Judges' Commentary: The Keep Right Papers

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Introduction

The questions for the Keep Right Problem of the 2014 MCM required teams to examine traffic rules and determine a way to assess different rules and practices. The specific task was to examine the "keep right except to pass" rule. Teams were asked to determine how to balance different concerns such as safety and traffic flow.

The majority of teams developed computational models to simulate traffic. The teams generally extended and adapted standard models to use multiple lanes, with the majority of drivers respecting various general rules. Many teams examined a variety of different rules, and one of the primary difficulties was to determine ways to analyze the results of the team's simulations.

This commentary is an overview of the approaches used by the teams on this problem. I first examine the problem itself and discuss the different approaches. Next, I discuss issues associated with presenting the results of a model. Finally, I provide an overview of various topics.

I do not provide an overview of the judging process. This is an important topic discussed in the commentaries from previous years [Black 2009; 2011; 2013]. I highly recommend that both advisors and students read some of the commentaries from previous years that do include information about the judging process, since that information can help to motivate the importance of different aspects of a report.

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The Modeling Problem

I discuss the questions posed as part of the Keep-Right Problem and offer an overview of the modeling approaches employed.

Prior to reading papers, the judges read the problem statement carefully. After reading the statement, each judge reads a random sample of papers. This is done so that the judges get an idea of what the teams are able to address in the short time allotted. We are acutely aware that the time and resource constraints make it difficult to address the problem, and we make every effort to let the teams' efforts—rather than the judges' expectations—drive the event.

The majority of papers made use of either a physics-based model or a cellular automata model. It was not uncommon to read papers that included both approaches, and many teams examined a variety of rules and made comparisons between the differences in their models.

The Questions

There are three parts in the original problem statement.

- The first part of the problem requires teams to "build and analyze a mathematical model to analyze the performance" of a traffic rule, the rule that requires drivers to stay to one side of the road and change lanes only to pass another car. The important aspect, which is open to interpretation, is to "analyze" the model. A wide range of approaches was attempted. At the extremes, some teams focused only on flow rates, and some teams focused only on safety issues. The best papers examined a combination and examined the relationship between these two aspects.
- The second part of the problem required the teams to make an argument as to whether or not the rule made sense in a country where drivers stay on the left side of the road. This was a question that received less attention. The vast majority of teams mentioned it or examined a few simulations, and most teams provided broad insight into the problem.
- The final part was to examine the impact of intelligent systems. The teams were asked to determine what would happen if all of the driving was automated, and the teams were asked to determine how the analysis might change. This part of the question was wide open to interpretation. The wording was vague, and it was interpreted in a wide variety of ways. This was an opportunity for a team to excel and do something that could set their paper apart.

Models

The majority of teams constructed a computational model and conducted simulations that made use of one of two approaches.

- The first common approach was a mechanics-based physics model.
- The second approach was a cellular automata model based on either the Biham–Middleton–Levine model [1992] or the Nagel–Schreckenberg traffic model [1992]. A small number of teams attempted to model traffic flow using a continuous model that results in a partial differential equation [Lighthill and Whitham 1955; Richards 1956].

In both the physics and the cellular automata approaches, the teams constructed computational models and ran multiple simulations under a variety of conditions. Many of the higher-ranked papers first presented a simple model and then looked at a succession of more complicated models. The teams often discussed the relative shortcomings of the models and proposed fixes to address the issues. In doing so, the teams constructed a sequence of models, with analysis, provided a critical review of their models, and posed adaptations to improve their models.

The teams often proposed a set of rules and governing equations for how to react over discrete time steps in their computational model. These rules could be adapted to examine different circumstances with respect to passing other cars and changing lanes. The models had a wide range of features such as how to handle the inflow and the outflow, as well as on- and off-ramps.

Presenting rules in a structured way is a challenge. Many teams simply provided a list of rules and relationships with little or no discussion or motivation. Reading these papers and trying to understand the models was difficult, and it was not clear what the teams actually did and what they simply repeated from what they found in the literature. The papers that were most warmly received by the judges included a narrative that described the relationships, discussed the motivation for the relationships, and offered insights into the individual terms. Additionally, many teams included a flowchart with a brief discussion of the chart in their narrative. Such aids were immensely helpful in trying to understand a team's model.

Finally, a smaller number of teams attempted to construct continuous models based on partial differential equations. Such models were problematic:

- First, it was difficult to incorporate multiple lanes in the resulting models.
- Second, the resulting models often led to nonlinear hyperbolic equations, and they often admit shocks in their solutions. Trying to approximate or find analytic solutions to the resulting equations is problematic.

Presenting Results

One of the biggest challenges in the problem was for the teams to present their results in a coherent, structured way. The majority of teams conducted Monte Carlo simulations. That is, they had to examine the results of multiple simulations, and those simulations were conducted using probabilistic models. Assembling the resulting data, analyzing it, and presenting the results is a difficult task.

The primary way that results were given was to provide sample means from multiple runs. Additionally, it was common to present the results from a single simulation. The results were often given in the forms of tables and graphical representations. Again, the tables often provided sample means, but it is was rare to provide any indication of the variation and distribution in the sample data.

With respect to graphical representation, a common figure was of a "time-space" chart representing the traffic density in both space and time. Common issues associated with such figures is that they were presented with little discussion or were poorly annotated. Teams that were able to provide good descriptions of every figure with proper annotation of the plots had a considerable advantage with respect to how a judge reacted to their paper. The kind of figures necessary to convey the important features of the complex data sets generated require detailed descriptions. A team that simply presented a figure with no discussion or had figures that lacked labels on the axes and titles was at a severe disadvantage.

Finally, most teams generated results that were sample data based on a probabilistic model. Few teams provided adequate statistical measures of their data. Even fewer teams provided any sense of the distribution of their data, with histograms and boxplots an exceedingly rare occurrence. This has been the case for as long as I have been a judge! Such basic practices to communicate and interpret stochastic data were acutely missing this year. It is clear to me that we are failing to provide for our students with respect to developing their understanding about what data is, thinking about sampling, and how to analyze data.

The few teams that recognized that they had data and provided the most basic of statistical analyses had an immediate advantage. Simply reporting a sample standard deviation was enough to make a paper stand out from the rest of the field! The few teams that discussed the distribution of their results in the slightest way immediately demonstrated an understanding of the nature of their work in a way that very few other teams were able to do.

Other Themes

In this section, I discuss a number of topics that are always important—and require discussion every year. First, I give a few notes on the summary and the introduction. Next, I discuss strengths and weaknesses, followed by sensitivity. Finally, I offer a discussion on writing in general.

The Summary and Introduction

Every year, the summary is given a nontrivial weight in the judging. The summary is the first thing that a judge will read, and it is the first impression. In the early rounds of the judging, the relative importance of the summary is magnified. By the later rounds, it is not as important; but that is partly because most of the papers still being read in the later rounds have well-written summaries.

My overall impression is that this year the summaries appeared to be better than in previous events. A good summary should do three things:

- It should provide a context and overview of the problem.
- It should give the reader a good idea of the general approach.
- Finally, it should include an explicit statement of specific results.

The next thing that a judge will read is the introduction. Many teams borrow heavily from the summary for their introduction, and that is a good thing. The introduction should include the same things that are in the summary, and we understand that the teams operate under difficult time constraints. There are some important additions, though, that should be in the introduction:

- First, the introduction should provide more *context and background* information about the problem. The best papers provided background information about the different modeling approaches and gave the reader a sense of the history of the problem. A team can set the tone for the paper by immediately demonstrating a basic understanding of the problem and letting the reader know what they think are the core ideas.
- The introduction should give an explicit statement of the *contents and structure* of the paper. The team should tell the reader what to expect and mitigate any surprises. After reading the introduction, the reader should know what to expect and understand the structure of the whole paper.

The Conclusion

One aspect of the paper that is often overlooked is the conclusion. It is not uncommon, even in the best papers, to have a short conclusion that provides little insight into the problem or the paper itself. The writing of the conclusion is an opportunity for the team to wrap up loose ends and to remind the reader of the full spectrum of activities that have been discussed in the paper.

When a paper ends with a weak statement and no overview of the results or approach, it is a let-down. The conclusion is the last chance impression on the person reading the paper. The teams should take advantage of the conclusion to remind the reader about the tremendous effort required to do the work they are presenting.

Strengths and Weaknesses

A critical part of the modeling process is to stop and engage in a critical view of the model. A team should identify the things that are best and the things that need to be improved, so that the team can demonstrate that they understand and can perform this aspect of the modeling process. No model is perfect, and each model can provide insights due to specific strengths.

Identifying the strengths and weaknesses of a model is something that is stated in the requirements and other materials for the MCM. Every year, the judges assign a relative weight for this part of a paper; and when we pick up a paper we expect to see insights into a model's strengths and weaknesses. Most teams include a separate section in which a bulleted list is given, although few provide an adequate introduction and transition for their lists.

We understand that there are enormous time constraints, so a bulleted list is good. The strengths and weaknesses is not just a bulleted list, though. There should be some commentary within the narrative itself that also discusses these issues. In this year's contest, a large number of teams developed multiple models. The best teams provided transitions that included a discussion of their motivation for their improvements, provided an explicit acknowledgment of the role of model refinement, and identified their model's relative strengths and weaknesses.

Sensitivity

Another aspect of modeling that is given a heavy weight is sensitivity of a model. This is an extremely important idea, yet it is the one part of the modeling process that is given the least attention. A team that performs a structured and detailed sensitivity analysis will stand out. Few teams do it, though.

Some teams have a section on sensitivity, but most of the discussion about sensitivity is either superficial or does not adequately demonstrate an understanding of what sensitivity is. It is important for a team to look at individual terms or parts of a model and ask what happens to the measured responses under some small change. That change can be as simple as looking at a small change in the value of a parameter or a more-nuanced look at what happens to a small change in the model itself. A team can perform a small bit of relatively easy analysis, and that can make a big difference in how a paper is received.

Writing

Writing and grammar are important. Every year, we see teams that appear to have created excellent models and probably performed an excellent set of analyses on their models. Unfortunately, in the eyes of someone reading a paper, the work cannot be better than the writing submitted. A paper with poor grammar or a poor overall structure will not receive a high rating.

Students need to have experience writing mathematics. We want our students to be able to share their ideas. Writing mathematics, though, is a skill, and it is a different skill from mathematics. Small things can make a big difference:

- All equations should be numbered. (This makes it easier to discuss the ideas in the paper with others.)
- A table of contents makes it easier for the reader to understand how a paper is structured.
- Equations and expressions should be part of a sentence and have proper punctuation.
- Incomplete sentences. Really bad.
- Spell-checkers can suggest the wrong word. Spell-checkers are not your friend.
- Transitions are absolutely vital.
- There is a difference between a citation and a reference. Both are necessary.

Students do not get many opportunities to engage in the full process of creating and bringing together complex mathematical ideas, performing an analysis of their ideas, and writing a complete report of all of their work. It is not something that they will do outside of a mathematics course. The inclusion of an expression in a sentence is not intuitive, and students will not know how to do it simply by reading or picking it up on the street. Most students have no issue about including both citations and a list of references in papers for their other courses, but they will not automatically bring that skill into mathematical writing unless explicitly reminded that it is still important.

Students need practice writing in a mathematical context. Writing a full report such as those submitted in the MCM is fundamentally different from almost all of the writing that they normally do. Advisers who take the time to have students practice this skill will be doing an enormous favor for their students, and that practice will aid them well beyond just this event.

Conclusions

To address the Keep-Right Problem, teams were required to develop a model to describe traffic flow. The majority of teams made use of a computational model and examined the results of Monte Carlo simulations. Most teams extended existing models, and the primary differences among entries was in the way that the teams interpreted and presented the results of their simulations.

This was a difficult task in that for most cases the data generated is stochastic in nature. Few teams examined their data using formal statistical techniques, and few were able to present the nature of the variation in their data in a formal manner. Presenting the results of this kind of data in a formal manner is a difficult task, and those who were able to make good use of figures and tables and were able to discuss them in a structured manner had an advantage in the way that their paper was perceived.

In addition to the problems associated with discussing and presenting their model, the teams had to address other important tasks. As is the case every year, the importance of the summary, conclusion, and writing cannot be over-stated. Also, the importance of providing a critical view of the model is vital, and this year the importance of determining the sensitivity of the model had a larger weight than usual.

References

Biham, O., A.A. Middleton, and D. Levine. Self-organization and a dynamical transition in traffic-flow models. *Physical Review A* 46 (10) (1992): R6124–R6127. `http://arxiv.org/pdf/cond-mat/9206001.pdf` .

Black, Kelly. Judge's Commentary: The Outstanding Traffic Circle papers. *The UMAP Journal* 30 (3) (2009): 305–311.

________. Judges' Commentary: The Outstanding Snowboard Course papers. *The UMAP Journal* 32 (2) (2011): 123–129.

________ Judges' Commentary: The Ultimate Brownie Pan papers. *The UMAP Journal* 34 (2–3) (2013): 141–149.

Lighthill, M.J., and G.B. Whitham, On kinematic waves. II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society of London* 229 (1178) (1955): 317–345. `https://amath.colorado.edu/sites/default/files/2013/09/1710796241/PRSA_Lighthill_1955.pdf` .

Nagel, Kai, and Michael Schreckenberg, A cellular automaton model for freeway traffic. *Journal de Physique I France* 2 (12) (December 1992): 2221–2229. `http://hal.archives-ouvertes.fr/docs/00/24/66/97/PDF/ajp-jp1v2p2221.pdf` .

Richards, P.I. Shock waves on the highway. *Operations Research* 4 (1) (1956): 42–51.

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Kelly Black is a faculty member in the Dept. of Mathematics and Computer Science at Clarkson University. He received his undergraduate degree in Mathematics and Computer Science from Rose-Hulman Institute of Technology and his master's and Ph.D. degrees from the Applied Mathematics program at Brown University. He has wide-ranging research interests, including laser simulations, ecology, and spectral methods for the approximation of partial differential equations.