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Abstract

After deep analyses of the problem, we conclude that it falls into Re-Balancing Human-Influenced Ecosystems problem based on food chain.

In Task I, based on Volterra Model, we establish Bait-Predator Model Derived from Three Biological Populations, and specify the steady state numbers of the three populations. Then, based on Analytic Hierarchy Process and Competition Model, we obtain the ratio of different species in Population II, predict the steady state level of water quality is not high, and make the water quality satisfying through respectively adjusting the numbers of six species,

In Task II, when the milkfish farming suppresses other animal species, we set up the Logistic Model, and predict the water quality of steady state is very awful, the same to that in the fish pen, insufficient for the continued healthy growth of coral species. When other species are not totally suppressed, with improved Bait-Predator Model derived from three biological populations, we simulate the water quality of Bolinao, making it match the one observed currently, obtain the predicted numbers of populations, and discuss some changes to the Bait-Predator Model which are aimed at making the numbers of populations into closer agreement with observations.

In Task III, we establish Polyculture Model, which is able to reflect the polyculture system made up of an interdependent set of species, introduce mussels and , with seaweed growing on the sides of the pens, and obtain the numbers of populations in steady state and the outputs of our model.

In Task IV and V, we differentiate the values of edible biomass and define the value of edible biomass as the sum of the values of each species harvested, minus the cost of milkfish feed. Under the circumstances of acceptable water quality, we build the Nonlinear Equilibrium Optimization Model, receiving optimal strategy and harvest.

In Task VI, we put forward a strategy to improve the water quality in Bolinao. With the ratio between feeding cost and net income as the index, the index value of the model is smaller than that of Bolinao area, which signifies the leverage of the strategy. Also, we analyze polyculture system in terms of ecology.

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Re-Balancing Human-Influenced Ecosystems

Introduction

Consider an area in the Philippines located in a narrow channel between Luzon Island and Santiago Island in Bolinao[1], Pangasinan, that used to be filled with coral reef and supported a wide range of species. The once plentiful biodiversity of the area has been dramatically reduced with the introduction of commercial milkfish (Chanos chanos) farming in the mid 1990’s.It's now mostly muddy bottom, the once living corals are long since buried, and there are few wild fish remaining due to over fishing and loss of habitat. While it is important to provide enough food for the human inhabitants of the area, it is equally important to find innovative ways of doing so that allow the natural ecosystem to continue thriving; that is, establishing a desirable polyculture system that could replace the current milkfish monoculture.

In order to improve the situation in Bolinao, we need to establish a practicable polyculture system, and we make it through gradual effort.

So our goal is pretty clear:

- Model the original Bolinao coral reef ecosystem before fishfarm introduction.
- Model the current Bolinao monoculture milkfish.
- Model the remediation of Bolinao via polyculture.
- Discuss the outputs and economic values of species.
- Write an information paper to the director of the Pacific Marine Fisheries Council summarizing the relationship between biodiversity and water quality for coral growth.

Our approach is:

- Deeply analyze data in the problem, gradually establish the Model of Coral Reef Foodweb.
- With available data in the problem as evaluation criteria, confirm the water quality based on elements in the sediment.
- Establish models, and interpret actual situation with data, with the purpose of improving water quality.
- Do further discussion based on our work.

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Solutions

Task 1

Aimed to the Coral Reef Foodweb Model, we assume all the species grow in the same fish pen, and firstly we divide these species into three populations: one algae species(Population I)， one herbivorous fish、one mollusc species、one crustacean species、one echinoderm species(Population II) and the sole predator species, i.e. milkfish (Population III). The interrelationship between them is presented as Figure 1-1:

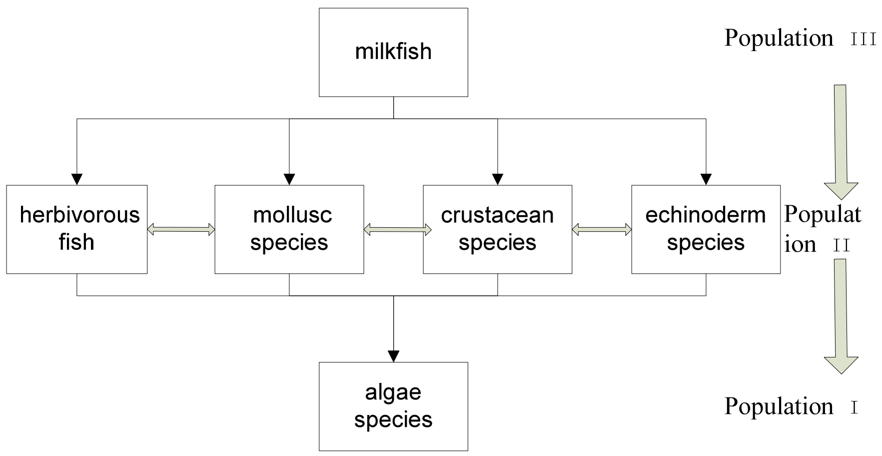

Figure 1-1 Figure of Interrelationship between Three Populations

On this basis, we can establish the Volterra Bait-Predator Model[2] Derived from Three Populations, assume the numbers of the three populations respectively as $x_1(t)$, $x_2(t)$, $x_3(t)$. If not taking restrictions of natural resources on the algae species into consideration, the algae species follow the exponential growth law when growing dependently, assume the relative growth rate as r_1, then $\dot{x}_1(t) = r_1 x_1$, but the species of Population II feeding on the algae species will decrease the growth rate of the algae. So the model of the algae species is:

$$\dot{x}_1(t) = x_1(r_1 - \lambda_1 x_2) \tag{1-1}$$

Proportionality coefficient λ_1 reflects the feeding capability of species in Population II for the algae species. Assume the death rate of the species in Population II as r_2

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when existing dependently, then $\dot{x}_2(t) = -r_2 x_2$, so based on the foodweb we conclude:

$$\dot{x}_2(t) = x_2(-r_2 + \lambda_2 x_1) \tag{1-2}$$

Proportionality coefficient λ_2 reflects the support capability of the algae species for Population II, which provide food for the milkfish, so the existence of milkfish reduce the growth rate of the species in Population II, the right side of Equation (1-2) should subtract the blocking effect, and then we obtain the model of Population II:

$$\dot{x}_2(t) = x_2(-r_2 + \lambda_2 x_1 - \mu x_3) \tag{1-3}$$

Likewise, the model of milkfish is:

$$\dot{x}_3(t) = x_3(-r_3 + \lambda_3 x_2) \tag{1-4}$$

At last, we obtain the interdependent and mutually-restricting mathematical model of the three populations:

$$\begin{cases} \dot{x}_1(t) = x_1(r_1 - \lambda_1 x_2) \\ \dot{x}_2(t) = x_2(-r_2 + \lambda_2 x_1 - \mu x_3) \\ \dot{x}_3(t) = x_3(-r_3 + \lambda_3 x_2) \end{cases} \tag{1-5}$$

Since System of Differential Equations (1-5) has no analytic solution, we use MATLAB to get its numerical solution, and then speculate the construction of the analytical solution through the numerical solution. The ecologists point out that the periodic solution cannot be observed in most balanced ecosystems, which tend to a certain balanced state, that is, there exists an equilibrium point in each balanced ecosystem. In addition, some ecologists think the structure of long-existing and periodically-changing balanced ecosystems in the nature is steady, that is to say, if diverging from the former periodic track because of disturbance; the internal control mechanism will restore it. However, the periodically-changing state described by Volterra Model is non-structure stability, and even subtle adjustments to the parameters will change the periodic solution. So we improve the model, and still set the numbers of the three populations as $x_1(t)$, $x_2(t)$, $x_3(t)$. Also, the number evolution follows Logistic Law. When the algae species grow independently, the model is:

$$\dot{x}_1(t) = x_1 r_1 (1 - \frac{x_1}{N_1}) \tag{1-6}$$

In Equation (1-6), r_1 refers to the intrinsic growth rate of the algae species; N_1

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refers to the maximum number of the algae species which is allowed by the environmental resources. The algae species provide food for the species of Population II, so the model of the algae species is:

$$\dot{x}_1(t) = r_1 x_1 (1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}) \tag{1-7}$$

In Equation (1-7), N_2 is the maximum capacity of the species in Population II; σ_1 refers to the quantity of the algae (compared to N_1) eaten by the unit quantity species in Population II (compared to N_2).

Without the algae species, the species in Population II will perish, set its death rate as r_2, then when existing dependently, we will have:

$$\dot{x}_2(t) = -r_2 x_2 \tag{1-8}$$

The algae species provide food for Population II, the right side of Equation (1-8) should add the auxoaction, and the growth of the species in Population II is also influenced by internal blocking action, then we have:

$$\dot{x}_2(t) = r_2 x_2 (-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1}) \tag{1-9}$$

In Equation (1-9), σ_2 is similar to σ_1 appearing in (1-7), we can get the model of the species in Population II:

$$\dot{x}_2(t) = r_2 x_2 (-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1} - \sigma_3 \frac{x_3}{N_3}) \tag{1-10}$$

In Equation (1-10), σ_3 is similar to σ_1 and σ_2. Without the species in Population II, milkfish will disappear, set its death rate as r_3, then when the milkfish exists independently, we will have: $\dot{x}_3(t) = -r_3 x_3$.

The species in Population II provide food for the milkfish, and the growth of milkfish is also restricted by the internal blocking action. Here the model is:

$$\dot{x}_3(t) = r_3 x_3 (-1 - \frac{x_3}{N_3} + \sigma_4 \frac{x_2}{N_2}) \tag{1-11}$$

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Simultaneous Equations (1-7), (1-10), (1-11) constitute the interdependent mathematical model between the three populations:

$$\begin{cases} \dot{x}_1(t) = r_1 x_1 (1 - \dfrac{x_1}{N_1} - \sigma_1 \dfrac{x_2}{N_2}) \\ \dot{x}_2(t) = r_2 x_2 (-1 - \dfrac{x_2}{N_2} + \sigma_2 \dfrac{x_1}{N_1} - \sigma_3 \dfrac{x_3}{N_3}) \\ \dot{x}_3(t) = r_3 x_3 (-1 - \dfrac{x_3}{N_3} + \sigma_4 \dfrac{x_2}{N_2}) \end{cases} \tag{1-12}$$

So far, we have established the Bolinao Coral Reef Ecosystem Model before putting farming into practice. After consulting relevant materials, we obtain the values of some parameters in the model, and through nonlinear data fitting of the original data of the local three populations[2][3][4], we get their natural growth rates: $\sigma_1 = 0.6$, $\sigma_2 = 0.5$, $\sigma_3 = 0.5$, $\sigma_4 = 2$, $r_1 = 1$, $r_2 = 0.5$, $r_3 = 0.6$, $N_1 = 150000$, $N_2 = 30000$, $N_3 = 2200$. According to the volume of local fish pens and relevant materials, we get the original numbers of the three populations: $x_1(0) = 121500$, $x_2(0) = 27000$, $x_3(0) = 2000$. Then we use MATLAB to simulate the model, and the changes are illustrated in Figure 1-3:

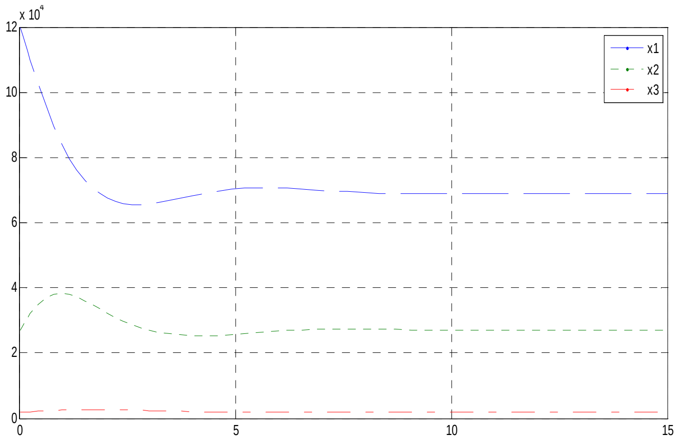

Figure 1-2 Figure of Numerical Solutions to $x_1(t)$, $x_2(t)$, $x_3(t)$

From Figure 1-2, we can see, with the passage of time, the changes of $x_1(t)$, $x_2(t)$ and $x_3(t)$ tend to steady values. Use simulation to get the numerical solution.

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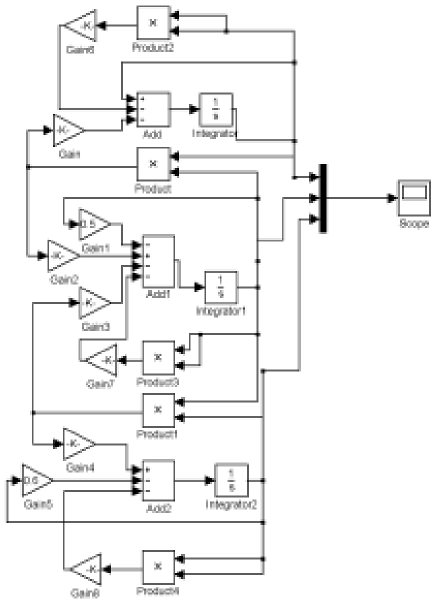

Figure 1-3 Simulation Diagram

From numerical solution, we can approximately get the stable value (69027，27015，1760.3). Then the numbers of the three populations are respectively 69027, 27015, 1760.3. Here the number 27015 of the species in Population II is made up of herbivorous fish, mollusks, crustaceans and echinoderm.

Now we will confirm the numbers of all the species in Population II, which stay at the same trophic level, coexisting and mutually competing. Here we apply Expert System and Group Decision Theory to confirm the weight between all the species in Population II. The so-called Expert System and Group Decision refers to the multi-attribute Decision Problem which aims to select the optimal solution from many alternatives or sort the available alternatives. Assume the finite solution set as $Y = \{y_1, y_2, \cdots, y_n\}$, of which $n \geq 2$, y_i signifies the i th solution, here $i = 1, 2, \cdots n$ .Attribute set is $C = \{c_1, c_{2,} \cdots c_q\}$,of which $q \geq 2$, c_j refers to the j th attribute, here $j = 1, 2, \cdots q$. Decision expert set is $E = \{e_1, e_2, \cdots e_m\}$, of which $m \geq 2$, e_k means the k th decision-making expert, here $k = 1, 2, \cdots m$. $S = \{s_1, s_2, \cdots s_g\}$, here S is a pre-defined set constituted by odd-chain elements, and $S = g + 1$. Decision-making Expert e_k select one element from S as the evaluation value of Solution y_i under Attribute c_j, set it as p_{ij}^k, $p_{ij}^k \in S$, $p^k = (p_{ij}^k)_{n \times q}$ refers to the

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judgment matrix made by Decision-making Expert e_k on all the solutions under all the attributes; the attribute weight vector in evaluating information form given by Decision-making Expert e_k is $W^k = (w_1^k, w_2^k, \cdots w_q^k)^T$,of which w_j^k means the weight of Attribute c_j selected by Decision-making Expert e_k from Set S , $w_j^k \in S$. This theory can be actualized through Analytical Hierarchy Process.

Analytical Hierarchy Process (shortened as AHP) was first put forward by American operational researcher A.L.Saaty in the 70s in 20th century. AHP is a kind of method for decision-making analysis combining qualitative and quantitative methods. Using this method, decision makers can separate complex problems into several levels and factors and compare and find the weight of different programs and provide the basis for the optimum program.

Basic principle: AHP first classify the problem into different levels based on the nature and the purpose of the problem, construct a multi-level structure model ranked as the lowest level(program for decision making, measures etc), compared with the highest level (the highest purpose).

Based on AHP, we can establish the stratification diagram shown in Figure 1-4:

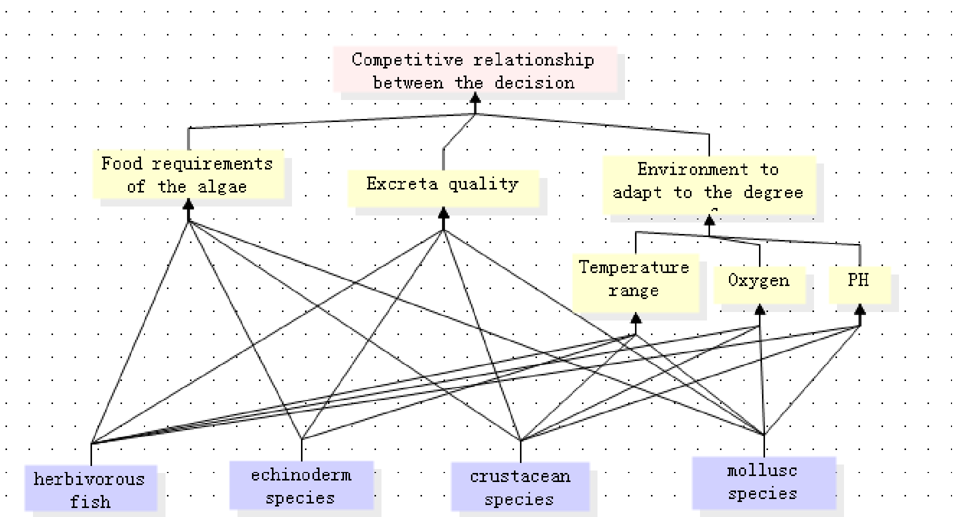

Figure 1-4 AHP Stratification Diagram

At last, we make consistency check of the result, finding that the consistency ratio of each expert's judgment matrix is below 1, so the consistency of judgment matrix is acceptable. Finally we figure out the weight of the numbers of all the species in Population II, as shown in Table 1-1:

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Table1-1 Weight of Each Species Measured by AHP

Alternatives	Weight
Herbivorous fish	0.2087
Crustaceans	0.2334
Molluscs	0.3140
Echinoderm	0.2438

Here we adopt population competition model to confirm the weight of each species in Population II

$$\begin{cases} \frac{dN_1}{dt} = N_1(\varepsilon_1 + \gamma_1 N_2) \\ \frac{dN_2}{dt} = N_2(\varepsilon_2 + \gamma_2 N_1) \end{cases} \tag{1-13}$$

Here, ε_i: birth-rate, γ_i :coefficients of species interaction.

According to (1-13), we find that the ratio between different species is almost consistent with that obtained by AHP, which also confirms the correctness of our method.

In this way, we can find: herbivorous fish, crustaceans, molluscs and echinoderm can coexist and also compete. So the number of each species can be figured out based on the data in steady state from the previous models, as shown in Table 1-2:

Table 1-2 The Number of Each Species in Steady State

Organism	Number (per pen)
The algae	69027
Herbivorous fish	5638
Crustaceans	6305
Molluscs	8483
Echinoderm	6589
milkfish	1760.3

Now we use the model to check the water quality, and make clear whether it is suitable for the continued healthy growth of the coral.

Firstly, we should calculate the current concentration of chlorophyll in the fish pen. With help of relevant materials, we find out the regression equation between the algae and chlorophyll:

$$N = 0.7568C + 1.2785$$

The unit of N :10^4/mL, the unit of $C : ug/L$ Since $N = 6.90273$, $C = 7.43155$.

Here we obtain the content of chlorophyll. Obviously the concentration of chlorophyll is far beyond 0.25 ug/L, suitable concentration for the growth of coral.

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Now with the available data in the problem, we figure out the mass of organic particles in the fish pen, and then work out the mass of each element.

- With the available data in the problem, figure out the dry weight of echinoderm in the pen is 45464.1g, the dry weight of milkfish excrement is 425992.6 ~ 867827.9 mg, and then get total dry weight of excrement in the pen is 948829.75 ~ 1390665.05mg.
- The volume of the pen is 10m*10m*8m. Finally we get the concentration of organic particles is 1186.037 ~ 1738.331 ug/L. Based on the percentage of elements given in the problem, we figure out then concentration of C(10%), N(0.4%), P(0.6%)

Table 1-3 The Concentration of Elements in the Pen

Element	Concentration(ug/L)
C（10%）	118.604 ~ 173.833
N（0.4%）	4.744 ~ 6.953
P（0.6%）	7.116 ~10.430

Comparing the water quality in Site A, B, C and D, we find the concentration of organics is between A and B, which is suitable for the growth of coral (here the concentration of elements is calculated only based on the excrement of milkfish and echinoderm), so the concentration of the microbe meets the multiplication needs of the coral. But the concentration of chlorophyll is seriously out of limits. So we have to adjust the numbers of some species to make the concentration of chlorophyll reach the standard.

Counter-reasoning from the concentration of chlorophyll (0.25 ug/L) suitable for the growth of the coral, together with the regression equation

$$N = 0.7568C + 1.2785,$$

With estimated steady state number, we can assume the initial introducing value as: (10000，5500，350). With relevant literature, we get the maximum volume of fish pens (N_1, N_2, N_3) : (30000，6000，400), and through re-simulation finally find out the positive corrected results of steady state number: (13732，5432，320).

we work out the estimated steady state number of the algae N=1.4677, and then counter-reason the numbers of three populations: (14677，5744，350).

After correction, we get the actual steady number of the algae: N=1.3732. Put is back into the regression equation, we get: C=0.125, that is, the concentration of chlorophyll is 0.125ug/L, which means the water quality after adjustment completely meets the demand. Moreover, the total number of milkfish and echinoderm is smaller than that before correction, so the index of the organics can certainly reach the growing demands of the coral, as shown in Figure 1-6:

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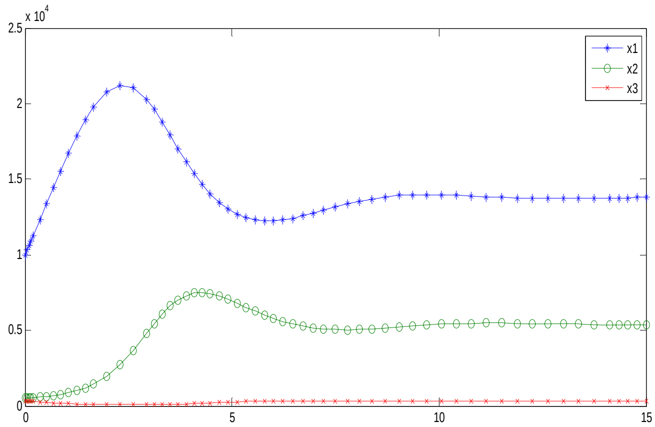

Figure 1-6 The Numbers of Species Meeting the Demands after Adjustment

In the retroegulation process, which is the feedback mechanism of this model, with known water quality, we counter-reason the estimated steady state number of all the species, make positive simulation after estimating the initial introducing value of all the species, and get the corrected steady state value. With this mechanism, we can find out the steady state number of each species based on water quality, which provides great convenience to the solution to the following problems.

Task 2

Task 2.a Establishment of Logistic Model

In this task, with all the herbivorous fish, crustaceans, molluscs and echinoderm excluded, it is required to find out the changes to the species and the circumstances of water quality. Based on analyses, we make clear the reasons why the growth rate will decrease when the milkfish increase to a certain amount. It is noticed that such factors as natural resources, environmental conditions restrict the growth of milkfish, and with its growth, the blocking effect will become greater and greater. The blocking effect is expressed in terms of the influence on the growth rate of milkfish r, making r decrease with the increase in the number of milkfish x. If r is expressed as the function of $x : r(x)$, it should be a decreasing function, so we have:

$$\frac{dx}{dt} = r(x)x, x(0) = x_0 \qquad (2\text{-}1)$$

The simplest assumption of $r(x)$ is: assume $r(x)$ as the linear function of x, that is,

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$$r(x) = r - sx\text{ , }(r > 0, s > 0) \qquad (2\text{-}2)$$

Here r is intrinsic growth rate, that is, the growth rate when the number of the milkfish is very small. In order to confirm the meaning of Coefficient s, we introduce the maximum quantity x_m which is allowed by natural resources and environmental conditions, which is regarded as the milkfish capacity. When $x = x_m$, x will stop increasing, that is, growth rate $r(x_m) = 0$, put it into （2-2）, we will get $s = \frac{r}{x_m}$, then:

$$r(x) = r(1 - \frac{x}{x_m}) \qquad (2\text{-}3)$$

Another interpretation to Equation (2-3): the growth rate $r(x)$ is in direct proportion to the unsaturated part of the milkfish capacity $x = (x_m - x)/x_m$, the proportionality coefficient is intrinsic growth rate r.Put Equation (2-3) into Equation (2-1), we will get:

$$\frac{dx}{dt} = rx(1 - \frac{x}{x_m}), x(0) = x_0 \qquad (2\text{-}4)$$

The factor rx on the right side of Equation (2-4) expresses the internal growing tendency of the milkfish, and the factor $(1 - \frac{x}{x_m})$ expresses the blocking effect of resources and environment on the milkfish growth. Obviously, the bigger x is, the bigger rx is, the smaller $(1 - \frac{x}{x_m})$ is. The growth of milkfish is the result of the coaction of the two factors.

It is easy to find that Equation (2-4) can be solved with the method of Separation of Variables:

$$x(t) = \frac{x_m}{1 + (\frac{x_m}{x_0} - 1)e^{-rt}} \qquad (2\text{-}5)$$

With Equation (2-5), we can know: the increase of x is fast and then full,

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when $t \to \infty$, $x \to x_m$. Now we use LLS linear least squares to estimate the parameter r and x_m of this model, and express Equation (2-4) as:

$$\frac{dx/dt}{x} = r - sx, s = \frac{r}{x_m} \tag{2-6}$$

We consult relevant data[3] (here we should point out the amount of milkfish refers to the one harvested all over the Philippines), and calculate these data with MATLAB, and then we get: r=0.5, x_m =190050.00. Put them into Equation (2-5), we get the changes to the function as shown in Figure 2-1. Further, we get the weight and number of the milkfish respectively as 172375346.966kg and 34475069 ~ 24625050. (Reference: The land area of the Philippines is 300,000 km^2, the sea area 27.6 mi^2. The Philippines is surrounded by the sea, and has lots of islands; the depth of the sea between islands is mostly within 50 m.)

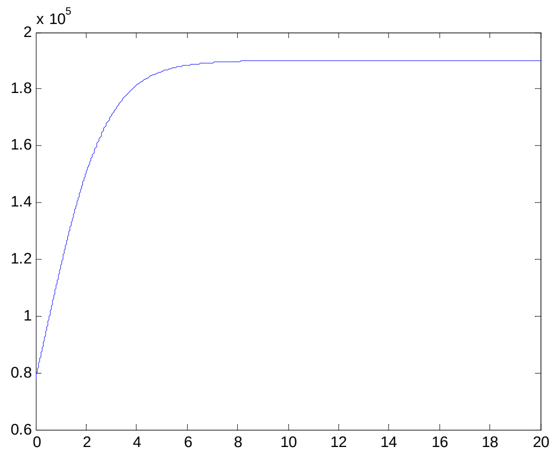

Figure 2-1 Milkfish Changes

We get the sediment per m^2 based on the sea area: 0.1165849 ~ 0.3325079 g/m^2. Since the sediment is usually not very thick, we assume it as 0.1m, then we get the sediment per m^3: 1.165849 ~ 3.325079 g/m^3. Then based on the information given in the problem, we get:

Table 2-1 Element Content

Element	Content(ug/L)
C(10%)	116.5849 ~ 332.5079
N(0.4%)	46.63396 ~ 133.00316
O(0.6%)	69.95094 ~ 199.50474

From Table 2-1, we can see eutrophy is very serious, and the coral cannot grow. The water quality is very poor, which almost matches the environment in the pen.

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Task 2.b Simulating Comparison of the Current Situation

In Task 2.a, we discuss the independent farming of the milkfish, but actually in the pen, there are more than the milkfish and the algae. So here we have to introduce the removed species as the middle strata, and according to the requirements of the problem, only adjust the numbers of the species in the middle strata to simulate the water quality in the Bolinao area until the water quality matches the one currently observed.

The concrete practices are as follows: take simulating the water quality in Site D for example, solve the problem according to the model in Task 1. Since the model bears the following characteristics: it is easier to find out the water quality in the area of stability when knowing the initial introducing values of the algae, milkfish and other species than to feedback the numbers of the algae, milkfish and other species when knowing the water quality.

Adopt violence random search method:

- Set the initial values of the algae, other species and the milkfish as $N_1 = 100000$, $N_2 = 10000$, $N_3 = 1300$. Other parameters have nothing to do with the introducing amount, the same to the model in Task 1.
- According to the introducing ratio between the milkfish and the algae and the requirements for the capacity of the pen obtained from Task 2.a, we introduce the algae and the milkfish respectively as 72000 and1300, and at the same time have the introducing amount of other species in random distribution between 8000~10000, with the aim of searching for the theoretical value matching the observed water quality.
- Simulate the model in Task 1, recycle 1,000 times, and finally put out the water quality in steady state which is consistent with the actually observed value.

Now give out the criteria for judging water quality consistency: (parameters: x1 is the steady state value of the algae, x2 the steady state value of other species, x3 the steady state value of the milkfish):

Chlorophyll a：ca=x1*0.0001-1.2785)/0.7568
Total content of organics：K=x2*0.2438*6.9*[0.2,11.5]+x1*[242,493]
Percentage of Different Elements in the Excrement：POC 10%，PON 0.4%， P 0.6%
POC meets： abs(c(1)-c1(1))<=100
abs(c(2)-c1(2))<=100
PON meets： abs(n(1)-n1(1))<=10
abs(n(2)-n1(2))<=10
Chlorophyll a：abs(ca-4.5)<=0.15

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Sort out results meeting the above requirements, that is, the numbers of three species when the water quality obtained through simulation similar to the observed one:

Table 2-2 Simulating Results

Simulating Results	Initial Introducing Amount：70000 [8008,8995] 1100 Amount in the Steady State：46062 8989 1040
Estimated Observing Data	Amount in the Steady State：45700 9325 890

In order to make the numbers of the species close to those predicted in the model, we compare the numbers of existing species with those observed in Bolinao area. Here we take into account that the added feedstuff can correct the model in Task 1, that is, we can add a constant λ to the model in Task 1 to express the influence of feedstuff on the numbers of the species, and the newly-built model is shown as:

$$\begin{cases} \dot{x}_1(t) = r_1 x_1 (1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}) \\ \dot{x}_2(t) = r_2 x_2 (-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1} - \sigma_3 \frac{x_3}{N_3}) \\ \dot{x}_3(t) = r_3 x_3 (-1 - \frac{x_3}{N_3} + \sigma_4 \frac{x_2}{N_2}) + \lambda \end{cases} \tag{2.7}$$

Then introduce the initial values (70000，[8008,8995]，1100), and figure out the steady state numbers of all the species: (46062，8989，1051), as shown in Figure 2-2:

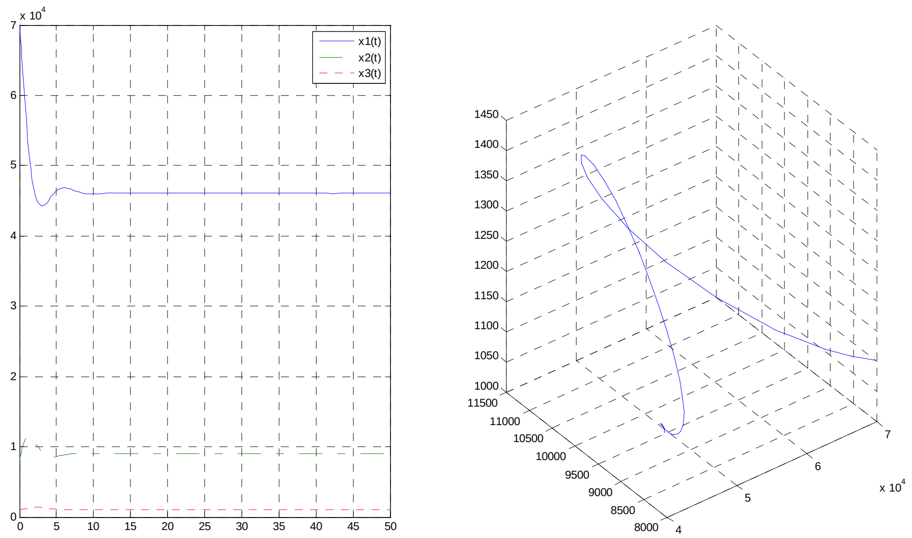

Figure 2-2 Comparison between Observed Values and Simulated Values

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Task 3

Task 3.a Develop a commercial polyculture to remediate Bolinao

Based on the Bolinao Coral Reef Ecosystem Model before farming in Task 1: after introducing filter feeders, correct the model in Task 1, and set the numbers of the four species respectively as: $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$. Also, the number evolution follows Logistic Law. When the algae species grow independently, the model is:

$$\dot{x}_1(t) = x_1 r_1'(1-\frac{x_1}{N_1}) \tag{3-1}$$

In Equation (3-1), r_1' refers to the intrinsic growth rate of the algae species, N_1 the maximum number of the plants allowed by the environment and resources. The algae species provide food for the filter feeders and the herbivores, so the model of the algae species is:

$$\dot{x}_1(t) = r_1' x_1(1-\frac{x_1}{N_1} - \sigma_{12}' \frac{x_2}{N_2} - \sigma_{13}' \frac{x_3}{N_3}) \tag{3-2}$$

In Equation (3-2), N_2 signifies the maximum capacity of the filter feeders, N_3 the maximum capacity of the herbivores; σ_{12}' refers to σ_{13}' times unit amount algae(compared to N_1) eaten by unit amount filter feeders (compared to N_2), σ_{13}' refers to σ_{13}' times unit amount algae(compared to N_1) eaten by the unit amount herbivores(compared to N_3).

Without the algae species, the filter feeders will perish, assume its death rate as r_2', then when it exists independently, we will have:

$$\dot{x}_2(t) = -r_2' x_2 \tag{3-3}$$

The algae species provide food for the filter feeders and the herbivores, the right side of Equation (3-3) should add the auxoaction of the algae species to the filter feeders and the herbivores, and the growth of filter feeders is also influenced by the internal

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blocking action, so we get the model of the filter feeders:

$$\dot{x}_2(t)=r_2'x_2(-1-\frac{x_2}{N_2}+\sigma_2'\frac{x_1}{N_1}-\sigma_7'\frac{x_4}{N_4}) \tag{3-4}$$

Likewise, we infer the models of the herbivores and the milkfish:

The herbivore: $$\dot{x}_3(t)=r_3'x_3(-1-\frac{x_3}{N_3}+\sigma_3'\frac{x_1}{N_1}-\sigma_8'\frac{x_4}{N_4}) \tag{3-5}$$

The milkfish: $$\dot{x}_4(t)=r_4'x_4(-1-\frac{x_4}{N_4}+\sigma_4'\frac{x_2}{N_2}+\sigma_6'\frac{x_3}{N_3}+\sigma_5'k) \tag{3-6}$$

Here k refers to the amount of feedstuff cast.

In this way, we get the model of polyculture with simultaneous equations (3-2), (3-5) and (3-6):

$$\begin{cases}
\dot{x}_1(t)=r_1'x_1(1-\frac{x_1}{N_1}-\sigma_{12}'\frac{x_2}{N_2}-\sigma_{13}'\frac{x_3}{N_3}) \\
\dot{x}_2(t)=r_2'x_2(-1-\frac{x_2}{N_2}+\sigma_2'\frac{x_1}{N_1}-\sigma_7'\frac{x_4}{N_4}) \\
\dot{x}_3(t)=r_3'x_3(-1-\frac{x_3}{N_3}+\sigma_3'\frac{x_1}{N_1}-\sigma_8'\frac{x_4}{N_4}) \\
\dot{x}_4(t)=r_4'x_4(-1-\frac{x_4}{N_4}+\sigma_4'\frac{x_2}{N_2}+\sigma_6'\frac{x_3}{N_3}+\sigma_5'k)
\end{cases} \tag{3-7}$$

In the simultaneous equations, $x(1),x(2),x(3),x(4)$ respectively signifies the numbers of the algae, the filter feeders, the herbivorous fish and the milkfish, r_1',r_2',r_3',r_4' their respective growth rate, and k the constant of feedstuff casting.

σ_{12}'： the algae eaten by the unit amount filter feeders，

σ_{13}'： the algae eaten by the unit amount herbivorous fish，

σ_2'： the filter feeders fed by the unit amount algae，

σ_3'： the herbivorous fish fed by the unit amount algae，

σ_4'： milkfish/the milkfish fed by the unit amount filter feeders，

σ_6'： milkfish/the milkfish fed by the unit amount herbivorous fish，

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$\sigma_7{}'$: the filter feeders preyed by the unit amount milkfish,

$\sigma_8{}'$: the herbivorous fish preyed by the unit amount milkfish.

So, Equation (3-7) can be solved with MATLAB to obtain the numbers of the algae, the filter feeders, the herbivorous fish and the milkfish: (14314, 6092, 6129, 6979). The diagram when reaching the steady state is shown as Figure 3-1:

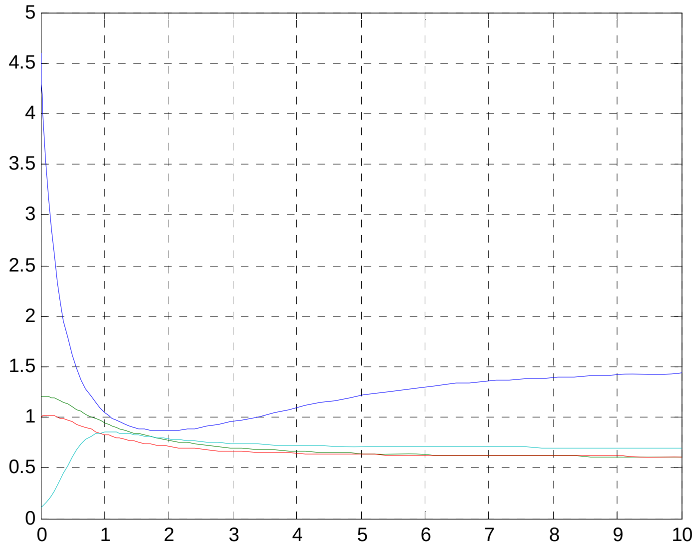

Figure 3-1 The Changes to the Numbers of the Algae, the Filter Feeders, the Herbivorous Fish and the Milkfish

Task 3.b Report on the outputs of the model.

Based on Equation (3-7), all the farming is carried out in the pen, and we can easily find out:

- This model optimize the water quality, since only when the water quality reaches a certain standard, can it provide the ideal growing environment for a species, and only in the viable environment, is it meaningful to talk about the number of each species.
- We establish a newly-born coral reef habitat without the help of man, that is, without feedstuff casting, with least leftover nutriment and particles (foodstuff and excrement) sediment.
- According to Task 3.a, we get the steady state numbers of the algae, the filter feeders, the herbivorous fish and the milkfish: (14314, 6092, 6129, 6979), and

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then with the calculating method in Task 1, we regard (14314，6092，6129，6979)as the initial values to figure out the concentration of chlorophyll is 0.2022 ug/L. Based on the information about the elements percentage given in problem, we calculate the content of different elements, as shown in Table 3-1:

Table 3-1 The Content of Elements in the Fish Pen

Element	Content(ug/L)
C(10%)	34.6604 ~ 71.7465
N(0.4%)	1.3864 ~ 2.8699
P(0.6%)	2.0796 ~4.3048

The water quality can be reflected from Table 3-1.

- Assume the total income $K=\sum x(i)t(i)$, $i=1,2,3,4$, here $x(i)$ respectively signifies the number of the algae, the molluscs, the herbivorous fish and the milkfish, and $t(i)$ their respective market values.
- Based on market investigation and relevant online data, we get the average weight and price of each species, and finally figure out the income: $K=\sum x(i)t(i)$ =\$$1.1357\times 10^5$/pen.
- In order to calculate the cost on improving water quality, assume that we introduce 1,000 mussels into the pen, investigate such factors as weight and market price of mussels, and put them into the model in Task 1 to figure out all the indexes.

Table 3-2 The Steady State Number of Each Species before Adjustment

	the algae	the molluscs (mussels)	the herbivorous fish	milkfish
Before Adjustment($\times 10^4$)	1.4314	0.6092	0.6129	0.6979
After Adjustment($\times 10^4$)	1.3726	0.6187	0.6149	0.7011

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Table 3-3 The Content of Each Element Before and After Adjustment

	Chlorophyll a(ug)	C (ug/L)	N(ug/L)	P(ug/L)
Before Adjustment	0.2022	34.6604-71.7465	1.3864-2.8699	2.0796-4.3048
After Adjustment	0.1245	33.2377-68.8661	1.3295-2.7546	1.9943-4.1320

According Table 3-3, it is easy to find the water quality has been improved, since the introduced mussels feed on the algae for one thing, and decompose the organic particles for another, which has been validated by the data.

- Calculate the cost on improving the water quality of a fish pen: the 1,000 introduced mussels \$361.2 or so, the total income is finally adjusted as $K=\sum x(i)t(i)=1.1407\times10^5$ \$, and the cost accounts for 0.317% of the income.

Task 4

According to the model in Task 3.a, we can figure out the number of each species after reaching the steady state. From Task 3.a, we know the numbers of the algae, the filter feeders, the herbivorous fish and the milkfish as (14314，6092，6129，6979). Here we can see the number of the algae is the biggest, and the numbers of the other species are similar. In such a steady state, we will discuss:

- First of all, according to the relationship between market supply and demand and price, we cannot think the amount of milkfish is as important as that of seaweed harvested. Obviously, the price of milkfish is higher that that of seaweed. In addition, although the amount of seaweed is large, but it is light, so we cannot pursue the maximum weight.
- Secondly, if measuring harvest with the price of each species harvested, we have to differentiate the values of the species. Since there are various costs on feeding the milkfish, we should take these costs into consideration when calculating the values of each species. We should define the value of edible biomass as the sum of the values of each species harvested, minus the cost of milkfish feed.
- Finally, we point out, when people expect the outputs of edible biomass to reach the maximum income, we should subtract the cost of milkfish feedstuff from its total value.

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Task 5

When confirming commercial polyculture scheme, we usually not only consider the economic benefits of farming, but also ensure to reach win-win between economy and environment under the premise of keeping the ecological environment and water quality in good condition. Hence, we establish the following optimal model to pursue the maximum commercial benefits, with the premise of not having water quality worse. Combined with the previous Polyculture System Model, we establish the following Non-linear Optimization Model of Balance to maximize the total values of harvest. This model falls into Optimization Model, but it belongs to complex Nonlinear single-objective optimization model since non-linear differential equations are embedded into its constraint conditions. And the specific model is as follows:

Objective function： $\max f = ax_1 + bx_2 + cx_3 + dx_4 - \mu$

a、b、c、d are respectively the unit market price of each species, and μ the feedstuff price.

Constraint conditions: the following three are constraint conditions of water quality, that is, the result must satisfy the water quality condition required by the coral growing.

- the content of chlorophyll below 0.28mg/mL
- the content of POC below196ug/L
- the content of PON below 39 ug/L

so the final model is： $\max f = ax_1 + bx_2 + cx_3 + dx_4 - \mu$

$$
s.t
\begin{cases}
\dfrac{0.0001x_1 - 1.2785}{0.756} \le 0.28 \\
1.68222x_2\left[0.2,11.5\right] + 0.1x_4\left[242,493\right] \le 196 \\
1.68222x_2\left[0.2,11.5\right] + 0.004x_4\left[242,493\right] \le 39 \\
\begin{cases}
\dot{x}_1(t) = r_1x_1(1 - \dfrac{x_1}{N_1} - \sigma_{12}\dfrac{x_2}{N_2} - \sigma_{13}\dfrac{x_3}{N_3}) \\
\dot{x}_2(t) = r_2x_2(-1 - \dfrac{x_2}{N_2} + \sigma_2\dfrac{x_1}{N_1} - \sigma_7\dfrac{x_4}{N_4}) \\
\dot{x}_3(t) = r_3x_3(-1 - \dfrac{x_3}{N_3} + \sigma_3\dfrac{x_1}{N_1} - \sigma_8\dfrac{x_4}{N_4}) \\
\dot{x}_4(t) = r_4x_4(-1 - \dfrac{x_4}{N_4} + \sigma_4\dfrac{x_2}{N_2} + \sigma_6\dfrac{x_3}{N_3} + \sigma_5 k)
\end{cases}
\end{cases}
\qquad (5\text{-}1)
$$

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The above is Non-linear optimization model of balance. From the equality constraint constrained by the nonlinear equations in the latter part of constraint conditions, we can see the dereferencing of variables must keep the balance between ecosystem stability and environmental stability.

Such complex optimal model cannot be solved directly with any software, so firstly we make Cycle Simulation Search (actually still Violence Search) for the equality constraints in the latter part to find out enough solutions meeting water quality conditions, and obtain the intervals of the steady numbers of the species which meet the demands of water quality, as shown in Table 5-1:

Table 5-1 The Steady State Number of Each Species

	Algae	Molluscs	herbivorous fish	milkfish
MAX($\times 10^4$)	1.3922	0.6249	0.6233	0.7061
MIN($\times 10^4$)	1.3286	0.6152	0.6174	0.7018

Therefore, we can replace the constraint conditions of nonlinear equations with the intervals of the steady numbers of the four species. If so, Equation (5-1) can be simplified as:

$$\max\ f = ax_1 + bx_2 + cx_3 + dx_4$$

$$s.t$$

$$\begin{cases} \dfrac{0.0001x_1 - 1.2785}{0.756} \le 0.28 \\ 1.68222x_2\left[0.2, 11.5\right] + 0.1x_4\left[242, 493\right] \le 196 \\ 1.68222x_2\left[0.2, 11.5\right] + 0.004x_4\left[242, 493\right] \le 39 \\ 1.3286 \le x_1 \le 1.3922 \\ 0.6152 \le x_2 \le 0.6249 \\ 0.6174 \le x_3 \le 0.6233 \\ 0.7018 \le x_4 \le 0.7061 \end{cases} \tag{5-2}$$

And we can use LINGO to solve the equivalent model, and the results are as follows:

Table 5-2 The Steady State Number of Each Species Solved by LINGO

	Algae	Molluscs	herbivorous fish	milkfish
($\times 10^4$)	1.3922	0.6249	0.6233	0.7061

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Corresponding to this solution, the maximum harvest is: max=$115189.9

Analyses of the model results: according to the optimal results, the current water quality can be obtained as shown in Table 5-3:

Table 5-3 Content of Each Element after Optimization

Chlorophyll a(ug/L)	C(ug/L)	N(ug/L)	P(ug/L)
0.1504	17.1086-36.0196	0.6843-1.4408	1.0265-2.1612

Compared to the water quality required by coral growth, the obtained water quality here is obviously satisfactory, and we reap relatively high economic benefits at the same time.

Task 6

Dear the director,

Less than 1% of the ocean floor is covered by coral. Yet, 25% of the ocean's biodiversity is supported in these areas. Thus, conservationists are concerned when coral disappears, since the biodiversity of the region disappears shortly thereafter.

Consider an area in the Philippines located in a narrow channel between Luzon Island and Santiago Island in Bolinao, Pangasinan, that used to be filled with coral reef and supported a wide range of species. The once plentiful biodiversity of the area has been dramatically reduced with the introduction of commercial milkfish (*Chanos chanos*) farming in the mid 1990's. It's now mostly muddy bottom, the once living corals are long since buried, and there are few wild fish remaining due to over fishing and loss of habitat.

Through modeling and discussion of the single farming mode of milkfish in Bolinao, we find out, based on the results of the model, the defects of existing farming mode and its negative influence on the water quality, and then, directed to the defects, we put forward new farming mode, according to which, we establish rational milkfish farming model and receive ideal results. At last, taking the economic benefits and water quality into consideration, we establish the farming mode with optimization target and draw reasonable conclusions.

According to the above models and conclusions, we have got the following findings when studying the relationship between biodiversityand water quality for coral growth:

1. Strategy to Restore the Ecosystem in Bolinao

1.1 Existing single farming mode

The milkfish farming programs in the mid 1990s promote the locals in Bolinao extensively adopt single farming mode, farming milkfish only in the fish pens, which led to poor water quality, the algae multiplying greatly, sludge covering the sea floor

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and coral buried, eventually reducing the biodiversity.

The milkfish appearing in the model is a kind of predator, which hardly restricts the multiplication of the algae, and lead to the accumulation of chlorophyll; the number of herbivorous fish is small in this area, especially that of filter feeders. As a result, the excrement of the milkfish cannot be normally filtered and thus accumulated, eventually worsening the water quality. These can be objectively reflected by the simulated results of Site D.

Table 6-1 Microbial Abundances and Particle Characteristics of Site Water

Site	Dissolved Organic Carbon(DOC) (uM)	Total Nitrogen (Dissolved, uM)	Chl *a* (ug/L)	Particulate Organic Carbon (POC) (ug/L)	Total Nitrogen (Particulate, ug/L)
A	69.7± 1.3	7.4±0.4	0.25± 0.03	106± 4	9±15
B	80.4± 2.9	8.0± 0.2	0.28± 0.03	196± 57	39± 15
C	89.6± 1.7	14.2± 0.7	0.38± 0.03	662± 68	54± 17
D	141± 2.9	30.5± 1.3	4.5± 0.2	832± 338	86± 45
Fish Pens	162± 18.5	39.8± 2.7	10.3± 0.2	641± 60	86± 18

1.2 Polyculture system with restoring function

Through the above analyses of the defects of single farming mode, we find out the key problem lies in the shortage of herbivorous fish. So, we take the importance of the middle strata of the foodweb into account when establishing the new farming mode, in which we adopt polyculture system by introducing the herbivorous fish. To be specific, we raise both milkfish and mussels in the available waters. Mussels have not only economic values, can provide food for the milkfish, and can also suppress the multiplication of the algae and filter-feed the microbe, thus the water quality improved.

Through modeling the polyculture system and solving the model, we get the initial introducing numbers, the improved water quality and the time required for restoring, as shown in Table 6-2,Table 6-3:

Table 6-2 The Stocking and Output in the Polyculture System

	Algae	Molluscs（mussels）	Herbivorous Fish	milkfish
The volume of the initial stocking（$\times 10^4$）	4.6007	0.6002	0.4002	0.1040
Stabilitynumber（$\times 10^4$）	1.4314	0.6092	0.6129	0.6979

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Table 6-3 Content of Elements in the Pen

Element	Content(ug/L)
C(10%)	34.6604 ~ 71.7465
N(0.4%)	1.3864 ~ 2.8699
P(0.6%)	2.0796 ~4.3048

Time needed for restoring: 3.2 years.

2. Optimal harvesting/feeding strategy

2.1 Optimal harvesting

First of all, according to the relationship between market supply and demand and price, we cannot think the amount of milkfish is as important as that of seaweed harvested. Obviously, the price of milkfish is higher that that of seaweed. In addition, although the amount of seaweed is large, but it is light, so we cannot pursue the maximum weight.

Secondly, if measuring harvest with the price of each species harvested, we have to differentiate the values of the species. Since there are various costs on feeding the milkfish, we should take these costs into consideration when calculating the values of each species. We should define the value of edible biomass as the sum of the values of each species harvested, minus the cost of milkfish feed.

Finally, we point out, when people expect the outputs of edible biomass to reach the maximum income, we should subtract the cost of milkfish feedstuff from its total value.

That is to say, we finally confirm the optimal harvest is the net income of the milkfish.

2.2 Optimal harvesting feeding strategy

According to the above optimal harvesting, regarding the net income of farming as the highest goal, we establish optimal farming strategy model under the premise of satisfying water quality constraint conditions. We obtain the greatest income and the numbers of all the species when reaping the greatest benefits, as shown in Table 6-4:

Table 6-4 The Steady State Number of Each Species Solved by LINGO

	Algae	Molluscs	herbivorous fish	milkfish
$(\times 10^4)$	1.3922	0.6249	0.6233	0.7061

Maximum benefit:$115189.9.

In order to prove the results of our model are correct, we define:

fishing/harvest index=feeding cost/net income

Then the result we obtained is: fishing/harvest index=0.06%

The actual result is : fishing/harvest index=2.8%

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Based on analyses of the model, when obtaining the optimal solution, we find the feeding cost of unit net income is obviously less than the actual one, so our feeding strategy can produce better harvest.

3. Comments on the polyculture system from perspective of ecology

According to analyses of 1.1 and 1.2, if actualizing our polyculture system, the join of the herbivorous fish as the middle strata contributes to the decomposition of solid particles, and can suppress the over-multiplication of the algae, improve water quality, enable the coral to grow normally, and then restore the current ecosystems and biodiversity.

However, in our model we don't take the dissoluble POC released by the algae into account, the accumulation of which is likely to hinder the improvement of water quality. In view of this, someone may doubt the restoring ability of our polyculture system. But the microbe such as bacteria in the waters can process POC, rational measures can be taken to control the content of microbiology, thus ensuring the improvement of water quality. So, in terms of ecology, our polyculture system bears the potential of improving water quality and promoting the development of ecosystems.

The above are our findings in the process of studying the relationship between biodiversity and water quality suitable for coral growth.

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Appendix

No.1 The function named "fun1"

```
function f=fun1(t,x);
x=[1,1,1];
r1=1;r2=0.5;r3=0.6;lambda1=0.1;lambda2=0.02;lambda3=0.06;mu=0.1;
f=[x(1)*(r1-lambda1*x(2)),x(2)*(-r2+lambda2*x(1)-mu*x(3)),x(3)*(-r3+lambda3*x(2))];
[t,x]=ode45('fun1',[0,20],[100,40,6]);
subplot(1,2,1)
plot(t,x(:,1),'-',t,x(:, 2),'-.',t,x(:, 3),':')
legend('x1(t)','x2(t)','x3(t)')
grid
subplot(1,2,2)
plot3(x(:,1),x(:,2),x(:,3))
grid
```

No.2 The function named "fun2"

```
function f=fun2(t, x);
sigma1=0.6;sigma2=5;sigma3= 0.5;sigma4=2;
r1=1;r2=0.5;r3=0.6;
N1=100000;N2=10000;N3=1300;
f=[r1*x(1)*(1-x(1)/N1-sigma1*x(2)/N2);r2*x(2)*(-1-x(2)/N2 +
sigma2*x(1)/N1-sigma3*x(3)/N3);r3*x(3)*(-1-x(3)/N3 + sigma4*x(2)/N2)];
```

No.3 The function named "fun3"

```
function f=fun3(t, x);
r1=1;r2=0.4;r3=0.6;r4=0.6;omic12= 0.4; omic13 =0.6 ; omic2 = 1; omic3 = 1.2;omic4
=0.4;omic6=1.5;omic5=0.8;omic7=0.1;omic8=0.05;
n1=15000;n2=7106*0.6;n3=7106*0.4;n4=1100;k=6.58;
f =
[r1*x(1)*(1-x(1)/n1-omic12*(x(2)/n2)-omic13*(x(3)/n3));r2*x(2)*(-1-x(2)/n2+omic2*(x(1)/n1)-o
mic7*(x(4)/n4));r3*x(3)*(-1-x(3)/n3+omic3*(x(1)/n1)-omic8*(x(4)/n4));r4*x(4)*(1-x(4)/n4+omi
c4*(x(2)/n2)+omic6*(x(3)/n3)+omic5*k)];
```

No.4 The M-file named "funnl1.m"

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```
for k=1:1000
        x0=[70000,rand*1000+8000,1100];
        [t,x]=ode45('fun2',[0, 15],x0);
        ca=(x(53,1)*0.0001-1.2785)/0.7568;
        k=x(53,2)*0.2438*6.9*[0.2,11.5]+x(53,1)*[242,493];%
        c=0.1*k;
        n=0.004*k;
        p=0.006*k;
        c1=[338,832];
        n1=[45,86];
        if abs(ca-4.5)<=0.15%(abs(c(1)-c1(1))<=100) && (abs(c(2)-c1(2))<=100) && (abs(n(1)-n1(1))<=10) && (abs(n(2)-n1(2))<=10)
            x0
            [x(53,1),x(53,2),x(53,3)]
        end
end
```

No.5 The M-file named “funnl2.m”

```
for k=1:1000
        x0=[rand*20000+45000,rand*2000+4002.4,rand*1500+2501.6,rand*2000+9540];
        [t,x]=ode45('fun3',[0, 15],x0);
        ca=(x(53,1)*0.0001-1.2785)/0.7568;
        k=x(53,2)*0.2438*6.9*[0.2,11.5]+x(53,4)*[242,493];
        c=0.1*k;
        n=0.004*k;
        p=0.006*k;
        c1=[338,832];
        n1=[45,86];
        if   ca<=0.25 % (abs(c(1)-c1(1))<=100) && (abs(c(2)-c1(2))<=100) && (abs(n(1)-n1(1))<=10) && (abs(n(2)-n1(2))<=10)

            [x(53,1),x(53,2),x(53,3),x(53,4)]
        end
end
```

No.6 The function named “fun24”

```
function f=fun24(t, x);
sigma1=0.6;sigma2=5;sigma3= 0.5;sigma4=2;
r1=1;r2=0.5;r3=0.6;
```

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```
%a1=evalin(funnl1,'b1');a2=evalin(funnl1,'b2');a3=evalin(funnl1,'b3');
for b1=1000:100:10000
    for b2=3500:100:7500
        for b3=150:10:350
            N1=b1+200;N2=b2+50;N3=b3+10;
            %N1=30000;N2=6000;N3=400;
            f=[r1*x(1)*(1-x(1)/N1-sigma1*x(2)/N2);r2*x(2)*(-1-x(2)/N2 +
sigma2*x(1)/N1-sigma3*x(3)/N3);r3*x(3)*(-1-x(3)/N3 + sigma4*x(2)/N2)];
            %assignin(fun23,'a1',b1);assignin(fun23,'a2',b2);assignin(fun23,'a3',b3);
            [t,x]=ode45('fun23',[0, 15],[b1,b2,b3]);
            if (x(53,2)*4>1008333) && (x(53,2)*15<2054167)
               [b1,b2,b3]
               x(53,1)
               x(53,2)
               x(53,3)
            end
        end
    end
end
```

No.7 Simulation data

x1	x2	x3	x4	x1	x2	x3	x4
1.349	0.0208	0.022	0.7052	1.3668	0.0176	0.021	0.7042
1.3527	0.0206	0.0215	0.7048	1.3593	0.0201	0.0208	0.7042
1.3709	0.0185	0.0199	0.7036	1.3704	0.0189	0.02	0.7036
1.3448	0.0204	0.023	0.7058	1.3639	0.0199	0.0202	0.7038
1.38	0.0156	0.0197	0.7032	1.3754	0.0171	0.0199	0.7034
1.347	0.0208	0.0225	0.7055	1.3607	0.0203	0.0204	0.7041
1.338	0.0241	0.0221	0.7056	1.3621	0.0205	0.0201	0.704
1.3557	0.0212	0.0208	0.7045	1.3576	0.0205	0.0209	0.7045
1.3333	0.0232	0.0232	0.706	1.3717	0.0179	0.0202	0.7037
1.3718	0.0191	0.0193	0.7031	1.3332	0.0235	0.023	0.7059
1.3339	0.0236	0.0228	0.7058	1.3651	0.0193	0.0203	0.7039
1.3485	0.0226	0.0213	0.705	1.3599	0.0188	0.0214	0.7046
1.3689	0.0181	0.0204	0.7037	1.3737	0.018	0.0197	0.7034
1.3771	0.0176	0.0191	0.7031	1.3726	0.0193	0.0191	0.7032
1.3598	0.0217	0.0201	0.7041	1.3672	0.0183	0.0206	0.7041
1.3487	0.0216	0.0216	0.7048	1.3549	0.0203	0.0215	0.7048
1.3736	0.0173	0.0198	0.7033	1.3598	0.0209	0.0204	0.7042

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1.37	0.0182	0.0203	0.7038	1.3542	0.0198	0.0217	0.7049
1.3457	0.0225	0.0218	0.7052	1.3565	0.0204	0.0213	0.7047
1.368	0.0192	0.02	0.7037	1.3437	0.0212	0.0227	0.7057
1.3668	0.0185	0.0205	0.704	1.3787	0.0177	0.0187	0.7029
1.3419	0.0231	0.022	0.7054	1.3803	0.0173	0.0189	0.7027
1.3667	0.0182	0.0207	0.7039	1.3541	0.0213	0.0212	0.7047
1.3592	0.0215	0.0202	0.7041	1.3775	0.019	0.0184	0.7027
1.3634	0.018	0.0213	0.7042	1.3556	0.0211	0.0209	0.7045
1.377	0.0185	0.0186	0.7027	1.3544	0.0218	0.0208	0.7045
1.3764	0.0182	0.019	0.703	1.3464	0.0222	0.0219	0.7052
1.3699	0.0186	0.02	0.7037	1.358	0.0194	0.0216	0.7048
1.3646	0.0198	0.0198	0.7036	1.3608	0.0217	0.02	0.704
1.373	0.018	0.0198	0.7033	1.3751	0.0181	0.0194	0.703
1.3633	0.0201	0.0201	0.7039	1.3738	0.0182	0.0197	0.7033
1.3501	0.0232	0.0209	0.7047	1.369	0.0184	0.0202	0.7036
1.3795	0.0173	0.019	0.7029	1.3687	0.0187	0.0201	0.7038
1.3706	0.0174	0.0204	0.7038	1.3671	0.0191	0.0205	0.7039
1.3816	0.0172	0.0187	0.7027	1.3586	0.0204	0.0206	0.7043
1.3575	0.0188	0.0219	0.7049	1.3775	0.0166	0.0198	0.7033
1.381	0.0171	0.019	0.7029	1.3491	0.0227	0.0215	0.705
1.3513	0.0224	0.0211	0.7048	1.368	0.0191	0.0199	0.7035
1.3738	0.0175	0.02	0.7034	1.3561	0.0206	0.0208	0.7043
1.3505	0.0205	0.0224	0.7053	1.3452	0.021	0.0228	0.7057
1.3483	0.0202	0.0227	0.7055	1.3652	0.0188	0.0207	0.7042
1.3625	0.0187	0.021	0.7041	1.3771	0.0179	0.0192	0.703
1.3601	0.0213	0.0203	0.7042	1.3516	0.0209	0.0218	0.7051
1.3734	0.0174	0.0201	0.7035	1.36	0.0204	0.0207	0.7043
1.3886	0.0168	0.0176	0.702	1.3718	0.0187	0.0197	0.7035

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