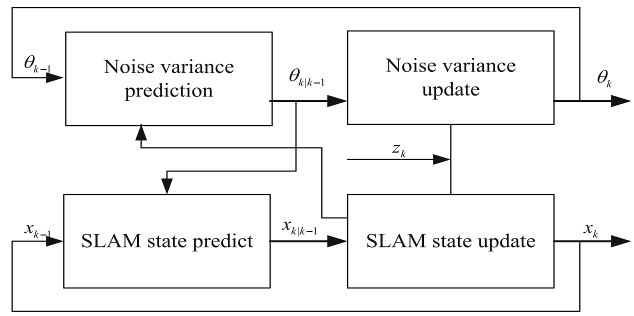

Fig. 1 CKF SLAM estimation model with unknown noise parameter

3.2 Adaptive CKF SLAM algorithm with maximum a posteriori estimation

Three maximum posterior probability density estimation algorithms are designed for the motion noise and the observation noise. For the state estimation of mobile robot, an adaptive CKF SLAM algorithm is designed to realize the noise estimation with constant noise parameter and slow time-varying noise, and the state estimation of mobile robot.

3.2.1 SLAM state estimation of mobile robot

1. Initialize mobile robot SLAM system state.

$$\hat{x}_0 = E[x_0] \tag{6}$$

$$P_0 = E\left[(x - \hat{x}_0)(x - \hat{x}_0)^{\mathrm{T}}\right] \tag{7}$$

2. Third-order CKF algorithm [12] is used to construct sampling points based on SLAM state estimation.

$$\begin{cases} \xi_i = \bar{x} + \sqrt{nP_x} e_i \\ \omega_i = \dfrac{1}{2n} \\ \xi_{i+n} = \bar{x} - \sqrt{nP_x} e_i \\ \omega_{i+n} = \dfrac{1}{2n} \end{cases}, i = 1, 2, \cdots, n \tag{8}$$

For Eq. (8), let $\bar{x} = \hat{x}_{k-1}$, $P_x = P_{k-1}$, you can get the sampling point $\xi_{i,k-1}$, $i = 1, 2, \ldots, L = 2n$, and its corresponding weight.

3. Mobile robot SLAM state prediction: The sampling point prediction, SLAM state prediction and the covariance matrix of SLAM state prediction error are respectively calculated by formulas (9)–(11) to predict SLAM state of mobile robot.

$$X_{i,k|k-1} = f(\xi_{i,k-1}, \theta_{k-1}), i = 1, 2, \ldots, L \tag{9}$$

$$\hat{x}_{k|k-1} = \sum_{i=1}^{L} \omega_i X_{i,k|k-1} \tag{10}$$

$$P_{k|k-1} = \sum_{i=1}^{L} \omega_i \left(X_{i,k|k-1} - \hat{x}_{k|k-1}\right)\left(X_{i,k|k-1} - \hat{x}_{k|k-1}\right)^{\mathrm{T}} + \theta_{k|k-1} \tag{11}$$

4. Mobile robot SLAM state update:Using the third-order CKF algorithm, for the formula (8), let $\bar{x} = \hat{x}_{k|k-1}$, $P_x = P_{k|k-1}$ to obtain the sampling points $\xi_{i,k-1}$, $i = 1,2,\ldots,L = 2n$ and its corresponding weights and calculate the sampling points for the state prediction. The observation vector is predicted and the covariance of the innovation is calculated with formula (12) and formula (13). The mutual covariance matrix between the SLAM state and the observation vector and the Kalman gain matrix are calculated by formula (14) and (15). The state of SLAM and its error covariance matrix are estimated and the SLAM state of mobile robot is updated by formula (16) and (17).

$$\hat{z}_{k|k-1} = \sum_{i=1}^{L} \omega_i h\left(\xi_{i,k|k-1}, \theta_{k|k-1}\right) \tag{12}$$

$$\begin{aligned} P_{zz,k|k-1} = \sum_{i=1}^{L} \omega_i &\left(h\left(\xi_{i,k|k-1}, \theta_{k|k-1}\right) - \hat{z}_{k|k-1}\right) \\ &\left(h\left(\xi_{i,k|k-1}, \theta_{k|k-1}\right) - \hat{z}_{k|k-1}\right)^{\mathrm{T}} + \theta_{k|k-1} \end{aligned} \tag{13}$$

$$P_{xz,k|k-1} = \sum_{i=1}^{L} \omega_i \left(\xi_{i,k|k-1} - \hat{x}_{k|k-1}\right)\left(h\left(\xi_{i,k|k-1}, \theta_{k|k-1}\right) - \hat{z}_{k|k-1}\right)^{\mathrm{T}} \tag{14}$$

$$G_k = P_{xz,k|k-1} P^{-1}_{zz,k|k-1} \tag{15}$$

$$\hat{x}_k = \hat{x}_{k|k-1} + G_k\left(z_k - \hat{z}_{k|k-1}\right) \tag{16}$$

$$P_k = P_{k|k-1} - G_k P_{zz,k|k-1} G_k^{\mathrm{T}} \tag{17}$$

3.2.2 Unknown parameter estimation

1. Parameter state initialization.

$$\hat{\theta}_0 = E[\theta_0] \tag{18}$$

$$P_0 = E\left[\left(\theta_0 - \hat{\theta}_0\right)\left(\theta_0 - \hat{\theta}_0\right)^{\mathrm{T}}\right] \tag{19}$$

2. Parameter State Prediction: Predict the state of the parameter and its error covariance matrix.

$$\hat{\theta}_{k|k-1} = \hat{\theta}_{k-1} \tag{20}$$

$$P_{k|k-1} = P_{k-1} + E_k \tag{21}$$

3. The parameter state update based on the maximum posteriori:The covariance of innovations, mutual covariance matrix between state and observation vector, and the Kalman gain matrix can be estimated using the spherical-radial rule, unscented transformation or second-order Stirling interpolation, Finally, the Eq. (38) is used to update the state of the parameters.

   ① Spherical-radial rule [13]: First, the third-order CKF algorithm is used. For the formula (8), let $\bar{x} = \hat{x}_{k|k-1}$, $P_x = P_{k|k-1}$ to obtain the sampling points $\xi_{i,k-1}$, $i = 1,2,\ldots,L = 2n$ and its corresponding weight, and calculate the sampling points for the parameter estimation prediction. we use formula (22) and formula (23) to predict the covariance between the observation vector and innovations. Then, the covariance matrix between the state and the observation vector, the Kalman gain matrix, and the covariance matrix of the state estimation error are calculated using Eqs. (24)–(26).

$$\hat{z}_{k|k-1} = \sum_{i=1}^{L} \omega_i h\left(\xi_{i,k|k-1}\right) \tag{22}$$

$$\begin{aligned} P_{zz,k|k-1} = \sum_{i=1}^{L} \omega_i &\left(h\left(\xi_{i,k|k-1}\right) - \hat{z}_{k|k-1}\right) \\ &\left(h\left(\xi_{i,k|k-1}\right) - \hat{z}_{k|k-1}\right)^{\mathrm{T}} + R_{k|k-1} \end{aligned} \tag{23}$$

$$\begin{aligned} P_{xz,k|k-1} = \sum_{i=1}^{L} \omega_i &\left(\xi_{i,k|k-1} - \hat{\theta}_{k|k-1}\right) \\ &\left(h\left(\xi_{i,k|k-1}\right) - \hat{z}_{k|k-1}\right)^{\mathrm{T}} \end{aligned} \tag{24}$$

$$G_k = P_{xz,k|k-1} P^{-1}_{zz,k|k-1} \tag{25}$$

$$P_k = P_{k|k-1} - G_k P_{zz,k|k-1} G_k^{\mathrm{T}} \tag{26}$$

② Unscented transformation [14]: First, we use the formula (27) to calculate the predicted sampling points of the parameter estimation. Then we use Eqs. (28) and (29) to predict the covariance between the observation vector and innovations. The covariance matrix of the state and the observation vector, the Kalman gain matrix, and the covariance matrix of the state estimation error are then calculated using Eqs. (30)–(32).

$$\xi_{k|k-1} = \left[\hat{\theta}_{k|k-1}, \hat{\theta}_{k|k-1} + \sqrt{(n+\lambda)P_{k|k-1}}, \hat{\theta}_{k|k-1} - \sqrt{(n+\lambda)P_{k|k-1}}\right] \tag{27}$$

$$\hat{z}_{k|k-1} = \sum_{i=0}^{2n} \omega_i^{(m)} h\left(\xi_{i,k|k-1}\right) \tag{28}$$

$$P_{zz,k|k-1} = \sum_{i=0}^{2n} \omega_i^{(c)} \left(h\left(\xi_{i,k|k-1}\right) - \hat{z}_{k|k-1}\right) \left(h\left(\xi_{i,k|k-1}\right) - \hat{z}_{k|k-1}\right)^{\mathrm{T}} + R_k \tag{29}$$

$$P_{xz,k|k-1} = \sum_{i=0}^{2n} \omega_i^{(c)} \left(\xi_{i,k|k-1} - \hat{\theta}_{k|k-1}\right) \left(h\left(\xi_{i,k|k-1}\right) - \hat{z}_{k|k-1}\right)^{\mathrm{T}} \tag{30}$$

$$G_k = P_{xz,k|k-1} P^{-1}_{zz,k|k-1} \tag{31}$$

$$P_k = P_{k|k-1} - G_k P_{zz,k|k-1} G_k^{\mathrm{T}} \tag{32}$$

③ Second-order Stirling interpolation [15]:First, define $E_k = S_{E,k} S_{E,k}^{\mathrm{T}}$, $R_k = S_{R,k} S_{R,k}^{\mathrm{T}}$, $P_{k|k-1} = S_{k|k-1} S_{k|k-1}^{\mathrm{T}}$, $P_k = S_k S_k^{\mathrm{T}}$. Then the covariance of the observation vector and the innovations is predicted with formulae (33) and formula (34). Then, the covariance matrix of the state and observation vectors, the Kalman gain matrix and the covariance matrix of the state estimation error are calculated by the formula (35)–(37).

$$\hat{z}_{k|k-1} = \frac{h^2 - n_\theta}{h^2} h\left(\bar{\theta}_k\right) + \frac{1}{2h^2} \sum_{p=1}^{n_x} \left[h\left(\bar{\theta}_k + d\bar{s}_{\theta,p}\right) + h\left(\bar{\theta}_k - d\bar{s}_{\theta,p}\right)\right] \tag{33}$$

$$S_{z,k} = \left[S_{z\theta,k}^{(1)} S_{z\theta,k}^{(2)}\right] \tag{34}$$

$$P_{\theta z,k} = \bar{S}_{\theta,k} S_{z\theta}^{\mathrm{T}} \tag{35}$$

$$G_k = P_{\theta z,k} \left[S_{z,k} S_{z,k}^{\mathrm{T}}\right]^{-1} \tag{36}$$

$$P_k = \left(\bar{S}_{\theta,k} - G_k S_{z\theta,k}^{(1)}\right)\left(\bar{S}_{\theta,k} - G_k S_{z\theta,k}^{(1)}\right)^{\mathrm{T}} + G_k S_{z\theta,k}^{(2)} \left(S_{z\theta,k}^{(2)}\right)^{\mathrm{T}} \tag{37}$$

4. Parameter state update. Finally, the estimation of the noise parameters is as following:

$$\hat{\theta}_k = \hat{\theta}_{k|k-1} + G_k\left(z_k - h\left(x_k, \hat{\theta}_{k|k-1}\right)\right) \tag{38}$$

For the unknown parameters of the noise variance of motion model and observation model of mobile robot, based on the three nonlinear approximation methods of second-order Stirling interpolation, unscented transformation and spherical-radial rule, three kinds of maximum posteriori estimators and corresponding CKF SLAM algorithms are constructed, which are CKF-SOI SLAM algorithm, CKF-UT SLAM algorithm and CKF-SRR SLAM algorithm respectively. The three approximate estimation algorithms of unknown noise parameters designed in this section are applied to the noise parameters of Gauss distribution and nonlinear systems. The algorithm has wide application scope. In addition, the three numerical approximation methods have their own advantages. According to the different requirements of the practical system in terms of calculation speed, estimation accuracy and numerical stability, choose different forms of CKF SLAM, to meet the practical system unknown noise parameter estimation, and to achieve the stability of mobile robot SLAM estimation.

4 Simulation experiment and result analysis

The simulation environment for mobile robot SLAM experiment is 100 m × 100 m, with 27 landmark features. Assuming that the mobile robot moves in a plane, a simplified motion model is used to approximate it, as in Eq. (39).

$$x_{r,k} = \begin{bmatrix} x_{rx,k-1} + \Delta T v_{k-1} \cos\left(x_{rx,k-1} + \alpha_{k-1}\right) \\ x_{ry,k-1} + \Delta T v_{k-1} \sin\left(x_{rx,k-1} + \alpha_{k-1}\right) \\ x_{ry,k-1} + \dfrac{\Delta T v_{k-1} \sin(\alpha_{k-1})}{WB} \end{bmatrix} + \begin{bmatrix} w_x \\ w_y \\ w_\psi \end{bmatrix} \tag{39}$$

Where $x_{r,k}$ is the position and pose of the robot; ΔT is the sampling time interval; v_{k-1} is the speed; α_{k-1} is the rotation angle; WB: Wheelbase between two axes of mobile robot.

The observation $z_{i,k}$ of the sensor is approximated by a simplified model, such as formula (40).

$$z_{i,k} = \begin{bmatrix} r \\ \theta \end{bmatrix} = \begin{bmatrix} \sqrt{\left(x_{Mi,k} - x_{rx,k}\right)^2 + \left(y_{Mi,k} - x_{ry,k}\right)^2} \\ \arctan \dfrac{y_{Mi,k} - x_{ry,k}}{x_{Mi,k} - x_{rx,k}} \end{bmatrix} + \begin{bmatrix} v_r \\ v_\theta \end{bmatrix} \tag{40}$$

It is assumed that the statistical characteristics of the system noise and the motion noise will change during the SLAM process, but the statistical characteristics of each segment noise are Gauss distribution, and the motion states x, y, z noises of the mobile robot motion model and the ranging noise and angle noise of the observation model have different variation regular and amplitude, as shown in Fig. 2.

The performance differences of each filtering algorithm are compared with the RMSE in the direction of x and y, which are defined as following:

$$RMSE = \sqrt{\frac{\sum_{k=1}^{n} \left(\hat{x}_k - x_{real,k}\right)^2}{\text{n}}} \tag{41}$$

Where $\hat{x}_k$ is the estimated position at step k, and $x_{real,k}$ is the actual position at step k.

Simulation experiments are carried out on four algorithms of CKF-SLAM,CKF-SOI SLAM, CKF-UT SLAM and CKF-SRR SLAM. During the experiment, the mobile robot explores the map in one circle, the blue trajectory is the preset path of the mobile robot, the blue"." indicates the map landmarks, and the green localization trajectory is estimated by the four algorithms with an unknown noise parameter of the mobile robot SLAM system, and the red " + " are the map landmarks estimated by the four algorithms respectively, as shown in Fig. 3.

It can be seen from Fig. 3 that under the unknown model noise parameters given in Fig. 2, there are several sections of localization trajectory deviates from the preset path of the mobile robot estimated by the CKF SLAM, whereas the localization trajectory of the CKF-SOI SLAM estimation is consistent with the preset path of mobile robot, which shows that the localization estimation of CKF SLAM is not effective when the model noise parameters are unknown.

The localization estimation of CKF SLAM, CKF-SOI SLAM, CKF-UT SLAM and CKF-SRR SLAM is in Figs. 3, 4 shows the noise variances estimation of motion and observation model using the three maximum posteriori forms, the horizontal axis is the iterations times of the three maximum a posteriori estimation algorithms, and the vertical axis is the noise variance estimation of pose parameter x, y, ψ and landmark observation ranging and angle of the mobile robot, the black line is the theoretical value.

We can see from Fig. 4, when the noise variance of the mobile robot motion and observation changes, there is some error between the estimation and the true noise variance at the beginning with the three maximum posteriori forms of CKF algorithm. But after a relatively short period of time the estimations converge to the real values.

Taking two moments of noise change for motion and observation state respectively in the estimation of the noise variance of Fig. 4, Tables 1, 2 are the comparison of noise variance estimation and tracking convergence time using the three kinds of maximum posteriori form of CKF algorithm.

It can be seen from Tables 1, 2, in the two changing moments, that the CKF-SOI SLAM algorithm and the CKF-UT SLAM algorithm converge the fastest of the noise estimation in x state.The CKF-SOI SLAM algorithm converge the fastest of the noise estimation in y state. The CKF-SOI SLAM algorithm and the CKF-SRR SLAM algorithm converge the fastest of the noise estimation in ψ state. The CKF-SOI SLAM algorithm and the CKF-UT SLAM algorithm converge the fastest of the noise estimation in observation angle state respectively. The CKF-SOI SLAM algorithm converge the fastest of the noise estimation in ranging state. Therefore, the CKF-SOI SLAM algorithm is the best in the three maximum posteriori noise estimation.

From the above noise parameter estimation and analysis, we can see that in the mobile robot SLAM, when the system motion noise and the observation noise variance are unknown, we can initialize them. With the SLAM process of mobile robot, the initialization variance will track the actual variance value. Once the noise variance is not accuracy, the estimated noise variance with the maximum posteriori algorithm will not be consistent with the true

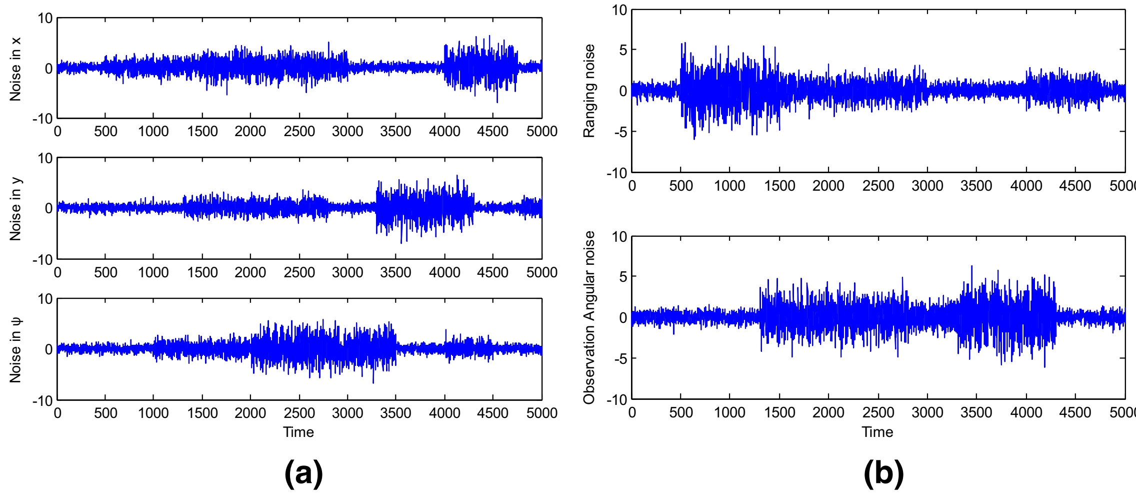

Fig. 2 The noise characteristics used in the experiment. **a** motion noise **b** observation noise

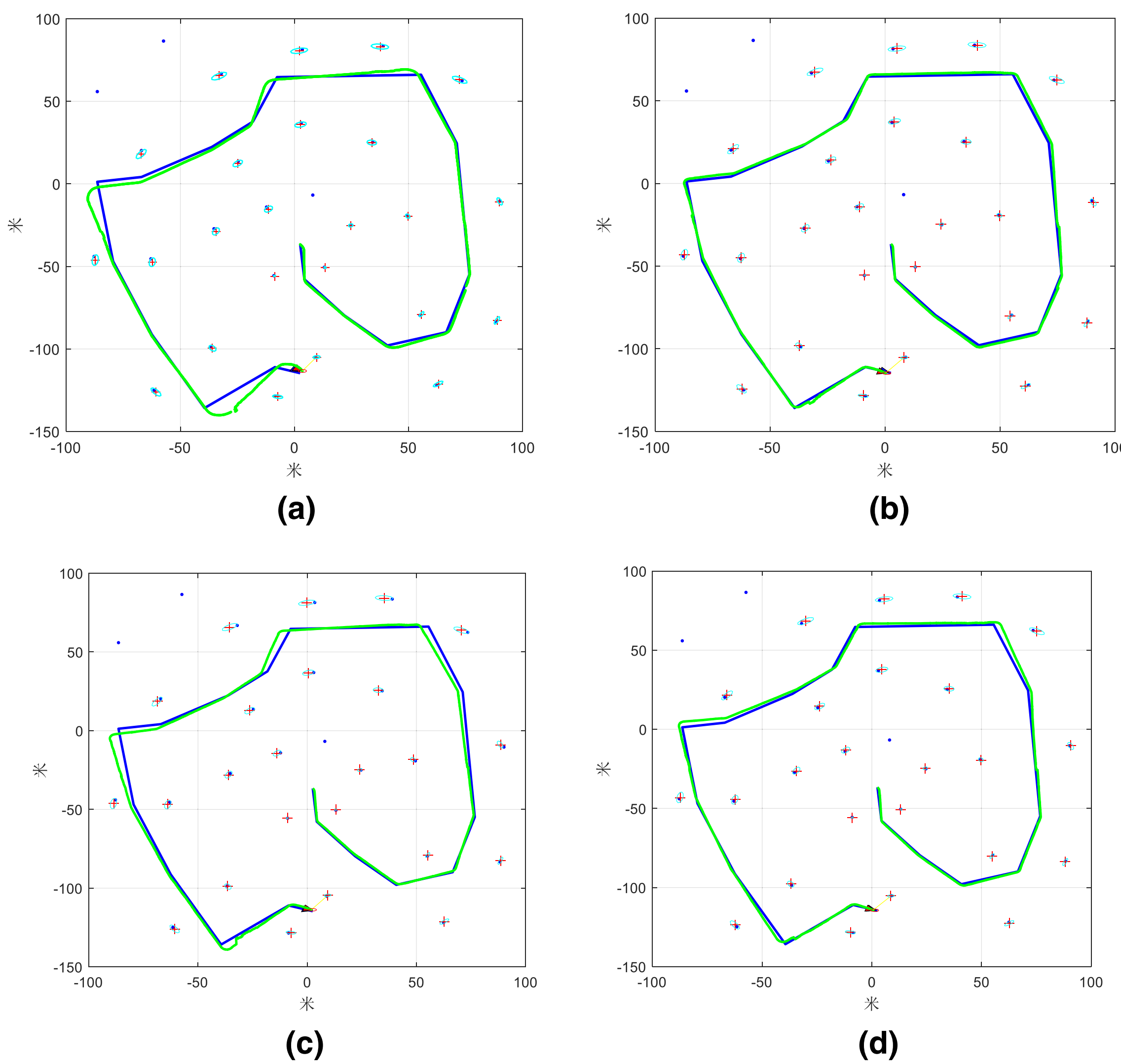

Fig. 3 Comparison of experimental simulation results of four filtering SLAM methods. **a** CKF SLAM **b** CKF-SOI SLAM **c** CKF-UT SLAM **d** CKF-SRR SLAM

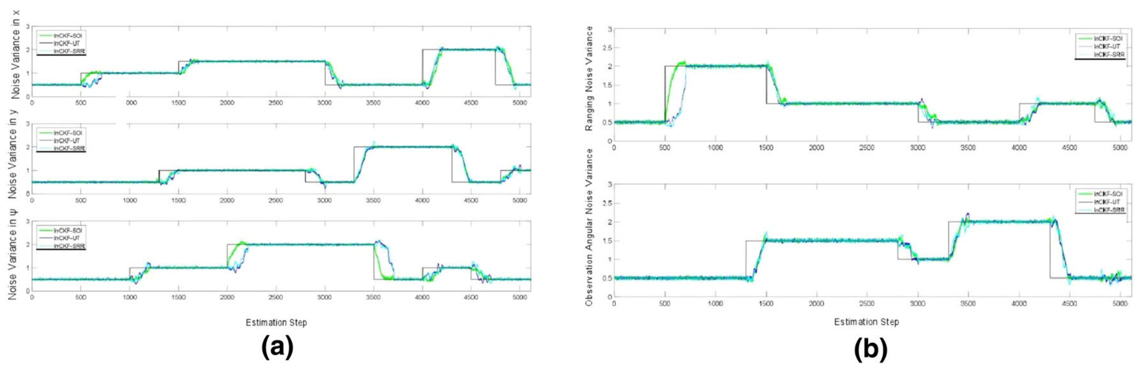

Fig. 4 Noise variance estimation. **a** Variance estimation of motion noise **b** Variance estimation of observation noise

Table 1 Convergence time contrast of three variance estimation algorithms for motion noise

State	Variance changes	Theoretical time/step	Estimation convergence time/step		
			CKF-SOI SLAM algorithm	CKF-UT SLAM algorithm	CKF-SRR SLAM algorithm
x	0.5–1	500	583	702	739
	2–0.5	4750	4901	4895	4912
y	0.5–1	1300	1452	1497	1460
	2–0.5	4300	4482	4490	4486
Ψ	0.5–1	1000	1227	1209	1205
	2–0.5	3500	3620	3712	3711

Table 2 Convergence time contrast of three kinds of variance estimation algorithm for observation noise

State	Variance changes	Theoretical time/step	Estimated convergence time/step		
			CKF-SOI SLAM algorithm	CKF-UT SLAM algorithm	CKF-SRR SLAM algorithm
Observation angle	0.5–2	500	604	718	718
	2–1	1500	1630	1629	1635
Ranging	0.5–1.5	1300	1491	1499	1498
	2–0.5	4300	4496	4502	4497

noise. The three types of maximum posteriori estimators will re-estimate the noise variance according to the changing of innovations, and track the real noise statistics characteristic, so as to ensure the accuracy of the mobile robot localization and mapping.

The State localization estimation error of numerical simulation with CKF SLAM, CKF-SOI SLAM, CKF-UT SLAM, and CKF-SRR SLAM in Fig. 3 is shown in Fig. 5.

Under the unknown noise parameters, Fig. 5 compares the localization performance of the four algorithms of CKF SLAM, CKF-SOI SLAM, CKF-UT SLAM and CKF-SRR SLAM from the localization estimation errors in x and y directions. From the comparison of the amplitude and the curve changing in Fig. 5, it can be seen that the localization error of the CKF algorithm is the largest, indicating that standard CKF SLAM algorithm can not accurately estimate the localization if noise statistical parameters are unknown.

The localization estimation errors with four SLAM algorithms in Fig. 5 are statistically analyzed. The mean absolute error and the root mean square error of localization estimation in x axis is contrasted in Table 3. The comparison of The mean absolute error and the root mean square error of localization estimation in y axis is contrasted in Table 4.

Under the condition of unknown model noise parameters, we can see from the numerical simulation comparison

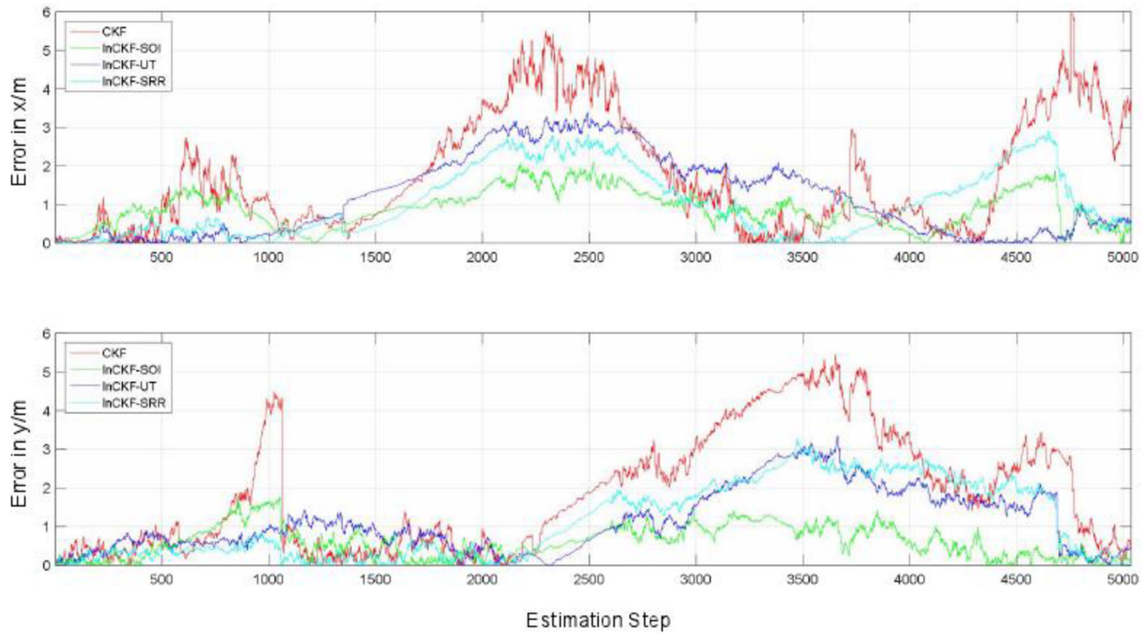

Fig. 5 Comparison of state localization estimation errors of four filtering SLAM algorithms

Table 3 State estimation error comparison in *x* axis with four filtering SLAM algorithms

Algorithm	Average absolute error/m	RMSE/m
CKF algorithm	1.7528	2.3055
CKF-SOI SLAM algorithm	0.8872	1.0106
CKF-UT SLAM algorithm	1.1581	1.5512
CKF-SRR SLAM algorithm	1.0331	1.3477

Table 4 State estimation error comparison in *y* axis with four filtering SLAM algorithms

Algorithm	Average absolute error/m	RMSE/m
CKF algorithm	1.8595	2.3974
CKF-SOI SLAM algorithm	0.5925	0.7229
CKF-UT SLAM algorithm	1.1272	1.3837
CKF-SRR SLAM algorithm	1.1254	1.4832

of average absolute error and the root mean square error of localization estimation in Tables 3, 4, that the localization error of CKF algorithm is the largest, which indicates that CKF SLAM algorithm is not suitable for state estimation under unknown noise parameters. The three forms of CKF algorithm estimate in real time and correct the statistical characteristics of system noise through the time-varying noise estimator, with which SLAM is avoid being affected by that the system model does not exactly match the real model and unknown of noise statistical characteristics. Therefore, the localization estimation accuracy of the mobile robot and mapping accuracy have been greatly improved compared with the CKF SLAM algorithm.

In conclusion, the CKF-SOI SLAM algorithm has the highest accuracy under unknown noise parameters. Although the theoretical accuracy of the three forms is the same, the CKF-SRR SLAM algorithm can not take advantage in the low-dimensional system model. The above simulation experiments shows that under the uncertainty noise parameters, if the noise variance is Gaussian white noise distribution, the designed CKF SLAM with three maximum posterior can well estimate the state, and can handle the problem of nonlinear systems.

5 Conclusion

Aiming at the problem that the model noise variance is unknown, the noise variance estimation model and the estimation criterion are constructed, and the noise statistical variance is estimated based on three kinds of maximum posteriori. A CKF SLAM algorithm is designed to solve the problem of localization and mapping state estimation of SLAM system model noise statistic is constant but unknown. The feasibility and validity of the proposed SLAM estimation method with unknown noise variance are verified by noise estimation and SLAM estimation simulation experiments.

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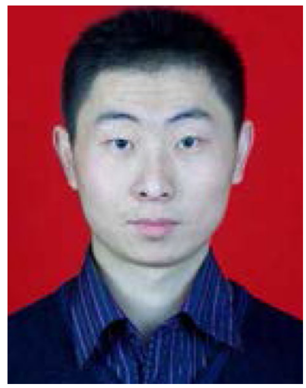

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