Judges' Commentary: Multi-Hop HF Radio Propagation

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Introduction

The questions for Problem A of the 2018 MCM required student teams to examine high-frequency (HF) radio waves that can hop between the ionosphere and the surface of the Earth. They were asked to build a model for the multi-hop process, compare the reflectivity and number of hops on calm and turbulent oceans, consider the case of reflections off rugged terrain vs. smooth terrain, and calculate signal coverage for the case of a shipboard receiver moving on a turbulent ocean. The ultimate goal was to analyze how long a ship can remain in contact using multi-hop communication.

The majority of teams developed models that were based on existing formulas for energy loss of energy waves propagating both in free space and through the ionosphere. They also considered losses for reflected signals on both turbulent and smooth ocean water and rough and smooth terrain and were expected to determine the number of hops that HF radio waves can make between the Earth and the ionosphere. Most teams combined a series of existing models to work toward the most difficult question of how long a ship can maintain communication if it is traveling across a turbulent ocean.

We first discuss the problem as stated and then give an overview of

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approaches used. Next, we comment on issues associated with how results for the models were presented. Finally, we address a variety of important topics highlighted by this year's judges.

This article is not intended to provide an overview of the judging process. For more information about this important aspect of the contest, please see commentaries from previous years [Black 2009; 2011; 2013]. We highly recommend that both advisors and potential participants read some of the commentaries from previous years that do include information about the judging process, since that information can help motivate the importance of different aspects of a report.

The Modeling Problem

Prior to judging entries, judges carefully read the problem statement and read a random sample of entries to get an idea of what level of detail and complexity that student teams are able to achieve in the short time frame available for the contest. We are aware of time and resource constraints that teams face and understand the difficulty of the contest. We make every effort to make sure that the judging is focused on teams' efforts rather than on judges' individual expectations.

The Questions

The first part of the problem asked teams to develop a model for an HF radio signal, at 100 watts of power and below the maximum usable frequency (MUF), that reflects first off the ionosphere and then reflects off the ocean. Teams were asked to consider the signal strength after reflection off both a smooth and turbulent ocean. Finally, teams were asked to predict "the maximum number of hops the signal can take before its strength falls below a usable signal-to-noise ration (SNR) threshold of 10 dB." This last part of the question was one of the more important outcomes of this initial modeling process.

Part II asked teams to compare their findings from part I with the case of ground reflections off mountainous, or rugged terrain, and smooth terrain.

Part III of the problem asked teams to extend the model from part I to answer the question of how long a ship moving on a turbulent ocean can remain in contact using the same multi-hop path.

Finally, teams were asked to prepare a 1–2 page *IEEE Communications Magazine* synopsis of the results.

Models

Part I

To address the questions in part I, the majority of teams used existing models for basic radio wave loss. Most teams assumed that radio waves are single rays and that transmission loss comes from free-space loss, ionosphere absorption loss, and reflection loss. Many models utilized the Fresnel equation, which describes reflection (and refraction) of light, and Snell's law of reflection. This approach was used both in the attempt to model the first reflection off the ionosphere and the subsequent reflection off the ocean.

The level of detail in modeling the ionosphere varied from simple constant reflection losses to more-detailed models including ionosphere layers and the curved path taken during reflection. Many teams assumed that the ionosphere is isotropic, some considered different ionosphere layers, some considered the number of sun spots given the time of day/year and location on earth. The weaker models chose fixed parameters and never tested the sensitivity of that choice.

To address the question of rough vs. smooth ocean, a wider variety of approaches were considered. A large number of teams used a rough correction factor to approximate additional losses of the rough sea [Hendrick and Anderson 2008; Rice 1951], but there were a few unique and interesting approaches that included simulating the rough sea using a Phillips spectrum [Phillips 1957] or the Pierson-Moskowitz [Pierson and Moskowitz 1964] spectrum and estimating effects of wave height, wind speed, sea foam, and/or back-scattering.

After modeling the possible losses of one hop, teams could make a straightforward calculation to predict the maximum number of hops before the signal becomes unusable. It is important to note that top teams directly compared rough and smooth conditions, considered the impact of the level of roughness, and made sure that their final number was plausible.

Part II

Most teams directly extended to the question of terrain the approach that they had used to compare rough and smooth oceans, and most considered just permeability and conductivity given different media (wet soil, dry soil, or concrete). In the simplest form, this approach included revising the roughness correction factor; however, some top approaches considered more complex interactions, including peak diffraction (the Epstein-Peterson method [Epstein 1953]), shadowing, and large obstacle effects. Regardless of the approach, it was important to compare the results from the analysis in part II directly to the results in part I and discuss how changes to the terrain decrease signal strength and thus the number of hops.

Part III

Finally, the third part of the problem was the most broadly-stated. Teams interpreted the problem in different ways. Some teams considered the movement of the ship away from the point source, and a few included the small effect of the Doppler shift. Other teams considered waves on the ocean causing shaking or rocking of the ship's receiver and increasing signal losses or frequency shifts due to the swaying of the ship. Top teams included ship speed, maximum number of hops, ideal launch angle, and a specific calculation discussing the amount of time that a ship on a turbulent ocean can remain in contact.

Part IV

Part IV asked teams to write a synopsis of the results as a note for *IEEE Communications Magazine*. The teams that scored highest in this effort shared some important characteristics:

- The synopsis was not a simple restatement of the one-page summary, although it included some of the same introductory information.
- The synopsis included more-detailed information, including some of the equations and figures from the modeling process.
- Finally, the synopsis contained clearly-stated results.

Model Development and Presenting Results

One challenge that many teams faced in presenting their model and results was in clear citing of equations or ideas from source material. Because of the nature of the problem, many teams were able to utilize and combine existing models for HF wave propagation; however, many papers did not clearly differentiate which equations were their own creation vs. which equations were taken directly from a source. It is also important to clearly cite within the text the particular source used, with an exact citation to the page on which it appears.

The importance of citing sources also extends to figures, physical constants, diagrams, and code used in describing the model. It is very important that teams clearly cite and reference the source of *any* content they they did not directly create. Teams that gave credit to their sources had a distinct advantage in this year's competition.

Another important part of the contest is presenting results in a clear and coherent way. It is important to convey clearly the meaning of the model equations, figures, and results. Simple things such as making sure graphs are readable and well-explained, and clearly discussing the meaning of each figure, were an important feature of top papers. Even some very intricate models with seemingly advanced mathematics did not make the final rounds of judging, simply due to a lack of clear descriptions of logic and results. Another important aspect of clarity in results includes stating what parameter values are used for each graph or number generated by your model.

Along with discussing the sensitivity of the results, top teams also readdressed the question of the number of hops and signal distance for each type of terrain. Comparing the main result—the maximum number of hops—across terrains and adding a variety of outcomes from the model helped to argue the model's validity. These top papers also often plotted energy loss, number of hops, or signal power as a function of one of the other parameters, such as launch angle, surface type, or frequency, to demonstrate the effects of the parameters.

Other Comments

We discuss a number of topics that were subtle but important during the judging process. These themes came up during final judging and should serve to add to the important themes from past judges' commentaries [Black 2009; 2011; 2013; 2014].

General Modeling Comments

This year, very few teams took a truly novel approach to the modeling process. Most teams used equations and models from existing literature. One way to stand out is to add your own unique aspect to the model and to go beyond combining existing equations. This approach might include: computational simulations, data-fitting for parameters, or adding real-world complications. For example, this year a few teams stood out because they considered wind speeds and sea roughness in an interesting way, or simulated hops off a randomly-generated ocean surface in order to account for variations in surface waves. It is an extra challenge to make such a creative effort, especially given the limited resources; but developing a unique approach, if it is well explained, can help your paper rise to the top.

Another aspect of the modeling process is to introduce parts of the model. Ideally, each important piece of your model is introduced or explained. However, in some cases, either parts of the model appeared in the text with no explanation, or detailed explanations were given for mathematics that was never actually used in the paper. In both cases, the paper suffers and the mathematical model becomes less clear to the reader.

Finally, teams should be careful not to place important results in graphs and data in the appendix; these should remain in the main text, and the appendix should be reserved for code written or more-extensive data tables. Remember that *the appendix is not technically part of your paper*, so judges should not need to look at the appendix to understand your conclusions or see your results.

The Summary

The executive summary is a non-trivial component of your final submission. Remember, this is the first impression that you give to the judges reading your paper. A good summary should provide an overview of the problem, give the reader a good idea of the modeling approach, and should contain specific results. A reader should be able to read your summary and see a true overview of your modeling approaches and most important findings.

Sensitivity

Sensitivity analysis is an extremely important part of the modeling process. It helps to argue not only the validity of the model but also the extent to which your parameter estimates might affect your results. This year, as in past years, this section was given very little attention in most entries; therefore, teams that perform a careful sensitivity analysis will stand out. Most teams had a small section on sensitivity, but one lacking in clarity and depth. A somewhat quick analysis of how much changing an input parameter affects your final results can go a long way in how the paper is received. Also adding a discussion about how the sensitivity of the parameters affect the big modeling questions being asked can have a big impact.

Strengths and Weaknesses

Many teams this year included a section about their models' strengths and weaknesses, but few did an outstanding job in truly assessing their model. Going beyond a simple bulleted list and writing a critical commentary about your model is important. You should discuss where your model might have errors, overestimates, or underestimates, and how your assumptions might be refined. You should also clarify what your model does that other models may not. This is where you can talk about what you would do if you had more time to work on your model, along with motivations for those refinements. Also, remember that a more-complicated model is not necessarily a better model.

Writing and Formatting

Formatting, writing, and communicating mathematics is an extremely important part of mathematical modeling. It is always important to have proper grammar and spelling, along with clear overall structure. Here is a quick list of some *things to avoid* when writing a mathematical paper:

- Misspellings. Remember that sometimes spell check will auto-correct to the wrong word.
- Incomplete sentences or poor structure. Often this can be avoided by reading your paper out loud before submitting it, to make sure it sounds correct.
- Equations without a clear written description of the meaning.
- Inconsistent fonts throughout the paper.
- Putting too many of your words in bold font.
- Making your figures too small or labeling the scales and axes in a font that is to tiny as to be illegible.

Conclusion

The problem asked teams to develop a model to predict both the number of hops that an HF radio wave can make and the amount of time that a ship can remain in communication. Most teams combined existing equations and models, but the top teams worked toward a unique approach, used proper citing of sources, and were very clear in their model development and presentation of results.

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