Judges' Commentary: The Goodgrant Challenge Papers

David H. Olwell
Dean and Professor
Hal and Inge Marcus School of Engineering
Saint Martin's University
Lacey, WA
dolwell@stmartin.edu

Carol Overdeep
Mathematics Dept.
Saint Martin's University
Lacey, WA

Katie Oliveras
Mathematics Dept.
Seattle University
Seattle, WA

Introduction: New Problem Type

The 2016 MCM introduced a new modeling challenge—Problem C—that is best described as data insights. Problem C focuses on mathematical modeling challenges associated with large, messy data sets. In this sense, techniques stemming from statistics and pattern classification will play a larger role in creating a mathematical model on this problem than in Problems A and B.

While not a "big data" challenge, in the sense that teams need to develop specialized computer science-based data-handling algorithms and analysis techniques, or have access to high performance computing platforms, Problem C provides teams with an opportunity to encounter real-world, challenging data sets that have interesting characteristics.

Naturally-occurring complicating factors such as data set size, blend of data types, breadth of representation in data elements, cross-disciplinary

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sources, time series dependencies, censored or missing data, and others present themselves, depending on the specifics of the problem.

Problem C was intended to motivate the same high levels of team exploration, discussions, and decisions as have occurred in past MCM challenges.

The Goodgrant Challenge Problem

(The text of the problem is given in the report on the 2016 contest, and we do not repeat it here.)

Overall, this year's problem appeared to present a challenge for teams. While there is an extensive literature on collegiate success, several of the questions posed in the problem are not directly represented in these sources. The data set was large, and as with most data sets, contained missing data. Additionally, the problem required students to define two key elements:

- "improve(d) undergraduate performance of undergraduates in colleges and universities in the United States," and
- "an estimated return on investment (ROI) defined in a manner appropriate for a charitable organization."

The most successful teams carefully addressed the problem of defining what constituted improved performance. They prepared their data, addressing missing data and in many cases reducing the dimensionality of the problem. Once they defined their response, they used mathematical and statistical modeling techniques to determine which predictors influence the response, and how. They then further modeled how funding affected these predictors, as necessary. Once they had a model that allowed funding to predict the improvement in student success, they then used optimization techniques to allocate the available funding to schools over the five years. The most successful teams carefully defined the return on the Goodgrant investment, and showed how they had maximized it.

As always, judges valued well-written and well-illustrated papers that carefully followed the contest directions.

The Judging Process

For this inaugural problem, many new judges were introduced, including a new head judge (Dr. Olwell), a new regional judging site (Saint Martin's University), and a new final judging team. The judging process for Problem C followed the usual scheme of triage, screening rounds, and final judging. That process is described well in earlier judges' commentaries [Black 2009; Black 2011; Black 2013].

Defining the Problem

The instructions for the contest require teams to restate and clarify the problem. The best papers submitted for Problem C identified the required elements of the problem first. The best papers carefully established the goals of their model before moving on to actual data analysis or model fitting. They thought about the context of the problem, possible motivations of the Goodgrant Foundation, and whether the focus of the model would be on improving students or institutions. Of these issues, defining Return on Investment presented the greatest challenge. There were many interesting and creative approaches in the better papers.

The team's proposal was required to have the "highest likelihood of producing a strong positive effect on student performance." Very few papers considered the probabilistic elements of the problem or its solution; those that did so tended to be ranked higher. Again, the problem statement identified that student performance was an important element. How did teams model student performance? Economically? Academically? Did they consider the quality of the student at entry, or just the value added by the collegiate experience?

One of the design criteria for MCM problems is to avoid presenting a problem where there is a single "correct" solution. In the inaugural Problem C, the teams were left the task of defining the objective of their modeling effort. Accordingly, there was no "correct" answer, but there were many good analyses.

The problem was worded so that the extensive literature on measuring educational performance would not provide an apparent analysis template. Many of the better papers did review the literature in their papers, which is always helpful, and used elements of it in their modeling.

Preparing the Data Set for Analysis

The data set was taken from U.S. government statistics and had many of the challenging aspects often associated with such data. There were missing fields in many of the institutional records. Some were missing purposefully: For example, a school might use SAT scores or ACT scores for admission, but not always both. Other data might be modeled as missing at random. Very few of the institutional records were complete.

The better papers attempted to deal with missing data. Some imputed values for the missing data using means or medians, while others attempted to predict the missing values directly based on other variables. Papers that merely dropped institutions with incomplete data from consideration were not evaluated highly.

There was also a large number of variables, with extensive linear correlation as well as nonlinear patterns. A large number of papers attempted some dimension-reduction techniques. These frequently included principal component analysis and factor analysis, as well as other techniques.

These dimension reduction efforts produced new variables for the modeling effort. Teams that attempted to interpret these new variables in the context of the problem were viewed very favorably by the judges.

Depending on the definition of the problem, some variables could be considered responses and others predictors. It was considered better technique to separate responses and predictors when doing dimension reduction.

Judges did not assign any extra weight to teams that attempted to supplement the provided data set with information from other sources, either longitudinally or with extra variables for each school. The expectation was that teams would use the provided data. Two reasons drove this:

- The first was equity— having everyone with the same starting point; and
- to avoid the challenge of judges having to verify and validate external data sources.

Building the Model(s)

There were a variety of techniques possible for modeling, and even more were used. Judges had no preferred technique, but rather looked for clarity of rationale, internal coherence in the techniques chosen, and thoroughness in the validation and interpretation of the model.

An important element of this was the careful listing of assumptions. The best papers not only included the assumptions that they had made about the problem and data, but also addressed the assumptions implicit in the techniques they were using. For example, those using regression addressed the corresponding standard assumptions; these in turn were checked during the model validation.

The data set provided for the problem did not include many inherently financial variables. Teams that developed a model that said a non-financial predictor variable improved student performance were expected to develop a second model that showed how funding affected that predictor. Many teams identified candidates for funding without explicitly modeling how the funding would affect the predictors that in turn affect performance.

Judges looked for a model that explained how funding affected whatever measure(s) of student performance the team developed, and how the team explicitly calculated return on investment for its proposed funding.

Model Analysis

Teams that used statistical modeling methods were expected to include standard diagnostic plots, and to use those plots to explore if the assumptions of the models looked reasonable.

Careful consideration of ways to illustrate the methodology and results made it easier to use visualization to analyze and validate the model. Two of the Outstanding papers were illustrated particularly well.

The problem instructions included the required element of identifying strengths and weaknesses. Higher-ranked papers carefully identified both strengths and weaknesses. Many papers mitigated shortcomings in their approach or models by frankly identifying them in the weaknesses, and were ranked higher as a result.

Model analysis also includes model interpretation. What does the recommended solution mean in the context of the original problem?

Model Justification/Validation

When a team selects its final model, it should carefully justify why that model is appropriate and valid. That justification can be based on the literature, on the analysis of the model and its results, or first principles from the underlying context of the problem. Whatever the justification, it needs to be explicit.

Sensitivity analysis is a useful way to build confidence in the model and to identify the effects of departure from the model assumptions. Most statistical techniques have standard diagnostic tests that can be used to support arguments for model validity.

Communicating Results

As papers pass through the various stages of judging, the importance of good communication increases. A highly-ranked paper is well organized, well illustrated, well documented, and includes all the required elements.

The summary of results should present the results, not just a description of the methods to be used. Even this year's Outstanding papers could have been improved by explicitly including the return on investment in their results.

Even papers with brilliant mathematics cannot overcome poor communication of results. Previous judges' commentaries have identified specific ways to improve communication when writing an MCM paper [Olwell 2013]. They apply as well to Problem C!

Conclusions

The inaugural Problem C proved to be challenging, and the 1,875 student teams that submitted papers produced many interesting approaches. Of those 1,875 teams, four were recognized as Outstanding, and another eight as Finalists.

The reader will see that the commentary for this year's Problem C addresses many of the contest topics and issues from MCM problems in previous years. There are also some new issues that arise from the data based nature of this problem.

The judges anticipate that Problem C will provide a challenge in the future. As Problem C develops its own history, we expect team performance to grow even better.

References

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About the Authors

David H. Olwell is Professor and Dean, Hal and Inge Marcus School of Engineering, at Saint Martin's University. He earned a B.S. at the United States Military Academy, where he studied mathematics, and an M.S. in Mathematics, M.S. in Statistics, and Ph.D. in Statistics from the University of Minnesota. He has previously been on the faculty of the U.S. Military Academy and the Naval Postgraduate School. Dr. Olwell has been a problem author for both the MCM and HiMCM, and has been an MCM judge for a decade. He was the principal author of Problem C as well as the Head Judge.

Carol Overdeep is an Associate Professor of Mathematics at Saint Martin's University. She earned a B.S. in Mathematics at the University of Puget Sound, then began her professional career in the ADCAP MK-48 torpedo software validation group at Hughes Aircraft Company Ground Systems Group in Buena Park, CA. While working for Hughes, Dr. Overdeep earned an M.A. in applied mathematics from California State University–Fullerton. After 15 years as a torpedo systems engineer for Hughes Aircraft Company and the Naval Undersea Warfare Center Division Newport, Rhode Island, Dr. Overdeep earned an M.B.A. and then a Ph.D. in Applied Mathematics at the University of Rhode Island.

Katie Oliveras is an Assistant Professor of Mathematics at Seattle University. She earned a B.S. in Systems Science and Engineering from Washington University in St. Louis, and then M.S. and Ph.D. degrees in Applied Mathematics from the University of Washington. As an undergraduate, she participated in the MCM competition; and she has served as a team advisor for MCM teams at Seattle University.