ASSIGNMENT : 05

- NAME: MUHAMMAD ADEEL AKHTAR
- SUBJECT: Complex Variable and Transforms
- ROLL NO: I20-1025
- DATE: DECEMBER, 2021

Q# 01

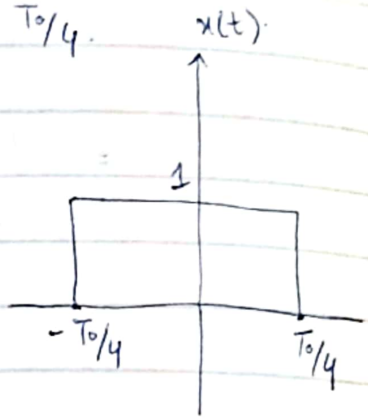
$$x(t) = x\,(t+T_0).$$

$-T_0 \leq t \leq T_0/2$

$t_c < T_0/2$

$$\begin{cases} 1, & \text{for } t < H1 < t_c. \\ 0, & \text{for } t_c < t < T_0/2. \end{cases}$$

- Interval $-2T_0 < t < 2T_0$ for $t_c = T_0/4$.



→ $\boxed{\text{PERIODIC FUNCTION}}$

- $$x(t) = \begin{cases} 1, & \text{for } t < T_0/4. \\ 0, & \text{for } T_0/4 < HK\ T_0/2 \end{cases}$$

- DC coefficient:

$$X_0 = \frac{1}{T_0}\int_0^t x(t)\,dt.$$

$$= \frac{1\cancel{2}}{{}_2\cancel{4}\cancel{T_0}}\left[\frac{\cancel{T_0}}{4}\right]$$

$$\boxed{X_0 = \frac{1}{8}}$$

- Formula For Fourier Series

$$X_K = \frac{4}{4T_0}\int^{T_0/4} x(t)\, e^{-jK\frac{2\pi t}{4T_0}}\ t\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0/4} e^{-jK\frac{2\pi}{4T_0}t}\,dt.$$

Scanned with CamScanner

$$= \frac{1}{T_0}\left[\frac{e^{-jk\frac{2\pi}{4T_0}t}}{-jk\frac{2\pi}{4T_0}}\right]_0^{T_0/4}$$

$$= \frac{1}{T_0}\left(\frac{e^{-jk\,2\pi/4T_0\,(T_0/4)}}{-jk\,2\pi/4T_0} - \frac{e^0}{-jk\,2\pi/4T_0}\right)$$

$$= \frac{1}{T_0}\left(\frac{e^{-jk\,\pi/8}}{-jk\,2\pi/4T_0} - \frac{1}{-jk\,2\pi/4T_0}\right)$$

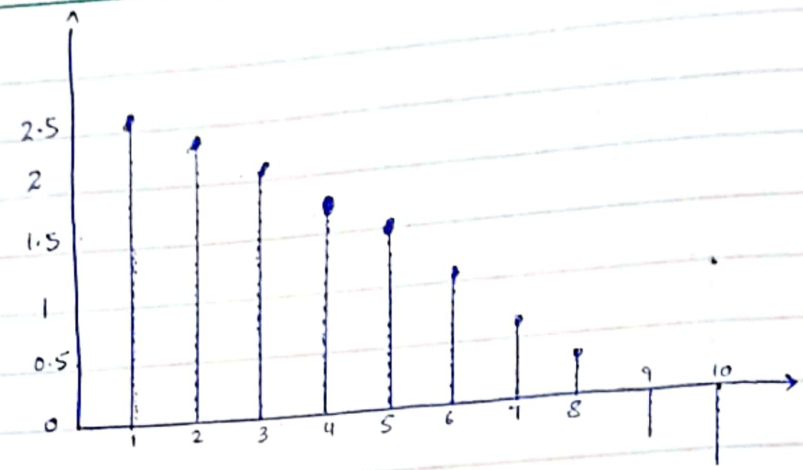
$$= \frac{1}{T_0}\left(\frac{e^{-jk\,\pi/8}-1}{-jk\,\frac{2\pi}{4T_0}}\right)$$

$$= \frac{2\cancel{4}\cancel{T_0}}{\cancel{T_0}}\left(\frac{e^{-jk\,\pi/8}-1}{jk\,\cancel{2}\pi}\right)$$

$$X_K = \frac{2j}{k\pi}\,e^{-jk\frac{\pi}{8}t}.$$

$$\boxed{x(t) = \frac{1}{8} + Re\left\{\frac{j2}{k\pi}\,e^{-jk\frac{\pi}{8}t}\right\}}$$

Spectrum of $x(t)$ for $\omega_0 = 2\pi(100)$ and $t_c = T_0/4$ frequency range from $-10\,\omega_0$ to $10\,\omega_0$.



Spectrum of $x(t)$ for $\omega_0 = 2\pi(100)$ and $t_c = T_0/10$.

frequency range from $-10\omega_0$ to $10\omega_0$.

$$x(t) = \begin{cases} 1, & |t| < T_0/10 \\ 0, & T_0/10 < |t| < T_0/10 \end{cases}$$

→ <u>DC coefficient:</u>

$$X_0 = \frac{1}{T_0}\int_0^t x(t)\,dt = \frac{2}{4T_0}\int_0^{T_0/10} 1\,dt$$

$$= \frac{1}{2T_0}\left[\frac{T_0}{10}\right] = \frac{1}{20}$$

→ <u>Fourier Series.</u>

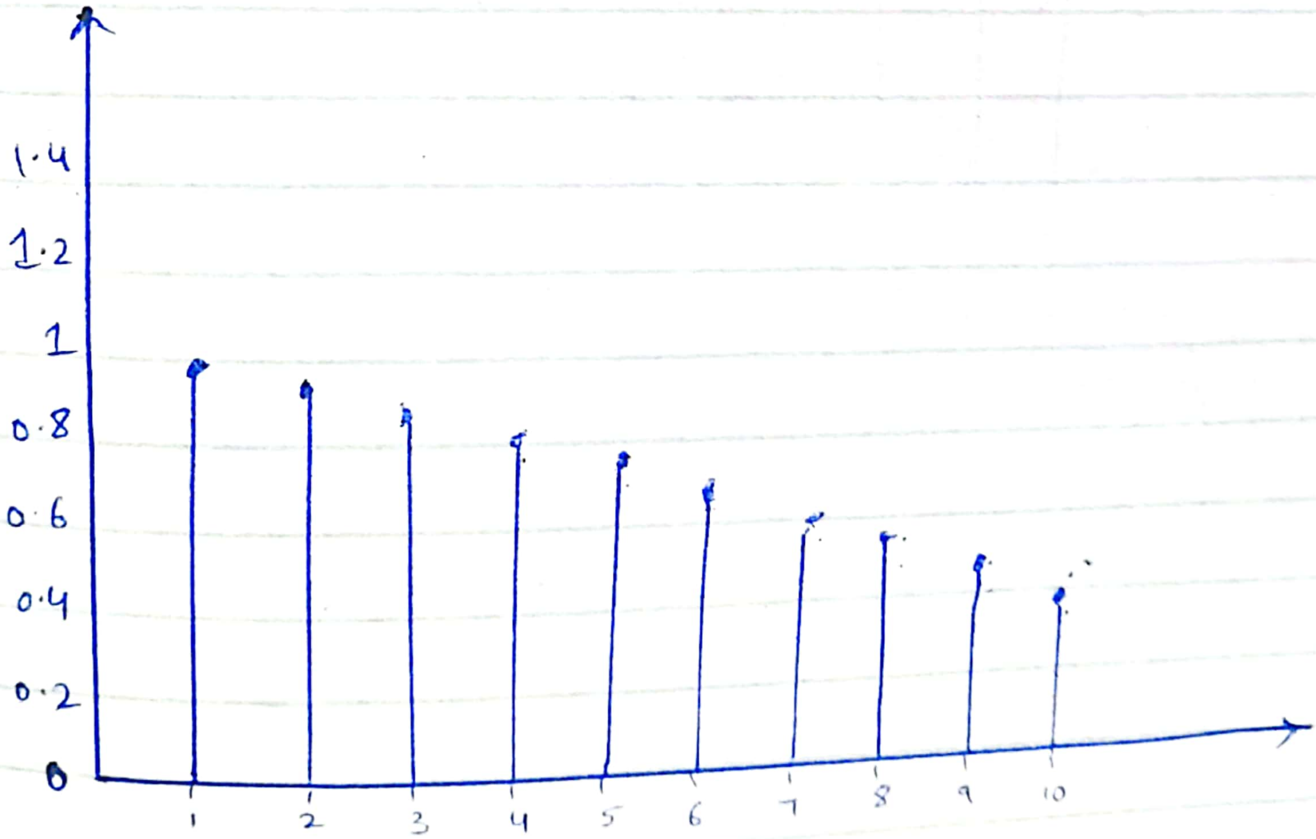
$$X_t = \frac{\cancel{4}}{\cancel{4}T_0}\int_0^{T/10} x(t)\,e^{-jk\frac{2\pi}{4T_0}t}\,t\,dt$$

$$= \frac{1}{T_0}\int_0^{T_0/10} e^{-jk\frac{2\pi}{4T_0}t}\,dt$$

$$= \frac{1}{T_0}\left[\frac{e^{-jk\frac{2\pi}{4T_0}t}}{-jk\frac{2\pi}{4T_0}}\right]_0^{T_0/10}$$

$$X_k = \frac{2j}{k\pi} e^{-jk \frac{\pi}{20} t}$$

$$x(t) = \frac{1}{20} + \mathrm{Re}\left\{ \frac{2j}{k\pi} e^{-jk \frac{\pi}{20} t} \right\}$$



Q # 02

$x(t) = 10 \cos(800\pi t + \pi/4) + 7\cos(1200\pi t - \pi/3) - 3\cos(1600\pi t)$

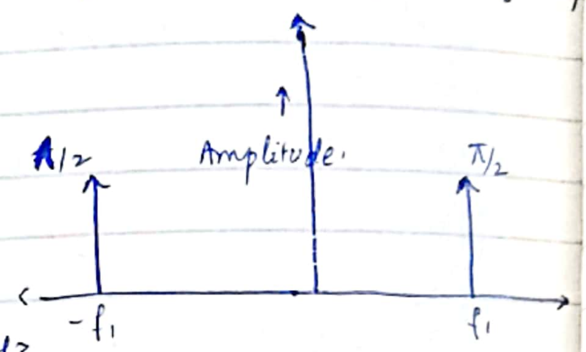
$M(f) = \frac{A}{2}\delta(f - f_1) + \frac{A}{2}\delta(f + f_1)$ $\quad \therefore \delta \rightarrow$ Impulse function

$x(t) = 10 \cos[2\pi(400)t + \pi/4] + 7\cos[2\pi(600)t - \pi/3] - 3\cos[2\pi(800)t]$

$\therefore \quad 2\pi(400)t \rightarrow 2\pi f_1 t$

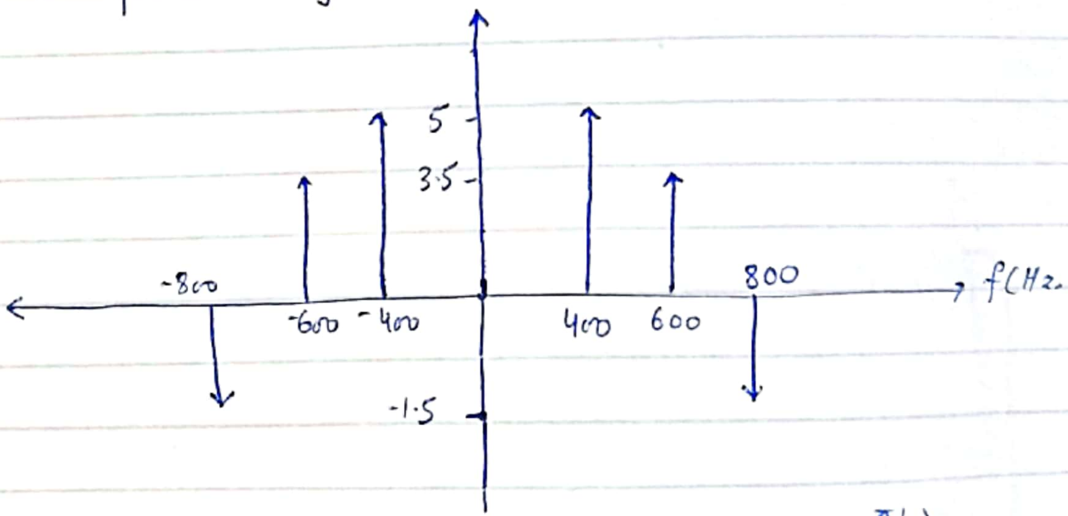
$2\pi(600)t \rightarrow 2\pi f_2 t$

$2\pi(800)t \rightarrow 2\pi f_3 t$



$f_1 = 400$, $f_2 = 600$, $f_3 = 800$ Hz.

The spectrum of $x(t)$ is drawn below-



- New signal $y(t) = x(t) + \cos(1000\pi t + \pi/2)$.

$y(t) = x(t) + \cos(2\pi(500)t + \pi/2)$.

$\therefore 2\pi(500t)t \longrightarrow 2\pi f_4 t$ $\quad \boxed{f_4 = 500 \text{ Hz}}$

The spectrum of $x(t)$ has already components at $\pm f_1$, $\pm f_2$ and $\pm f_3$.

Now $y(t)$ will have a component at

$\pm f_4$ i-e, ± 500 Hz

Thus,

$$12 \cos 13\pi t - \frac{7}{2} \cos\left(14\pi t + \frac{\pi}{6}\right) - \frac{7}{2} \cos\left(12\pi t - \frac{\pi}{6}\right)$$

$$A_1 \cos(\omega_1 t + \phi_1) + A_2 \cos(\omega_2 t + \phi_2) + A_3 \cos(\omega_3 t + \phi_3)$$

$$-\frac{7}{2} \cos\left(12\pi t - \frac{\pi}{6}\right) + 12 \cos 13\pi t - \frac{7}{2} \cos\left(14\pi t + \frac{\pi}{6}\right)$$

Therefore, $\boxed{A_1 = -\frac{7}{2}}$ $\boxed{A_2 = 12}$ $\boxed{A_3 = -\frac{7}{2}}$

$\boxed{\omega_1 = 12\pi}$ $\boxed{\omega_2 = 13\pi}$ $\boxed{\omega_3 = 14\pi}$

$\boxed{\phi_1 = -\pi/6}$ $\boxed{\phi_2 = 0}$ $\boxed{\phi_3 = \pi/6}$

Q # 05

$$y(n) + \frac{1}{2} y(n-1) - \frac{1}{4} y(n-2) = 0.$$

$$y(-1) = y(-2) = 1$$

$$Y(z) + \frac{1}{2}\left[z^{-1} Y(z) + 1\right] - \frac{1}{4}\left[z^{2} Y(z) + z^{-1} + 1\right] = 0.$$

$$Y(z)\left[1 + \frac{1}{2} z^{-1} - \frac{1}{4} z^{-2}\right] + \frac{1}{2} - \frac{1}{4} z^{-1} - \frac{1}{4} = 0$$

$$Y(z)\left[1 + 1/2\, z^{-1} - 1/4\, z^{-2}\right] = -1/2 + 1/4\, z^{-1}.$$

$$Y(z)\left[\frac{4z^2 + 2z - 1}{4z^2}\right] = \frac{1 - 2z}{4z}$$

$$\boxed{Y(z) = \frac{z(1-2z)}{4z^2 + 2z - 1}}$$