Information Retrieval

WS 2017 / 2018

Lecture 3, Tuesday November 7th, 2017
(Efficient List Intersection)

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Overview of this lecture

- Organizational
  - Your experiences with ES2 — Ranking and Evaluation
  - This lecture — Recording from WS 16/17 with up-to-date beginning
- Contents
  - List Intersection — Recap, Time Measurement
  - Non-algorithmic improvements — Arrays, Branching, Sentinels
  - Algorithmic improvements — Galloping Search, Skip Pointers

Exercise Sheet 3: implement list intersection and make it as fast as possible on a small benchmark we have prepared

Experiences with ES2 1/2

- Summary / excerpts
  - Nice and useful practical exercise; time-extensive for some
  - Technically not hard, but must get **all the details** right
  - Hints and tests from TIP file were much appreciated
  - Not easy to beat the simple baseline
  - Some of you felt that without a lot of training data, (manual) tuning of k and b is just guessing
  - One of you exhaustively tried (almost) all combinations
  - In Python, doctests painful for floats and dicts

    Not that painful really, see the elegant master solution
  - Many expressed happiness about comments from tutor

Experiences with ES2 2/2

- Results
  - Small differences in the implementation can make a significant difference in the results
  - Changes improve some queries and make others worse
  - Improvements that helped: specific stopwords ("movie"), popularity, boost keywords in title, …
  - Baseline: P@3 ≈ 60%, P@R ≈ 45%, MAP ≈ 45%
  - Best results: P@3 ≈ 63%, P@R ≈ 48%, MAP ≈ 48%

  Bottom line: tuning a ranking algorithm is super important (for result quality) but also super hard

  In particular, it is very hard to understand / predict the effect of changes in the parameters / implementation

List Intersection 1/4

- Recap and motivation for today
  - In Lecture 1, we have intersected the inverted lists
  - In Lecture 2, we have merged the inverted lists
  - For efficiency reasons, many search engines only return results which contain all the query words

    Apache's Lucene, the most widely used open-source search engine, supports intersect (AND) and merge (OR)

    In most applications, intersect is used by default
  - Today we will focus on **efficiency** and therefore on list intersection

List Intersection 2/4

- Time measurement
  - Trickier than it may seem at first, because there can be significant variation between runs, for example due to:

    Other jobs running on your machine

    The Java garbage collector running unpredictably

    Data is partly in disk cache / L1-cache / TLB cache
  - Therefore, always repeat your time measurements, and take the average over all these

    For ES3, **repeat 5 times** for each measurement

    Note: repetition itself can also distort the truth because of caching effects ... but not an issue for us today

List Intersection 3/4

- Time measurement in **Java**
  - For **milli**second resolution

```
long time1 = System.currentTimeMillis();
// whatever code you want to time
long time2 = System.currentTimeMillis();
long millis = time2 – time1;
```

  - For **micro**second resolution

```
long time1 = System.nanoTime();
// whatever code you want to time
long time2 = System.nanoTime();
long micros = (time2 – time1) / 1000;
```

List Intersection 4/4

- Time measurement in **C++**
  - For **milli**second resolution (C-Style) #include <time.h>

```
clock_t time1 = clock();
// whatever code you want to time
clock_t time2 = clock();
size_t millis = 1000 * (time2 – time1) / CLOCKS_PER_SEC;
```

  - For **micro**second resolution (C++11) #include <chrono>

```
auto time1 = std::chrono::high_resolution_clock::now();
// whatever code you want to time
auto time2 = std::chrono::high_resolution_clock::now();
size_t micros = std::chrono::duration_cast
                        <microseconds>(…).count();
```

Non-algorithmic improvements 1/4

- Motivation
  - Implementation details can have a great impact on performance (even with the same underlying algorithm)
  - Let us implement the basic "**zipper**" algorithm for list intersection from Lecture 1 and look at a few variations
  - We make a part of the code (reading from file and the basic algorithm) available to you in both **Java** and **C++**

    This should make ES3 easier / less work for you
  - During the lecture, I will implement in Java today

    Note that using **Python** makes little sense when studying efficiency issues: the overhead of its internal data types (i.p. Python's lists/arrays) weighs too heavy

Non-algorithmic improvements 2/4

- Native arrays
  - **Java:** ArrayList much worse than native [] array

    Elements of an ArrayList cannot be basic data types (e.g. int), but have to be objects (e.g. Integer)

    This causes inefficient byte code / machine code

  - **C++:** std::vector is as good as [] with option –O3

    Elements of an std::vector can be basic data types as well as objects

    Due to C++'s templating mechanism, machine code for std::vector<int> is almost the same as for int[]

Non-algorithmic improvements 3/4

- Predictable branches
  - Branches = all **conditional** parts in your code

    In particular, **if … then … else** parts
  - Modern processors do pipelining = speculative execution of future instructions before the current ones are done
  - For conditional parts they have to guess the outcome
  - So good to **minimize** amount of conditional parts and/or improve the predictability of conditionals

    A conditional has good predictability if it evaluates to the same Boolean value most of the time

Non-algorithmic improvements 4/4

- Sentinels
  - Special elements to avoid testing for index out of bound

    Less code + further reduction in number of branches
  - For list intersection: id ∞ at the end of both lists

    For Java, take: Integer.MAX_VALUE

    For C++, take: std::numeric_limits<int>::max()

Algorithmic improvements 1/8

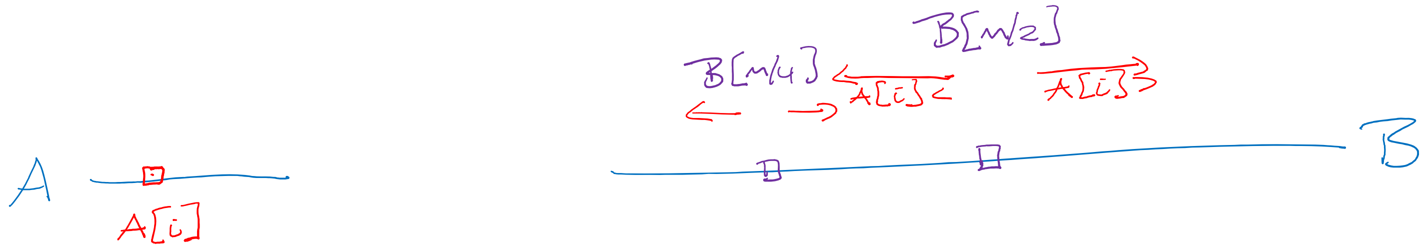
- Preliminaries
  - We have two lists, which we want to intersect
  - Let **A** be the smaller list, with **k** elements
  - Let **B** be the longer list, with **n** elements

    List intersection is commutative, so we can always assume that the first list is A, and the second is B
  - Recall that both lists are **sorted** ... this is crucial for the basic algorithm and all the algorithms in the following

Algorithmic improvements 2/8

- Binary search in the longer list
  - Search each element from A in B, using binary search
  - This has time complexity $\Theta(k \cdot \log n)$

    Good for small k … but for $k = \Theta(n)$ this is $\Theta(n \cdot \log n)$, and hence slower than the "zipper"-style linear intersect



Algorithmic improvements 3/8

- Binary search in remainder of longer list
  - Time complexity in the best case **Θ(k + log n)**

    First element from A towards the end of list B
  - Time complexity in the worst case **Θ(k · log n)**

    All elements of A at the beginning of list B
  - Time complexity in the "typical" case **Θ(k · log n)**

    Elements of A "evenly distributed" over list B

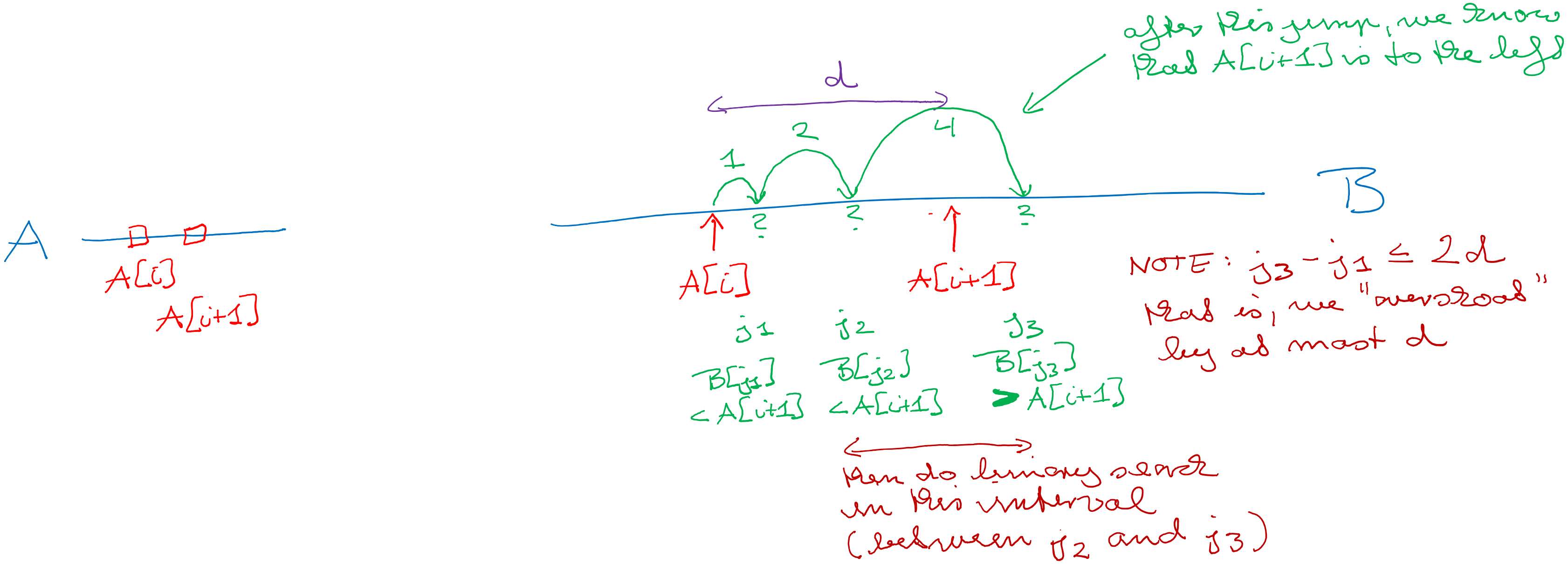
because $\log \frac{n}{k} \approx \log n$ if $k << n$

suffices to binary search A[i+1] in this part because A[i+1] ≥ A[i]

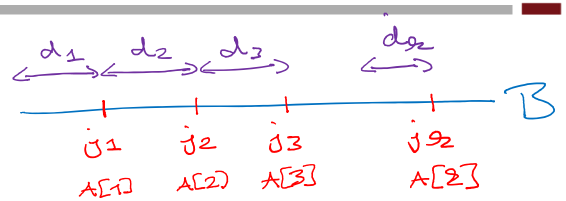
Algorithmic improvements 4/8

- Galloping search
  - **Goal:** when elements A[i] and A[i+1] are located at positions j_1 and j_2 in B, then, with $d := j_2 - j_1$ ("gap"):

    spend only time **Θ(log d)** to locate element A[i+1]
  - **Idea:** first do an exponential search, to get an upper bound on the range, then a binary search as before



Algorithmic improvements 5/8



- Galloping search, time complexity
  - Let $j_1, \ldots, j_k$ the positions of the elements of A in B
  - Let $d_i = j_i - j_{i-1}$ for $i > 1$ and $d_1 = 1$ (the "gaps")

    Note that $\Sigma_i\ d_i \leq n$ = the number of elements in B
  - Then the time complexity is $O(\Sigma_i \log d_i)$

    Not a nice formula, so let's find the maximum value, independent of the particular $d_1, \ldots, d_k$

    in case $k = n$ $\log \frac{n}{k} = 0$
  - Lemma: $\Sigma_i \log d_i$ is maximized when all $d_i = n / k$
  - Galloping search therefore takes time $\mathbf{O(k \cdot \log (1 + n/k))}$

    This is always O(n) and hence never worse than "Zipper"

    VERIFY YOURSELVES !

Algorithmic improvements 6/8

NOTE: $\max \sum_{i=1}^{k} \log d_i \hat{=} \max \sum_{i=1}^{k} \ln d_i$

- Proof of Lemma … max $\Sigma_i \ln d_i$ under constraint $\Sigma_i d_i \leq n$
  - This is an instance of **Lagrangian optimization**:
    1. Write constraint as equation: $\mathbf{\Sigma_i\, d_i - n' = 0}$ … $n' < n$
    2. Define $\mathbf{L(d_1, \ldots, d_k, \lambda)} = \Sigma_i \ln d_i - \lambda \cdot (\Sigma_i d_i - n')$
    3. Set partial derivatives = 0 to find all local optima and check the objective function at the borders

$$\frac{\partial L}{\partial d_i} = \frac{1}{d_i} - \lambda \overset{!}{=} 0 \quad \Rightarrow \quad d_i = \frac{1}{\lambda}$$

$\Rightarrow$ d_i are all equal (*)

$$\frac{\partial^2 L}{\partial d_i^2} = -\frac{1}{d_i^2} < 0 \quad \Rightarrow \quad \text{we have a MAX at } d_i = \frac{1}{\lambda}.$$

$$(*) \Rightarrow d_i = \frac{n'}{k} \Rightarrow \sum_{i=1}^{k} \log d_i \leq \sum_{i=1}^{k} \log \frac{n'}{k} \leq k \cdot \log \frac{n}{k} \quad \blacksquare$$

Algorithmic improvements 7/8

- Comparison-based lower bound
  - Recall the lower-bound for comparison-based sorting

    There are n! possible outputs, we have to differentiate between all of them, and only two choices per step

    Hence #steps required $\geq \log_2 (n!) = \mathbf{\Omega(n \cdot \log n)}$
  - We can use a similar argument for intersection / union:

    There are n+k over k ways how the k elements from A can be placed within the n elements from B, …

    Hence #steps required $\geq \log_2 (n/k)^k = \mathbf{k \cdot \log_2 (n/k)}$

    Galloping search is hence asymptotically optimal

Algorithmic improvements 8/8

- Skip Pointers
  - **Idea:** potentially skip large parts of longer list B
  - Skip pointer = special element in list B with a value x and the index j of the first element in B with B[j] ≥ x

    When intersecting, follow pointer if current A[i] ≥ x

    Placement of skip pointers is heuristic … for ES3 you can investigate good placements experimentally
  - Advantage: **very simple** to implement

    In particular, simpler than galloping search and thus often more effective in practice, even if not "optimal"

References

- Textbook

  Section 2.3: Faster intersection with skip pointers

- Literature

  A simple algorithm for merging two linearly ordered sets

  F.K. Hwang and S. Lin SICOMP 1(1):31–39, 1980

  A fast set intersection algorithm for sorted sequences

  R. Baeza-Yates CPM, LNCS 3109, 31–39, 2004

- Wikipedia

  http://en.wikipedia.org/wiki/Lagrange_multiplier