The UMAP Journal

Vol. 26, No. 2

Subscription Rates for 2005 Calendar Year: Volume 26

Membership Plus

Individuals subscribe to *The UMAP Journal* through COMAP's Membership Plus. This subscription also includes print copies of our annual collection *UMAP Modules: Tools for Teaching*, our organizational newsletter *Consortium*, on-line membership that allows members to download and reproduce COMAP materials, and a 10% discount on all COMAP purchases.

(Domestic)	**#2520**	**$ 99**
(Outside U.S.)	**#2521**	**$111**

Institutional Plus Membership

Institutions can subscribe to the *Journal* through either Institutional Plus Membership, Regular Institutional Membership, or a Library Subscription. Institutional Plus Members receive two print copies of each of the quarterly issues of *The UMAP Journal*, our annual collection *UMAP Modules: Tools for Teaching*, our organizational newsletter *Consortium*, on-line membership that allows members to download and reproduce COMAP materials, and a 10% discount on all COMAP purchases.

(Domestic)	**#2570**	**$456**
(Outside U.S.)	**#2571**	**$479**

Institutional Membership

Regular Institutional members receive print copies of *The UMAP Journal*, our annual collection *UMAP Modules: Tools for Teaching*, our organizational newsletter *Consortium*, and a 10% discount on all COMAP purchases.

(Domestic)	**#2540**	**$198**
(Outside U.S.)	**#2541**	**$220**

Web Membership

Web membership does not provide print materials. Web members can download and reproduce COMAP materials, and receive a 10% discount on all COMAP purchases.

(Domestic)	**#2510**	**$41**
(Outside U.S.)	**#2510**	**$41**

To order, send a check or money order to COMAP, or call toll-free
1-800-77-COMAP (1-800-772-6627).

The UMAP Journal is published quarterly by the Consortium for Mathematics and Its Applications (COMAP), Inc., Suite 210, 57 Bedford Street, Lexington, MA, 02420, in cooperation with the American Mathematical Association of Two-Year Colleges (AMATYC), the Mathematical Association of America (MAA), the National Council of Teachers of Mathematics (NCTM), the American Statistical Association (ASA), the Society for Industrial and Applied Mathematics (SIAM), and The Institute for Operations Research and the Management Sciences (INFORMS). The Journal acquaints readers with a wide variety of professional applications of the mathematical sciences and provides a forum for the discussion of new directions in mathematical education (ISSN 0197-3622).

Periodical rate postage paid at Boston, MA and at additional mailing offices.

Send address changes to: info@comap.com

COMAP, Inc. 57 Bedford Street, Suite 210, Lexington, MA 02420

Vol. 26, No. 2 2005

Table of Contents

Editorial

Where Else to Publish

Paul J. Campbell
Dept. of Mathematics and Computer Science
Beloit College
700 College St.
Beloit WI 53511–5595
campbell@beloit.edu

Kunio Mitsuma
Dept. of Mathematics and Computer Science
Kutztown University of Pennsylvania
Kutztown, PA 19530
mitsuma@kutztown.edu

Introduction

Our Guide for Authors (Vol. 26, No. 1, pp. 91–92) advises that

> *The UMAP Journal* focuses on **mathematical modeling and applications of mathematics at the undergraduate level**.

The editor also welcomes

- expository articles for the On Jargon column,
- reviews of books and other materials, and
- guest editorials on new ideas in mathematics education or on interaction between mathematics and application fields.

Major vehicles for achieving the goals of the *Journal* are

- **UMAP Modules:** A UMAP Module is a teaching/learning module with exercises and often a sample exam (with solutions) and in particular precise statements of

The UMAP Journal 26 (2) (2005) 93–114.

  - the target audience,
  - the mathematical prerequisites, and
  - the time frame for completion.

  UMAP Modules are designed for class use to learn about applications of mathematics but are also often useful for independent study and student projects.

- **ILAP Modules:** An ILAP (Interdisciplinary Lively Application Project) is an *interdisciplinary* student *group* project, jointly authored by faculty from mathematics and a partner department. The project usually includes
  - some instructional material,
  - requirements that the student teams must fulfill, including preparing a report.
- **Minimodules:** While a UMAP Module or ILAP may run from 12 to over 50 pages, a Minimodule is usually 6 pages or less.
- the annual undergraduate **Mathematical Contest in Modeling** and **Interdisciplinary Contest in Modeling**. The 2% or so of submitted papers rated by the judges as Outstanding are published in special sections of the *Journal*, including an entire issue devoted to the Mathematical Contest in Modeling.
- **Articles:** Articles do not need to be tightly focussed toward direct classroom use, nor must they include exercises (though they may). Simply being in a field that has traditionally been designated as "applied mathematics," e.g., numerical analysis, differential equations) does not meet the needs of our readers; articles, like the modular materials, must treat mathematical modeling or applications of mathematics.
- **On Jargon columns:** An On Jargon column exposits a mathematical term or concept (which need not be related directly to modeling or applications). (The *Notices of the American Mathematical Society* recently adopted this idea in the form of their "What Is..." column (which treats concepts that arise at a much higher level). On Jargon columns have appeared in the *Journal* since its first issue but have been relatively rare in recent issues. The editor especially encourages readers to submit material for this department of the *Journal*.

Most UMAP Modules and ILAP Modules appear in the *Journal*, and all Modules (including ones too long for a regular issue of the *Journal*) appear in the *Journal*'s annual supplement *UMAP Modules: Tools for Teaching*.

For UMAP Modules, ILAP Modules, and Minimodules, the ideals are as follows:

- The occasion is a real-world situation (with real data), usually outside the mathematical sciences, that is described in some detail.

- The problems are solvable by undergraduates, using mathematical techniques and modeling aids (graphing calculators, computer algebra systems, statistical packages, simulation, differential equations solvers, etc.).
- The modeling concludes by *returning to the application* to discuss how the modeling leads to insight about the application.

The corpus of UMAP Modules is available on a CD-ROM with a companion database; purchasers are authorized to duplicate Modules for their students only ($299 PROD #7954; see the COMAP Website at `www.comap.com` or call (800)–77–COMAP).

Where Else to Publish?

The major reason for this *Journal* to reject a manuscript is that it does not emphasize modeling and grounding in applications.

The editor tries to steer authors of unsuitable manuscripts to other journals. Below we give a list of journals, including each journal's focus and editor's address, completely updated—and with numerous additions and deletions, including electronic journals—from a list published in Vol. 17, No. 1, 1–14.

Our aim in providing this list is not to dissuade authors from submitting manuscripts to the *Journal*! Far from it! The *Journal* can always use more good papers to help realize its mission. Rather, there are relatively few outlets for mathematical exposition at the undergraduate level compared with the plenitude of research journals. This list exhibits more opportunities than most authors realize; we offer it so that they can more readily find suitable outlets for their work.

We welcome suggestions from readers for additions or revisions to this list.

Criteria

We list only journals that

- are familiar to us,
- publish articles in English, and
- specifically focus on mathematics (including computing aspects and statistics) at the undergraduate college/university level.

We exclude departmental journals that usually include authors from only the home institution (e.g., *Eureka* at Cambridge University, or *Journal of Undergraduate Mathematics at Puget Sound* at the University of Puget Sound).

Appearance of any journal in this list, or failure to list any particular journal, does not imply any endorsement or judgment by us about the quality of the journal, nor about the suitability of that journal for any author's manuscript. Descriptions of the journals are derived largely from their own published statements of purpose.

More journals from throughout the world are listed in the Source Journal Index (Zeitschriften-verzeichnis) of *Zentralblatt für Didaktik der Mathematik (International Reviews on Mathematical Education)*, available from

Gerhard Koenig
Fachinformationszentrum Karlsruhe
Gesellschaft für wissenschaftlich-technische Information mbH
D-7514 Eggenstein-Leopoldshafen 2
Germany

Links to many journals on computer science education and the uses of computers and information technology in education can be found at

`http://www.cs.washington.edu/research/edtech/pubs_orgs/` .

List of Journals

(∗): indicates electronic distribution only.

AMATYC Review
The AMATYC Review
Barbara S. Rives, Editor
Lamar State College–Orange
410 Front Street
Orange, TX 77630
`rivesbs@gt.rr.com`
`http://www.amatyc.org/Review/index.html`

A semi-annual publication of the American Mathematical Association of Two-Year Colleges. Its purpose is to provide an avenue of communication for all mathematics educators concerned with the views, ideas, and experiences pertinent to two-year college teachers and students.

American Mathematical Monthly
Bruce Palka, Editor
Department of Mathematics
University of Texas at Austin
1 University Station
Austin, TX 78712–1082
`monthly@math.utexas.edu`
`http://www.maa.org/pubs/monthly.html`

Problems or Solutions to:
Doug Hensley, *Monthly* Problems
Department of Mathematics
Texas A&M University
College Station, TX 77840
`hwaldman@maa.org`

Articles, as well as notes and other features, about mathematics and the profession. Its readers span a broad spectrum of mathematical interests, and include professional mathematicians as well as students of mathematics at all collegiate levels. Authors are invited to submit articles and notes that bring interesting mathematical ideas to a wide audience of *Monthly* readers.

Bulletin of Mathematics Books and Computer Software
Steven Roman, Publisher
The Roman Press
8 Night Star
Irvine, CA 92715

Disseminates publishers' and reviewers' information about mathematics books and software.

Chance
Dalene Stangl, Editor
Box 90251, ISDS
Duke University
Durham, NC 27708
`dalene@stat.duke.edu`
`http://www.stat.duke.edu/chance/`

Jointly published by the American Statistical Association and Springer-Verlag, about statistics and the use of statistics in society. It is "intended for everyone who has an interest in the analysis of data. *Chance* features articles that showcase the use of statistical methods and ideas in the social, biological, physical, and medical sciences. It also presents material about statistical computing and graphical presentation of data. Through its regular departments and columns, *Chance* will keep its readers informed about developments and ideas in a variety of areas including government statistics and sports. The goal is to promote the field of statistics and make its contributions accessible to a broad audience."

($*$) **Chance News**
J. Laurie Snell, Editor
`jlsnell@dartmouth.edu`
`http://www.dartmouth.edu/~chance/`

Reviews articles in the news that teachers of probability and statistics might want to use in their classes. Please send suggestions to `jlsnell@dartmouth.edu`. You are encouraged to include your comments on the article.

College Mathematics Journal
Lowell Beineke
Mathematics Department
Indiana University–Purdue University Ft. Wayne
Ft. Wayne, IN 46805–1499
`http://www.maa.org/pubs/cmj.html`

Seeks lively, well-motivated articles that can enrich undergraduate instruction and enhance classroom learning. The *CMJ* also invites expository papers that stimulate the thinking and broaden the perspectives of those who teach undergraduate-level mathematics, especially the first two years. Articles involving all aspects of mathematics are welcome: history, philosophy, problem solving, applications, computer-related mathematics, and so on.

Classroom Capsules
Michael Kinyon
Dept. of Mathematical Sciences
Indiana University South Bend
South Bend, IN 46634

Fallacies, Flaws, and Flimflam
Ed Barbeau
Department of Mathematics
University of Toronto
Toronto, Ontario
Canada M5S 1A1

Problems and Solutions
James Bruenin
Department of Mathematics
Southeast Missouri State University
Cape Girardeau, MO 63702

Student Research Projects
Brigitte Servatius
Dept. of Mathematical Sciences
Worchester Polytechnic Institute
Worchester, MA 01609–2280

Media Highlights
Warren Page
Department of Mathematics
New York City Technical College
300 Jay Street
Brooklyn, NY 11201

Software Reviews
L. Carl Leinbach
Dept. of Mathematics and
Computer Science
Gettysburg College
Gettysburg, PA 17325

Proofs without words, letters to the editor, quotations, verse, cartoons, mathematical facetiae, and all other material
Lowell Beineke
Mathematics Department
Indiana University-Purdue University Ft. Wayne
Ft. Wayne, IN 46805–1499

Computers & Graphics: An International Journal of Systems & Applications in Computer Graphics

JosÈ L. Encarnac„o
Technical University Darmstadt, Gris
Fraunhoferstrasse 5
D–64283 Darmstadt
Germany
`jle@igd.fhg.de`
`http://authors.elsevier.com/JournalDetail.html?PubID=371&Precis=DESC`

Research and applications, tutorial papers, state-of-the-art papers, and information on innovative uses of computer graphics, including computers and graphics in education.

Computers & Mathematics with Applications

E.Y. Rodin
Dept. of Systems Science and Mathematics
Box 1040
Washington University
St. Louis, MO 63130
`http://www.elsevier.com/wps/find/journaldescription.cws_home/301/description#description`

The journal pays particular attention to applications in "non-classified" fields, such as environmental science, ecology, biology, urban systems and also to appropriate papers in applied mathematics.

(∗) **Convergence**

Victor Katz, Editor
The Mathematical Association of America
1529 18th St. N.W.
Washington, DC 20036–1385
`vkatz@udc.edu`
`http://convergence.mathdl.org/jsp/index.jsp`

"Sponsored by the Mathematical Association of America with the cooperation of the National Council of Teachers of Mathematics, *Convergence* is intended to be a resource and forum for mathematics teachers of grades 9–14 mathematics who are interested in using mathematics history as a learning/teaching tool.

- Expository articles on aspects or concepts from the history of mathematics that the author feels possess a special pedagogical or learning appeal.
- A sharing of classroom experiences.

- Animated mathematical demonstrations that can be downloaded for classroom use.
- Translations and commentaries of mathematical works that shed particular light on mathematical discovery and understanding.
- Discussions of particular problems from an historical context.
- Reviews of materials, books, websites and teaching aids that lend themselves to historical enrichment."

Crux Mathematicorum with *Mathematical Mayhem*
James Totten, Editor-in-Chief
Department of Mathematics & Statistics
University College of the Cariboo
Kamloops, BC V2C 5N
`crux-editors@cms.math.ca`
`http://journals.cms.math.ca/CRUX/`

A problem-solving journal at the secondary and university undergraduate levels.

Educational Studies in Mathematics
Editor Social Sciences Division
Kluwer Academic Publishers
P.O. Box 17
3300 AA Dordrecht
The Netherlands
`http://www.kluweronline.com/issn/0013-1954/contents`

"Presents new ideas and developments of major importance to those working in the field of mathematical education. It seeks to reflect both the variety of research concerns within this field and the range of methods used to study them. It deals with didactical, methodological and pedagogical subjects, rather than with specific programmes for teaching mathematics."

Elemente der Mathematik
Jürg Kramer, Managing Editor
Humboldt-Universität zu Berlin
Institut für Mathematik
Unter den Linden
D-10099 Berlin
`kramer@mathematik.hu-berlin.de`
`http://www.springeronline.com/sgw/cda/frontpage/`
`0,10735,5-10042-70-1176268-detailsPage%253Djournal`
`%257Cdescription%257Cdescription,00.html`

Survey articles about important developments in the field of mathematics; stimulating shorter communications that tackle more specialized questions; and papers that report on the latest advances in mathematics and applications in other disciplines.

L'enseignement mathématique
Case postale 240
CH-1211 Geneva 24
Switzerland
`http://www.unige.ch/math/EnsMath/EM_en/welcome.html`

Articles on teaching mathematics, in French, English, German, or Italian.

Experimental Mathematics
Rafael de la Llave, Editor-in-Chief
A K Peters, Ltd.
888 Worcester Street
Suite 230
Wellesley, MA 02482
`expmath@akpeters.com`
`http://www.expmath.org`

Formal results inspired by experimentation, conjectures suggested by experiments, descriptions of algorithms and software for mathematical exploration, surveys of areas of mathematics from the experimental point of view, and general articles of interest to the community.

Fibonacci Quarterly
Gerald E. Bergum, Editor
South Dakota State University
Box 2201
Brookings, SD 57007–1596
`bergumg@mg.sdstate.edu`
`http://www.mathpropress.com/problemColumns/fq/fqInfo.html`

Articles that are intelligible, yet stimulating, to its readers, most of whom are university teachers and students. These articles should be lively and well motivated, with new ideas that develop enthusiasm for number sequences or the exploration of number facts. Illustrations and tables should be wisely used to clarify the ideas of the manuscript. Unanswered questions are encouraged, and a complete list of references is absolutely necessary.

Elementary Problems
Stanley Rabinowitz
12 Vine Brook Rd.
Westbrook, MA 01886–4212
`Fibonacci@MathPro.com`

Advanced Problems
Raymond E. Whitney
Mathematics Dept.
Lock Haven University
Lock Haven, PA 17745
`rwhitney@LHUP.edu`

(∗) **Furman University Electronic Journal of Undergraduate Mathematics**
Mark Woodard, Editor
Department of Mathematics
Furman University
Greenville, SC 29613–0448
`mark.woodard@furman.edu`
`http://math.furman.edu/~mwoodard/fuejum/content/toc.html`

> "The *Journal* accepts papers of significant mathematical interest written by students containing work done prior to the students' obtaining a Bachelor's degree. Papers of all types will be considered, including technical, historical, and expository papers. Each paper must be sponsored by a mathematician familiar with the student's work, a full-time faculty member willing to endorse the student's work. The sponsor is largely responsible for ensuring the quality and veracity of the student's work."

Historia Mathematica
Editorial Office
525 B Street, Suite 1900
San Diego, CA 92101–4495
`hm@elsevier.com`
`http://authors.elsevier.com/JournalDetail.html?PubID=622841`
`&Precis=DESC`

> Historical scholarship on mathematics and its development in all cultures and time periods. In particular, the journal encourages informed studies on mathematicians and their work in historical context, on the histories of institutions and organizations supportive of the mathematical endeavor, on historiographical topics in the history of mathematics, and on the interrelations between mathematical ideas, science, and the broader culture.

Humanistic Mathematics Network Journal
Sandra and Philip Keith, Managing Editors
St. Cloud State University
St. Cloud, MN 56301
`szkeith@stcloudstate.edu`
`http://www2.hmc.edu/www_common/hmnj/`

> Essays, book reviews, syllabi, and letters on mathematics as a humanistic endeavor.

International Journal of Mathematical Education in Science and Technology
M.C. Harrison, Editor
Department of Mathematical Sciences
Mathematics Education Centre
Loughborough University

Loughborough
Leicestershire
LE11 3TU
U.K.
`m.c.harrison@lboro.ac.uk`
`http://www.tandf.co.uk/journals/titles/0020739X.asp`

"A medium by which a wide range of experience in mathematical education can be presented, assimilated and eventually adapted to everyday needs in schools, colleges, polytechnics, universities, industry and commerce. Contributions will be welcomed from lecturers, teachers and users of mathematics at all levels on the contents of syllabuses and methods of presentation. Increasing use of technology is being made in the teaching, learning, assessment and presentation of mathematics today; original and interesting contributions in this new area will be especially welcome. Mathematical models arising from real situations, the use of computers, new teaching aids and techniques also form an important feature. Discussion will be encouraged on methods of widening applications throughout science and technology. The need for communication between teacher and user will be emphasized and reports of relevant conferences and meetings will be included."

Journal for Research in Mathematics Education
Steven R. Williams, Editor
JRME
Department of Mathematics Education
Brigham Young University
P.O. Box 26537
Provo, UT 84602–6537
`williams@mathed.byu.edu`
`http://my.nctm.org/eresources/journal_home.asp?journal_id=1`

Promotes and disseminates disciplined scholarly inquiry into the teaching and learning of mathematics at all levels, including research reports, book reviews, and commentaries.

Journal of Computers in Mathematics and Science Teaching
Association for the Advancement of Computing in Education
P.O. Box 3728
Norfolk, VA 23514
`pubs@aace.org`
`http://www.aace.org/pubs/jcmst/default.htm`

" Offers an in-depth forum for the interchange of information in the fields of science, mathematics, and computer science. *JCMST* is the only periodical devoted specifically to using information technology in the teaching of mathematics and science."

Journal of Educational Computing Research
Robert H. Seidman, Editor
New Hampshire College Graduate School
2500 North River Road
Manchester, NH 03106
`seidmaro@nhc.edu`
`http://www.epicent.com/journals/journals/j_ed_comp_research.html`

> "Articles of value and interest to the educator, researcher, scientist. Designed to convey the latest in research reports and critical analyses to both theorists and practitioners."

($*$) **Journal of Online Mathematics and its Applications**
David A. Smith, Editor-in-Chief
`das@math.duke.edu`
`http://www.joma.org/about.html`

> Takes advantage of the World Wide Web as a publication medium for materials containing dynamic, full-color graphics; internal and external hyperlinks to related resources; applets in Java, Flash, Shockwave, or other languages; MathML, SVG, and other XML markups; audio and video clips; and other Web-based features.

Journal of Recreational Mathematics
Baywood Publishing Company, Inc.
26 Austin Ave.
P.O. Box 337
Amityville, NY 11701
`http://www.ashbacher.com/jrecmath.stm`

> Articles, book reviews, alphametics problem section, and problem and conjectures section.

Book Reviews
Charles Ashbacher
Charles Ashbacher Technologies
Box 294
119 Northwood Drive
Hiawatha, IA 52233

Problems and Solutions
Steven Kahan
41 St Quentin Drive
Sheffield
S17 4PN
U.K.

($*$) **Journal of Statistics Education**
W. Robert Stephenson, Editor
327 Snedecor Hall
Dept. of Statistics
Iowa State University
Ames, IA 50011–1210
`wrstephe@iastate.edu.`
`http://www.amstat.org/publications/jse/`

"The intended audience includes anyone who teaches statistics, as well as those interested in research on statistical and probabilistic reasoning.

"Possible topics for manuscripts include, but are not restricted to: curricular reform in statistics, the use of cooperative learning and projects, innovative methods of instruction, assessment, and research (including case studies) on students' understanding of probability and statistics, research on the teaching of statistics, attitudes and beliefs about statistics, creative and tested ideas (including experiments and demonstrations) for teaching probability and statistics topics, the use of computers and other media in teaching, statistical literacy, and distance education. Articles that provide a scholarly overview of the literature on a particular topic are also of interest. Reviews of software, books, and other teaching materials will also be considered, provided these reviews describe actual experiences using the materials.

"In addition, JSE also features departments called 'Teaching Bits: A Resource for Teachers of Statistics' and 'Datasets and Stories.' 'Teaching Bits' summarizes interesting current events and research that can be used as examples in the statistics classroom, as well as pertinent items from the education literature. The 'Datasets and Stories' department not only identifies interesting datasets and describes their useful pedagogical features, but enables instructors to download the datasets for further analysis or dissemination to students."

(∗) **The MAA Online Book Review**
Fernando Gouvêa, Editor
Department of Mathematics
Colby College
Waterville, ME 04901
`fqgouvea@colby.edu`
`http://www.maa.org/reviews/reviews.html`

Book reviews in the following categories:

- Books on mathematics intended for the "general public".
- Books intended for a general mathematical audience. For example, expository works on mathematical subjects, particularly if they are accessible to people who have an undergraduate background in mathematics.
- Books designed or usable as supplements to classroom instruction in mathematics. For example, this includes problem books and books with mathematics-related readings.
- Books on the history and philosophy of mathematics, if they are of broad appeal. This includes books on the mathematical community, biographies of mathematicians and scientists whose work is closely related to mathematics, etc.

- Books on mathematics teaching, especially those focusing on undergraduate teaching. We are particularly interested in seeing more books in this area.
- Innovative textbooks, especially those covering topics not usually taught at the undergraduate level.
- Science books whose topics are closely related to mathematics, especially if they can help mathematics professors learn about new points of contact between mathematics and other disciplines.

Math Horizons
Art Benjamin, Co-Editor
Harvey Mudd College
1250 N. Dartmouth Ave
Claremont, CA 91711
`benjamin@hmc.edu, jquinn@oxy.edu`
`http://www.maa.org/mathhorizons/`

Intended primarily for undergraduates interested in mathematics. "Our purpose is to introduce students to the world of mathematics outside the classroom. Thus, while we especially value and desire to publish high quality exposition of beautiful mathematics we also wish to publish lively articles about the culture of mathematics. We interpret this quite broadly–we welcome stories of mathematical people, the history of an idea or circle of ideas, applications, fiction, folklore, traditions, institutions, humor, puzzles, games, book reviews, student math club activities, and career opportunities and advice."

Mathematical Gazette
Gerry Leversha, Editor
The Mathematical Association
259 London Road
Leicester, LE2 3BE
U.K.
`gazette@m-a.org.uk`
`http://www.ma.org.uk/resources/periodicals/the_mathematical_gazette/`

Articles about the teaching and learning of mathematics, with a focus on the 15–20 age range, and expositions of attractive areas of mathematics. Regular sections include letters, extensive book reviews, and a problem corner.

Mathematical Scientist
Executive Editor
School of Mathematics and Statistics
University of Sheffield
Sheffield S3 7RH
U.K.
`http://www.shef.ac.uk/uni/companies/apt/tms.html`

- Research papers of general interest in the mathematical sciences, particularly those where the use of mathematical theory, methods and models provides insight into phenomena studied in the engineering, physical, biological and social sciences.
- Review papers, and historical surveys.
- Expository papers on any branch of mathematics which is of general interest.
- Reports on applications of the mathematical sciences to problems in real life.
- Abstracts and proceedings of appropriate conferences.
- Unsolved problems, letters to the editor and readers' comments on any branch of mathematics of general interest.

Mathematical Spectrum
David W. Sharpe, Editor
Hicks Building
University of Sheffield
Sheffield S3 7RH
U.K.
`http://www.appliedprobability.org/ms.html`

Articles from all branches of mathematics, as well as regular features on mathematics in the classroom, a computer column, letters, problems and solutions, book and software reviews.

Mathematics and Computer Education
George M. Miller, Editor-in-Chief
P.O. Box 158
Old Bethpage, NY 11804
`http://www.macejournal.org/index.html`

- Critical evaluation and dissemination of articles.
- Development of materials for the improvement of classroom effectiveness in the first years of college.
- Encouragement of high academic standards.

Mathematics & Informatics Quarterly
Jordan Tabov
Bulgarian Academy of Sciences
Institute of Mathematics with Computer Center
bl. 8, Acad. G. Bontchev Str.
1113 SOFIA
Bulgaria
`banmath@bgearn.bitnet`
`http://olympiads.win.tue.nl/ioi/misc/miq.html`

> Articles, notes, problems and solutions in school mathematics and informatics.

Mathematics Magazine
Allen J. Schwenk, Editor
Western Michigan University
Department of Mathematics
Kalamazoo, MI 49008-3899
`http://www.maa.org/pubs/mathmag.html`

> "Articles submitted to the *Magazine* should be written in a clear and lively expository style. The *Magazine* is not a research journal; papers in a terse "theorem-proof" style are unsuitable for publication. The best contributions provide a context for the mathematics they deliver, with examples, applications, illustrations, and historical background. We especially welcome papers with historical content, and ones that draw connections among various branches of the mathematical sciences, or connect mathematics to other disciplines."

Mathematics Today
(formerly **Bulletin of the Institute of Mathematics and Its Applications**)
Gayna Leggott, Editorial Officer
The Institute of Mathematics and its Applications
Catherine Richards House
16 Nelson Street
Southend-on-Sea
Essex SS1 1EF
U.K.
`http://www.ima.org.uk/institute/mathstoday.htm`

> "*Mathematics Today* is a general interest mathematics publication aimed primarily at Institute members. It contains articles, reviews, reports and other news on developments in mathematics and its applications. Authors are encouraged to discuss proposed articles with Gayna Leggott (`gayna.leggott@ima.org.uk`) before submission."

The Missouri Journal of Mathematical Sciences
Shing So, Coordinating Editor
Department of Mathematics and Computer Science
Central Missouri State University
Warrensburg, MO 64093
`so@cmsu1.cmsu.edu`
`http://www.math-cs.cmsu.edu/~mjms/mjms.html`

- "Commentaries on issues pertaining to mathematics/computer science or the teaching/learning of mathematics/computer science.
- Articles concerning the teaching/learning of mathematics or computer science.
- Research or survey articles in any of the mathematical sciences.
- Interesting mathematical problems and solutions."

The Pentagon
Steve Nimmo, Editor
Morningside College
1501 Morningside Ave.
Sioux City, IA 51106
`sdn001@alpha.morningside.edu`
`http://www.kme.eku.edu/pentagon.html`

Articles of interest to undergraduate mathematics students are included, assisting the Society in achieving its objectives.

Philosophia Mathematica
R.S.D. Thomas, Editor
Department of Mathematics
The University of Manitoba
Winnipeg, Manitoba
Canada R3T 2N2
`thomas@cc.umanitoba.ca`
`http://www.umanitoba.ca/pm/`

Work in the philosophy of pure and applied mathematics including computing.

Pi Mu Epsilon Journal
Brigitte Servatius, Editor
Department of Mathematics
Worcester Polytechnic Institute
Worcester, MA 01609
`bservat@wpi.edu`
`http://www.pme-math.org/journal/overview.html`

Research or expository papers by undergraduates, with occasional articles by faculty members.

PRIMUS
"Problems, Resources and Issues in Mathematics Undergraduate Studies"
Brian J. Winkel, Editor
Department of Mathematical Sciences
United States Military Academy
West Point, NY 10996
`ab3646@usma2.usma.edu`
`http://www.dean.usma.edu/math/pubs/primus/`

A forum for the exchange of ideas in mathematics education at the college level.

(∗) **Rose-Hulman Institute of Technology Undergraduate Math Journal**
Editor, *Rose-Hulman Undergraduate Mathematics Journal*
Rose-Hulman
Terre Haute, IN 47803
`mathjournal@rose-hulman.edu`

"Devoted entirely to papers written by undergraduates on topics related to mathematics; the work must have been completed before graduation. Although the paper need not contain original research in mathematics, it must be interesting, well-written, and at a level that is clearly beyond a typical homework assignment. Readers of the journal should expect to see new results, new and interesting proofs of old results, historical developments of a theorem or area of mathematics, relationships between areas of mathematics and/or other fields of study, or interesting applications of mathematics."

SIAM Review
Margaret H. Wright, Editor-in-Chief
Computer Science Department
Courant Institute of Mathematical Sciences
New York University
New York, NY 10012
`mhw@cs.nyu.edu`
`http://www.siam.org/journals/sirev/sirev.htm`

Consists of five sections, all containing articles of broad interest:

- *Survey and Review* features papers with a deliberately integrative and up-to-date perspective on a major topic in applied or computational mathematics or scientific computing.
- *Problems and Techniques* contains focused, specialized papers about interesting problems, techniques, and tools, including descriptions of mathematical formulations, solution methods, and open questions.

- *SIGEST* highlights a recent paper from one of SIAM's nine specialized research journals, chosen on the basis of exceptional interest to the entire SIAM community and revised and condensed as needed for greater accessibility.
- *Education* consists primarily of modules that are self-contained presentations of specific topics in applied mathematics, scientific computation, or their applications; each module provides the primary material needed to teach a given topic as well as supplementary material. Editor: Bobby Schnabel, `bobby@cs.colorado.edu` .
- The *Book Reviews* section contains a featured review that provides an overview of several books in a subject area. Shorter reviews of individual books are also included.

Significance
Helen Joyce, Editor
Royal Statistical Society
12 Errol Street
London
EC1Y 8LX
U.K.
`significance@rss.org.uk`
`http://www.blackwellpublishing.com/journal.asp?ref=1740-9705`

> "New quarterly magazine for anyone interested in statistics and the analysis and interpretation of data. Its aim is to communicate and demonstrate in an entertaining and thought-provoking way the practical use of statistics in all walks of life and to show how statistics benefit society. Articles will be largely non-technical and hence accessible and appealing, not only to members of the profession, but also to all users of statistics. As well as promoting the discipline and covering topics of professional relevance, *Significance* will contain a mixture of statistics in the news, case-studies, reviews of existing and newly developing areas of statistics, the application of techniques in practice and problem solving, all with an international flavour."

Stats: The Magazine for Students of Statistics
Allan Rossman and Beth Chance
Department of Statistics
Cal Poly State University
San Luis Obispo, CA 93407
`arossman@calpoly.edu, bchance@calpoly.edu`
`http://www.amstat.org/publications/stats/`

> Contributions of statisticians to important and interesting problems. This generally includes presenting a scientific problem and the nature of interaction between the statistician and others working on the problem.

Symmetry: Art and Science
(continuation of *Symmetry: Culture and Science*)
Denes Nagy
Institute for the Advancement of Research
Australian Catholic University
Locked Bag 4115
Fitzroy, Victoria 3065
Australia
`d.nagy@patrick.acu.edu.au`
or
George Lugosi
ISIS-Symmetry Melbourne Centre (Symmetrion)
2 Union Street
Kew 3101
Victoria
Australia
`g.lugosi@hfi.unimelb.edu.au`
`http://www.mi.sanu.ac.yu/vismath/isis6.htm`

> "Symmetry, or the lack of symmetry, fulfils an important methodological function in modern art and science. Inspired by various cultural traditions, from Europe to Africa and from the Far-East to America, symmetry can bridge different branches of science and art, as well as different human cultures, and thus avoid overspecialization and some related problems. This process, matured by the end of the 1980s, became the starting point of a remarkable intellectual movement."

Teaching Statistics
Gerald Goodall, Editor
Royal Statistical Society
12 Errol St.
London EC1Y 8LX
U.K.
`Gerald.Goodall@brunel.ac.uk`
`http://science.ntu.ac.uk/rsscse/TS/`

> "Aimed at teachers and students aged up to age 19 who use statistics in their work. The emphasis is on teaching the subject and addressing problems which arise in the classroom. The journal seeks to support not only specialist statistics teachers but also those in other disciplines, such as economics, biology and geography, who make widespread use of statistics in their teaching. *Teaching Statistics* seeks to inform, enlighten, stimulate, correct, entertain and encourage. Contributions should be light and readable. Formal mathematics should be kept to a minimum."

The UMAP Journal
"Undergraduate Mathematics Applications Project"
Paul J. Campbell, Editor
Campus Box 194
Beloit College
700 College St.
Beloit, WI 53511–5595
`campbell@beloit.edu`
`http://cs.beloit.edu/campbell/umap`

Articles on mathematical modeling and applications of mathematics at the undergraduate level. The editor also welcomes expository articles for the *On Jargon* column, reviews of books and other materials, and guest editorials on new ideas in mathematics education or on interaction between mathematics and application fields.

Undergraduate Mathematics Journal
Roger Lautzenheiser, Editor-in-Chief
Rose-Hulman Institute of Technology
Terre Haute, IN 47803
`mathjournal@rose-hulman.edu`
`http://www.rose-hulman.edu/mathjournal/`

Papers written by undergraduates on topics related to mathematics.

(∗) **Visual Mathematics**
Slavik Jablan, Co-editor
Knez Mihailova 35, P.O. Box 367
YU–11001 Belgrade
Yugoslavia (Serbia)
`jablans@mi.sanu.ac.yu`
`http://www.mi.sanu.ac.yu/vismath/vm.htm`

"A forum for the dialogue between artists and scientist. *VM* publishes original works in the following sense:

- mathematical research papers with new results and some attractive illustrations,
- artistic papers with new pieces of visual information and some mathematical links,
- mathematical-educational papers with new methods or approaches,
- mathematical-historic papers with new facts or new interpretations,
- survey papers with new approaches.

The main goal of *VM* is to show the beauty of mathematics in a broad artistic-scientific context. As a secondary aim, *VM* tries to correct the negative tendency that led to the unpopularity of mathematics in school and the lack of public understanding of this field. ”

This online journal supplements the printed journal *Symmetry: Art and Science* (see above). Some papers are published in both electronic form and in print.

About the Authors

Paul Campbell graduated summa cum laude from the University of Dayton and received an M.S. in algebra and a Ph.D. in mathematical logic from Cornell University. He has been at Beloit College since 1977, where he served as Director of Academic Computing from 1987 to 1990. He is Reviews Editor for *Mathematics Magazine*. He has been editor of *The UMAP Journal* since 1984.

Kunio Mitsuma was born in Tokyo, Japan. After finishing college as a mathematics major, he moved to the U.S. for his master’s (West Virginia University) and Ph.D. (Pennsylvania State University) degrees in mathematics. He is currently on the mathematics faculty at Kutztown University of Pennsylvania.

Modeling Forum

Results of the 2005 Interdisciplinary Contest in Modeling

Chris Arney, ICM Co-Director
Division Chief, Mathematical Sciences Division
Program Manager, Cooperative Systems
Army Research Office
PO Box 12211
Research Triangle Park, NC 27709–2211
`David.Arney1@arl.army.mil`

Introduction

A total of 164 teams of undergraduates, from 88 institutions in 4 countries, spent a weekend in February working on an applied mathematics problem in the 7th Interdisciplinary Contest in Modeling (ICM).

This year's contest began at 8:00 P.M. (EST) on Thursday, Feb. 3, and ended at 8:00 P.M. (EST) on Monday, Feb. 7. During that time, the teams of up to three undergraduates or high-school students researched, modeled, analyzed, solved, wrote, and submitted their solutions to an open-ended complex interdisciplinary modeling problem involving the depletion of a nonrenewable or exhaustible resource. After the weekend of challenging and productive work, the solution papers were sent to COMAP for judging. Three of the top papers, which were judged to be Outstanding by the panel of judges, appear in this issue of *The UMAP Journal*. Results and winning papers from the first six contests were published in special issues in 1999 through 2004.

COMAP's Mathematical Contest in Modeling and Interdisciplinary Contest in Modeling are unique among modeling competitions in that they are the only international contests in which students work in teams to find a solution. Centering its educational philosophy on mathematical modeling, COMAP supports the use of mathematical tools to explore real-world problems. It serves society by developing students as problem solvers in order to become better informed—and prepared—citizens, consumers, workers, and leaders.

This year's nonrenewable resource problem was particularly challenging. It required teams to select a nonrenewable or exhaustible resource and model its depletion over time. The problem contained economic, demographic, political, environmental, security, and technological issues to be analyzed, along with several challenging requirements needing scientific and mathematical analysis. The problem also included the ever-present requirements of the ICM to use thorough data analysis, creativity, approximation, precision, and effective communication. The authors of the problem were Paul Campbell, editor of *The UMAP Journal* and Professor of Mathematics and Computer Science at Beloit College, and geoscientist and civil engineer Ted Hromadka, who served on the panel of final judges. The problem originated from their research and interest in resource management. Commentary from both Dr. Campbell and Dr. Hromadka appear in this issue of *The UMAP Journal*.

All 164 of the competing teams are to be congratulated for their excellent work and dedication to scientific modeling and problem solving. This year's judges remarked that the quality of the papers was high and the modeling very robust. The 2005 ICM was managed by COMAP via its information system connected to the World Wide Web, where teams registered, obtained contest materials, and downloaded the problem at the appropriate time through COMAP'S ICM Website.

Start-up funding for the ICM was provided by a grant from the National Science Foundation (through Project INTERMATH) and COMAP. Additional support is provided by the Institute for Operations Research and the Management Sciences (INFORMS).

The Exhaustible Resource Problem

Select a vital nonrenewable or exhaustible resource (water resources, mineral, energy source, food source, etc.) for which your team can find appropriate worldwide historic data on its endowment, discovery, annual consumption, and price.

The modeling tasks are to:

1. Using the endowment, discovery, and consumption data, model the depletion or degradation of the commodity over a long horizon using resource modeling principles.

2. Adjust the model to account for future economic, demographic, political and environmental factors. Be sure to reveal the details of your model, provide visualizations of the models output, and explain the limitations of the model.

3. Create a fair, practical "harvesting/management" policy, that may include economic incentives or disincentives, which sustains the usage over a long

period of time while avoiding severe disruption of consumption, degradation or rapid exhaustion of the resource.

4. Develop a "security" policy that protects the resource against theft, misuse, disruption, and unnecessary degradation or destruction of the resource(s). Other issues that may need to be addressed are political and security management alternatives associated with these policies.

5. Develop policies to control any short or long-term "environmental effects" of the harvesting. Be sure to include issues such as pollutants, increased susceptibility to natural disasters, waste handling and storage, and other factors you deem appropriate.

6. Compare this resource(s) with any other alternatives for its purpose. What new science or technologies could be developed to mitigate the use and potential exhaustion of this resource? Develop a research policy to advance these new areas.

The Results

Solution papers were coded at COMAP headquarters so that names and affiliations of authors were unknown to the judges. Each paper was then read preliminarily by at least two "triage" judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary, the model description, and overall organization are the primary elements in judging a paper. If the judges' scores diverged for a paper, the judges conferred; if they still did not agree on a score, additional triage judges evaluated the paper.

Final judging by a team of modelers, analysts, and subject-matter experts took place on March 4 and 5, again at West Point, NY. The judges classified the papers as follows:

			Honorable	Successful	
	Outstanding	Meritorious	Mention	Participation	Total
IT Security	3	26	89	46	164

The three papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with commentaries by the author and a final judge. We list those teams and the Meritorious teams (and advisors) below; the list of all participating schools, advisors, and results is in the **Appendix**.

Outstanding Teams

Institution and Advisor	Team Members
"The Coming Oil Crisis" East China University of Science and Technology Shanghai City, China Ni Zhongxin	Wei Deling Chen Jie Xu Hui
"Preventing the Hydrocalypse: A Model for Predicting and Managing Worldwide Water Resources" Franklin W. Olin College of Engineering Needham, MA (INFORMS Prize winner) Burt Tilley	Steven Krumholz Frances Haugen Daniel Lindquist
"The Petroleum Armageddon" Maggie Walker Governor's School Richmond, VA John Barnes	Jonathan Giuffrida Palmer Mebane Daniel Lacker

Meritorious Teams (26 teams)

Beijing Language and Cultural University, China (Rou Song)
Central University of Finance and Economics, China (Weihong Yu)
Dalian Maritime University, China (Guoyan Chen)
Dalian Nationalities University, China (Xiangdong Liu)
Dalian University of Technology, China (Mingfeng He)
East China University of Science & Technology, China (Lu Yuanhong)
Jinan University, China (Daiqiang Hu)
Maggie Walker Governor's School, Richmond, VA (John Barnes)
Nanjing University of Posts & Telecommunications, China (LiWei Xu)
National University of Defense Technology, China (Mengda Wu)
Ningbo Institute of Technology, China (Jufeng Wang)
Päivölä College, Finland (Esa Lappi)
Peking University, China (3 teams) (Yulong Liu: 2 teams; Huang Hai)
Shandong University, China (2 teams) (Jiahua Ma) (Baodong Liu)
Sun Yat-Sen University, China (Qi-Ru Wang)
United States Military Academy, West Point, NY (2 teams) (Bart Stewart) (Michael Smith)
University of Science & Technology of China, Hefei, China (Qiang Meng)
University of Virgina, Charlottesville, VA (Robert Hirosky)
Wuhan University, China (Zhong Liuyi)
Xidian University, China (Xiaogang QI)
Zhejiang University City College, China (2 teams) (Xusheng Kang) (Waibin Huang)

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Directors and by the Head Judge. Additional awards were presented to the Olin College of Engineering team advised by Burt Tilley from the Institute for Operations Research and the Management Sciences (INFORMS).

Judging

Contest Directors

Chris Arney, Mathematical Sciences Division, Army Research Office,
Research Triangle Park, NC

Gary W. Krahn, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Associate Director

Richard Cassady, Dept. of Industrial Engineering, University of Arkansas,
Fayetteville, AR

Judges

Laura Hromadka, Hromadka and Associates, Costa Mesa, CA

Theodore V. Hromadka, Hromadka and Associates, Costa Mesa, CA

V. Frederick Rickey, Dept. of Mathematical Sciences, U.S. Military Academy
West Point, NY

Triage Judges

Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY:
Scott Billie, Mason Crow, David Ellison, Andrew Glen, Alex Heidenberg, John Jackson, Michael Johnson, Gary Krahn, Gary Lambert, Amy Lin, Keith McClung, Barbara Melendez, Fernando Miguel, Joe Myers, Mike Phillips, Jack Picciuto, Frederick Rickey, Tyge Rugenstein, Bart Stewart, Rodney Sturdivant, Frank Wattenberg, and Brian Winkel.

Materials Division, Army Research Laboratory, Aberdeen, MD: William de Rosset.

Source of the Problem

The Exhaustible Resources Problem was contributed by Paul J. Campbell (Dept. of Mathematics and Computer Science, Beloit College, WI) and Ted Hromakda (Hromadka and Associates, Costa Mesa, CA).

Acknowledgments

We thank:

- the Institute for Operations Research and the Management Sciences (INFORMS) for its support in judging and providing prizes for the winning team;
- all the ICM judges and ICM Board members for their valuable and unflagging efforts;
- the staff of the Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY, for hosting the triage and final judgings.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the student papers here is the result of undergraduates working on a problem over a weekend; allowing substantial revision by the authors could give a false impression of accomplishment. So these papers are essentially au naturel. Light editing has taken place: minor errors have been corrected, wording has been altered for clarity or economy, style has been adjusted to that of *The UMAP Journal*, and the papers have been edited for length. Please peruse these student efforts in that context.

To the potential ICM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Editor's Note

As usual, the Outstanding papers were longer than we can accommodate in the *Journal*, so space considerations forced me to edit them for length. It was not possible to include all of the many tables and figures.

In editing, I endeavored to preserve the substance and style of the paper, especially the approach to the modeling.

—Paul J. Campbell, Editor

Appendix: Successful Participants

KEY:
P = Successful Participation
H = Honorable Mention
M = Meritorious
O = Outstanding (published in this special issue)

INSTITUTION	CITY	ADVISOR	I
CALIFORNIA			
California State Univ., Monterey Bay	Seaside	Jeffrey Groah	P
		Hongde Hu	H
Harvey Mudd College	Claremont	Hank Krieger	H
COLORADO			
Regis University	Denver	Jim Seibert	H
University of Colorado	Colorado Springs	Radu Cascaval	H
	Denver	Lynn Bennethum	H
INDIANA			
Earlham College	Richmond	Charlien Peck	H
IOWA			
Simpson College	Indianola	James Bohy	H
KENTUCKY			
Asbury College	Wilmore	David Coulliette	H
		Kenneth Rietz	H
		Duk Lee	H
Thomas More College	Crestview Hills	Steven Lameier	P
MARYLAND			
Villa Julie College	Stevenson	Eileen McGraw	P
MASSACHUSETTS			
Olin College of Engineering	Needham	Burt Tilley	O
MONTANA			
Carroll College	Helena	Mark Parker	H,P
NEW YORK			
United States Military Academy	West Point	Bart Stewart	M
		Michael Smith	M
NORTH CAROLINA			
Duke University	Durham	David Kraines	H
OHIO			
Youngstown State University	Youngstown	George Yates	H,H
		Scott Martin	P

INSTITUTION	CITY	ADVISOR	I
PENNSYLVANIA			
Clarion University of Pennsylvania	Clarion	John Heard	P
		Curt Foltz	P
SOUTH CAROLINA			
Midlands Technical College	West Columbia	Richard Bailey	P
VIRGINIA			
Maggie Walker Governor's School	Richmond	John Barnes	O,M
		Martha Hicks	H
James Madison University	Harrisonburg	David Walton	H
University of Virginia	Charlottesville	Robert Hirosky	M
WASHINGTON			
University of Washington	Seattle	Sara Billey	H
		Sandor Kovacs	H,H
CHINA			
Anhui			
Anhui University	Hefei	Wang Jian	P
Hefei University of Technology	Hefei	Bao Chaowei	H
		Liang Weizhong	H
		Gong Kun	H
University of Science and Technology of China	Hefei	Cheng Yezeng	H
		Meng Qiang	M
		Wang Huiwen	H
Beijing			
Beihang University	Beijing	Wu Sanxing	P
Beijing Institute of Technology	Beijing	Li Bingzhao	H,P
		Chen Yihong	H
Beijing Jiaotong University, School of Science	Beijing	Wang Xiaoxia	P
		Feng Guochen	P
		Ren Liwei	H
Beijing Language and Culture University	Beijing	Song Rou	M
Beijing University of Chemical Technology	Beijing	Liu Hui	H
		Huang Jinyang	H
		Cheng Yan	P
Beijing Univ. of Posts and Telecommunications	Beijing	Ding Jinkou	H
		Sun Hongxiang	H
		Wu Yunfeng	H
		Zhang Wenbo	H
Beijing University of Technology	Beijing	Xue Yi	H,P
		Chang Yu	H
Central University of Finance and Economics	Beijing	Yu Weihong	M

INSTITUTION	CITY	ADVISOR	I
Peking University	Beijing	Wang Ming	H
		Liu Yulong	M,M
		Huang Hai	M
(Earth and Space Science)		Liu Chuxiong	H
(Health Science Center)		Shu Xue	H,H
Tsinghua University	Beijing	Ye Jun	H
		Xie Jinxing	P
(School of Science)		Huang Hongxuan	P
Chongqing			
Chongqing University	Chongqing	Li Zhiliang	H
		Gong Qu	H
Guangdong			
Jinan University	Guangzhou	Hu Daiqiang	M
		Fan Suohai	P
		Zhang Chuanlin	P
South-China Normal University	Guangzhou	Wang Henggeng	P
South China University of Technology	Guangzhou	Tao Zhi Sui	P
		Pan Shao Hua	P
		Liang Man Fa	H
Sun Yat-Sen University	Guangzhou	Wang Qi-Ru	M
		Bao Yun	P
		Li Cai Wei	H
Heilongjiang			
Harbin Engineering University	Harbin	Gao ZhenBin	H,P
		Zhang XiaoWei	H
		Yu Fei	H
		Yu Tao	P
		Shen JiHong	H
Harbin Institute of Technology	Harbin	Zhang Yunfei	H,P
Harbin University of Science and Technology	Harbin	Wang Shuzhong	H
		Chen Dongyan	P
		Li Dongmei	H
Northeast Agricultural University	Harbin	Tang Yanya	H
Hubei			
Wuhan University	Wuhan	Zhong Liuyi	M
		Liu Dichen	H
Hunan			
Central South University	Changsha	Hou Muzhou	H,H
Hunan University	Changsha	Li Xiaopei	H
National University of Defense Technology	Changsha	Wu Mengda	M,H

INSTITUTION	CITY	ADVISOR	I
Jiangsu			
China University of Mining and Technology	Xuzhou	Zhou Shengwu	H
Nanjing University of Post and Telecommunication	Nanjing	Xu LiWei	M,H
Nanjing University of Science & Tech.	Nanjing	Zhang Haifei	H
		Huang Zhenyou	P
Southeast University	Nanjing	Chen Enshui	H
		Wang Feng	H,P
Jilin			
Jilin University	Changchun	Huang Qingdao	P
		Ji Youqing	P
		Cao Chunling	H
Liaoning			
Dalian Maritime University	Dalian	Chen Guoyann	M
Dalian Nationalities University	Dalian	Liu Xiangdong	M
(Economics and Management)		Zhang Hengbo	P
		Ge Rendong	P
Dalian University (Information and Engineering)	Dalian	Gang Jiatai	H
(Physics)		Wang Yanchun	H
Dalian University of Technology	Dalian	He Mingfeng	M
		Wang Yi	H
(Institute of University Students' Innovation)		He Mingfeng	H,H
		Pan Qiuhui	P
(School of Software)		Yu Changliang	H,P
Shaanxi			
North University of China	Taiyuan	Le Yingjie	H
Northwestern Polytechnical University	Xi'an	Lv Quanyi	H
Xi'an Jiaotong University	Xi'an	Zhou Yicang	P
		Dai Yonghong	H
Xidian University	Xi'an	Zhou Shuisheng	H
		Qi Xiaogang	M
		Feng Hailin	H
Shandong			
Shandong University	Jinan	Liu Baodong	M
		Ma Jianhua	M
Shanghai			
Donghua University	Shanghai	Ma Biao	H
		Ma Yu-fang	H
East China University of Science and Technology	Shanghai	Lu Yuanhong	M
		Ni Zhongxin	O
Fudan University	Shanghai	Cao Yuan	H
		Cai Zhijie	H
Shanghai Jiao Tong University	Shanghai	Song Baorui	H

INSTITUTION	CITY	ADVISOR	I
Sichuan			
Univ. of Electronic Science and Tech. of China	Chengdu	Zhang Yong	H
		Du Hongfei	H,H
Tianjin			
Tianjin University	Tianjin	Lin Dan	H
		Song Zhanjie	H
Zhejiang			
Academy of Science	Hangzhou	Shi Guosheng	P
Zhejiang Gongshang University	Hangzhou	Zhu Ling	P,P
		Zhao Heng	P
Zhejiang University (Applied Mathematics)	Hangzhou	Yang Qifan	P
(Mathematics)		Tan Zhiyi	P
(College of Science)		Jiang Yiwei	H
(City College)	Hangzhou	Kang Xusheng	M
		Wang Gui	H
		Huang Waibin	M
(Chu Kechen Honors College)	Hangzhou	Wu Jian	H
(Ningbo Institute of Technology)	Ningbo	Li Zhening	P
		Wang Jufeng	M,P
FINLAND			
Päivölä College	Tarttila	Jukka Ilmonen	P
		Esa Lappi	M,H
INDONESIA			
Institut Teknologi Bandung	Bandung	Edy Soewono	P
		Agus Gunawan	H

Editor's Note

Unless otherwise specified, the sponsoring department is the Dept. of Mathematics, Applied Mathematics, Mathematical Sciences, or Mathematics and Computer Science.

For team advisors from China, we have endeavored to list family name first.

The Coming Oil Crisis

Wei Deling
Chen Jie
Xu Hui
East China University of Science and Technology
Shanghai City, China

Advisor: Ni Zhongxin

Summary

We model depletion of oil, a typical vital nonrenewable resource. Based on the theory of supply and demand, we establish a differential equation system that includes demand, supply, and price, and derive explicit formulas for the three variables. We modify the model to reflect exponetially increasing worldwide oil demand.

We fit the modified model to worldwide oil demand data 1970–2003. We conclude that all oil will be used up in 2032 without countermeasures. We then take economic, demographic, political, and environmental factors into account.

To meet the needs of people today without compromising those of future generations, we establish a criterion of rational oil allocation between generations and model optimal oil allocation under this criterion, with an illustration.

We provide a strategy for oil exploitation to reduce the possibility of disasters in the short term.

Finally, according to marginal utility replacement rules, we study the trade-off between oil and its alternatives. Since our model is based on demand-supply theory and the intrinsic law of nonrenewable resources, it can be applied to general nonrenewable resources.

Task 1: Modeling the Depletion of Oil

Under the following assumptions, no restriction is made to protect oil, so it will be exhausted in the fastest way.

The UMAP Journal 26 (2) (2005) 127–146.

Assumptions

- Oil refining capacity is adequate.
- All undiscovered oil is available when necessary—as long as there is demand, there is supply, until all the oil on Earth is completely used up.

Notations

- $U(t)$: Oil undiscovered in year t.
- $R(t)$: Oil discovered but has not been used (reserves) in year t.
- $D(t)$: Worldwide oil demand in year t (in thousands of barrels per day (*bpd*))
- $S(t)$: Worldwide oil supply in year t.
- $P(t)$: Oil price in year t.
- P_0: Equilibrium price of oil.

Modeling

From the above definitions, $U(t) + R(t)$ is the total remaining oil on Earth in year t, and $\sum_{i=t}^{n} D(i)$ is the total demand from year t through year n.

To learn when the total remaining oil will be used up, we find n such that

$$\sum_{i=1}^{n} D(i) \leq U(t) + R(t) < \sum_{i=1}^{n+1} D(i); \qquad \textbf{(1)}$$

then oil will be depleted between year n and year $n+1$.

Data

- Estimated undiscovered oil worldwide in 1997 was 180 billion barrels, that is, $U(1997) = 180$ ($\times 10^9$ bbl) [Campbell 1997].
- Worldwide oil reserve in 1997 was 1,018.5 billion barrels, that is, $R(1997) = 1{,}018.5$ ($\times 10^9$ bbl) [Energy Information Administration 2004].
- The worldwide oil demand from 1980 to 2003, $D(i)$ ($i = 1980, \ldots, 2003$), is shown in **Table 1** [Energy Information Administration 2004].[1]

To predict future demand, we consider the following system of first-order linear ordinary differential equations that express "supply-demand" principles:

[1]EDITOR'S NOTE: Subsequent to the contest, the EIA revised the 2003 demand to 79,892 bpd and posted the 2004 demand as 82,631 bpd.

Table 1.
World-wide oil demand, 1970–2003 (thousands of barrels/day (bpd)).
Source: Energy Information Administration [2004].

1970	46,808	1980	63,108	1990	66,443	2000	76,954
1971	49,416	1981	60,944	1991	67,061	2001	78,105
1972	53,094	1982	59,543	1992	67,273	2002	78,439
1973	57,237	1983	58,779	1993	67,372	2003	79,813
1974	56,677	1984	59,822	1994	68,679		
1975	56,198	1985	60,087	1995	69,955		
1976	59,673	1986	61,825	1996	71,522		
1977	61,826	1987	63,104	1997	73,292		
1978	64,158	1988	64,963	1998	73,932		
1979	65,220	1989	66,092	1999	75,826		

$$\frac{dS}{dt} = a\tilde{P}, \tag{2}$$

$$\frac{d\tilde{P}}{dt} = -b(S - D), \tag{3}$$

$$\frac{dD}{dt} = -c\tilde{P}, \tag{4}$$

where $\tilde{P} = P(t) - P_0$ and a, b, c are positive constants.

Eq. **(2)** means that if the oil price is greater than its equilibrium price, the output will increase accordingly, and vice versa. Eq. **(3)** says that if oil supply exceeds demand, the price will decline. Eq. **(4)** indicates when the price goes up or down, demand of shrinks or expands accordingly.

After careful calculation, we get the solution of the system:

$$\tilde{P}(t) = k \sin(\omega t + \phi), \tag{5}$$

$$S(t) = S_0 - \frac{ak}{\omega}\cos(\omega t + \phi), \tag{6}$$

$$D(t) = D_0 + \frac{ck}{\omega}\cos(\omega t + \phi), \tag{7}$$

where

- $k = \sqrt{\tilde{c}_1^2 + \tilde{c}_2^2}$,
- $\phi = \arctan(\tilde{c}_1/\tilde{c}_2)$,
- $\omega = \sqrt{b(a+c)}$, and
- $S_0 = D_0$, $\tilde{c}_1$, and $\tilde{c}_2$ are parameters to be determined.

We are particularly interested in **(7)**. It implies that oil demand is periodic. However, as time passes, the world population is expanding exponentially, and the demand of oil increases accordingly. Therefore, we modify **(7)** to reflect this intrinsic tendency to increase. We add to the right-hand side of **(7)** an exponential term $k_1 \exp\big(k_2(t - t_0)\big)$, where k_1, k_2, t_0 are constants), getting

$$D(t) = a_1 + a_2\cos(a_3 t + a_4) + a_5\exp(a_6 t). \tag{8}$$

Fitting **(8)** to the data in **Table 1**, we get the curve in **Figure 1**, for the function

$$D(t) = 31950 + 556.7\cos(1.605t - 3159.659) + 1.239 \times 10^{-16}\exp(0.02366t).$$

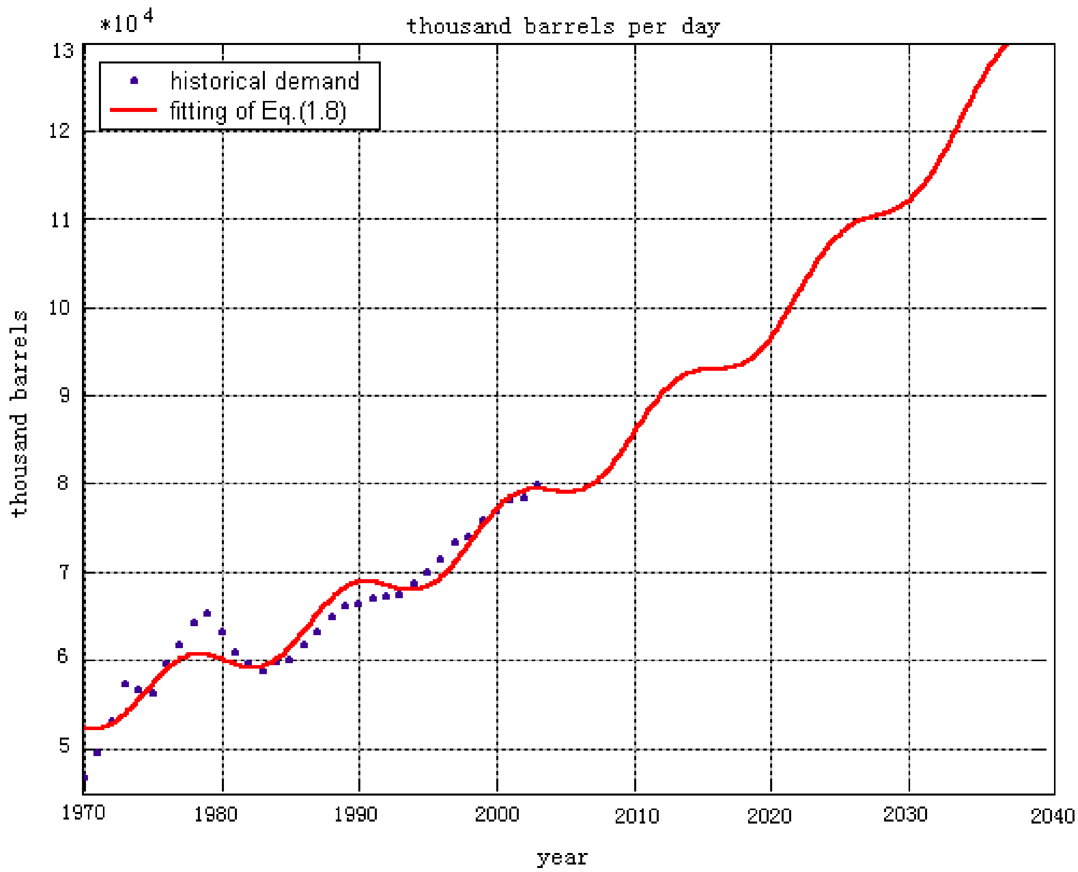


Figure 1. Data and fitted curve for oil demand per day; vertical scale is in 10^7 bpd.

With the passage of time, the third term $[a_5\exp(a_6t)]$ on the right-hand side of **(8)** will play a more important role and the second term $[a_2\cos(a_3t + a_4)]$ can be neglected. Thus, for the sake of convenience, we reduce **(8)** to

$$D(t) = a_1 + a_5\exp(a_6t).$$

For comparison, we also do linear fitting plus an unvarying-demand case in which future demand is the same as in 2003. Fitting to **Table 1** gives for $t \geq 2004$:

Exponential fit	$D(t) = 29820 + 2.265 \times 10^{-15}\exp(0.02223t)$
Linear fit	$D(t) = 771.2t - 1.467 \times 10^6$

The predicted demand is shown in **Figure 2** as average daily demand.

All oil on Earth will be used up by 2032 and 2033 according to the exponential and linear fits, and by 2037 if future demand remains at the level of 2003.

With an increase in oil demand, its price will accordingly rise. As shown by the broken lines in **Figure 2**, the rising price will lead to a decline in demand; we discuss this phenomenon in detail later.

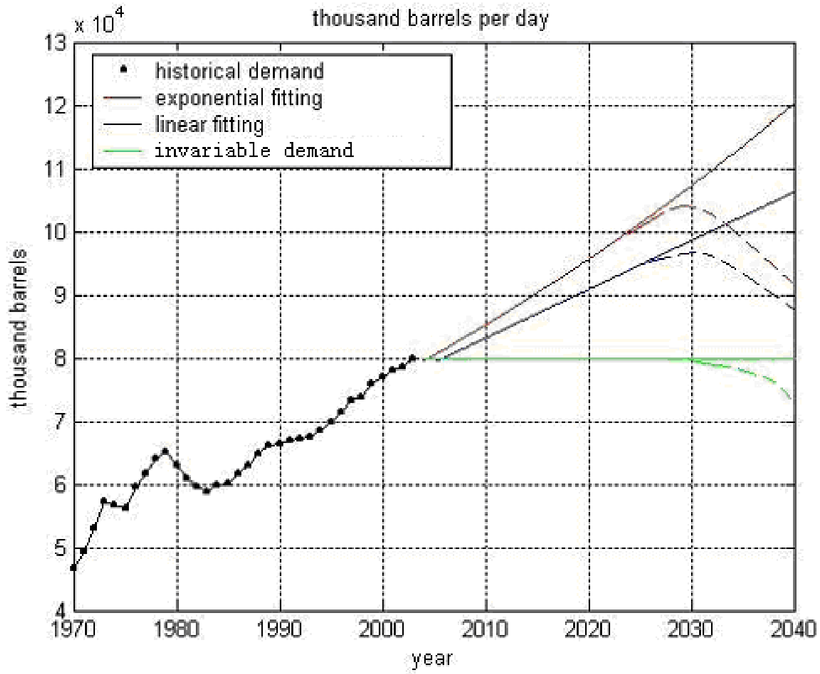


Figure 2. Estimated future oil demand, for several scenarios; vertical scale is in 10^7 bbl/d.

Sensitivity Analysis

It is difficult to get an accurate value for $U(t)$, undiscovered oil on Earth in year t. But we can estimate and vary the estimate to see whether the variation vastly changes n. Varying $U(1997)$ by $\pm 10\%$, for each demand model, the change in n is less than one year.

Task 2: Other Factors

We modify the exponential model to include other factors.

Assumptions

- Annual demand for oil reflects annual consumption.
- We do not take into account interactions between factors.
- We ignore small fluctuations in the future consumption of oil.

Economic Factors

We use GDP as the measure of economy. **Table 2** shows recent data for world total GDP and the corresponding oil consumption.

The correlation between world GDP and oil consumption is .9930, with linear regression equation

$$\text{consumption} = 1183 \times \text{GDP} + 38140. \qquad \textbf{(9)}$$

Table 2.
World total GDP ($\$10^8$) and world oil consumption (10^3 bpd), 1995–2003.

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003
GDP	27.134	28.247	29.433	30.257	31.377	32.85	33.64	34.6487	36
Oil	69955	71522	73292	73932	75826	76954	78105	78439	79813

Using **(9)**, we predict cumulative consumption. We take 2001 as the starting point, when total remaining oil (undiscovered plus known reserves) was $U(2001) + R(2001) = 1.1178 \times 10^{12}$ bbl. We calculate the time to oil exhaustion under different cases: GDP growing at 10%, 5%, 3%, and 1% (**Figure 3**).

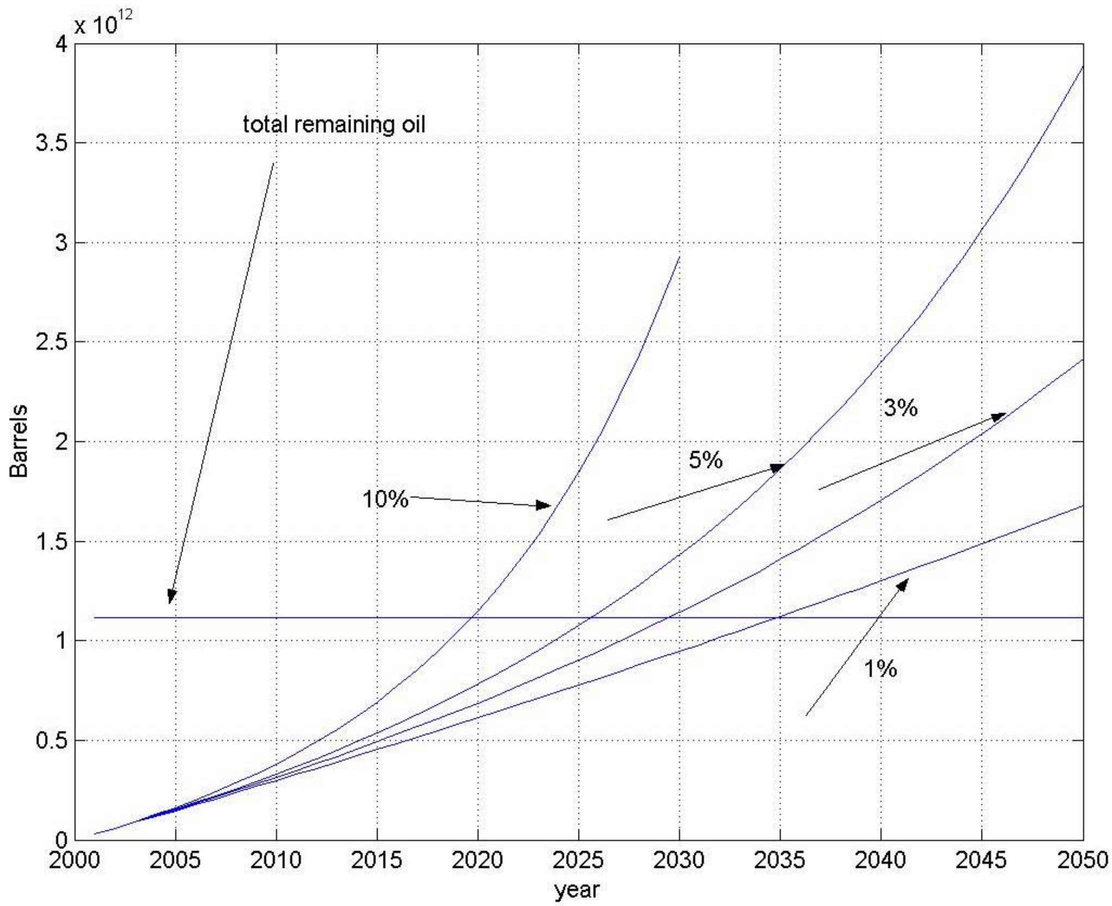


Figure 3. Cumulative oil consumption under some rates of GDP growth; vertical scale is in 10^{12} bbl.

The horizontal line denotes the total remaining oil in 2001. The x-axis coordinate of the intersection of the horizontal line and a curve denotes oil exhaustion time. The faster the GDP growth rate, the larger the oil consumption and the sooner the time of exhaustion. For 10%, oil will be depleted in 2020; for 5%, in 2026; for 3%, in 2029; and finally, for 1%, oil will be used up in 2035.

Demographic Influence

We resort to a logistic model to predict the world population $x(t)$:

$$x(t) = \frac{k}{1 + \left(\dfrac{k}{x_0} - 1\right) e^{-(t-t_0)r}},$$

where

- t is time, with initial time $t_0 = 1980$;
- $x(t)$ is the population, in billions of people, with $x_0 = x(1980) = 4.4585$;
- k is the environment capacity—the maximum population that the Earth can accommodate—in billions of people, and we take $k = 10$; and
- r is the intrinsic growth rate of the population, determined from data.

We use population data from 1980, 1990, and 2000 to fit the equation and get

$$x(t) = \frac{10}{1 + \left(\dfrac{10}{4.4585} - 1\right) e^{-0.0327(t-1980)}}, \qquad \textbf{(10)}$$

Using **(10)**, we predict the future population, as shown in **Table 3**.

Table 3.
World population estimated from the logistic model.

1980	1990	2000	2010	2020	2030	2040	2050	2060
4.4585	5.2736	6.0744	6.8212	7.4849	8.0495	8.5126	8.8811	9.1560

Similarly, we obtain a relationship between consumption and total population. The correlation coefficient is .9877, with linear regression

$$\text{consumption} = 1443 \times \text{population} - 11170. \qquad \textbf{(11)}$$

The time of oil exhaustion, based on the logistic growth of the population, is 2033.

Political Influence

Here, we mainly discuss the influence of wars.

$$D(t) = 9.14 \times 10^{-11} \times e^{0.01718t}.$$

The annual rate of growth of consumption is

$$r = \frac{D(t+1)}{D(t)} - 1 = e^{0.01718} - 1 = 1.73\%.$$

Figure 4 shows the annual growth rate for oil consumption during the past decades, plus a horizontal line at r. The growth rate declined sharply in 1974, 1980, and 1990, coinciding with the fourth Middle East War (1973), the Iran-Iraq War (1980), and the Gulf War (1990), all in the Middle East, the center of oil production. So wars strongly impact the price of oil, and consequently demand.

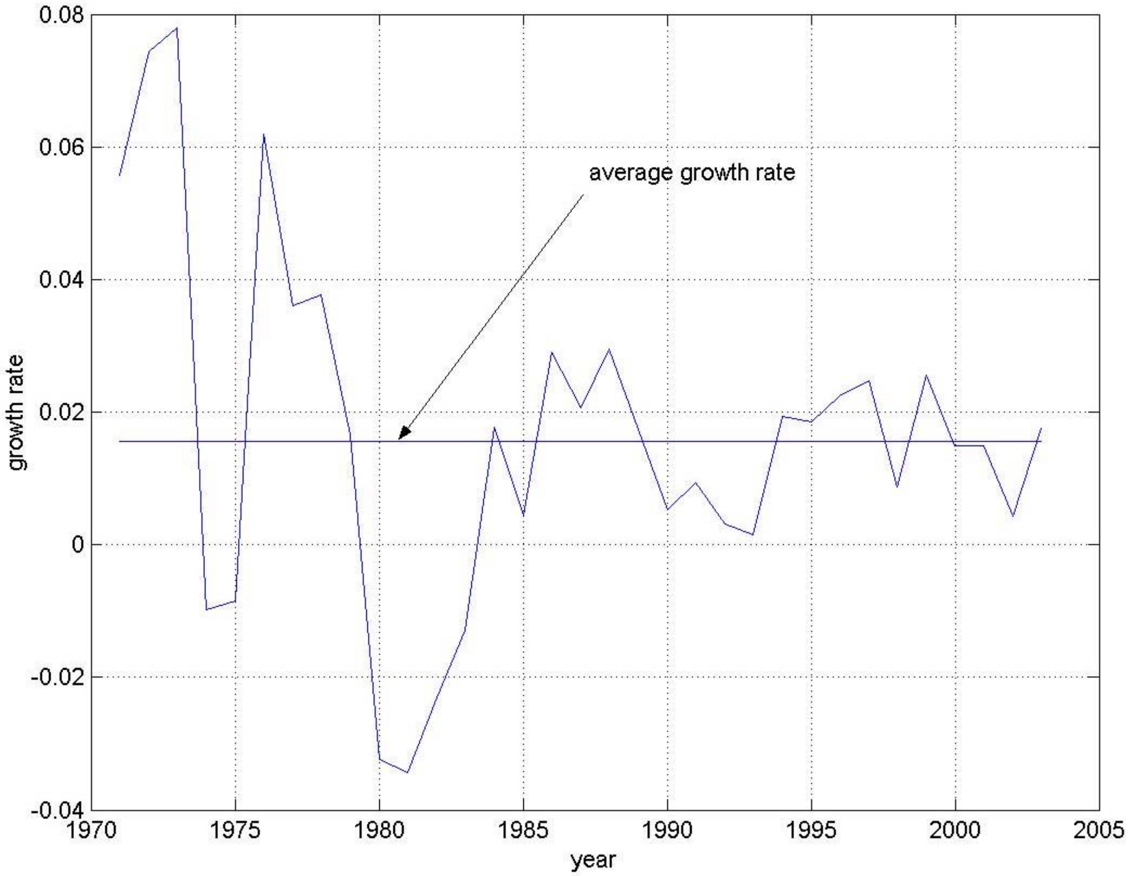


Figure 4. Historic growth of oil consumption.

Environmental Influence

The use of oil inevitably leads to environmental pollution. To protect the environment against excessive pollution, governments can adopt measures to limit the use of oil, thus curbing oil demand. We take the amount of carbon dioxide discharged by oil consumption as the scale to measure environment pollution; world data are shown in **Table 4**.

Table 4.

World carbon dioxide emissions from the consumption of oil (10^6 metric tons).

1993	1994	1995	1996	1997	1998	1999	2000	2001
9220	9284	9388	9586	9691	9766	9939	10138	10292

The correlation between oil consumption and carbon dioxide emission from oil consumption is 0.9937, with the regression

$$\text{consumption} = 10.09 \times (\text{CO}_2 \text{ from oil}) - 25320. \qquad \textbf{(12)}$$

Using **(12)**, we determine the amount of consumption to allow under different controlled annual emission growth rates. **Figure 5** shows the results and the corresponding dates for exhaustion of oil, for emission growth rates of 1% and 3%.

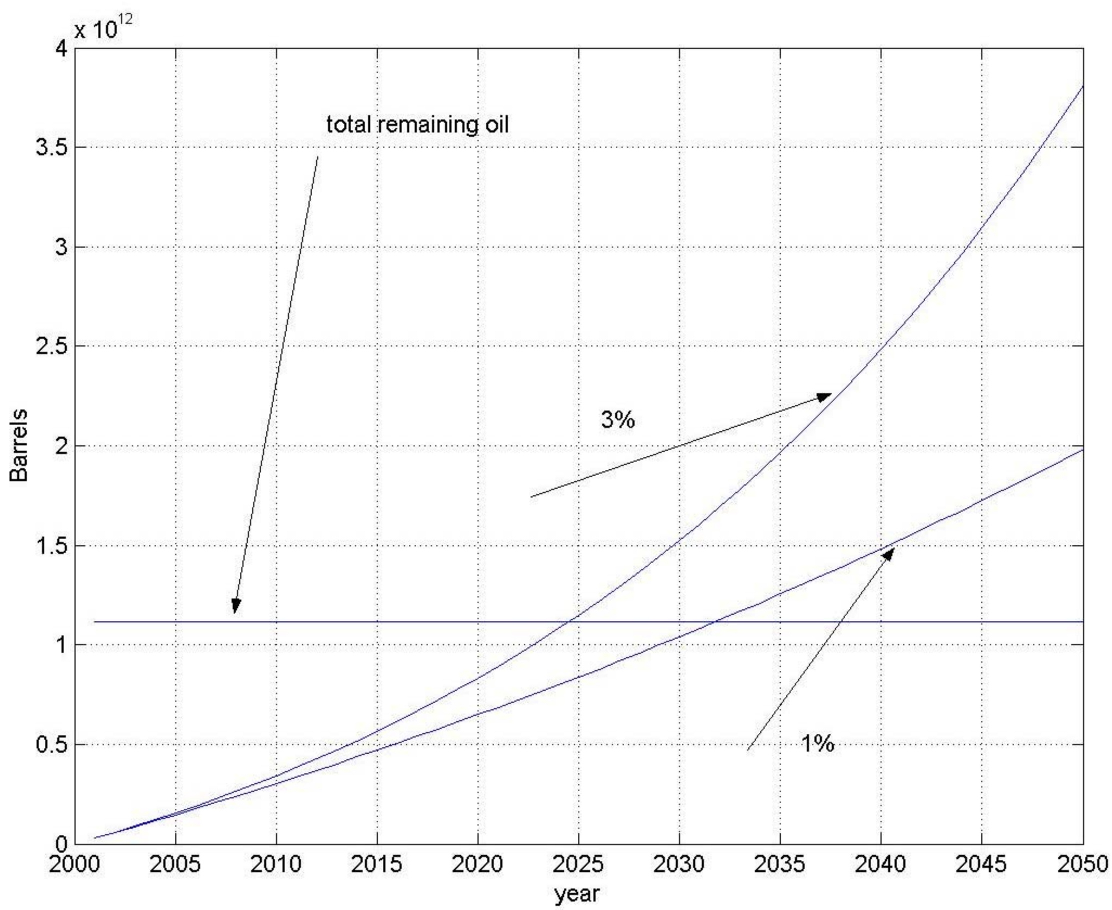


Figure 5. Cumulative oil consumption under various rates of CO_2 emission growth rates.

Limitations

The above models are based on the assumption that all the other factors are fixed when modeling for a specific factor. But this cannot be true in reality, because one factor may interact with others. Thus, interactions should be taken into account in further study.

Task 3: Sustainable Use

To prevent excessive consumption and rapid depletion, and to take into account our offspring's interests, we should allocate consumption rationally between generations.

Assumptions

- Annual demand for oil truly reflects oil consumption.
- Oil consumption in year t cannot be far less than that in year $t-1$.
- We must provide a rational consumption allocation between generations.
- A generation consists of n years.

Allocation of Oil Between Generations

The total remaining oil in year t is $U(t)+R(t)$, so

$$U(t)+R(t)=m_1+m_2,$$

where m_1 is the amount of oil for people today and m_2 is the amount of oil left for offspring. We define *the degree of rational consumption allocation for oil* as

$$\eta=\frac{m_1}{m_2}\times 100\% \qquad (0\le\eta\le\infty).$$

If the value of η is too high, the amount of oil for contemporary human beings is too small to meet their needs.

Modeling the Rational Consumption Allocation

We expect that future oil demand will not undergo a sharp decline, and we want oil to be allocated among generations fairly. Meanwhile, we want the resource to be used in the most efficient way.

We model an interval of n years, i.e., one generation. We have the following linear programming optimization problem:

$$\max\sum_{i=1}^{n} c_i d_i \tag{13}$$

such that

$$\begin{aligned}
\frac{r}{\sum_{i=1}^{n} d_i} &\ge \eta', \\
d_i &\ge \alpha d_{i-1}, \qquad i=1,2,\ldots,n, \\
d_i &\ge 0, \qquad\quad\;\; i=1,2,\ldots,n,
\end{aligned}$$

where

- c_i is the *utilization rate* of oil (crude oil available divided by refinery capacity) at year i;
- η' = degree of rational consumption allocation of oil between generations;
- d_i is oil consumption in year i (with d_0 oil consumption at the initial time);
- r is the total remaining oil in the first year of one generation;
- α is a set percentage such that the oil consumption in a given year must not be less than α times the consumption in the previous year, with α close to 1.

The objective is maximum utilization over n years. The first constraint assures a rate of allocation between generations, while the second assures that oil consumption in year i is not less than α times the consumption in year $i-1$. When η', c_i, r, α are given, we can obtain the optimal consumption allocation of oil over n years by solving the linear programming problem **(14)**. As for estimating c_i, we believe that the utilization rate should increase as time passes but should always be smaller than 1. Thus, we should have

$$c_i = 1 - a_1 e^{a_2 t}, \tag{14}$$

where a_1 and a_2 are constants determined by fitting historical data.

We give an illustration. We set $n = 20$, $\alpha = 1$, $\eta' = 1.67$, $r = 1.0 \times 10^{12}$, and year 2004 as the base year, so that d_0 = oil consumption in 2004. **Figure 6** shows oil consumption under optimal allocation from 2005 on. Oil consumption under optimal allocation is far less than under exponential growth. The optimal consumption varies smoothly until the late phase of the prediction, when it jumps sharply. This may be because we chose an inappropriate η'-value. However, choosing the η'-value is rather difficult, because it should incorporate many factors such as population, price, specific economic environment, etc.

Implementation

- We could levy a relatively heavy tax on oil compared to other resources.
- We could encourage the development of alternatives to oil.

Task 4: The "Security" Policy for Oil

We believe that the problem of oil security arises mainly due to the different utilization rates among countries. If a country with low oil utilization is assigned a redundancy of oil, whereas a country with high utilization gets an insufficient, there will be great waste. We can establish a model to find the optimal distribution of oil among countries with different utilization rates.

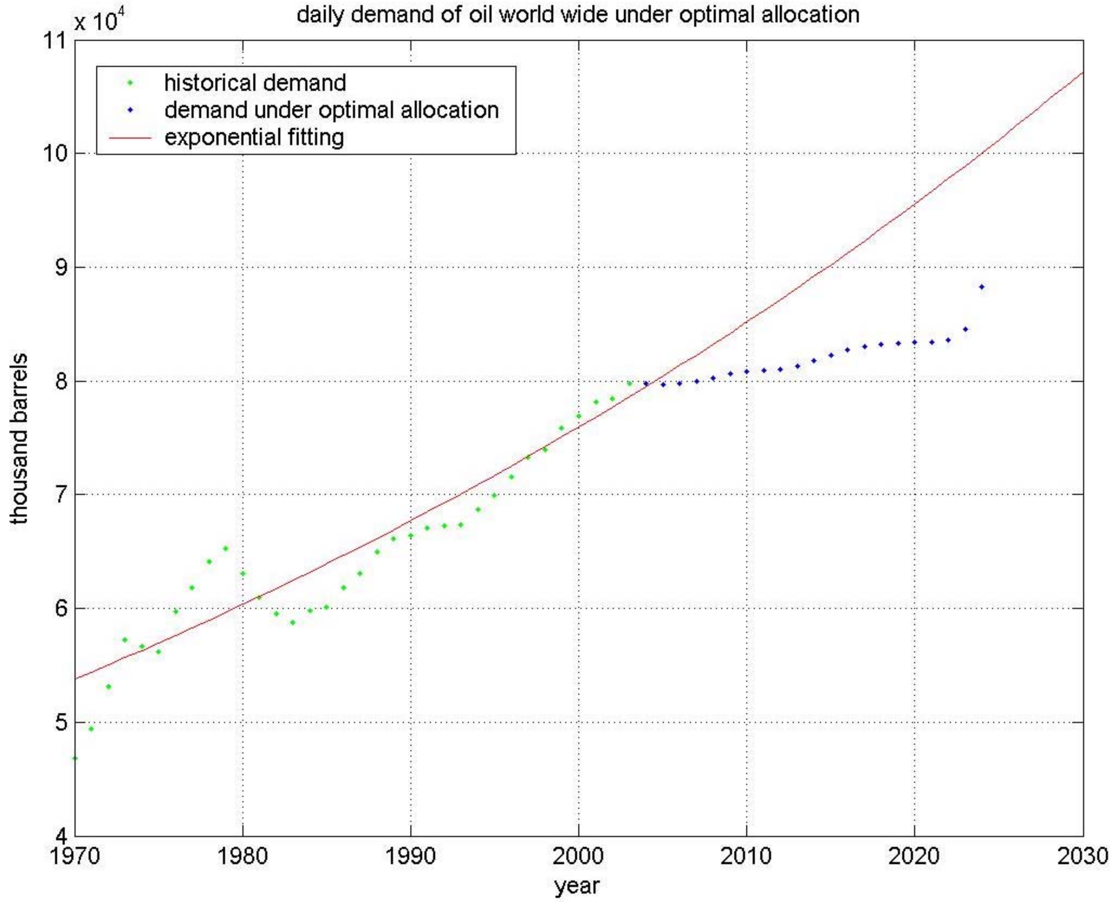


Figure 6. Oil consumption under optimal allocation (dotted line), compared with exponential growth (solid line).

Assumptions

- The annual oil consumption of the whole world is according to the optimal oil allocation model in Task 3.
- We do not take trade barriers into account, hence assume that reallocation of oil among countries is feasible.

Modeling

Suppose there are n main oil-consuming countries in the world. Given the year t, we can establish the linear programming as follows:

$$\max \sum_{i=1}^{n} l_i(t)x_i(t) \tag{15}$$

such that

$$\begin{aligned}
&\sum_{i=1}^{n} x_i(t) = d(t), \\
&\qquad x_i(t) \geq \alpha_i(t) x_i(t-1), \quad i = 1, 2, \ldots, n, \\
&\qquad x_i(t) \geq 0, \qquad\qquad\quad\;\; i = 1, 2, \ldots, n,
\end{aligned}$$

where

- $l_i(t)$ is the oil utilization rate of country i in year t;
- $x_i(t)$ is the oil use of country i in year t; ;
- $d(t)$ is worldwide oil consumption allocation in year t, which can be obtained using the model in Task 3; and
- $\alpha_i(t)$ is the the minimum ratio of $x_i(t)$ to $x_i(t-1)$, as a percentage.

The first constraint means that total oil consumption by all countries in a given year should equal total oil consumption under optimal oil allocation. The second constraint means that oil consumption of a country in a particular year is not less than a certain proportion of the previous year's consumption.

Limitations of the Model

In reality, countries would more likely consider their own interests, leading to trade barriers and making it impossible to get the optimal distribution.

Implementation

For countries with low utilization rates, we could levy a relatively heavy tax on oil or set a limit on annual oil consumption.

Task 5: Natural Disasters

An oil field occupies a large area, destroys vegetation in the vicinity, changes the components of the soil, and deteriorates the environment nearby, to the detriment of animals' habitat. Exploitation of the field influences the groundwater and causes desertification. And then there are oil spills, which often lead to the pollution of nearby waters. However, we mainly consider short-term effects of natural disasters on oil exploitation.

Assumptions

- All oil produced is consumed.
- The total amount of oil produced meets the needs of economic development.

Short-term Effects

Let n be the number of oil fields on Earth. For sustainable development of the economy, we must keep the total output of all oil fields the same as worldwide oil consumption under the optimal allocation of Task 3. Thus, we have

$$\sum_{i=1}^{n} x_i(t) = d(t),$$

where $x_i(t)$ is the output of the oil field i in year t, and $d(t)$ is the worldwide oil consumption under the optimal oil allocation of Task 3.

We believe that the susceptibility of an oil field to disasters is a function of its output in a given year and the ratio of its cumulative output to its initial oil reserve. Naturally, the more the oil produced, the greater the likelihood of disasters. We believe that the relationship is linear. And of course, a new oil field and an old one will have different effects on the environment. This difference is given by the ratio of the cumulative output to the initial oil reserve. We introduce a penalty function $e^{-a[1-\lambda_i(t)]}$ and set

$$p_i(t) = k_i x_i(t) e^{-a[1-\lambda_i(t)]},$$

where

- p_i is the susceptibility to disasters;
- k_i is the proportion coefficient, which is determined by the age and exploitation method of the oil field—a small value of k_i represents a young field with an advanced exploitation method;
- $\lambda_i(t)$ is the ratio for oil field i of its cumulative output until year t to its initial oil reserve.

We wish to minimize the total susceptibilities of different oil fields under the condition that worldwide oil demand is satisfied. That is:

$$\min \sum_{i=1}^{n} p_i(t) \qquad \text{such that} \qquad \sum_{i=1}^{n} x_i(t) = d(t).$$

In the solution, oil fields with small k_i will tend to have larger outputs, and vice versa. We also increase the value of n tentatively, i.e., increase the number of oil fields, and find that the total susceptibility decreases. This is mainly due to the fact that during early exploitation of an oil field the penalty function damps down the risk of disaster, thus favoring the development of new oil fields and decreasing the likelihood of disasters.

Implementation

We increase the output of old oil fields with small k_i (young fields with advanced methods of extraction) and reduce the output of those with large k_i

(older fields with obsolete methods of extraction). Also, if possible, we should exploit as many new oil fields as possible and decrease the exploitation of old oil fields, so as to control the susceptibilities to disaster.

Task 6: The Development of Alternatives

Even if we control the use of oil, its use can be prolonged only 4–5 years beyond an exhaustion horizon of 30 years. We urgently need an alternative to oil. For the sake of sustainable development, we must gradually accelerate consumption of the alternative as oil is depleted. The question is how to develop the alternative to keep the economy stable during the transition period.

Assumptions

- We consider a single kind of alternative.
- Oil and its alternative are interchangeable as energy resources.
- The measure of oil and its alternative is their contributions to GDP.
- The quantity of oil to produce a unit of energy does not change with time.

Analysis

Let the cost for oil to produce a unit of energy be C_1, and that of its alternative be C_2, with $C_1 \leq C_2$ (the cost for oil to produce a unit of energy is the lowest compared with any other resources [Vernon 1976]). But as the total amount of remaining oil declines, the price of oil will correspondingly increase. On the other hand, with advances in the technology for the alternative, its price will fall. The general tendency is shown in **Figure 7**.

With rising cost, oil consumption will decrease, while demand for the alternative will increase, until the day when oil is completely replaced. The question is, How fast should oil be replaced?

Modeling

From our model for optimal oil distribution among countries, we concluded that the consumption in future years will increase slowly. Let G be the value of GD, x the consumption of oil, y the consumption of the alternative, and t time. Let $G = G(x, y)$, so that

$$\frac{dG}{dt} = \frac{\partial G}{\partial x}\frac{dx}{dt} + \frac{\partial G}{\partial y}\frac{dy}{dt}. \quad \textbf{(16)}$$

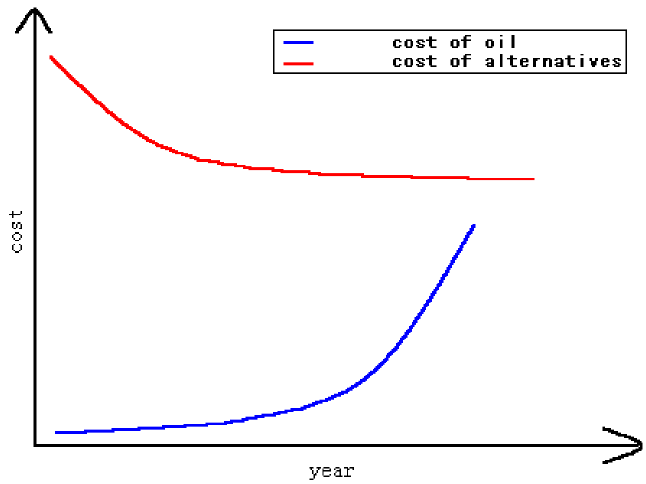


Figure 7. Trend of cost of oil (lower curve) and of its alternative (upper curve), per unit of energy.

We think that a smooth exponential decline of use of oil is reasonable, so we let

$$x(t) = x(t_0)e^{-b(t-t_0)} \quad (t > t_0,\ b > 0),$$

so that

$$\frac{dx}{dt} = -x(t_0)be^{-b(t-t_0)}, \tag{17}$$

where t_0 is the year when oil demand begins to decline. Substituting **(17)** into **(16)**, we get:

$$\frac{dy}{dt} = \frac{\dfrac{dG}{dt} - x(t_0)be^{-b(t-t_0)}\dfrac{\partial G}{\partial x}}{\dfrac{\partial G}{\partial y}},$$

where

- dy/dt is the replacement rate,
- $\partial G/\partial x$ is the contribution rate of oil to GDP, and
- $\partial G/\partial y$ is the contribution rate of the alternative to GDP.

Knowing the other quantities, we can determine dy/dt and hence the consumption of the alternative to guarantee a stable economy.

We choose 2010 as t_0 and simulate the consumption of oil and the alternative. The result is shown in **Figure 8**.

Potential Oil Substitutes

Potential oil substitutes include solar energy, wind power, geothermal energy, hydroelectricity, and tides, as well as oil substitutes such as compressed natural gas, biofuels (biodiesel and ethanol), and gas hydrates.

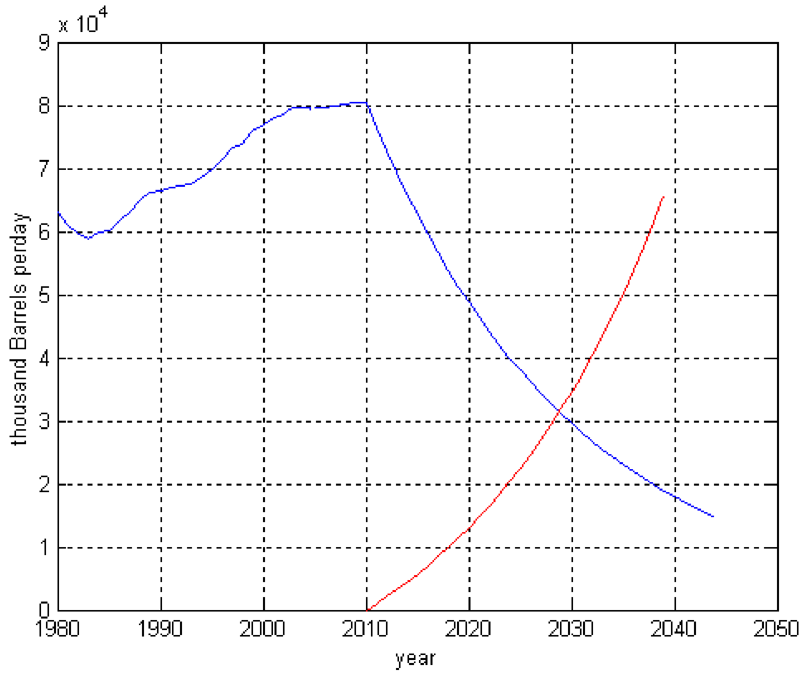


Figure 8. Consumption of oil (curve from upper left) and of its alternative (starting in 2010).

Biofuels are produced from agricultural crops that assimilate carbon dioxide from the atmosphere. The carbon dioxide released this year from burning these fuels will, in effect, be recaptured next year by crops grown to produce more biofuel. Also, biofuels contain no sulfur, aromatic hydrocarbons, or metals. Absence of sulfur means reduction of acid rain; lack of carcinogenic aromatics (benzene, toluene, xylene) means reduced impact on human health.

Gas hydrate is an ice-like crystalline solid; its basic unit is a gas molecule surrounded by a cage of water molecules. Gas hydrates are found in suboceanic sediments in the polar regions (shallow water) and in continental slope sediments (deep water).

Although these substitutes may provide some relief from the oil crisis, whether any of them—or all together—can solve the problem completely is unknown.

Conclusion

Considering the concurrent problems of population size and the adjustment of economies and lifestyles, the challenge of conversion to alternative energy resources is both urgent and formidable. A realistic appraisal should encourage people to prepare for the future. Delay in dealing with the issues will surely result in unpleasant surprises. Let us get on with the task of moving orderly into the post-petroleum paradigm.

References

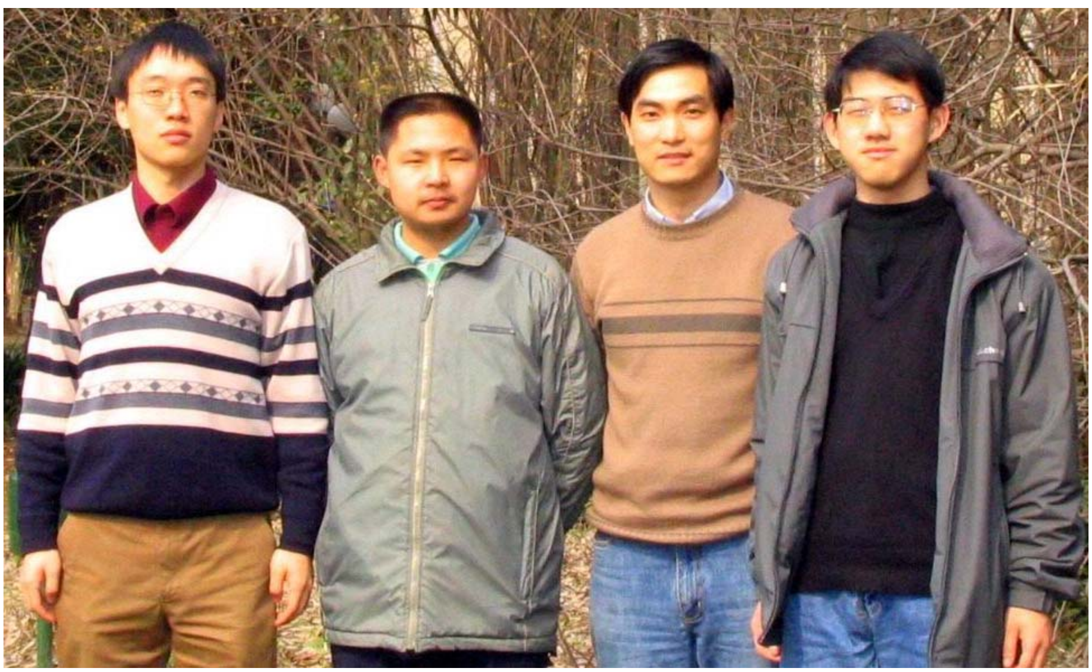
Campbell, C.J. 1997. *The Coming Oil Crisis*. Multi-science Publishing Co.

Energy Information Administration, U.S. Dept. of Energy. 2004. Table 4.6 World Oil Demand, 1970–2003. http://www.eia.doe.gov/emeu/ipsr/t24.xls .

Keeling, C.D., T.P. Whorf, and the Carbon Dioxide Research Group. 2002. Atmospheric CO2 concentrations (ppmv) derived from in situ air samples collected at Mauna Loa Observatory, Hawaii. http://serc.carleton.edu/files/introgeo/interactive/examples/mlco2.doc .

Ramirez, Vincent. 1999. Oil crises delay—A world price forecast. http://www.betterworld.com/getreallist/article.php?story=20040214014558571 .

Vernon, Raymond. 1976. *The Oil Crisis*. Toronto, Canada: George J. McLeod Limited.



From left to right: Chen Jie, Xu Hui, advisor Ni Zhongxin, and Wei Deling.

Preventing the Hydrocalypse: A Model for Predicting and Managing Worldwide Water Resources

Steven Krumholz
Frances Haugen
Daniel Lindquist
Franklin W. Olin College of Engineering
Needham, MA

Advisor: Burt Tilley

Summary

We examine and model trends in water withdrawal throughout the world and develop plans to prevent using water beyond its renewable capacity.

We look at the three major components of water consumption: agricultural, industrial, and municipal uses. We formulate a differential model to account for the rates of change of these uses, and how this change would affect the overall consumption of water within the studied region. We also incorporate feedback based on economic and political stimuli that force a decrease in water usage as it approaches dangerous consumption levels.

Using historical data from the United States, we determine initial conditions for our model and project U.S. water usage to the year 2025. The model simulates how a country could react to water scarcity without drawing from nonrenewable water sources.

In addition to the model, we also discuss policies for effective water management by reducing freshwater usage and preventing tapping into nonrenewable resources. By being able to predict problem areas and suggesting methods of improving water usage in those areas, we can hope to prevent the "hydrocalypse."

The UMAP Journal 26 (2) (2005) 145–162.

Introduction

Two-thirds of the Earth is covered in water, but only 2.5% of it is freshwater. Fortunately, each year some seawater is naturally desalinated by evaporating from the ocean and precipitating on continents and islands. Globally, this supply of freshwater from rain is plentiful—humanity does not use it all, and most of it simply washes back into the oceans—but locally, water can be very scarce. Some regions use vastly more water than is naturally supplied to them each year. To make up for this deficit, fossil water sources are tapped and exploited. Many communities are walking a dangerous road, as they may exhaust their fossil water sources within the next 20 to 50 years.

The Basic Model

We model past data for water use with linear and with logistic functions. The two models correspond to continuing withdrawal vs. eventual plateauing, due to factors such as availability, population growth, and arable land. A more complex model follows that considers these factors, as well as possible political and economic influences.

For simplicity, we model net water withdrawal in the United States.

Table 1 shows data for net water withdrawal of the United States from 1950–2000 [Shiklomanov 1999].

Table 1.
U.S. water withdrawal 1950–2000.

Year (after 1900)	Water withdrawal (km^3/yr)
50	247
60	347
70	470
80	538
90	492
95	503
100	512

The models show trends but have two major limitations.

- Most importantly, they fail to incorporate any external factors, such as population and technological growth, as well as economic and political influences. The models are likely to fail if a country's water withdrawal rates approach the amount of renewable water available, as increased prices and political regulation drive down the amount of additional water consumed.
- The models assume that the area modeled has a stable enough past for the trend to predict the future accurately. Even for the U.S., there is enough variability in the data that we cannot convincingly extrapolate.

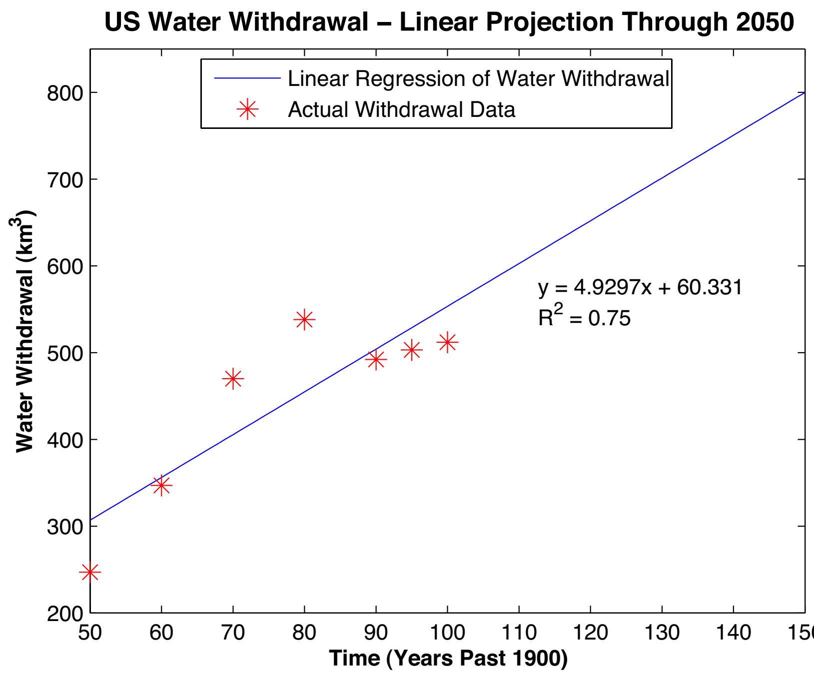


Figure 1. Linear regression of U.S. water withdrawal 1950–2000, extrapolated to 2050.

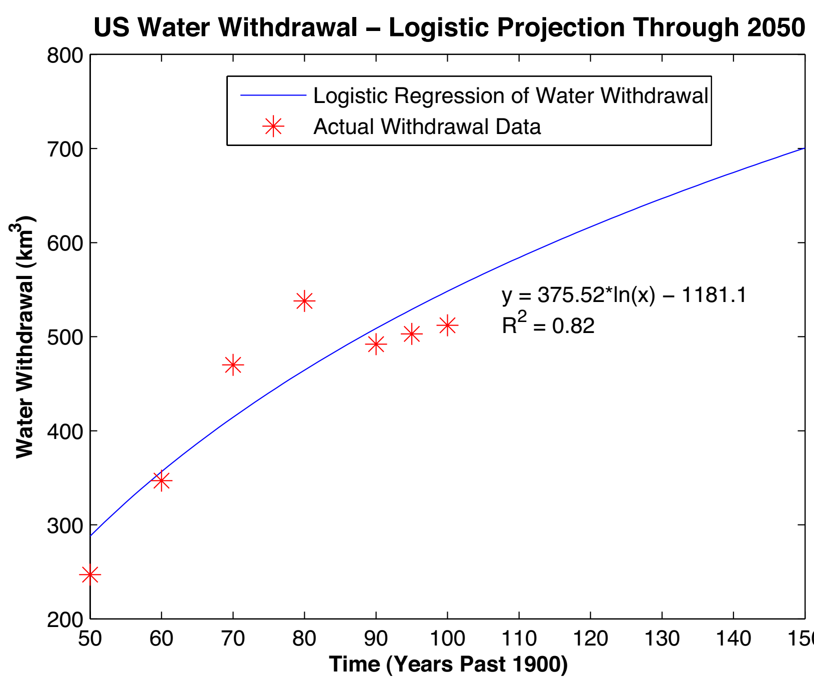


Figure 2. Logistic fit of U.S. water withdrawal 1950–2000, extrapolated to 2050.

A Better Model

While a simple data fit may indicate trends, a better model should take into account how the water is used, how the use of water is changing over time, and what other influences affect the use of water. The three major categories of human water use are agricultural, industrial and municipal. Each has its own trends in growth and water use and can be affected differently by the economy or by political influences. Thus, it is important to consider them separately.

Table 2 shows the variables that we use, their definitions, and measurement units.

Table 2.
Variables used in the model.

Symbol	Definition	Unit
R	Renewable water	km^3/yr
C	Water withdrawn	km^3/yr
A	Size of agriculture	10^3 Ha
I	Size of Industry (GDP in 1990)	\$US/person
M	Size of the municipality	10^3 people
C_A	Water withdrawn by agriculture	km^3/yr
C_I	Water withdrawn by industry	km^3/yr
C_M	Water withdrawn by the municipality	km^3/yr
t	Time	yr

Agriculture

We discuss the influence on agriculture and extrapolate our conclusions to industry and municipality. As with the simple model, we use the U.S. as an example.

First, we consider the rate at which agriculture changes. We quantify agriculture as the net irrigation area within the region. We employ a logistic model, since there is a fixed amount of arable land available, and as land use approaches that, the net increase in agriculture will tend towards zero. We arrive at the logistic model by linear regression of change in irrigation area on time. Using data for the U.S. 1970–1995 [Shiklomanov 1999], we find

$$\frac{dA}{dt} = 15326 \ln t - 49039 \qquad (R^2 = .889).$$

To calculate the additional demands for water that increased agriculture will place on the U.S. We multiply current water consumption due to agriculture, C_A, by the rate of change of agriculture, and normalize by dividing by the current amount of agriculture:

$$\frac{dC_A}{dt} = \frac{dA}{dt}\frac{C_A}{A}. \tag{1}$$

Finally, we must adjust this for political and economic factors.

- Consider the case when net consumption of water (from all three categories) does not approach the amount of renewable resource. There should be little, if any, political or economic inhibition of water use.
- Now consider the case when water use approaches available resources. In this scenario, the price of water will increase, and the government will likely place restrictions on each of the three sectors to help keep water consumption within limits of the resource.
- Finally, consider the scenario when consumption exceeds resource. Ideally, in this scenario, political and economic factors will actively drive the use of water down over time, decreasing net use, and returning the region to a stable state.

This set of circumstances can be modeled by factoring the following coefficient into **(1)**:

$$\text{Political influence} = \left(1 - \frac{C}{R}\right)^{P_A},$$

where R is the amount of renewable water, C is the amount of water withdrawn, we define the parameter P_A to be the economic and political influence on the agricultural sector of the region. This parameter can be easily scaled to simulate the economy and government's response to changes in environmental factors, such as a drought. When $C > R$, the influence on water usage is negative (the government inhibits further water usage), and when $C < R$, it is positive (the government encourages further water usage).

Combining these two equations, the change in water consumption over time due to agriculture is

$$\frac{dC_A}{dt} = \left(1 - \frac{C}{R}\right)^{P_A} \frac{dA}{dt}\frac{C_A}{A}. \qquad \textbf{(2)}$$

Industry

The reasoning for the industrial use of water falls along similar lines as agriculture. We quantify industry as the region's Gross Domestic Product (GDP) per person. Working from GDP data from the Groningen Growth & Development Centre [2005] and population data from UNESCO [Shiklomanov 1999] for 1970–1995, we fit a linear regression of GDP/person for the U.S. to time, since industry tends to grow at a steady rate, despite the nonlinear dynamics of economy and population. We find

$$\frac{dI}{dt} = 392.99t - 12929 \qquad (R^2 = .998).$$

Since the rate of change of water consumption by industry is likely to follow the same trends as agriculture, just with a different power of the political and economic scaling factor, we use the same differential equation, replacing P_A with P_I. Thus:

$$\frac{dC_I}{dt} = \left(1 - \frac{C}{R}\right)^{P_I} \frac{dI}{dt}\frac{C_I}{I}. \tag{3}$$

Municipality

Water consumption of the municipality is probably the easiest of the three models; population is the best indicator of the size of a municipality. Further, population growth tends to be logistic—it will plateau at a certain level. Fitting a logistic model to U.S. population 1970–1995 gives

$$\frac{dM}{dt} = 194163 \ln t - 616608 \qquad (R^2 = .991).$$

Similarly, only the power of the political and economic scaling factor will differentiate consumption of the municipality from that of the other two sectors, so

$$\frac{dC_M}{dt} = \left(1 - \frac{C}{R}\right)^{P_M} \frac{dM}{dt}\frac{C_M}{I}. \tag{4}$$

Bringing it Together

We combine the rates of change of water consumption for each of the three sectors into one governing equation. Since total consumption is the sum of the consumption of these sectors, the rate of change of the total consumption is also simply a sum of the rates of change of the three sectors, or the equations **(2, 3, 4)**. Therefore, our final governing equation is:

$$\frac{dC}{dt} = \left(\frac{dC_A}{dt} + \frac{dC_I}{dt} + \frac{dC_M}{dt}\right). \tag{5}$$

Derivation of the Values of the Parameters P

We use initial conditions to identify the values of the parameters P (powers of the political influence). We rearrange **(2)** to get:

$$P_A = \frac{\ln\left(\dfrac{\dfrac{dC_A}{dt}}{\dfrac{dA}{dt}\dfrac{C_A}{A}}\right)}{\ln\left(1 - \dfrac{C}{R}\right)}.$$

Using data for the U.S. 1990–1995 [Shiklomanov 1999], we solve for the country's three political and economic constants. We find dC_A/dt by calculating the change in water consumption due to agriculture over the five years; C and R are also known for the U.S. in 1990 ($C = 492\,\text{km}^3$, $R = 2930\,\text{km}^3$) and dA/dT can be found from the logistic fit for agriculture; and C_A and A are both known for 1990 [Shiklomanov 1999]. Plugging these values (and their counterparts for industry and municipality), we find:

$$P_A = 1.052, \qquad P_I = 7.36, \qquad P_M = 3.45.$$

Running the Model

To solve the differential equations in our model, we use the ODE45 numerical integrator in MATLAB on **(5)** to find the results in **Figure 3**. Agricultural, industrial, and municipal withdrawal rates each increase steadily, as does the total withdrawal rate. The economic and political scaling factor makes virtually no attempt to curb the increasing water usage, since the U.S. has a significant surplus of renewable water sources.

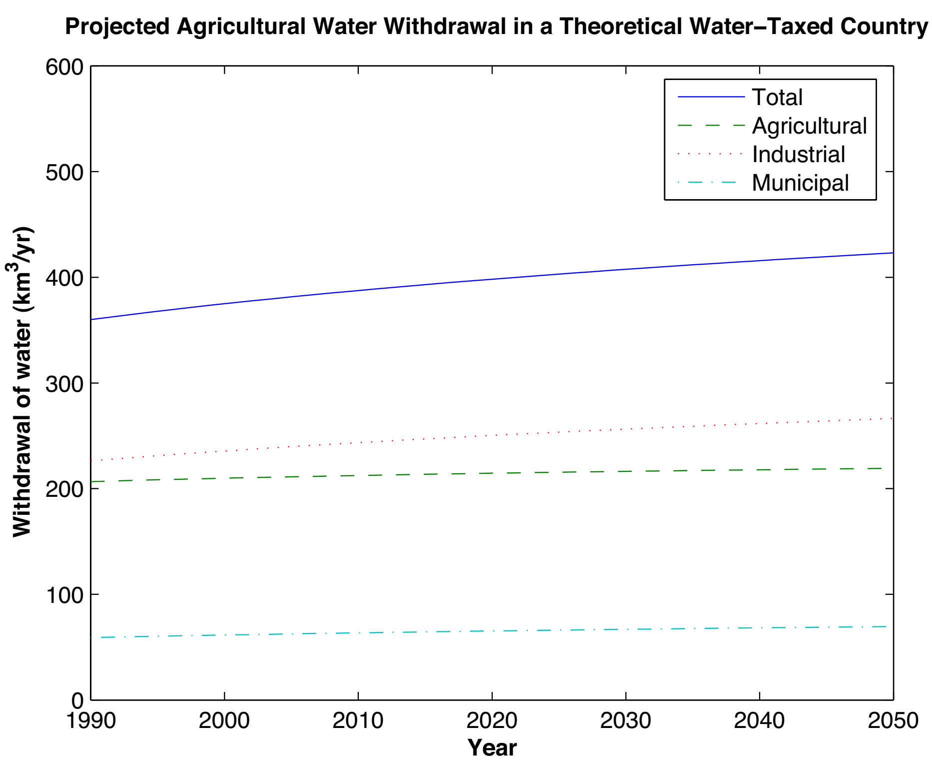


Figure 3. Projected water usage in the U.S. 1990–2050.

The second example, seen in **Figure 4**, is a simulation of a water-taxed country that is currently OK but approaching dangerously low levels of renewable water resources. As total withdrawal approaches total available, the economic and political scaling factor becomes negative and forces reduction of water use, even though the population is still increasing. This fictional country is similar in agricultural, industrial, and municipal trends to the U.S.; but because it has greatly decreased water renewability, the outlook is particularly bleak.

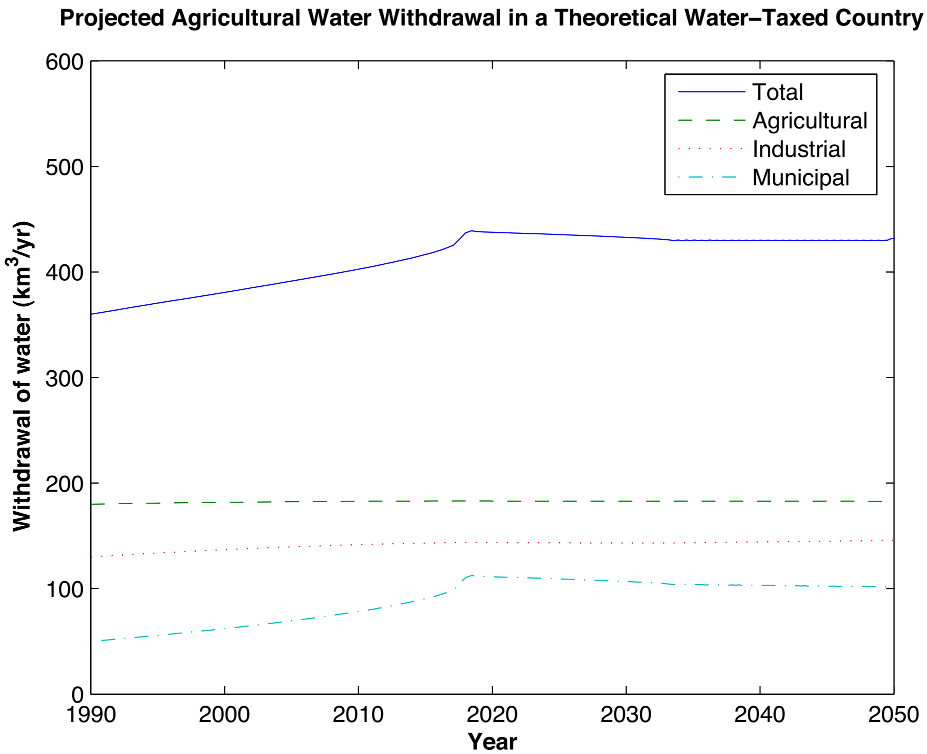


Figure 4. A theoretical country on track for water problems. Initial withdrawal is 360 km^3/yr with total renewable resources 430 km^3/yr.

In our model, we treat each of the values of the parameters P as a regional constant derived from past data. However, it is much more likely that each value changes dynamically. The assumption of constant P prevents our model from adapting to radically changing times.

Figure 5 shows how changing P_A has drastic effects on how water scarcity is affected by political and economic factors. For our fictional country, P_A is 1.05, based on our initial values. Different values do very little to stymie agricultural withdrawal until it is essentially too late. The larger values exhibit a very large dampening effect, keeping agricultural withdrawal low. P_I and P_M affect industrial withdrawal and municipal withdrawal in similar ways.

Limitations of the Model

- The model will have difficulty adjusting to a drastic change in one of the three sectors that instantaneously throws the region from stable to unstable. This is because we chose our political and economic factors to be a constant property of the region. If these values were adjustable, the model would likely be able to adjust for a catastrophe.

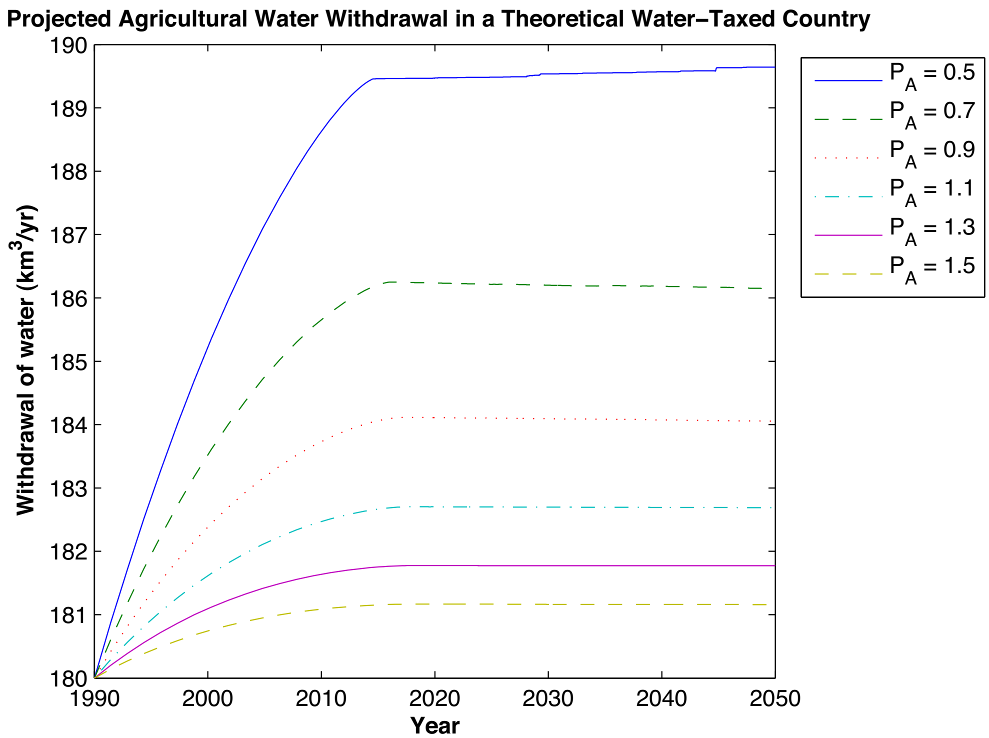


Figure 5. Graph illustrating the damping effects of deliberate variation of the P_A constant

- Since the model relies on trends in population, GDP, and agricultural data, a country with an unstable past might not be able to extrapolate domestic trends accurately enough for the model to be useful.

- We must consider the scope of the region being modeled. More often than not, applying aggregated data for a country to all regions in the country will result in inaccurate assumptions about those regions. For example, even though the U.S. as a whole, has an overabundance of water, the Southwest uses more water than is naturally renewed. A smaller region can apply its own historical data to this model to attain a more accurate representation of its current and projected water situation.

New Approaches to Water Harvesting

Only 2.5% of the Earth's water is freshwater and less than 1% of that amount is renewed each year by natural means [Sayegh 2004]. Growing communities, when faced with the need for freshwater, rely more and more heavily upon fossil water sources, aquifers, and wells to remedy their water deficits. While these sources can naturally refill over long periods of time, people are drawing from them at a rate that is far too fast for the sources to regenerate.

Localities must change how they conceptualize water acquisition. In the status quo, too much emphasis is placed on finding the cheapest water source to fuel economic growth. Instead, communities should focus on strategies to use naturally renewable water sources efficiently and prefer those over nonrenewable sources.

Individual Responsibility

To reduce the strain on centralized water acquisition and distribution systems, the responsibility for water harvesting must first be shifted from the community to the individual. Domestic rainwater harvesting systems would provide a feasible alternative to the inefficiencies of centralized water systems or well-drilling, by allowing individual households to supply a substantial fraction of their water needs. Furthermore, implementing such rain-harvesting technologies is not as far fetched as one might think. Modern rainwater-harvesting systems range in complexity from inexpensive rain barrels to contractor-designed and -installed systems costing thousands of dollars. By using locally harvested water, individuals can mitigate water's growing environmental and economic costs and avoid health concerns regarding its source and treatment [Texas Water Development Board 1997]. In addition, by collecting their own water, citizens can further appreciate the efforts that are necessary to harvest water, and hopefully be more willing to embrace the concept of water conservation. Already, island states such as Hawaii and entire continents such as Australia promote rainwater harvesting as the principal means of supplying household water. Throughout the Caribbean, public buildings, private houses, and resorts collect and store rainwater. Rainwater harvesting can even be used in urban areas with high population densities. In Hong Kong, skyscrapers collect and store rainwater to help supply the buildings' water requirements [Jiwarajka 2002]. Municipalities should require the installation of rainwater harvesting devices in all new construction and encourage the retrofitting of older properties through subsidies or tax incentives.

Municipal Strategies

While residential and commercially-based rain harvesting systems will take significant pressure off of municipal water grids, it is likely that there will not be a sufficient or reliable source of water year round. Municipalities should begin positioning their current water supplies as the "fallback" for when individual water harvesting is not sufficient, and as a result charge more for municipal water. Prices should be intentionally set at a premium over the cost of harvesting the water, to encourage people to conserve and to promote improvements in water use efficiency by returning the excess capital to the community in the form of grants.

Similarly, municipalities must also work to improve the efficiency of their water use. The City of New York has a long tradition of investing in the protection and improvement of much of the watershed from which it receives the 1.3 billion gallons of water it needs every day. As result of this careful planning, the city uses no fossil water, relying solely on its network of 19 reservoirs in a 1,969 square-mile watershed that extends 125 miles north and west of New York City [City of New York . . . 2002].

Some cities, however, may not have an abundant supply of renewable water. When naturally renewable water sources seem to be exhausted, municipalities should prioritize investing in additional technology to account for their shortfall of water, instead of turning to fossil-water pumping. This may be more expensive in the short term; but investing in efficiency programs, improving water purification and desalination techniques, or buying and rerouting water from other regions with abundant natural sources will save in the long run, by lowering dependence on fossil water and preventing exhaustion of nonrenewable water sources.

Agricultural Rain Harvesting

In agriculture, rainfall can be captured, diverted, and stored for plant use. If fields are plowed so that the plowing contours wrap around the terrain rather than run down inclines, a higher fraction of the water can infiltrate into the ground. This method also reduces water pollution by preventing soil erosion, preventing contamination of usable water downstream. Similar effects can be achieved by deploying precision leveling of fields, eliminating inclines and thus the means for wasteful runoff. Both of these techniques do not require sophisticated machinery but instead simply modify current practices. Improving agriculture water efficiency in the United States alone would save over half a cubic kilometer of water per year, enough to satisfy the needs of 3.6 million people [Pacific Institute 2002].

Last Resort: Fossil Water

Communities and individuals should turn to fossil water as a last resort and should take steps to protect and maintain the aquifers or wells they harvest from. In addition, drawing from these wells should come at an added premium, to further discourage use. Fortunately, many regions with seasonal water scarcities also have a "rainy season." By installing new methods of groundwater management such as artificial recharge or injection of surface waters during seasonal wet periods, it is possible to extend the life of many freshwater aquifers. Such practices have already been successful in the U.S. [Slattery 2004].

For illustration of the ideal progression of water use, see **Figure 6**.

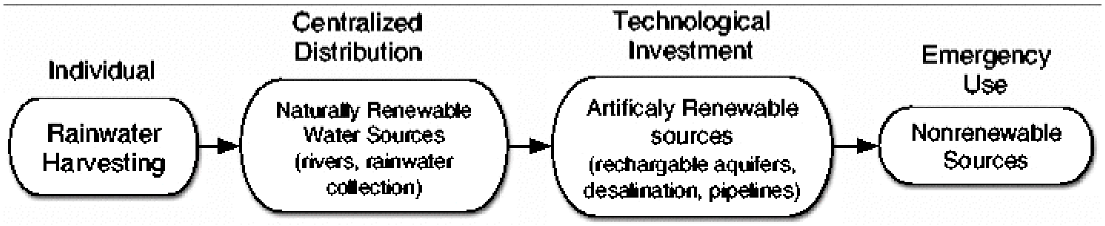


Figure 6. Ideal progression of water usage.

Developing Alternatives to Water

Salt Water

Though salt water has limited applications in agriculture and industry, due to the corrosive nature of salt, China already uses some salt water to conserve freshwater. A major domestic use of water is flushing toilets (30–35% of domestic water). However, there is little reason that toilets must contain freshwater. Instead, some Asian cities have begun experimenting with parallelizing the plumbing in houses in an attempt to replace the freshwater in toilets with seawater. Using seawater in this way not only allows freshwater supplies to be stretched over a longer period of time, it is also more cost-effective. In China, residents pay for processed seawater only 0.5 yuan ($U.S. 0.06) per ton, about one-eighth the price of fresh tap water [A liter here . . . 2004].

Greywater

Greywater—wastewater except toilet wastes and food wastes from garbage grinders—can serve as a substitute in some applications; 30–50% of all water used domestically is greywater, most of which can be easily reused in a variety of applications. Industrially, greywater can be used for air conditioning, cooling, general washdown, and street cleaning. Fire protection is another potential use, as are construction activities such as making cement [Emmerson 1998]. Strategies to reuse greywater do not have to change radically how people interact with water. For example, washing industrial parts can be done in stages starting with greywater and using progressively cleaner water [Accepta 2005]. Greywater, after filtering, can also be used for landscaping or agriculture irrigation. Though it may require more upfront infrastructure to use, greywater reuse can ultimately save municipalities money by reducing sewage flows and reducing the demand on potable water supplies [Martin 1997].

Desalination of Salt Water

Regrettably, desalination of seawater is not the answer to the world's freshwater needs, because it is highly energy-intensive. Since most of the world's energy is generated by fossil fuels, intensive desalination would simply replace one nonrenewable resource (water) with another (coal, oil, gas). However, desalination is still an option for locations that desperately need additional water.

Regulation of Harvesting

Almost every body of water in the world has been negatively impacted by human water harvesting, whether from over-harvesting, nutrient enrichment, agricultural runoff, or toxic pollution. Mexico's Lake Chalapa, the country's largest body of freshwater, lost 75% of its original volume as a result of over-harvesting water from its tributaries, for irrigation purposes and for the water supply of Guadalajara. [Living Lakes 2003] In Central Asia, the freshwater Aral Sea, which lies astride Kazakhstan and Uzbekistan, was once the fourth largest lake in the world. Today, the lake is only 20% of its original volume as result of Soviet diversion for agriculture of water from its tributaries. In Africa, Lake Chad, which spanned 25,000 km^2 of surface area in 1963, has shrunk to 1,350 km^2 today as a result of aggressive expansion of irrigated agricultural projects [Coe 1998]. In each case, local wildlife paid the price of the water diversions as the salinity of the lakes increased and available habitats were destroyed.

Large lakes are natural gauges of water use: Use a lot of water, and they go down; use too much water, and they die. In Los Angeles, simple water reduction and reclamation measures were implemented to compensate for the water that would have been used from Mono Lake, saving it from annihilation.

Water storage is an important issue in many regions throughout the world. During dry seasons, such localities often experience water shortages, while during the rainy season, usable water is lost to flooding. Aquifer recharging provides a simple and effective way to store water during the plentiful rainy season. Excess rain water is channeled into recharge basins where it naturally filters through hundreds of feet of earth before entering the groundwater aquifer. In this way, groundwater supplies can be naturally recharged and available for future use during the dry season [City of Peoria . . . 2003]. Actively maintaining these natural freshwater storage regions is essential to securing freshwater for much of the world. Underground aquifers store 97% of the world's unfrozen freshwater, and they provide drinking water to almost one-third of the world's people. In Asia alone, more than a billion people rely on groundwater for drinking, and in Europe it is estimated that 65% of public water supplies come from groundwater sources [Ramsar Convention . . . 1995].

Another danger of excessive water harvesting is greater susceptibility to droughts. As communities harvest more water from renewable sources, their members grow accustomed to elevated levels of water availability. However, during a drought, the amount of naturally renewable water is much less than typical. If the drought drops renewable water levels below the threshold of water needed by the community or nation, a water crisis may result [Dykstra 1999]. Minimizing water needs (and water-harvesting) through conservation or efficiency improvements can insulate communities from droughts, since communities will not have grown accustomed or dependent on unnecessarily generous water use policies. Beyond basic water dependency, excessive water-harvesting also increases vulnerability to droughts by altering the water table and distribution of water.

Protecting the World's Water

Unfortunately, water is not as abundant as it may seem—by 2025, the United Nations projects that 1 in 3 people in the World will not have adequate freshwater for life [CNN.com 2000]. Thus, it is imperative that the governments of the world take steps to help prevent degradation of the world's freshwater supply. Such measures can be taken by focusing efforts in three different areas:

- effective international allocation of water,
- building consciousness among those who may not be aware of the need to preserve water, and
- prevention of lost water due to pollution.

International Allocation

Probably the area of most contention, and the one that requires the most governmental regulation, is allocation of international waters. Over the past 50 years, there have been are 1,800 international incidents concerning the use of international freshwaterways. More than 500 were conflicts, and 21 resulted in military action [Cosgrove 2003, 68–70]. Generally, what causes these disputes is one country monopolizing a waterway, preventing the flow of some water to neighboring countries. Thus, it is crucial that the international community help countries work together to solve disputes over water.

Another way to solve international water allocation disputes is purification technology. One example of technology resolving a conflict is Israel's use of desalination. Of the freshwater for the West Bank of the Palestinian Territory, 80% is owned by Israel. To preempt conflict, Israel (with U.S. help) will construct a large desalination plant to purify water from the Mediterranean Sea and pump it to regions in the West Bank [Pearce 2004].

Other water purification technologies can drastically help prevent degradation of the world's freshwater supply. Currently, many countries discharge waste directly into freshwater sources. While this is the easiest and least expensive solution to wastewater disposal, every cubic meter of waste discharged in this way pollutes about 8–10 cubic meters of consumable freshwater [Rosegrant et al. 1995, 255]. This unnecessary pollution could be prevented by building basic sewage treatment systems, perhaps with international help.

Building Consciousness

Governments can provide some answers to water conservation, but it is also important for individual citizens to be educated about the world's water problems. Generally, when people feel as though a resource is abundant enough to last forever, they use it with reckless abandon. However, if people were to realize that water is a resource to be conserved, drastic improvements in the amount of wasted consumable water can be seen. In the U.S. despite a growing population, per capita use of water has decreased steadily since 1995; and net water use in many countries, including highly populated ones such as China, is beginning to plateau [Gleick 2000, 290–293]. These changes come from domestic and economic reforms of the countries' governments but also from increased societal awareness about water conservation. From domestic improvements such as low-flush toilets to improved agricultural water-saving techniques, countries are beginning to conserve water in daily life.

Avoiding Water Pollution

Governmental regulations have helped control negative side-effects of industrial water use. For more drastic reductions, new regulations or social initiatives need to "shift the corporation's thinking from [pollution] compliance to pollution prevention" [Greer et al. 1999]. The company itself can save substantially with reduced resource consumption and waste [Greer et al. 1999]. An initiative to support such organizations, and universalize their scientific waste-reduction procedures, would serve as yet another mechanism to decrease the continuing abuse of the world's freshwater supplies.

References

Accepta. 2005. Water efficiency in the textile & leather Industry. `http://www.accepta.com/Industry_Water_Treatment/Water_efficiency_textile_industry.asp` . Accessed: 4 Feb 2005

Bokek, E. 2002. Proceedings of the two parallel conferences: "The Dead Sea—Between Life and Death" and "Learning from Other Lakes." `http://www.livinglakes.org/images/200210proceedings.pdf` . Accessed: 4 Feb 2005

A liter here, a liter there, is water saved. 2004. *China Daily* `http://www.chinadaily.com.cn/english/doc/2004-03/23/content_317082.htm` .

City of New York Department of Environmental Protection. 2002. New York City's water supply system: Watershed agreement. http://www.ci.nyc.ny.us/html/dep/html/agreement.html . Accessed 4 Feb 2005.

City of Peoria, AZ. 2003. *Recharging Groundwater*. Water Report 2003. http://www.peoriaaz.com/Water03/Water-Report-recharge.asp . Accessed: 4 Feb 2005

CNN.com. 2000. Global water supply central issue at Stockholm conference. cnn.com http://archives.cnn.com/2000/NATURE/08/14/water.shortage.reut/ . Accessed 4 Feb 2005

Coe, M. 1998. Lake Chad has shrunk into "a ghost." *Afrol News* http://www.afrol.com/Categories/Environment/env062_lake_chad.htm . Accessed: 4 Feb 2005.

Cosgrove, William J. 2003. *Water Security and Peace: A Synthesis of Studies Prepared under the PCCP-Water for Peace Process*. Paris, France: UNESCO.

Dunne, George W., and Roland F. Eisenbeis. 1973. Droughts. Nature Bulletin No. 488-A, 7 April 1973. Forest Preserve District of Cook County (Illinois). http://www.newton.dep.anl.gov/natbltn/400-499/nb488.htm .

Dykstra, P. 1999. Droughts come and go, but growing demand for water remains. http://www.cnn.com/SPECIALS/views/y/1999/08/dykstra.water.aug12/ .

Emmerson, G. 1998. *Every Drop Is Precious: Greywater as an Alternative Water Source.* Brisbane, Australia: Queensland Parliamentary Library Publications. http://www.parliament.qld.gov.au/Parlib/Publications_pdfs/books/rb0498ge.pdf . Accessed 4 Feb 2005

Engelman, R., P. LeRoy, and T. Gardner-Outlaw. 1993. *Sustaining Water Population and the Future of Renewable Water Supplies*. Washington, DC: Population Action International.

Farley, Maggie. 2001. Report predicts thirstier world. *Los Angeles Times* (August, 2001). http://www.commondreams.org/cgi-bin/print.cgi?file=/headlines01/0814-03.htm . Accessed: 2/05/2005.

Food and Agriculture Organization of the United Nations. 2003. *Review of World Water Resources by Country.* Water Reports 23. Rome, Italy: United Nations. http://www.fao.org/documents/show_cdr.asp?url_file=/DOCREP/005/Y4473E/Y4473E00.HTM .

Gleick, Peter H. 2000. *The World's Water, 2000–2001: The Biennial Report on Freshwater Resources*. Washington, DC: Island Press.

Greer, Linda. 1999. Preventing industrial pollution at its source: A final report of the Michigan Source Reduction Initiative. National Resources Defense Council. http://www.nrdc.org/water/pollution/msri/msriinx.asp .

Groningen Growth & Development Centre. 2005. Total Economy Database: Real Gross Domestic Product. University of Groningen. http://www.ggdc.net/dseries/gdp.shtml . Accessed: 2/07/2005.

Japan Aerospace Exploration Agency. 2004. *Shrinking sea in the desert: The Aral Sea*. http://www.eorc.jaxa.jp/en/imgdata/topics/2004/tp040217.html . Accessed 4 Feb 2005.

Jiwarajka, Shri Sushil Kumar. 2002. Rainwater harvesting in urban habitats: Issues & implications. Welcome address, Seminar on Rainwater Harvesting, 7 November 2002. http://www.cleantechindia.com/eicnew/MUMBAI/jiwarajka’sSpeechforRainwaterharvesting\%20seminar.htm . Accessed: 4 Feb 2005.

Living Lakes. 2003. Mexico’s largest lake is shrinking fast—International Living Lakes network supports protection of Lake Chapala. Global Nature Fund Press Releases. http://www.globalnature.org/docs/02_vorlage.asp?id=13873\&sp=E\&m1=11089\&m2=13812\&m3=13873\&m4=\&domid=1011\&newsid=1168 .

Martin A., G. Ho, and K. Mathew. 1997. Greywater reuse: Some options for Western Australia. Permaculture Association of Western Australia. http://www.rosneath.com.au/ipc6/ch08/anda/ . Accessed: 4 Feb 2005.

Nelson, L.C., W.K. Sawyer, and L.Z. Shuck. 1999. Recharging Appalachian aquifers using watershed specific technology and methodology and technical considerations relative to mountain top removal. In *1999 Appalachian Rivers II Conference Proceedings*.

Pacific Institute for Studies in Development, Environment, and Security. 2002.]Water: Facts, trends, threats, and solutions. http://www.pacinst.org/reports/water_fact_sheet/water_factsheet.pdf .

Pearce, Fred. 2004. Israel lays claim to Palestine’s water. *New Scientist Online* http://www.newscientist.com/article.ns?id=dn5037 . Accessed: 2/05/2005.

Ramsar Convention on Wetlands. 1995. Groundwater replenishment—Fact sheet 2 of 10 on wetland values and functions. Background papers on Wetland Values and Functions. http://www.ramsar.org/values_groundwater_e.htm . Accessed: 4 Feb 2005

Rosegrant, M.W. 1995. Dealing with water scarcity in the next century. 2020 Vision Brief. http://www.ifpri.org/2020/briefs/number21.htm . Accessed: 5 Feb 2005

________, X. Cai, and S.A. Cline. 2002. *World Water and Food to 2025: Dealing with Scarcity* Appendix A: Model Formulation and Implementation: The Business-as-Usual Scenario. Washington, DC: International Food Policy Research Institute and the International Water Management Institute.

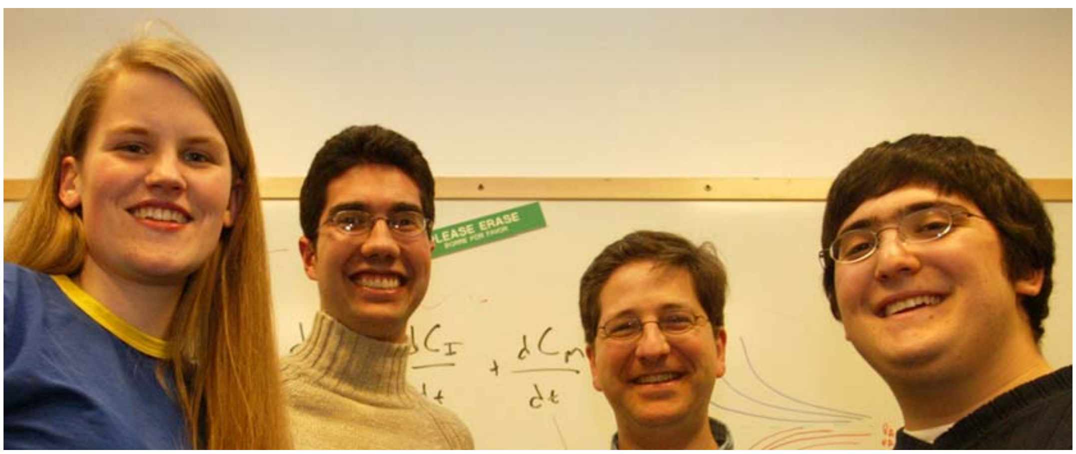
Sayegh, J. 2004. Hydropolitics and geopolitics in Africa: Impact of water on communities, food security, transnational relationships, and development. http://www.sas.upenn.edu/African_Studies/Current_Events/hydro0405.html .

Shiklomanov, Igor A. 1999. *World Water Resources and Their Use*. State Hydrological Institute. St. Petersburg, Russia. http://espejo.unesco.org.uy/part'3/_read'me.html .

________. 2000. Appraisal and assessment of world water resources. *Water International* 25(1): 11–3.

Slattery, R.N. 2004. Recharge to and discharge from the Edwards Aquifer in the San Antonio Area, Texas, 2003. U.S. Geological Survey. http://tx.usgs.gov/reports/dist/dist-2004-01 . Accessed 4 Feb 2005

Texas Water Development Board. 1997. *Texas Guide to Rainwater Harvesting*. 2nd ed. Austin,TX http://www.twdb.state.tx.us/publications/reports/RainHarv.pdf .



Frances Haugen, Daniel Lindquist, Burt Tilley, and Steven Krumholz

The Petroleum Armageddon

Jonathan Giuffrida
Palmer Mebane
Daniel Lacker
Maggie Walker Governor's School
Richmond, VA

Advisor: John Barnes

Summary

We describe the depletion of petroleum, a vital nonrenewable resource, over the next few decades. Petroleum is a fossil fuel that fuels our industries, heats our buildings, and powers our automobiles; plastics and fertilizers are also derived from oil. The production of nonrenewable resources that cannot be returned to the environment is generally considered to follow a bell-shaped curve; as interest and demand increase, the rate of production likewise increases until the world is producing at capacity, whence the rate decreases as the resource is slowly exhausted.

Our model assumes that production and consumption are equivalent; this does not account for stockpiles of oil or the delay caused by shipping and distribution. We also assume that total discoveries of new reserves and total production follow logistic curves; this is heavily supported by professional opinion and by our data.

Our approach includes four major functions of time: total production, total known oil, total remaining oil, and total demand. "Total" means cumulative; the derivatives of these functions are the production, discovery, and demand of oil at any time (except for total remaining oil). The equations include parameters for production, discovery, and demand, which allow our functions to follow historical data. The function for demand is based heavily on total production, in accordance with the law of supply and demand, which in turn depends on total known oil (as a carrying capacity) and on demand.

By varying the parameters, the model is flexible enough to provide for technological advance, economic limit/incentive, natural or manmade disaster, and increase or decrease in demand. The model also includes a management policy for future production, involving government limits on production to enable

The UMAP Journal 26 (2) (2005) 163–174.

production at a nearly constant rate well into the 22nd century, at which point a decent alternative should be available. Policies for increasing the security of the oil supply, decreasing the impact on the environment, and developing an alternative to oil are suggested as part of this management policy.

Strengths of the model include its ability to adjust to virtually any factors influencing production, even when those factors overlap. Prominent among its weaknesses is its dependence on the assumptions. Another weakness is the possibility of a change in total recoverable oil, which would severely affect all four functions, although the model could easily be adjusted.

Introduction

The United States per capita GNP (gross national product) rose by a factor of 7.5 between 1870 and 1980. The fuel for such unparalleled growth was petroleum. Nonrenewable fuels now supply almost 90% of the energy produced domestically. But petroleum is a nonrenewable resource, with a limited supply. And we have used almost half of the world's total supply, demand is increasing, and world production will soon peak.

In 1956, near the height of the growth rate of the U.S. oil industry, geologist M.K. Hubbert drew a bell-shaped curve to depict production of oil in the U.S. over the coming decades. With remarkable accuracy, even despite a large unforeseen find in Alaska, Hubbert correctly predicted domestic oil production would peak in 1970. In 1989, the United States imported more fuel than it produced domestically; currently, it gets 60% of its fuel from imports. However, the lack of recent discoveries is even more disconcerting. We have already located over 1,600 billion barrels of oil in the world, whether already produced or still underground. If we accept the preferred estimate of 1,800 billion barrels as the total amount of oil recoverable for a profit, then we have only 200 billion barrels left to discover, which will only add about 20% to our current reserves.

Many people believe that once oil prices rise high enough due to scarcity, it will be profitable for oil companies to harvest reserves that are not yet economically viable However, there is something else more prevalent here than the price in dollars: the price in energy. In Hubbert's own words, "When the energy cost of recovering a barrel of oil becomes greater than the energy content of the oil, production will cease no matter what the monetary price may be" [Ecosystems 2005].

The fundamental question is, How long will our oil supply last? World production is predicted to peak between 2000 and 2020. The world supply can be expected to run out between 40 and 60 years from now.

Our model addresses depletion of oil over a long horizon by using historical data from 1930 onward. This model is flexible enough to account for almost any economic, political, and natural factors.

Historical Data

Figure 1 shows oil discovered in the last 70 years, together with a logistic curve fitted by least-squares.

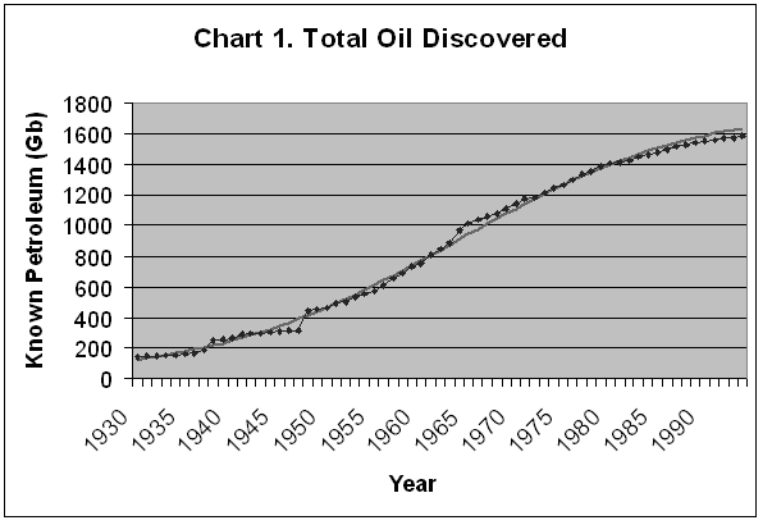


Figure 1. Total oil discovered with fitted logistic curve. Data source: Campbell [n.d.].

This logistic curve has a carrying capacity of 1,800 Gb (gigabarrels = billion barrels), the total of known oil (harvestable at a profit of both money and energy with modern technology). This number is one of the most disputed among scientists in this field; 1,800 Gb is approximately the median estimate [Campbell 1997].

The logistic curve models cumulative production; its derivative models actual production, or the rate of harvest. The derivative of a logistic curve is a normal distribution (bell curve). **Figure 2** shows a bell curve fit by least-squares to production data. The rise from expected levels in the early 1970s is due to the OPEC price increase. The decline from the peak at 1979 is due to concern over oil supplies following the Iranian revolution. In the data set, the post-WWII economic boom and the early-1980s recession are clearly visible.

Assumptions

- Production and consumption are identical, that is, there is no delay between production and consumption.
- The demand function must obey the economic laws of supply and demand: that supply and price are inversely proportional, and that demand and price are directly proportional.
- The model year 0 ($t = 0$) is 2000.
- Oil cannot be artificially produced.

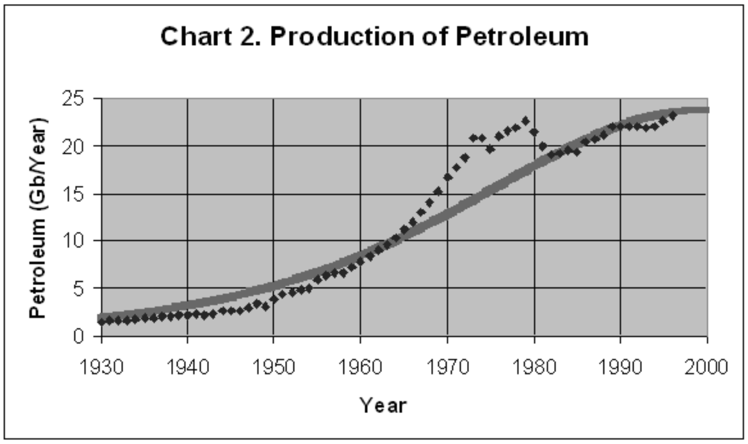


Figure 2. Oil production with fitted bell curve. Data source: Ramirez [1999].

- Harvesting oil will follow the bell curve based on past data, and discovery the logistic curve indicated, although actual past production and discoveries do not fit the data exactly, nor can future figures be expected to. According to our model, the midpoint of cumulative oil production will occur in early 2006, with peak oil production per year of 28 Gb/year. By the end of 2072, 99% of the total oil will have been produced.

Model

Our model has four main functions:

- $S(t)$, the cumulative amount of oil discovered by time t;
- $H(t)$, the cumulative amount of oil harvested by time t;
- $D(t)$, the cumulative amount of oil demanded by time t; and
- $M(t)$, the total amount of untapped oil at time t.

Both $S(t)$ and $H(t)$ correspond to data and both $D(t)$ and $M(t)$ depend on $H(t)$.

We model the discovery function, $S(t)$, as growing logistically toward a carrying capacity M_0 and also depends on demand:

$$S' = \frac{dS}{dt} = kDS\left(1 - \frac{S}{M_0}\right),$$

where k is a constant.

The total amount of oil ever harvested by time t, $H(t)$, also follows a logistic curve. It too should increase with demand.

The carrying capacity to which $H(t)$ levels off is $S(t)$, since oil harvested cannot exceed oil discovered. Thus, we have the following differential equation:

$$H' = \frac{dH}{dt} = bDH\left(1 - \frac{H}{S}\right),$$

where b is a constant.

Total world oil, $M(t)$, is given by

$$M(t) = M_0 - H(t).$$

However, considering natural disasters and outside manipulations, expressing $M(t)$ as follows is more relevant and more practical:

$$\frac{dM}{dt} = M' = -\frac{dH}{dt} = -bDH\left(1 - \frac{H}{S}\right).$$

By the basic economic laws of supply and demand, supply and price are inversely proportional, and price and demand are directly proportional. Transitively, supply and demand are inversely proportional, or $D(t) = c/H(t)$ for some constant of proportionality c. From this relationship, we get

$$D' = \frac{dD}{dt} = \frac{c}{bDH\left(1 - \dfrac{H}{S}\right)}.$$

We used these four functions and the improved Euler's method to create a spreadsheet to project estimates from known initial values. The tangent at the initial value is calculated; then, the tangent at a point some distance h along the x-axis from the initial value is calculated using the differential equation. As $h \to 0$, the estimate becomes increasingly accurate.

Figure 3 illustrates the depletion and cumulative discovery, harvest, and demand of oil. For many purposes, the derivatives of these functions are more relevant: At time t, H' is production rate, D' is demand rate, and S' is discovery rate. The interplay between these rates is illustrated in **Figure 4**.

Production noticeably lags behind discovery but follows a similar bell curve. Due to the sensitivity of the demand function , it takes a very low production to cause a perceptible demand increase.

To customize the model, we add several more factors.

- We implement a limiting factor $L(t)$ for H' in the simple linear form $L(t) = mt + r$, where m is the limit of H' and r is a constant. WHen $H'(t) > L(t)$, we use the the value of $L(t)$ instead of $H'(t)$. Doing so allows the model to simulate governmental or other external restrictions on the rate of harvesting.

- We make the constant b in the differential equation for H' more flexible by dividing it into two different factors: b and a second harvesting constant. The new harvesting constant comes into effect at a certain starting time. This implementation allows the model to be modified easily to simulate the effects of a future technological innovation or other external change in harvesting rate. The difference between the limiting factor and the harvest constant is that while the limiting factor caps the rate of harvesting, the harvest constant sets no such limit but simply changes the rate of harvesting at a certain time.

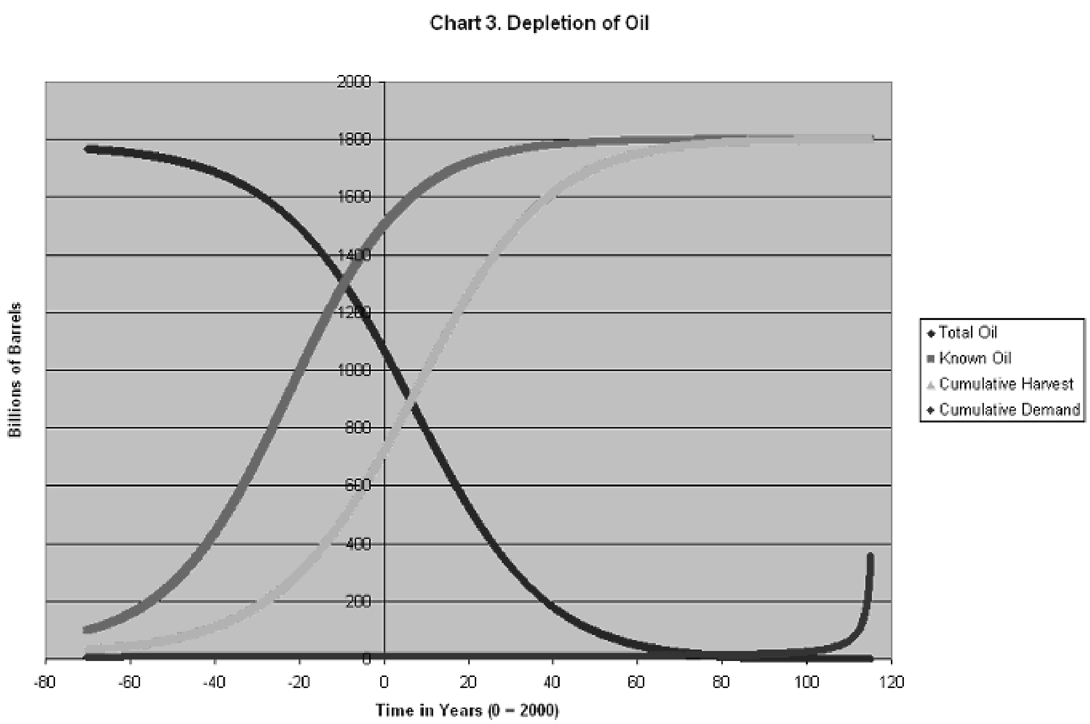


Figure 3. Depletion of oil (year 0 = 2000).

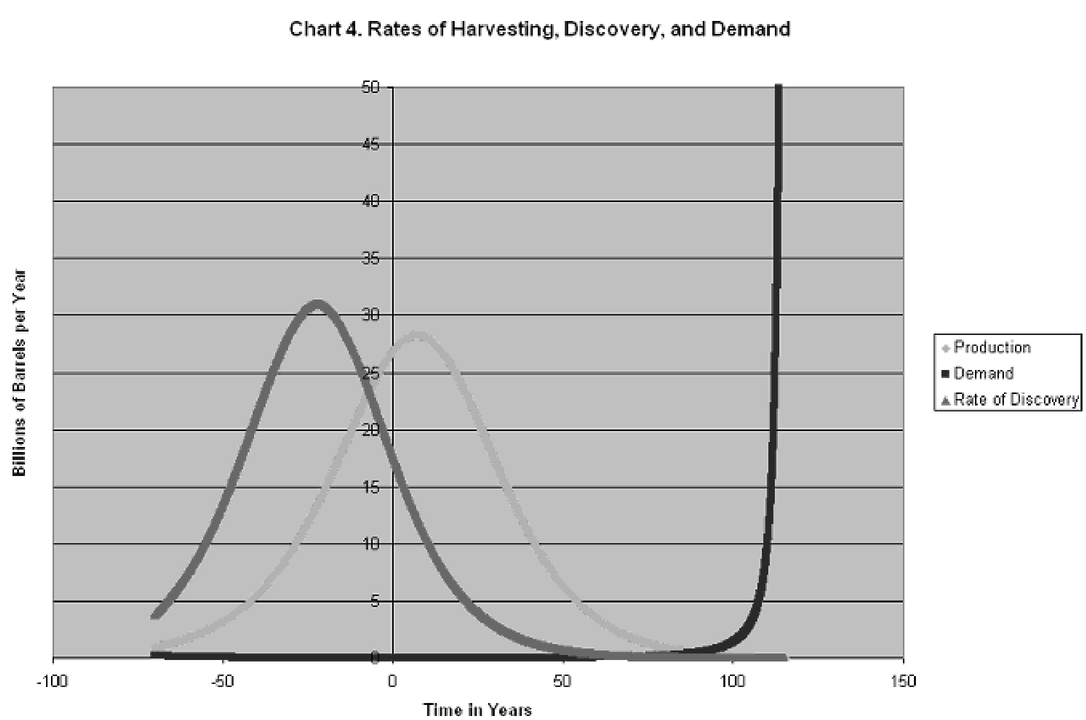


Figure 4. Rates of harvesting, demand, and discovery (year 0 = 2000).

Manipulations

To apply this model to hypothetical real-world situations, we manipulate the customization parameters. First, we imagine a moderate limit of 12 Gb/year on the consumption rate, beginning in 2010. **Figure 5** shows the result.

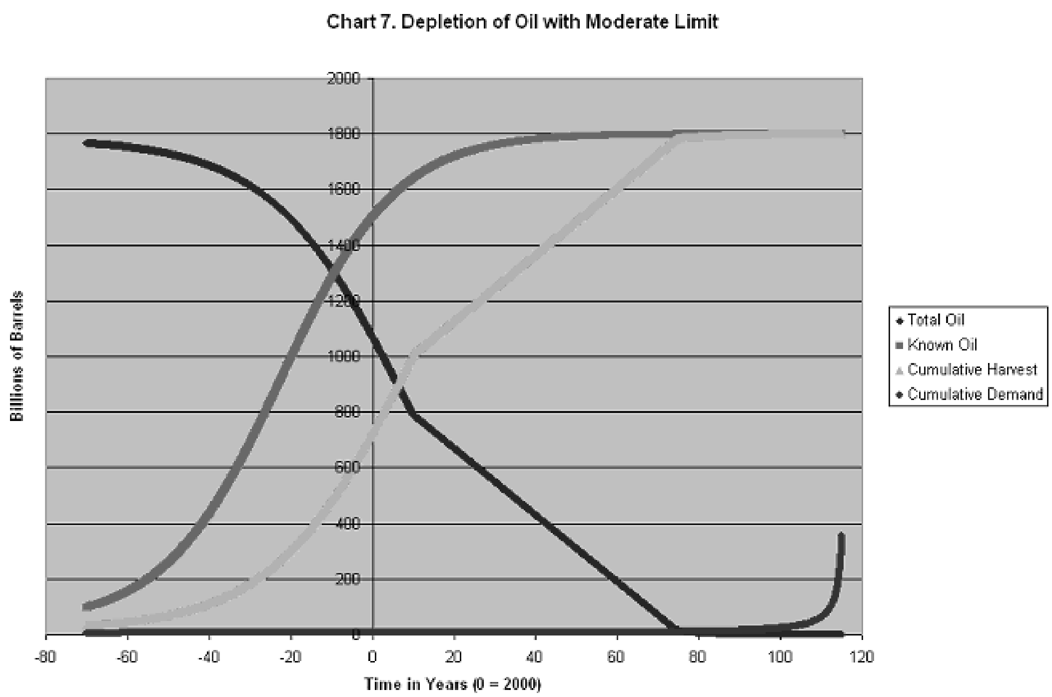


Figure 5. Depletion of oil with moderate annual limit of 12 Gb/yr.

Such a worldwide limit would be difficult to implement. Eventually, no matter how production is limited, the oil supply will run out (unless of course production is completely halted); in this scenario, the oil is depleted just about as quickly without a limit. All that can be manipulated is short-term versus long-term satisfaction of demand. A harsh limitation on harvesting would satisfy less of the demand for a longer period of time, while a less restrictive limitation would satisfy more of the demand but for a shorter period of time. Additionally, the sharp drops in rate of production in 2010 and 2075 would damage the world economy and deprive a large percentage of the population of the oil it needs. Thus, the problem of oil depletion cannot be mitigated, only manipulated.

An alternative, but less effective, policy would be a 60% downgrade in efficiency of oil-harvesting methods or technologies that occurs or is imposed suddenly in 2010. Mathematically, we decrease the harvest constant, b, by a factor of 0.4. Such a restriction would conserve oil for a longer period of time while causing a sharp drop in current production; however, the effect would be more gradual, producing an economic recession rather than economic collapse. A corresponding increase in efficiency of oil harvesting, due to technological innovation, would accelerate depletion.

To simulate a natural disaster, we make manual adjustments to $S(t)$ and $M(t)$. **Figure 6** illustrates the effects of a disaster in 2010 that destroys 400 billion barrels of known but unharvested oil (imagine wiping out a very productive oil field). After the natural disaster, the supply approaches a new carrying capacity of $M_0 - 400$.

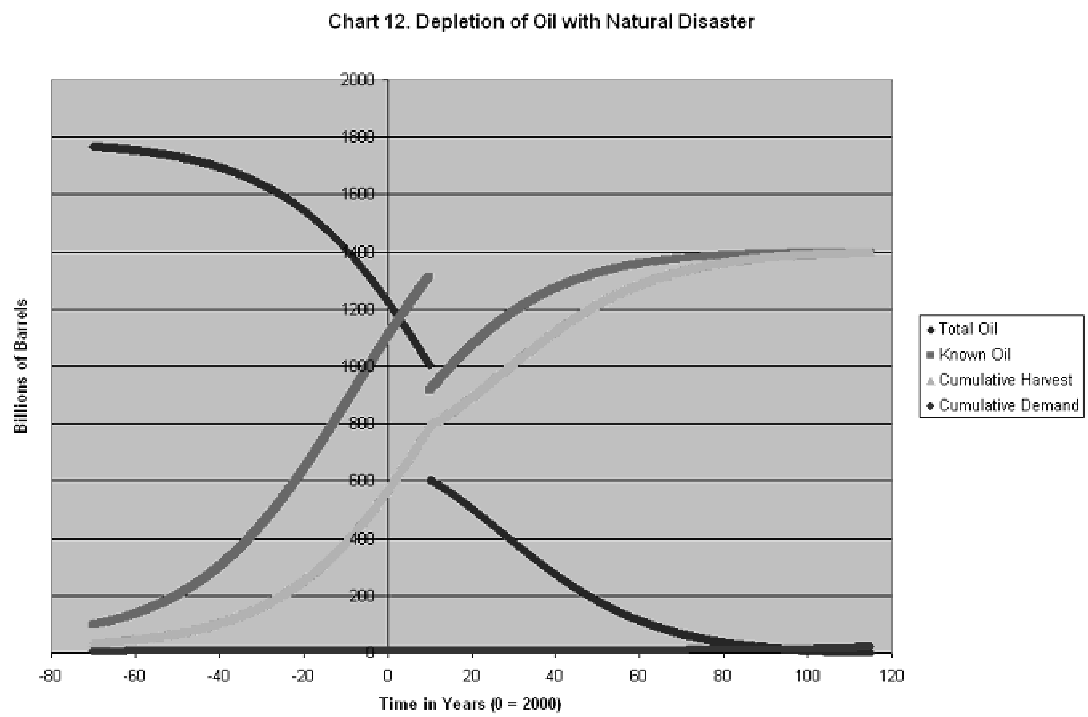


Figure 6. Depletion of oil with natural disaster occurring in 2010 to unharvested oil.

Theft, terrorism, or any oil-wasting (like an oil spill) would have a similar effect but on already-harvested oil rather than unharvested oil. Thus, H would decrease by the same amount as did S in the previous example, but $M(t)$ would not change. This kind of disaster would not cause the world's oil supply to deplete more rapidly, as the natural disaster did. In fact, it decreases the rate of depletion. However, it eventually reaches the same result.

For a future technological development necessitating more oil, that is, a sudden increase in demand, the model reflects the expected effect of shortening the horizon to oil exhaustion (**Figure 7**); the opposite is seen with a development, such as introduction of an oil substitute, that reduces demand.

Future Alternatives

We must develop another fuel source, or combination of sources, to replace oil. This fuel source need not be renewable; the U.S. has enough coal reserves to last for centuries. But there must ultimately be a switch to renewable fuel sources (such as nuclear, hydroelectric, solar, and wind). We assume that a

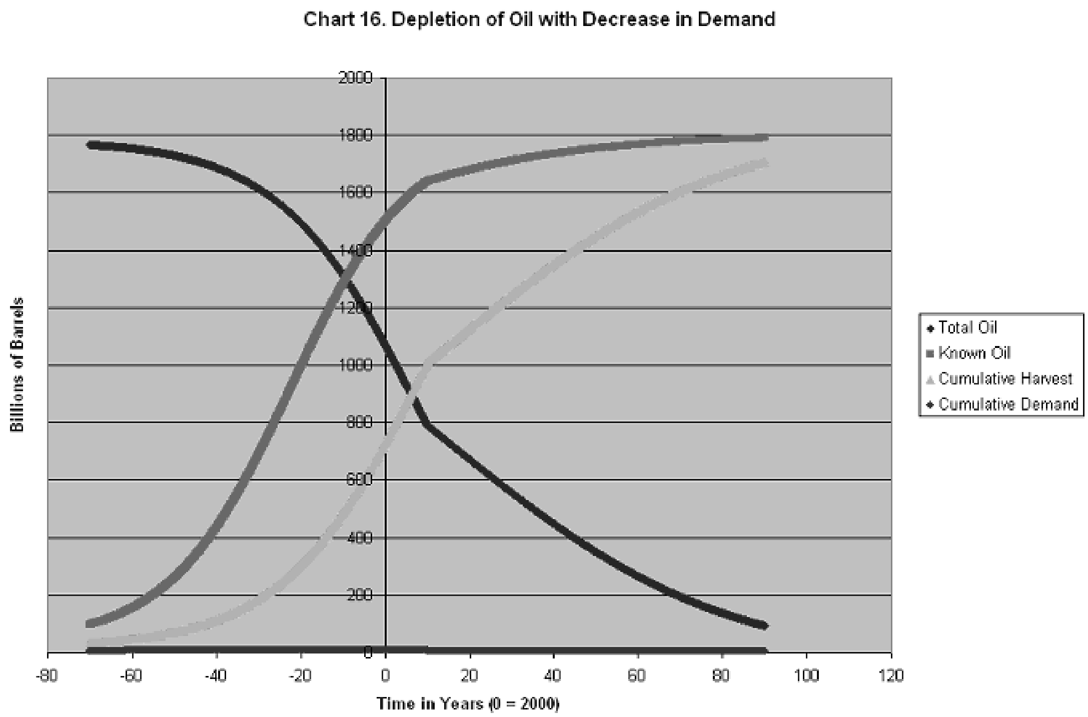


Figure 7. Depletion of oil with sudden increase in demand.

conglomeration of renewable and nonrenewable fuel sources ultimately completely replaces oil, long after the model's range.

We create a complex management policy to govern the harvesting of oil for the next century, starting in 2010. From then on, scientific alternatives to oil would be encouraged by any means necessary: government funding, taxes, etc. The world energy crisis would be given precedence above all other projects. Hoping that by the turn of the century this scientific development would be near completion, the model projects to conserve oil so that in 2100 no less than 10% of the initial world oil supply would remain: $0.1M_0$. With $M_0 = 1800$, the policy would provide that $H(100) = 1620$.

We manipulate the harvest constant rather than enforce a production limit. The total deficit is the same; the difference is when the deficit occurs and how quickly it grows. Imposing a rate limit fixes production until the rate limit exceeds the default H'; thus, short-term deficit and future deficit are equal. By decreasing the harvest constant, the short-term deficit is less than the future deficit, since production decreases over time. We assume a preference for short-term production over long-term production, since the sudden drop in oil production in 2010 caused by a limit would be much harder to cope with than a gradual drop caused by a decreased harvest constant.

We find that the optimal reduction of the harvest constant is by 70.5%, causing $H(100)$ to be as close as possible to 1620.

As soon as an alternative to oil is available and marketable, demand for oil will drop (we assume by a factor of 20). As long as production is not too low by this point, the limited oil supply will have sustained the demand. For example, **Figure 8** displays the result if in 2050 an alternative to oil is introduced and widely accepted, making the management policy obsolete.

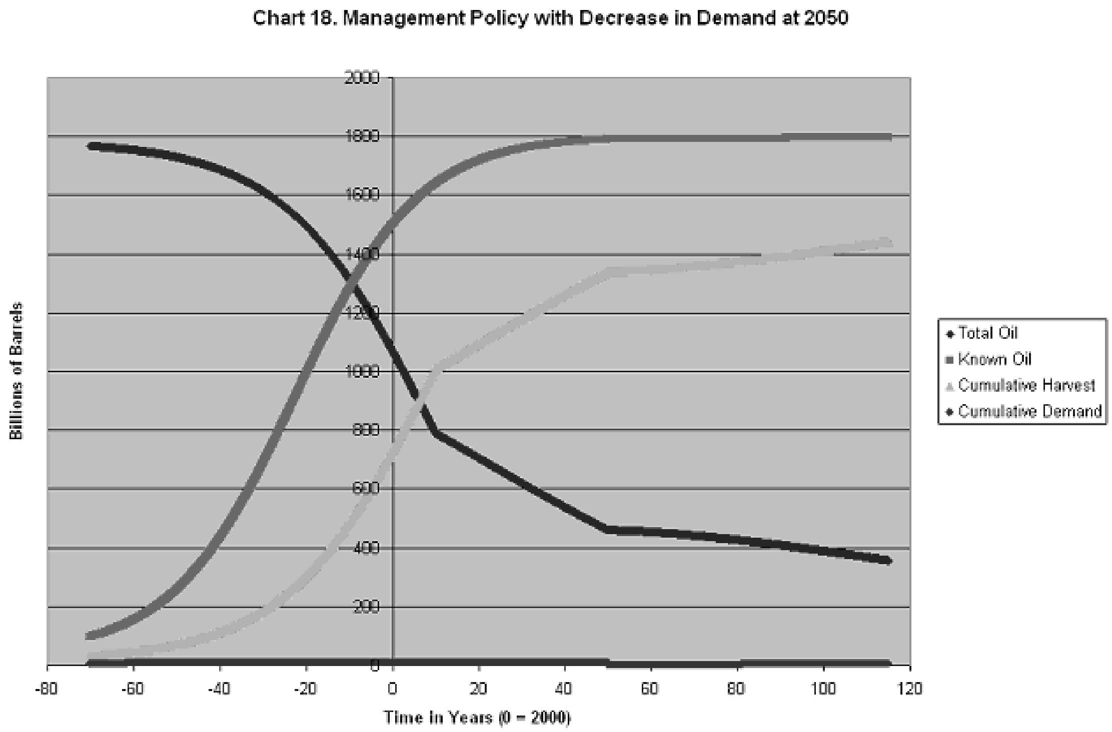


Figure 8. Effects of management policy with 95% decrease in demand in 2050.

Figure 9 shows the corresponding production scenario. Production drops suddenly by more than 75% at the beginning of this management policy until the appearance of an oil substitute. The drops caused by the change in the harvest rate constant and by the drop in demand are visible in 2010 and 2050.

The sudden drop in 2010 could be mitigated by severe conservation without reducing harvesting, so that at the time that production plummets there are large storehouses of unused petroleum that can appear on the market during the next few years and alleviate the economic crash and bankruptcies. Because it affects actual use instead of production, such a policy cannot be shown through the model, which is unable to distinguish between production and consumption. This sudden drop in production is necessary if any management policy is to be followed with any haste; even a gradual drop in production is bound to be accompanied by failing industries and economic recessions.

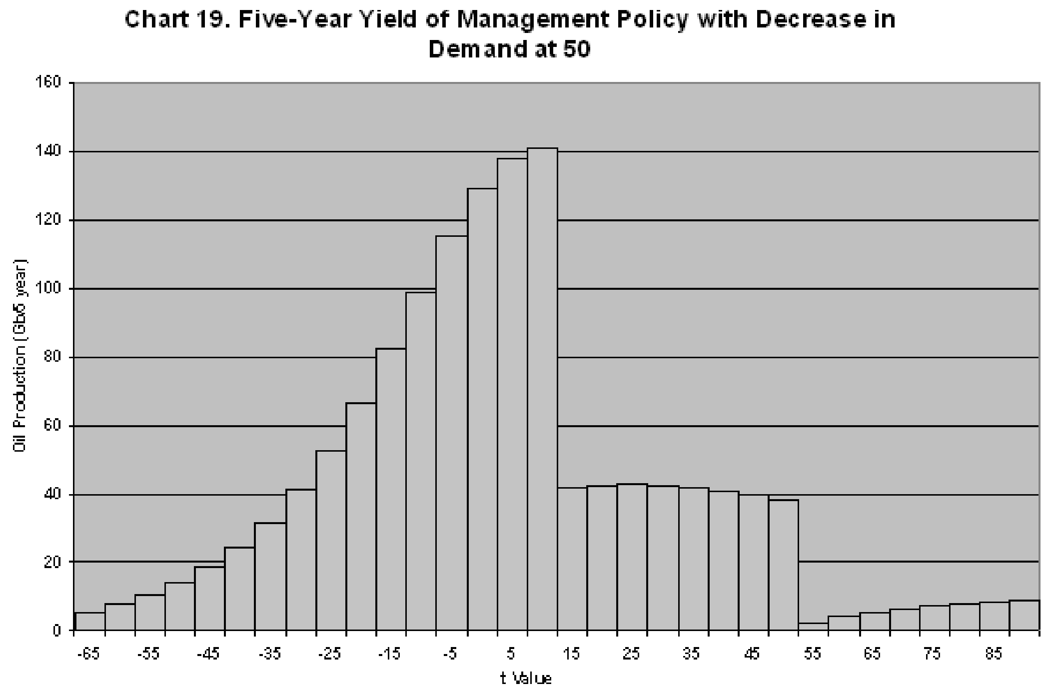


Figure 9. Production under management policy that decreases demand 95% in 2050.

References

Campbell, C.J. 1997. *The Coming Oil Crisis*. Multi-science Publishing Co.

________. n.d. Discovery trend: Cumulative discovery by region. `http://www.hubbertpeak.com/campbell/images/983fig5.gif` .

Ecosystems. 2005. M. King Hubbert. `http://www.hubbertpeak.com/hubbert/` .

Gever, John, Robert Kaufmann, David Skole, and Charles Vorosmarty (eds.). 1986. *Beyond Oil: The Threat to Food and Fuel in the Coming Decades*. Cambridge, MA: Ballinger. 1991. Boulder, CO: University of Colorado Press.

Ramirez, Vincent. 1999. Oil crises delay—A world oil price forecast. `http://members.aol.com/vrex/oil/price_forecast.htm` .

Daniel Lacker, Palmer Mebane, and Jonathan Giuffrida
with advisor John Barnes behind middle .

Judge's Commentary: The Outstanding Exhaustible Resource Papers

Ted Hromadka II
Hromadka & Associates
3151 Airway Ave, Suite H-2
Costa Mesa, CA 92626
`thromadka@hromadka-associates.com`

Introduction

The Interdisciplinary Contest in Modeling (ICM) provides an exciting and competitive environment for time-constrained innovative thinking. The judging of the papers was accomplished by several stages of review, culminating with a final set of papers that were reviewed for placement in the top two categories of Outstanding Winners and Meritorious Winners. Consequently, each paper ranked in the top two categories was refereed by at least 7 reviewers.

This year's ICM problem examined the eventual depletion of a nonrenewable or exhaustible resource, with the team selecting the resource to be analyzed. Consequently, several different topics were considered, ranging from oil resources to the availability of lumber, among other topics. Participating teams prepared several exciting investigations that demonstrated innovative thinking and good topic research into the underpinnings of the resource selected. To rank the contributions across the many selected topics analyzed, judges assessed the following qualities:

Summary: Adequacy of the one-page summary in describing the paper, its results, and its methodology. The summary was deemed to be a very important factor in the overall paper's scoring.

Science: Thoroughness of research into the literature regarding the nature and handling of the selected resource, alternatives to the subject resource, and new technology in increasing use efficiency or providing alternatives.

Modeling: Assumptions used, documentation of assumed parameter values, appropriateness of the governing mathematical equations, and conceptual model construct.

Analysis: Adequacy of model calibration to historic data and trends, analysis of model strengths and weaknesses, sensitivity analysis of modeling components, variations in modeling predictions due to changes in various types of society reactions to continuing depletion of resource.

Presentation: Quality of report text, graphics, and mathematical development. Clarity in paper presentation. Use of proper references and citations.

Each judge independently scored each paper on these qualities and then determined an overall score. There was little variability among judges' scores. A large majority of the papers demonstrated an in-depth investigation of the selected resource, and frequently innovative thinking was applied towards solving the resulting governing mathematical equations. The judges were uniformly impressed with the quality of research presented in the papers and the amount of work achieved in the very short time frame.

The Problem

The 2005 ICM Problem considered a highly relevant issue, the fate of a nonrenewable or exhaustible resource, with the problem incorporating linkages of consumption to economic, political, environmental, security, demographic, emerging technologies, and other factors. The first task for a participating team was to choose a resource and to understand the underpinnings of its nature and interdependencies on the factors that affect its rate of consumption. For example, such a resource typically has associated alternatives which, although possibly more expensive to utilize, could extend the life of the resource.

Oil was the most popular resource selected among the teams. Other resources considered included potable water, lumber, natural gas, minerals, uranium, and living space.

Modeling Approach

Ideally, once a conceptual model of supply and demand is developed and calibrated to the history of discovery, development, and consumption, the model can be used to predict the future of the resource under various conditions. Teams also did research on emerging technologies that provide alternatives or more efficient use of the resource. Using historical data, teams used regression to assess demand and supply trends.

Teams typically noted that the historic consumption trend was increasing with time and at an increasing rate. The recommended mathematical analogs were typically of the exponential type with parameters calibrated by a least-squares fit to data. Interestingly, although teams considered different resources, the resulting mathematical equations tended to be similar.

Many teams examined world population trends and developed relationships between resource consumption and world population. They paid little attention, however, to the resource-relevant distribution of the population growth; most consumption has occurred in developed nations, which have a different population growth trend than developing countries do. In any case, teams readily noted that at some point demand will exceed supply. For oil, this "undersupply" was estimated to occur between 2015 to 2050.

Many teams went no further; they did not focus on how their model's predictions would change under different global conditions and reactions to decreasing availability of the resource. In other words, they assumed that the future will reflect the past and that the world will not react to decreasing availability of the resource. However, a few teams did examine global influences on their model. For example, one team quantified the effects of an oil embargo by correlating the impacts of past embargos on oil consumption.

Many teams researched their resource and investigated alternatives methods to improve efficiency. Probably due to the limited time frame, however, they paid little attention to modeling the effect of implementation of these alternatives or efficiency improvements in delaying a possible "undersupply point in time."

Presentation of Results

The judges were impressed with the hard work that went into the paper write-ups. Excellent graphs and presentation of equations were typical. However, the presentation of the model development and modeling results varied greatly. In a few cases, the equations presented were not appropriate for the model description in the text of the paper; possibly, these incidents were simply carelessness or typographical errors. The top-ranked papers were of the highest quality in research into the literature, development of the mathematical model, approximation or solution of the governing equations, analysis of the recommended model, and presentation of the results.

Conclusions and Recommendations

For myself, being involved with the review and judging of the 2005 ICM Problem was an enjoyable experience. The judging demonstrated to me once again the continuing potential for young people to absorb new technology and to accept challenges to improve themselves by independent work. It gives me comfort to know that perhaps a few of the 2005 ICM Problem teams will be interested and challenged by this very relevant problem, and may one day discover new technology or policies that will postpone the so-called "undersupply point in time," or find an alternative technology that does not use nonrenewable or exhaustible resources.

The following are recommendations for future ICM Problem solvers:

- Write-ups: Check your equations to avoid a typographical error resulting in a relationship that is inconsistent with the relevant written description.
- Clearly state modeling assumptions and their limitations, and cite references to justify specific choices (such as ranges for modeling parameter values).
- Provide a relevant list of references that are clearly used in the text. Don't list references that you don't cite in the report.
- Do sensitivity testing of your model and discuss your testing results.
- Evaluate your modeling results and discuss their implications. If your results agree with the literature, say so and cite references; if not, state the disagreement and cite references.
- Double-check your grammar and do a spell-check of the report.

About the Author

Ted Hromadka II has three Ph.D. degrees, in the fields of applied mathematics, civil engineering (water resources emphasis), and advanced computational modeling. He is a Certified Hydrologist in both surface and groundwater, a registered civil engineer in the States of California, Nevada, Arizona and Hawaii, and a licensed Geoscientist in the State of Texas. His background includes concurrently holding both academic and consulting positions since 1973, with positions such as Research Hydrologist at the USGS, Research Associate at Princeton University, and Professor in the Departments of Mathematics, Geological Sciences, and Environmental Studies at California State University, Emeritus. He currently holds an adjunct position at the Wessex Institute of Technology, England, and is a Principal Hydrologist at the consulting firm Hromadka & Associates. Prior to this position, Dr. Hromadka founded and was Practice Director of the Hydrology & Atmospheric Sciences practice at Exponent Failure Analysis & Associates. You can learn more about Ted at `http://www.hromadka.net` .

Author's Commentary: The Outstanding Exhaustible Resource Papers

Paul J. Campbell
Mathematics and Computer Science
Beloit College
Beloit, WI 53511
campbell@beloit.edu

Introduction

This modeling problem was inspired by revisions that I was making in the last chapter in COMAP's overwhelmingly successful college-level textbook in applied mathematics for liberal arts students [Garfunkel et al. 2006]. That chapter, "The Economics of Resources," applies concepts and formulas from simple finance to assess how large the Earth's population may become, how long nonrenewable resources can last, and why renewable resources are extinguished in pursuit of economic gain. The revision includes M.K. Hubbert's model for exhaustion of oil and ends with a retelling of the ecological and human tragedy of the despoilment of Easter Island [Diamond 1995; Diamond 2005].

The compound interest formula serves as a basic model for growth of a biological population. The chapter also considers the logistic model, used by many teams in this year's ICM. The other main formula, for savings at interest, provides a way to estimate cumulative usage of a nonrenewable resource whose rate of use is increasing at a fixed rate.

The chapter introduces concepts and terminology of Michael Olinick (Middlebury College) [1991]: The *static reserve* of a resource is how long a fixed supply S will last at a constant annual rate of use U, namely, S/U years. The *exponential reserve* is how long the supply will last at an initial rate of use U that is growing at rate r per year (that is, growing exponentially), namely,

$$\frac{\ln\left(1 + \dfrac{S}{U}\right)}{\ln(1+r)}.$$

The UMAP Journal 26 (2) (2005) 179–184.

In keeping with the spirit of the book, the chapter illustrates these concepts with data on real resources. For example, as several teams in this year's ICM noted, the U.S. has recoverable reserves of coal that would last about 250 years—at the current rate of use. However, the coal will last only 85 years if the rate of use increases at 2.25% per year, as it did from 2002 to 2003. Exercises ask students to calculate the exponential reserves for oil and natural gas under the curious projections of the U.S. Geological Survey that world consumption will increase at respectively 1.9% and 2.2% per year through 2025.

Perhaps the most eye-opening exercise for students is the exercise that asks, "Can our energy problems be solved by increasing the supply?" This exercise asks students to compare the exponential reserve for an amount S of a resource with those for $10S$ and $100S$, under a rate of consumption that is increasing 2.5% per year (as U.S. oil use has been doing since 1973) [Nering 2001].

> **Exercise.** U.S. oil consumption in 2004 was 7.5 billion barrels (bbl), of which almost 60% was imported. As I write this in spring 2005, the U.S. House of Representatives has approved harvesting oil from the Alaska National Wildlife Refuge (ANWR) and the Senate is to take up the matter. Seismic estimates put the amount of recoverable oil in the ANWR at 5 to 8 billion bbl [Korpela 2002], which can become available starting in 5 years or so (so perhaps in 2010). How many months of supply of oil for the U.S. will the entire output of ANWR oil amount to in 2010, if U.S. usage continues to increase at 2.5% per year? (Predicts Korpela [2002], "That it will be drilled one day is a foregone conclusion, for when shortages appear every argument against drilling it will be dismissed by the public's clamour for oil. . . . The most appealing argument is to save it as long as possible, for once all efforts have been made to shift into a thrifty living, any oil from it would go further.")

Searching for data for the book's examples and exercises led me to the literature on models for exhaustion of nonrenewable resources.

The Outstanding Papers

The Outstanding papers go into these matters far more deeply than a book that cannot presume knowledge of calculus or differential equations.

The paper from the Olin School of Engineering examines data on U.S. water withdrawals and correctly concludes that the data are too variable to allow extrapolation. It then uses a common parametric family of models to model water withdrawal in the agricultural, industrial, and municipal sectors. This model incorporates a factor of "economic and political influence"—the fraction of under/over-draw of water relative to the renewable amount—taken to a power that is a parameter for the sector. This feature is a clever idea that avoids directly considering pricing, lets the three sectors "compete" for the same water, and (via different powers of the fraction) lets the model adapt to differential "pressures" in the three sectors. Using linear regressions of change in irrigation area and of population growth on log time and of GDP on time, plus data on U.S. water usage 1990–1995, the paper identifies the power parameters. These values vary considerably in size but the paper does not go into why this is the case. The paper then projects water usage for the U.S. and (with the same power parameter values) for an imaginary country with different initial withdrawal amounts and a different amount of renewable water. The projections are point projections, without any specification of range of variability. The paper concludes with a thorough roundup of literature on alternatives to drawing down nonrenewable water sources.

The other two papers treat oil. The paper from the Maggie Walker Governor's School follows the ideas of Hubbert [Laherrère 2000; Deffeyes 2001; 2005], fitting a logistic curve to oil discovery and a normal-distribution curve to oil production (a Hubbert curve is the derivative of a logistic curve). The paper uses a spreadsheet and Euler's method to project estimates for cumulative discovered, harvested, demanded, and remaining untapped oil. A key assumption that avoids considering price (and its volatility) is simple inverse proportionality of cumulative oil discovered and cumulative demand for oil, in the form $y = c/x$. The rationale offered for this assumption is simply the law of supply and demand; but the paper does not try to argue for the functional form used, nor for why past discoveries and past demand should be so affected. The paper concludes with applications of the model to various alternative scenarios, including disasters and alternative fuels.

The paper from East China University of Science and Technology begins with a simple system of linear first-order differential equations involving oil supply, demand, and price over time. Surprisingly, the (analytic) solution for demand fluctuates; with addition of an exponential forcing term, the demand function fits historical data well. The paper then fits demand as just an exponential plus a constant but does not return to the system to examine the consequences for supply and price (they are exponentially driven, too). The paper uses data for 1995–2003 and finds linear regressions of demand on world GDP, on population, and (rather obviously) on carbon dioxide emission from oil consumption. The linear fits are excellent in part because the time interval is short enough to mask the exponential trend in demand fitted earlier. The paper projects the date of oil exhaustion under various assumptions of growth in GDP and under a logistic model of population growth, as well as the allowable oil consumption under various rates of growth of carbon dioxide emissions. Utterly innovative is the paper's idea to allocate oil between generations (setting aside some oil for the future, by smoothly decreasing the amount of oil used) and between countries in terms of refinery capacity. Balancing conservation with development also suggests optimizing for GDP produced per barrel of oil. Countries vary enormously in the energy used per dollar of GDP; China uses 3 times as much as the U.S., and Ukraine uses 17 times as much. But some differences are unavoidable because natural resources (e.g., aluminum) help determine industries (energy-intensive smelting of ore, as in Jamaica). Countries also differ in how severely changes in the price of oil would affect their GDP (growth or decrease) [Bacon 2005, 48–52].

The two Outstanding papers on oil model the world as a whole; a finer analysis would disaggregate the world into geographical sectors, as the Olin paper did with water and economic sectors.

One concept only implicit in the Outstanding papers is *price elasticity of demand* [Nievergelt 1987]. How elastic is the demand for oil, water, or other exhaustible resources? For example, by how much did the 2004 increase in oil price from $30 to over $50 per barrel lessen demand in the U.S.? How much would a gasoline tax of $4/gallon (as in Europe) affect demand? For price P and demand Q, the *price elasticity of demand* is the relative change in price quantity divided by the relative change in demand:

$$\epsilon = \frac{dQ/Q}{dP/P} = \frac{dQ}{dP}\frac{P}{Q}.$$

The quantity is always nonpositive; values farther from 0 correspond to greater elasticity of demand; values above -1 indicate inelastic demand. For the U.S., elasticity of demand for oil is -0.06, about the same as for coffee or cigarettes; demand for oil is more elastic than for pet food or breakfast cereal, but less elastic than for ice cream, beer, or wine. Elasticity is less when there is no good substitute (not yet for oil in transport), when consumers spend only a little on it at a time (as for gasoline), and when it is seen as a necessity (as gasoline and home heating oil are) [Besanko and Braeutigam 2005, 44–52].

The elasticity of -0.06 is for the short term, during which consumers don't have time to adjust completely to the price change. In the long run, U.S. price elasticity for oil is -0.45, reflecting opportunity to plan for reduced use of oil.

Another consideration that the Outstanding papers do not handle (and it would be very difficult to do so) is that the market for oil is not completely subject to supply and demand principles. The market is partly manipulated by OPEC, whose members account for about 40% of production and (if they cooperate) can adjust their output to meet targets for world supply and price. Despite the run-up in oil prices from 2002 ($20/bbl) through 2005 (over $50/bbl), OPEC revenues today are far less than in 1980 ($66/bbl in 2004 dollars). One result—about whose other consequences one can speculate—is that per capita income in Saudi Arabia declined by 70% from 1980 to 1999 [Wikipedia 2005], the worst ever for any nation in history. Oil prices are denominated in U.S. dollars; the decline of the dollar since 2002 would have required a 30% increase in the price of oil just to maintain purchasing power of the producers in other currencies. A key question for producers is how to optimize present value of future revenues from oil—and over how long a time frame. As I explain in the chapter in *For All Practical Purposes*, if economic returns from other investments are expected to be higher, it is more profitable to pump oil now (all of it, if possible) and invest the proceeds (e.g., buy the U.S. economy). On the other hand, if the cost of oil can be expected to rise faster than the returns on other investments, it pays to keep it in the ground as long as possible.

Action?

The U.S. faces no countrywide shortage of water, but over 30 years ago it received a "wake-up" call about oil. America has adjusted to rising energy costs (gas, coal, and wood all go up with oil) by gradually improving efficiencies of industrial production, home heating, and home appliances—but not cars.

Americans feel they have a right to cheap gasoline and are highly averse to increased taxes on it; the American Automobile Association (AAA) demands that such taxes go for highways. The $0.50/gal energy conservation tax recommended in 1992 by presidential candidates Paul Tsongas and Ross Perot helped sink their candidacies, and Bill Clinton settled in 1993 for a $0.04/gal increase.

Last fall, I estimated the cost and economic benefit of putting photovoltaic cells or solar water heating on the roof of our house in Wisconsin. Despite enough sun and some incentives from Wisconsin, neither is a "good investment," and fewer than 10 of each are installed each year in the state. Is economics the only basis for economic decision-making? Are there peculiar economic, political, and particularly religious [Moyers 2005] considerations in U.S. culture that lead us to focus on short-term profits, economies, and pleasures?

But we are not alone in our indifference and in our ambivalence about providing for the future. I am currently living in a building in a Western European country completely dependent on oil imports, where gasoline and electricity cost almost three times as much as in the U.S. This country produces half as much solid waste per person, and uses half as much energy per dollar of GDP, as the U.S. An enormous solar collector field built by farmers south of town is economically viable because the government encourages expansion of non-fossil-fuel electric capacity by requiring electric companies to buy such power at twice what they can sell it at (there is probably a governmental subsidy to the utilities). Yet in our brand-new "energy-efficient" building (it has amazingly good insulation), the hall lights are not on timers, the light bulbs in our apartment are not compact fluorescents but incandescents and halogens (120 to 180 W—I won't go into their short lifetimes), and the environmental organizations on the floor below do not always completely separate waste office paper from trash.

Meanwhile, are Americans still debating the question, from my mother's generation or earlier, and long since definitively settled [Greenfield 2004; Rea et al. 1987]: Should they shut off the light if they leave a room for a few minutes?

References

Bacon, Robert. 2005. The impact of higher oil prices on low income countries and on the poor. International Bank for Reconstruction and Development, United Nations Development Program, World Bank Energy Sector Management Assistance Programme (ESMAP) Formal Report 299/05. `http://wbln0018.worldbank.org/esmap/site.nsf/files/299-05_HigherOilPrices_Bacon.pdf/$FILE/299-05_HigherOilPrices_Bacon.pdf` .

Besanko, David, and Ronald R. Braeutigam. 2005. *Microeconomics*. 2nd ed. New York: Wiley.

Deffeyes, Kenneth S. 2001. *Hubbert's Peak : The Impending World Oil Shortage*. 2nd ed. 2003. Princeton, NJ: Princeton University Press.

________. 2005. *Beyond Oil: The View from Hubbert's Peak*. New York: Hill and Wang.

Diamond, Jared. 1995. Easter's end. *Discover* 16 (8) (August 1995): 63–69.

________. 2005. *Collapse: How Societies Choose to Fail or Succeed*. New York: Viking.

Garfunkel, Solomon, et al. 2006. *For All Practical Purposes: Mathematical Literacy in Today's World*. 7th ed. New York: W.H. Freeman.

Editions of this book have been used by more than half a million students since the first edition in 1988, and translations have appeared in German, Greek, and Spanish.

Greenfield, Alan. 2004. A state of enlightenment. *Actuary Australia* (June 2004) 4–6. `www.actuaries.asn.au/PublicSite/pdf/actaust0406.pdf` .

Korpela, Seppo A. 2002. Oil depletion in the United States and the world. `http://greatchange.org/ov-korpela.US_and_world_depletion.html` .

Laherrère, J.H. 2000. The Hubbert curve: Its strengths and weaknesses. Oil and Gas Journal (18 February 2000). `http://dieoff.org/page191.htm` .

Moyers, Bill. 2005. There is no tomorrow. *Minneapolis Star Tribune* (30 January 2005) AA1, AA5. `http://www.informationclearinghouse.info/article7960.htm` . Correction noted by the *Washington Post*: "Although that statement ["after the last tree is felled, Christ will come back"] has been widely attributed to Watt [James Watt, first Secretary of the Interior under Ronald Reagan], there is no historical record that he made it."

Nering, Evar D. 2001. The mirage of a growing fuel supply. *New York Times* (4 June 2001) Op-Ed page.

Nievergelt, Yves. 1987. Price elasticity of demand: Gambling, heroin, marijuana, whiskey, prostitution, and fish. UMAP Modules in Undergraduate Mathematics and Its Applications, Unit 674. *The UMAP Journal* 8 (1987) 31–61. Reprinted in *UMAP Modules: Tools for Teaching 1987*, edited by Paul J. Campbell, 151–181. Lexington, MA: COMAP, Inc., 1987.

Olinick, Michael. 1991. Modelling depletion of nonrenewable resources. *Mathematical Computer Modelling* 15(6): 91–95.

Rea, M.S., R.F. Dillon, and A.W. Levy. 1987. The effectiveness of light switch reminders in reducing light usage. *Lighting Research & Technology* 19 (3): 81–85.

Wikipedia. 2005. Saudi Arabia. `http://en.wikipedia.org/wiki/Saudi_Arabia` .