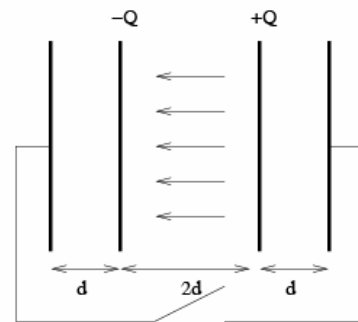# Physics Qualifier—Spring 2009 Part I

1.   Consider this edge-on view of a set of wide metal plates. The wires connecting the outer plates through a switch have non-zero resistance. The inner plates are initially charged as shown. Find the steady-state charge distribution when the switch is closed.

2.   An electron makes a radiative transition from the n=3 level to the n=2 level of hydrogen. Neglecting fine structure, find the frequency, in Hz, and the wavelength, in nm, of emitted radiation resulting from this transition.

3.   A force F is applied to a solid cylindrical object of radius R and mass M as shown. What is the acceleration of the center of mass of the object (when it rolls without slipping)?
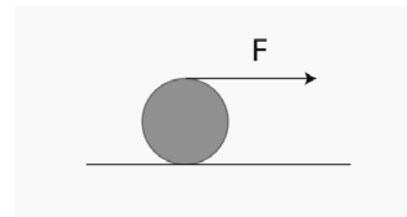
4.   On a still winter night, the temperature over Lake Mendota is -10°C. The ice thickness is X= 0.2 m. How fast is the ice growing, i.e., what is the value of dX/dt ? Ice has a density ρ= 0.9 X10^3 kg/m^3, a thermal conductivity of 2.1 W/(Km), and a latent heat of fusion L = 3.3 X 10^5 J/kg.

5.   Write down the time independent Schrodinger equation for the s-wave states of hydrogen. Guess a one parameter form for the ground state wave function and demonstrate it is a solution by finding the value of the parameter and the energy in terms of fundamental constants.

6. A simple radio transmitter consists of a perfect LC circuit freely oscillating at high frequency. The oscillator is coupled to an antenna that radiates electromagnetic waves.

      a. Suppose the inductance in the LC circuit is 22 μH. What capacitance do we need to produce an oscillation frequency of 1500 kHz?

      b. If the capacitor is initially fully charged at 2.0×10^{-4} C what is the energy stored in the system?

      c. Is initially charging a freely oscillating LC circuit a good way to run a 500 W radio station? Justify your answer.

7. A well-known type of insect called a "water strider" is able to walk on water (see photo). Its feet leave depressions ('dimples') in the water's surface. Suppose the water strider is on the surface of a pond and is illuminated by the sun overhead. (The strider's legs are so skinny that they do not block a significant amount of light) The dimples in the water's surface can be seen as patterns of light on the sand at the bottom of the pond. Consider a single dimple, whose cross section is illustrated by the diagram on the right. Does the dimple appear as a bright spot or a dark spot on the bottom of the pond, and why?

8. State (and explain why) which statistics (Maxwell-Boltzmann, Fermi-Dirac, or Bose-Einstein) would be appropriate to describe:

      a. the density of He^4 gas at room temperature

      b. the density of electrons and holes in semiconducting Ge at room temperature (the Ge band-gap is about 1 volt)

      c. the density of electrons in Cu at room temperature.

9. How much power would a 50% efficient wind turbine generate if it had a radius of 2m and the wind was blowing through it at a velocity of 5m/s? Assume the air density to be 1 kg m^{-3}.

10. Two identical spin 1/2 fermions are confined in a one-dimensional harmonic oscillator potential. The potential has energy eigenstates $E_n = (n+1/2)\hbar\omega, n = 0,1,2...$ with corresponding wavefunctions $\psi_n(x)$. The fermions interact with each other via a spin-spin coupling $V = -J\vec{S}_1 \cdot \vec{S}_2 / \hbar^2$ where J is positive. Find the ground state wavefunction and ground state energy of the 2-particle system for (a) $J \gg \hbar\omega$ and (b) $J \ll \hbar\omega$ (If the ground state is degenerate you need give only one wavefunction.)

Physics Qualifier—Spring 2009 Part II

11. Consider a three-dimensional charged oscillator placed in a homogeneous static magnetic field B. The frequency of this oscillator without the field is ω_0 for oscillation in any direction. Find the new resonance frequencies in the presence of the field, for oscillations parallel and perpendicular to the field direction.

12. Two thin, parallel rods are separated by a distance d and move with the same velocity $\bar{v}$, not small relative to the speed of light c. The velocity is parallel to the rods. Assume that the rods are infinitely long and non-conducting with the same charge density per unit length q.

    a. Calculate the force per unit length between the rods in the lab frame.

    b. Repeat the calculations in a reference frame moving with the rods.

    c. Compare the two results

13. An object of mass M is falling at a constant velocity under the influence of a uniform gravitational force (with gravitational acceleration g = constant) and a drag force $F_d = \alpha A v$ that is proportional to the velocity v and the surface area A. At some instant the object spontaneously breaks into N smaller objects. For simplicity, assume that each daughter object has a mass and volume equal to 1/N times the mass and volume of the original object. Assume that the daughter objects have the same shape as the original object.

    a. What is the asymptotic velocity (terminal velocity) of the daughter objects relative to the velocity of the original object at breakup?

    b. What is the change in net kinetic energy due to the break up?

    c. Estimate the time it takes for the daughter objects to reach their new terminal velocity.

14. A large, horizontal disk of radius R, shown to the right, starts to rotate from rest with an angular acceleration of α. The rotation is about a vertical axis through the center of the disk. The disk contains a narrow channel of length 2R and rectangular cross section. Gravity acts in the vertical direction with an acceleration of g. There is a small rectangular puck that just fits easily in the aforementioned channel, as shown. The puck is situated a distance r from the axis rotation.
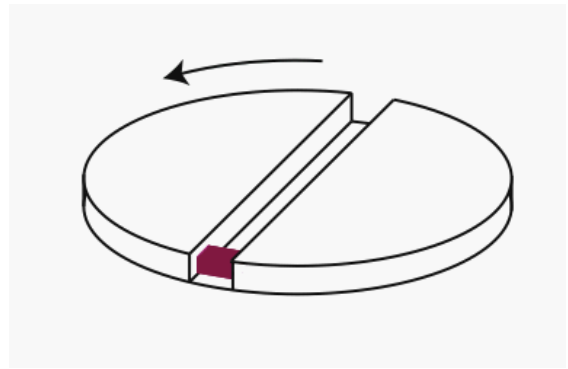
a. If the sides of the channel are frictionless but the bottom of the channel has a static coefficient of friction μ, then how long, in terms of any or all of the aforementioned constants, will it be before the puck begins to slide?

b. Now, instead, the situation is that the bottom of the channel is frictionless but the walls have a static coefficient of friction μ. How long, in terms of any or all of the aforementioned constants, will it be before the puck begins to slide?

15. The lowering, or annihilation, operator of the linear harmonic oscillator can be written in units where the ground state wavefunction is proportional to $e^{-x^2/2}$, as

$a = \dfrac{1}{\sqrt{2}}\left(x + \dfrac{d}{dx}\right)$. An eigenfunction of this operator is termed a "coherent state."

Solve the eigenfunction problem $a\varphi(x) = \dfrac{ik}{\sqrt{2}}\varphi(x)$ and interpret the wave function

$\varphi(x)$.

16. The Hamiltonian describing a non-relativistic approximation to the weak interaction between an electron and an atomic nucleus can be written as

$H_\omega = \omega \bar{s} \bullet \left[\bar{p} \delta(\bar{r}) + \delta(\bar{r}) \bar{p} \right]$ where ω is a constant, $\bar{s}$ is the electron spin, $\bar{p}$ is the electron momentum, $\bar{r}$ is the electron position, and $\delta(\bar{r})$ is a three dimensional delta function.

    a. Does H_ω conserve parity? Give a reason for your answer.

    b. Calculate the matrix element of H_ω between opposite parity Hydrogen atom states $|1s, m_l = 0, \uparrow\rangle$ and $|2p, m_l = 0, \uparrow\rangle$ ($\uparrow$ denotes the electron spin up). The spatial parts of the wavefunctions are
$\psi_{1s}(r) \equiv \langle r | 1s, m_l = 0 \rangle = c_1 e^{-r/a}$ and
$\psi_{2p}(r) \equiv \langle r | 2p, m_l = 0 \rangle = c_2 z e^{-r/2a}$ with a the Bohr radius and c_1, c_2 being normalization constants.

17. Atoms of the alkali metals have spectral lines resembling transitions among Hydrogen-like energy levels, $E(n,l) \simeq -\dfrac{13.6eV}{(n - \Delta(l))^2}$, where n is the principal quantum number, and $\Delta(l)$ are corrections that depend on the orbital quantum number l (such corrections are absent for the Hydrogen atom). Propose qualitative answers to the following questions:

    a. Why does the Hydrogen-like formula work for these atoms?

    b. Why do the energy levels depend on the orbital quantum number l?

    c. Do the corrections $\Delta(l)$ increase or decrease when l increases?

18.   a. Is the following reaction allowed, according to the standard model of particle physics (if not, cite a specific rule violated and why): $p \rightarrow e^+ + \gamma$

        b. Assuming that this reaction does occur (whether forbidden or not) and that the proton is initially at rest, what is the energy of the photon? Use m_p = 938.8 MeV/c^2 and m_e = 0.511 MeV/c^2.

        c. What is the momentum of the photon?

        d. What fraction of the speed of light is the speed of the positron? (Assume perfect accuracy of the input numbers).

19. Suppose that we build a laser by placing a thin laser crystal (gain medium) inside a cavity formed by two mirrors. Energy is added to the crystal so that at low light intensities it amplifies light that goes through it by a factor e^{gL}, where L=1 mm is the length of the crystal and g is the gain coefficient. One of the mirrors is perfectly reflecting, while the other reflects 99% of the light incident on it and transmits the rest.

        a. What is the minimum gain coefficient g for the laser crystal such that light will be continuously generated by the laser?

        b. If more energy is added to the crystal, thereby temporarily increasing g over the value from part a, what keeps the laser intensity from increasing without bound?

        c. If the output power of the laser is 1 Watt, what is the circulating power inside the cavity?

20. This question is about the angular resolution of the human eye. The surface of the retina is covered with cells that sense light. The centers of the cells (rod cells) are spaced apart by a distance of 2 microns.   Is the angular resolution of the eye limited by diffraction, or by the spacing of these receptor cells?   You will need to make some assumptions about the dimensions of the eye. Please state your assumptions clearly.

21. In a simplified view of disk galaxies, stars follow circular orbits in the gravitational field of a spherically symmetric halo. It is observed that the orbital velocity V(r) is nearly constant with r. Find M(r), the halo mass interior to r, which produces this rotation curve as well as the mass density rho(r) in the halo which gives this mass distribution. Derive a numerical value for M(r) for r = 10 kiloparsecs and V = 250 km/s (1 kiloparsec = 3.1 10^{21} cm).

22. The free energy of a crystal due to vibrational modes (phonons) can be written in the form $F = k_B T \sum_{i=1}^{3N} \ln(1 - \exp(-\hbar\omega_i / k_B T))$ where the sum runs over all vibrational modes ω_i, and N is the total number of atoms in the crystal.
Calculate the vibrational energy of the crystal $E = F - T(\partial F / \partial T)$ at high temperatures, $k_B T \gg \max\{\hbar\omega_i\}$, and find the resulting heat capacity C. Use the approximation $\ln(1 - e^{-x}) \approx \ln x - x/2$. Explain your result in terms of the equipartition theorem.

23. Consider a pion generated in a cosmic ray air shower close to sea level. You are given: Pion mass=135 MeV, Density of atmosphere: 10^{-3} g/cm^3, hadronic interaction length: 87 g/cm^2, Pion lifetime: 10^{-8} s. Above what energy is it more likely that the pion will undergo a hadronic interaction than a decay?

24. The input circuit of an oscilloscope often has a switch that allows only the ac component of a voltage to be observed, accomplished by switching in a coupling capacitor. The equivalent circuit of the oscilloscope is shown below. The oscilloscope is connected to an ac source with an internal impedance of 1000 ohms. Draw the two equivalent circuits representing the high and low frequency limits, including the source, and determine the lower and upper 3 dB frequency roll off points.
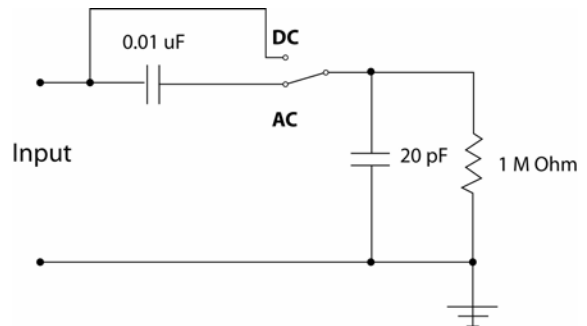
25. A mechanically isolated system has only two states with energies E$_1$ and E$_2$. It is in contact with a temperature reservoir of temperature T.
   a. What value does the mean energy of the system approach as $T \to 0$? Explain
   b. What value does the mean energy of the system approach as $T \to \infty$? Explain
   c. What value does the entropy approach as $T \to \infty$?
   d. Calculate the T-dependence of the heat capacity at constant volume in the low-temperature limit $(kT \ll E_2 - E_1)$.
   e. What value does the heat capacity approach as $T \to 0$?

26. A monotonic ideal gas is used in the PV diagram shown at the right. Assume that $V_A = 0.25$ m^3, $P_A = 10^5$ N/m^2, and $T_A = 300$ K. The gas expands from A to B, increasing its volume by a factor of 4, in contact with a thermal reservoir with temperature T_A. The gas is then cooled at a constant volume from B to C by placing it in contact with a thermal reservoir at temperature T_C. Finally, the gas is adiabatically compressed from C to A, with its volume decreasing by a factor of 4.

Ideal Gas

Pressure

isothermal

adiabatic

B

C

A
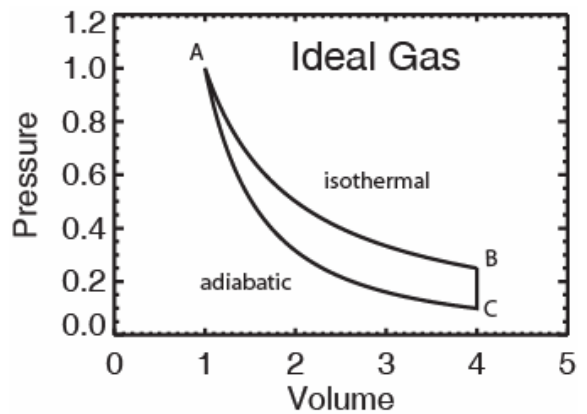
Volume

a. Fill in the following table, finding the temperature and pressure at B and C.

	Pressure (N-m^{-2})	Temperature (K)	Volume (m^3)
A	10^5	300	.25
B			1.0
C			1.0

b. Fill in the table, finding the change in internal energy of the gas, the work done by the gas, and the heat transferred to the heat.

Process	ΔU	Work	ΔQ
AB			
BC			
CA			

c. What is, if any, the change in entropy for one cycle? Is this system reversible or irreversible?

d. What is the efficiency of this engine? How does this efficiency compare to an optimal engine?