EM SOLUTIONS:

1 A) $$V(z) = \frac{\sigma}{4\pi\varepsilon_0}\iint \frac{r'dr'd\varphi'}{\sqrt{r'^2+z^2}} = \frac{\sigma}{4\pi\varepsilon_0}\cdot 2\pi\int_0^R \frac{r'dr'}{\sqrt{r'^2+z^2}}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\sqrt{r'^2+z^2}\right]_0^R = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2} - |z|\right)$$

B) $$V(z) = \frac{\sigma}{2\varepsilon_0}\left(|z|\sqrt{1+\frac{R^2}{z^2}} - |z|\right)$$

USE BINOMIAL EXPANSION, $\frac{R}{z} \ll 1$

$$\simeq \frac{\sigma}{2\varepsilon_0}\left[|z|\left(1+\frac{1}{2}\frac{R^2}{z^2}\right) - |z|\right]$$

$$= \frac{\sigma R^2}{4\varepsilon_0 z} = \frac{\sigma\cdot\pi R^2}{4\pi\varepsilon_0 z} = \frac{Q}{4\pi\varepsilon_0 z}$$

LOOKS LIKE POINT CHARGE WITH Q = TOTAL CHARGE OF DISC.

C) FAR FROM DISC, V CAN BE EXPANDED AS:

$$V(r,\theta) = \sum_\ell \frac{B_\ell}{r^{\ell+1}} P_\ell(\cos\theta)$$

MATCH THIS TO EXACT RESULT ALONG Z-AXIS ($\theta = 0$)

$$\sum_\ell \frac{B_\ell}{r^{\ell+1}} \underbrace{P_\ell(\cos 0)}_{=1} = \frac{\sigma}{2\varepsilon_0}\left[r\sqrt{1+\frac{R^2}{r^2}} - r\right]$$

USE 3 TERMS OF BINOMIAL EXPANSION

$$\sum \frac{B_\ell}{r^{\ell+1}} \simeq \frac{\sigma}{2\varepsilon_0}\left[r\left(1+\frac{1}{2}\frac{R^2}{r^2} + \frac{\frac{1}{2}(-\frac{1}{2})}{2}\left(\frac{R^2}{r^2}\right)^2\right) - r\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[\frac{R^2}{2r} - \frac{R^4}{8r^3}\right]$$

$$\Rightarrow B_0 = \frac{\sigma R^2}{4\varepsilon_0}, \quad B_2 = \frac{-\sigma R^4}{16\varepsilon_0}$$

$$V(r,\theta) \simeq \frac{\sigma R^2}{4\varepsilon_0 r} - \frac{\sigma R^4}{16\varepsilon_0}\frac{\frac{1}{2}(3\cos^2\theta - 1)}{r^3}$$

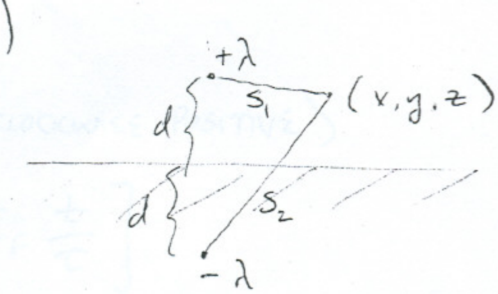
(2) A) APPLY IMAGE METHOD.

POTENTIAL DUE TO SINGLE LINE CHARGE CAN BE WRITTEN:

$$V_{\substack{SINGLE \\ LINE}} = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{d}{s}\right)$$

WHERE $s = d$ IS MY CHOICE OF REFERENCE POINT $(V=0)$

$$V_{TOTAL} = \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{d}{s_1}\right) - \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{d}{s_2}\right)$$



$$= \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{s_2}{s_1}\right)$$

$$= \frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{\sqrt{x^2+(y+d)^2}}{\sqrt{x^2+(y-d)^2}}\right)$$

$$= \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{x^2+(y+d)^2}{x^2+(y-d)^2}\right)$$

B) $$C = \frac{Q}{\Delta V} = \frac{Q}{V(0,d-a,0) - V(0,0,0)}$$

$$= \frac{\lambda L}{\left(\frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{(2d-a)^2}{a^2}\right) - 0\right)}$$

$$\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{2d-a}{a}\right)}$$

TAKE $d = 0.1\,m$, $a = 0.001\,m$

$$\frac{C}{L} = \frac{2\pi(8.85\times10^{-12})}{\ln(199)} = 1.05\times10^{-11}\ F/m$$

C) $$\vec{F} = q\vec{E}_{IMAGE}$$

$$= (\lambda L)\left(\frac{-\lambda}{2\pi\varepsilon_0 s}\hat{s}\right) = (\lambda L)\left(\frac{-\lambda}{2\pi\varepsilon_0(2d)}\hat{y}\right)$$

$$\frac{\vec{F}}{L} = \frac{\lambda^2}{4\pi\varepsilon_0 d}(-\hat{y})$$

(3) A) FOR A LONG SOLENOID, THE FIELD IS NEARLY UNIFORM INSIDE SO $\Phi_B = \pi a^2 |\vec{B}|$

IF $\ell >> b$, NEARLY ALL THE FLUX THROUGH THE COIL ALSO GOES THROUGH THE LOOP (I.E. NO CLOSED LINES OF $\vec{B}$ THAT ARE ENTIRELY WITHIN THE LOOP)

B) $B = \mu_0 n I = \frac{\mu_0 N I}{\ell}$

$I(t) = -\frac{t}{\tau} I_F$ (TAKING COUNTERCLOCKWISE POSITIVE)

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{-\pi a^2 \mu_0 N}{\ell} I_F \frac{t}{\tau}\right]$$

$$= +\frac{\mu_0 N I_F \pi a^2}{\ell \tau}$$

POSITIVE SIGN MEANS EMF TRIES TO DRIVE CURRENT CCW IN THE LOOP, SO POSITIVE CHARGES "PILE-UP" AT THE POSITIVE TERMINAL GIVING A $\boxed{\text{POSITIVE}}$ READING

C) AN IDEAL VOLTMETER DRAWS NO CURRENT, SO THERE IS NO SELF-EMF, BUT FOR FINITE INTERNAL RESISTANCE THE SELF-EMF IS:

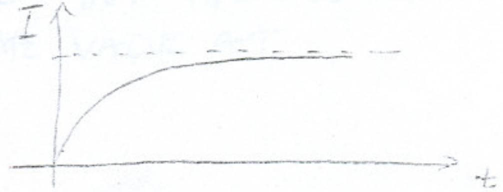
$$\mathcal{E}_{SELF} = -L_{LOOP} \frac{dI_{LOOP}}{dt}$$

THE TOTAL EMF IN THE LOOP IS: $\mathcal{E}_{TOT} = \mathcal{E} + \mathcal{E}_{SELF}$

THE KIRCHHOFF LOOP EQN. IS:

$$I_{LOOP} R = \mathcal{E} - L_{LOOP} \frac{dI_{LOOP}}{dt}$$

CURRENT WILL EXPONENTIALLY APPROACH A STEADY STATE CONDITION WHERE $\mathcal{E}_{SELF} = 0$, GIVING THE <u>SAME</u> READING AS FOR PART (B)



④ A) $V = 0$ (WIRE ASSUMED NEUTRAL)

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I(\vec{r}', t_r)}{\mathscr{r}} d\vec{l}'$$

$$= \frac{\mu_0}{4\pi}\left[2\int_R^\infty \frac{I(t_r)\,\hat{x}}{x'} dx' + \int_\pi^0 \frac{I(t_r)\,\hat{\varphi}\, d\varphi'}{R} \right]$$

$I = I_0$ FOR $t_r > 0$

$$t - \frac{\mathscr{r}}{c} > 0$$

$$\mathscr{r} < ct$$

$$\vec{A} = \frac{\mu_0}{4\pi}\left[2\hat{x}\int_R^{ct} \frac{I_0\, dx'}{x'} + \frac{I_0}{R}\int_\pi^0 \left((-\sin\varphi)\hat{x} + \cos\varphi\,\hat{y}\right) R\, d\varphi' \right], \quad t > \frac{R}{c}$$

$$= \frac{\mu_0}{4\pi}\left[2\hat{x}\ln\left(\frac{ct}{R}\right) + \hat{x}\left[-\cos\varphi\right]_0^\pi \right], \quad t > \frac{R}{c}$$

$$\vec{A} = \begin{cases} \hat{x}\,\frac{\mu_0 I}{2\pi}\left[\ln\left(\frac{ct}{R}\right) + 1\right], & t > \frac{R}{c} \\ 0, & t < \frac{R}{c} \end{cases}$$

B) $$\vec{E} = -\vec{\nabla}V - \frac{\partial \vec{A}}{\partial t}$$

$$= \begin{cases} -\hat{x}\,\frac{\mu_0 I}{2\pi}\cdot\frac{R}{ct}\cdot\frac{c}{R}, & t > \frac{R}{c} \\ 0, & t < \frac{R}{c} \end{cases}$$

$$= \begin{cases} -\hat{x}\,\frac{\mu_0 I}{2\pi t}, & t > \frac{R}{c} \\ 0, & t < \frac{R}{c} \end{cases}$$

c) $\vec{B} = \vec{\nabla}\times\vec{A}$

IF $\vec{A}$ WERE KNOWN AS A FUNCTION OF x, y, z WE COULD DETERMINE ITS CURL BUT THE RESULT OF PART (A) ONLY TELLS US THE VALUE AT THE ORIGIN

(5) THIS IS ESSENTIALLY AN L-R CIRCUIT. FIND L BY EQUATING THE TOTAL MAGNETIC ENERGY TO $\frac{1}{2}LI^2$.

FIRST DETERMINE $\vec{H}$

$$\oint \vec{H}\cdot d\vec{l} = \int \vec{J}_f \cdot d\vec{a}, \quad J_f = \frac{I}{\pi a^2}$$

$$H_\varphi(s)\cdot 2\pi s = \int \frac{I}{\pi a^2}\, da$$

$$H_\varphi(s) = \frac{1}{2\pi s}\cdot\frac{I}{\pi a^2}\cdot \pi s^2, \quad s<a$$

$$\vec{H} = \begin{cases} \dfrac{Is}{2\pi a^2}\hat{\varphi}, & s<a \\ \dfrac{I}{2\pi s}\hat{\varphi}, & b>s>a \end{cases} \quad \left(\vec{H}, \vec{B} = 0 \text{ FOR } s>b\right)$$

$$\vec{B} = \begin{cases} \dfrac{\mu I s}{2\pi a^2}\hat{\varphi}, & s<a \\ \dfrac{\mu_0 I}{2\pi s}\hat{\varphi}, & b>s>a \end{cases}$$

$$\frac{1}{2}LI^2 = U = \int \frac{1}{2}\vec{H}\cdot\vec{B}\, d\tau$$ VOLUME ELEMENT

$$= \frac{1}{2}\cdot 2\pi l\left[\int_0^a \frac{\mu I^2 s^2}{4\pi^2 a^4}\, s\, ds + \int_a^b \frac{\mu_0 I^2}{4\pi^2 s^2}\, s\, ds\right]$$

$$= \frac{\pi l I^2}{4\pi^2}\left[\frac{\mu}{a^4}\cdot\frac{1}{4}s^4\Big|_0^a + \mu_0 \ln(s)\Big|_a^b\right]$$

$$L = \frac{l}{2\pi}\left[\frac{\mu}{4} + \mu_0 \ln\left(\frac{b}{a}\right)\right]$$

$$R = \frac{l\rho}{A} = \frac{l\rho}{\pi a^2}$$

$$I = \frac{V_0}{R}\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{OR} \quad \frac{V_0 \pi a^2}{l\rho}\left(1 - e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R} = \frac{a^2}{2\rho}\left[\frac{\mu}{4} + \mu_0 \ln\left(\frac{b}{a}\right)\right]$$