Physics Qualifying Examination – Part I 7-Minute Questions

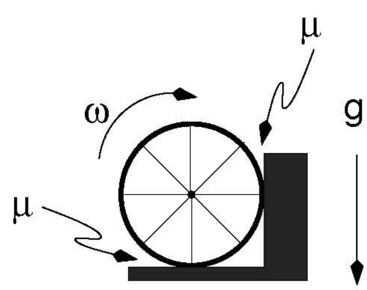
September 11, 2010

1. A string that has a linear mass density of 4.00×10^{-3} kg/m is under a tension of 360 N and is fixed at both ends. One of its resonance frequencies is 375 Hz. The next higher resonance frequency is 450 Hz.

   a. What is the fundamental frequency of this string?
   b. Which harmonics have the given frequencies?
   c. What is the length of the string?

2. A wheel of mass m, rotational moment of inertia $I = mR^2$, and radius R initially spins at angular velocity ω while in contact with a vertical barrier (see figure below). Both the horizontal and vertical contact surfaces have a coefficient of kinetic friction μ. Gravity acts in the vertical direction.

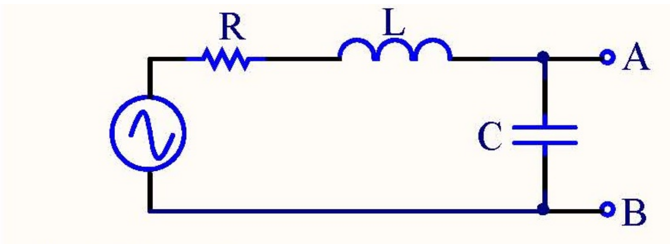
In terms of the above parameters, what is the resulting angular acceleration of the wheel?



3. Find the average kinetic energy and average potential energy of a particle of mass *m* in the ground state in a simple harmonic oscillator with frequency ω_0. Hint:

$$\psi_0 = N \exp(-\alpha x^2 / 2)$$
$$\alpha = (m\omega_0 / \hbar)$$

4. Consider the LCR circuit shown below. The circuit is driven by a sinusoidal voltage source $V(t) = V_0 \exp(j\omega t)$, where $\omega = \omega_0 \equiv 1/\sqrt{LC}$ is the resonant frequency of the circuit.



$$V(t) = V_0 \exp(j\omega_0 t)$$

a. Solve for the current $I(t)$ in the circuit.
b. Find the voltage $V_{AB}(t)$ where $V_{AB} \equiv V_A - V_B$.

5. A neutral Na atom has eleven electrons. Ten electrons are removed to form a Na^{+10} ion. Calculate the frequency in Hz and the wavelength in nm of the $n = 2 \rightarrow n = 1$ transition of the Na^{+10} ion.

6. A beam of light with an intensity of 10^8 W/m^2 traveling in free space hits normally on a lossy mirror. This mirror reflects 60% of the light and absorbs 40%.

What is the resulting radiation pressure on the surface?

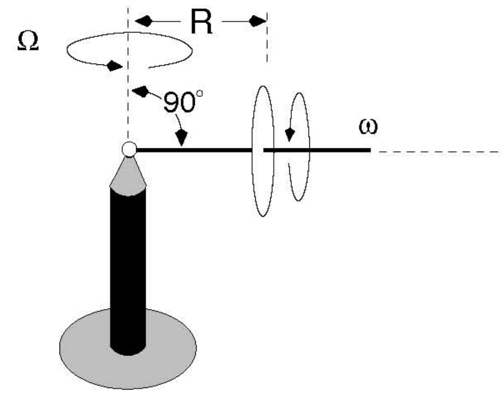
7. You fill a bicycle tire with air using a manual bicycle pump. The volume of the tire is 1 x 10^{-3} m^3. The initial pressure (absolute pressure) of the air in the tire is 1 atm and when you are done inflating the tire the pressure is 7 atm.

Assuming that you pump so slowly that the air remains at room temperature throughout the process, estimate how much work you do on the gas.

8. A thin conducting wire with resistivity ρ forms a circular loop. The wire is being stretched so that the radius of the loop increases linearly in time at a rate α from an initial value r_0. The volume of copper remains fixed at V_0, so the cross sectional area of the wire decreases as the length of the wire increases. A constant magnetic field, B_0 points along the axis of the loop.

Find the induced current in the wire as a function of time.

9. A thin disk top of mass m has a massless axle running through its center. The disk is situated a distance R away from the pivot. This top is rotating about the horizontal axis shown with angular velocity ω. The rotational inertia of the top about the ω-axis is I. The top is also precessing slowly.



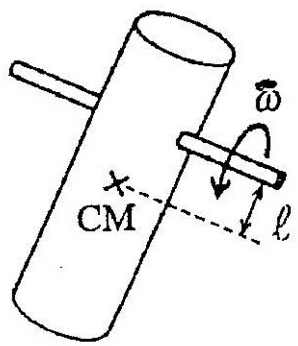
What is the angular velocity Ω of the precession in terms of the given parameters?

10. Through some coincidence, the Balmer lines from single ionized helium in a distant star happen to overlap with the Balmer lines from hydrogen in the Sun. Assuming that the relative motion is along the line of sight, how fast is that star receding from us?

Physics Qualifying Examination – Part II 12-Minute Questions

September 11, 2010

1. A physical pendulum consists of a solid cylinder which is free to rotate about a transverse axis displaced by a distance ℓ along the symmetry axis from the center of mass, as illustrated. Find the value of ℓ for which the period of small amplitude oscillations is a minimum. Express the result in terms of the mass M and moment of inertia I about a transverse axis through the CM.



2. A microscopic spherical dust grain of radius R is inserted into an electron-proton plasma at temperature T. Assume that the electrons and the protons stick to the grain when they collide with it. It is observed that the dust grain becomes charged.

   a. What is the sign of the net charge on the dust grain? Explain your answer.
   b. Estimate the magnitude of the charge.

3. A non-relativistic particle of charge e and mass m moves with velocity $\vec{\upsilon}$ in a constant uniform magnetic field along the z direction $\vec{\mathbf{B}} = B\,\hat{z}$ and is subject to a drag force $\vec{F}_d = -m\vec{\upsilon}/\tau$ where τ is a constant. At time $t = 0$ the position and velocity are $\vec{R}_0$ and $\vec{\upsilon}_0$.

   a. Write Newton's equations of motion for the velocity components υ_x, υ_y and υ_z.
   b. Construct an equation of motion for $\upsilon_x + i\upsilon_y$ and solve it.
   c. Describe in words the shape of the trajectory in **three** dimensions.

4. A system has two types of angular momentum, $\vec{L}$ and $\vec{S}$, which interact and couple to form $\vec{J} = \vec{L} + \vec{S}$. The eigenvalues of L^2 are $\ell(\ell+1)\hbar^2$ and of S^2 is $\frac{1}{2}\left(\frac{3}{2}\right)\hbar^2$.

a. Find the eigenvalues of J^2.
b. Find the corresponding eigenvalues of $\vec{L} \bullet \vec{S}$.

5. Many metals behave as if there is one free electron per atom. Based on this statement, and assuming classical particles, a calculation of the molar specific heat at constant volume of such a metal would be the following:

$$C_v = 3R + 3R/2 = 9R/2, \qquad \text{(Eq. 1)}$$

where R is the molar gas constant. In fact, the correct value for C_v for such metals is very close to 3R.

a. The first term in Eq. 1 represents the contribution from the ion cores. Why is this term equal to 3R and not to some other value?
b. The second term in Eq. 1 is meant to represent the contribution from the electrons themselves. Why does this term significantly overestimate the electron contribution to C_v?

6. Concisely explain the operating principles of two of the four pumping methods: (1) an oil diffusion pump, (2) a turbomolecular pump, (3) a cryopump, and (4) a sputtering ion pump. Include a neat, labeled drawing of each (suitable for a lab notebook) as part of your description and describe the relative merits of the method.

Calculate how long will it take to pump down a 1 m^3 vacuum vessel to 10^{-7} torr beginning at 1 torr using a 100 liter/second turbomolecular pump. (You may assume the walls of the vessel do not outgas and you do not need to consider the ultimate base pressure of the pump.)

7. Dark matter particles can strike nuclei in a detector with considerable energy because they move with a velocity of 220 km/s relative to our laboratories. Assume that the dark matter particles have a mass of order the weak boson mass, i.e. M_W = 100 GeV/c^2, and that the detector is made out of nuclei of mass M_N.

a. Calculate the kinetic energy of a dark matter particle entering the detector.

b. If a nucleus in the detector is struck, what is its recoil energy as a function of the center-of-mass angle θ of the collision.

c. Estimate the maximum recoil energy in a Germanium detector with atomic mass number 76.

Mass of the nucleon = 0.938 GeV/c^2.

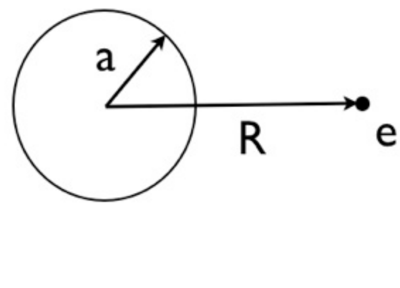
8. Assume that a hadron (neutron, proton, pion etc.) is a "bag", with quark matter all confined inside. The leading contributions to its self-energy are given by

$$E(R) = \frac{\alpha}{R} + \frac{4\pi}{3} R^3 \rho + 4\pi R^2 \sigma ,$$

where R is the average radius of the hadronic bag and α, ρ, and σ are positive constants. The three terms are a repulsive term (α), a volume energy density (ρ), and a surface energy density (σ).

a. Derive and equation for a stable radius R_0 of the hadronic bag.
b. In the low density limit $R^3\rho \ll R^2\sigma$, find the hadronic radius R_0 and the hadron mass $m_h = E(R_0)$.
c. If $\alpha \to 0$, what would happen to the hadronic bag?

9. A point charge e is placed at a distance R from the center of a metallic sphere of radius a, with R > a, as shown in the figure below. The sphere is isolated and electrically neutral. Find the electrostatic potential on the surface of the sphere (relative to infinity).



10. Given the arrival times and energies of two neutrinos of the same flavor and the distance to a supernova that produced them, derive an expression for the mass, m, of the neutrino in terms of the time difference, the energies, E_1 and E_2, and the distance, d, assuming they were produced at the same time. Taylor expand keeping terms to order mc^2/E squared.

Evaluate m in units of eV for a time difference of 5 seconds, energies of E_1 = 22MeV and E_2 = 38MeV and a distance of d = 150,000 light years. These values are from supernova 1987A.

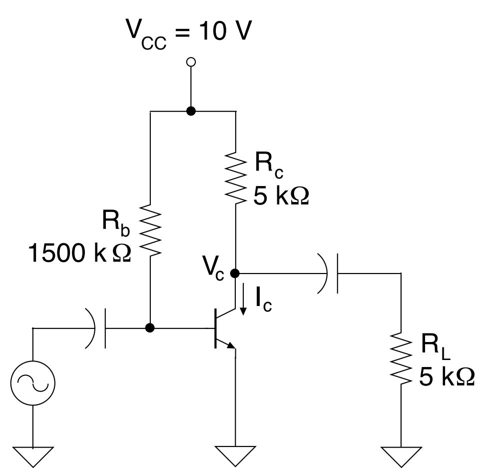
11. The ground state wavefunction of the hydrogen atom is given by

$$\psi(r,\theta,\phi) = C\exp(-r/a_0)$$

Here a_0 is the Bohr radius; $a_0 = \frac{4\pi\varepsilon_0}{e^2}\frac{\hbar^2}{\mu} = 0.0529$ nm and μ is the reduced mass of the electron. The ground state energy is $E_0 = -\hbar^2/2\mu a_0^2 = -13.6$ eV.

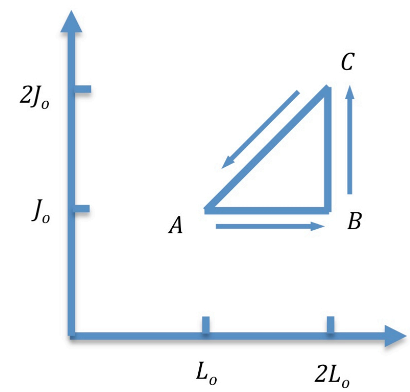
a. What is the value of C?
b. Suppose the potential energy between an electron and a proton had a term $V_0(a_0/r)^2$ in addition to usual electrostatic potential energy. To the first order V_0, where V_0=0.01 eV, by how much would the ground state energy be changed?

12. This amplifier uses a standard silicon transistor with a beta of 180.



a. What is the value of I_C?
b. What is the value of V_C?

13. A rubber band is used to make a heat engine. It has an equation of state $J = \alpha LT$ where J is the tension in the rubber band, L is the length of the rubber band, T is the temperature, and α is a constant. The heat capacity of the rubber band at constant length is c_L, independent of temperature. Like the ideal gas, the internal energy of the rubber band depends only on temperature.



The engine cycle $A \to B \to C \to A$ is, with quasi-static changes:

$A \to B$: Cooled by cold reservoir, temperature T_{cold}, while keeping the tension constant until rubber band reaches T_{cold}.
$B \to C$: Heated by hot reservoir, temperature T_{hot}, while keeping the length constant until rubber band reaches T_{hot}.
$C \to A$: Stay in contact with hot reservoir and decrease the length.

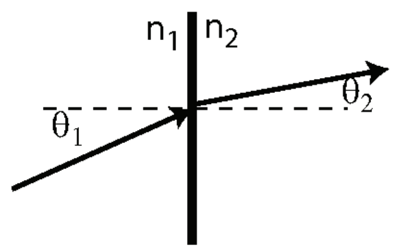
a. What is the ratio of the temperature of the cold reservoir to the temperature of the hot reservoir (T_{cold} / T_{hot})?
b. How much work is done by the heat engine when it moves through one cycle $A \to B \to C \to A$?
c. During one cycle $A \to B \to C \to A$, how much heat is extracted from the hot reservoir, and how much heat is exhausted to the cold reservoir?

14. Consider a system of N identical relativistic particles with spin $1/2$ occupying three-dimensional volume V. The relation between energy and momentum of a particle is linear, $\varepsilon(p) = c|p|$.

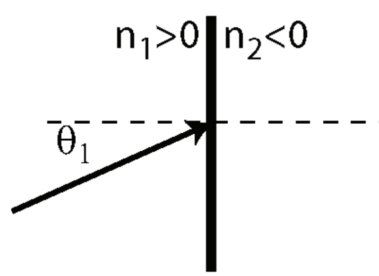
a. Explain why the Fermi momentum of this gas of relativistic particles coincides with the Fermi momentum of non-relativistic fermions of the same number density, $p_F = \hbar(3\pi^2 N / V)^{1/3}$.
b. What is the Fermi energy ε_F of the gas?
c. Calculate the internal energy U of the gas at zero temperature.
d. Using U, calculate the pressure at zero temperature.

15. For the purposes of this problem assume that Snell's Law of Refraction applies with negative as well as positive indices of refraction.

a. Light is transmitted across a planar interface separating media with refractive indices n_1 and n_2. Write down an expression for θ_2 in terms of n_1, θ_1, n_2 with $n_1 > 0, n_2 > 0$.



b. Assume $n_1 > 0$ and $n_2 < 0$. Complete the figure showing the direction of the transmitted ray.



c. Consider a point source located a distance L_1 in front of a planar slab of thickness d with index $n_2 = -1$. Find the condition on d such that a real image is formed to the right of the slab and find the position L_2 of the image.