Assignment #4

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Ans. 1

a) Show ω is subspace in $\mathbb{R}^4$

$$\omega = \left\{ \begin{bmatrix} a \\ b \\ -b \\ a \end{bmatrix}, \quad a, b \in \mathbb{R} \right\}$$

1) $a = b = 0$ $\quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \omega$

2) $\omega_1 = \begin{bmatrix} a_1 \\ b_1 \\ -b_1 \\ a_1 \end{bmatrix}$ $\quad \omega_2 = \begin{bmatrix} a_2 \\ b_2 \\ -b \\ a_2 \end{bmatrix}$

$$\omega_1 + \omega_2 = \begin{bmatrix} a_1 + a_2 \\ b_1 + b_2 \\ -b_1 - b_2 \\ a_1 + a_2 \end{bmatrix}$$

lets say

$a_1 + a_2 = r$

$b_1 + b_2 = m$

$\downarrow$

$$\omega_1 + \omega_2 = \begin{bmatrix} r \\ m \\ -m \\ r \end{bmatrix}$$

(closed under addition)

$$cw_1 = \begin{bmatrix} a_1 c \\ b_1 c \\ -b_1 c \\ a_1 c \end{bmatrix}$$

$a = a_1 c$

$b = b_1 c$

so

$$cw_1 = \begin{bmatrix} a \\ b \\ -b \\ a \end{bmatrix}$$

$\rightarrow$ satisfies scaler multiplication.

thus we can conclude that w is subspace in $\mathbb{R}^4$

b) show $a + bx - bx^2 + ax^3$ subspace of P_3

1) $a = 0$ $\quad$ $b = 0$

$w = 0 + x - 0x^2 + 0x^3$

$w = 0 + 0 - 0 + 0$

satisfied 0 vector.

2) w_1 , w_2

$w_1 + w_2 = (a_1 + a_2) + (b_1 + b_2)x - (b_1 + b_2)x^2 + (a_1 + a_2)x^3$

$a_1 + a_2 = a$

$b_1 + b_2 = b$

$w_1 + w_2 = a + bx - bx^2 + ax^3$

Closed under addition.

3) $cw_1 = ca_1 + cb_1x - cb_1x^2 + ca_1x^3$

$ca_1 = a$ $\quad$ $cw_1 = a + bx - bx^2 + ax^3$

$cb_1 = b$

$\Downarrow$

closed under scaler multiplication.

c) show that $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is subspace.

1) $a=0$ $\quad$ $b=0$

$$M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2) $M_1 + M_2 = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ -(b_1+b_2) & a_1+a_2 \end{bmatrix}$

$a_1+a_2=a$ $\quad$ $M_1+M_2 = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

$b_1+b_2=b$

↳ closed under addition.

3) $cM_1 = \begin{bmatrix} ca_1 & cb_1 \\ -cb_1 & ca_1 \end{bmatrix}$

$ca_1=a$

$cb_1=b$ $\quad$ $cM_1 = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$

$\downarrow$

closed under scaler multiplication.

Ans.2

no it is not a subspace
as it does not hold the first condition
of having a 0 vector.

$a = b = 0$ $\quad w = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Ans.3

Such matrices are not subspaces
as condition 2 is not met.

e.g $\quad A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \det(A) = 0$

$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \det(B) = 0$

$A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \det(A+B) = 1 \neq 0$

thus not closed under addition.

Ans. 4

a) ω is not subspace
does not hold condition 1

$a = b = 0$ $\qquad \omega = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

b) Is a subspace as it satisfies
all 3 conditions.