Physics Qualifying Examination – Part I 7-Minute Questions

March 1, 2014

1. A typical person has a surface area about 1.4 m², and an average skin temperature of 33°C.

    a. Determine the net power radiated per unit area if the person's total emissivity is 97% and the environment is at room temperature T=20°C.
    b. How much energy does the body radiate per second?

2. Three capacitors have identical area and plate separation as shown. The first is filled with vacuum and has a capacitance C_0. The second is half filled with dielectric ε all across the bottom plate and the third half filled with the same dielectric halfway from bottom to top as shown. Find the capacitance of the (a) second and (b) third capacitors in terms of C_0 and ε.

3. Two particles of mass m_1 and m_2 are attracted to each other by a central force $\vec{F} = (-\alpha/r^4)\hat{r}$, where α is a positive constant and $\vec{r}$ is the separation vector. They move in a circular orbit at constant speed v about each other, separated by a distance r_0. Derive a relation between v and r_0.

4. a. What is the lowest possible energy (in electron volts) of an electron in hydrogen if its orbital angular momentum is $\sqrt{12}\hbar$?

    b. What are the largest and smallest values of the z-component of the orbital angular momentum in terms of $\hbar$ for the electron in part (a)?

    c. What are the largest and smallest values of the spin angular momentum, in terms of $\hbar$, for the electron in part (a)?

5. In charged pion decay $\pi^+ \rightarrow \mu^+ + v_\mu$, compute the fraction of the energy taken by the neutrino compared to the pion energy (rest mass), working in the rest frame of the pion.

6. The angular wavefunction of a system is

$$\psi(\theta,\phi) = \sqrt{\frac{1}{4}}Y_5^3(\theta,\phi) + \sqrt{\frac{3}{4}}Y_3^2(\theta,\phi),$$

where $Y_l^m(\theta,\phi)$ are spherical harmonics.

    a. If $\vec{L}^2$ is measured, what are the possible outcomes, and with what probability do they occur? What is the expectation value?

    b. If L_z is measured, what are the possible outcomes, and with what probability do they occur? What is the expectation value?

7. A cylindrical tank of diameter $2R$ contains water of depth d. A small hole of diameter $2r$ is opened in the bottom of the tank. $r \ll R$, so the tank drains slowly. Find the expression for the time it takes to drain the tank completely.

8. A large cube of glass has a metal reflector on one face and water on the adjoining face (the index of refraction of water is 1.33). A light beam strikes the reflector, as shown. You observe that as you gradually increase the angle of the light beam, if $\theta \geq 59.2°$ no light enters the water. What is the speed of light in this glass?
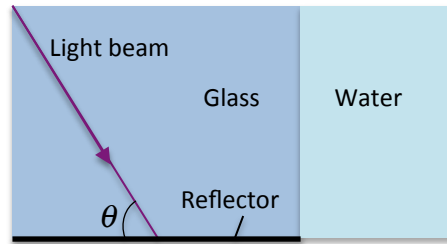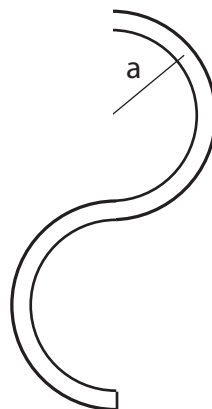

9. a. What are the resonant frequencies of sound waves inside a narrow circular pipe bent to a radius a? The speed of sound is c.
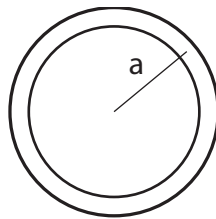
   b. A narrow cut is taken out of the pipe, as shown, and one end is sealed. What are the resonant frequencies?
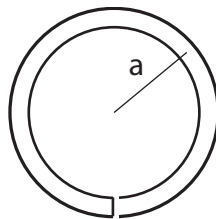
   c. The pipe is re-bent as shown. What are the resonant frequencies?

10. Your latest invention is a car alarm that produces sound at a particularly annoying frequency of 3500 Hz. To do this, the car alarm circuitry must produce an alternating electric current of the same frequency. That's why your design includes an inductor and a capacitor in series. The maximum voltage across the capacitor is to be 12.0 V (the same voltage as the car battery). To produce a sufficiently loud sound, the capacitor must store 0.0160 J of energy. What values of capacitance and inductance should you choose for your car alarm circuit?

# Physics Qualifying Examination – Part II   12-Minute Questions

March 1, 2014

1. The luminosity L of an astrophysical object is the power it emits in light. Extreme astrophysical objects such as quasars are powered by accretion, sucking massive quantities of matter into a deep gravitational well and converting the gravitational potential energy of the matter into light. Assume the infalling matter is ionized hydrogen.

    a. What is the gravitational force attracting each ionized hydrogen atom (mass $m_p + m_e \approx m_p$ and distance r) to a quasar of mass M ?
    b. Assume each photon emitted by the quasar has energy E. What is the momentum of the photon?
    c. What is the radiation force on each free electron, in terms of the quasar luminosity L? The radiation force is exerted via Thomson scattering of photons on free electrons (cross section σ_T). After each scatter, a photon is equally likely to recoil forward or backward.
    d. The free electrons and protons in the plasma are mutually attracted by the Coulomb force, so if the electrons are pushed outward by the radiation pressure they also push the protons outward. If the luminosity is too high, the radiation force is greater than the gravitational force and accretion cannot occur. In terms of M and other relevant quantities, what is the maximum luminosity before this occurs (known as Eddington luminosity)?

2. For the circuit below, $V_{OUT} = A(V^+ - V^-)$, $A > 10^5$, and the current into V^+ and V^- is negligible. Find an approximate expression for $V_{OUT}(V_1,V_2)$ good to 1 part in 10^4. Show your work.
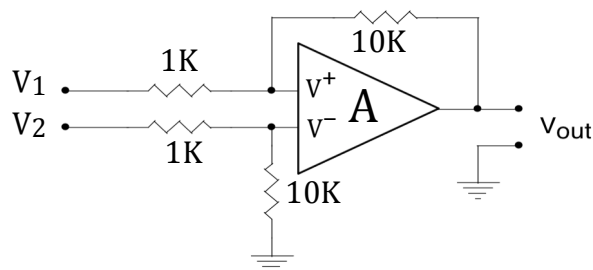

3. Find the r.m.s. fluctuations $\left\langle \Delta N^2 \right\rangle^{\frac{1}{2}}$ in the number of particles N occupying a fermion orbital when the probability it is occupied is $P(1) = a$.

4. Two fixed parallel conducting rods are connected by a resistor R and there exists a uniform and constant magnetic field perpendicular to the plane of the rods. A bar of mass m , making frictionless contact with the rods is made to move along them with constant velocity v_0 .
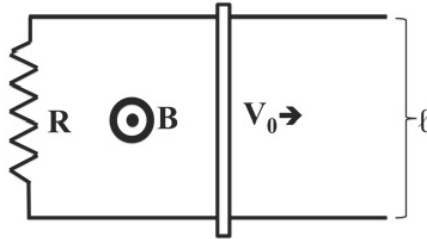
   a. Find the current that flows in the circuit (assume R is large enough that all other resistance in the circuit is small compared to it and so that the magnetic field produced by the current is negligible relative to B).
   b. Now, at time t=0, the applied force that has kept the bar moving at the constant velocity is removed. Find the subsequent velocity of the bar as a function of time.

5. A fully ionized hydrogen plasma is placed in a uniform magnetic field $B = 0.1\,T$. The plasma is initially in thermal equilibrium with the temperature T_0 . The magnetic field is then slowly (compared to the particle gyro-frequencies) increased to $1\,T$, in such a way that particle collisions can be neglected during this process. The plasma is then allowed to relax to a new thermal equilibrium due to particle collisions. What is the plasma temperature now?

6. In another universe, the electron is a spin-$\frac{3}{2}$ rather than a spin-$\frac{1}{2}$ particle, but all other physics are the same as in our universe.

   a. What are the atomic numbers of the lightest two inert gases?
   b. What is the ground-state electron configuration of sodium (Z=11)?

7. The supernova 1987A is located about 170,000 light years from earth. The interactions of 10 neutrinos from the supernova were observed in a tank of 1000 tons of water within an interval of 1 second. The average neutrino energy was 10 MeV, and the energies varied from 5 to 20 MeV. The experimental signature of each neutrino interaction included recoil nucleons as well as a charged lepton.

   a. What are the weak interactions most likely involved in the creation and subsequent detection of the neutrinos?
   b. Estimate an upper limit on the neutrino mass from the observed data.

8. A Hall probe is used to measure the magnetic field in a particle detector. The Hall probe consists of a piece of n-doped silicon with a carrier density $n = 10^{18}$ cm^{-3} and the dimensions $\Delta x = 6$ mm, $\Delta y = 4$ mm, $\Delta z = 0.5$ mm. A magnetic field B_z is applied in the z-direction and a current $I_x = 0.1$ A flows in the x-direction.
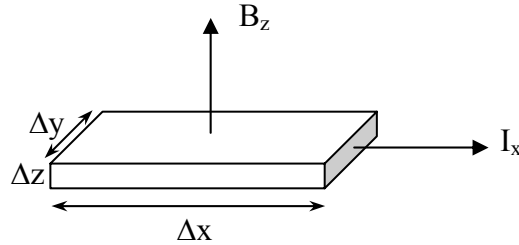
a. Which quantity X is measured to determine the magnetic field B_z?
b. Write down the equilibrium condition that determines this quantity.
c. Calculate the current density j_x from I_x and the electron velocity v_x from j_x. Give numerical values.
d. Calculate the quantity X measured for a magnetic field $B_z = 1$ T.
e. What would be the result for p-doped Si compared to n-doped Si?

9. A neutron is a spin-½ particle with magnetic moment μ_N. A neutron finds itself in a magnetic field $\vec{B}$ that lies in the x-y plane so that $\vec{B} = B_0(\cos\phi\hat{x} + \sin\phi\hat{y})$. The Hamiltonian is

$$H = -\frac{1}{2}\mu_N \vec{B} \cdot \vec{\sigma},$$

where the Pauli matrices are given by

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

a. Assuming that $\vec{B}$ is independent of time, what are the spin eigenvalues E and spin eigenfunctions ψ of H?
b. At time $t = 0$, the field is in the x-direction $\vec{B}(t=0) = B_0\hat{x}$ (i.e., $\phi = 0$) and the neutron is in its ground spin state. The direction of the field is suddenly changed to $\phi_0 \neq 0$, but the magnitude is unchanged. What is the wavefunction?

10. The average lifetime of muons at rest is $\tau_\mu = 2.2$ μs. A laboratory measurement on the decay in flight of muons in a beam emerging from a particle accelerator yields an average lifetime of $\tau_\mu = 6.6$ μs, as measured in the lab frame S.

    a.   What is the speed of these muons in the laboratory frame S?

    b.   The rest mass of the muon is $m_\mu \approx 100$ MeV/c^2 ($=10^8$ eV/c^2). What is the energy of the muons in their rest frame S'?

    c.   What is the kinetic energy of the muons in the laboratory frame S?

    d.   How far on average do the muons travel before decaying in the laboratory frame S?

11. A 50 g sample of Selenium-82 is observed for 100 days for neutrinoless beta decay. The apparatus has a detection efficiency of 20%. No events with the expected signature for neutrinoless beta decay are observed. Set an upper limit (at 90% confidence level) on the lifetime for this decay mode.

12. Consider a system of two quantum particles in a harmonic potential. The single particle energy eigenvalues are given by

$$E_n = \hbar\omega(n+1/2),$$

where $n = 0,1,2,3....$

    a.   Assume that the particles are distinguishable. Show that the canonical partition function factorizes and calculate the free energy.

    b.   Assume now that the particles are identical fermions. Evaluate the canonical partition function (no chemical potential). Enumerate the possible occupations of the single particle states by first assuming the lowest state occupied and summing over the possible occupations by the second particle. Then assume the state $n=1$ occupied and sum over the available occupations for the second particle and so on. Calculate the free energy and compare with the results obtained in (a) at high and low T. Explain the difference.

13. Suppose the TE_{11} mode of a rectangular wave guide has an electric field with $\hat{y}$ component

$$E_y(t,\vec{x}) = E_{0y}\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos(k(\omega)z - \omega t)$$

where a and b are constants and the waveguide is vacuum inside. What is its group velocity $d\omega/dk$ as a function of ω in the region in which $k = k(\omega)$ is real?

14. A pencil, initially standing vertically on its tip on a flat horizontal table, starts to fall under gravity. The coefficient of static friction between the pencil tip and the table surface is $\mu_s = 0.25$. Will the tip begin to slip before the pencil falls to 25 degrees from vertical? Treat the pencil as a uniform rod of mass M and length L ($I = (1/3)ML^2$) and support your answer with arguments.
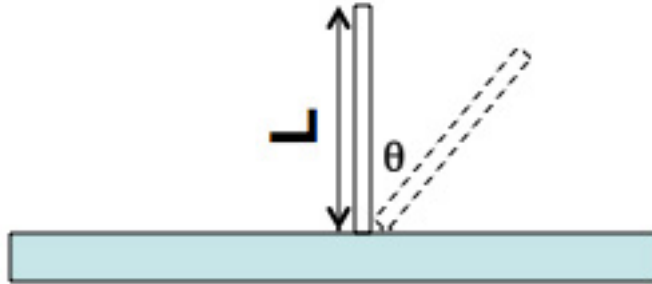
15. A lens is to be coated with a thin film with an index of refraction of 1.2 in order to reduce the reflection from its surface at λ=5000 Å. The glass of the lens has an index of refraction of 1.4, as shown in the figure below.

    a.   What is the minimum thickness of the coating that will minimize the intensity of the reflected light?
    b.   In the above case the intensity of the reflected light is small but not zero. Explain. What needs to be changed, and by how much, to make the intensity of the reflected light zero?