諧振電路

大綱

- 諧振電路的概念與定義
  - 阻抗並聯與串聯
  - 串聯諧振(Series Resonance)
  - 並聯諧振(Parallel Resonance)
- 電阻轉換

諧振電路的概念與定義

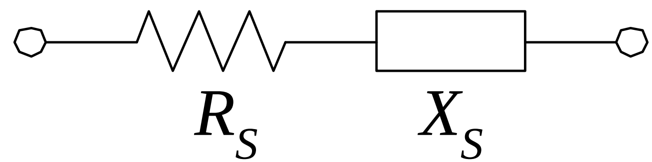
- 所謂諧振電路是指有選擇頻率能力的電路，亦即能使某頻率的信號源傳送到負載，讓其他頻率的信號衰減，例如濾波器。
- 諧振電路工作的頻寬(Bandwidth)定義為通帶(Passband)降$3dB$的頻率範圍。
- 諧振電路的電路Q值定義為

$$Q=\frac{\text{中心頻率}}{3dB\text{頻寬}}=\omega\times\frac{\text{電抗所儲存之能量}}{\text{電阻所消耗之功率}}$$

諧振電路的概念與定義

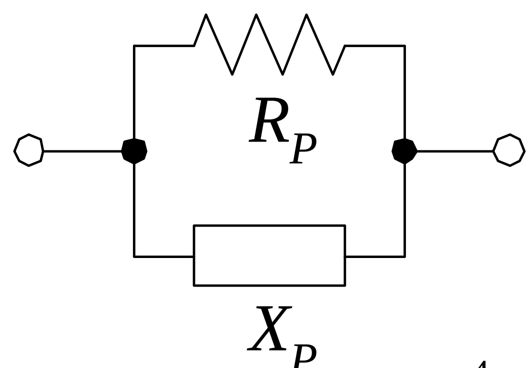
- 串聯諧振，等效阻抗為$Z_S=R_S+jX_S$，若為電感L_S電阻R_S串聯，則

$$Q_S=\frac{\frac{1}{2}|I|^2\omega L_S}{\frac{1}{2}|I|^2 R_S}=\frac{\omega L_S}{R_S}=\frac{X_S}{R_S}$$



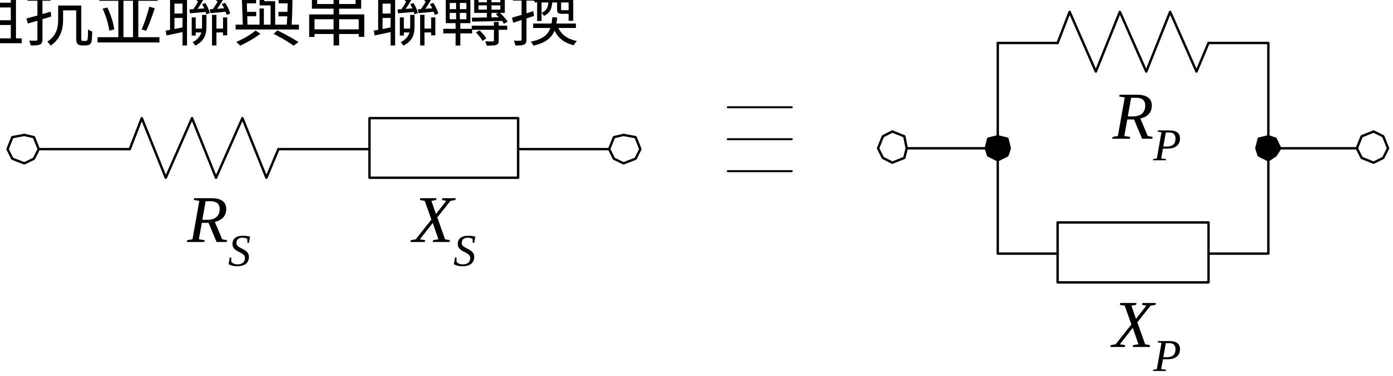
- 並聯諧振，等效導納為$Y_P=G_P+jB_P$，若為電容C_P與電阻R_P並聯，則

$$Q_P=\frac{\frac{1}{2}|V|^2\omega C_P}{\frac{1}{2}|V|^2 G_P}=\frac{\omega C_P}{G_P}=\frac{B_P}{G_P}=\frac{R_P}{X_P}$$



諧振電路的概念與定義

- 阻抗並聯與串聯轉換



在阻抗並聯串聯轉換時，其Q值不變，即

$$Q = Q_S = Q_P = \left(\frac{R_P}{R_S} - 1\right)^{1/2}$$

其中 $Q_S = \dfrac{X_S}{R_S} =$串聯阻抗的Q值

$Q_P = \dfrac{R_P}{X_P} =$ 並聯阻抗的Q值

諧振電路的概念與定義

❑ 阻抗並聯與串聯轉換(續)

由上述關係可以求得串聯與並聯之轉換公式

$$R_P = (Q_S^2 + 1)R_S, \qquad X_P = \frac{R_P}{Q_P}$$

若Q_S>>1，則

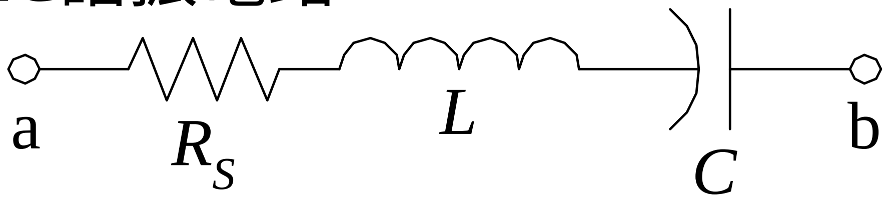
$$R_P \approx Q_P^2 R_S, \qquad X_P \approx X_S$$

同理， 若Q_P>>1，則

$$R_S \approx \frac{R_P}{Q_P^2}, \qquad X_S \approx X_P$$

串聯諧振

- 串聯的RLC諧振電路



ab兩端的等效阻抗為

$$Z_S = R_S + jX_L - jX_C = R_S + j\omega L - j\frac{1}{\omega C}$$

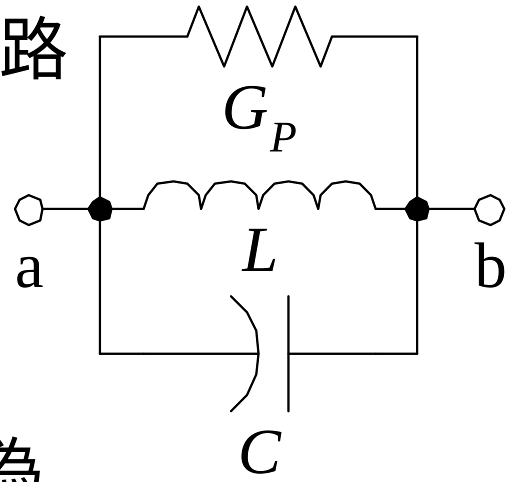
- 在共振頻率 $\omega = \omega_r$ 時，電抗部份等於零，使Z_S等於一純電阻R_S，稱為串聯諧振，$\omega_r = 2\pi f_r$為串聯諧振的共振頻率，即

$$\omega_r L - \frac{1}{\omega_r C} = 0 \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$$

$\omega = \omega_r$ 時，Z_S值為最小。

並聯諧振

- 並聯的RLC諧振電路



ab兩端的等效導納為

$$Y_P = G_P - jB_L + jB_C = G_P - j\frac{1}{\omega L} + j\omega C$$

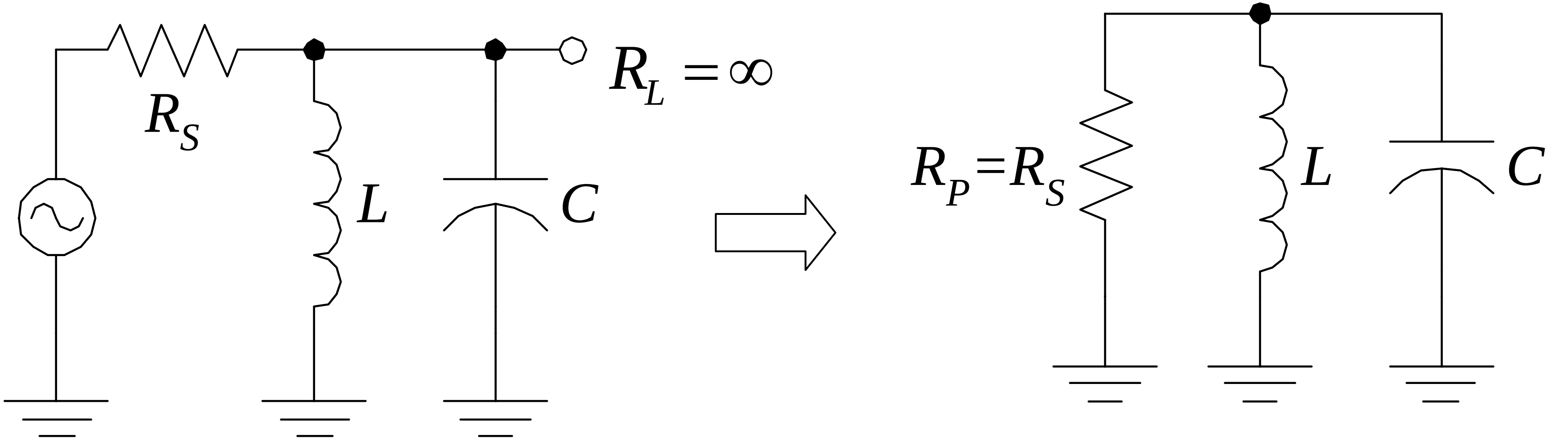
- 在共振頻率 $\omega = \omega_r$ 時，電納部份等於零，稱為並聯諧振，即

$$-\frac{1}{\omega_r L} + \omega_r C = 0 \Rightarrow \omega_r = \frac{1}{\sqrt{LC}}$$

$\omega = \omega_r$ 時，Y_P值為最小或等效阻抗Z_P為最大。

諧振電路的Q值

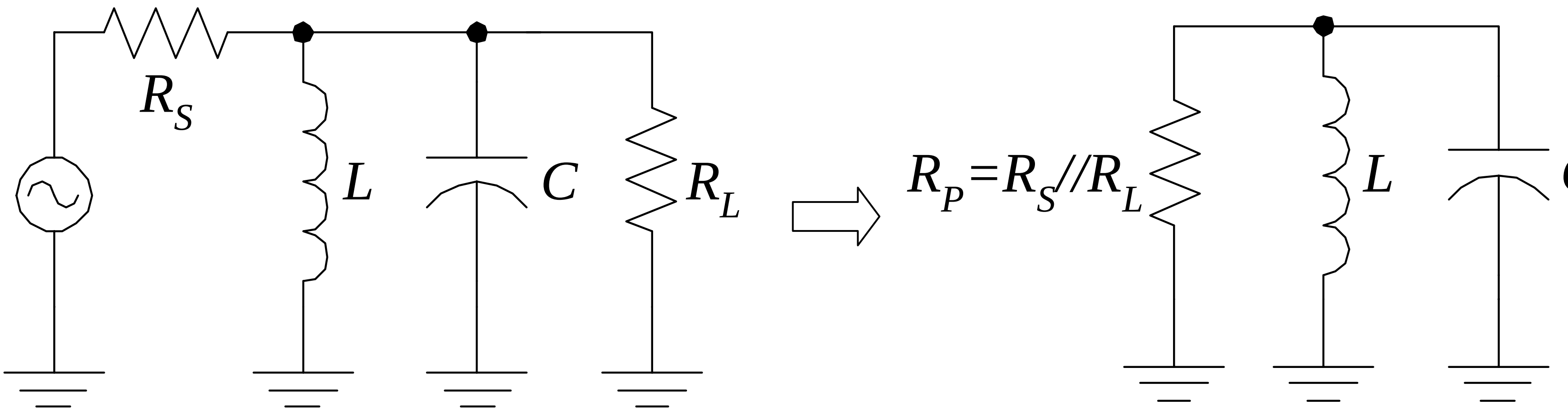
- 諧振電路的Q值定義為中心頻率與$3dB$頻寬的比，通常稱之為負載Q值(Loaded Q)；若諧振電路負載R_L=∞時的電路Q值則稱為無載Q值(Unloaded Q)。
  - 無載Q值



$$Q_U = \frac{R_P}{X_P} = \frac{R_P}{X_L} = \frac{R_P}{X_C}$$

諧振電路的Q值

- 負載Q值

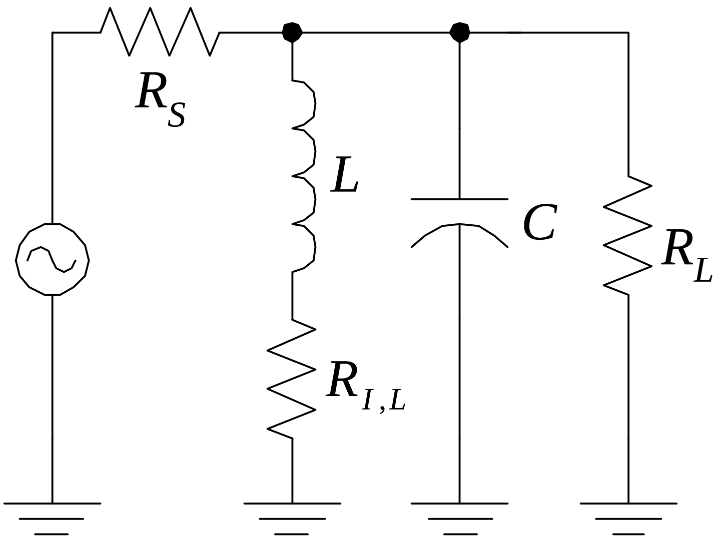


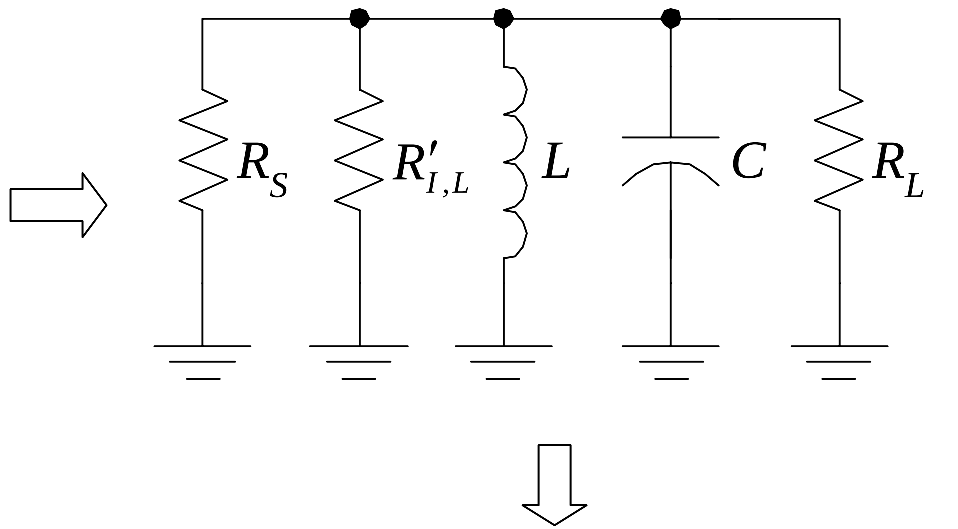
$$Q_L = \frac{R_P}{X_P} = \frac{R_P}{X_L} = \frac{R_P}{X_C}$$

$$R_P = R_S // R_L < R_S \Rightarrow Q_U > Q_L$$

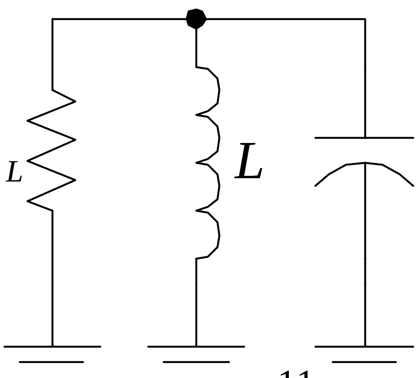
諧振電路的Q值

- Poor Component (Lossy)





$$R_P = R_S \,//\, R'_{I,L} \,//\, R_L$$



$$\therefore R_P = R_S \,//\, R'_{I,L} \,//\, R_L < R_S \,//\, R_L < R_S$$

諧振電路的Q值

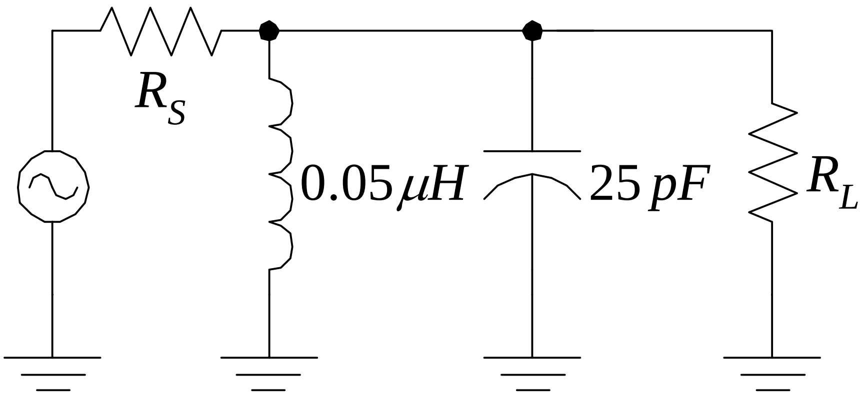
❑例題1

(1)試求中心頻率。

(2)當R_S=50Ω及R_S=1KΩ時，試求無載Q值。

(3)當$R_S=R_L$=1KΩ時，並且$Q_I=Q_C=\infty$，試求負載Q值。

(4)若$R_S=R_L$=1KΩ，並且Q_I=10，$Q_C=\infty$，試求負載Q值。



諧振電路的Q值

[解](1)

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.05\times10^{-6})(25\times10^{-12})}} = 142.4(MHz)$$

(2)

$$Q_{U,50} = \left.\frac{R_P}{X_P}\right|_{f=f_r} = \frac{50}{\omega_r L} = 1.1$$

$$Q_{U,1K} = \left.\frac{R_P}{X_P}\right|_{f=f_r} = \frac{1K}{\omega_r L} = 22.4$$

(3)

$$Q_{L,1K} = \left.\frac{R_P}{X_P}\right|_{f=f_r} = \frac{R_S // R_L}{\omega_r L} = \frac{500}{\omega_r L} = 11.2$$

諧振電路的Q值

[解] (4)

$$Q_I = \frac{X_L}{R_{I,L}} \Rightarrow R_{I,L} = \frac{X_L}{Q_I} = 4.55$$

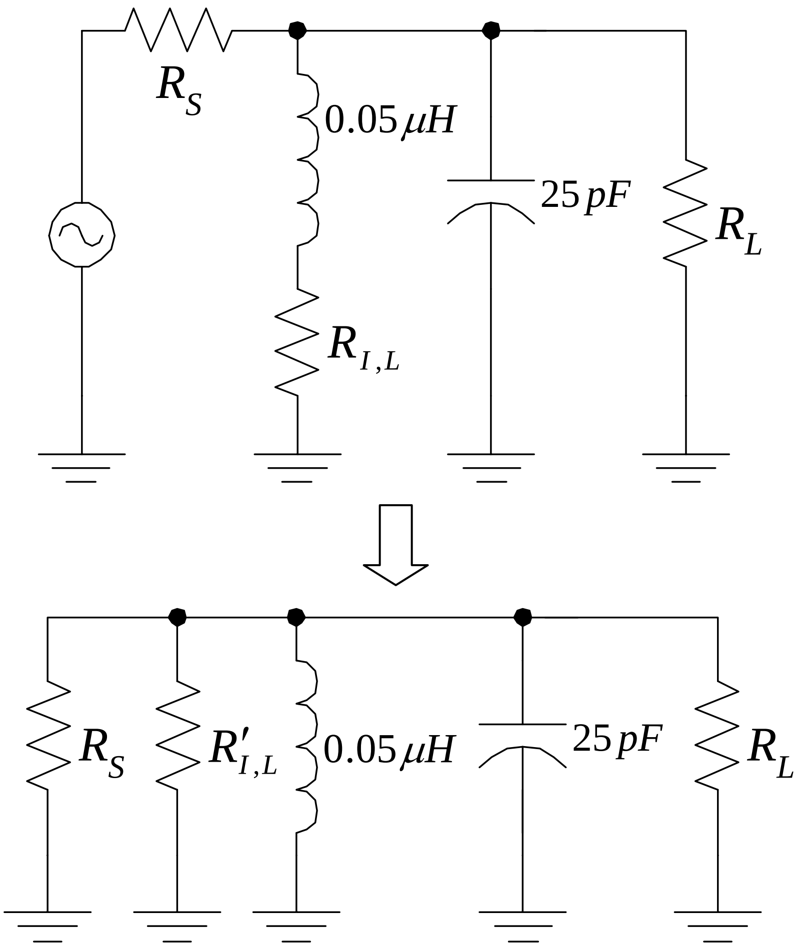
$$R'_{I,L} \approx Q_I^2 R_{I,L} = 455\Omega$$

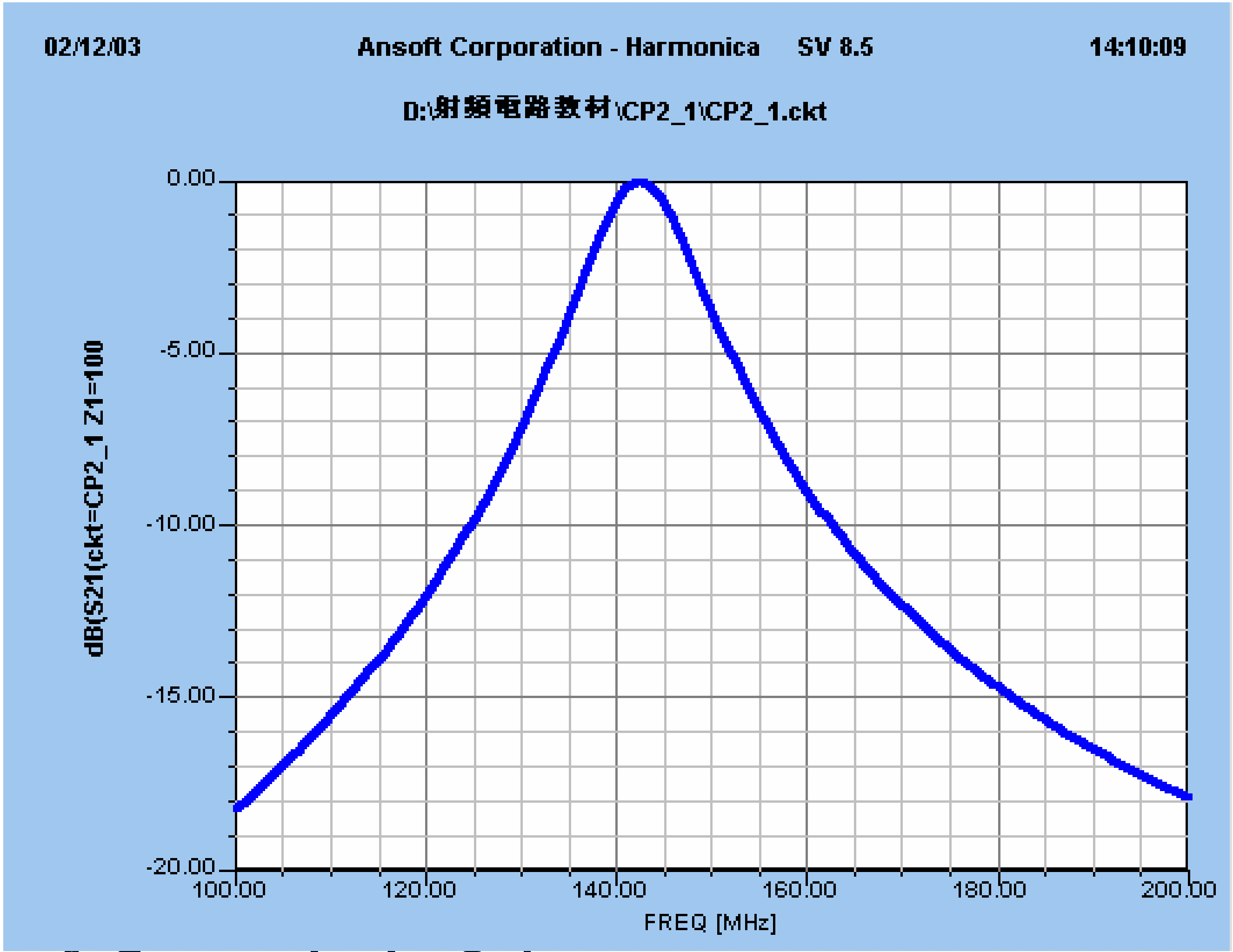
$$\Rightarrow X'_L = \frac{R'_{I,L}}{Q_I} = 45.5\Omega$$

$$R_P = R_S \,//\, R'_{I,L} \,//\, R_L$$

$$= 1K \,//\, 455 \,/\, 1K = 237.4\Omega$$

$$\therefore Q_L = \frac{R_P}{X'_L} = 5.3$$





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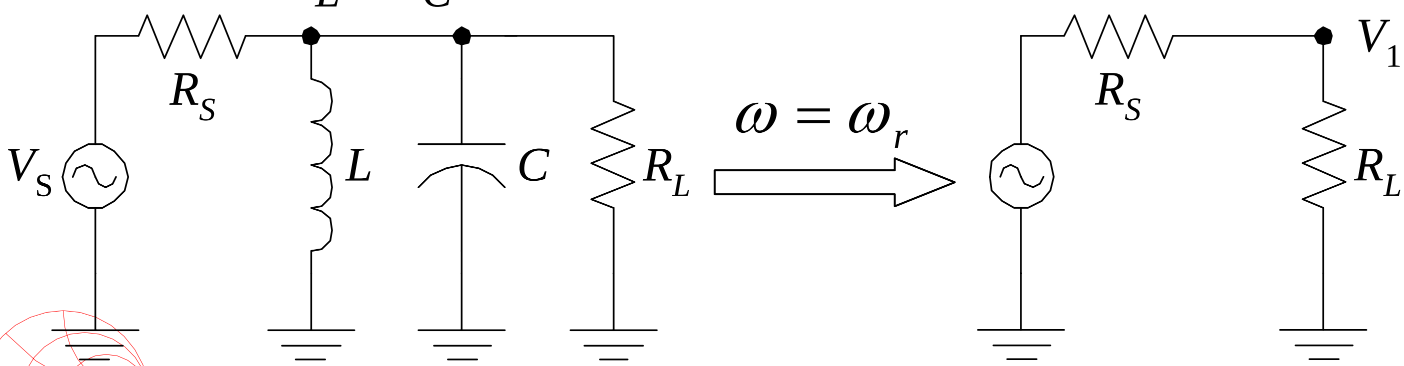
-20.00
100.00
120.00
140.00
160.00
180.00
200.00
FREQ [MHz]

諧振電路的Q值

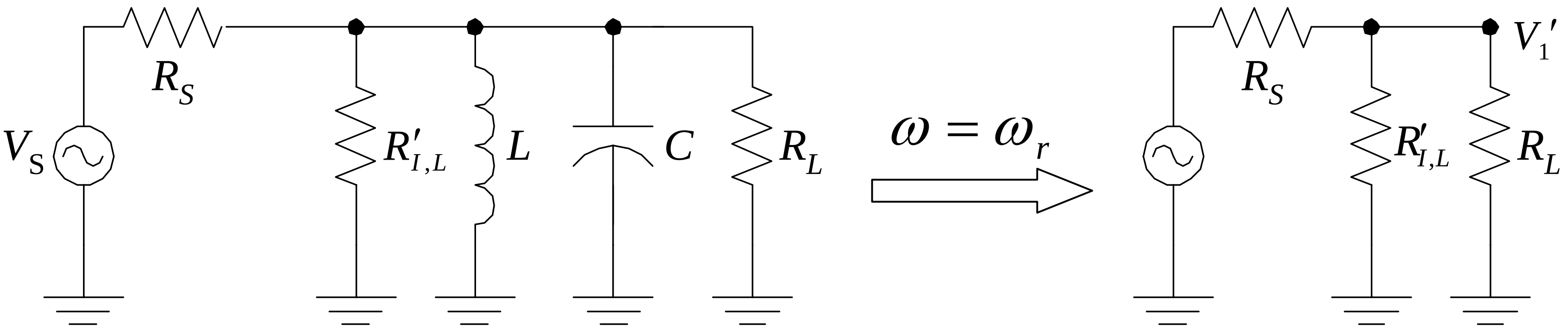
- 若欲提高負載Q值Q_L，即提升電路的選擇性，可以增加R_P或減少X_P，所以在設計較佳選擇性的諧振電路時，通常選取:
  - 較大值得信號源內阻R_S及負載阻抗R_L。
  - 低值的電感並配合高值的電容器。
- 電路中電感L及電容C為理想元件，當達共振頻率ω_r時，X_L=X_C，



$$\therefore V_1 = \frac{R_L}{R_S + R_L} V_S$$

諧振電路的Q值

- 但若電感L為非理想元件，當達到共振頻率 ω_r 時，$X'_L = X_C$



$$\therefore V_1' = \frac{R'_{I,L} \,//\, R_L}{R_S + (R'_{I,L} \,//\, R_L)} V_S$$

則插入損耗 $IL = 20\log\left(\dfrac{V_1'}{V_1}\right)$

諧振電路的Q值

❑例題2

一LC並聯電路其3dB頻寬為10MHz，中心頻率為100MHz，R_S=R_L=1$K\Omega$，Q_I=85，Q_C=∞，試求插入損耗。

[解]

$$Q_L = \frac{f_0}{BW} = \frac{100MHz}{10MHz} = 10$$

$$Q_L = \frac{R_P}{X_L} = \frac{R_S // R'_{I,L} // R_L}{X_L} = 10$$

$$Q_I = \frac{R'_{I,L}}{X_L} = 85$$

$$\therefore Q_L = \frac{500 // 85X_L}{X_L} = 10 \Rightarrow X_L = 44.1\Omega$$

諧振電路的Q值

[解] (續)

$$\because \frac{1}{\omega C} = X_L \quad \& \quad \omega L = X_L$$

$$\Rightarrow C = 36.1pF, L' = 70.2nH, R'_{I,L} = 3.75K\Omega$$

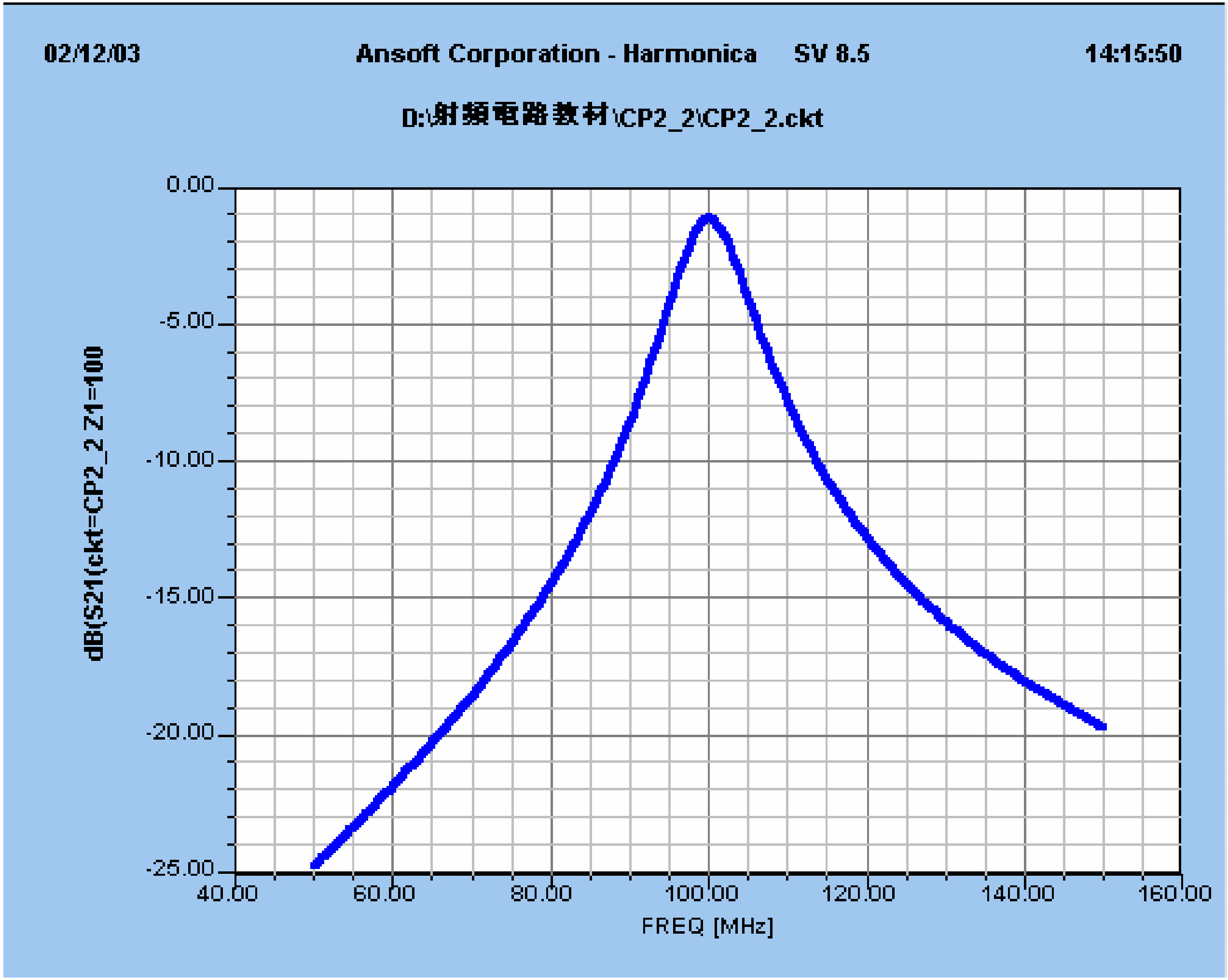
若無損耗之電路 $Q_I = Q_C = \infty$

$$V_1 = \frac{R_L}{R_S + R_L} V_S = 0.5V_S$$

實際之電路 $Q_I = 85 \quad \& \quad Q_C = \infty$

$$V_1' = \frac{R'_{I,L} // R_L}{R_S + (R'_{I,L} // R_L)} V_S = 0.44V_S$$

$$\Rightarrow IL = 20\log\left(\frac{V_1'}{V_1}\right) = 20\log\left(\frac{0.44V_S}{0.5V_S}\right) = -1.1 \quad dB$$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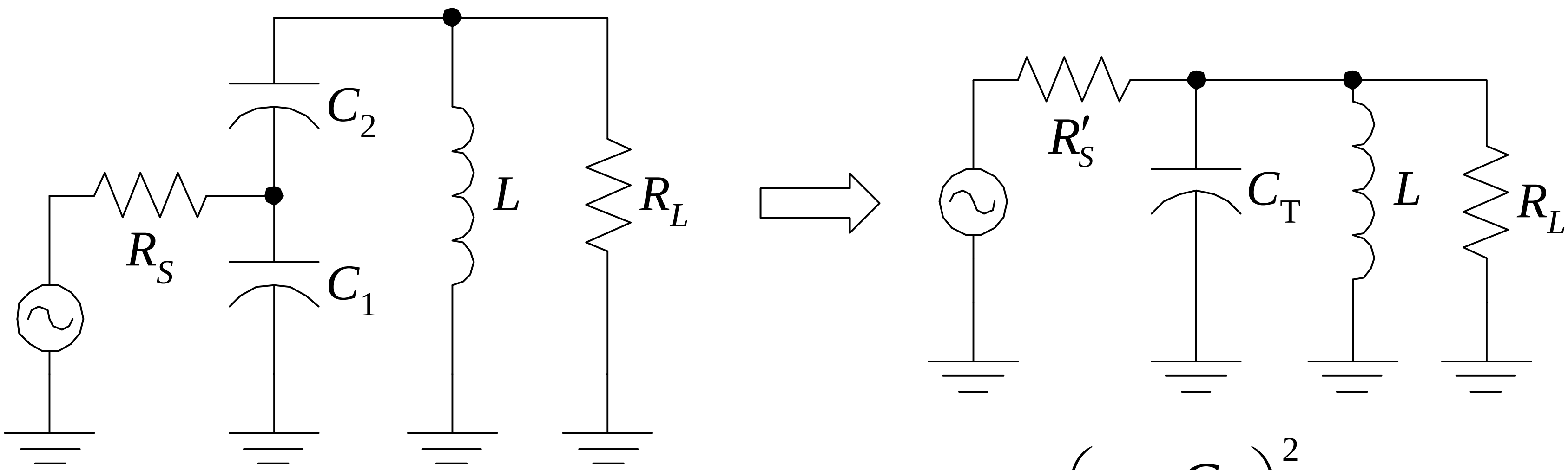
FREQ [MHz]

電阻轉換

- 阻抗轉換的目的：
  - 諧振電路之負載Q值Q_L決定電路之頻寬，影響對頻率之選擇性。
  - 低信號源阻抗或低負載阻抗會降低電路之負載Q值。
  - 經由電阻轉換將阻抗提高以達到高負載Q值、窄頻寬之要求。
- 常用阻抗轉換轉換的方法有:
  - 電容抽頭(Tapped-C)諧振電路
  - 電感抽頭(Tapped-L)諧振電路

電阻轉換

❑ 電容抽頭諧振電路

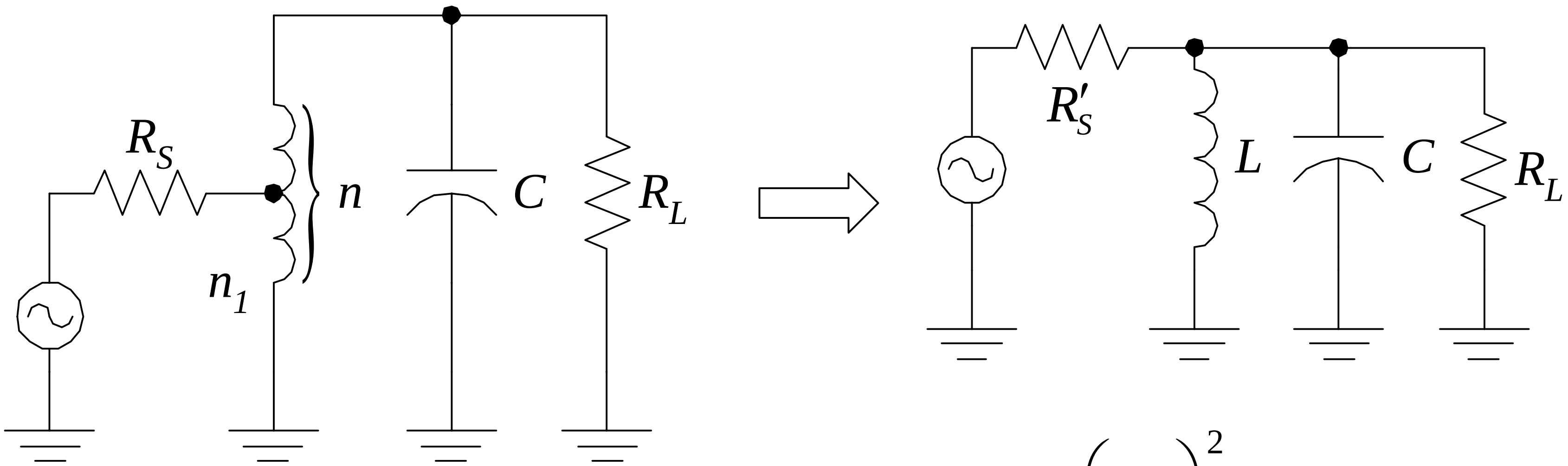


$$R_S' = R_S\left(1+\frac{C_1}{C_2}\right)^2$$

$$C_T = \frac{C_1C_2}{C_1+C_2}$$

電阻轉換

- 電感抽頭諧振電路



$$R_S' = R_S\left(\frac{n}{n_1}\right)^2$$

$$\because L \propto n \Rightarrow \left(\frac{R_S'}{R_S}\right)^{\frac{1}{2}} = \frac{L}{L_1}$$

電阻轉換

❑例題3

設計一諧振電路Q_L=20，中心頻率為100MHz，R_S=50Ω，R_L=2KΩ，且$Q_I=Q_c=\infty$。

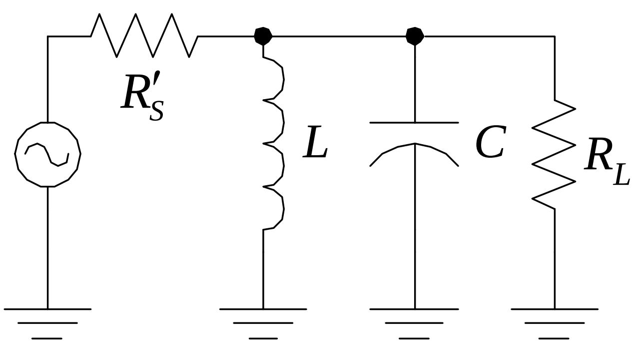
[解]為了使有最大功率傳輸$\Rightarrow R_S' = R_L = 2K\Omega$

$$Q_L = \frac{R_S' // R_L}{X_P} = 20$$

$$\Rightarrow \frac{1K}{X_P} = 20 \therefore X_P = 50$$

$$X_L = \omega L = 50 \Rightarrow L = 79.5nH$$

$$X_C = \frac{1}{\omega C} = 50 \Rightarrow C = 31.8\,pF$$

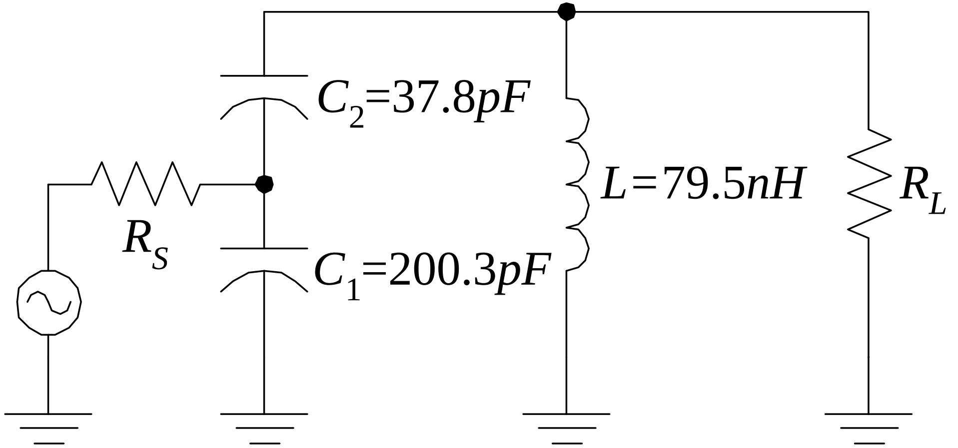


電阻轉換

[解](續)電容抽頭阻抗轉換

$$\frac{C_1}{C_2}=\sqrt{\frac{R_S'}{R_S}}-1=5.3\Rightarrow C_1=5.3C_2$$

$$C=\frac{C_1C_2}{C_1+C_2}=31.8pF\Rightarrow C_1=200.3pF,\quad C_2=37.8pF$$

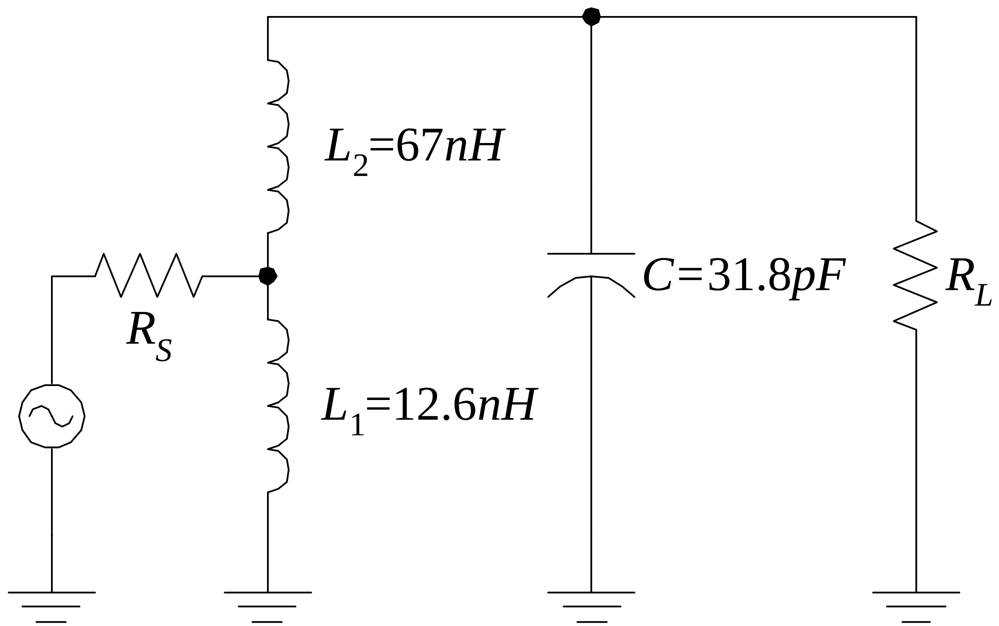


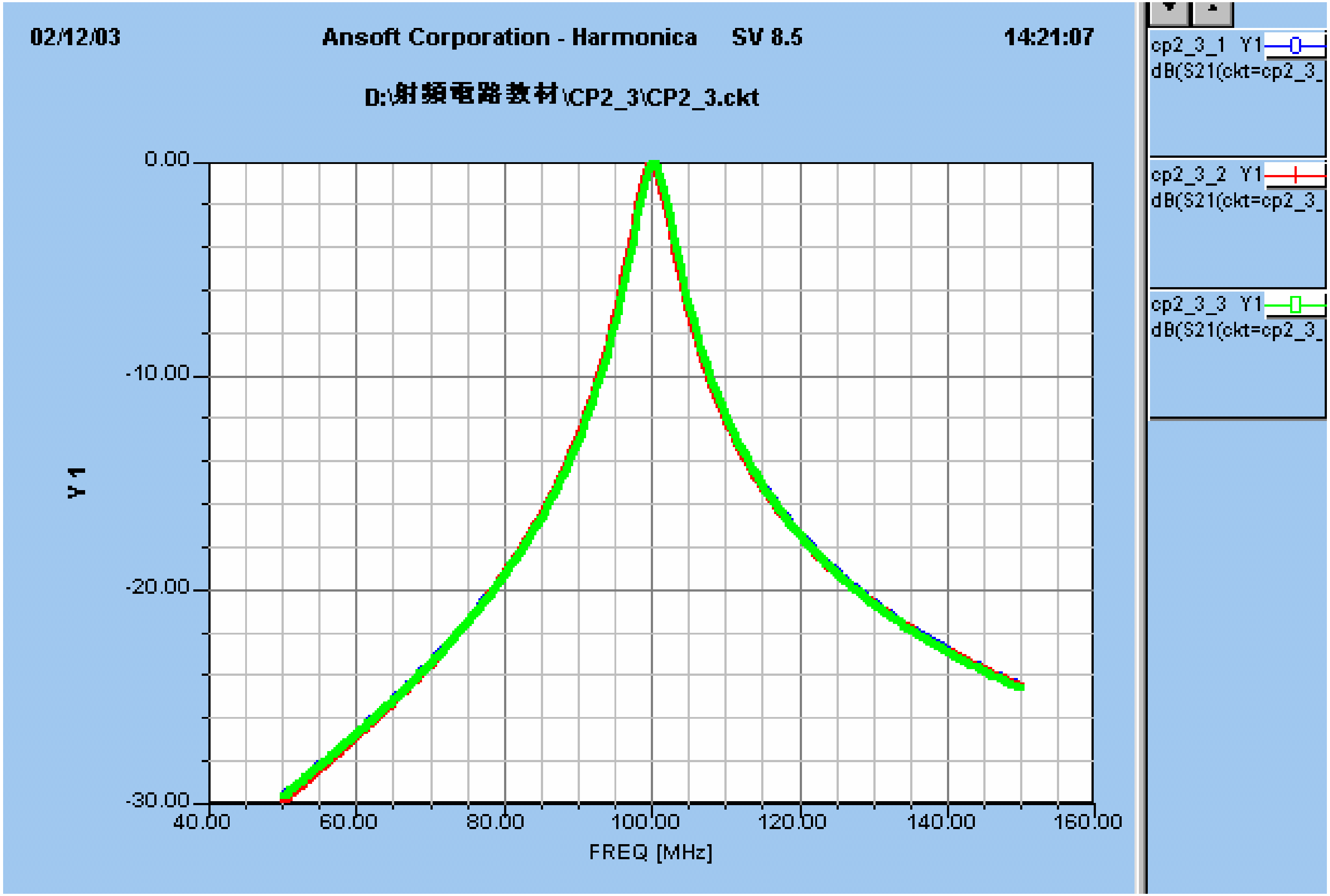
電阻轉換

[解](續)電感抽頭阻抗轉換

$$\left(\frac{R_S'}{R_S}\right)^{\frac{1}{2}}=\frac{L}{L_1}\Rightarrow(40)^{\frac{1}{2}}=\frac{L}{L_1}\Rightarrow L=6.32L_1=79.6nH\Rightarrow L_1=12.6nH$$

$$\Rightarrow L=L_1+L_2=79.6nH\Rightarrow L_2=67nH$$

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