

# Information Retrieval

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Lecture 2, Tuesday October 24<sup>th</sup>, 2017  
(Ranking, Evaluation)

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# Overview of this lecture

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## ■ Organizational

- Your experiences with ES1      Inverted index
- No lecture next week      Thanks Martin Luther

## ■ Contents

- Ranking      tf.idf and BM25, other tricks
- Evaluation      Ground truth, Precision, AP, MAP, nDCG, BPref, Overfitting
- **ES2:** implement BM25, tune your ranking, and then evaluate on a small benchmark  
    There will be a small competition (table on the Wiki)

## ■ Summary / excerpts

- For some of you, it took some time to "get started"

Linux, SVN, out of programming practice, ...

- The task itself was (deliberately) easy and nice

Quite a few implemented extra features, like highlighting

- Suggestions: GIT instead of SVN, terminal recording

GIT has no access control, video recording not sufficient?

- Why is it called "inverted" index?

The movies.txt provides the words for each record, the inverted index provides the records for each word, so in this sense, it's an "inverse" representation of the data

## ■ Results

- Examples for queries that work well:

james bond

sufficiently specific keywords,  
works well even without ranking

guadalquivir

maximally specific keyword

- Examples for queries that don't work well

titanic 1997

numbers not indexed

NCIS

series not in movies.txt

space balls

in movies.txt it's spaceballs

forest gump

typo, it's forrest gump

redemption

The Shawshank Redemption listed  
late (at line 284) in movies.txt

## ■ Linear construction time?

- Quite a few of you implemented something like this:

for line in file:

...

```
if record_id not in self.inverted_lists[word]:  
    self.inverted_lists[word].append(record_id)
```

- Then index construction on movies.txt takes **very** long:

the "not in" takes linear time, not constant time

which means the whole loop take **quadratic time**

Super-important piece of advice: **never** use built-in functions without understanding their time-complexity

## ■ Motivation

- Queries often return many hits
- Typically more than one wants to (or even can) look at  
For web search: often millions of documents  
But even for less hits a proper ranking is **key** to usability
- So we want to have the most "relevant" hits first
- **Problem:** how to measure what is how "relevant"

## ■ Basic Idea

- In the inverted lists, for each **doc id** also have a **score**

university 17 0.5 , 53 0.2 , 97 0.3 , 127 0.8

freiburg 23 0.1 , 34 0.8 , 53 0.1 , 127 0.7

- While merging, **aggregate** the scores, then **sort** by score

MERGED 17 0.5 , 23 0.1 , 34 0.8 , 53 0.3 , 97 0.3 , 127 1.5

SORTED 127 1.5 , 34 0.8 , 17 0.5 , 53 0.3 , 97 0.3 , 23 0.1

- The entries in the list are referred to as **postings**

Above, it's only doc id and score, but a posting can also contain more information, e.g. the position of a word

## ■ Getting the top-k results

- A full sort of the result list takes time  $\Theta(n \cdot \log n)$ , where  $n$  is the number of postings in the list
- Typically only the top- $k$  hits need to be displayed
- Then a **partial sort** is sufficient: get the  $k$  largest elements, for a given  $k$

Can be computed in time  $\Theta(n + k \cdot \log k)$

$k$  rounds of HeapSort yield time  $\Theta(n + k \cdot \log n)$

For constant  $k$  these are both  $\Theta(n)$

For ES2, you can ignore this issue



## ■ Meaningful scores

- How do we precompute good **scores**

university      17 0.5 , 53 0.2 , 97 0.3 , 127 0.8

freiburg      23 0.1 , 34 0.8 , 53 0.1 , 127 0.7

- **Goal:** the score for the posting for doc  $D_i$  in the inverted list for word  $w$  should reflect the **relevance** of  $w$  in  $D_i$

In particular, the larger the score, the more relevant

- **Problem:** relevance is somewhat subjective

But it has to be done somehow anyway !

## ■ Term frequency (**tf**)

- The number of times a word occurs in a document
- **Problem:** some words are frequent in many documents, regardless of the content

university	...	57	5	,	...	...	...	,	123	2	,	...
of	...	57	14	,	...	...	...	,	123	23	,	...
freiburg	...	57	3	,	...	...	...	,	123	1	,	...
SCORE SUM	...	57	22	,	...	...	...	,	123	26	,	...

- A word like "of" should not count much for relevance

Some of you observed that already while trying out queries for ES1

## ■ Document frequency (**df**)

- The number of documents containing a particular word

$$\mathbf{df}_{\text{university}} = 16.384, \mathbf{df}_{\text{of}} = 524.288, \mathbf{df}_{\text{freiburg}} = 1.024$$

For simplicity, number are powers of 2, see below why

- Inverse document frequency (**idf**)

$$\mathbf{idf} = \log_2 (N / \mathbf{df}) \quad N = \text{total number of documents}$$

For the example **df** scores above and  $N = 1.048.576 = 2^{20}$

$$\mathbf{idf}_{\text{university}} = 6, \mathbf{idf}_{\text{of}} = 1, \mathbf{idf}_{\text{freiburg}} = 10$$

Understand: without the **log<sub>2</sub>**, small differences in the value of **df** would have too much of an effect

# Ranking 7/14

*idf university = 6*  
*idf of = 1*  
*idf freiburg = 10*

## ■ Combining the two (**tf.idf**)

- Reconsider our earlier **tf** only example

university	...	57	5	, ...	123	2	, ...
of	...	57	14	, ...	123	23	, ...
freiburg	...	57	3	, ...	123	1	, ...
SCORE SUM	...	57	22	, ...	123	26	, ...

- Now combined with **idf** scores from previous slide

university	...	57	30	, ...	123	12	, ...
of	...	57	14	, ...	123	23	, ...
freiburg	...	57	30	, ...	123	10	, ...
SCORE SUM	...	57	74	, ...	123	45	, ...

## ■ Problems with tf.idf in practice

- The **idf** part is fine, but the **tf** part has several problems
- Let **w** be a word, and **D<sub>1</sub>** and **D<sub>2</sub>** be two documents
- **Problem 1:** assume that **D<sub>1</sub>** is longer than **D<sub>2</sub>**  
  
Then **tf** for **w** in **D<sub>1</sub>** tends to be larger than **tf** for **w** in **D<sub>2</sub>**,  
because **D<sub>1</sub>** is longer, not because it's more "about" **w**
- **Problem 2:** assume that **D<sub>1</sub>** and **D<sub>2</sub>** have the same length,  
and that the **tf** of **w** in **D<sub>1</sub>** is twice the **tf** of **w** in **D<sub>2</sub>**  
  
Then it is reasonable to assume that **D<sub>1</sub>** is more "about" **w**  
than **D<sub>2</sub>**, but just a little more, and not twice more

## ■ The **BM25** (best match) formula

- This **tf.idf** style formula has consistently outperformed other formulas in standard benchmarks over the years

**BM25 score** =  $tf^* \cdot \log_2 (N / df)$ , where

$$tf^* = tf \cdot (k + 1) / (k + \underbrace{(1 - b + b \cdot DL / AVDL)}_{\alpha} + tf)$$

**tf** = term frequency, **DL** = document length, **AVDL** = average document length

- Standard setting for **BM25**:  $k = 1.75$  and  $b = 0.75$

Binary:  $k = 0, b = 0$ ; Normal tf.idf:  $k = \infty, b = 0$

$$\begin{aligned} b = 0 &\Rightarrow \alpha = 1 \\ tf^* &= tf \cdot \frac{\alpha + 1}{\alpha + tf} \\ &= \frac{tf}{tf} = 1 \quad \square \end{aligned}$$

$$\begin{aligned} b = 0 &\Rightarrow \alpha = 1 \\ tf^* &= tf \cdot \frac{\alpha + 1}{\alpha + tf} \\ &= tf \cdot \frac{1 + 1/\alpha}{1 + tf/\alpha} \xrightarrow{\alpha \rightarrow \infty} tf \quad \square \end{aligned}$$

## ■ Plausibility argument for BM25, part 1

- Start with the simple formula  $tf \cdot idf$
- Replace  $tf$  by  $tf^*$  such that the following properties hold:

- $tf^* = 0$  if and only if  $tf = 0$

$$tf \cdot \frac{q+1}{q+tf} = 0 \iff tf = 0 \quad \checkmark$$

- $tf^*$  increases as  $tf$  increases

$$tf^* = \frac{q+1}{q/tf + 1} \quad \checkmark$$

- $tf^* \rightarrow \text{fixed limit}$  as  $tf \rightarrow \infty$

$$tf^* = \frac{q+1}{q/tf + 1} \xrightarrow{tf \rightarrow \infty} q+1 \quad \checkmark$$

- The "simplest" formula with these properties is

- $tf^* = tf \cdot (k+1) / (k+tf) = \frac{tf \cdot (q+1)}{q+tf} = \frac{q+1}{q/tf + 1}$

## ■ Plausibility argument for BM25, part 2

- So far, we have  $tf^* = tf \cdot (k + 1) / (k + tf)$
- Normalize by the length of the document
  - Replace  $tf$  by  $tf / \alpha$
  - Full normalization:  $\alpha = DL / AVDL$  ... too extreme
  - Some normalization:  $\alpha = (1 - b) + b \cdot DL / AVDL$
- This gives us  $tf^* = tf / \alpha \cdot (k + 1) / (k + tf / \alpha)$
- And hence  $tf^* = tf \cdot (k + 1) / (k \cdot \alpha + tf)$
- The final BM25 score is then  $tf^* \cdot idf$

no  
normalization  
↑  
 $b=0: 1$   
 $b=1: DL/AVDL$   
↑  
full  
normalization

Lots of "theory" behind this formula, but to me not really more convincing than these simple plausibility arguments



## ■ Implementation advice

- First compute the inverted lists with **tf** scores

We already did that (implicitly) in Lecture 1

- Along with that compute the document length (DL) for each document, and the average document length (AVDL)

You can measure DL (and AVDL) via the number of words

- Make a second pass over the inverted lists and replace the **tf** scores by **tf\*** · **idf** scores

$$tf \cdot (k + 1) / (k \cdot (1 - b + b \cdot DL / AVDL) + tf) \cdot \log_2 (N / df)$$

Note that the **df** of a word is just the length (number of postings) in its inverted list

## ■ Further refinements

- For ES2, play around with the BM25 parameters **k** and **b**
- Boost results that match each query word at least once

Warning: when you **restrict** your results to such matches, you might miss some relevant results

For example: steven spielberg **movies**

- Somehow take the popularity of a movie into account

In the file on the Wiki, movies are sorted by popularity

Popularity scores also have a Zipf distribution, so you might take  $\sim N^{-\alpha}$  as popularity score for the N-th movie in the list

- Anything else that comes to your mind and might help ...

## ■ Advanced methods

- There is a multitude of other sources / approaches for improving the quality of the ranking
- **Example 1:** Using query logs and click-through data  
Who searches what and clicked on what ... main pillar for the result quality of big search engines like Google
- **Example 2:** Learning to rank  
Using machine learning (more in a later lecture) to find the best parameter settings in ranking function

## ■ Ground truth

- For a given query, the ids of all documents relevant for that query

Query:                   matrix movies

Relevant:               10, 582, 877, 10003

- For ES2, we have built a ground truth for 10 queries

Building a good and large enough ground truth is a common (and time-consuming) part in research in IR

## ■ Precision (**P@k**)

- The P@k for a given result list for a given query is the percentage of relevant documents among the top-k

Query: matrix movies

Relevant: 10, 582, 877, 10003

Result list: 582, 17, 5666, 10003, 10, 37, ...

**P@1:** 1/1 = 100%

**P@2:** 1/2 = 50%

**P@3:** 1/3 = 33%

**P@4:** 2/4 = 50%

**P@5:** 3/5 = 60%

← P@R

- **P@R** = P@k, where k = #relevant documents

## ■ Average Precision (**AP**)

- Let  $R_1, \dots, R_k$  be the sorted list of positions of the relevant document in the result list of a given query
- Then AP is the average of the  $k$   $P@R_i$  values

Query: matrix movies

Relevant: 10, 582, 877, 10003

Result list: 582, 17, 5666, 10003, 10, ..., 877  
REL NOT NOT REL REL REL  
1. 2. 3. 4. 5. 40.

$R_1, \dots, R_4$ : 1, 4, 5, 40

$P@R_1, \dots, P@R_4$ : 100% , 50% , 60% , 10%

**AP:**  $(100\% + 50\% + 60\% + 10\%) / 4 = 55\%$

Note: for documents not in result list, just take  $P@R_i = 0$

## ■ Mean Precisions (**MP@k**, **MP@R**, **MAP**)

- Given a benchmark with several queries + ground truth
- Then one can capture the quality of a system by taking the **mean** (average) of a given measure over all queries

**MP@k** = mean of the  $P@k$  values over all queries

**MP@R** = mean of the  $P@R$  values over all queries

**MAP** = mean of the  $AP$  values over all queries

These are very common measures, which you will find in a lot of research papers on information retrieval

## ■ Discounted Cumulative Gain (DCG, nDCG)

- Sometimes relevance comes in more than one shade, e.g.  
 $0$  = not relevant,  $1$  = somewhat rel,  $2$  = very relevant
- There should be a "discount" in the score if very relevant documents come after only somewhat relevant documents
- **Cumulative gain:**  $CG@k = \sum_{i=1..k} rel_i$
- **Discounted CG:**  $DCG@k = rel_1 + \sum_{i=2..k} rel_i / \log_2 i$
- Problem: CG and DCG are larger for larger result lists
- Solution: normalize by maximally achievable value
- **Ideal DCG:**  $iDCG@k = DCG@k$  of ideal ranking
- **Normalized DCG:**  $nDCG@k = DCG@k / iDCG@k$



## ■ Discounted Cumulative Gain (DCG, nDCG), example

- Consider the following result list and relevances, assuming only 3 relevant documents overall (for this query)

Hit #1: very relevant 2

Hit #2: relevant 1

Hit #3: not relevant 0

Hit #4: very relevant 2

Hit #5: not relevant 0

– Then **DCG@5** =  $2 + \frac{1}{\log_2 2} + \frac{2}{\log_2 4}$  *the "discount" for position 4* =  $2 + 1 + 1 = 4$

– And **iDCG@5** =  $2 + \frac{2}{\log_2 2} + \frac{1}{\log_2 3}$  =  $2 + 2 + 0.63 = 4.63$

– Hence **nDGC@5** =  $4 / 4.63 = 0.86 = 86\%$

## ■ Binary preference (bpref)

- Sometimes we have relevance judgements only for a subset of all the documents

Typically for very large datasets, where it is infeasible to check all the documents for relevance

- Then measure whether judged relevant documents come before judged non-relevant documents
- $\text{bpref} = 1/|R| \cdot \sum_{r \in RR} (1 - |NR(r)| / \min(|R|, |N|))$  where

$R$  = judged relevant docs

$N$  = judged non-relevant docs

$RR$  = docs from  $R$  in result list

$NR(r)$  = docs from the  $|R|$  top-ranked from  $N$  ranked before  $r$

# Evaluation 8/9

$R$  = all docs judged relevant  
 $N$  = ——— " ——— not relevant  
 $RR$  = docs in  $R$  and result list

## ■ Binary preference (bpref), example

- Consider the following result list and relevances, assuming 7 judged documents overall *for a particular query*

#1: judged relevant

#2: not judged

#3: not judged

#4: judged not relevant

#5: judged relevant

Not in result list:

one doc (A) judged relevant

three docs (X,Y,Z) judged not relevant

$$\min \{ |R|, |N| \} = 3$$

–  $R = \{\#1, \#5, A\}$ ,  $N = \{\#4, X, Y, Z\}$ ,  $RR = \{\#1, \#5\}$

–  $NR(\text{Hit } \#1) = \emptyset$ ,  $|NR(\#1)| = 0$

–  $NR(\text{Hit } \#5) = \#4$ ,  $|NR(\#5)| = 1$

–  $\text{bpref} = \frac{1}{3} \left( \underset{\text{for } \#1}{\left(1 - \frac{0}{3}\right)} + \underset{\text{for } \#5}{\left(1 - \frac{1}{3}\right)} \right) = \frac{1 + \frac{2}{3}}{3} = \frac{5}{9}$   *$\rightarrow = 55,5\%$*

## ■ Overfitting

- Tuning parameters / methods to achieve good results on a given benchmark is called **overfitting**

In an extreme case: for each query from the benchmark, output the list of relevant docs from the ground truth

- In a realistic environment (real search engine or competition), one is given a **training** set for development

The actual evaluation is on a **test** set, which must not be used / was not available during development

For ES2, do the development / tuning on some queries of your choice, then evaluate without further changes

# References

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- In the Manning/Raghavan/Schütze textbook

  - Section 6: Scoring and Term Weighting

  - Section 8: Evaluation in Information Retrieval

- Relevant Papers

  - Probabilistic Relevance: BM25 and Beyond **FnTIR 2009**

  - Test Collection Based Evaluation of IR Systems **FnTIR 2010**

- Relevant Wikipedia articles

  - [http://en.wikipedia.org/wiki/Okapi\\_BM25](http://en.wikipedia.org/wiki/Okapi_BM25)

  - [https://en.wikipedia.org/wiki/Information\\_retrieval#Performance\\_and\\_correctness\\_measures](https://en.wikipedia.org/wiki/Information_retrieval#Performance_and_correctness_measures)