Spring 2008 Qualifier - Part I **7 minute questions**

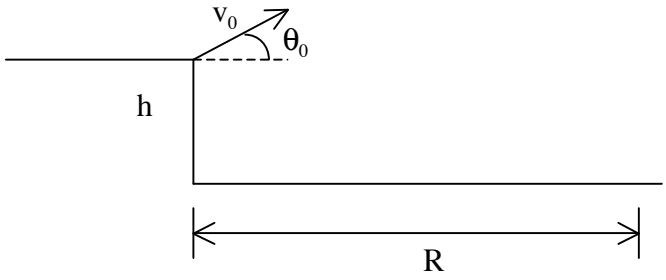
1) Standing at a crosswalk, you hear a frequency of 560 Hz from the siren of an approaching police car. After the police car passes, the frequency you hear is 480 Hz. What is the car's speed in m/s? The speed of sound is 343 m/s.

2)

a) What is the root mean square speed, in m/s, of nitrogen molecules in air at room temperature (295 K)?

b) Use the 225 nm mean free path of these molecules at 1 atm pressure and room temperature to estimate the mean time between collisions, in seconds.

3) A projectile is shot from the edge of a cliff of height h at an angle of θ_0 above horizontal and with speed v_0. What horizontal distance R will it move before it hits the ground?



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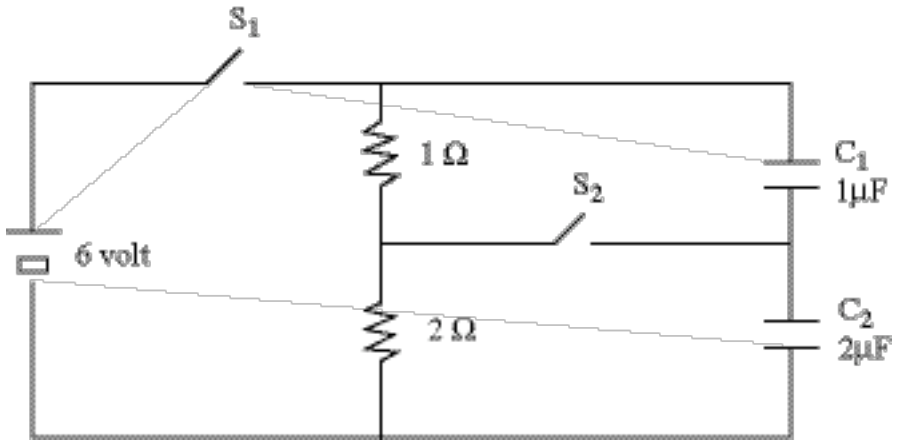


a) With S_1 and S_2 are both closed, what are the steady-state charges on C_1 and C_2?

b) Now S_2 and then S_1 are opened. What are the final charges (magnitude and polarity) on C_1 and C_2?

5) A parallel-plate capacitor consists of two circular disks of radius a and separation $d << a$. The capacitor is connected to an AC voltage, and the charge on the positive plate is $q = q_0 \cos \omega t$. The lines of magnetic field induced by the displacement current are circles centered on the axis of symmetry. Find the strength of the magnetic field as a function of position (axial and radial) between the plates.

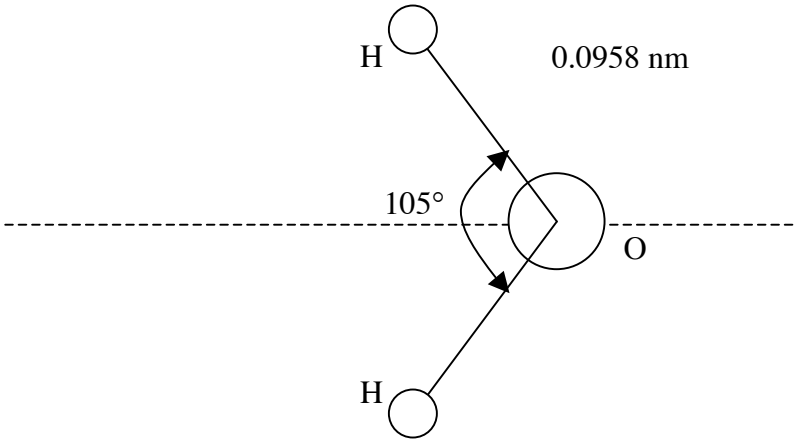
6) Some people believe that the dark matter in our galaxy is made up of weakly interacting massive particles (WIMPs). The mass density of galactic WIMPs at the position of Earth is estimated to be $\rho c^2 \approx 0.3\,\text{GeV cm}^{-3}$. Assume that WIMPs move around the galaxy with the same speed as stars, about $\frac{v}{c} \approx 10^{-3}$ and that the WIMP mass is 50 GeV/c^2.

a) What is the WIMP kinetic energy?

b) Suppose that the WIMP scattering cross section is 10^{-35}cm^2 on typical nuclei. Estimate the mean free path of WIMPs passing through ice.

c) Are the WIMPs energetic enough to excite a typical nucleus and produce a gamma ray?

7) Consider the model of the H_2O molecule shown here:



Numerical answers are required for the following:

a) Find the rotational moment of inertia of H_2O about the dashed line.

b) Estimate the rotational energy of the first excited rotational level (J = 1) of the molecule.

c) What is the wavelength of a photon required to excite a transition from J= 0 to J = 1?

8) Consider the one-dimensional Schrodinger equation for a particle of mass m moving in the potential $V(x) = \frac{1}{2} m\omega_0^2 x^2$ for $x > 0$ and $+\infty$ for $x < 0$. Find the energy eigenvalues.

9) In recent years, it has become possible to prepare artificial materials with negative indices of refraction ($n < 0$). Using Snell's Law, show with a sketch the path of a light beam passing from vacuum to a transparent material with a negative index of refraction. Compare your answer with the path of a light beam entering a material with a positive index of refraction of the same magnitude. Explain how a negative index material makes a perfect lens.

10) A particle moves in one dimension. It has total energy E and mass m and is subject to a force $F(x) = qx^4$, $q>0$. Using classical mechanics:

- **a)** Find the velocity v of the particle as a function of the position x and energy E.
- **b)** Sketch the phase diagram for the motion; plot v vs. x for various values of E.