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ASSIGNMENT : 1

BCS-2D

20P-0101

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Q no: 4 $(x-y)\,dx + x\,dy = 0$

let $x = vy$

$dx = v\,dy + y\,dv$

$\Rightarrow (x-y)\,dx + x\,dy = 0$

Substitute $x = vy$

$(vy - y)(v\,dy + y\,dv) + vy\,dy = 0$

$= v^2 y\,dy + v y^2\,dv - y^2\,dv = 0$

$y^2(v-1)\,dv + v^2 y\,dy = 0$

Seperate Variables,

$y(1-v)\,dv = v^2\,dy$

$\frac{(1-v)}{v^2}\,dv = \frac{dy}{y}$

$$\Rightarrow \int \frac{(1-v)}{v^2}\,dv = \int \frac{dy}{y}$$

$$\Rightarrow \int \frac{1}{v^2}\,dv - \int \frac{1}{v}\,dv = \int \frac{1}{y}\,dy$$

$$\Rightarrow \frac{-1}{v} - \ln v\,dv + c = \ln y$$

Put Back,

$$\frac{-y}{x} - \ln \frac{x}{y} + c = \ln y$$

$$y = \cancel{\ln x}\; x\left(\ln y + \ln \frac{x}{y}\right) + cx$$

$\Rightarrow$ Log Property:

$$y = -x \ln x + cx \quad \boxed{\text{Ans}}$$

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(2)

$(x+y)\,dx + x\,dy = 0$

Let $y = ux$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$\Rightarrow (x + ux)\,dx + x(u\,dx + x\,du) = 0$

$\Rightarrow x\,dx + ux\,dx + xu\,dx + x^2\,du$

$\Rightarrow x\,dx + 2ux\,dx + x^2\,du$ [illegible]

$\Rightarrow x\,dx(1+2u) + x^2\,du = 0$

Separate Variables.

$$\Rightarrow \frac{x\,dx}{x^2} = -\frac{du}{(1+2u)}$$

$$\Rightarrow \frac{1}{x}\,dx = \frac{-du}{(1+2u)}$$

Taking integration.

$$\Rightarrow \int \frac{1}{x}\,dx = -\int \frac{du}{1+2u}$$

$$\Rightarrow \ln x = -\frac{1}{2}\ln(1+2u) + c$$

$$\Rightarrow e^{\ln x} = e^{\ln(1+2u)^{-\frac{1}{2}} + c}$$

$$\Rightarrow x = (1+2u)^{-\frac{1}{2}}$$

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Back Substitute....

$\Rightarrow \quad x = c(1+2)\left(\frac{y}{x}\right)^{-\frac{1}{2}}$

$= 2\left(\frac{y}{x}\right)+1 = cx^{-2}$

$\Rightarrow y = \frac{c}{2x} - \frac{x}{2}$ Ans

(iii)

$x\,dx + (y-2x)\,dy = 0$

let $y = vx \qquad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$x\,dx + (vx-2x)(v\,dx + x\,dv) = 0$

$x\,dx + v^2x\,dx + vx^2\,dv - 2x\,dx\,v - 2x^2\,dv$

$\frac{x\,dx}{x^2} = -\frac{(v-2)}{(v^2-1)}\,dv$

$\frac{-1}{x}\,dx = +\frac{(v-2)}{(v^2-1)}\,dv$

$\Rightarrow \frac{v-1}{(v-1)^2} - 1\,dv = -\int \frac{1}{x}\,dx$

$\Rightarrow \int \frac{v-1}{(v-1)^2} - \frac{1}{(v+1)^2}\,dv = -\ln x + c$

$\Rightarrow \ln(v-1) + \frac{1}{v-1} = -\ln x + c$

$\Rightarrow \ln\left(x\left(\frac{y}{x} - 1\right)\right) + \frac{x}{y-x} = c$

$\Rightarrow \ln(y-x) + \frac{x}{y-x} = c$

$\Rightarrow (y-x)\ln(y-x) + x = c(y-x)$ [Ans]

(iv)

$y\,dx = 2(x+y)\,dy$

$y\,dx - 2(x+y)\,dy = 0$

let $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow dy = dx\,v + x\,dv$

Put

$vx\,dx - 2(x+vx)(v\,dx + x\,dv)$

$= vx\,dx - 2\left[x\,dx + x^2\,dv + v^2\,dx + vx^2\,dx\right]$

$vx\,dx - 2xv\,dx - 2x^2\,dv - 2v^2x\,dx - 2vx^2\,dv$

$x\,dx\,(v - 2v - 2v^2) = 2x\,dv\,(1-v)$

$x\,dx\,(-v - 2v^2) - 2x^2\,dv\,(1-v)$

$x\,dvx(-1-2v) - 2x^2\,dv\,(1-x)$

Seperate the Variable,

$\frac{1}{2}x^{-1}\,dx = \frac{v+1}{-v-2v^2} - dv$

Take Integration,

$+\frac{1}{2}\ln x = -\int \frac{v+1}{2v^2+v}$

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Do partial fraction,

$$\int \left(\frac{A}{x} + \frac{B}{2x+1} \right) dx$$

$\Rightarrow \quad A(2x+1) + B(x) = x+1$

$\Rightarrow \quad 2Ax + A + Bx = x+1$

$\Rightarrow \quad 2A+B = 1$ same way

$\Rightarrow \quad A = 1 \qquad B = -1$

$\Rightarrow \frac{1}{2}\ln(x) = \int \frac{1}{2x+1}\,dx - \int \frac{1}{x}\,dx$

$\Rightarrow \quad \int \frac{1}{x} + \frac{-1}{2x+1}$

$\Rightarrow$ let $v = 2x+1 \qquad dv = 2dx$

$\frac{1}{2}dv = dx$

$= \frac{1}{2}\ln(x) = \frac{1}{2}\ln(2x+1) - \ln|x|$

$\Rightarrow y\,dx - 2(x+y)\,dy = 0$

$\Rightarrow y(x\,dy + y\,dx) - 2(xy+y)\,dy = 0$

$\Rightarrow y^2\,dv = (v+2)\,y\,dy$

$$\Rightarrow \int \frac{1}{v+2}\,dv = \int \frac{1}{y}\,dy$$

$\Rightarrow \ln(y) = \ln|v+2| + c$

$\Rightarrow y = (v+2)\cdot e^c$

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$= e^c = C_2$

$\Rightarrow x = vy$

$\Rightarrow v = \frac{x}{y}$

$\Rightarrow y = \left(2\left(\frac{x}{y} + 2\right)\right)$ $\boxed{\text{Ans}}$

(2)

$$(y^2+yx)dx - x^2dy = 0$$

let $y = (u(x))$

$dy = u\,dx + x\,du$

$\Rightarrow ((ux)^2 + ux^2)dx - x^2(u\,dx + x\,du) = 0$

$\Rightarrow (u^2x^2 + ux^2)dx - x^2(u\,dx + x\,du) = 0$

$\Rightarrow (u^2x^2dx + \cancel{ux^2dx} - \cancel{x^2u\,dx} - x^3\,du = 0$

$\Rightarrow u^2x^2dx = +x^3du$

$\Rightarrow u^2dx = x\,du$

$\Rightarrow \frac{dx}{x} = \frac{du}{u^2}$

$\Rightarrow \int \frac{dx}{x} = \int \frac{du}{u^2}$

$\Rightarrow \ln(x) + c = -u^{-1}$

$\Rightarrow \ln(x) + c = \frac{-1}{(y/x)}$

$\Rightarrow \ln(x) + c = \frac{-x}{y}$

$\Rightarrow \frac{y}{x} = \frac{+1}{c - \ln(x)}$

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(vi)

$(y^2+yx)dx + x^2dy = 0$

let $y = ux$

$\Rightarrow \frac{dy}{dx} = u + x\frac{du}{dx} \Rightarrow dy = udx + xdu$

$\Rightarrow (y^2+yx)dx + x^2dy = 0$

$\Rightarrow ((ux)^2 + (ux)x)dx + x^2[udx + xdu] = 0$

$\Rightarrow (u^2x^2 + ux^2)dx + x^2(udx + xdu) = 0$

$u^2x^2dx + ux^2dx + x^2udx + x^3du = 0$

$(u^2x^2 + 2ux^2)dx = -x^3du$

$u\cancel{x^2}(u+2)dx = -x^3du$

$u(u+2)\,d\cancel{x} = -xdu$

$-\frac{dx}{x} = \frac{du}{u(u+2)}$

$-\int\frac{dx}{x} = \int\frac{du}{u(u+2)}$

$-\ln(x)+c = \int\left(\frac{\frac{1}{2}}{u} - \frac{\frac{1}{2}}{u+2}\right)du$

$-\ln(x)+c = \frac{1}{2}\ln(u) - \frac{1}{2}\ln(u+2)$

$\Rightarrow -\ln(x)+c = \frac{1}{2}\ln\left(\frac{u}{u+2}\right)$

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$= -2\ln(x) + c = \ln\left(\frac{u}{u+2}\right)$

$\ln(x)^{-2} + c = \ln\left(\frac{u}{u+2}\right)$

$\ln\left(\frac{1}{x^2}\right) + c\ \ln\left(\frac{u}{u+2}\right)$

$\ln\left(\frac{c}{x^2}\right) = \ln\left(\frac{u}{u+2}\right)$

$= \frac{c}{x^2} = \frac{u}{u+2}$

$= \frac{c}{x^2} = \frac{\left(\frac{y}{x}\right)}{\left(\frac{y}{x}\right)+2}$

$\Rightarrow \frac{c}{x^2} = \frac{y}{y+2x} \Rightarrow c(y+2x) = yx^2$

$= cy + 2(x = y^2) \Rightarrow 2(x = y\,x^2 - cy$

$2(x = y(x - c) \Rightarrow y = \frac{2cx}{x^2 - c}$

(vii)

$\frac{dy}{dx} = \frac{y-x}{y+x}$

let $y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$

$\frac{dy}{dx} = \frac{y-x}{y+x}$

$$x + x\frac{dx}{dx} = \frac{xx - x}{xx + x}$$

$$\Rightarrow x\frac{dx}{dx} = \frac{x-1}{x+1} - x$$

$$x\frac{dx}{dx} = \frac{x-1}{x+1} - x$$

$$x\frac{dx}{dx} = \frac{\cancel{x} - 1 - x^2 - \cancel{x}}{x+1}$$

$$\Rightarrow x\frac{dx}{dx} = -\frac{(x^2+1)}{x+1}$$

$$\Rightarrow \frac{(x+1)\,dx}{(x^2+1)} = -\frac{dx}{dx}$$

$$\Rightarrow \int \frac{(x+1)\,dx}{(x^2+1)} = -\int \frac{dx}{x}$$

$$\Rightarrow \int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx = -\ln(x) + c$$

$$= \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \tan^{-1}x = -\ln(x) + c$$

$$\frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \tan^{-1}x = -\ln(x) + c$$

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$$\frac{1}{2}\ln\left(\frac{y^2}{x^2}+1\right)+\tan^{-1}(y/x) = -\ln x + c$$

$$\frac{1}{2}\ln\left(\frac{y^2}{x^2}+1\right)+\tan^{-1}(y/x) = -\ln x + c$$

$$\frac{1}{2}\ln\left(\frac{y^2+x^2}{x^2}\right)+\tan^{-1}\left(\frac{y}{x}\right) = -\ln x + c$$

$$= \frac{1}{2}\ln(y^2+x^2) - \frac{1}{2}\ln(x^2) + \tan^{-1}\left(\frac{y}{x}\right) = \ln(x) + c$$

$$= \frac{1}{2}\ln(y^2+x^2) - \cancel{\ln(x)} + \cancel{\ln(x)} + \tan^{-1}(y/x) + c$$

$$\Rightarrow \frac{1}{2}\ln(y^2+x^2) + \tan^{-1}\left(\frac{y}{x}\right) = c$$

(viii)

$$\frac{dy}{dx} = x + 3y/3x+y$$

let $y = ux$

$$\frac{du}{dx} = u + x\frac{du}{dx}$$

$$\Rightarrow \left[\frac{dy}{dx} = \frac{x+3y}{3x+y}\right.$$

$$\left[u + \frac{x\,du}{dx} = \frac{x+3ux}{3x+ux}\right.$$

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$$u + x\frac{du}{dx} = \frac{1+3u}{3+u}$$

$$\Rightarrow x\frac{du}{dx} = \frac{1+3u}{3+u} - u$$

$$x\frac{du}{dx} = \frac{1+3u-3u-u^2}{3+u}$$

$$x\frac{du}{dx} = \frac{1-u^2}{3+u}$$

$$\frac{(3+u)\,du}{(1-u^2)} = \frac{dx}{x}$$

$$\int \frac{3+u}{(1-u^2)}\,du = \int \frac{dx}{x}$$

$$\Rightarrow \int \left(\frac{2}{1-u} + \frac{1}{1+u}\right) du = \int \frac{dx}{x}$$

$$\int \frac{2}{1-u}\,du + \int \frac{1}{1+u}\,du = \ln x + \ln c$$

$$-2\ln(1-u) + \ln(1+u) = \ln(x)$$

$$\ln(1-u)^{-2} + \ln(1+u) = \ln(x)$$

$$\ln\left(\frac{1+u}{(1-u)^2}\right) = \ln(x)$$

$$\frac{1+u}{(1-u)^2} = cx$$

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$$\Rightarrow \frac{1+\left(\frac{y}{x}\right)}{\left(1-\left(\frac{y}{x}\right)^2\right)} = cx$$

$$\frac{(x+y)x}{(x^2-y^2)x} = cx$$

$$(x+y) = (x-y)^2 c$$

$$\Rightarrow c(x+y) = (x-y)^2$$

9x

$$-y\,dx + (x+\sqrt{xy})\,dy = 0$$

let $x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$dx = v\,dy + y\,dv$$

$$-y\,dx + (x+\sqrt{xy})\,dy = 0$$

$$-y[v\,dy + y\,dx] + (|xy| + \sqrt{|xy|y})\,dy = 0$$

$$= -yx\,dy - y^2dx + xy\,dy + \sqrt{xy^2}\,dy = 0$$

$$= -y^2dx + y\sqrt{x}\,dy = 0$$

$$\Rightarrow y^2dv = y\sqrt{v}\,dy$$

$$\Rightarrow \frac{dv}{\sqrt{v}} = \frac{dy}{y}$$

$$\int \frac{dv}{\sqrt{v}} = \int \frac{dy}{y}$$

$$\Rightarrow 2\sqrt{v} = \ln y + c$$

$$\Rightarrow 2\sqrt{\frac{x}{y}} = \ln y + c$$

$$\Rightarrow 4\left(\frac{x}{y}\right) = (\ln y + c)^2$$

$$4x = y(\ln y + c)^2$$

(2)

$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2}$$

$$y = vx$$

$$dy = v\,dx + x\,dv$$

$$x\,dy = (y + \sqrt{x^2 - y^2})\,dx$$

$$\Rightarrow x[v\,dx + x\,dv] = [vx + \sqrt{x^2 - v^2x^2}]\,dx$$

$$vx\,dx + x^2\,dv = vx\,dx + x\sqrt{1 - v^2}\,dx$$

$$x^2\,dv = x\sqrt{1 - v^2}\,dx$$

$$\int \frac{dv}{\sqrt{1 - v^2}} = \int \frac{dx}{x}$$

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$\sin^{-1} u = \ln(x) + c$

$\sin^{-1}\left(\frac{y}{x}\right) = \ln x + c$

$\frac{y}{x} = \sin(\ln x + c)$

$y = x \sin(\ln x + c)$

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$xy^2 \frac{dy}{dx} = y^3 - x^3, \; y(1) = 2$

$xy^2 dy = (y^3 - x^3) dx$

let $y = ux$

$dy = u dx + x du$

$x = (ux)^2 [u dx + x du] = ((ux)^3 - x^3) dx$

$u^2 x^3 [u dx + x du] = (u^3 x^3 - x^3) dx$

$\cancel{x^3}(u^3 dx + u x^2 du) = \cancel{x^3}(u^3 - 1) dx$

$u^3 dx + u^2 x du = (u^3 - 1) dx$

$u^3 dx + (1 - u^3) dx = -u^2 x du$

$(u^3 - u^3 + 1) dx = -u^2 x du$

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$$dx = -x^2 x\, dx$$

$$-\frac{dx}{x} = x^2 dx$$

$$-\int \frac{dx}{x} = \int x^2 dx$$

$$-\ln x + c = \frac{x^3}{3}$$

$$-\ln(x) + c = \left(\frac{y/x}{3}\right)^3$$

$$-\ln(x) + c = \left(\frac{\frac{y}{x}}{3}\right)^3$$

$$-\ln(x) + c = \frac{y^3}{3x^3}$$

$$y^3 = -3x^3 \ln(x) + 3x^3 c$$

$$y(1) = 2$$

$$(2)^3 = 3(1)^3 \ln(1) + 3(1)^3 c$$

$$8 = -3(0) + 3c$$

$$8 = 3c \Longrightarrow \boxed{c = \frac{8}{3}}$$

$$y^3 = -3x^3 \ln(x) + 8x^3$$

$$\Longrightarrow y^3 + 3x^3 \ln(x) = 8x^3$$

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