1

QM I

final Review

Schrodinger Eqn. $H|\alpha\rangle = i\hbar \frac{\partial}{\partial t}|\alpha(t)\rangle$

$H|\alpha\rangle = E|\alpha\rangle$ <u>TISE</u>

$$H = T+V = \left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right)$$

Feynman Propagator: $\mathbb{K}(x't', xt) = \int_{i\to f} d[x]\, e^{iS/\hbar}\, P[x]$ functional

(see 11/17 Bohm/Amnov $= \int d[x]\, e^{iS_{classical}/\hbar + \frac{i}{\hbar}\frac{e}{c}\int \vec{A}\cdot d\vec{x}}$

$$= \Psi_0\, e^{\frac{ie}{\hbar c}\left(\int_1 \vec{A}\cdot d\vec{x} + \int_2 \vec{A}\cdot d\vec{x}\right)}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Bohm Aranhov effect:

$$\mathbb{K}(x't'|xt) = \int_{i\to F} d[x]\, e^{iS/\hbar}\, P_\Phi[x]$$ for $P_\Phi[x]$ is a path dependent phase

$$* \to P_{\Phi_{i,j}}[x] = e^{\frac{i}{\hbar}\frac{e}{c}\int_{i,j}\vec{A}\cdot d\vec{x}}$$ Phase from solenoids

$$\mathbb{K}(x't'|xt) \simeq \int d[x]\, e^{iS_{cl}/\hbar + \frac{i}{\hbar}\frac{e}{c}\int_{i,j}\vec{A}\cdot d\vec{x}}$$

then
$$\Psi_i \simeq \Psi_0\, e^{\frac{ie}{\hbar c}\int_i \vec{A}\cdot d\vec{x}}$$

$$\Psi_{F_{i,j}} = \Psi_i + \Psi_j$$

$$\left|\frac{\Psi_{F_{ij}}}{\Psi_0}\right|^2 = \left| e^{\frac{ie}{\hbar c}\int_i \vec{A}\cdot d\vec{x}} + e^{\frac{ie}{\hbar c}\int_j \vec{A}\cdot d\vec{x}} \right|^2$$

$$= \left| 1 + e^{\frac{ie}{\hbar c}\left[\int_i \vec{A}\cdot d\vec{x} - \int_j \vec{A}\cdot d\vec{x}\right]} \right|^2$$

$$= 2 + e^{-\{\quad\}} + e^{\{\quad\}}$$

$$= 2 + 2\cos\left[\frac{e}{\hbar c}\left(\int_i \vec{A}\cdot d\vec{x} - \int_j \vec{A}\cdot d\vec{x}\right)\right]$$

$$= 2 + 2\cos\left[\frac{e}{\hbar c}\oint \vec{A}\cdot d\vec{x}\right)$$ if i & j paths make a closed loop

$$= 4\cos^2(\underline{\gamma_{ij}})$$

where $\frac{\gamma_{ij}}{2} = \frac{1}{2}\frac{e}{\hbar c}\oint_{ij} \vec{A}\cdot d\vec{x}$

$$= \frac{\pi e}{\hbar c}\int \vec{B}_{ij}\cdot d\vec{a}$$

$$\frac{\gamma_{ij}}{2} = \pi\frac{\phi_{ij}}{\phi_0} \qquad \phi_0 = \frac{\hbar c}{e} \text{ fundamental quantum flux}$$

phase

So for Feynman path integral propagator going around long solenoids we have

$$\left|\frac{\psi_{F_{ij}}}{\psi_0}\right|^2 = 4\cos^2(\text{phase})_{ij}$$

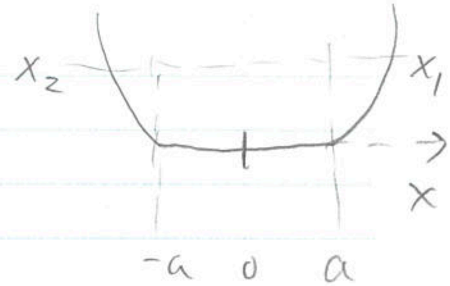
where the phase induced by the path's i,j is

$$\pi\frac{\phi_{ij}}{\phi_0}, \quad \phi_{ij} = \int \vec{B}_{ij}\cdot d\vec{a}$$

$$p = \sqrt{2m\left(E - V(x)\right)}$$

WKB

$$\oint \vec{p}\cdot d\vec{x} = \overbrace{2\pi\hbar}^{h}\left(n+\tfrac{1}{2}\right)$$



$$\cancel{2}\int_1^2 p\,dx = \cancel{2}\pi\hbar\left(n+\tfrac{1}{2}\right)$$

try
$$V(x) = \begin{cases} \frac{1}{2}m\omega^2(x-a)^2 & x > a \\ 0 & |x| < a \\ \frac{1}{2}m\omega^2(x+a)^2 & x < a \end{cases}$$

$$E = \tfrac{1}{2}m\omega^2\left(x_{1,2} \mp a\right)^2$$

$$x_{1,2} \mp a = \pm\sqrt{\frac{2E}{m\omega^2}}$$ ← sign depends on side

$$\boxed{x_{1,2} = \pm\sqrt{\frac{2E}{m\omega^2}} \pm a}$$

$$p = \sqrt{2m(E-V)}$$

$$\pi\hbar\left(n+\tfrac{1}{2}\right) = \int_{x_2}^{-a}\left(2mE\left(1 - \frac{m\omega^2}{2E}(x+a)^2\right)\right)^{1/2}dx$$

$$+ \int_{-a}^{a}\left(2mE\right)^{1/2}dx$$

$$+ \int_{a}^{x_1}\left(2mE\left(1 - \frac{m\omega^2}{2E}(x-a)^2\right)\right)^{1/2}dx$$

right
left

$$y = \sqrt{\frac{m\omega^2}{2E}}(x \mp a)\ , \quad dy = \sqrt{\frac{m\omega^2}{2E}}\,dx \qquad \begin{matrix} y = 0,\ 1 \\ y = -1,\ 0 \end{matrix}$$

$$dx = \sqrt{\frac{2E}{m\omega^2}}$$

$$\pi\hbar(n+\tfrac{1}{2}) = \sqrt{2mE}\left(\int_{-1}^{0} \frac{\sqrt{\frac{2E}{m}}}{\omega} \sqrt{1-y^2}\,dy + 2a + \int_{0}^{1} \frac{\sqrt{\frac{2E}{m}}}{\omega} \sqrt{1-y^2}\,dy \right)$$

$$= \frac{2E}{\omega}\left[\int_{\underset{\pi}{-1}}^{\underset{0}{1}} \cos^2\theta\,d\theta \right] + 2a\sqrt{2mE}$$

$$\frac{2E}{\omega}\frac{\pi}{\cancel{2}} + 2a\sqrt{2mE} = \pi\hbar(n+\tfrac{1}{2})$$

for large a its just $\boxed{\left(\frac{\pi\hbar(n+\frac{1}{2})}{2a\sqrt{2m}}\right)^2 = E_n}$ = box with sm. +1/2 g.s. zero point

Small a is just $\boxed{E_n = \hbar\omega(n+\tfrac{1}{2})}$ = SHO

as expected

(4)

$$B = \vec{\nabla} \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{y} + \ldots$$

$$B = \hat{z}\frac{\partial A_y}{\partial x} = B\hat{z}$$

3D landau problem, choose gauge $\vec{A} = xB\hat{y}$

$$\hat{H} = \frac{(\vec{\Pi})^2}{2m} \left(\pm \vec{\mu}\cdot\vec{B}\right) \quad H\Psi = E\Psi \quad , \quad \Pi_y = \frac{\hbar}{i}\frac{\partial}{\partial y} - \frac{e}{c}x B$$

TISE: $$\left(\frac{\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2}{2m} + \frac{\left(\frac{\hbar}{i}\frac{\partial}{\partial y} - \frac{e}{c}xB\right)^2}{2m} + \frac{\left(\frac{\hbar}{i}\frac{\partial}{\partial z}\right)^2}{2m}\right)\Psi(\vec{r}) = E\,\Psi(\vec{r})$$

$$\vec{k} = k_y\hat{y} + k_x\hat{x} \quad , \quad \Psi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}_\perp}\phi(x)$$

$$\left(\frac{-\hbar^2\partial_x^2}{2m} + \frac{1}{2m}\left(\hbar k_y - \frac{exB}{c}\right)^2 + \frac{\hbar^2 k_z^2}{2m}\right)\phi(x) = E\,\phi(x)$$

$$E' = E - \frac{\hbar^2 k}{2m}$$

$$\left(\frac{-\hbar^2\partial_x^2}{2m} + \frac{1}{2m}\left(\hbar k_y - \frac{exB}{c}\right)^2\right)\phi(x) = E'\phi(x)$$

$$\left[\frac{-\hbar^2}{2m}\partial_x^2 + \frac{1}{2}m\omega^2\left(x - \bar{x}\right)^2\right]\phi(x) = E'\phi(x)$$

with $\omega = \frac{eB}{mc}$, $\bar{x} = \frac{\hbar k_y}{\omega m}$

So $E_n' = \hbar\omega(n + \frac{1}{2})$ by analogy with the SHO.

So $E_{n_z} = \hbar\omega(n + \frac{1}{2}) + \frac{\hbar^2 k_z^2}{2m} \quad (\pm \mu B)$

for a box of sidelength L

$$K_y = n_y \frac{2\pi}{L} \quad , \quad K_z = n_z \frac{2\pi}{L}$$

$$0 < |\bar{x}| < L \longrightarrow 0 < \frac{\hbar c}{eB} |K_y| < L$$

$$\text{so} \quad 0 < \frac{\hbar c}{eB} \frac{2\pi}{L} |n_y| < L$$

total degeneracy

$$0 < |n_y| < \frac{L^2 eB}{2\pi\hbar c} \leq \frac{\phi}{\phi_0} = N$$

$$\psi(x,y,z) = e^{i\vec{K}\cdot\vec{r}_\perp} H_n(x+\bar{x}_n) \cdot e^{-\frac{i}{\hbar}E_n t} \cdot e^{\frac{1}{2}\frac{(x+\bar{x})^2}{x_0^2}} , \quad x_0 = \frac{\hbar}{m\omega}$$

$$\frac{\hbar c}{e} = \phi_0$$

$$\text{BCH rule} \quad e^A e^B = e^B e^A e^{[A,B]}$$

$$e^a e^{a^\dagger} = e^{a^\dagger} e^a e^{\underset{1}{[a,a^\dagger]}}$$

Coherent States

$$\langle\lambda|\lambda\rangle = |\text{const}|^2 \langle 0| e^{\lambda^* a} e^{\lambda a^\dagger} |0\rangle$$

$$= |\phi|^2 \langle 0| \underset{1}{e^{\lambda a^\dagger}} \underset{1}{e^{\lambda^* a}} |0\rangle \, e^{\lambda^*\lambda \overset{1}{[a,a^\dagger]}}$$

$$1 = |\phi|^2 e^{|\lambda|^2}$$

$$\text{so} \quad |\phi| = e^{-\frac{|\lambda|^2}{2}} , \quad |\lambda\rangle = e^{-\frac{|\lambda|^2}{2}} e^{\lambda a^\dagger} |0\rangle$$

9

$$|\lambda\rangle = e^{-\frac{|\lambda|^2}{2}} \sum_{n=0}^{\infty} \frac{\lambda}{\sqrt{n!}} \frac{a^{+n}}{\sqrt{n!}} |0\rangle$$

$$= e^{-\frac{|\lambda|^2}{2}} \sum_{n=0}^{\infty} \frac{\lambda^n}{\sqrt{n!}} |n\rangle$$

for $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$ with $|n\rangle = \frac{a^{+n}}{\sqrt{n!}} |0\rangle$

Show this is coherent state : if $a|\alpha\rangle = \alpha|\alpha\rangle$

$$a|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} a \sum_{n=0}^{\infty} \frac{\alpha^n a^{+n}}{n!} |0\rangle$$

$$= e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \alpha \alpha^{n-1} \frac{a a^{+n}}{n(n-1)!} |0\rangle$$

$$a a^{+n} = a a^{+n} - a^{+n} a + a^{+n} a$$

$$= [a, a^{+n}] + a^{+n} a$$

$$a|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha \alpha^{n-1}}{n(n-1)!} \left([a, a^{+n}] + \underbrace{a^{+n} a}_{0} \right) |0\rangle$$

but $[a, a^{+n}] = \underbrace{[a, a^{+}]}_{1} a^{+(n-1)} + a^{+}[a, a^{+n-1}] + \dots$

$$= n\, a^{+(n-1)}$$

$$a|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum^{\infty} \frac{\alpha \alpha^{n-1}}{\cancel{n}(n-1)!} \cancel{n}\, a^{+(n-1)} |0\rangle = \alpha|\alpha\rangle$$

Same with $n \to n-1$ ✓

arbitrary spin direction matrix

$$|\hat{n};+\rangle = \cos(\beta/2)\,|+\rangle + e^{i\alpha}\sin(\beta/2)\,|-\rangle$$

$$|\hat{n};-\rangle = \sin(\beta/2)\,|+\rangle + e^{i\alpha}\cos(\beta/2)\,|-\rangle$$