MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF PHYSICS

SEPTEMBER 4, 1996

DOCTORAL GENERAL EXAMINATION

PART I

FIVE HOURS

INSTRUCTIONS

1. This examination is divided into four sections, each containing four problems worth 10 points each. Answer ALL of the problems. Although the problems in each section tend to concentrate on one area of physics, this does not mean that the subject matter of any problem will fit entirely, or even mostly, into that one area.

2. Use a separate fold of paper for each problem, and write your name on each fold. Include the problem number with each solution. A diagram or sketch as part of the answer is often useful, particularly when a problem asks for a quantitative response.

3. Calculators may be used.

4. No Books or Reference Materials May Be Used.

Mechanics I

A planet orbits a massive star in a highly elliptical orbit, i.e., the total orbital energy, E, is close to zero ($|E| << KE$). The initial distance of closest approach is R_o. Energy is dissipated through tidal motions until the orbit is circularized with a final radius of R_f. Assume that orbital angular momentum is conserved during the circularization process.

(1) Find and make a sketch of the effective one-dimensional potential in the frame of reference that is rotating with the instantaneous angular velocity of the planet. Indicate on this sketch the initial and final energy states of the orbit.

(2) Compute the final radius of the circular orbit, R_f, in terms of the initial distance of closest approach, R_o.

Mechanics II

A long belt of a conveyer is moving horizontally with a constant speed v_o. A cylinder of mass M and radius R is rotating with an angular velocity ω_o and is quietly dropped on the belt. Find the relative distance the cylinder skips on the belt before it starts to rotate without slipping. The kinetic friction between the cylinder and the belt is μ_k.
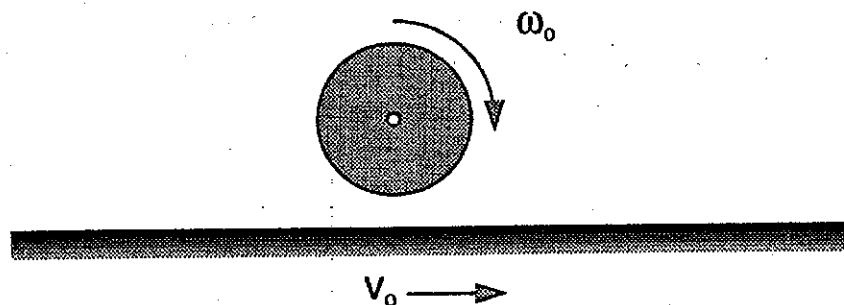
Mechanics III

A cosmic-ray proton collides head-on with a photon of energy $h\nu = 10^{-3}\ eV$ (from the cosmic microwave background), and a π-meson (with rest energy $140\ MeV$) is produced. What is the minumum energy that a cosmic-ray proton must have in order for this reaction to take place?

Mechanics IV

A string of length L and mass M is under tension T with its two ends fixed on the wall. The two points, $L/4$ from both ends, are pulled and slightly displaced from equilibrium by distance h as shown in the diagram. Suddenly the string is released. Assume $h \ll L$.
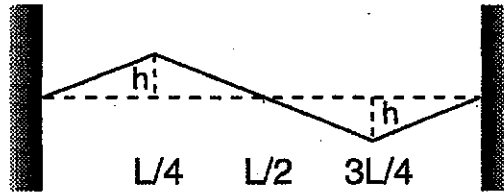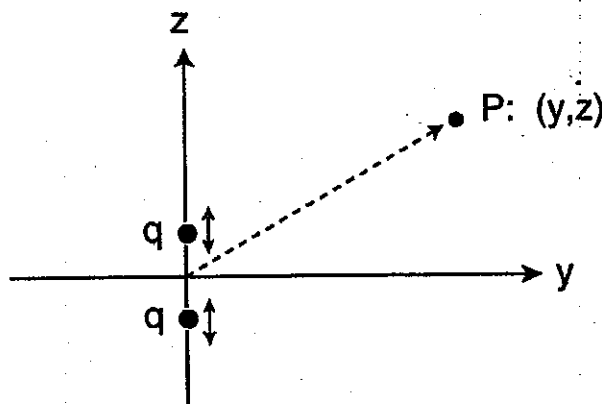

(1) Write down the normal modes of the string.
(2) Which normal modes have non-zero amplitudes?
(3) Find the displacement $u(x, t)$ indicating the vertical position of the string.

Electromagnetism I

Two positive charges, $+q$, oscillate in nonrelativistic motion along the z axis, separated by a mean distance s. They oscillate $180°$ out of phase with each other, with an amplitude of $s/2$ and an angular frequency of ω. Derive the electric field at a distant point P: $(0,y,z)$. Assume $1/k = c/\omega \gg s$ and $y^2 + z^2 \gg s^2$. Find the dependence of the Poynting flux on ω.
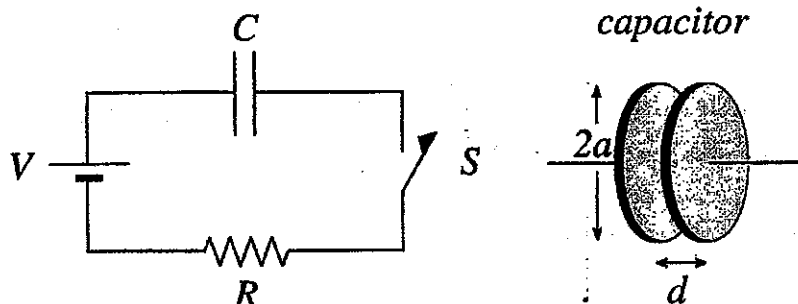
(Assume that at P, the difference in the directions of the E fields from the two charges, as well as the difference in the $1/r$ dependence, are both negligible.)
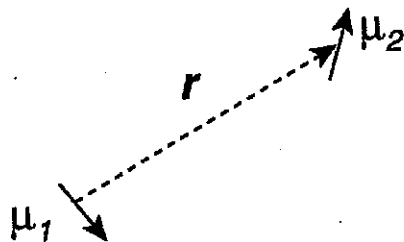
Electromagnetism II

Consider the current flowing in a circuit containing a capacitor C made of two circular plates of radius a and distance d. At time $t = 0$, the switch is turned on. Find the displacement current density and the magnetic field inside the capacitor. Assume d<<a.

Electromagnetism III

Two permanent magnetic dipoles μ_1 and μ_2, are at the ends of a rod whose length and direction are fixed and are described by the vector r. Derive an expression for the energy of interaction between the dipoles for arbitrary orientations of the dipoles. Describe the minimum-energy configuration.
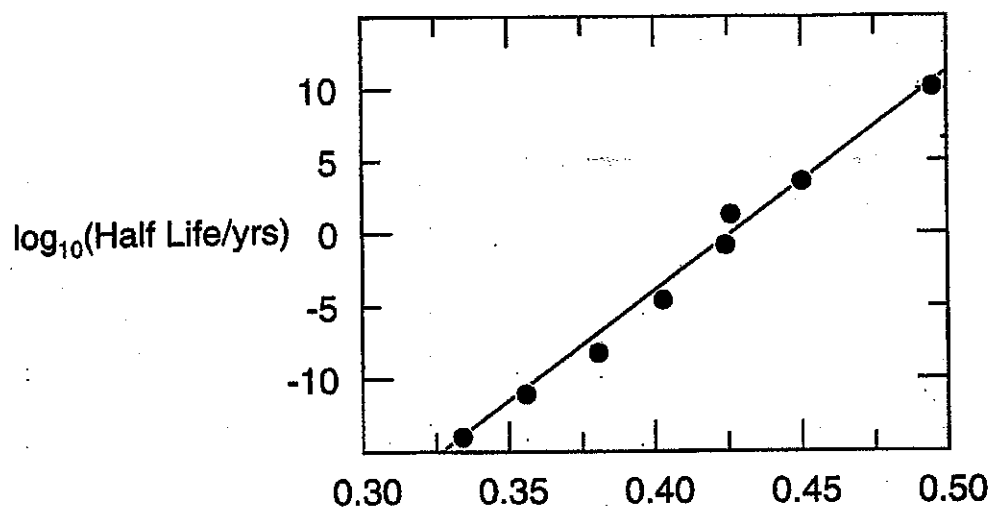
SECTION II
Electromagnetism IV

What is the charge distribution when the potential in space is given by Yukawa potential:

$$\phi(r) = \frac{q}{4\pi\varepsilon_o r} \exp\left(-\frac{r}{r_o}\right)$$

Quantum Mechanics I

The following plot shows the logarithm of the decay time, τ, for α particles to be emitted from various nuclei (all with $Z \cdot 90$). Specifically, the quantity $ln(\tau)$ is plotted vs. the reciprocal square root of the energy of the emitted α particles. Note that the relation between these two quantities is close to linear. Use a simple one-dimensional quantum mechanical model, involving only a Coulomb barrier, to show that the observed proportionality is to be expected. You may assume that the "classical turning radius" r_0 is much greater than the radius of the nucleus, R.
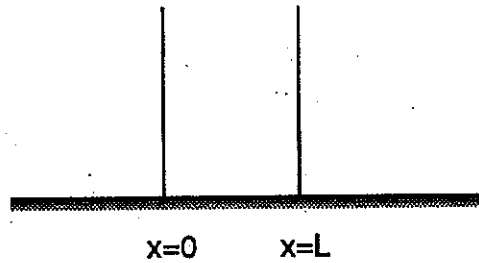
Quantum Mechanics II

A particle of mass m with energy E is incident from the left in one dimensional onto a double potential wall given by

$$V(x) = A\{\delta(x) + \delta(x - L)\}$$

Find the condition that the particle is not reflected by the wall and fully transmitted.

Quantum Mechanics III

The nucleus of a tritium atom ($^3H_1$) undergoes beta decay to helium-3 ($^3He_2$). Assume that the nuclear decay and the subsequent exit of the beta-decay products (e^-, ν) are effectively instantaneous; i.e., the original single atomic electron remains bound to the $^3He_2$ atom and its wavefunction is initially unperturbed. Compute the probability of finding the electron in the ground state of He immediately after the decay of the tritium.

The normalized ground-state wavefunction for a hydrogen-like atom is:

$$\Psi = 2\left(\frac{Z}{a_o}\right)^{3/2} \exp\left(-\frac{Zr}{a_o}\right) Y_o^o$$

where Z is the nuclear charge, a_o is the Bohr radius, and $Y_o^o = \frac{1}{4\pi}$.

Quantum Mechanics IV

Estimate the ground-state energy of a particle in a one-dimensional potential well given by:

$$V(x) = \beta x \qquad \text{for } x \geq 0$$
$$V(x) = \infty \qquad \text{for } x < 0,$$

where β is a constant.

Statistical Mechanics I

Suppose that a particle has only five states with energies given by $E_n = nE_o$, where n = 0, 1, 2, 3, or 4. A collection of these particles forms a non-interacting gas in thermal equilibrium with temperature T.

(1) Write down the single-particle partition function.
(2) Compute the average energy of a particle in this gas.
(3) Compute and sketch the specific heat of this gas as a function of T.

Statistical Mechanics II

Consider a gas consisting of N Boson particles each of mass m in a volume V. Find the temperature at which a Bose-Einstein condensation occurs.

Statistical Mechanics III

A blackbody radiation field, with temperature T, is established inside of a perfectly insulated cavity whose volume is V. Consider the interior of the cavity to be evacuated of all particles. The walls of the cavity are slowly expanded until a final volume V' is reached. No energy leaks into or out of the cavity, during or after the expansion. Derive the temperature of the radiation field inside the cavity after the expansion.

Statistical Mechanics IV

Consider one dimensional motions of two equal masses M which are connected by three equivalent springs of spring constant k and relaxed length L. The two springs are attached to walls as shown in the diagram. Find the thermal fluctuation of the position of one of the masses from its equilibrium position at temperature T. Use k_B for Boltzmann constant.