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Problem Chosen

## B

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### 2013 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to your solution paper.)

Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

We build a network model to simulate Chinese water transferring systems and determine the strategies of water storage, movement, desalination, and conservation with “minimum-cost maximum flow” and “simulated annealing” algorithms.

**Model Structure:** Our network is made up of 30 nodes, each of which represent a province. The node has its own attributes: coordinate, altitude, water demand, water maximum supply and desalinating-viability. These nodes are connected by two directed edges if they are contiguous. These two edges represent the possible watercourse between these two provinces. The edges are directed because of different cost considering. We build more sub-models to quantify the cost. Besides, each node have a dummy node linked because of water conservation. The water conservation have the same effects of supplying water at the conservation cost. Also, if the node is desalinating viable, there is an extra dummy node linked, performing the task of supplying desalinated sea water at the desalination cost.

**Main Findings:** In the economic-focusing scenario, we find that constructing the water transfer and production system will cost  $9.344 \times 10^{10}$  RMB per year in the coming 50 years, which is only 0.197% of China’s GDP in 2011. We also find water desalination unfavorable and should be the last choice unless its cost reduces dramatically. In the environment-and-society-focusing scenario, we find that if desalination becomes cheaper and does little harm to the environment than water transfer, it can solve nearly 50% of the water shortage problem in China. Finally we give four suggestions to Chinese government: investing on desalination, promoting water conservation, constructing infrastructures of water transferring, and doing more research on water arrangement and planning.

There are several features of our study that may make it distinguished:

**Future Predicting and Parameter Estimation:** All of the parameters are quantified by our various sub-models, either using multivariate regression to forecast the water demand in 2025 or a hydraulics engineering model to estimate the energy consumption cost of water transfer.

**Sensitivity analysis:** We allow several key parameters to change to test the stability of our prediction as well as provide some insight of the implication of our results

**Multi-targets:** We consider two scenarios: one is economy-focused and the other is environment-and-society-focused.



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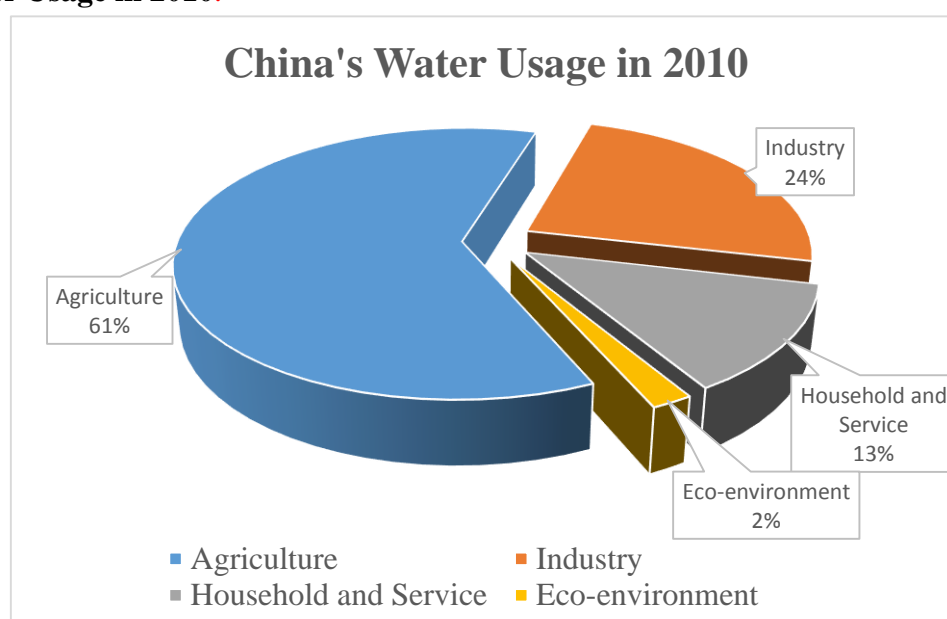
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## Introduction

Fresh water is a scarce resource which confines the development of economy, society and environment and the importance of arranging a country's water resource cannot be exaggerated.

In 2010, China's total amount of water use reached  $6.022 \times 10^{11} \text{ m}^3$ , with an average annual growth rate of 1.61% since 2003. The data of total amount of water use are calculated separately in four sectors, namely Agriculture, Industry, Household and Service and Eco-environment, and then added up by National Bureau of Statistics of China<sup>①</sup>. The ratio of each component in 2010 is shown in **Figure 1 China's Water Usage in 2010**.



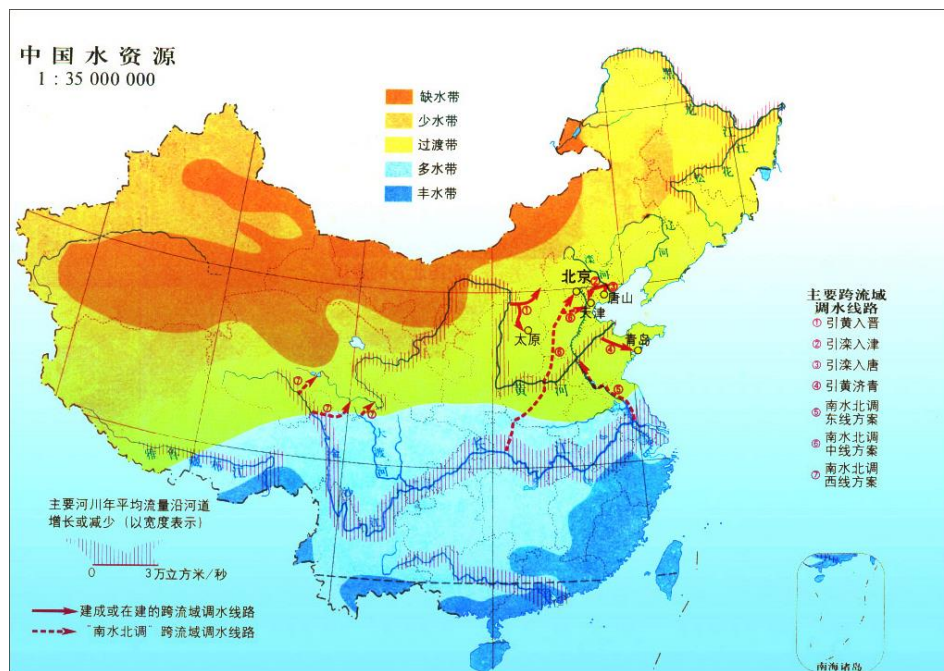
**Figure 1 China's Water Usage in 2010<sup>②</sup>**

However, the spatial distribution of China's water resource is extremely uneven (See **Figure 2**). The southern part of China possesses much more water resources than the northern part, so it is not rare for the south suffers from floods while the north droughts at the same time to happen.

<sup>①</sup> *Environmental Statistics Summary of China, 2010*

<sup>②</sup> Data are from *Environmental Statistics Summary of China, 2010*





**Figure 2 Regional Distribution of Water Source in China<sup>①</sup>**

Our study purports to provide a water arrangement strategy for China in 2025 with economic, social and environmental elements taken into account. We focus on how different provinces, with distinct endowment of water resource, act and interact to satisfy their demand for water use. The result of our model provides the amount of water transferred from one province to another, how much water are produced by new technologies such as desalination, the total cost of the whole transferring program and the price of water in different provinces.

The main results of our study are presented in the study of two scenarios: In the economic-focusing scenario, we find that constructing the water transfer and production system will cost  $9.344 \times 10^{10}$  RMB per year in the coming 50 years, which is only 0.197% of China's GDP in 2011. We also find water desalination unfavorable and should be the last choice unless its cost reduces dramatically. In the environment-and-society-focusing scenario, we find that if desalination becomes cheaper and does little harm to the environment than water transfer, it can solve nearly 50% of the water shortage problem in China. At last we give four suggestions to Chinese government: investing on desalination, promoting water conservation, constructing infrastructures of water transferring, and doing more research on water arrangement and planning.

## Model Conceptualization

Each province is abstracted to be a point, which jointly form a network. Islands

<sup>①</sup> <http://www.zge21.cn:8083/zyzy/czpd/kczy/shang/dl/1/03/rj-kebiao/1/mtzs/szy.jpg>



isolated from mainland China, namely Hainan, Hong Kong, Macau and Taiwan are excluded from this network because the cost of transferring water across the sea is too high. We adopt the province-based partition considering three facts: i) in China's command-and-control economy, provinces are the basic atoms for economy regulation and planning, including the planning of water use and related infrastructure construction; ii) most of the statistics are on the provincial level; and iii) we need to keep our analysis manageable. What's more, we assume two provinces that are not contiguous are not connected in the network so they cannot directly transfer water with each other, which obviously makes sense.

Water inflow is the amount of water that can be utilized, and water outflow is the amount of water that are utilized. Water inflow consists water supply, i.e. available water resource from natural occurrences, such as rainfall, melting glacier and underground water that can be utilized by human, seawater desalination, and the water transfer from other provinces. There are other ways to generate water inflow, such as pumping underground water. However, since underground water are highly over-withdrawn in China, especially in the northern regions(Wang, Meng, Su and Guo 2012), we do not allow for producing additional water resource by pumping underground water in our model. Water outflow includes the domestic demand for water, i.e. water used for agriculture, industry, household and service and eco-environment maintenance, the water transferred to other provinces, and the unused water supply. Used water supply is the water supply are actually used, and unused water supply is the rest of water supply, and the sum of used and unused water supply must equal to total water supply.

Water balance suggests the water inflow equals to the water outflow in every province. This is analogous to your cash flow: the inflow of cash may consist your income, the money you borrow from your friends, and the outflow may consist your daily consumption, money used for paying the mortgage and saving; however, every cent coming into your wallet will eventually go out.



**Figure 3 Water Balance**

The cost of allocating water resources come from two parts: direct costs, i.e. the



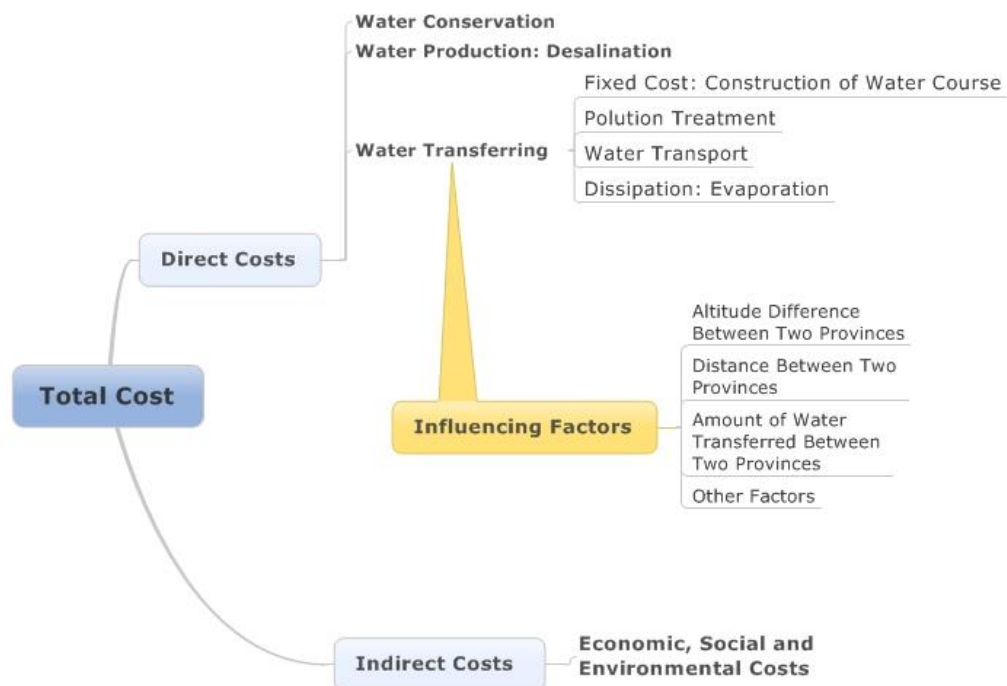
cost of producing and rearranging the water and indirect costs, i.e. the other cost incurred because of this very production and rearrangement. Direct costs includes the cost of water transferring, water production and water conservation. The cost of water transferring includes i) fixed cost, i.e. the cost of digging the watercourses, construction of related facilities, buying the land and resettlement, etc., ii) transferring water from one province to another, i.e. the cost of energy of lifting up the water in order to overcome the resistance and height difference between these two provinces, iii) the cost of pollution treatment, i.e. cost of dealing with the pollution occurred during the transfer to maintain the water quality, and iv) the cost of water loss during the transfer, i.e. water dissipated ,by, for instance, evaporation. What's more, we assume the cost of using local water supply is always lower than other methods, so a province will neither desalinate seawater or have water transfer from other provinces if its water supply is not used up.

The cost of water production is the cost of desalination, and we impose restrictions that only coastal provinces can desalinate sea water.

Indirect costs includes the project's economic, social and environmental impact, which are often negative. For example, water transferring program of Michigan Lake in Chicago in 1948 also brings epidemic typhus to the city.(Dong 2012) What's more, water transfer program may have adverse impact on environment, such as soil salinization, alien species invasion, etc. The damage to environment will in turn lead to economic losses.

We also take the time value of money into consideration. We assume the project will last for 50 years and discount the costs each year to the present value in 2025 to calculate the total cost of the project.





**Figure 4 Total Cost of Water Arrangement**

Since there are so many components of indirect costs, which are often hard to measure, our model finds two solutions to minimize direct costs and indirect costs respectively. In our first model, we only minimize the direct cost in our model. In our second model, since the longer watercourses are, the higher indirect costs will be, we minimize the length of water transferring, so the indirect cost will be reduced to the furthest. In both of our models, we solve for the amount of water produced by desalination and transferred between provinces, the marginal cost of water transfer, the total length of water transfer and the total direct cost of the project.

## Some Concepts and Definitions

Some basic definitions are listed in **Table 1**:

**Table 1 Some Concepts and definitions**

Terms	Definition
Province	Including provinces, provincial level cities and autonomous regions.
Water Supply	Water generated from natural occurrence.
Used/Unused Water Supply	Water supply used by a province/Water supply unused by a province
Water Demand	Water needed for agriculture, industry, household and service, and eco-environment use in a province.
Desalination	The process of removing minerals from seawater so that it can





	satisfy daily uses
Water Transfer	The whole process of transferring water from one province to another with the water's quality guaranteed.
Fixed Cost	The cost of digging the watercourses, construction of related facilities, buying the land and resettlement, etc.
Pollution Treatment	The process of cleaning polluted water so that it can satisfy daily uses.
Water Transport	The physical process of transporting water from one province to another
Evaporation	The amount of water loss during water transfer due to evaporation
Water Inflow	The amount of water that can be utilized in each province
Water Outflow	The amount of water that are utilized in each province
Water Balance	Water Inflow=Water Out
Direct Costs	The costs of producing and rearranging the water
Indirect Costs	The other costs incurred because of the producing and rearranging of water

## Model

Let  $P_j, (j = 1, 2, \dots, 30)$  be the nodes representing the 30 provinces in China. For each  $j$ , let  $\theta_j = (alt_j, lat_j, lon_j, D_j, S_j, \vec{c}_j, coa_j)$  be the information vector of  $P_j$ , where  $alt_j, lat_j, lon_j$  are the altitude, latitude and longitude of  $P_j$ ,  $D_j$  and  $S_j$  are province  $j$ 's water demand and water supply;  $\vec{c}_j = (c_{j1}, c_{j2}, \dots, c_{j30})_{1 \times 30}$ , in which  $c_{ji} = 1$  if province  $j$  and  $i$  are contiguous and  $c_{ji} = 0$  otherwise ( $c_{jj} = 0$ ),  $coa_j = 1$  if province  $j$  is coastal and  $coa_j = 0$  otherwise. Let  $\varphi_j = (s_j, E_j, \vec{O}_j)$  be the water flow vector of province  $j$ , in which  $s_j \in \mathbb{R} \setminus \mathbb{R}^-$  is the used water supply,  $E_j \in \mathbb{R} \setminus \mathbb{R}^-$  is the amount of water produced by desalination in province  $j$  and  $\vec{O}_j = (O_{j1}, O_{j2}, \dots, O_{j30})_{1 \times 30} \in \mathbb{R}^{30}$ , in which  $O_{ji}$  is the net amount of water transfer from province  $j$  to province  $i$ :  $O_{ji} > 0$  if province  $j$  transfers its water to province  $i$ ,  $O_{ji} < 0$  if province  $i$  transfers water to province  $j$ , and  $O_{ji} = 0$  if there is no transfer between them, and  $O_{jj} = 0$ . By our





assumption,  $O_{ij} = 0$  if  $c_{ij} = 0$ .

Let  $d_{ij}$  be the distance of province  $j$  and  $i$  and  $d_{ij} = d(lat_i, lat_j, lon_i, lon_j)$  non-negative, and the function  $d(\cdot)$  will be given later.  $fix_{ij}, pol_{ij}, tran_{ij}, C_{ij}$  are the fixed cost, pollution treatment cost, water transferring cost and total cost of transporting water from province  $j$  to province  $i$ , and later we will show that dissipation cost can be neglected. Let  $des_j$  be the cost of sea water desalination in province  $j$ . The direct cost of the project is  $C$ . By definition, all the costs are non-negative and

$$C_{ij} = fix_{ij} + pol_{ij} + tran_{ij}, i, j = 1, 2, \dots, 30, \quad C = \sum_{i,j=1,i \leq j}^{30} C_{ij} + \sum_{j=1}^{30} des_j. \quad \text{The total length of}$$

$$\text{water transferring } L = \sum_{i,j=1,i \leq j}^{30} \delta_{ij} d_{ij}, \text{ where } \delta_{ij} = 1 \text{ if } O_{ij} \neq 0 \text{ and } \delta_{ij} = 0 \text{ if } O_{ij} = 0.$$

Water supply constraints suggest:

$$s_j \leq S_j, j = 1, 2, \dots, 30 \quad (1)$$

and water balance suggest:

$$s_j + E_j = D_j + \sum_{i=1}^{30} O_{ji}, j = 1, 2, \dots, 30 \quad (2)$$

, since only contiguous provinces can transfer water.

The first solution of our model is minimizing direct costs, i.e. find

$$\vec{\varphi}_1 = (\varphi_j)_{j=1}^{30} \in (\mathbb{R} \setminus \mathbb{R}^-)^2 \times \mathbb{R}^{30} \text{ to minimize } C \text{ subject to (1) and (2), and the second}$$

solution is to find  $\vec{\varphi}_2 = (\varphi_j)_{j=1}^{30} \in (\mathbb{R} \setminus \mathbb{R}^-)^2 \times \mathbb{R}^{30}$  to minimize  $L$  subject to (1) and (2).

Following is a theorem illustrating there will be no loop water transferring in both of our two solutions.

**No Loop Transferring Theorem:** Let  $j_1, j_2, \dots, j_k \in \{1, 2, \dots, 30\}$  and  $j_m \neq j_n$  if

$m \neq n$  and both  $C$  and  $L$  strictly increase with  $|O_{ij}|, i, j = 1, 2, \dots, 30$ , then

$O_{j_1 j_2}, O_{j_2 j_3}, \dots, O_{j_{k-1} j_k}, O_{j_k j_1}$  cannot be all positive or negative in  $\vec{\varphi}_1$  and  $\vec{\varphi}_2$ .



**Proof:** Take  $\vec{\varphi}_1$ . If  $O_{j_1 j_2}, O_{j_2 j_3}, \dots, O_{j_{k-1} j_k}, O_{j_k j_1}$  in  $\vec{\varphi}_1$  are all positive for example.

Let  $O = \min\{O_{j_1 j_2}, O_{j_2 j_3}, \dots, O_{j_{k-1} j_k}, O_{j_k j_1}\}$  and  $O'_{j_i j_{i+1}} = O_{j_i j_{i+1}} - \frac{O}{2}$ , then

$|O'_{j_i j_{i+1}}| < |O_{j_i j_{i+1}}|$ , and  $C$  becomes lower. What's more,

$$\begin{aligned} 0 &= D_{j_k} - s_{j_k} - E_{j_k} + \sum_{i=1}^{30} O_{j_k i} \\ &= D_{j_k} - s_{j_k} - E_{j_k} + \left( \sum_{i=1}^{30} O_{j_k i} - O_{j_k j_{k+1}} - O_{j_k j_{k-1}} \right) + O_{j_k j_{k+1}} - O_{j_{k-1} j_k} \\ &= D_{j_k} - s_{j_k} - E_{j_k} + \left( \sum_{i=1}^{30} O_{j_k i} - O_{j_k j_{k+1}} - O_{j_k j_{k-1}} \right) + O'_{j_k j_{k+1}} - O'_{j_{k-1} j_k} \end{aligned}$$

So (1) and (2) still hold, and  $\vec{\varphi}_1$  is not the solution that minimizes  $C$ . The proof of the other part of the theorem is in a similar way.

## Estimation of Parameters and Costs

### Data

All the data, if not specially mentioned, are from the database of National Bureau of Statistics of China. Population, GDP deflators, GDP of secondary sectors, *Statistics Almanac of China* from 2002 to 2011, and the other are from *Environmental Statistics Summary of China*, from 2002 to 2010. GDP is adjusted to real GDP by GDP deflators. To estimate the coefficients of the models, we use the data from 2002 to 2010, and data from 2002 to 2011 are used to estimate the effective irrigation area, real GDP of secondary sectors and population in 2025.

### Distance

We use the latitude and longitude of the main water source near the center of each province to represent the location of each province, and the data are obtained from Google Earth. We assume the earth is an ideal sphere, with a mean radius of 6,371.0 km (Wikipedia). The distance between point  $i$  and point  $j$  is (Veness):

$$d(lat_i, lat_j, lon_i, lon_j) = 6371 \arccos(\sin(lat_i) \sin(lat_j) + \cos(lat_i) \cos(lat_j) \cos(lon_i - lon_j))$$

, from which we calculate  $d_{ij} (i, j = 1, 2, \dots, 30)$ .



## Water Supply

Water resources are constituted by surface water resource and under water resource. These two approaches are partly overlapped and closely connected, so we'd better not to distinguish them(Zuo 2011). We assume there will be no disasters or wars in the next 12 years and water resources keep stable in the period. Thus we can use the average water resources for the last 10 years to estimate the water resources for 2025 of each province.

However, not all the water resources can be utilized by human beings. According to(Zuo 2011), the reasonable utilization rate should be 30% of the water resources and the maximization utilization rate is 40%, which is the environmental limit.

We provide two estimations of supply. In the baseline model, we assume China will exploit its water resource to the best and set the utilization rate 40%; In the other model, we set the utilization rate 30%, which suggests China will not use its water resource to the full potential and thus face a higher shortage of water resource.

## Water Demand

We use multivariate regression to estimate water demand in 2025. Let  $D_{it}$  denote the total water demand of province  $i$  in year  $t$ . Since the total water demand can be decomposed into water demand in agriculture,  $D_{1it}$ , industry,  $D_{2it}$ , household and service and eco-environment,  $D_{3it}$ , and the water use on eco-environment hardly ever consists over 5% of total water use, we only consider the first three part and hence

$$D_{it} = \sum_{j=1}^3 D_{jit}$$

Effective irrigation area,  $A_{it}$ , is the area arable lands equipped with adequate irrigation facilities and water source, which is positively correlated with agricultural water usage. Besides, real gross domestic product of the secondary sector,  $G_{it}$ , is positively correlated with industrial water usage, and population  $P_{it}$ , moves along with water usage of households and services. Hence, we write  $D_{it}$  as a function of



$A_{it}$ ,  $G_{it}$ ,  $P_{it}$ ,  $i$  plus a stochastic term  $\varepsilon_{it}$ . Using second-order Taylor expansion, we have a quadratic form of panel data models with fixed coefficients and variable intercepts with fixed effect:

$$D_{it} = \alpha_i + \beta_1 A_{it} + \beta_2 G_{it} + \beta_3 P_{it} + \beta_4 A_{it}^2 + \beta_5 G_{it}^2 + \beta_6 P_{it}^2 + \beta_7 A_{it} P_{it} + \beta_8 A_{it} G_{it} + \beta_9 G_{it} P_{it} + \varepsilon_{it}$$

$$\varepsilon_{it} \sim N(0, \sigma^2) \quad (3)$$

The underlying assumption of this model setting up is that  $A$ ,  $G$ ,  $P$  affect different provinces in the same pattern and invariant of time. Although this assumption may not be always true, however, the length our data does not support varying coefficient models; In spite of this, this estimation is a good approximation of the reality.

The task following this regression is to use the estimated model to predict total water use in 2025. We have to estimate  $A$ ,  $G$  and  $P$  of different provinces in 2025. We assume  $G$  and  $P$  will grow exponentially with time, and  $A$  linearly. We regress the  $A$  and the logarithm form of  $P$  to year for each province to get their average growth rates  $i$ ; for the growth rate of real GDP in secondary sections, we adopt the proposition of OECD(2012): a growth rate of 8.9% from 2012 to 2017 and 5.5% from 2017 to 2025, from which we get the average growth rate:

$$r = (1.089^5 \times 1.055^8)^{\frac{1}{13}} - 1 = 6.79\%$$

We assume the growth rate of real GDP in different provinces the same. Although in reality this assumption is often violated, any other means to predict the growth rate of GDP for each provinces lack either reliable data or economic sense. With the estimated  $A$ ,  $G$  and  $P$  in 2025, we can thus predict the water demand in each province in 2025.

Another thing worth to mind is that we do not take desalination and water transferring into account in the estimation, hence these estimations serve as baseline water demand in 2025 in our model.

## Result of Estimation

We use ordinary least square methods to estimate equation (3). Akaike information criterion (AIC) (Akaike 1973) is widely used to screen out the variables be leave in the regression models. The most proper model should have the lowest AIC. With this criterion, we find out 5 terms of equation (3) should be kept in the equation, so the model becomes:

$$Y_{it} = \alpha_i + \beta_1 A_{it} + \beta_2 G_{it} + \beta_3 P_{it} + \beta_4 P_{it}^2 + \beta_5 A_{it} P_{it} + \varepsilon_{it}, \varepsilon_{it} \sim N(0, \sigma^2) \quad (4)$$



As is discussed above, we expect  $\beta_1, \beta_2, \beta_3$  to be positive and  $\beta_4$  negative, for an increasing of population will make the competition for water fiercer and lead water use per capita to drop. We also expect  $\beta_5$  to be negative. To prove this, suppose the production per square meter of farmlands,  $f$ , is an increasing function of water usage per square meter  $k$ , i.e.  $f = f(k)$ . We assume  $\frac{\partial f}{\partial k} > 0$  and  $\frac{\partial^2 f}{\partial^2 k} < 0$  due to the diminishing marginal returns of watering. We further assume the demand for crops,  $D$ , is an increasing function of population,  $p$ , and perfect inelastic to price, because crops are necessities for consumers. Let  $A$  be effective irrigation area and in equilibrium, crops supply equals to crops demand, hence  $Af(k) = D(p)$ . Since  $\frac{\partial f}{\partial k} > 0$ , let  $g$  be the inverse function of  $f$  and  $k = g(\frac{D(p)}{A})$ . It is easy to show that  $g' > 0, g'' > 0$ . Total agricultural water use should be  $K = Ak$ .

Taking derivatives, we have  $\frac{\partial K}{\partial p \partial A} = g''(\frac{D(p)}{A}) D'(p) \cdot \frac{-D(p)}{A^2} < 0$  and since  $K$  is a part of total water use, we expect  $\beta_5$  to be negative.

The result is shown in **Table 2**. The signs of coefficient is coherent with our expectations:

**Table 2 Estimated Coefficients<sup>①</sup>**

$Y_{it}$	Coefficient	p-value
$A_{it}$	0.0825679	0.000
$G_{it}$	0.0052399	0.000
$P_{it}$	0.009511	0.368
$A_{it} \cdot P_{it}$	$-9.47 \times 10^{-6}$	0.004
$P_{it}^2$	$-1.95 \times 10^{-7}$	0.867

Based on the estimation above, we can calculate the water supply and water demand of each province in China in 2025, the result is shown in **Table 3**:

**Table 3 Estimated Water Supply and Demand in 2025<sup>②</sup>**

<sup>①</sup> The units of effective irrigation area, real GDP of secondary sectors and population are 1000 hectares, 100 million RMB in 2000 and 10 thousand people, respectively.

<sup>②</sup> Measurement:  $10^8 m^3$



Province	Water Supply( $S_j$ )	Water Demand ( $D_j$ )	Province	Water Supply( $S_j$ )	Water Demand ( $D_j$ )
Beijing	9.269	65.727	Hubei	173.053	340.841
Tianjin	4.750	66.941	Hunan	391.604	359.371
Hebei	54.135	245.072	Guangdong	676.361	394.774
Shanxi	38.848	124.140	Guangxi	694.642	504.610
Inner Mongolia	160.367	313.827	Hainan	712.266	346.429
Liaoning	115.703	221.109	Chongqing	210.404	121.502
Jilin	156.434	179.065	Sichuan	952.611	298.618
Heilongjiang	273.793	707.434	Guizhou	374.449	232.874
Shanghai	12.355	176.209	Yunnan	769.636	182.686
Jiangsu	164.496	653.675	Tibet	1767.216	59.514
Zhejiang	368.831	304.748	Shaanxi	178.558	126.121
Anhui	295.121	332.343	Gansu	84.255	194.094
Fujian	443.322	255.121	Qinghai	277.354	53.843
Jiangxi	578.803	252.012	Ningxia	4.004	93.939
Shandong	128.399	319.070	Xinjiang	367.684	630.183

## Estimation of Costs

Now we are going to focus on calculating the cost of the watercourses. First we estimate the discounting coefficient to take time value of money into consideration, then the cost of water transferring, including fixed costs, water transport and pollution treatment, and the cost of desalination. We will also estimate the effect of evaporation, the result of which shows that we can safely ignore it. In all calculations, we assume the density of water is  $1.0 \times 10^3 \text{ kg/m}^3$  and gravitational acceleration coefficient

$10 \text{ m/s}^2$ .



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## Discount coefficient

The water transfer program is a long-term benefit process, thus we assume its overall useful life is 50 years. There will be an annual cost each year of the 50 years. Money have its time value, we cannot simply add all the variable costs together, and rather we should use the concept of present value.

According to the yield curve published by the Chinese government (2013), the annual government bond yield of the 10 year-term and 50 year-term are 3.6004% and 4.3553% respectively on Feb. 1<sup>st</sup>,2013, hence we can calculate the average forward risk-free rate between 10 year and 50 year(2023~2063) by the no-arbitrage principle in finance. The average forward risk-free rate is  $(\frac{1.043553^{50}}{1.036004^{10}})^{1/40} - 1 = 4.54\%$ , which we

use as the average discount rate during 2025~2075. Let  $VC_i$  denote the annual cost

of the  $i^{\text{th}}$  year, and the present value of  $VC_i$  should be  $\frac{VC_i}{1.0454^i}$ . Thus, the present

value of the overall-50-year cost should be  $VC_1 + \frac{VC_2}{1.0454} + \dots + \frac{VC_{50}}{1.0454^{49}}$ . We assume the annual costs in every year,  $VC$ , are the same, so the present value of total annual cost is  $VC(1 + \frac{1}{1.0454} + \dots + \frac{1}{1.0454^{49}}) = 20.5255VC$ . We thus define 20.5255 as the discount coefficient and total present value of any annual-equally annual cost is 20.5255VC

## Fixed costs

It is extremely difficult to estimate the construction cost, but fortunately the Chinese South-to-North water diversion project provide an available estimation for our model.

According to XinhuaNet (Gu and Liu 2012), the official media in China, The projected construction cost of the overall “South-to-North water transferring project” is 486 billion RMB. The overall length of the watercourses is 4350 kilometers. Hence, the fixed cost of one kilometer is  $4860 \times 10^8 / 4350 = 111724137.9$  RMB, and  $fix_{ij} = 111724137.9d_{ij}$ . This number is too large to be true at a first glance, but further research reveals that: apart from the basic courses cost, this projected cost also includes removal cost, resident relocation payment, pump station cost, pollution treatment cost, maintenance cost, and etc. Hence, our fixed cost can be estimated by the number





suitably.

## Water Transfer Cost

### Energy loss due of resistance

When water moves through pipes or watercourses, there exists resistance. Therefore, we need pump stations to provide kinetic energy for water so that they can move to the destination.

In hydraulics, the engineer transforms the resistance effect into a height (meter). The height is called ‘head loss’, representing some sort of gravitational potential energy, which is the equivalent energy loss during the resistance process. Darcy's formula says

the head loss is  $h_f = \lambda \frac{l}{d} \cdot \frac{v^2}{2g}$ , where  $h_f$  is the head loss,  $\lambda$  is the resistance coefficient,  $l$  is the transfer distance,  $d$  is the inner diameter of the pipe,  $v$  is the speed of water,  $g$  is the acceleration of gravity.

We take  $\lambda \approx 0.01$  (Li, Tian and Wang 2008), volume flowrate  $Q=100m^3/s$  and inner diameter  $d=10m$  (Gu 2003).

The estimated water speed should be  $v = \frac{Q}{\pi(d/2)^2} \triangleq 1m/s$ .

Hence the head loss for every kilometer is:

$$h_f = 0.01 \times \frac{1}{10} \times \frac{1^2}{2 \times 10} = 0.05 \text{ meter}.$$

This head loss indicates that transferring 1 kilogram of water 1 kilometer will consume  $1 \times 10 \times 0.05 = 0.5$  Joule of energy.

### Equivalent altitude difference between two province

When the altitude of origin province is higher than the destination's, so gravitational potential energy itself can provide part of the energy water transfer needs; however, if the opposite is true, more energy is needed to pump water to the destination.

If we transfer water from province  $i$  to province  $j$ , the nature can provide  $mg(alt_i - alt_j)$  of energy, where  $m$  is the mass of water transferred and  $g$  is the



acceleration of gravity. The energy loss due to resistance is  $0.05mg \cdot d_{ij}$

If  $mg(alt_i - alt_j) + 0.05mg \cdot d_{ij} \leq 0$ , i.e.,  $(alt_i - alt_j) + 0.05d_{ij} \leq 0$ , we need exert extra energy to transport water to the destination; if the opposite is true, we assume we need no extra energy for water transport.

Define the equivalent altitude difference from province i to j:

$$\Delta H \triangleq 0.05d_{ij} - (alt_i - alt_j), .$$

If  $\Delta H < 0$ , there is no need for pump station. Actually, we may be able to build hydroelectric generating station, but in our model we simply do not consider this.

However, if  $\Delta H > 0$ , transferring water needs energy of  $mg\Delta H$ .

### Total water transport cost

The pump station cannot perfectly transfer the electricity energy into kinetic energy, so we must take the efficiency into consideration. The efficiency of water pump to turn electricity energy into gravitational potential energy is around 70%(Zhang 2004). Thus,

our estimation for the electricity energy consumption is  $\frac{mg\Delta H}{70\%} \text{ J / (kg} \cdot \text{km)}$

The industrial electricity fee is within the range of 0.4~1 RMB per kilowatt hour in China. We assume the average electricity fee is 0.7RMB/kWh for the pump station in the future.

Considering 1 kilowatt hour =  $3.6 \times 10^6$  Joule, the electricity expenditure should be

$$\frac{mg\Delta H}{70\%} \cdot \frac{1}{3.6 \times 10^6} \cdot 0.7 \text{ RMB}.$$

Besides, the electricity expenditure only consists of 60%~70% of the total cost of a pump station(Zhang 2004). Assuming it 65%, the water transporting cost per kilogram

of water per kilometer should be  $\frac{mg\Delta H}{70\%} \times \frac{1}{3.6 \times 10^6} \times 0.7 \times \frac{1}{65\%} = 4.2735 \times 10^{-6} m\Delta H$

RMB if  $\Delta H \geq 0$ . Discounting the cost in the 50 years to year 2025, and when

$m = 1 \text{ ton}$  and  $\Delta H = 1 \text{ m}$ , the unit cost of water transporting will be  $8.772 \times 10^{-2}$  RMB.

The cost equals 0 if  $\Delta H < 0$ .

Thus, given the distance and altitude between province i and j, we can reckon whether we need to build pump stations and if we do, how much cost it will incur per cubic meter of water per kilometer.



## Pollution Treatment Costs

Water quality is one of the most severe problem of water transfer. For example, nearly all the water of east line of south-to-north water transfer program in Shandong province reaches level V, some part even V+, standing for not being able to be utilized for any domestic use according to Chinese water quality(Pang, et al. 2002). Thus, pollution treatment is a must for water transferring program and its cost should be included in the cost analysis.

Pollutants come from two sources: the first is from the nature, e.g. fallen leaves, muds washed away by rains or floods, etc.; the second is from human activities, e.g. industrial pollutants, domestic pollutants and agricultural pollutants, etc.

Total costs of pollutatns treatment comprises two parts: initial costs and variable costs. Initial costs are the cost of building sewage treatment plants, mediating closure of sewage, industry structure transferring and industrial management costs(Zhao and Chai 2006). Variable costs includes the cost of treatment of pollutants, equipment maintenance and replacement and labor cost, etc. (Yuan 2008)

Total costs of pollution treatment are correlated to the length and volume of water transfer. On the one hand, we assume the polluting source are evenly distributed along the canal, so that water in a longer transferring route is more severely polluted and thus needs more pollution treatment plants and capacities; on the other hand, the cost of pollution treatment is proportional to the amount of water treated, so a higher volume of water leads to a higher demand for treatment capacity.

We assume that the total cost of pollution  $pol$  is proportional to the length of transferring,  $L$ , and the volume of water transferred,  $V$ , i.e.  $pol = aLV$ , where  $a$  is a constant. We use the cost of east line of south-to-north water transfer program to estimate  $a$ . According to *the Planning of Sewage Treatment in the East Line of South-to-north Water Transferring Project*, the initial investment on pollution treatment is 23.84 billion RMB, building 135 sewage treatment plants, with treatment capacity of  $6.69 \times 10^6 \text{ m}^3/\text{day}$  along the canal. The transfer volume is  $1.48 \times 10^{10} \text{ m}^3$  per year and the total length is 1467 km(Yan 2011). So the initial cost of pollution treatment per cubic meter per kilometer is  $1.098 \times 10^{-3}$  RMB. For variable costs, we assume the water transferred is evenly withdrawn and the sewage treatment plants are evenly distributed along the canal, and the rate of open capacity is 80%. The marginal cost (including depreciation, labor cost, maintenance fee are also) of sewage treatment ranges from 0.6 to 3 RMB per  $\text{m}^3$ .(Yuan 2008). We assume the marginal cost will drop to 0.4 RMB in 2025 due to technology development. Based on assumptions above, the total variable cost per year is  $0.4 \times 80\% \times 6.69 \times 10^6 \times 365 = 8.791 \times 10^8 \text{ RMB}$ ,



so the variable cost a cubic meter a kilometer per year is  $4.049 \times 10^{-5}$  RMB. Using the discounting coefficient we have calculated, the present value of variable cost a cubic meter a kilometer a year is  $8.311 \times 10^{-4}$  RMB, so  $a = 8.311 \times 10^{-4} + 1.098 \times 10^{-3} = 1.929 \times 10^{-3}$ .

## Evaporation

Water will evaporate when being transferred in the course, which lowers the efficacy of water transferring. We have to examine whether the rate of evaporation is so high that we cannot ignore its effect.

Annual water evaporation in the courses of the south-to-north water transfer program in Jiangsu province is  $1.36 \times 10^8 \text{ m}^3$  (Qiu, Wang, Feng, Huang and Yang 2011), and the total length of the course in Jiangsu is 404 km, which is nearly 1/3 of the whole length of the course, 1467 km. (Chen 2012)

Based on the assumptions above, we can safely ignore the effect of evaporation. Assume the amount of evaporation remains the same along the course, the total amount of evaporation is  $1.36 \times 10^8 \times \frac{1467}{404} = 4.938 \times 10^8 \text{ m}^3$ , which is only

$\frac{4.938 \times 10^8}{1.48 \times 10^{10}} = 3.336\%$  of the total amount of water transferred per year. What's more,

since the water are withdrawn along the transfer course, the true amount of evaporation will be even less than our estimation.

## Desalination Cost

Seawater reverse osmosis (SWRO), multiple effect distillation (MED) and multi stage flash (MSF) are three methods widely used in water desalination, and SWRO has the least marginal cost among these technologies, (OCN 2011), so we just assume all the desalination are done by SWRO. The marginal cost of desalination is 4-5 RMB per year in 2011 in China. If we assume the marginal cost will decrease 3% per year, the marginal cost in 2025 will be  $4.5 \times (1/1.03)^{14} = 2.975 \text{ RMB} / \text{m}^3$ . Using the discounting coefficient we have calculated, the present value of desalination cost is  $61.063 \text{ RMB} / \text{m}^3$ .

We have estimated all the parameters we need in our model. **Table 4** summarizes all the relating costs.



**Table 4 Cost Estimation**

Items	Costs
Fixed Cost, from province i to j	$1.1172 \times 10^8 d_{ij}$
Water Transport Cost, from province i to j	$8.772 \times 10^{-2} O_{ij} \Delta H$
Pollution Treatment Cost, from province i to j	$1.929 \times 10^{-3} O_{ij} d_{ij}$
Desalination in Province j	$61.063 E_j$

\*Here the measurement of  $d_{ij}$ ,  $\Delta H$ ,  $O_{ij}$  and  $E_j$  are km, m,  $m^3$  and  $m^3$ .

## Solving the Model

### Basic Set-ups Before Solving the Model

Denote the 30 provinces  $C = \{c_0, c_1, \dots, c_{29}\}$  and water supply and demand  $s_i$  and  $d_i$  for provinces  $i$ , ( $i = 0, 1, \dots, 29$ ). We construct a transferring cost matrix  $A_{30 \times 30}$ , where  $A_{i,j}$  denotes the average cost of water transferring of water of  $1 m^3$  from  $c_i$  to  $c_j$ , and fixed cost matrix  $B_{30 \times 30}$ , where  $B_{i,j}$  denotes the basic cost of building water course from  $B_i$  to  $B_j$ . Water Desalination Cost:

$$w_i = \begin{cases} \infty & , c_i \text{ is not a coastal province} \\ DC & , c_i \text{ is a coastal province} \end{cases}, \text{ where } DC \text{ is the constant cost of Water}$$

Desalination per cubic meter;

Our Goal is to minimize the total direct cost of water or the total length of watercourse subject to all constraints. One thing worth noticing is that the model we are solving is NOT a linear program, since we have a fixed cost of constructing water courses. Considering this, we develop an algorithm combining simulated annealing and Min-cost-Max-flow to solve our model.



## Description of the Algorithm's Model

We will basically adopt simulated annealing process to solve our model. If we have decided that watercourse we build (which can be well decided by simulated annealing expressed below), we can construct a minimum cost maximum flow model to solve the original task.

Let the graph be  $G = \{V, E, Cap, Cost, s, t\}$ , where  $V = \{s, t, v_d, v_0, v_1, \dots, v_{29}\}$  are the vertices of the graph, and

$$E = \{e = (s, x) | x \in V\} \cup \{e = (x, t) | x \in V\} \cup \{(s, v_d)\} \cup \{e = (v_d, x) | x \in V, x \text{ is a coastal province}\} \cup \{e = (x, y) | x, y \in V, \text{ there is a water course between } x \text{ and } y\}$$

are the edges of the graph.

Let capacity function be:

$$Cap[(x, y)] = \begin{cases} s_i & , \quad x = s \& y \in \{v_0, \dots, v_{29}\} \\ d_i & , \quad x \in \{v_0, \dots, v_{29}\} \& y = t, \\ \infty & , \quad \text{otherwise} \end{cases}$$

and cost function:

$$Cost[(x, y)] = \begin{cases} DC & , \quad (x, y) = (s, v_d) \\ A_{i,j} & , \quad (x, y) = (v_i, v_j) \\ 0 & , \quad \text{otherwise} \end{cases}$$

According to the No Loop Transferring Theorem we proved, the total inflow will exactly equal to the total outflow.

Our ultimate goal is to find the minimum cost while the flow from  $s$  to  $t$  is maximized. To illustrate the structure of our model, a simple example of 4 cities is shown in **Figure 5**. Dashed lines represent edges that will generate costs while solid lines represent edges that will not.

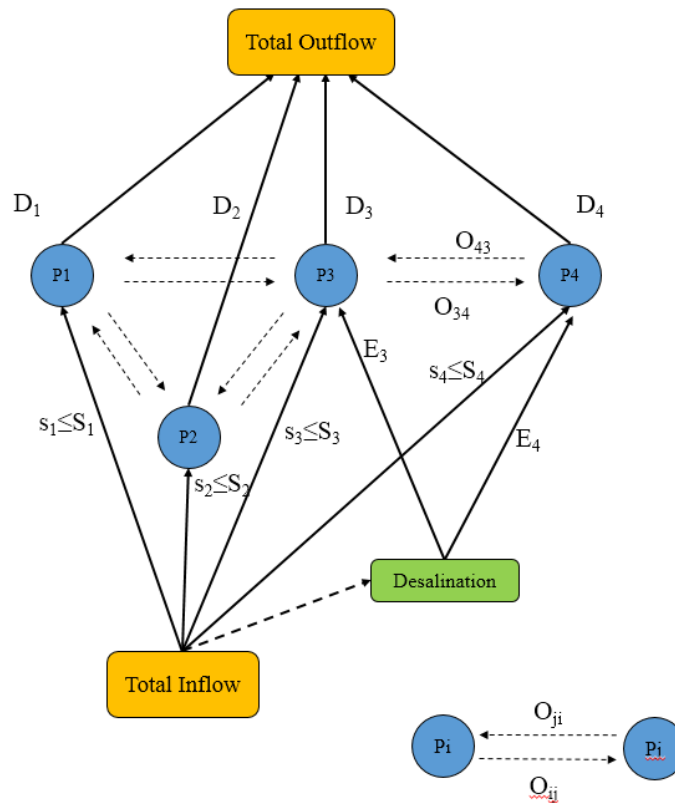
## Relationship between the Model and the Original Task

Solving this optimization problem, we can get the flow from  $s$  to  $v_i$ , which is  $s_i$ ;

the flow from  $v_i$  to  $v_j$ , which is  $O_{ij}$ , and the flow from  $v_d$  to  $v_j$  is the amount



of water produced by seawater desalination,  $E_j$ . These are all the components of  $\vec{\varphi}_1$  and  $\vec{\varphi}_2$ , so the outcome will solve our problems. Total cost of the network is exactly the minimum total direct cost we need.



**Figure 5** The Structure of Our Algorithm's Model: An illustration

### Algorithm: A Combination of Simulated Annealing and Min-cost-Max-flow

“Simulated Annealing” and “Minimum Cost Maximum Flow” are two existing algorithms. The inspiration of our study is to combine these two algorithms and to fit our original task into them.

The state of simulated annealing in this task is the water course we choose to build. For each state, we construct a minimum cost maximum flow model to get the minimum cost under this method of water course. The energy function we choose is just the total cost under this state.

After the well-performed simulated annealing (with plenty times of min-cost-max-





flow calculation), we can get almost a best answer for the original task.

## Model Algorithm Design

### Simulated Annealing

To decide where we should build watercourse, we use simulated annealing algorithm instead of brute force, which is time consuming and inefficient. The main idea of simulated annealing is to replace the local maximum by a bit worse state with some probability. The algorithm can be shown as following:

```

s ← s0; e ← E(s)           // Initial state, energy.
sbest ← s; ebest ← e       // Initial "best" solution
k ← 0                       // Energy evaluation count.
while k < kmax and e > emax // While time left & not good enough:
    T ← temperature(k/kmax) // Temperature calculation.
    snew ← neighbour(s)     // Pick some neighbor.
    enew ← E(snew)          // Compute its energy.
    if P(e, enew, T) > random() then // Should we move to it?
        s ← snew; e ← enew // Yes, change state.
    if enew < ebest then     // Is this a new best?
        sbest ← snew; ebest ← enew // Save 'new neighbor' to 'best found'.
    k ← k + 1               // One more evaluation done
return sbest               // Return the best solution found

```

The energy is explained as before. The neighbor is made by changing a construct or not condition of the “best” solution.

### Minimum Cost Maximum Flow Algorithm

The way we get the minimum cost of  $G$  while the flow is maximized is by finding the cost-minimized augmentable chain from  $s$  to  $t$  time by time. In this way, we can reach the maximum flow with the minimum cost gradually.

In the minimum cost maximum flow model there are edges with negative cost, which is to say that when finding the minimum augmentable chain there will be negative weight of edge. Thus, we need to choose the algorithm that can deal with this situation. We choose SPFA (Shortest Path Faster Algorithm) here.



## Result

### The First solution

Our first solution intends to find an approach to minimize the cost of water producing and transporting. The result is shown in **Table 5** and a map showing the amount of water desalination and the route of water transferring is shown in **Figure 6**.

**Table 5 Result: Amount of Water Transferred**

From	To	Amount( $10^8$ $m^3$ )	From	To	Amount( $10^8m^3$ )
Tibet	Qinghai	1392.298	Ningxia	Inner Mongolia	1110.015
Tibet	Xinjiang	262.499	Inner Mongolia	Shanxi	394.877
Qinghai	Gansu	1615.81	Inner Mongolia	Jilin	561.678
Gansu	Ningxia	1199.95	Jilin	Heilongjiang	433.641
Gansu	Shaanxi	306.021	Jilin	Liaoning	105.406
Hunan	Hubei	78.944	Hebei	Tianjin	62.19
Jiangxi	Anhui	514.993	Hebei	Beijing	56.458
Fujian	Jiangxi	188.202	Shanxi	Hebei	309.585
Zhejiang	Shanghai	64.084	Henan	Shandong	190.67
Anhui	Jiangsu	588.949	Jiangsu	Shanghai	99.77
Hubei	Anhui	111.178	Shaanxi	Henan	358.458
Total Amount		10005.676			

Basically, water are transformed from the south to the north, which is consistent with the regional distribution of water in China. Another tendency is that water are transferred from the west to the east, which can be attributed to a lower transporting cost due to high altitude of the western China.

The present value total cost of this project in 50 years is  $1.918 \times 10^{12}$  *RMB*, which is equivalent to an annual cost of  $9.344 \times 10^{10}$  *RMB*, which is 0.197% of China's GDP in 2011. Considering its benefits, the cost of developing the water transferring system is very low.



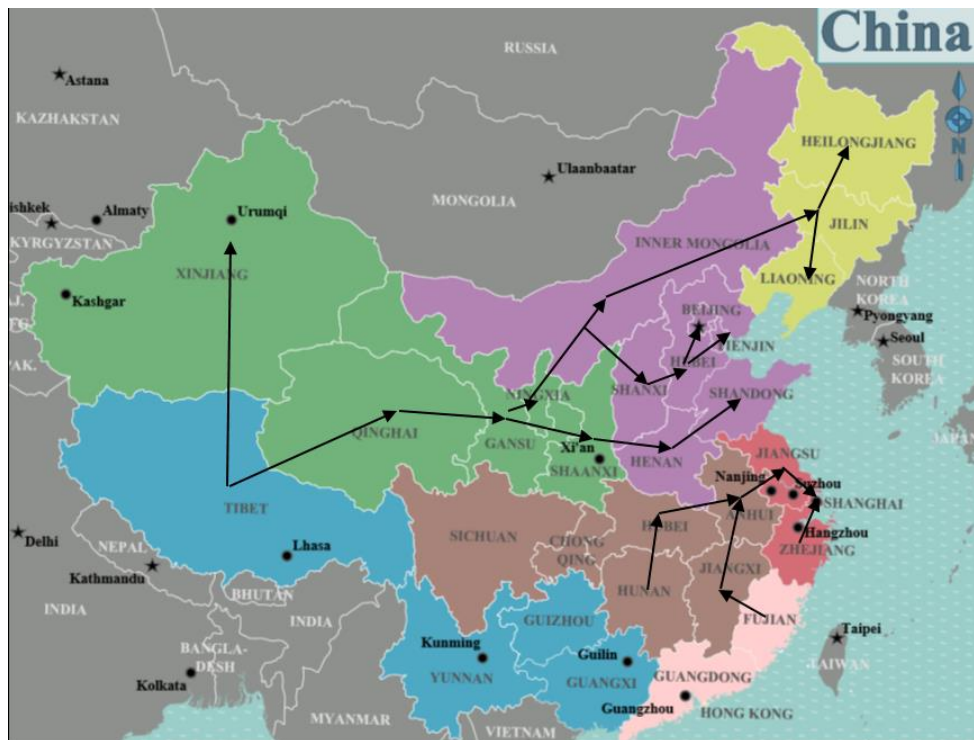


Figure 6<sup>①</sup> Result: Water Desalination and Routes of Water Transferring

## No Need for Desalination?

One interesting feature of the outcome is that there is no provinces conducting desalination. This is mainly due to our baseline assumption that China will exploit its water resource to the full. In our estimation, the total water supply exceeds total water demand, so there is enough water source to be transported among provinces. Another factor contributing to the absence of desalination is the cost of desalination is much higher than transporting.

If desalination is so costly, why there are some provinces are producing water by desalination? One answer is that the water transferring system in reality is different from our proposal. If the construction of facilities of water transferring has not completed, there is no other ways for coastal cities to cater its own demand of water.

Despite this, it may also arise because of the assumptions we adopt and lack of accuracy of our estimation. So in the following part we will change some of our assumptions to gain some insight of seawater desalination.

<sup>①</sup> The territory of China should include Taiwan and South China Sea Islands



## Less Water Supply

As is discussed earlier, our baseline assumption postulates that China will use its water resource extensively. However, this may lead to profound damage to the environment. Thus here we assume China will only utilize 30% of its available water resource while keeping other assumptions unchanged. In this case, total water supply is less than total water demand.

The result is shown in **Table 6** and **Figure 7**. The total amount of water transported increases by nearly 7% of the original one. We also find there are three provinces desalinate seawater to produce water now, which all locates in the northeastern of China. This is because transferring water from the south or the west of China to the northeastern will generate the highest transportation and pollution treatment cost, so it is economic to produce water by desalination in the northeastern China while transferring water in the other parts.

Another change is that there are more watercourses built in this scenario. This is because when more provinces become short of water, they will transfer water from the rest provinces. Considering the high cost of desalination, it is uneconomic to desalinate seawater and transfer it into inland provinces.

The total cost of the whole project in 50 years is  $4.995 \times 10^{12}$  RMB, which is four times of the baseline scenario. The additional cost is mainly due to the high cost of desalination.

**Table 6 The Amount of Water Transferring, utilization rate=30%**

From	To	Amount ( $10^8\text{m}^3$ )	From	To	Amount ( $10^8\text{m}^3$ )
Tibet	Qinghai	911.478	Zhejiang	Shanghai	49.247
Tibet	Xinjiang	354.42	Anhui	Shandong	356.882
Qinghai	Gansu	1065.651	Anhui	Jiangsu	647.998
Gansu	Ningxia	934.749	Hubei	Anhui	933.792
Sichuan	Shaanxi	211.826	Shaanxi	Henan	211.051
Sichuan	Chongqing	204.015	Shaanxi	Ningxia	8.572
Yunnan	Guizhou	394.541	Ningxia	Inner Mongolia	852.384
Guangxi	Guangdong	187.771	Inner Mongolia	Shanxi	95.004
Guangdong	Hunan	204.142	Inner Mongolia	Jilin	563.829
Chongqing	Hubei	682.82	Jilin	Heilongjiang	502.09



Guizhou	Chongqing	442.504	Hebei	Beijing	58.775
Hunan	Hubei	316.639	Jiangsu	Shanghai	117.695
Jiangxi	Anhui	182.09	Shandong	Hebei	134.112
Fujian	Zhejiang	77.371	Total Amount		10701.448

Table 7     Amount of Seawater Desalination, utilization rate=30%

Provinces	Amount (10 <sup>8</sup> m <sup>3</sup> )
Hebei	129.314
Tianjin	63.378
Liaoning	134.332

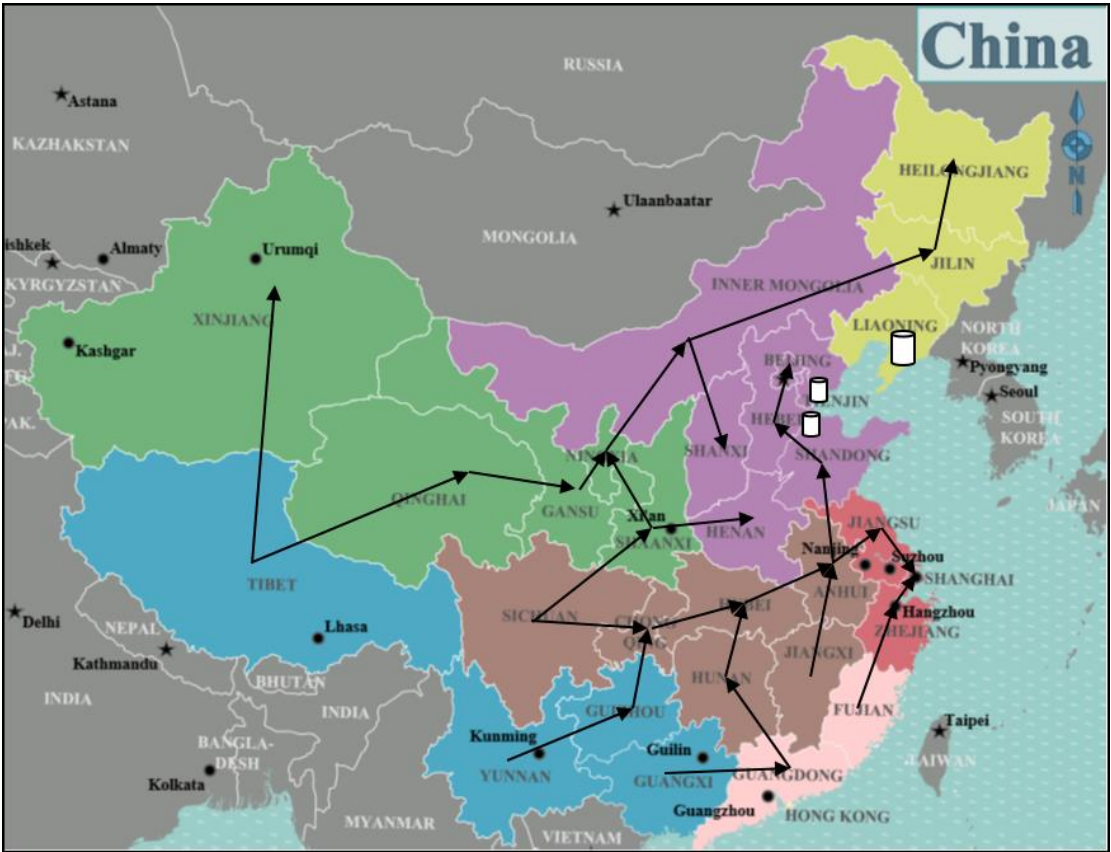


Figure 7<sup>①</sup>     Water Desalination and Routes of Water Transfer, utilization rate=30%

The lessons we learn from this part is that when desalination is very costly, it will dramatically affect the total direct cost of water transferring and it is the last choice to meet the high water demand.

<sup>①</sup> The territory of China should include Taiwan and South China Sea Islands.    The volume of column represents the amount of seawater desalination.



## Lower Desalination Cost?

In our model, we assume the cost of desalination is much higher than the cost of water transferring. However, what if there is a breakthrough which may dramatically reduce the cost of desalination? In this part, we will briefly study this possibility.

We let the marginal cost of desalination vary between 0.05 to 3 RMB per ton and the utilization rate is 40%. The results are shown in **Table 8** and **Figure 8**.

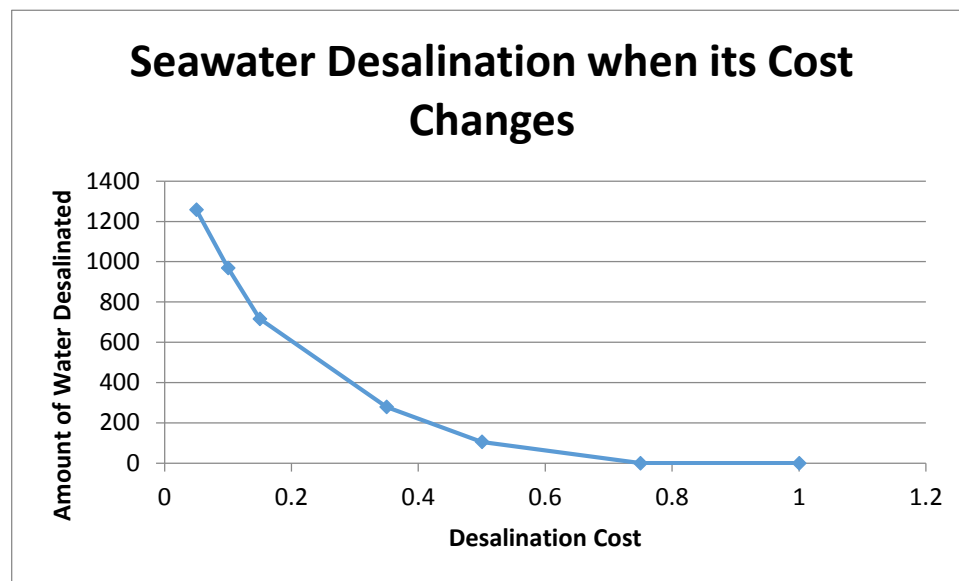
**Table 8 Outcomes when marginal cost of desalination varies**

Marginal Cost of Desalination (RMB/t)	Amount of Water Transfer ( $10^8\text{m}^3$ )	Amount of Water Desalinated ( $10^8\text{m}^3$ )	Total Direct Cost ( $10^8$ RMB) <sup>①</sup>
3	10680.36	0	19279.34
2.5	10112.16	0	19310.09
2	10680.36	0	19279.34
1.5	9572.035	0	19340.14
1	10005.68	0	19176.8
0.75	9496.551	0	19618.41
0.5	9501.407	105.4059	19238.11
0.35	8536.837	278.7744	18499.89
0.15	6214.468	716.8394	15768.9
0.1	5963.314	969.1248	14696.52
0.05	4521.365	1258.694	13676.69

Unsurprisingly, total direct cost increases when marginal cost of desalination increases; however, when the cost rises up to 0.75 RMB/t, which is only a quarter of our estimation, 2.975RMB/t, there will be no desalination and its further increase will not affect the total cost. This suggests unless there will be profound breakthrough on desalination, it will still remain to be the last choice.

<sup>①</sup> Due to the random feature of simulated annealing, the solution may not be exactly the same even the very same program is run for twice. This can explain why the total direct costs when the desalination cost exceeds 0.75 RMB/t are not exactly the same. However, the variation is little compared to the amount of total cost, thus the error is negligible.





**Figure 8 Desalination when its Cost Changes**

## The Second Solution, and Maybe the Third...

As is discussed, we will also carry out a solution concerning minimizing the indirect cost of water transferring, that is, minimizing the total length of watercourse. In this part, we will shift our focus onto achieving this goal.

Using a similar algorithm model, we solve the required result, which is shown in **Table 9** and **Figure 9**.

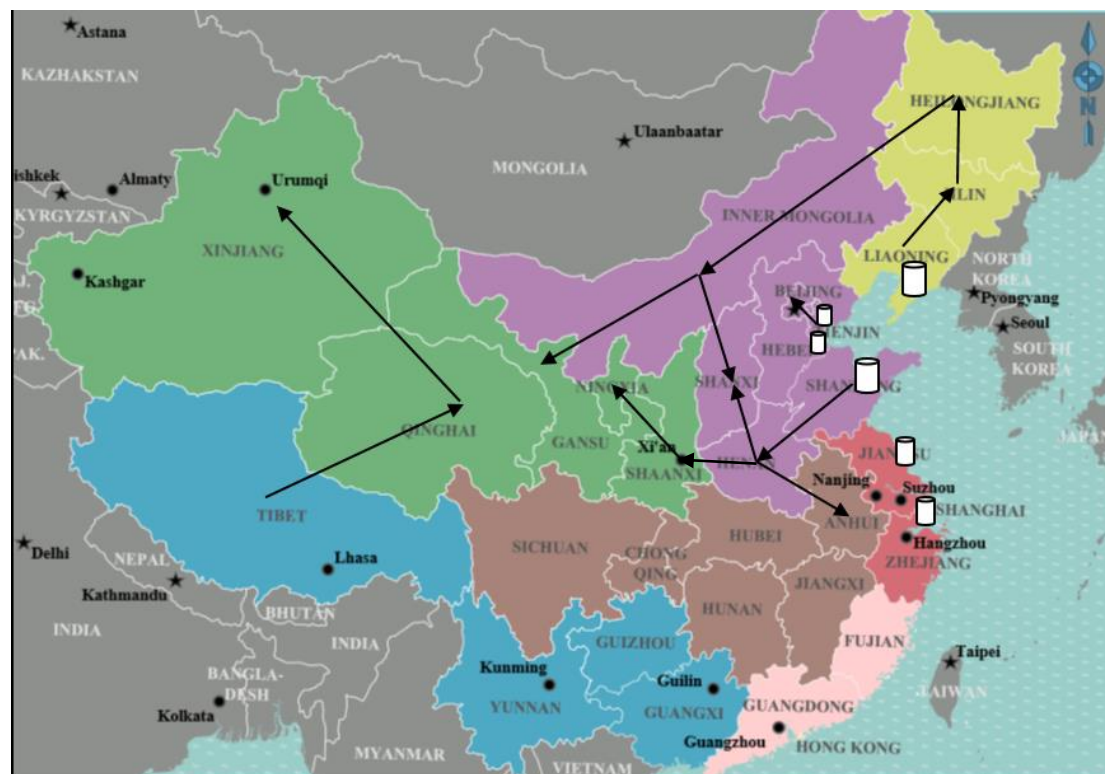
**Table 9 Water Transfer and Seawater Desalination: Minimizing total length of watercourse**

Water Transfer					
From	To	Amount (10 <sup>8</sup> m <sup>3</sup> )	From	To	Amount (10 <sup>8</sup> m <sup>3</sup> )
Tibet	Qinghai	38.987	Jilin	Heilongjiang	782.232
Qinghai	Xinjiang	262.499	Heilongjiang	Inner Mongolia	348.59
Shaanxi	Ningxia	89.935	Tianjin	Beijing	56.458
Inner Mongolia	Shanxi	85.291	Henan	Shaanxi	37.499
Inner Mongolia	Gansu	109.839	Henan	Anhui	37.222
Liaoning	Jilin	804.863	Shandong	Henan	242.508
Total amount		2895.923			
Seawater Desalination					
Province	Amount (10 <sup>8</sup> m <sup>3</sup> )		Province	Amount (10 <sup>8</sup> m <sup>3</sup> )	





Shandong	433.178	Tianjin	118.648
Jiangsu	489.179	Liaoning	910.269
Hebei	190.937	Shanghai	163.854
Total amount	2306.065		



**Figure 9<sup>①</sup> Desalination and Routes of Water Transfer**

Unsurprisingly the amount of desalination increases to 79.63% of the amount of water transfer. Since the direct cost of producing or transferring water is neglected, desalination becomes more favorable and many coastal cities desalinate water and transport them to inland provinces.

Another feature of our result is its distinctiveness to our solution focusing on reducing the total direct cost. Water are transferred from the east to west and most of the provinces in the south are self-sufficient.

One will never forget that our target is only a partial measure to address the social and environmental problems in water transferring. For example, desalination will change the concentration of seawater and sometimes lead to indwelling, which is also harmful. However, since the indirect cost of water transfer is very hard, if possible, to measure, we can bring up with other targets, such as minimizing the sum of water transferred and produced, or getting the least number of provinces influenced, etc. to take environmental and other indirect costs into consideration.

<sup>①</sup> The territory of China should include Taiwan and South China Sea Islands. The volume of column represents the amount of seawater desalination.



## Discussion

### Letter to the Governmental Leadership: China's Water Strategy in 2025

Water resource is one of the most important limitation to the development of a country and its security cannot be overstated. In China, along with the rapid development of economy and steady increasing of population, the demand for water will keep increasing in the foreseeable future. However, since the total water supply is a natural process and unlikely to increase in the following years, the water problem China is facing is becoming imminent. Our study suggests in 2025-2075, the demand of water are likely to exceed its supply and China has to start to use seawater desalination to quench its thirsty demands. We strongly believe Chinese government should deal this problem in the following 4 aspects: i) investing on technologies of desalination, ii) promoting water conservation, iii) constructing infrastructures of water transferring, and iv) doing more systematic research on water arrangement and planning.

As is discussed in our study, the high cost of desalination makes it the last choice of providing water, and whenever water demand exceeds water supply and seawater desalination has to be used, the cost of water allocation will skyrocket. We also have shown that unless there is major breakthrough on seawater desalination, which can dramatically reduce its costs, desalination will always be an uneconomic choice. As a result, Chinese government must put more weight on making desalination economically in order to reduce the shock of water shortage on the economy in a future not so far from now.

Water conservation is the only way to put off the coming of the day of water shortage without compromising economy growth. What's more, water conservation will do good to protecting the eco-system and bring China further benefit. Hence, promoting water conservation is a must for China in the future.

Our study provides a methodology and also a plan, although might be accurate, to arrange water resource all over China. The government could make the water arrangement based on our study and extant infrastructures. The government should also accelerate the construction of the network of water transfer, because the sooner this network is built, the less China will suffer from the imbalance of water supply and demand and the better China can allocate its water resources.

Our last suggestion is that Chinese government should encourage studies on water



resource planning. Our study, albeit taking many important factors into consideration, misses many essential elements and is largely simplified. Systematic and technical studies on this topic will do a lot of good to improving China's water security.

## Strengths and limitations

In the end we will discuss some strength and limitations of our work. The most prominent strength is that we take many elements, including economic, social and environmental factors, into consideration at the same time, catering to the need of sustainable development. For example, besides finding a solution that minimizes the economic costs, we also seek solutions that address social and environmental problems at the same time; another example is that we have spent a lot of time on estimating the cost of pollution treatment, which comprises a large part of the cost of water transferring. What's more, considering the fact that underground water is over withdrawn in China, we refute the idea of allowing for pumping underground water in our model

Another strengths is that our model is not confined to this modeling competition; rather it is a general model to solve similar problem. For example, our estimate of the water demand in 2025 is not based on simple regression with time but a multivariate regression model taking major influencing factors into account. The water demand is stemmed from complex components such that using the historical demand to forecast the future is unappropriated. Even though the factors we estimate (GDP, population and effective irrigation area), may also be inaccurate, the government itself can have a long-term plan for these factors and thus can forecast their own data of water demand in 2025, based on our model.

Last but not least, the flexibility of the network model we worked on is also a strength of our study. If one subdivides the provinces into more accurate nodes, he can approach the reality infinitely. In this study, we only have 30 provincial nodes due to the limitation of data, but actually, our algorithm, with combining simulated annealing and min-cost-Max-flow, can support over one thousands nodes. The longer the program run, the more accurate the result get. In addition, we can choose different "objective function" under different requests. As the cases in our paper, to change the "cost expression" of the edges, we can achieve goals like minimum water courses built or minimum amount of water transferred, beyond the basic minimum cost goal.

There are some limitations of our work as well. For example, some of our parameters estimated are not accurate, such as the cost of construction, the cost of pollution treatment. This is partly due to the lack of more accurate data. However, this is not a fatal problem, since the most important target of modeling is not providing the most accurate outcome, which is often impossible, but bringing up insights and enlighten our thoughts; the calculation in our paper are just performing the task of demonstration.



## Appendix

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