2020 年美国大学生数学建模竞赛(MCM/ICM) B 题 中英版

翻译：最长的沙堡

世界上任何一个有休闲沙滩的地方，似乎都有儿童（和成人）在海岸上建造沙堡。游客们利用工具、玩具和想象力创造了沙堡，从简单的沙堆到带有墙壁、塔、护城河和其他模仿真实城堡特征的复杂城堡复制品。在所有这些中，一个典型的建造方式是从一个由单一的、普通的湿润沙堆组成的初始雏形开始，然后继续切割和塑造这个基础，形成一个可识别的3维几何形状，从而塑造出更多的城堡特征。

不可避免地，海浪的涌入加上涨潮侵蚀了沙堡。然而，似乎并非所有的沙堡对波浪和潮汐的反应都是一样的，即使这些沙堡的建造规模大致相同，到海水的距离也相同且坐落在同一片海滩上。因此，人们怀疑是否存在一个最好的三维几何形状用于沙堡初始雏形。

要求

1. 构造一个数学模型来识别最佳三维几何形状，以作为沙堡初始雏形，它将在经历下列条件的海浪和潮汐的海岸上持续最长的一段时间：
   - 建造在同一片海滩上，且到海水的距离大致相同，
   - 使用相同类型、大致相同数量、相同水沙比例的沙子建造。
2. 使用你的模型，确定一个最佳的水沙比例来建造沙堡初始雏形，假设你不使用其他添加剂或材料（如塑料或木制支架，石头等）。

3. 根据需要调整您的模型，以确定您在需求 1 中识别出的最佳三维沙堡雏形会如何受降雨影响，以及它是否仍然是下雨时用作城堡雏形的最佳三维几何形状。
4. 如果有的话，你还可以用什么策略来延长你的沙堡寿命？
5. 最后，写一篇内容丰富的一到两页的文章，描述你的模型及其结果，发表在《假期》杂志上：*阳光下的乐趣*，读者主要是不懂模型的小白。

您的提交应包括：

- 一页摘要表
- 目录
- 一页至两页的杂志文章
- 您的解决方案不超过20页，包含摘要，目录和文章时最多24页。

注意：参考列表和任何附录均不计入页数限制，应在完成解决方案后显示。您不应使用受版权法限制使用的未经授权的图像和材料。要确保您引用的想法的来源和报告中使用的材料。

原文：The Longest Lasting Sandcastle(s)

Wherever there are recreational sandy ocean beaches in the world, there seem to be children (and adults) creating sandcastles on the seashore. Using tools, toys, and imagination, beach goers create sandcastles that range from simple mounds of sand to complicated replicas of actual castles with walls, towers, moats, and other features that mimic real castles. In all these, one typically forms an initial foundation consisting of a single, nondescript mound of wetted sand, and then proceeds to cut and shape this base into a recognizable 3-dimensional geometric shape upon which to build the more castle-defining features.

Inevitably, the inflow of ocean waves coupled with rising tides erodes sandcastles. It appears, however, that not all sandcastles react the same way to waves and tides, even if built roughly the same size and at roughly the same distance from the water on the same beach. Consequently, one wonders if there exists a best 3-dimensional geometric shape to use for a sandcastle foundation.

Requirements

1. Construct a mathematical model to identify the best 3-dimensional geometric shape to use as a sandcastle foundation that will last the longest period of time on a seashore that experiences waves and tides under the following conditions:

- built at roughly the same distance from the water on the same beach, and
- built using the same type of sand, roughly the same amount of sand, and the same water-to-sand proportion.

2. Using your model, determine an optimal sand-to-water mixture proportion for the castle foundation, assuming you use no other additives or materials (e.g. plastic or wooden supports, stones, etc.).

3. Adjust your model as needed to determine how the best 3-dimensional sandcastle foundation you identified in requirement 1 is affected by rain, and whether it remains the best 3-dimensional geometric shape to be used as a castle foundation when it is raining.

4. What other strategies, if any, might you use to make your sandcastle last longer?

5. Finally, write an informative, one- to two-page article describing your model and its results for publication in the vacation magazine: *Fun in the Sun*, whose readers are mainly non-technical.

Your submission should consist of:

- One-page Summary Sheet
- Table of Contents
- One- to Two-page Article
- Your solution of no more than 20 pages, for a maximum of 24 pages with your summary, table of contents, and article.

Note: Reference List and any appendices do not count toward the page limit and should appear after your completed solution. You should not make use of unauthorized images and materials whose use is restricted by copyright laws. Ensure you cite the sources for your ideas and the materials used in your report.