

Qualifying Examination - Part I

Time: 9:30-11:30 a.m.

Saturday, February 21, 2004

The examination is to be written on the single sheets that are provided; no more than one question is to be answered on a single sheet. Number each question. You are to answer all questions in Part I; however, if you do omit any questions, *cross out those numbers on your title page*. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from your title page) on the back of each sheet of paper.

Part I counts one-third (1/3) of the final grade.

Part II is in this same room at 1:00 p.m.

PHYSICAL CONSTANTS

Planck's constant

$$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$$

Vacuum speed of light

$$c = 3.00 \times 10^8 \text{ m/sec}$$

$$\hbar c = 197 \text{ MeV}\cdot\text{fm} = 1.97 \times 10^{-5} \text{ eV}\cdot\text{cm}$$

Electron charge

$$e = 1.60 \times 10^{-19} \text{ C} = 4.80 \times 10^{-10} \text{ esu}$$

Boltzmann constant

$$k = 1.38 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-5} \text{ eV/K}$$

Gas constant

$$R = 8.31 \text{ J/(mol}\cdot\text{K)}$$

Gravitational constant

$$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

Permeability of free space

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

Permittivity of free space

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

Electron mass

$$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV}/c^2$$

Proton mass

$$m_p = 1.67 \times 10^{-27} \text{ kg} = 938 \text{ MeV}/c^2$$

Bohr radius of hydrogen

$$a_B = 5.3 \times 10^{-11} \text{ m}$$

Ionization energy of hydrogen

$$13.6 \text{ eV}$$

Avogadro's number

$$6.02 \times 10^{23} / \text{mole}$$

Conversion Factors

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ erg}$$

$$1 \text{ m} = 10^{10} \text{ \AA} = 10^{15} \text{ fm} = 6.25 \times 10^{-4} \text{ miles}$$

$$1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2 = 760 \text{ Torr}$$

$$1 \text{ cal} = 4.186 \text{ J}$$

Divergence and curl in spherical coordinates

$$\nabla \cdot E = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta E_\theta) + \frac{1}{r \sin \theta} \frac{\partial E_\phi}{\partial \phi}$$

$$\begin{aligned} \nabla \times E = & r \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta E_\phi) - \frac{\partial E_\theta}{\partial \phi} \right] + \theta \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial E_r}{\partial \phi} - \frac{\partial (r E_\phi)}{\partial r} \right] \\ & + \phi \frac{1}{r} \left[\frac{\partial (r E_\theta)}{\partial r} - \frac{\partial E_r}{\partial \theta} \right] \end{aligned}$$

where $\mathbf{r}, \boldsymbol{\theta}, \boldsymbol{\phi}$ are the unit vectors associated with the spherical coordinates r, θ, ϕ .

Useful integrals

$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{2\alpha}$$

$$\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{\alpha^3}}$$

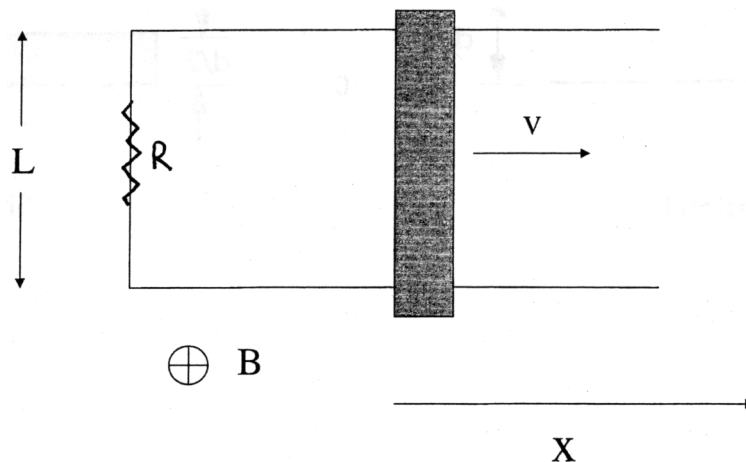
$$\int_0^x \sin^2 x \, dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{(1-x)}; \quad -\sum_{n=1}^{\infty} (-)^n \frac{x^n}{n} = \ln(1+x)$$

1. A thin rope has an overall length of 10 m and a total mass of 2 kg. The rope is stretched with a tension of 50 N. One end of the rope is fixed, and the other is moved up and down with a frequency of 4 Hz.
 - a. What is the speed of longitudinal waves on this rope? Assume that the effects of gravity can be neglected, and that the rope only undergoes small oscillations.
 - b. What is the wavelength for the frequency of 4 Hz?

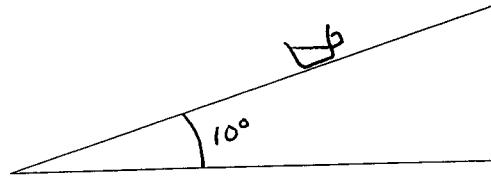
2. A cyclotron for accelerating protons has a magnetic field of 1 T and a maximum radius of 0.5 meters.
 - a. What is the frequency of microwave power needed to accelerate protons?
 - b. What is the energy (in eV) of the protons when they emerge?

3.

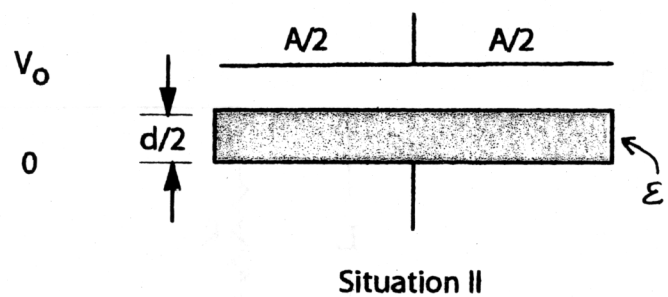
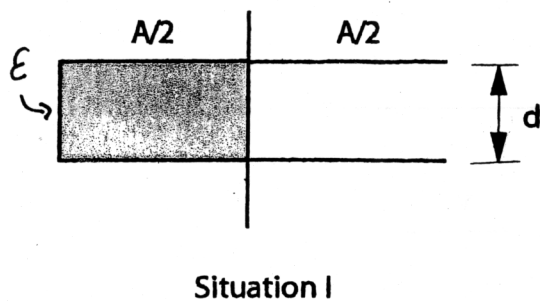


A conducting bar of mass m slides without friction on two conducting rails. There is a uniform magnetic field of strength B pointing in the direction perpendicular to the plane of the loop. The bar starts with a velocity v_0 . What is the velocity of the bar as a function of the time?

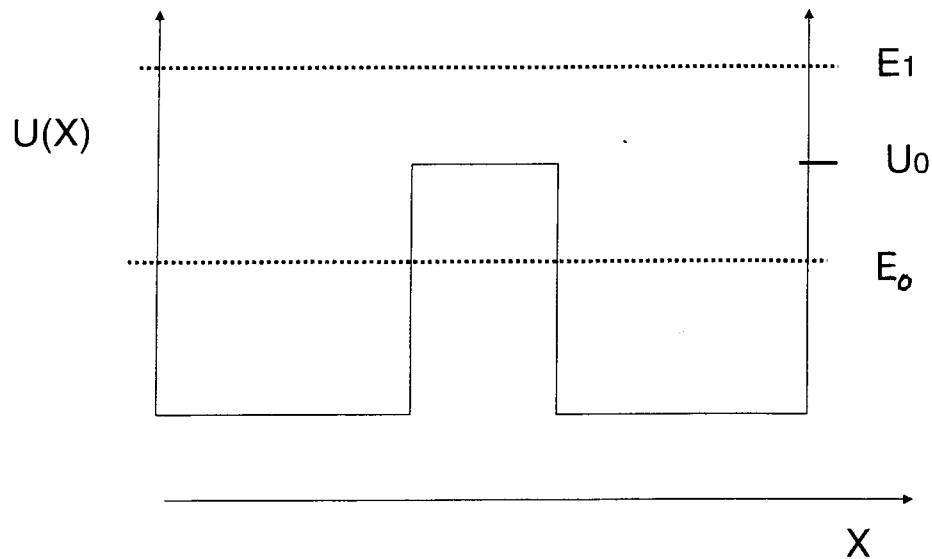
4. An empty cup, mass 0.1 kg sits on an inclined plane that makes an angle of $\theta = 10^\circ$ with the horizontal. The coefficient of static friction is $\mu_s = 0.20$.



- Will the cup slide?
 - What is the frictional force on the cup?
 - The cup is held still while it is filled with 0.20 kg of coffee. When it is released, will it now slide?
5. Two identical parallel plate capacitors (area = A ; separation = d) are each half filled with insulating material with dielectric constant $\epsilon = 2$. Both are charged to the same potential V_0 . Which stores the larger energy? Why?



6. The potential $U(x)$ in the figure approximates the potential of a “one-dimensional” H_2^+ ion. The potential is infinite for $x < 0$ and for $x > L$. A particle of mass m is confined in this potential well. The dotted lines indicate the energies E_0 and E_1 of the two lowest eigenstates. The height U_0 of the central barrier satisfies $E_0 < U_0 < E_1$.



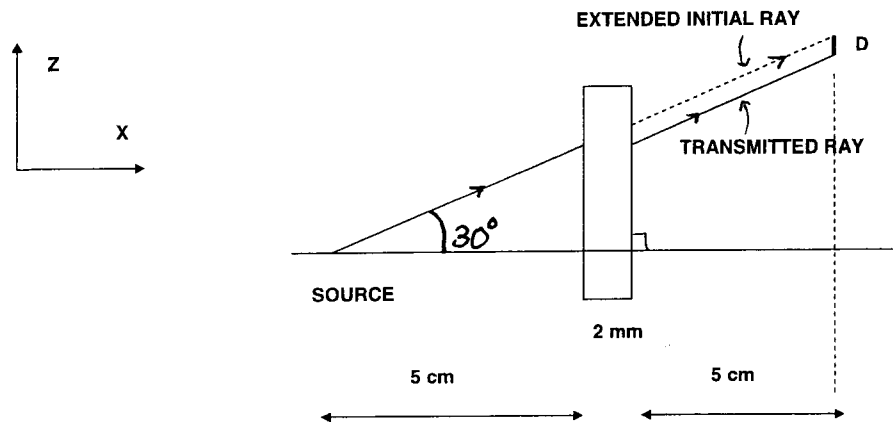
Give sketches of the wave functions of the two lowest states, paying particular attention to the local curvature. Indicate which wave function is the ground state, and which is the first excited state.

7. The electron in a hydrogen atom is in the $n = 3$ state. What is the maximum wavelength (in units of nm) of the radiation that can be absorbed by this atom? (Neglect the recoil of the atom.)
8. Two gas cylinders A and B each hold 100 cm^3 of an ideal gas at a temperature of 30°C and a pressure of 1 atm. Cylinder A undergoes an isobaric expansion to 300 cm^3 . Cylinder B undergoes an isothermal expansion that does the same work as cylinder A.

Calculate the final volume, temperature, and pressure of each cylinder.

9. A series RLC circuit has an input voltage $V_{\text{rms}} = 120 \text{ V}$ at a frequency of 60 Hz and contains a resistance $R = 200.0 \, \Omega$, an inductance with $X_L = 80.0 \, \Omega$ and a capacitance with $X_C = 120.0 \, \Omega$.
- What is the phase difference between the current and the voltage?
 - What is the average rate at which energy is dissipated in the circuit?

10. A light ray hits a slab of glass (refractive index = 1.5) from air as shown. Calculate the difference in vertical height D at 5.0 cm after the glass between the actual ray and the extended incident ray.



Qualifying Examination - Part II

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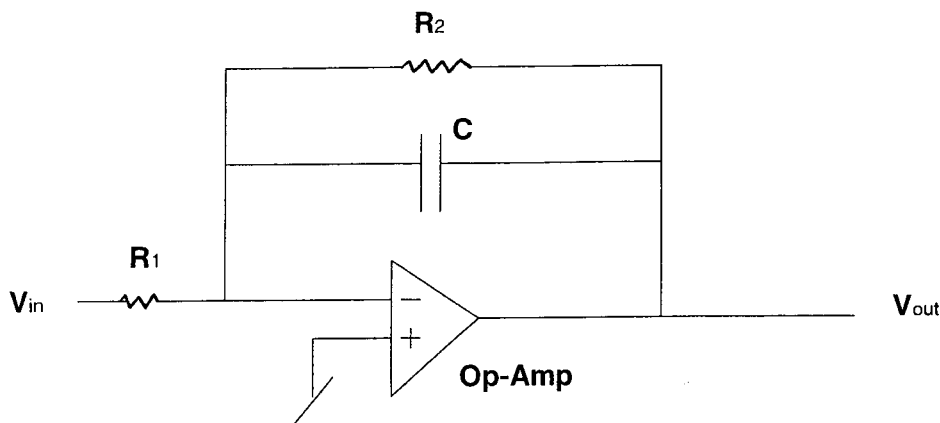
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$$\sum_{n=0}^{\infty} x^n = \frac{1}{(1-x)}; \quad -\sum_{n=1}^{\infty} (-)^n \frac{x^n}{n} = \ln(1+x)$$

$$\int_0^{2\pi} \cos n\theta \cos m\theta \, d\theta = \begin{cases} \pi, & \text{if } m = n \\ 0, & \text{if } m \neq n \end{cases}$$

$$\int_0^{2\pi} \cos n\theta \sin m\theta \, d\theta = 0$$

11.



Assume that the op-amp in the circuit shown above is ideal.

- a. Derive an expression for $V_{\text{out}}/V_{\text{in}}$ as a function of the frequency ω of the input signal.
 - b. Make a rough sketch of $\ln(V_{\text{out}}/V_{\text{in}})$ vs. $\ln \omega$.
12. a. In a helium atom, which dipole selection rules are violated by the transition $1s2s\ ^1S_0 \rightarrow 1s8s\ ^1S_0$?
- b. Is this transition more likely to be observed if the atoms are in an electric field of 2kV/cm or a magnetic field of 0.2 T? Explain.
13. A space vehicle in a near earth orbit is about to embark on a trip to Mars. An impulsive thrust inserts the vehicle into an elliptical orbit about the sun with its perigee at the earth's orbit and its apogee at Mars' orbit. The earth and Mars both have nearly circular orbits, with diameters of 2AU and 3AU, respectively.
- Estimate the time (in days) for the space vehicle to travel from earth orbit to Mars orbit if no further thrust is used.
14. Photons and neutrinos are emitted at the same time in an astrophysical burst of short time duration. The source is at distance 10^9 light years, the neutrino energy is 10^{11} eV, and the neutrino rest mass is assumed to be 1 eV.

Determine the expected time delay of the neutrino relative to the photon arrival time.

15. A thin disk floats at rest in outer space. It has $R=0.1$ m and the mass $m = 0.25$ kg.

a. An impulsive force $J\delta(t)$ is applied at the edge of the disk and directed perpendicular to the plane of the disk.

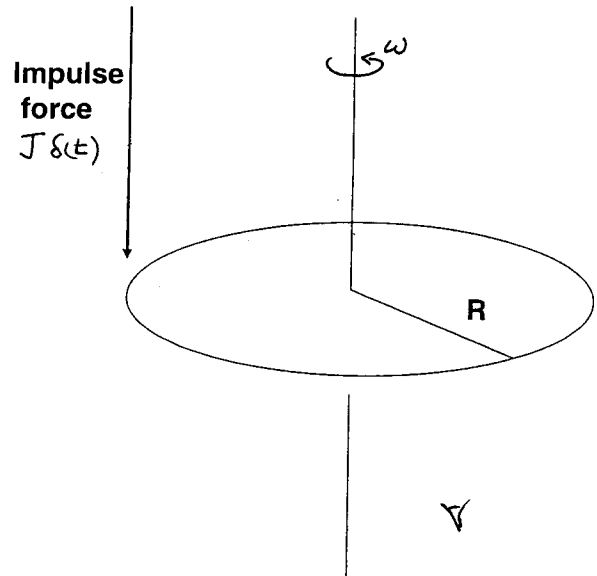
1) What impulse J is needed to make the disk rotate at one revolution per second?

2) What is the velocity of the center of mass?

b. Now suppose the disk is initially rotating with angular velocity ω about an axis (the "symmetry axis") through its center and perpendicular to the plane of the disk. The same impulse (direction, magnitude, and point of application) is applied to the disk.

What is the minimum value of ω so that only one side of the disk is ever visible from below? [See the sketch.]

The moments of inertia of the disk about a diameter and about the symmetry axis are $mR^2/4$ and $mR^2/2$, respectively.



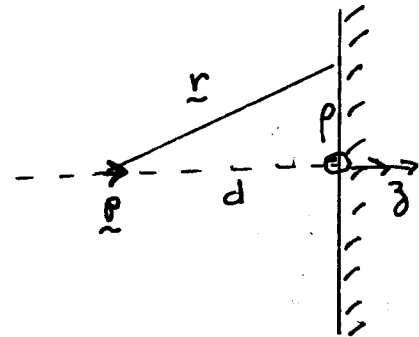
16. An electric dipole \vec{p} points normally toward a semi-infinite conducting plane as shown in the sketch and is located a distance d from it. The electrostatic potential Φ and electric field \vec{E} of a dipole \vec{p} in free space are

$$\Phi = \vec{p} \cdot \vec{r} / 4\pi\epsilon_0 r^2$$

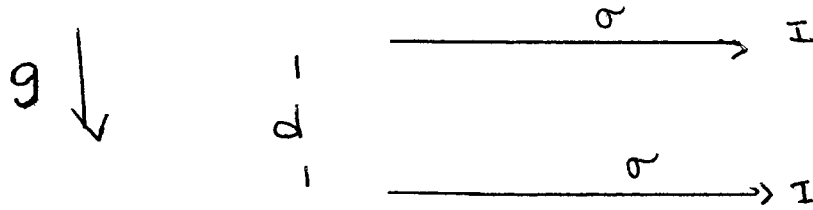
$$\vec{E} = [3(\vec{p} \cdot \vec{r}) \vec{r} - r^2 \vec{p}] / 4\pi\epsilon_0 r^3$$

a. Find the "image" dipole which keeps the $z = 0$ plane grounded.

b. Find the charge density on the plane.



17. Two long straight parallel wires are oriented horizontally one above the other in a uniform gravitational field. Each carries an electric current I in the same direction and has mass per unit length σ . The upper wire is fixed.



- For what separation will the lower wire be suspended force free?
 - Is the system stable with respect to small vertical perturbations of the lower wire? Assume the currents remain constant, the upper wire does not move, and the wires remain straight.
 - If the system is stable, calculate the frequency of small oscillations about the equilibrium. If not, calculate the growth rate of the perturbation.
18. In a tritium β -decay experiment ${}^3\text{H} \rightarrow {}^3\text{He}^+ + e^- + \nu_e$, the electron energy (E_e) is measured. The decay is in the ${}^3\text{H}$ -rest frame and the masses of ${}^3\text{H}$, ${}^3\text{He}$ and e^- are all known.

Assume that the neutrino is massless $m_\nu = 0$, calculate the maximum energy for the electron E_e^{max} in terms of the masses. (You may choose to take the natural units with speed of light $c = 1$.)

19. Assume that there are two flavors of neutrinos, the electron neutrino $|\nu_e\rangle$ and the muon neutrino $|\nu_\mu\rangle$. They are mixtures of the energy eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$. The energies are $m_1 c^2$ and $m_2 c^2$, where m_1 and m_2 are the masses of $|\nu_1\rangle$ and $|\nu_2\rangle$. Assume maximal mixing, i.e.,

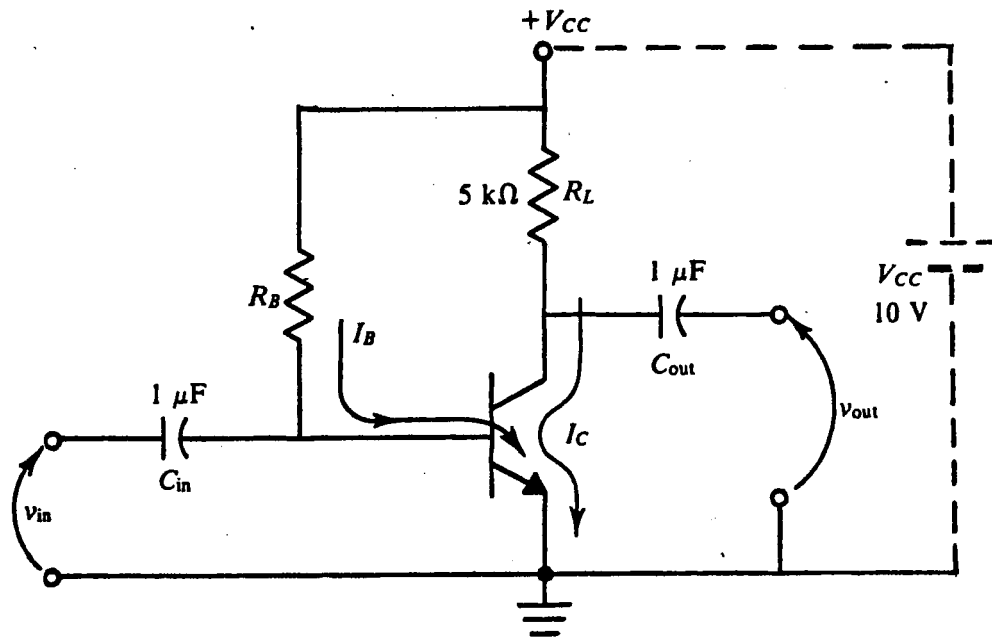
$$|\nu_e\rangle = \frac{1}{\sqrt{2}} (|\nu_1\rangle + |\nu_2\rangle)$$

and

$$|\nu_\mu\rangle = \frac{1}{\sqrt{2}} (|\nu_1\rangle - |\nu_2\rangle)$$

An electron neutrino starts at the sun with speed v and travels a distance R to the earth. What is the probability that it is observed to be a muon neutrino at the earth? Express your answer in terms of v , c , R , m_1 and m_2 .

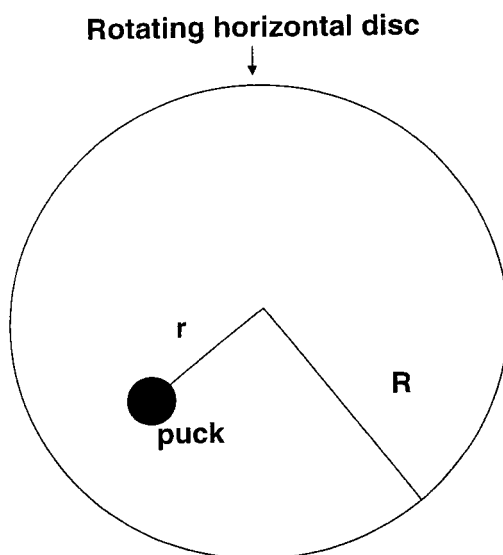
20.



In the above circuit, $V_{CC} = 10 \text{ V}$, $V_{RL} = 5 \text{ V}$, $R_L = 5 \text{ k}\Omega$, the transistor β is 100 and $r_{BE} = 1 \text{ k}\Omega$.

- Find I_C .
- Find I_B .
- Find R_B .
- Find the voltage amplification at mid-frequencies where the capacitive reactances are negligible. (Neglect transresistance.)

21. Hockey puck on merry-go-round.



The coefficient of static friction between the hockey puck and the horizontal surface is μ_s .

- What is the largest radius r from the center of the merry-go-round that the hockey puck can be placed without sliding?
- Let the puck break free at this radius and begin to slide. Assume no friction after the sliding begins.
 - Sketch the path of the puck as seen in the non-rotating lab frame.
 - Through what angle does the merry-go-round rotate while the puck is sliding off?

22. Consider a system at temperature T with only two energy levels, of energy $-E_0$ and $+E_0$.

- What is the mean energy in the limit $T \rightarrow 0$? [no calculation necessary]
- What is the mean energy in the limit $T \rightarrow \infty$? [no calculation necessary]
- Calculate the mean energy \overline{E} of the system in terms of E_0 and T .
- From your calculation, what temperature is required for $\overline{E} = +E_0$? Explain this result.

23. A charged quantum-mechanical particle of mass M is confined to move on a one-dimensional ring of radius R . The Schrodinger equation is

$$-\frac{\hbar^2}{2MR^2} \frac{d^2\psi(\theta)}{d\theta^2} = E\psi(\theta)$$

- Find the eigenstates and their energies.
- An electric field corresponding to a potential energy $V_0 \cos 2\theta$ is applied to the ring. Find the corrections to the energies of 3 lowest-lying states.

24. An optical (visible light) collimator is constructed using a slit 10 mm high and 1 mm wide with a good quality lens of focal length 1.0 m and of diameter 10 cm.

- Find the horizontal angular divergence of the collimated light.
- Find the vertical angular divergence of the collimated light.
- Answer (a) and (b) again for a slit width of $5 \mu\text{m}$ and a wavelength of $0.5 \mu\text{m}$.

25. Two FM Radio stations broadcast at the same signal strength from the same nearby location. The first broadcasts music you like at 91.3 MHz, the second broadcasts music you hate at 91.1 MHz. You decide to construct an RLC series circuit to select the music you like, using a fixed inductor of $1.00 \mu\text{H}$ and an adjustable capacitance and resistance to limit the power from the station you hate to 1% of that of the station you like.

- What value of capacitance do you choose?
- What is the maximum resistance you can use?