

Q : 4

$$y dx + x(\ln x - \ln y - 1) dy = 0, \quad y(1) = e$$

\Rightarrow Using the substitution

$x = vy$ and $dx = v dy + y dv$ we get,

$$y(v dy + y dv) + vy(\ln vy - \ln y - 1) dy = 0$$

$$\textcircled{1} \quad vy dy + y^2 dv + vy(\ln v - 1) dy = 0$$

$$\textcircled{2} \quad (vy + vy \ln v - vy) dy + y^2 dv = 0$$

$$\textcircled{3} \quad \cancel{vy} dy + y^2 dv = 0 \rightarrow \frac{v \ln v}{y} dy + dv = 0$$

$$\textcircled{4} \quad \frac{dy}{y} + \frac{dv}{v \ln v} = 0 \rightarrow \int \frac{dy}{y} + \int \frac{dv}{v \ln v} = c_1$$

$$\textcircled{5} \quad \ln|y| + \ln|\ln v| = c_1 \rightarrow \ln|y \ln v| = c_1 \rightarrow y \ln v = c$$

Now, substitute back by $v = \frac{x}{y}$ to get,

$$y \ln \left| \frac{x}{y} \right| = c$$

Now, apply the initial condition $y(1) = e$ to get,

$$e \ln(e^{-1}) = c \rightarrow -e = c \rightarrow c = -e$$

$$\boxed{\textcircled{6} \quad y \ln \left| \frac{x}{y} \right| = -e}$$

$$\boxed{\text{Result :-} \quad y \ln \left| \frac{x}{y} \right| = -e}$$