Qualifying Examination - Part I

Time: 9:30-11:30 a.m. **Saturday, September 18, 2004**

The examination is to be written on the single sheets that are provided; no more than one question is to be answered on a single sheet. Number each question. You are to answer all questions in Part I; however, if you do omit any questions, ***cross out those numbers on your title page***. When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from your title page) on the back of each sheet of paper.

Part I counts one-third (1/3) of the final grade.
Part II is in this same room at 1:00 p.m.

	PHYSICAL CONSTANTS
Planck's constant	$h = 6.63 \times 10^{-34}$ J•s $= 4.14 \times 10^{-15}$ eV•s
Vacuum speed of light	$c = 3.00 \times 10^{8}$ m/sec
	$\hbar c = 197$ MeV•fm $= 1.97 \times 10^{-5}$ eV•cm
Electron charge	$e = 1.60 \times 10^{-19}$ C $= 4.80 \times 10^{-10}$ esu
Boltzmann constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Gas constant	$R = 8.31$ J/(mol•K)
Gravitational constant	$G = 6.67 \times 10^{-11}$ N•m^2/kg^2
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12}$ F/m
Electron mass	$m_e = 9.11 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
Proton mass	$m_p = 1.67 \times 10^{-27}$ kg $= 938$ MeV/c^2
Bohr radius of hydrogen	$a_B = 5.3 \times 10^{-11}$ m
Ionization energy of hydrogen	13.6 eV
Avogadro's number	6.02×10^{23} / mole

Conversion Factors

1 eV $= 1.6 \times 10^{-19}$ J $= 1.6 \times 10^{-12}$ erg

1 m $= 10^{10}$ Å$= 10^{15}$ fm $= 6.25 \times 10^{-4}$ miles

1 atm $= 1.01 \times 10^{5}$ N/m^2 = 760 Torr

1 cal = 4.186 J

Divergence and curl in spherical coordinates

$$\nabla \cdot E = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta E_\theta) + \frac{1}{r\sin\theta}\frac{\partial E_\varphi}{\partial\varphi}$$

$$\nabla \times E = r\frac{1}{r sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta E_\varphi) - \frac{\partial E_\theta}{\partial\varphi}\right] + \theta\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial E_r}{\partial\varphi} - \frac{\partial(rE_\varphi)}{\partial r}\right] + \varphi\frac{1}{r}\left[\frac{\partial(rE_\theta)}{\partial r} - \frac{\partial E_r}{\partial\theta}\right]$$

where r, θ, φ are the unit vectors associated with the spherical coordinates r, θ, φ.

Useful integrals

$$\int_0^\infty e^{-\alpha x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{\alpha}}$$

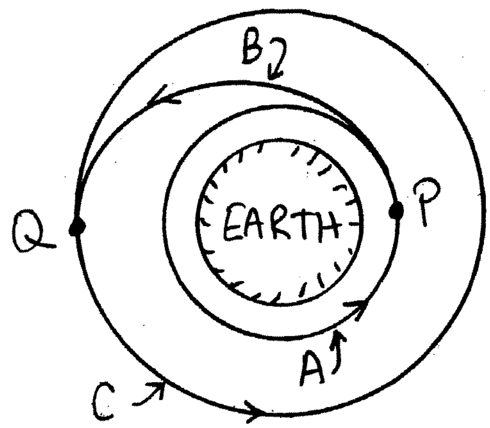
$$\int_0^\infty xe^{-\alpha x^2}\,dx = \frac{1}{2\alpha}$$

$$\int_0^\infty x^2 e^{-\alpha x^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\int_0^x \sin^2 x\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x$$

$$\Sigma_{n=0}^{\infty} x^n = \frac{1}{(1-x)};\quad -\Sigma_{n=1}^{\infty}(-)^n\frac{x^n}{n} = \ell n\,(1+x)$$

1. This question is about an elliptical "transfer orbit" B from an inner circular orbit A to an outer circular orbit C. The transfer starts at point P and is completed at point Q. The transfer orbit is an ellipse which is tangent to A at point P and tangent to C at point Q.



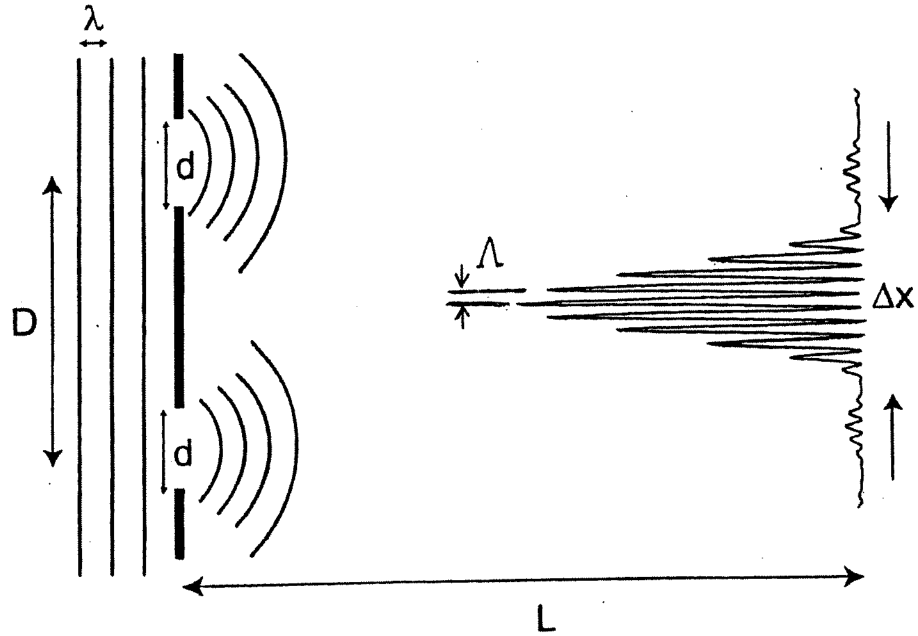
   a. Derive a formula for the relationship between v and r for circular orbits. Is the speed in orbit C greater or less than that in A ?

   b. Should the satellite be speeded up or slowed down at P ?

   c. Should it be speeded up or slowed down at Q ?

2. A surface is irradiated with monochromatic light whose wavelength can be varied. Above a wavelength of 500 nm, no photoelectrons are emitted from the surface. With an unknown wavelength, a stopping potential of 3 volts is necessary to eliminate the photoelectric current. What is the unknown wavelength?

3.



A monochromatic wave of wavelength λ illuminates an opaque mask with two slits as shown in the figure. The diffraction pattern is recorded on a screen a distance L from the mask. You may assume $\lambda \ll d,\ D \ll L$.

a. What is the distance Λ between adjacent interference fringes observed on the screen?

b. What is the width Δx of the central lobe of the interference pattern on the screen?

4. An object is placed 8 cm from a thin double convex lens of focal length 12 cm.

a. Find the image position, and determine whether the image is real or virtual by using the lens formula.

b. Repeat part (a) by using graphical construction instead of the lens formula.

5. A charge Q is distributed uniformly throughout the volume of the sphere of radius R.

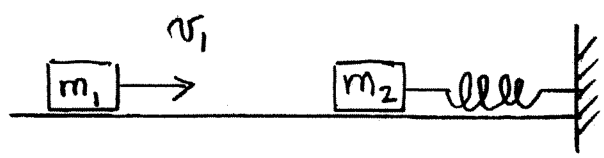
a. Find the electric potential at a point outside the sphere at distance r from the center.

b. Find the electric potential at the surface.

c. Find the electric field inside the sphere.

d. Find the electric potential inside the sphere.

6. A block of mass m_1 and initial velocity v_1 collides head on with a stationary block of mass m_2 . The m_2 mass compresses a spring of spring constant k. Neglect friction and assume the collision is elastic.



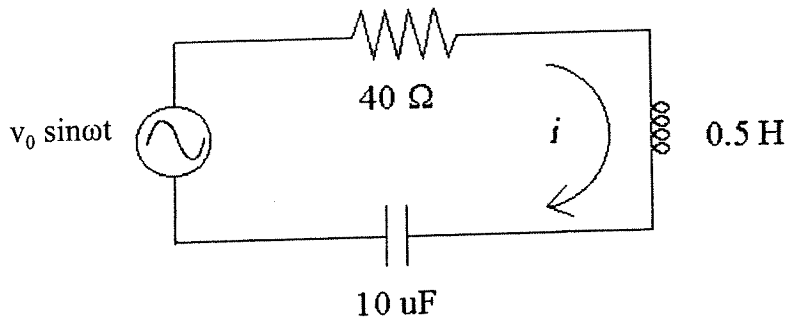
a. What is the velocity v_2 of m_2 just after the collision?

b. What is the maximum compression of the spring?

7. A Madison nursery uses natural gas heating to keep their greenhouses at 30°C all year. A very good engineer points out that water at the bottom of Lake Mendota is at a constant temperature of 5°C, and that he can build them an ideal heat pump that will work at the maximum possible efficiency to pump heat from this lake water into their greenhouses. He claims that they will come out ahead with his system, even though it uses electricity instead of natural gas at three times the cost per Joule.

Is he right? Neglecting capital and maintenance costs, by what factor would their energy bill be reduced? Show your work!

8. In the circuit below, the voltage source is sinusoidal, with $V_0 = 9$ volts, $\omega = 400/s$.



a. Calculate the peak value of the current i.

b. Calculate the phase of i relative to the applied voltage, and state whether $i(t)$ leads or trails v(t).

9. a. Consider molecules of mass m in the atmosphere above a spherical planet of mass M and radius R. Find the escape velocity of the molecules in terms of M, R, and the Newtonian gravitational constant G.

b. The escape velocity you found in part a) is independent of the molecular mass m. Why is it that the molecules with smaller mass are lost from the planetary atmosphere first?

10. A particle of mass m is confined in a one-dimensional harmonic oscillator well, $V = \frac{1}{2}kx^2$, with a wave function $\psi(x) = Nxe^{-\alpha x^2}$.

a. Sketch $\psi(x)$ and the probability distribution.

b. How many nodes does ψ have (not counting $\pm\infty$)?

c. Assuming $\psi(x)$ is a valid wave function, does it represent the ground state, the first excited state, or the second excited state?

d. Find the value of α for which $\psi(x)$ is a solution to the time-independent Schrodinger equation, and determine the energy of the state.

Qualifying Examination - Part II

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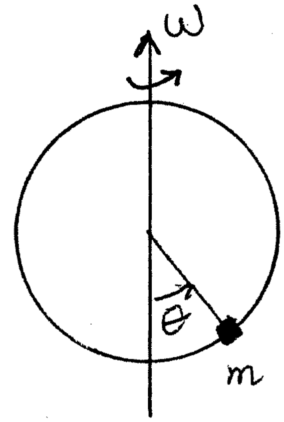
11. A circular hoop of radius r rotates at frequency ω about a vertical axis through the center of the hoop in the plane of the loop. A bead of mass m slides without friction around the hoop and is subject to earth gravity.

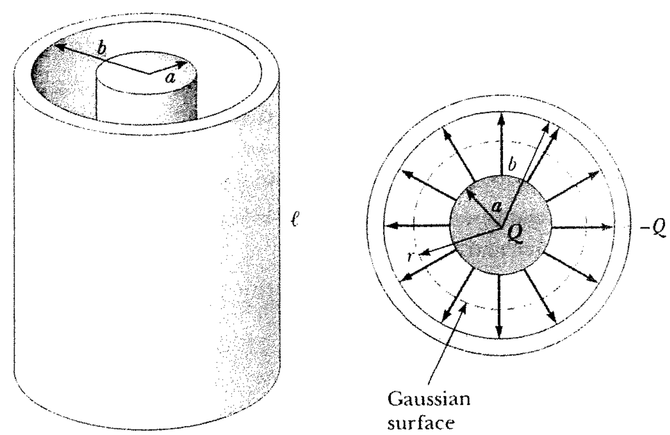


a. Give the Lagrangian in terms of the angle θ shown in the drawing.

b. Find the equation of motion in terms of this angle.

c. Find the values of θ for which the bead may be stationary relative to the hoop, and determine which of the stationary points are stable.

12. A cylindrical capacitor of length ℓ has an inner cylinder of radius a, and an outer cylinder of radius b. A potential difference V_0 is maintained between the cylinders.



a. Find the value of the electric field $\vec{E}(r)$ in the region $a \le r \le b$ in terms of V_0, a and b, assuming $\ell >>$ a, b.

b. Find the capacitance in terms of ℓ, a and b.

c. Find the radius of the inner cylinder "a" that minimizes the electric field at the surface r = a for a given V_0. (Useful in design of high-voltage capacitors to minimize field-emission effects)

13. The Rydberg constant $R_\infty = 109{,}737.568525$/cm is one of the most accurately known fundamental constants.

a. Find the wavenumber of Balmer alpha $n = 3 \rightarrow 2$ in atomic hydrogen (neglect fine structure).

b. Is Balmer alpha in atomic <u>deuterium</u> shifted towards blue or towards red compared to normal hydrogen?

c. Calculate the shift in wavenumber between deuterium and hydrogen.

14. Suppose you are driving at 60 mph into a police "X-band" radar speed trap. X-band radar works at $f = 10.525$ GHz. The speed of light is 186,000 mps.

a. What is the frequency f_{det} of the microwaves detected by an "antiradar" device mounted in your car?

b. The police officer detects the microwaves reflected from your moving car. What is the frequency difference between the reflected microwaves detected by the police officer and the microwaves transmitted by the police officer?

c. Your antiradar device tries to fool the police officer into thinking that your car is stationary. It does this by emitting microwaves at a new frequency f'. What should f' be?

15. A particle of mass m is given an initial velocity v_0. Assume that the particle is subject to a drag force $F = -bv^{1/2}$, and no other forces.

a. Find v as a function of time.

b. How far does the particle travel before coming to rest?

16. This is a question about the "electronic specific heat" of the conduction electrons in a metal.

a. What would one obtain for the heat capacity of a "gas" of N electrons if the electrons were treated as classical non-interacting particles?

b. Use qualitative arguments based on the Fermi distribution function to estimate the electronic heat capacity of a metal with N conduction electrons at temperatures $k_BT << E_F$ where E_F denotes the Fermi energy.

c. What is the order of magnitude of E_F in typical metals such as Cu or Au? A numerical answer is expected.

17. The charge density of a hydrogen atom in its ground state can be modeled as

$$\rho(\vec{r}) = e\left(\delta(\vec{r}) - \frac{\alpha^3}{8\pi}e^{-\alpha r}\right)$$

where e is the elementary charge, r is the distance from the origin, $\delta(\vec{r})$ is a Dirac delta function, and α is a constant.

a. Give a simple physical interpretation for the two terms in the charge density.

b. Calculate the total charge of the atom.

c. We define the Bohr radius, a_0 , as the value of r at which the net electronic charge in a spherical shell of thickness dr is maximized. Express the constant α in terms of a_o .

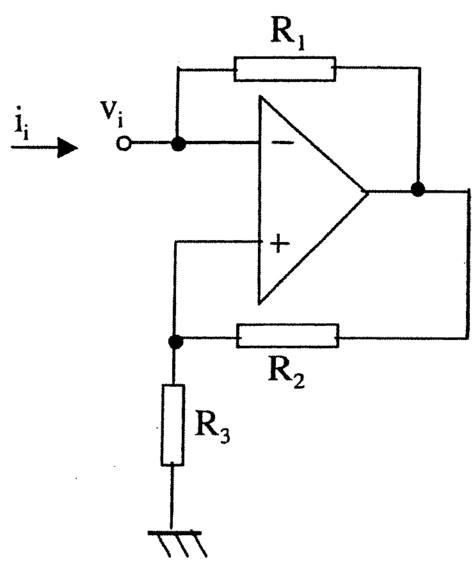
18. The Clausius-Clapyron equation, dP/dT = L/(T•ΔV), relates the slope of the coexistence line between two phases in the pressure vs. temperature plane to the latent heat of the phase transformation, L, and the volume difference of the two phases ΔV. Use this to find the equation for the variation with temperature of the equilibrium vapor pressure over a liquid. Assume: a) the vapor can be approximated as an ideal gas, b) the specific volume of the liquid is negligible compared to the specific volume of the gas, and c) the latent heat of vaporization is independent of temperature. Show all steps of your derivation clearly, and explain as necessary.

19. The Maxwell Equations inside a good conductor simplify to

$$\vec{\nabla} \cdot \vec{E} = 0$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\vec{\nabla} \times \vec{B} = \sigma \vec{E} + \frac{\partial \vec{E}}{\partial t}$$

Compute the skin depth of the conductor (the distance it takes for the amplitude of the electric field of a plane electromagnetic wave to decrease by a factor of 1/e) in the limit that $\sigma \to \infty$ to leading order in powers of $1/\sigma$.

20. For the circuit shown below, determine the magnitude of the input resistance $r_i = v_i/i_i$. The Operational Amplifier is ideal (infinite gain, zero input current).



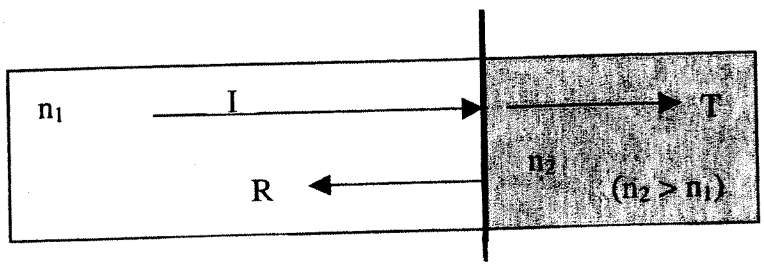
21. A gas of argon atoms is at pressure p = 1 atm = 1.01×10^5 Pascal and temperature 300K. The atoms have mass 66.3×10^{-27} kg and diameter 0.38 nm. [Under these conditions, this is nearly an ideal gas.]

a. Estimate the average spacing (in nm) of the atoms in the gas.

b. Estimate the distance (in nm) that an argon atom travels between collisions.

c. Assume that the main correction to the ideas gas approximation comes from the reduction of the free volume V by the excluded volume occupied by the atoms. Estimate the fractional correction to the pressure from this effect.

22.



An electromagnetic plane wave E_I traveling in medium n_1 as shown is incident normally on medium n_2. It produces a reflected wave E_R and a transmitted wave E_T.

a. Using the Poynting Flux $\vec{S} = \vec{E} \times \vec{H}$ (which gives the flux in W/m^2), derive an expression for S in terms of ε_0, c, n and E.

b. Using the expression derived in a), and the conservation of S, derive the flux relation between E_I, E_R and E_T; the incident, reflected and the transmitted fluxes.

c. Using the result from b) and the tangential boundary conditions on $\vec{E}$ at the interface, derive expressions for $r = \dfrac{E_R}{E_I}$ at $t = \dfrac{E_T}{E_I}$ in terms of n_1 and n_2 $(n_2 > n_1)$.

d. Rewriting the relation found in b) in the form R + T = 1 where R and T are the reflection and transmission coefficients, show that $R = r^2$ and $T = \dfrac{n_2}{n_1} t^2$.

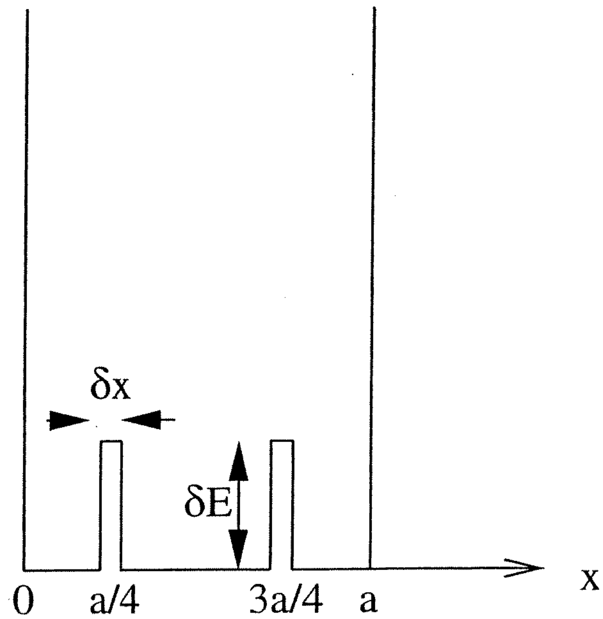
23. A particle of mass m is in an infinitely deep square well that has a potential $V = 0$ between $x = 0$ and $x = a$.

a. What is the normalized wave function for the first excited state?

b. Sketch this wave function inside the well.

Two barriers (see drawing below) are now added as perturbations to the well, each with width $\delta x = 0.01a$ and height $\delta E = +\varepsilon$. One barrier is centered at $x = a/4$, and the other is centered at $x = 3a/4$.

c. Estimate to 1st order the magnitude and sign of the energy shift this perturbation produces for the first excited state.



24. Consider a gas of diatomic molecules which remain in the gas state down to very low temperatures and which never dissociate at high temperature.

a. Plot the heat capacity per mole at constant volume as a function of temperature. Be sure to put units on the heat capacity axis.

b. Assuming the molecules are H_2, calculate the temperature at which rotational degrees of freedom become important, and label this point on the horizontal axis. Assume the distance between nuclei is 0.074 nm, and recall that the mass of a proton is 1.67×10^{-27} kg.

25. You are given a sample of silicon wafer in the shape of a rectangle, 1 mm wide by 10 mm long by 0.5 mm thick. You need to determine whether the carriers are electrons or holes and what their concentration (number per unit volume) is. You have any equipment that has ever been used in any physics experiment at your immediate disposal. How would you proceed?

A complete answer need not be too long. Simply make it clear what physics you will exploit to complete your task and what (major) equipment you will use.