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Distributed Systems, Summer Term 2019
Exercise Sheet 8

In both exercises we consider the synchronous message passing model.

1 MIS Application: Matching

A **matching** of a graph $G = (V, E)$ is a subset of edges $M \subseteq E$ such that no two edges in M are adjacent. A matching is maximal if no edge can be added without violating this property.

Give an algorithm that computes a maximal matching in $O(\log n)$ rounds w.h.p.

2 MIS Application: Dominating Set

A **dominating set** of a graph $G = (V, E)$ is a subset of the nodes $D \subseteq V$ such that each node is in D or adjacent to a node in D. A minimum dominating set is a dominating set containing the least possible number of nodes.

$G = (V, E)$ has **neighborhood independence** β if for every node $v \in V$ the largest independent set in the neighborhood $N(v)$ is of size at most β.

a) Show that for an MIS M and a minimum dominating set D of a graph it holds $|D| \leq |M|$.

b) Give a class of graphs each containing an independent set I and a dominating set D with $\frac{|I|}{|D|} = O(n)$.

c) Show that for graphs with neighborhood independence $\beta \geq 1$, a β-approximation to a minimum dominating set (that is a dominating set which is at most β times larger than a minimum dominating set) can be found in time $O(\log n)$ w.h.p.

d) A unit disc graph is a graph (V, E) with $V \subset \mathbb{R}^2$ and $E = \{\{u, v\} \mid \|u - v\|_2 \leq 1\}$. Show that one can compute a 5-approximation to a minimum dominating set in disc graphs in time $O(\log n)$ w.h.p.