Spring 2008 Qualifier - Part II **12 minute questions**

11) Consider a parallel *LCR* circuit maintained at a temperature $T >> \hbar/\left(k_B\sqrt{LC}\right)$. Use Maxwell-Boltzmann statistics to calculate the mean-square flux threading the inductor and the mean-square charge on the capacitor.

12) The index of refraction of glass n_2 is to be calculated using Snell's law: $n_2 = n_1 \dfrac{\sin\theta_1}{\sin\theta_2}$, where n_1 is the index of refraction of the external medium. The measured values and statistical errors are $\theta_1 \pm \delta\theta_1 = 61.0^\circ \pm 0.5^\circ$, $\theta_2 \pm \delta\theta_2 = 36.0^\circ \pm 0.2^\circ$, and $n_1 = 1.00$ (the index of air). The measurements of θ_1 and θ_2 are assumed to be uncorrelated. Determine $n_2 \pm \delta n_2$. (No credit will be given for calculating n_2).

13) Gravitational clustering of matter neglecting the effects of general relativity can be treated using force balance and Poisson's equation:

$$\frac{1}{\rho}\vec{\nabla}p + \vec{\nabla}\phi = 0$$

$$\nabla^2\phi = 4\pi G\rho$$

where ϕ is the gravitational potential, p is the pressure, and ρ is the mass density. Given the isothermal equation of state $\dfrac{p}{\rho} = \mathrm{v}_s^2 = \text{constant}$, and assuming a spherically symmetric density distribution $\rho = \rho(r) = \dfrac{C}{r^2}$, find C in terms of v_s and G.

14) A positron e^+ is moving with a kinetic energy equal to its rest energy. It annihilates with an electron e^- at rest, creating two photons. One of the outgoing photons (γ_1) emerges at a right angle to the incident positron direction in the lab frame.

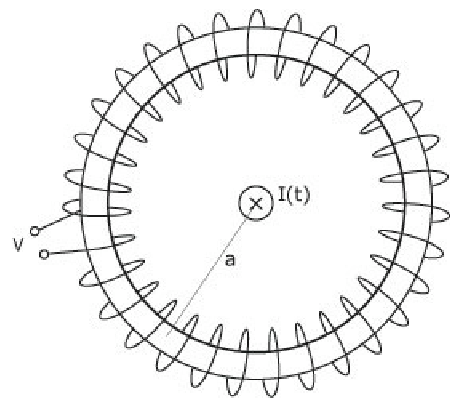
- **a)** Find the energies of the outgoing photons E1, E2 in the lab frame.
- **b)** Find the direction (θ_2) of the second photon (γ_2) with respect to the incident positron direction in the lab frame.
- **c)** What is the energy of one of the photons in the center-of-momentum frame?

15) Given the surface temperature T_0 of the Sun, its angular diameter θ_0 as seen from the Earth, and the assumption that both Earth and Sun radiate as black bodies, write an expression for the temperature of the Earth T_0? (Internal heat sources are negligible.) Show your steps clearly, and substitute the values $T_0 = 5700K, \theta_0 = \frac{1}{2}^\circ$.

16) A satellite in a circular orbit around the earth is affected by a viscous force $A\mathrm{v}^\alpha$ from the thin upper atmosphere; v is the speed of the satellite. The force results in a steady and slow reduction of the orbit with time; i.e. $dR = -C\,dt \ll v\,dt$. Find the values of A and α in terms of the gravitational constant G, the mass of the earth M, and C.

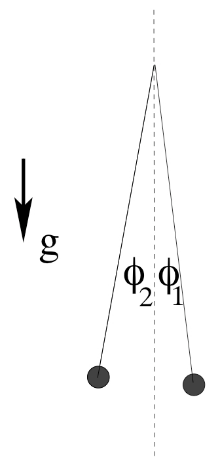
17) Show that the kinetic energy of three-dimensional gas of N free elections at $T = 0$ K is $U_0 = \frac{3}{5}N\varepsilon_F$, where ε_F is the Fermi energy.

18) A straight rod carries a variable current I(t). The current can be measured by surrounding it by a solenoid in the form of a torus. The solenoid has cross sectional area A, radius a, N turns of wire, and is open-circuited. Find the open circuit voltage V(t) in terms of I(t). Assume A is small, N is large, and the torus is nonmagnetic.



19) Two bodies with equal masses *m* are suspended in earth gravity by massless strings of length *R* as shown in the figure. The bodies have equal electric charges *q*. Consider only motion in the plane of the paper. Assume $\phi_1, \phi_2 << 1$ rad.

- **a)** Calculate ϕ_1 and ϕ_2 for the system in equilibrium.
- **b)** Derive the equations of motion in terms of angles $\phi_{1,2}$.
- **c)** Linearize the equations of motion for small oscillations near equilibrium and find the frequencies of the small oscillations. Qualitatively describe the modes of such oscillations. Hint: take a sum and a difference of the equations of motion.



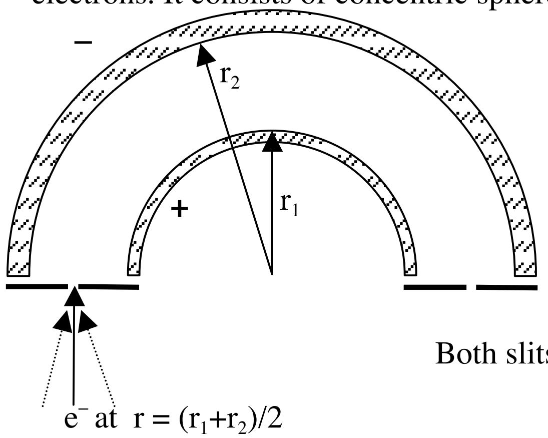
20) Two particles of mass *m* are in a rectangular box with side lengths $a < b < c$ (the potential is 0 inside the box and $+\infty$ elsewhere). The particles are in the lowest energy state of the system compatible with the conditions below. Assume the particles interact through the contact potential $V = V_0\delta(r_1 - r_2)$ where V_0 is a positive constant. Using first order perturbation theory calculate the energy shift due to *V* for the following three cases:

- **a)** Particles not identical
- **b)** Identical particles, each with spin zero
- **c)** Identical particles with spin 1/2 and spins parallel.

You may find the following integrals helpful:

$$\int_0^a dx \sin^2(\pi x/a) = a/2; \quad \int_0^a dx \sin^4(\pi x/a) = 3a/8$$

21) A hemispherical electron spectrometer is a device to measure the energy of electrons. It consists of concentric spheres as shown:



Both slits are at $r = (r_1+r_2)/2$ and grounded.

Draw the electric field lines and the constant potential surfaces between the two hemispheres (assuming that they are similar to those for full spheres).

a) Give the power law r^x for the electric force acting on an electron between the hemispheres.

b) For comparison, give the power law r^y for the gravitational force acting between the Sun and the planets.

c) What kinetic energy E_{kin} does an electron need to make it through the slit on the right side? Determine E_{kin} as a function of the electric field E.

d) At energy E_{kin} , which type of general orbits do the electrons follow? Include electrons that enter the slit at small oblique angles (dotted arrows).

22) Using the masses given below, answer these questions:

a) A free neutron decays into a proton, electron and antineutrino. Assuming the latter to be massless and the original neutron to be at rest, calculate the maximum momentum that could be carried off by the electron and compare this with the maximum momentum which the antineutrino could have. You may neglect the recoil energy of the proton.

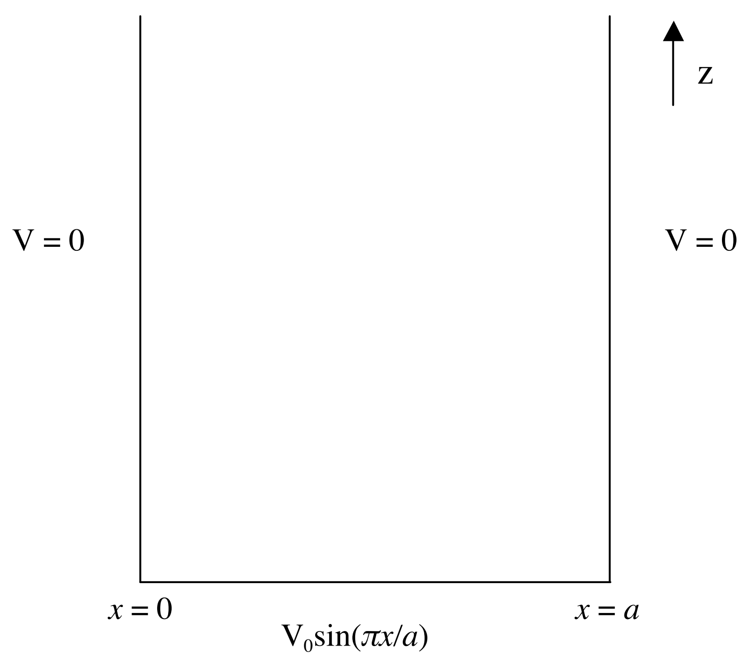
The isotope ${}^{14}_{8}\mathrm{O}$ is a positron emitter, decaying to an excited state of ${}^{14}_{7}\mathrm{N}$. The gamma rays from this latter have an energy of 2.313 MeV and the maximum kinetic energy of the positrons is 1.835 MeV.

b) Write the equation for the decay of the oxygen isotope.

c) Given that one unified mass unit (u) is equal to 931.502 MeV/c^2 find the mass of ${}^{14}_{8}\mathrm{O}$.

<u>Masses:</u> electron 511 keV/c^2, proton 938.272 MeV/c^2, neutron 939.566 MeV/c^2, ${}^{14}_{7}\mathrm{N}$ atom 14.003074 u.

23) Find the electrostatic potential in the region between conducting planes $x = 0$ and $x = a$ held at zero potential, and the plane $z = 0$ held at potential $V_0\sin(\pi x/a)$.



24) Consider two infinite parallel plates separated by a distance d. One plate is at ground potential, and the other is held at potential V. Consider that a uniform gas of electrons with number density n_o and temperature T is inserted between the plates. These metal plates have the unusual property of being highly reflective to electrons at temperature T.

a) The gas will redistribute because of the electric field between the plates. Assuming that the gas does not alter the electric field between the plates (e.g, the density is very low), calculate the density, n(x), of the electron gas (where x is the coordinate perpendicular to the plates) in the time-independent final equilibrium state.

b) Now consider that the electron gas does alter the potential between the plates. Write the equations that would permit solution for the potential $\varphi(x)$. Do not solve.

25) .An electric charge *Q* and a mass *M* are each spread uniformly over a spherical shell of radius *R*. The shell is rotating around a diameter with constant angular velocity *ω*. $\int_0^{\pi/2} \cos^3\theta d\theta = 2/3$

a) What is the value of the moment of inertia about a diameter?
b) What is the value of the angular momentum?
c) What is the value of the magnetic dipole moment?
d) What is the value of the ratio (the "g factor") of the magnetic moment to the angular momentum?

26) The quantum numbers for the lightest 4 quarks are shown below. Answer these questions:

a) Are baryons bosons or fermions? Explain.

b) Are mesons bosons or fermions? Explain.

c) Why is there no such thing as a $uu\bar{d}$ baryon?

d) Can a charge +2 baryon exist with all quarks in the same spin state? Explain why or why not.

Quark	Charge	Spin	Baryon Number	Strangeness	Charm
u	2/3	1/2	1/3	0	0
d	-1/3	1/2	1/3	0	0
s	-1/3	1/2	1/3	-1	0
c	2/3	1/2	1/3	0	1