

# Judges' Commentary:

## The Self-Driving Cars Problem

Carol Overdeep  
Mathematics Dept.  
Saint Martin's University  
Lacey, WA

Katie Oliveras  
Mathematics Dept.  
Seattle University  
Seattle, WA

David H. Olwell  
Dean and Professor  
Hal and Inge Marcus School of Engineering  
Saint Martin's University  
Lacey, WA  
dolwell@stmartin.edu

## Introduction

2017 marked the second year for an MCM Problem C—a problem designed to elicit data insights in the context of a large, potentially messy data set. Problem C is not intended to be a “big data” problem requiring specialized computer science-based tools and techniques; but rather an opportunity to encounter real-world, challenging data sets that have interesting characteristics.

The 2017 problem provided participants the opportunity to analyze the effects of allowing self-driving cars on various high-volume roads in the Seattle-Tacoma metropolitan area. Specifically, participants were asked to determine if there were any *equilibria* or *tipping points* as the percentage of self-driving cars was varied—terms that the participants had to define as part of the modeling process. The associated data set provided participants with average daily traffic at various mileposts along the roads of interest.

The problem-writing team continues to refine its expectations for Problem C. This year's problem and associated data set shifted the focus slightly from statistical techniques to model-based data insights.

---

*The UMAP Journal* 38 (3) (2017) 347–353. ©Copyright 2017 by COMAP, Inc. All rights reserved. Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice. Abstracting with credit is permitted, but copyrights for components of this work owned by others than COMAP must be honored. To copy otherwise, to republish, to post on servers, or to redistribute to lists requires prior permission from COMAP.



关注数学模型  
获取更多资讯

## The Problem: “Cooperate and Navigate”

We repeat the problem verbatim:

Traffic capacity is limited in many regions of the United States due to the number of lanes of roads. For example, in the Greater Seattle area drivers experience long delays during peak traffic hours because the volume of traffic exceeds the designed capacity of the road networks. This is particularly pronounced on Interstates 5, 90, and 405, as well as State Route 520, the roads of particular interest for this problem.

Self-driving, cooperating cars have been proposed as a solution to increase capacity of highways without increasing number of lanes or roads. The behavior of these cars interacting with the existing traffic flow and each other is not well understood at this point. The Governor of the State of Washington has asked for analysis of the effects of allowing self-driving, cooperating cars on the roads listed above in Thurston, Pierce, King, and Snohomish counties (see the provided map and Excel spreadsheet). In particular, how do the effects change as the percentage of self-driving cars increases from 10% to 50% to 90%? Do equilibria exist? Is there a tipping point where performance changes markedly? Under what conditions, if any, should lanes be dedicated to these cars? Does your analysis of your model suggest any other policy changes?

Your answer should include a model of the effects on traffic flow of the number of lanes, peak and / or average traffic volume, and percentage of vehicles using self-driving, cooperating systems. Your model should address cooperation between self-driving cars as well as the interaction between self-driving and non-self-driving vehicles. Your model should then be applied to the data for the roads of interest, provided in the attached Excel spreadsheet.

*Your MCM submission should consist of a 1-page Summary Sheet, a 1–2 page letter to the Governor’s office, and your solution (not to exceed 20 pages) for a maximum of 23 pages. Note: The appendix and references do not count toward the 23-page limit.*

## Summary

Once again, this year’s problem provided a challenge for the teams. The teams had no difficulty identifying existing models on traffic flow (e.g., cellular automata, microscopic / macroscopic models, etc.). Incorporating the given data into the models, however, is where many teams struggled. A surprising number of teams either did not mention the given data set in their papers or summarized the given data set but did not apply their models / results to the data. Regardless of how well these papers were written,



关注数学模型  
获取更多资讯

not including the given data in a meaningful way prevented them from further consideration beyond the triage phase of judging. The most successful teams provided detailed descriptions of both their model assumptions and how their model was constructed. They used the data to gain insight into regional traffic patterns and then applied their traffic model to the traffic data. Once the traffic data were applied to their model, they carefully investigated the impact of both self-driving cars along with dedicated lanes to the overall traffic flow throughout the entire system.

The top papers discussed both equilibrium and tipping points within this context, and used the ensuing results to provide policy recommendations in a clear, concise manner.

As always, judges value well-written, well-illustrated papers that carefully followed the contest directions. Judges valued papers that described and motivated the equations used, especially those adapted from other sources. Judges paid close attention to citations, both proper use of inline citations and using reputable sources. Repurposing images from external sources without credit detracted from a paper's overall rating.

There were four Outstanding papers. One was from the University of California–Berkeley (advised by John Harte) and the other three were from Shanghai Jiao Tong University (all advised by Xiaofeng Gao). Shanghai Jiao Tong University had an exceptional performance on Problem C, with 3 Outstanding papers, 8 Meritorious papers, 22 Honorable Mention papers, and 10 Successful Participants.

## The Judging Process

The judging process followed the usual scheme of triage, screening rounds, and final judging as described in previous Judges' Commentaries [Black 2009; Black 2011; Black 2013]. In 2016, the judging team met in early March near the end of the triage phase; this year, the judging team (a mix of judges from the previous year and new judges) met in mid-February at the start of the triage phase. Prior to the meeting, the judging team had been asked to read and evaluate five papers. This facilitated discussion about scoring criteria used in the evaluation process, leading to increased consistency among judges, as well as providing hands-on training for the new judges.

## Defining the Problem

Teams were asked to model the effects of introducing self-driving cars on traffic flow during peak and/or average volumes. In particular, teams were asked to analyze the impacts of self-driving cars as the percentage of these vehicles increased from 10% to 50% to 90%.



The teams had to determine how to measure the impacts of self-driving cars; unlike last year's problem, this was not a critical element in the development of the model but rather a byproduct of the type of model chosen. The best papers clearly outlined how they would measure the impacts to the traffic systems and what both an equilibrium and a tipping point would represent with regards to their model.

The problem also required that teams address the "cooperation between self-driving cars as well as the interaction between self-driving and non-self-driving vehicles." While many papers described the interaction between self-driving and non-self-driving vehicles, very few papers described or modeled cooperation between self-driving cars; those that did were rated higher.

The best papers also included a literature review that discussed both self-driving cars and various co-operation models. The best teams described how their models compared to the existing literature, and where they built upon it; the judges found this approach helpful.

## Insights from the Data

The data set came from the Washington State Dept. of Transportation and contained seven fields:

- route ID,
- start milepost,
- end milepost,
- average daily traffic counts for 2015,
- number of lanes in the milepost-decreasing direction,
- number of lanes in the milepost-increasing direction,
- the route type (state route or interstate), and
- comments.

One of the challenging aspects of the data is that it contained average daily counts, not hourly counts. The problem asked teams to address the effects on traffic flow during peak and/or average traffic volume. In the problem description, teams were told that on average, 8% of the daily traffic volume occurs during peak travel hours. Teams had to identify peak travel hours, and most teams chose two 2-hour windows (one for the morning commute, one for the evening commute) while others chose two 3-hour windows. The judges gave no preference to either selection. The best papers used the data to gain insight into the overall traffic flow and potential locations for bottlenecks in the overall traffic system. They then applied their traffic models to the data set and analyzed the resulting changes



关注数学模型  
获取更多资讯

throughout the entire region. They then compared their metrics against the given data to provide evidence for their proposed policy suggestions.

As noted earlier, papers that did not use the data set or did not clearly communicate how they used the data set were not considered after the triage phase.

## Building the Models

There is a wide variety of existing traffic models. Judges had no preferred model; rather, they looked for

- clarity in the description of the model(s),
- a complete list of assumptions used in the model, and
- how the given data set was incorporated into the model.

The key assumptions in the models included

- the reaction time of a self-driving vehicles,
- the reaction time of human-driven vehicles,
- the following distance of self-driving cars, and
- the following distance of human-driven vehicles.

Of paramount importance was the use of the given data set! A somewhat surprising percentage of papers were either “Here are several traffic flow models and here are our results” or “Here are several traffic flow models, here is a summary of the data, and here are our results” without connecting the data to the model and/or results.

Some of the models that fell into one of these two categories had another shortcoming: They included modeling of traffic lights. Traditional traffic lights are not part of the interstate system.

Similarly, many papers built on existing traffic flow models without clearly providing either motivation or the associated assumptions corresponding to these base models. Papers that simply presented equations or models without motivation or justification were not evaluated highly by the judges.

The highest-ranked papers included valid mathematics with sufficient details justifying the model's choice. They also defined interactions between vehicles and applied their model to the data. Both continuous and discrete models were successfully applied to the problem by the Outstanding papers. The paper by the University of California–Berkeley team included both a macro model based on a PDE flux-flow-density model and a discrete model to analyze lane-changing behavior. The three papers from Shanghai Jiao Tong University used a cellular automata approach, each with a different emphasis.



## Model Analysis and Validation

By design, there was no one “correct solution” to the problem, and this feature was borne out in the papers. Policy recommendations ran the gamut from not supporting self-driving cars up to advocating that 90% of the cars be self-driven. Consequently, judges did not read the papers in search of a “correct” answer; instead, judges looked for keen insights into the results generated by the model. Most teams justified their models by citing existing literature. Papers that also contained sensitivity analysis were viewed more favorably by the judges. Papers whose sensitivity analysis focused on the assumed values of the parameters (e.g., reaction times and following distances) and demonstrated an understanding of the purpose of sensitivity analysis were viewed even more favorably.

## Communicating Results

Teams produced two documents: the paper and a letter to the Governor of Washington outlining policy recommendations with respect to self-driving vehicles. For the paper itself, highly-ranked papers were well organized, contained proper in-line citations, and included all the required elements. For the letter, again, organization and the inclusion of all the required elements were hallmarks of highly-ranked papers. In addition, letters that were written in “plain English” for a non-technical audience were valued.

## Conclusions

The first Problem C, in 2016, attracted 1,875 student teams, with 4 recognized as Outstanding and 8 recognized as Finalists. This year’s Problem C had 1,527 submissions, with 4 recognized as Outstanding and another 7 as Finalists. The judges continue to be delighted with the quality of submissions to the contest.

## References

- Black, Kelly. 2009. Judge’s Commentary: The Outstanding Traffic Circle papers. *The UMAP Journal* 30 (3): 305–311.
- \_\_\_\_\_. 2011. Judges’ Commentary: The Outstanding Snowboard Course papers. *The UMAP Journal* 32 (2): 123–129.
- \_\_\_\_\_. 2013. Judges’ Commentary: The Ultimate Brownie Pan papers. *The UMAP Journal* 34 (2–3): 141–149.



关注数学模型  
获取更多资讯



Olwell, David H. 2013. Judge's Commentary: The Outstanding National Water Strategy papers. *The UMAP Journal* 34 (2–3): 189–195.

\_\_\_\_\_, Carol Overdeep, and Katie Oliveras. 2016. Judges' Commentary: The Goodgrant Challenge papers. *The UMAP Journal* (3): 325–331.

## About the Authors



Carol Overdeep is an Associate Professor of Mathematics at Saint Martin's University. She earned a B.S. in Mathematics at the University of Puget Sound, then began her professional career in the ADCAP MK-48 torpedo software validation group at Hughes Aircraft Company Ground Systems Group in Buena Park, CA. While working for Hughes, Dr. Overdeep earned an M.A. in applied mathematics from California State University–Fullerton. After 15 years as a torpedo systems engineer for Hughes Aircraft Company and the Naval Undersea Warfare Center Division Newport, Rhode Island, Dr. Overdeep earned an M.B.A. and then a Ph.D. in Applied Mathematics at the University of Rhode Island.



Katie Oliveras is an Assistant Professor of Mathematics at Seattle University. She earned a B.S. in Systems Science and Engineering from Washington University in St. Louis, and then M.S. and Ph.D. degrees in Applied Mathematics from the University of Washington. As an undergraduate, she participated in the MCM competition; and she has served as a team advisor for MCM teams at Seattle University.

David H. Olwell is Professor and Dean, Hal and Inge Marcus School of Engineering, at Saint Martin's University. He earned a B.S. at the United States Military Academy, where he studied mathematics, and an M.S. in Mathematics, M.S. in Statistics, and Ph.D. in Statistics from the University of Minnesota. He has previously been on the faculty of the U.S. Military Academy and the Naval Postgraduate School. Dr. Olwell has been a problem author for both the MCM and HiMCM, and has been an MCM judge for a decade.



关注数学模型  
获取更多资讯