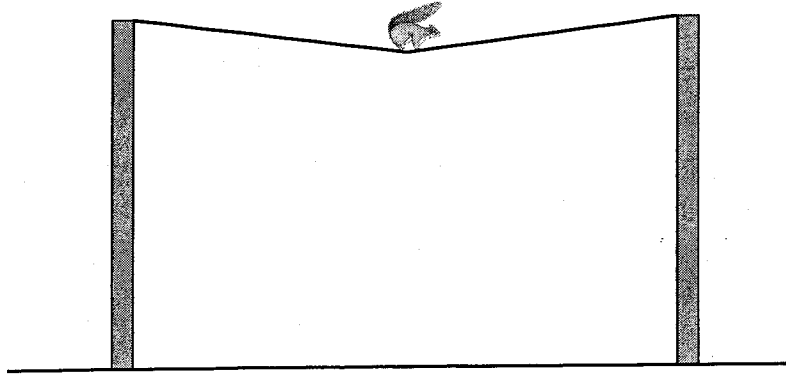


1. An infinite line of positive charge, with charge per unit length of (λ) , moves along the z-axis at a velocity v , much less than the speed of light. Find the magnitude and direction of the electric field and of the magnetic field at a distance (ρ) from the z-axis. Show the steps in the calculation.
2. A mole of helium is expanded slowly and adiabatically from the volume V_1 to the volume V_2 . The same experiment is then repeated with a mole of air. The initial pressure of the gases is the same in both experiments. Which gas has the lower final temperature? Explain your answer. Assume that the conditions of the experiments are close to normal conditions and both gases can be considered as ideal gases. Air consists largely of diatomic molecules.
3. The K^0 meson decays to π^+ plus π^- about two-thirds of the time. The rest energy of the K^0 is 498 MeV, and of the π^+ or π^- , 140 MeV. The lifetime of the K^0 is about 10^{-9} seconds. The quark contents are $K^0: d\bar{s}, \pi^+: u\bar{d}, \pi^-: d\bar{u}$.
 - a. What interaction causes this decay? Support your answer with two facts.
 - b. How much kinetic energy must each pion carry in the kaon rest frame to conserve energy and momentum?
 - c. How much momentum must each pion carry to conserve momentum?
4. Given a point charge q a distance d above an infinite grounded conducting plane:
 - a. Calculate the potential in the region above the plane.
Hint: The problem can be solved using the concept of image charges.
 - b. What is the amount of charge induced on the plane? Justify your answer.
5. A 9 volt battery is delivering 3 W to a resistive load. If an 18Ω resistor is placed in series with the load, the power delivered to the load drops to 0.75 W. What is the internal resistance of the battery?

6. A squirrel of mass $m = 1\text{kg}$ hangs on to the center of an initially horizontal, massless taut wire of length $L = 10\text{m}$ and tension $T = 100\text{N}$. Assume throughout that the magnitude of the tension T is constant.
- In the approximation of small angular deflection, what is the vertical sag caused by the weight of the squirrel?
 - What is the natural angular frequency of vertical oscillation of the squirrel on the wire?



7. A string, half of which has length L and linear mass density μ and the other half has the same length L and mass density 2μ is fixed at each end under a tension T . The tension T satisfies the relation: $\mu Lg \ll T$, where g is the acceleration of gravity.
- How are the values of the transverse wave speeds of the two halves related?
 - Carefully draw the shapes of the two lowest frequency modes of transverse vibration of this string.

Qualifying Examination – Part I

Time: 9:30-11:30 A.M.

Saturday, September 16, 2006

The examination is to be written on the single sheets that are provided: no more than one question is to be answered on a single sheet. Number each question. You are to answer all questions in Part I; however, if you do omit any questions, *cross out those numbers on your title page*. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from the title page) on the back of each sheet of paper.

Part I counts one-third (1/3) of the final grade.

Part II is in this same room at 1:00 P.M.

PHYSICAL CONSTANTS

Planck's constant	$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$
Vacuum speed of light	$c = 3.00 \times 10^8 \text{ m/sec}$
	$\hbar c = 197 \text{ MeV} \cdot \text{fm} = 1.97 \times 10^{-5} \text{ eV} \cdot \text{cm}$
Electron charge	$e = 1.60 \times 10^{-19} \text{ C} = 4.80 \times 10^{-10} \text{ esu}$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-5} \text{ eV/K}$
Gas constant	$R = 8.31 \text{ J/(mol} \cdot \text{K)}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
Electron mass	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV}/c^2$
Proton mass	$m_p = 1.67 \times 10^{-27} \text{ kg} = 938 \text{ MeV}/c^2$
Bohr radius of hydrogen	$a_B = 5.3 \times 10^{-11} \text{ m}$
Ionization energy of hydrogen	13.6 eV
Avogadro's number	$N_A = 6.02 \times 10^{23}/\text{mole}^{-1}$

CONVERSION FACTORS

$$\begin{aligned} 1 \text{ eV} &= 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ erg} \\ 1 \text{ m} &= 10^{10} \text{ \AA} = 10^{15} \text{ fm} = 6.25 \times 10^{-4} \text{ miles} \\ 1 \text{ atm} &= 1.01 \times 10^5 \text{ N/m}^2 = 760 \text{ Torr} \\ 1 \text{ cal} &= 4.186 \text{ J} \end{aligned}$$

Divergence and curl in spherical coordinates:

$$\nabla \cdot E = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta E_\theta) + \frac{1}{r \sin \theta} \frac{\partial E_\varphi}{\partial \varphi}$$

$$\begin{aligned} \nabla \times E = & r \frac{1}{\sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta E_\varphi) - \frac{\partial E_\theta}{\partial \varphi} \right] + \theta \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial E_r}{\partial \varphi} - \frac{\partial (r E_\varphi)}{\partial r} \right] \\ & + \varphi \frac{1}{r} \left[\frac{\partial (r E_\theta)}{\partial r} - \frac{\partial E_r}{\partial \theta} \right] \end{aligned}$$

where r, θ, φ are the unit vectors associated with the spherical coordinates r, θ, φ .

Useful integrals:

$$\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{\alpha}}$$

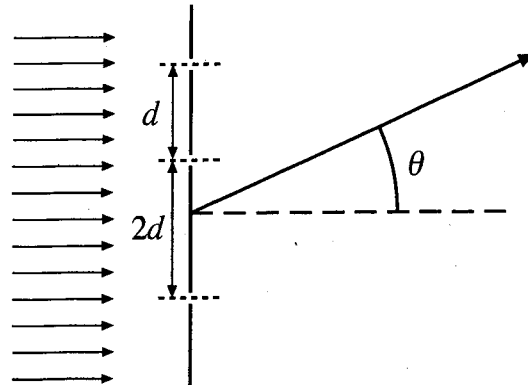
$$\int_0^\infty x e^{-ax^2} dx = \frac{1}{2\alpha}$$

$$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{\alpha^3}}$$

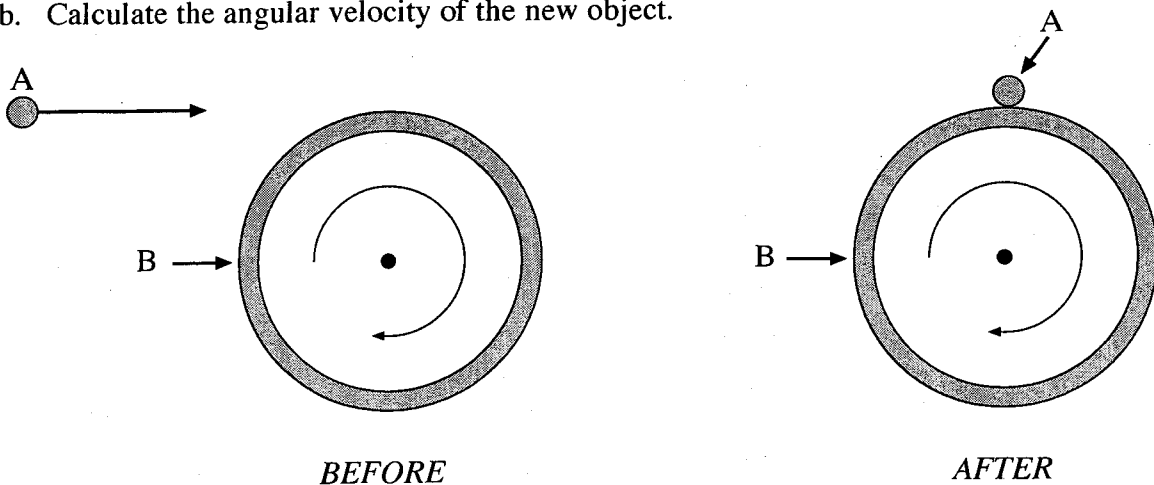
$$\int_0^x \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin^2 x$$

$$\sum_{n=0}^\infty x^n = \frac{1}{(1-x)}; - \sum_{n=1}^\infty (-)^n \frac{x^n}{n} = \ln(1+x)$$

8. Three very narrow slits, with spacings of d and $2d$ as shown in the drawing, are irradiated from the left with a plane wave of monochromatic light with a wavelength λ , with $d = 2.5\lambda$.
- Write down the condition that θ be a direction in which the waves from the top two slits interfere constructively.
 - When the condition of part (a) is satisfied, what will be the phase relation between the waves from the top and the bottom slit? Will the waves be in phase, out of phase, or what?
 - Find all the values of $\sin\theta$ for which the light from all three slits will be in phase, and so produce principal maxima in the intensity.



9. The ^{14}C in a living organism decays at 15 decays per minute per gram of carbon content. The half-life of ^{14}C is 5,739 years. An old piece of wood contains 10 grams of carbon and decays 5 times every 10 seconds. How old is it?
10. A hoop (B) of radius 1.5 meter and mass 50 kg is rotating about its center at an angular speed of 75 radian/second clockwise. A point mass (A) of 16 kg moves tangentially to the disk from left to right at a linear speed of 13 meter/second and sticks to the top of the disk.
- Calculate the moment of inertia of the new object, (A) plus (B).
 - Calculate the angular velocity of the new object.



Qualifying Examination – Part II

Time: 1:00-5:00 P.M.

Saturday, September 16, 2006

The examination is to be written on the single sheets that are provided: no more than one question is to be answered on a single sheet. Number each question. You are to answer all 10 questions in Part II; however, when you omit questions, *cross out those numbers on your title page*. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from the title page) on the back of each sheet of paper.

Part II counts two-thirds (2/3) of the final grade.

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Divergence and curl in spherical coordinates:

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$$\begin{aligned} \nabla \times E = & r \frac{1}{\sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta E_\varphi) - \frac{\partial E_\theta}{\partial \varphi} \right] + \theta \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial E_r}{\partial \varphi} - \frac{\partial (r E_\varphi)}{\partial r} \right] \\ & + \varphi \frac{1}{r} \left[\frac{\partial (r E_\theta)}{\partial r} - \frac{\partial E_r}{\partial \theta} \right] \end{aligned}$$

where \mathbf{r} , $\boldsymbol{\theta}$, $\boldsymbol{\varphi}$ are the unit vectors associated with the spherical coordinates r , θ , φ .

Useful integrals:

$$\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

$$\int_0^\infty x e^{-ax^2} dx = \frac{1}{2a}$$

$$\int_0^\infty x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}}$$

$$\int_0^x \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin^2 x$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{(1-x)}; - \sum_{n=1}^{\infty} (-)^n \frac{x^n}{n} = \ln(1+x)$$

11. Consider a system of N free classical particles, where the energy of each particle can be either E_0 or $E_1 > 0$. Denote by n_0 and n_1 the occupation numbers of levels with energies E_0 and E_1 . The temperature is $T > 0$.
- Find average values of n_0 and n_1
 - Find the total energy of the system.
 - Find the entropy of the system.
12. A simplified form of the Hamiltonian of an ellipsoidal rotor is:

$$H = \frac{L_x^2}{2I_x} + \frac{L_y^2 + L_z^2}{2I}$$

when L_x, L_y and L_z are the Cartesian components of total orbital angular momentum and $I \neq I_x$:

- What are the good quantum numbers?
- Find the eigenvalues of H .
- Find the expectation values $\langle L_x \rangle$, $\langle L_y \rangle$, and $\langle L_z \rangle$ in the energy eigenstates.

13. An electron (mass m) moves in a classical circular orbit around a proton (no Bohr angular-momentum restriction). The electron has acceleration a and so it loses energy to radiation at the rate:

$$\frac{dE}{dt} = -\frac{2}{3} \frac{e^2/4\pi\epsilon_0}{c^3} a^2$$

In such an orbit, the acceleration is related to the energy by:

$$a = \frac{4E^2}{(e^2/4\pi\epsilon_0)m}$$

Thus we have an ordinary differential equation for the energy:

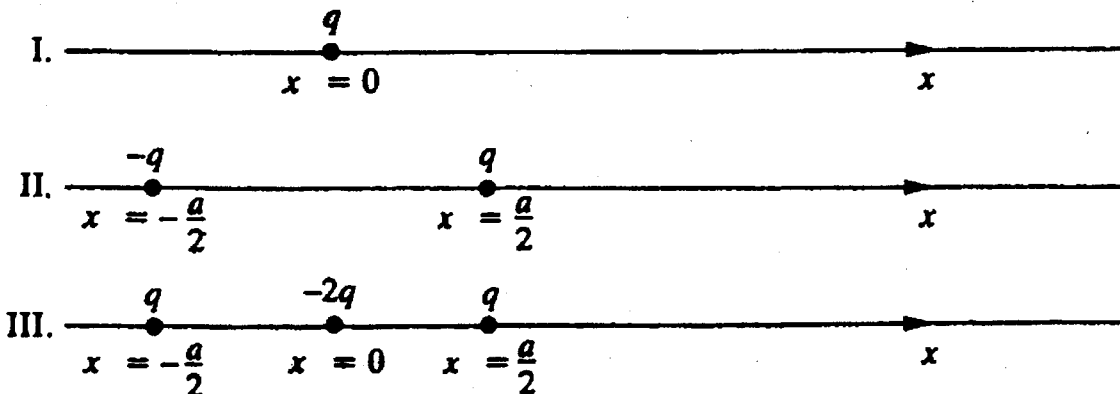
$$\frac{dE}{dt} = -kE^4 \quad \text{with} \quad k = \frac{32}{3(e^2/4\pi\epsilon_0)m^2c^3}$$

Find a solution in terms of k, E_0 , and t of the differential equation that describes the collapse of a classical orbit of initial energy E_0 . Sketch $E(t)$ as a function of time and interpret the results.

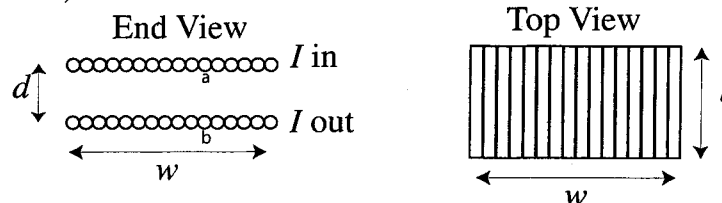
Hint: The energy is negative at all times.

14. For each of the charge configurations shown:

- Find the total charge $Q = \int \rho dx$
- Find the dipole moment $P = \int x\rho dx$
- Find the quadrupole moment $Q_{xx} = 2 \int x^2 \rho dx$
- Find the leading term (in powers of $1/x$) in the potential Φ at a point $x \gg a$.



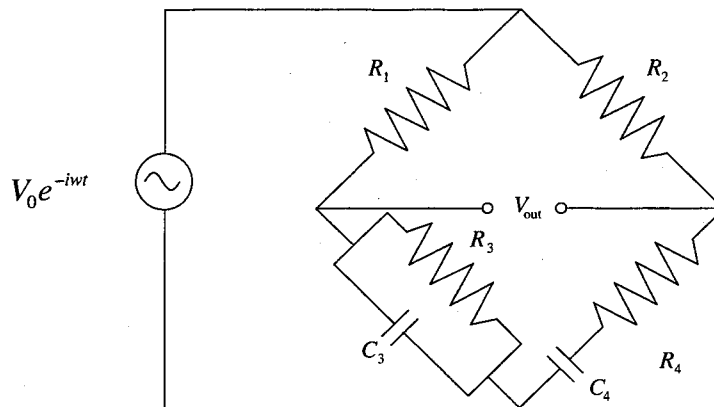
15. An electron is in a non-relativistic classical (macroscopic) orbit attracted by a very long insulating string. The string is along the z -axis and has a linear charge density of λ Coulomb/meter.
- How does the electric field depend on the radial distance ρ of the electron from the string?
 - How does the electrostatic potential depend on the cylindrical coordinates (ρ, ϕ, z) ?
 - What constants of the motion are to be expected?
 - For a circular orbit, how does the speed of the electron depend on the radius ρ ?
 - For the circular orbit, how does the speed depend on the linear charge density λ ?
 - For the circular orbit, how does the frequency depend on the area of the orbit?
16. A Fabry-Perot-like cavity consists of a 99% reflecting mirror and a 100% reflecting mirror, separated by a distance (L) . Laser light of frequency (ν) is incident on this cavity normal to the partially reflecting mirror.
- Use conservation of energy arguments alone to determine the transmitted and reflected intensities.
 - Find the intensity inside the cavity when (L) is adjusted to make the intensity maximum.
17. Shown below is a "parallel-plate" inductor, consisting of two sets of N wires, each wire carrying time-dependent current $I(t)$. For the top plate, the currents flow into the page, while for the bottom plate the currents flow out of the page. The currents loop from top to bottom at the ends; ignore edge effects. The width of the plates is w , the length l , and the distance between them $d \ll w, l$.



- Use Ampere's law to find the magnetic field (magnitude and direction) in between the two plates. You may assume the field is zero everywhere except between the plates.
- Calculate the induced emf for a single pair ab of the wires, and for the whole system.
- Calculate the inductance.
- Now relax the assumption of zero field outside the plates. Draw some field lines, and use them to estimate the magnetic field strength just above the plates.

18. Determine the relations between the circuit parameters that balance the AC bridge circuit shown below, i.e. derive the conditions that assure $V_{\text{out}} = 0$.

Hint: There are two balance conditions.



19. A 3 cm thick aluminum disk (density = 3.67 gram/cm³, Atomic Weight = 26.98) attenuates a flux of 662 keV gamma rays. Using counting intervals of 100 seconds, the detector registers 980 counts when the disk is in place between the source and detector, and 1,700 counts when the disk is not in place. The background count rate is also measured, yielding 35 counts in the same 100 second interval.

- Find the total attenuation cross section for 662 keV photons on aluminum. Give your answers in cm².
- Is the measurement accurate to $\sim \pm 50\%$, $\sim \pm 5\%$, or $\sim \pm 0.5\%$?

20. A quantum-mechanical particle of mass m moves one dimensionally along the x-axis, subject to a potential $V(x) = -A\delta(x)$ where $\{A > 0\}$.

Find the bound state energy and normalized wave function in terms of $\{m, A, \hbar\}$.

21. Let P be the operator of space reflection (parity), i.e.:

$$P\Psi(x, y, z) = \Psi(-x, -y, -z)$$

- Write down the operator for the z component of the orbital angular momentum.
- Does this angular momentum operator commute with P ? Support your answer with a calculation or reasoning.

22. Are the following transitions in carbon allowed or forbidden for electric dipole transitions? If they are forbidden, tell why.

a. $1s^2 2s^2 2p\ 3p\ ^1P_1 \rightarrow 1s^2 2s^2 2p^2\ ^1S_0$

b. $1s^2 2s^2 2p\ 5p\ ^1D_2 \rightarrow 1s^2 2s^2 2p\ 3s\ ^3P_1$

c. $1s^2 2s^2 2p\ 5d\ ^3P_0 \rightarrow 1s^2 2s^2 2p\ 3p\ ^3P_0$

d. $1s^2 2s^2 2p\ 5d\ ^3P_2 \rightarrow 1s^2 2s^2 2p\ 3p\ ^3P_1$

e. $1s^2 2s^2 2p\ 5d\ ^3P_2 \rightarrow 1s^2 2s^2 2p\ 3p\ ^3P_0$

23. The neutral pion (rest mass m) decays into two photons. A pion moving with velocity v along the z -axis decays in flight and emits one of the photons at an angle θ relative to the z -axis.

a. Write the Lorentz transformations that relate the photon momentum p and p' in the laboratory and rest frames, respectively.

b. Write the relationship between θ and θ' .

c. For what decay angle θ' in the rest frame is the angle between the two photons in the lab frame the smallest? Write the expression for this angle.

24. A uniformly-charged spherical shell with positive total charge Q is rotating about a diameter with a constant angular velocity $\vec{\omega}$. The magnetic field outside the shell is a pure dipole field (no lower or higher multipoles).

a. Make a sketch showing the direction of the angular velocity vector together with the shape and direction of the magnetic field outside the spherical shell.

b. Describe the electric field, in magnitude and direction, outside the spherical shell.

c. This system is not radiating, even though there is acceleration of charge. Prove that there is no radiation with a statement based on your answers in (a) and (b).

25. Suppose you want to design a container for one liter of liquid hydrogen. You decide to make it out of Styrofoam in the form of a closed box with walls and ends that are 5 cm thick. Here are some facts that may be useful:

- Boiling temperature of hydrogen at atmospheric pressure = 20.3K
- Heat of vaporization of hydrogen at atmospheric pressure = 31 J/cm³
- Thermal conductivity of Styrofoam = 0.01 W/mK (assume constant with temperature)
- Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$
- Styrofoam is essentially transparent to radiation at infrared wavelengths
- The average emissivity of liquid hydrogen is $\varepsilon = 0.05$
- Assume the hydrogen vapor escapes from the container as soon as it is formed.

- a. Determine what is the dominant source of heat flowing into the hydrogen.
- b. Approximately how long will a single liquid liter of hydrogen last in your container?
- c. How can you improve the “hold time” of your hydrogen storage vessel?