- Calculus Assignment # 4.
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$$y'' - (x+1)y' - y = 0$$

To find series solution $x=0$, let $y = \sum_{n=0}^{\infty} c_n x^n$

Then,

$$y' = \sum_{n=1} n c_n x^{n-1} \quad \& \quad y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

Substituting,

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - (x+1) \sum_{n=1}^{\infty} n c_n x^{n-1} - \sum c_n x^n$$

$$\sum_{n=2} n(n-1) c_n x^{n-2} - x \sum_{n=1} n c_n x^{n-1} - \sum_{n=1} n c_n x^{n-1} - \sum_{n=0}^{\infty} c_n x^n$$

$$\underbrace{\sum_{n=2}^{\infty} c_n n(n-1) x^{n-2}}_{a_2 \to x^0} - \underbrace{\sum_{n=1}^{\infty} c_n x^n}_{a_1 \to x^1} - \underbrace{\sum_{n=1}^{\infty} c_n x^n}_{a_1 \to x^0}$$

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$$-\underbrace{\sum_{n=0}^{\infty} C_n x^n}_{a_0 \rightarrow x^0}$$

$$f(y'', y', y) = \left[c_2 \cdot 2(2-1)\, x^0 + \sum_{n=3}^{\infty} C_n n(n-1) x^{n-2}\right]$$

$$-\sum_{n=1}^{\infty} C_n n x^n - \left[C_1 \cdot 1 \cdot x^0 + \sum_{n=2} C_n n x^{n-1}\right] -$$

$$\left[C_0 x^0 + \sum_{n=1} C_n x^n\right.$$

$$2C_2 + \sum_{n=3}^{\infty} C_n n(n-1)\, x^{n-2} - \sum_{n=1}^{\infty} C_n n x^n - C_1$$

$$-\sum_{n=2} C_n n x^{n-1} - C_0 - \sum C_n x^n$$

$$= \underbrace{\sum_{n=3} C_n n(n-1)\, x^{n-2}}\ \underbrace{-\sum_{n=1} C_n n x^n}_{a_1 \rightarrow x^1}\ \underbrace{-\sum_{n=2} C_n n x^{n-1}}_{a_2 \rightarrow x^1}$$

$$\underbrace{-\sum_{n=1} C_n x^n + 2C_2 - C_1 - C_0}_{a_1 \longrightarrow x^1}$$

Now, we do shift by replacing,

$$(y'', y', y) = \sum_{n=3} C_n n(n-1) x^{n-2} - \sum_{n=1} C_n n x^n -$$

$$\sum_{n=2} C_n n x^{n-1} - \sum_{n=1} C_n x^n + 2C_2 - C_1 - C_0$$

$$\sum_{n+2=1} c_{n+2}\left((n+2)(n+2)-1\right)x^{(n+2)-2} - \sum_{n=1} c_n x^n$$

$$= \sum_{n=1} c_{n+1}(n+1)x^n - \sum_{n=1} c_n x^n + 2c_2 - c_1 - c_0$$

$$= \sum_{n=1}\left[c_{n+2}(n+2)(n+1) - c_{n+1} - c_{n+1}(n+1) - c_n\right]x^n + 2c_2 \ldots$$

$$\sum_{n=1}\left[c_{n+2}(n+2)(n+1) - c_{n+1}(n+1) - c_n(n+1)\right]x^n$$

$$+ 2c_2 - c_1 - c_0$$

$$\sum_{n=1}\left[c_{n+2}(n+2) - c_{n+1} - c_n\right](n+1)x^n + 2c_2 - c_1 - c_0$$

$$= 0$$

$c_{n+2}(n+2) - c_{n+1} - c_n = 0 \quad \Sigma,\ 2c_2 - c_1 - c_0 = 0$

$$c_2 = \frac{c_1 + c_0}{2}$$

$$c_{n+2}(n+2) = c_{n+1} + c_n$$

$$c_{n+2} = \frac{c_{n+1} + c_n}{n+2},\quad \frac{c_{n+1}-1}{n+2} + \frac{c_n}{n+2}$$

By using (4) and (5)

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$n=1$, $c_3 = \frac{c_2}{3} + \frac{c_1}{3}$

$= \frac{\frac{c_1+c_0}{2}}{3} + \frac{c_1}{3}$

$= \frac{c_1}{6} + \frac{c_0}{6} + \frac{c_1}{3}$

$= \frac{c_0}{6} + \frac{c_1}{2}$

$n=2$, $c_4 = \frac{c_3}{4} + \frac{c_2}{4}$

$= \frac{1}{4}\left[\frac{c_0}{6} + \frac{c_1}{2}\right] + \frac{1}{4}\left[\frac{c_0}{2} + \frac{c_1}{2}\right]$

$= \frac{c_0}{6} + \frac{c_1}{4}$

$y = \sum_{n=0} c_n x^n$

$= c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 - c$

$= c_0 + c_1 x + \left[\frac{c_0}{2} + \frac{c_1}{2}\right] x^2 + \left[\frac{c_0}{6} + \frac{c_1}{2}\right] x^3$

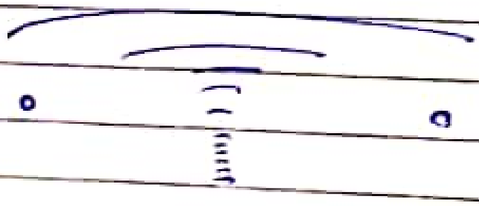
$+ \left[\frac{c_0}{6} + \frac{c_1}{4}\right] + \text{——}$

$$= c_0\left[1+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{6}x^4+\dots\right]$$

$$c_1\left[x+\frac{1}{2}x^2+\frac{1}{2}x^3+\frac{1}{4}x^4+\dots\right]$$

$$y_1 = c_0\left[1+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{6}x^4+\dots\right]$$

$$y_2 = c_1\left[x+\frac{1}{2}x^2+\frac{1}{2}x^3+\frac{1}{4}x^4+\dots\right]$$



$\boxed{\text{Question : 2}}$

$$(x^2+2)y''+3xy'-y=0$$

The general form,

$$y=\sum_{n=0} a_n x^n$$

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$$y'=\sum_{n=1} n c_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

Substitute $y, y'$ and $y''$ in eq (1)

$$(x^2+2)\sum_{n=2} n(n-1) c_n x^{n-2} + 3x \sum_{n=1} n c_n x^{n-1} -$$

$$\sum_{n=0} c_n x^n = 0$$

$$\sum_{n=2} n(n-1) c_n x^n + 2\sum_{n=2} n(n-1) c_n x^{n-2} + 3\sum_{n=1} n c_n x^n$$

$$-\sum_{x=0} c_k x^n = 0$$

$$\sum_{k=2} k(k-1) c_k x^k + 2.2.1 c_2 x^0 + 2.3.2.c_3 x^3$$

$$+2\sum_{k=2} (k+2)(k+1) c_{k+2} x^k + 3.1.c_1 x^1 +$$

$$3\sum_{k=2} k c_k x^k$$

$$- c_0 x - c_1 x - \sum_{k=2} c_k x^k = 0$$

$$4c_2 - c_0 + (12c_3 + 2c_1)x + \sum_{k=2} \Big[ 2(k+2)(k+1) c_{k+2}$$

$$+ (k(k-1) + 3k - 1)\, c_k \Big] x^k = 0$$

Compare the coefficients,

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$$4c_2 - c_0 = 0$$

$$c_2 = \frac{1}{4} c_0$$

$$12c_3 + 2c_1 = 0$$

$$c_3 = \frac{-1}{6} c_1$$

$$2(k+2)(k+1)c_{k+2} + [k(k-1) + 3k - 1] c_k = 0$$

$$2(k+2)(k+1)c_{k+2} + (k^2 + 2k - 1)c_k = 0, \quad k = 2, 3, 4$$

$$c_{k+2} = \frac{-(k^2 + 2k - 1)}{2(k+2)(k+1)} c_k, \quad k = 2, 3, 4$$

Now, $c_0 = 1$ and $c_1 = 0$

$$c_2 = \frac{1}{4}$$

$c_3 = 0$, $c_5$, $c_7$, $c_9 \ldots = 0$

$k = 2$ in $c_{k+2} = \frac{-(k^2 + 2k - 1)}{2(k+2)(k+1)} c_k$

$$c_4 = \frac{-(2^2 + 2.2 - 1)}{2(2+2)(2+1)} c_2$$

$$= \frac{-7}{24} \times \frac{1}{4}$$

$$= \frac{-7}{96}$$

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$= -\frac{7}{4 \cdot 4}$

$k = 4$ in $c_{k+2} = -\frac{(k^2+2k-1)}{2(k+2)(k+1)} c_k$

$c_6 = \frac{-(4^2+2\cdot4-1)}{2(4+2)(4+1)} c_4$

$= -\frac{16+8-1}{2\cdot6\cdot5} \times \frac{-7}{96}$

$= \frac{23\cdot7}{8\cdot6!}$

$y_1(x) = c_0\left[1 + \frac{1}{4}x^2 - \frac{7}{4\cdot4!}x^4 + \frac{23\cdot7x^6}{8\cdot6!} - \right.$

$c_0 = 0,$ and $c_1 = 1$

$c_3 = \frac{-1}{6}$

$c_2 = 0,\ c_4 = c_6 = c_8 = \dots = 0$

$k = 3 \Rightarrow c_5 = -\frac{(2^2+2\cdot2-1)}{2(2+2)(2+1)} c_3$

$= \frac{-7}{24} \times \frac{-1}{6}$

$= \frac{7}{\dots} = \frac{14}{2\cdot5!}$

$$y_2(x) = c_1 \left\{ x - \frac{1}{6}x^3 + \frac{14}{2.5!}x^5 - \frac{34.14}{4.7!}x^7 \dots \right]$$

Thus two solutions,

$$y_1(x) = c_0 \left\{ 1 + \frac{1}{4}x^2 - \frac{7}{4.4!}x^4 + \frac{23.7}{8.6!}x^6 \dots \right]$$

$$y_2(x) = c_1 \left\{ x - \frac{1}{6}x^3 + \frac{14}{2.5!}x^5 - \frac{34.14}{4.7!}x^7 \dots \right]$$