STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Graduate Placement Exam Part 2, August 27, 2008

General Instructions: This exam is for incoming graduate students who wish to demonstrate mastery in one or more areas of the graduate core curriculum, in order to skip one or more of the first-year courses.

Do any two of the three problems in either or both areas.

Each solution should typically take on the order of 45 minutes.

Use one exam book for each problem, and label it carefully with the problem topic and number and your name. Make sure to do every part of the problems you choose.

You may use a one page help sheet, a calculator, and with the proctor's approval, a foreign language dictionary. No other materials may be used.

Quantum Mechanics 1

Calculate the lifetime of the lowest localized state of a nonrelativistic particle in the three-dimensional "spherical shell" potential

$$U(r) = W\delta(r - R)$$

in the limit of large $W > 0$. Specify the condition of validity of your result in that limit.

Quantum Mechanics 2

Calculate the rate of spontaneous photon emission (to vacuum) by an electrically charged 2D rotator ($x^2 + y^2 = R^2 = \text{const}, \; z = 0$), initially in the state with $L_z = \hbar$. Assume that $R \gg \hbar/mc$.

Quantum Mechanics 3

Use (a) the perturbation theory, and (b) direct solution of the Dirac equation solution, to calculate the splitting of hydrogen atom's $2p$ level due to spin-orbit interaction, in the first nonvanishing order in $\alpha \equiv e^2/\hbar c$. Compare the results and comment.

Statistical Mechanics and Thermodynamics 1

The one-dimensional Ising model is defined by the Hamiltonian $\mathcal{H} = -J\sum_i \sigma_i\sigma_{i+1}$, where the classical spin variable $\sigma_i = \pm 1$ on each site i. Consider the thermodynamic limit of this system with periodic boundary conditions.

a) (4 points) Derive an exact expression for the specific heat.

(b) (6 points) Derive an exact expression for the spin-spin correlation function $\langle\sigma_0\sigma_n\rangle$.

b) (4 points) Is there a phase transition at any nonzero temperature? If you say yes, derive the value of this transition temperature; if you say no, give a proof of your answer.

c) (3 points) Derive the value of the transition temperature in the mean-field approximation.

d) (3 points) Give a brief explanation for the difference between the mean-field value of the transition temperature and the exact transition temperature. Apply your explanation to this particular model.

Statistical Mechanics and Thermodynamics 2

Consider a hypothetical insulating material in $d = 6$ spatial dimensions, in which the atoms are connected by harmonic springs. Derive expressions for the temperature dependence of the specific heat for this material in both the low temperature and high temperature limits (assuming the solid is not near melting). For any properties of the material that you need to incorporate in your analysis (density, speed of sound, etc.), be sure to clearly identify the symbol you use so that the grader can understand what you have done.

Statistical Mechanics and Thermodynamics 3

The van der Waals equation of state is given by

$$P = \frac{k_B T}{v - b} - \frac{a}{v^2}$$

where $v = V/N$ denotes the volume per molecule, with V and N the total volume and number of molecules.

a) (6 points) Explain the physical origin of the constants a and b, and give approximate values for a real gas such as nitrogen.

b) (6 points) Derive expressions for the critical temperature and pressure.

c) (8 points) If a gas expands without doing work or exchanging heat with its environment, its temperature may change. Find the appropriate dT/dV for a van der Waals gas in terms of the heat capacity C_V and constants in the equation of state. You may find one or more of the Maxwell relations useful:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V; \quad \left(\frac{\partial T}{\partial P}\right)_S = \left(\frac{\partial V}{\partial S}\right)_P; \quad \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T; \quad \left(\frac{\partial V}{\partial T}\right)_P = -\left(\frac{\partial S}{\partial P}\right)_T$$