CS 181
Problem Set 1
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1. (a) Here is the breakdown of the data:

	1	0
A = 1	2	2
A = 0	2	1
B = 1	1	1
B = 0	3	2
Total	4	3

Let Y represent the space of the label

$$\begin{aligned} Information gain &= H(Y) - H(Y|X) \\ &= H(Y) - \Sigma_{x,y} p(x|y) log_2 \frac{1}{p(x|y)} \end{aligned}$$

$H(Y) = \frac{4}{7} log_2 \frac{7}{4} + \frac{3}{7} log_2 \frac{7}{3} = 0.985$

For A:

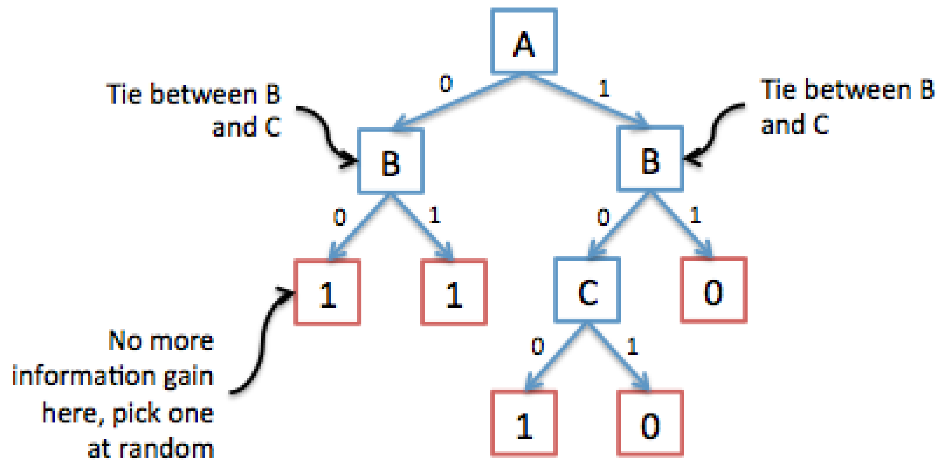
$H(Y|A=1) = 0.5 log_2 2 + 0.5 log_2 2 = 1$
$H(Y|A=0) = \frac{2}{3} log_2 \frac{3}{2} + \frac{1}{3} log_2 3 = 0.918$
$H(Y|A) = \frac{4}{7}(1) + \frac{3}{7} 0.918 = 0.965$
$I(A;Y) = 0.985 - 0.965 = \boxed{0.02}$

For B:

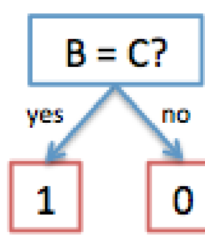
$H(Y|B=1) = 0.5 log_2 2 + 0.5 log_2 2 = 1$
$H(Y|B=0) = \frac{2}{5} log_2 \frac{5}{3} + \frac{2}{5} log_2 \frac{5}{2} = 0.971$
$H(Y|B) = \frac{4}{7}(1) + \frac{3}{7}(0.971) = 0.979$
$I(B;Y) = 0.985 - 0.979 = \boxed{0.006}$

Since A has more mutual information with the label, ID3 would split on attribute A. Splitting on A could be more useful because even though A = 1 does not give any extra information, when A=0, the distribution becomes split into 1/3 and 2/3. This is better than when B = 0, when the distribution becomes 2/5 and 3/5 since 1/3 is farther from 1/2 than 3/5. On the other hand, splitting on B could be more useful because 5/7 of the time, the split will provide something useful while splitting on A will only be useful 3/7 of the time. Since ID3 picked splitting on A, this means that ID3 has an inductive bias for splits that give better ratios for each value.

(b) The complete tree:



(c) We would get the same error if we split on whether or not B and C have the same attribute. This shows that ID3 cannot combine properties of more than one attribute when deciding how to split.



2. (a) The average ten-fold cross-validation score for the non-noisy data is 0.87 while the score for noise data is 0.78.

   (b) i.

      ii.

      iii.

      iv.

3. (a) We are defining our information gain criterion to be the mutual information between an attribute and the label. However, we will modify all the

calculated specific conditional entropies according to the weights of the examples. So instead of:

$$H(X|y) = \Sigma_x p(x|y) log_2 \frac{1}{p(x|y}$$

We will be replacing the probability distribution p with a weighted distribution based on the weights of the data. This way, data that is "more important" (the ones that tend to be predicted incorrectly) will be weighted more when the algorithm is trying to decide what attribute to split on. Attributes that split the more important data correctly will be favored over those that do not. For the example given where the label of the first example is $y_1 = 1$, etc, the weighted entropy of the set is

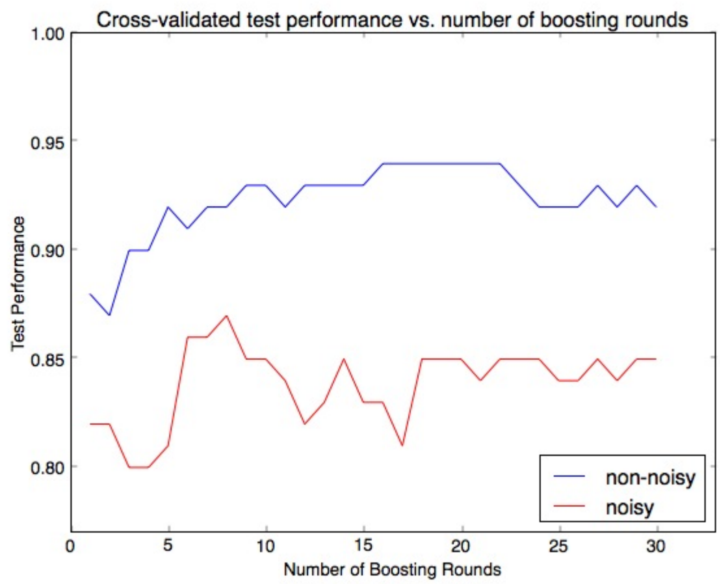
$$0.5 * log_2 2 + 0.5 * log_2 2 = 1.$$

(b) i. Table of how the depth of the tree affects boosting

	10 rounds	20 rounds
maxDepth = 1	0.93	0.92
maxDepth = 2	0.86	0.86

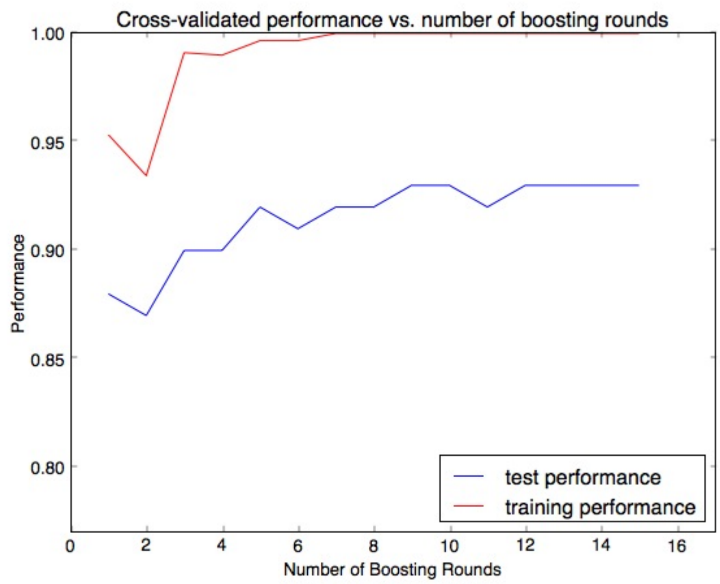
A larger maximum depth actually hurts the cross-validated test performance because when the individual trees are more complex, they are more susceptible to overfitting. This introduces trees with splits that are based more on noise than signal.

ii. Boosting performs consistently worse on noisy data compared to non-noisy data no matter how many rounds (from 1 to 30) are performed. This is because the algorithm gives highers weights to data points that are predicted incorrectly. Therefore, if there are inconsistencies within the data, the algorithm will focus on the inconsistent data more than the rest of the data.



iii.

iv. The algorithm improves with more rounds of boosting both on the training and on the test data. The performance on the test data continues to improve a bit after the performance on the training data plateaus at zero error. This is expected because each individual tree only splits once and thus avoids splitting on an attribute that is not actually important. Because of this, each additional tree does not mold the algorithm to the small nuances of the training data and the algorithm avoids overfitting.


Cross-validated performance vs. number of boosting rounds
1.00
0.95
0.90
0.85
0.80
Performance
0
2
4
6
8
10
12
14
16
Number of Boosting Rounds
test performance
training performance

4. We chose