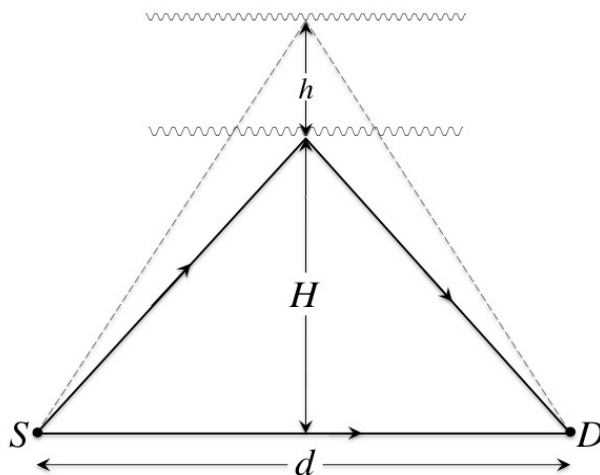


## Physics Qualifying Examination – Part I

## 7-Minute Questions

September 12, 2015

1. A moving kaon decays into two pions, one of which is left at rest. ( $m_K \approx 500 \text{ MeV}$ ,  $m_\pi = 140 \text{ MeV}$ ).
  - a. What is the total energy of the kaon?
  - b. What is the kinetic energy of the other pion?
2. A uniform spherical ball of mass  $m$  and radius  $R$  is initially spinning with an angular velocity  $\omega$ . The rotation axis is horizontal. The ball is carefully lowered onto a perfectly horizontal surface so that it does not bounce. When the sphere contacts the surface (at  $t = 0$  seconds) it is released and friction between the ball and the surface cause it to begin moving. The coefficients of kinetic and static friction are  $\mu$  and  $2\mu$  respectively. How far will the ball travel before it starts rolling?
3. A series RLC circuit has  $L=2.0 \text{ H}$ ,  $C=2.0 \mu\text{F}$  and  $R = 20 \Omega$ . It is driven with an ideal generator that has a peak emf of  $100 \text{ V}$  and variable frequency. Find:
  - a. the resonance frequency
  - b. the Q-factor
  - c. the width of the resonance
  - d. the maximum current at resonance
4. Calculate the capacitance between two concentric conducting spheres of radii  $R_1 = R_2 - \varepsilon$  where  $\varepsilon$  is small, when the left half of the space between is empty and the right half has an insulator of dielectric constant  $k$ .
5. A point source  $S$  and a detector  $D$  of high-frequency waves are a distance  $d$  apart on the ground. The direct wave from  $S$  is found to be in phase at  $D$  with the wave from  $S$  that is reflected from a horizontal layer at an altitude  $H$ . The incident and reflected rays make the same angle with the reflecting layer. When the layer rises a distance  $h < H$  a minimum signal is detected at  $D$ . Neglect absorption in the atmosphere and find the relation between  $d$ ,  $h$ ,  $H$ , and the wavelength  $\lambda \ll d$  of the waves.



6. Use Fermat's Principle (Principle of Least Time) to prove the law of refraction.
  
7. Very small solid particles, called grains, exist in interstellar space. They are continually bombarded by hydrogen atoms of the surrounding interstellar gas. As a result of these collisions, the grains execute Brownian movement in both translation and rotation. Assume that each grain is a sphere of diameter  $4.0 \times 10^{-6}$  cm with a uniform mass density  $1.0 \text{ g/cm}^3$ , and that the temperature of the gas is 100 K. Find
  - a. the root-mean-square speed of the grains between collisions and
  - b. the approximate rate (rev/s) at which the grains are spinning.
  
8. a. Show that when a substance of mass  $m$  having a constant specific heat  $C$  is heated from  $T_1$  to  $T_2$  the entropy change is
 
$$S_2 - S_1 = mC \ln \frac{T_2}{T_1}.$$
  - b. Does the entropy of the substance decrease on cooling?
  - c. If so, does the total entropy of the universe decrease in such a process?
  
9. What microwave frequency is required to excite a H atom from the 50s to the 51p state? Estimate the electric field required to make the transition in 1 msec.
  
10. A water tank of cross section  $A$  has at its bottom a small opening of area  $B$ . If initially the water level in the tank is  $H$  above the bottom, how long does it take to drain all the water out?

**Physics Qualifying Examination – Part II****12-Minute Questions****September 12, 2015**

1. Consider a spherical mass distribution with a radial density profile:

$$\rho(r) = \begin{cases} \rho_0 \frac{R^2}{r^2} & \text{if } r \leq R \\ 0 & \text{if } r > R \end{cases}$$

- How much mass,  $M(r)$ , is enclosed within a radius  $r$ ?
  - How does the average density within  $R$  compare to  $\rho_0$ ?
  - For which values of  $\rho_0$  and  $R$  would you expect a black hole to be implied? (Hint: Escape speed?)
  - Given that Newtonian gravity is fully described by the Poisson equation  $\nabla^2 \Phi = 4\pi G \rho(\mathbf{x})$ , formally derive the acceleration  $-\nabla \Phi$  for this mass distribution using the divergence theorem.
  - What is the velocity of an initially stationary test particle, released at  $R$ , when it arrives at  $r = R/10$ ?
2. A Penning trap confines a particle of charge  $q$  and mass  $m$  in a vacuum region with constant uniform magnetic field  $\mathbf{B} = B\hat{z}$  and electrostatic potential

$$\phi(\mathbf{x}) = \frac{1}{4}k(2z^2 - x^2 - y^2)$$

where  $k$  is a constant.

- a. Choose the vector potential  $\mathbf{A} = -(1/2)(\mathbf{x} \times \mathbf{B})$  and express the Lagrangian

$$L = \frac{1}{2}m\mathbf{v}^2 - q\phi + q\mathbf{v} \cdot \mathbf{A}$$

in Cartesian coordinates. (Here  $\mathbf{v} = \dot{\mathbf{x}}$  is the velocity.)

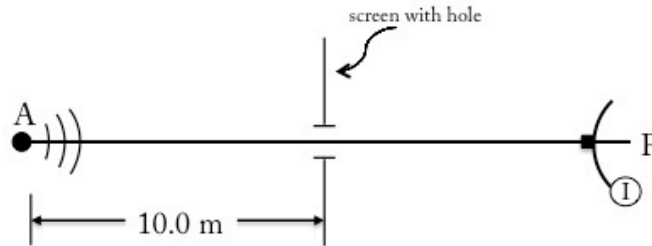
- Find the equation of motion.
- Show that the equations for  $x$  and  $y$  may be written in the form

$$\ddot{u} + i\omega_c \dot{u} - \frac{1}{2}\omega_k^2 u = 0$$

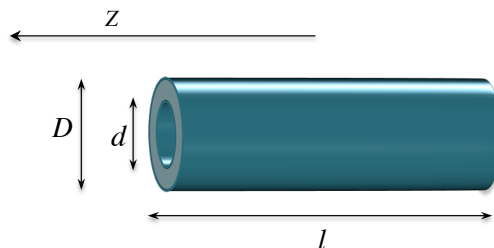
where  $u(t) = x(t) + iy(t)$ . Give an expression for the cyclotron frequency  $\omega_c$  and try solutions of the form  $u(t) = R_\omega e^{-i\omega t}$  to derive the general solution for  $u(t)$ .

- d. For what values of  $k$ ,  $q$ ,  $m$ , and  $B$  is the general motion bounded?

3. A point source of light ( $\lambda = 600 \text{ nm}$ ) is located at point A, 10 m from an opaque screen with a small circular hole of radius  $b$ . A very small photodiode is moved on axis from distant point P toward the screen. The first observation of maximum signal occurs at a distance 10.2 m.



- Calculate the radius  $b$  of the hole.
  - If the photodiode current is  $10 \mu\text{A}$ , what will this current be if the screen is removed?
4. Evaluate the momentum matrix element  $\langle 1 | \hat{p} | 2 \rangle$  for a particle moving in a 1-dimensional infinite square well from  $[0, L]$ .
5. Explain how you could use a voltage source of finite size (say 20 Volts) to construct a practical, constant-current source that will supply a load resistor,  $R$ . Specify what current,  $I$ , your device is designed to deliver. Estimate the approximate range of  $R$  over which the current will be constant. You may use as needed auxiliary batteries, transistors, transformers, op amps, capacitors, etc. (There is no single correct answer, but your constant-current source should be practical.)
6. A transmitter on a beach sends radio waves out to sea with a frequency  $\nu = 10^8 \text{ s}^{-1}$ . Find the frequency of the back-scattered radiation. The relation between angular frequency and wave number of water waves is  $\omega = \sqrt{gk}$ .
7. A uniformly magnetized ( $M = M\hat{z}$ ) extruded ring, as shown, has inner diameter  $d$  and outer diameter  $D$ , with thickness  $l$ . Calculate the magnetic field at the position of the center of the ring. Use Ampere's law to justify your result in the limit  $l \gg d, D$ .



8. Use dimensional analysis to determine how  $\hbar$ ,  $c$ , and  $k_B$  enter in the Stefan-Boltzmann radiation law.
9. The sun produces mostly neutrinos while nuclear reactors produce antineutrinos. In detecting the two types of (anti)neutrinos, for which would you use chlorine ( $^{37}\text{Cl}$ ) or water? Give the reactions detected in each case.
10. The single qubit Pauli group comprises the following four matrices:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The eigenstates of Pauli  $Z$  operator are the computational basis states  $|0\rangle$  and  $|1\rangle$ :

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Now consider a system consisting of two qubits, which we will label  $A$  and  $B$ . The computational basis states  $|00\rangle$ ,  $|01\rangle$ ,  $|10\rangle$ , and  $|11\rangle$  of the two-qubit system are formed from tensor products of the single-qubit basis states, e.g.

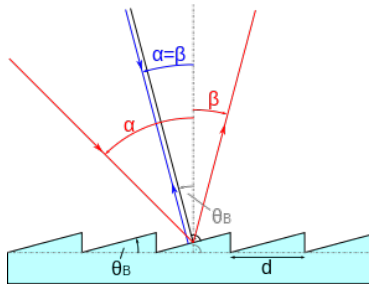
$$\begin{aligned} |00\rangle &= |0\rangle_A \otimes |0\rangle_B \\ |01\rangle &= |0\rangle_A \otimes |1\rangle_B \\ &\text{etc.} \end{aligned}$$

Similarly, the sixteen operators of the generalized two-qubit Pauli group are formed from tensor products of the single-qubit Pauli operators:

$$\begin{aligned} XI &= X_A \otimes I_B \\ XX &= X_A \otimes X_B \\ &\text{etc.} \end{aligned}$$

- a. Evaluate the commutators  $[X, Z]$  and  $[XX, ZZ]$ .
- b. Compute the expectation values of the operators  $XI$ ,  $ZI$ ,  $XX$ , and  $ZZ$  for the entangled Bell states  $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$  and  $|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ .

11. A type of diffraction grating known as an echelle has coarse (20 grooves/mm) rulings and is blazed at a  $\theta_B = 63.5^\circ$  angle as shown. The grating, 150 mm along the grooves by 300 mm wide, is typically rotated so that the incident and diffracted light beams are counter-propagating,  $\alpha = \beta$ .



- a. Find the optimum diffraction order for a wavelength of 355 nm.
  - b. Find the diffraction-limited resolving power  $R = \lambda/\Delta\lambda$  for this grating.
12. A particle of mass  $m$  moves in a potential  $V(x)$ . Suppose we define an operator  $A = ip/\sqrt{2m} + W(x)$ , where  $p$  is the momentum operator and  $W(x)$  is a function of  $x$  to be determined. Show that the Hamiltonian can be written in the form  $H = A^\dagger A$ , and find a 1<sup>st</sup> order differential equation relating  $W(x)$  and  $V(x)$ .
13. The radius of a proton is about  $R_0 = 10^{-15}$  m. Assuming a point-like potential for the proton, calculate the probability that the electron in the ground-state of the hydrogen atom is inside a sphere of radius  $R_0 = 10^{-15}$  m.
14. Two spaceships, each 100 m long in its own rest frame, pass each other traveling in opposite directions. Instruments on spaceship A determine that the front end of spaceship B requires  $5.00 \times 10^{-7}$  seconds to traverse the length of A.
- a. What is the relative velocity of the two spaceships?
  - b. A clock in front of B reads exactly 0 as it passes the front end of A. What will the clock read as it passes the rear end of A?

15. State which of the decays or reactions violate one or more of the conservation laws, and give the law or laws violated in each case.

- a.  $p \rightarrow n + e^+ + \bar{\nu}_e$
- b.  $\mu^+ \rightarrow e^+ + e^+ + e^-$
- c.  $\pi^0 \rightarrow e^+ + e^-$
- d.  $p + \bar{p} \rightarrow \gamma + \gamma$
- e.  $\nu_e + p \rightarrow n + e^+$
- f.  $p \rightarrow \pi^+ + e^+ + e^-$

16. Cold interstellar molecular clouds often contain the molecule cyanogen (CN). The first rotational excited state of CN has an energy of  $4.7 \times 10^{-4}$  eV above the ground state. the excited state has a degeneracy of 3. In 1940, astronomers found that for every 10 CN molecules that are in the ground state, there are about three in the first excited state (so an average of one in each of the three states). To account for this data, it was suggested that the molecules are in thermal equilibrium with some “reservoir” with a well-defined temperature.

- a. What is the temperature that this reservoir needs to have?
- b. Based on your answer to part (a), can you identify the reservoir?