Part I 7-Minute Questions

1. A tennis ball of mass m is held just above a basketball of mass $M \gg m$ and radius R. The bottom of the basketball is held a height h above the ground. With their centers vertically aligned, both are released from rest at the same time. Assume that when the basketball reaches the ground, the collision with the ground instantaneously reverses the velocity of the basketball while the tennis ball is still moving down. Immediately after this, the two balls meet in an elastic collision. To what height does the tennis ball rebound?

2. A copper wire has length L, mass M, cross sectional area A, and Young's modulus Y. The wire is placed under tension by elastically stretching it by a small amount to a length $L + \Delta L$. The wire (anchored at both ends) supports a standing transverse wave whose fundamental frequency is f. Find a formula for ΔL.

3. On cold, dark fall nights, why does frost preferentially form on the horizontal surfaces of cars and not on their vertical surfaces?

4. Given two light sources, red and green, and a thin layer of oil on top of a puddle on the ground, what experiment can a color-blind person do to tell which source is which?

5. A rigid rotor consists of two particles of mass m attached to the ends of a mass-less rigid rod of length a. The rod is free to rotate in three dimensions about its center, but the center point is fixed in space.

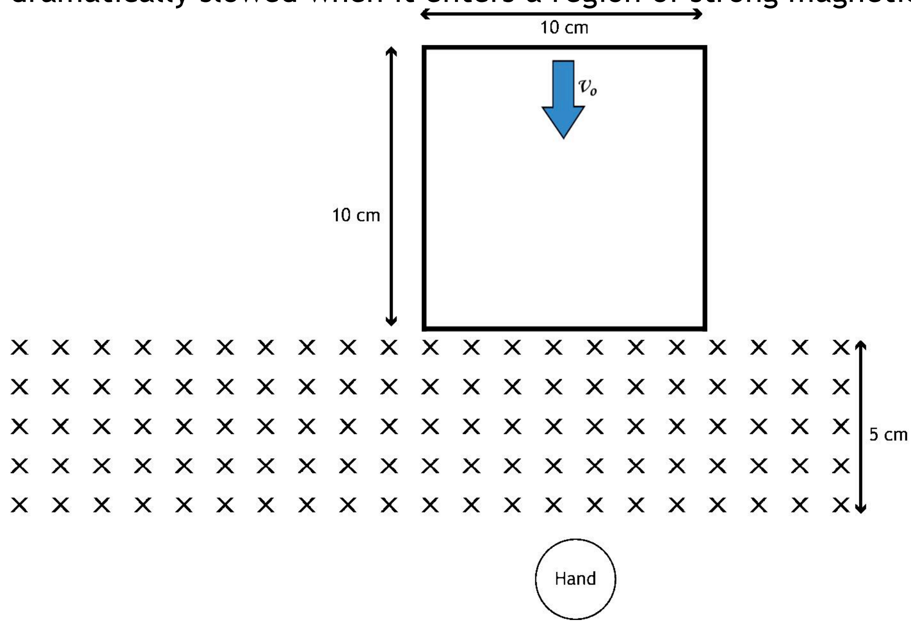
   a. Find the quantum mechanically allowed energy levels of this rigid rotor.
   b. What are the degeneracies of the energy levels?

6. A beam of particles of mass μ and momentum $p = \hbar k$ traveling along the x-axis approaches from the left $(x < 0)$ a potential

$$V(x) = \begin{cases} \lambda\delta(x+a) & x \le 0 \\ \infty & x > 0 \end{cases}$$

   where $a > 0$ and $\lambda > 0$. Write down the form of the wave function in the regions $x<-a$ and $-a<x<0$. Write down the boundary conditions relating the wave functions in the two regions.

7. A physics demonstration involves an “electromagnetic guillotine,” in which a conducting guillotine blade dropping toward the professor’s hand is dramatically slowed when it enters a region of strong magnetic field.

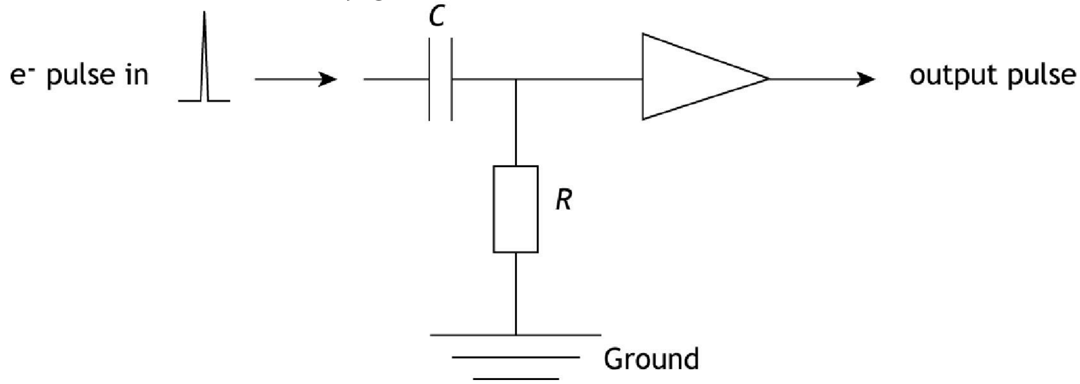


Model the blade as a square conducting ring of wire $\ell = 10cm$ on a side. It has a total resistance of $R = 0.002\Omega$, and a mass of 0.1 kg. The 2 T magnetic field is uniform in a region from the professor’s hand to 5 cm above his hand, and zero everywhere else. Find the velocity of the blade when it reaches the professor’s hand. Assume that the blade reaches terminal velocity.

8. A thin, uniformly charged disc has radius *R* and charge density σ. Find the electric field *E* at a point *A* located along the axis of the disc, at a distance *h* from its center.

9. Consider the reaction $^{235}_{92}U + n \rightarrow {}^{141}_{55}Cs + {}^{93}_{37}Rb + ?\,n$.
   a. If “?n” represents the number of neutrons, what is it?
   b. Given the binding energy per nucleon of ^{235}U, ^{141}Cs and ^{93}Rb are 7.6 MeV, 8.35 MeV, and 8.7 MeV, *estimate* the total energy released in this reaction.
   c. Given the atomic masses of ^{235}U, ^{141}Cs and ^{93}Rb are 235.043923 u, 140.920044 u, and 92.922033 u, what is the *precise* total energy released?

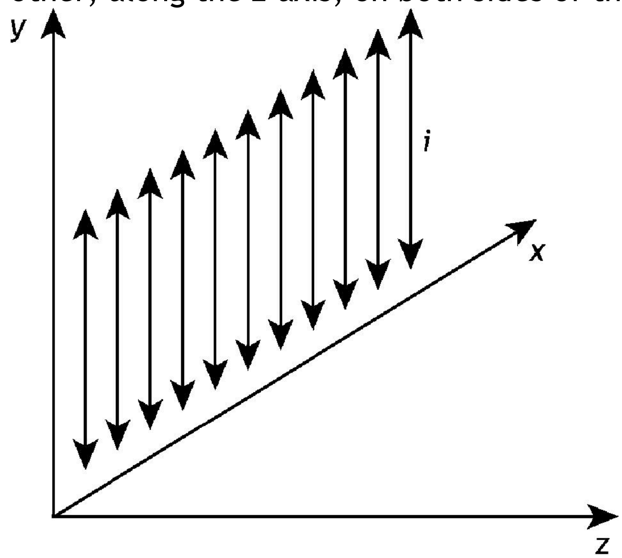
10. The following combination of a capacitor $C = 1\ nF$, resistor $R = 10\ k\Omega$, and an amplifier (triangle) is used to detect pulses of electrons. The amplifier draws no current and has unity gain.



If the input current pulse has the shape of a δ-function (very narrow pulse) but a total charge $Q=1\ nC$, find the output voltage as a function of the time t. Make a sketch of your result.

Part II 12-Minute Questions

11. A sheet of electric current oscillates in the *xy* plane as shown. The current density *i* (Amperes/m) oscillates between the +*y* direction and the –*y* direction at some frequency. Corresponding plane waves propagate, one way or the other, along the *z* axis, on both sides of the xy plane.



   a. If the current density and the electric field at $z=0$ are in phase, what is the direction of the vector $\vec{E}\times\vec{H}$ just to the +*z* side of the current sheet?
   b. Still with the electric field in phase with the current, what is the direction of $\vec{E}\times\vec{H}$ on the -*z* side of the sheet?
   c. Now, instead, if the electric field and the current are 180^0 out of phase, what is the direction of $\vec{E}\times\vec{H}$ for $z=0+$?
   d. Still with the field and the current of opposite phase, what is the direction of $\vec{E}\times\vec{H}$ for $z=0-$?
   e. Explain these results in terms of energy transfer.

12. A point-like electric dipole $\bar{p}$ is oriented normal to and at distance *d* from an infinite conducting plane. The plane is grounded (i.e., at zero potential). Calculate the magnitude and direction of the force exerted on the plane by the dipole in terms of *p* (i.e., $|\,\bar{p}\,|$), *d*, and any other fundamental constants.

13. A billiard ball is a homogeneous sphere of mass *M* and radius *R*. Find the height at which a billiard ball sitting at rest should be struck horizontally to roll with no initial slipping.

14. A mass M is hung from a mass-less spring (with spring constant K). The spring is oriented parallel to gravity, g. If at time t=0 the mass is at the spring's un-stretched position x=0 and has zero velocity, find its subsequent position as a function of time. Find a conserved quantity for the system. If now a very small amount of friction force $F = -\frac{\gamma}{M}\frac{dx}{dt}$ is present, again find the position as a function of time.

15. A police speed radar gun is basically a radio emitter and receiver. To measure the velocity u of an object that moves away from the radar gun, the gun sends out a radio signal at a frequency f_0 towards the moving object and measures the frequency f_1 of the reflected radiation received back at the source. The speed of the moving object can be determined from the frequency shift $\Delta f = f_1 - f_0$.
    a. Find a relativistically correct formula for the frequency shift.
    b. How large is the frequency shift for typical values f=10 GHz and u=120 km/h? Describe briefly how one could measure such a small frequency shift.

16. Suppose that a particle of mass m in a one-dimensional box of size L is in its ground state. The walls of the box are at $x = 0$ and at $x = L$. Suppose at $t = 0$, the wall at $x = L$ moves to $x = 2L$ *instantaneously*. What is the expectation value of energy as a function of time after $t = 0$?

17. The Hamiltonian of a spin S= 1/2 in a magnetic field B is

$$\hat{H} = \frac{1}{2}\mu\beta\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

At t=0 the spin is described by the wave function

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Write down the wave function $|\psi(t)\rangle$ of the spin at arbitrary time t and calculate the expectation values $\langle \hat{S}_i(t) \rangle$ of the spin projection operators $\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$; $\hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

18. The Hamiltonian for an electron in an atom, and also for a nucleon in a nucleus, includes a "spin-orbit" term proportional to the operator $\bar{l} \cdot \bar{s}$ where $\bar{l}$ is the orbital angular momentum operator and $\bar{s}$ is the spin operator. Of course, the total angular momentum operator $\bar{j} = \bar{l} + \bar{s}$ is conserved.
    a. What are the possible values of the orbital quantum number l?
    b. What are the possible values of the spin quantum number s?
    c. For a given quantum number l, what are the possible values of the quantum number j?
    d. Find a formula for the value of the operator $\bar{l} \cdot \bar{s}$ in terms of the quantum numbers l and j.
    e. List all the values of $\bar{l} \cdot \bar{s}$ for an *S* state, for a *P* state, and for a *D* state.

19. Suppose that you need to estimate the energy of full ionization of Lithium from the ground state ($Li \rightarrow Li^{3+}$). The only information you have is the energy of the first ionization of Helium from the ground state ($He \rightarrow He^{1+}$), which is $E \simeq 24.5 eV$. Explain how the estimate can be done and perform the computation with the accuracy within 10 percent.

20. As an extreme example of the greenhouse effect, assume a layer of gas in the atmosphere that is energetically affected only by radiation and is entirely transparent at the visible and near infrared wavelengths radiated by the Sun, but totally absorbing at the mid- to far-infrared wavelengths radiated by the Earth. By what factor will the absolute temperature of the Earth be increased, assuming the Earth's atmosphere is normally totally transparent?

21. Consider a non-interacting gas of spin zero bosons, whose energy-momentum relationship is given by $\varepsilon(\mathbf{p}) = A|\mathbf{p}|^s$, for some fixed positive numbers *A* and *s*. The dimensionality of the gas is the number *d*, i.e. the "volume" of the gas is $V=L^d$, for a system of length *L*. In the thermodynamic limit of $V \rightarrow \infty$, for what values of *s* and *d* can there exist Bose-Einstein condensation at sufficiently low temperatures?

22. A broken thermometer leaves a spherical droplet of mercury of radius 2 mm. Its density is 13.6 gm/cm^3 and the atomic weight is 201 gm/mole. The vapor pressure of Hg at 300 K is 0.3 Pascal (= 0.3 $Newton/m^2$). Assume that a mercury atom incident on the droplet sticks to it with probability equal to 1, and that the mercury pressure in the room remains at the vapor pressure. Estimate the time for the droplet to evaporate.

23. In many light-atom interaction experiments, researchers ignore the magnetic field of the light and assume that the dominant interaction is due to the electric field of light. In this problem, you will verify the validity of this assumption. We will assume that a plane electromagnetic wave incident on a hydrogen atom and calculate the electric and magnetic forces on the electron. Numerical answers are required for each part. Ignore vector effects.
    a. Model the hydrogen atom classically as an electron revolving around the nucleus at a distance of one Bohr radius away. Calculate the velocity (v) of the electron by equating the Coulomb forces between the electron and the nucleus to the centrifugal force.
    b. Consider a plane wave propagating in the $\hat{z}$ direction with electric and magnetic fields of the form $E(t) = \hat{x}E_0 \cos(\omega t - kz)$ and $B(t) = \hat{y}B_0 \cos(\omega t - kz)$ respectively. Assume that the wave has a time-averaged power flow per unit area of P/A = 1 Watt/cm^2. Find the values of E_0 and B_0 in SI units.
    c. Use your result in part b to find the values of the electric force $F_e = eE_0$ and magnetic force $F_m = evB_0$ on the electron. What is the ratio of the two quantities F_e/F_m?

24. Neglecting gravitational metric fluctuations of the standard cosmological homogeneous and isotropic metric and neglecting inhomogeneities of the dark matter, the collisionless Boltzmann equation (with k_B=1) governing the phase space density $f(t,\vec{p})$ of non-relativistic dark matter is given by

$$\frac{\partial f(t,\vec{p})}{\partial t} - \frac{|\vec{p}|}{a}\frac{da}{dt}\frac{\partial f(t,\vec{p})}{\partial |\vec{p}|} = 0$$

where $\vec{p}$ is the 3-momentum and $a(t)$ is the scale factor of the universe. Assuming a quasi-thermal distribution of the form

$$f \ \alpha \ \exp\left[-\frac{|\vec{p}|^2}{2mT(t)}\right] \quad ,$$

where $T(t)$ is a time dependent temperature and m is the mass of the dark matter particles, the temperature can be computed using the Boltzmann equation to satisfy a scaling law $T \propto a^n$. Find the number n.

25. The technique of Johnson Noise Thermometry uses Johnson noise from a resistor as a means for measuring absolute temperature. It relies on the fact that the variance σ_V^2 of the voltage fluctuations across a resistor of resistance R and temperature T is $\sigma_V^2 = 4k_B TR\Delta\nu$, where k_B is Boltzmann's constant and $\Delta\nu$ is the bandwidth of the measurement. Suppose you want to measure the Johnson noise from a resistor using an amplifier with voltage noise σ_{Vn} and current noise σ_{In} both measured at the input of the amplifier. In order to minimize the effect of noise from the amplifier on your Johnson noise measurement, what value of R should you choose?

26. The neutral pion (rest mass m) can decay into two photons. A pion moving with velocity v along the z -axis decays in flight and emits one of the photons at an angle θ relative to the z -axis.
    a. Write the Lorentz transformations that relate the photon momenta p and p' in the laboratory and rest frames, respectively.
    b. Write the relationship between θ and θ'.
    c. For what decay angle θ' is the angle between the two photons in the lab frame the smallest? Show your calculation or reasoning. Write the expression for this angle in the lab frame.