Classical Mechanics I:

1

* Assume: Thrust/per engine $= 56\,000$ pounds $\sim 25\,000$ kg

Speed ~ 0.6 sound velocity ~ 200 meter/s

power/per engine $\sim 5 \times 10^6 \ \frac{\text{kg-meter}}{\text{second}}$

$= \frac{5 \times 10^6}{75}$ horse power

$= \frac{2}{3} \times 10^5$ hp

Time of flight: $\sim$ houres

* A car of 100 hp takes 4 liter fuel to run ½ houre $\Rightarrow$ $\boxed{\text{4 liter fuel} \sim 50 \text{ hp} \times \text{houre}}$

* Fuel for one engine:

$4 \times \left(4 \times \frac{2}{3} \times 10^5 / 50\right) \sim 2 \times 10^4$ liter

$\boxed{\text{Fuel for four engine: } 8 \times 10^4 \text{ liter}}$ $\pm 100\%$

2

* Fuel of $\frac{1}{2}$ takeoff weight $= 180 \times 10^3$ kg can make 747 fly for 8000 mile

* York to London ~ 2000 mile

$\boxed{\text{Fuel needed: } 4 \times 10^4 \text{ kg} \sim 5 \times 10^4 \text{ liter}}$ $\pm 100\%$

a) Define effective mass $m^* = \dfrac{\text{Kinetic Energy} \times 2}{(\text{Velocity})^2}$

1. $K.E. = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$, $I = \frac{1}{2} a^2 m$, $\omega = \frac{v}{a}$

$= \frac{1}{2}\left(1 + \frac{1}{3}\right) m\, v^2$

$\Rightarrow \boxed{m^* = \frac{3}{2}\, m}$

2. $KE = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$, $I = a^2 m$

$\Rightarrow \boxed{m^* = 2m}$

3. $I = \frac{2}{5} m a^2$

$\Rightarrow \boxed{m^* = \frac{7}{5} m}$

4. $\boxed{m^* = 2m}$

Order of finish: (assum $a \ll h, \ell$)

fast ③ ① ② ④ slow if ④ starts as 

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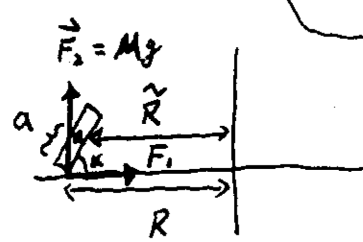
b) Angular momentum $L = I\omega = I \frac{v}{a}$



$\left|\frac{d}{dt}\vec{L}\right| = \dfrac{2\pi L \sin(\alpha)}{2\pi \tilde{R} / v} = \dfrac{L\, v \sin(\alpha)}{R - \cos(\alpha)\, a}$

$= a F_2 \cos(\alpha) - a F_1 \sin(\alpha)$

$= a Mg \cos(\alpha) - a F_1 \sin(\alpha)$

$\Rightarrow \boxed{I \frac{v}{a} \frac{v \sin(\alpha)}{R - \cos(\alpha)\, a} = a Mg \cos(\alpha) - a F_1 \sin(\alpha)}$ ①

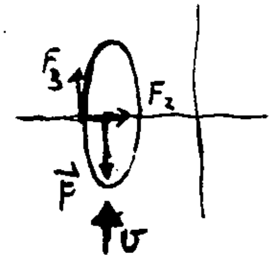
$\boxed{F_1 = M \dfrac{v^2}{R - \cos(\alpha)\, a}}$ ②

$\boxed{R = \dfrac{v^2}{g} \dfrac{\left(1 + \frac{1}{2}\right) \sin(\alpha)}{\cos(\alpha)} + a \cos(\alpha)}$

$\Rightarrow \frac{1}{2} M v^2 \dfrac{\sin(\alpha)}{\tilde{R}} = Mg \cos(\alpha) - \dfrac{M v^2}{\tilde{R}} \sin(\alpha)$

c) Assume friction to be $\boxed{\vec{F} = -A\vec{v}}$ act on the center of mass.

$$\frac{d}{dt} L = - F_3 a = -\frac{I}{a}\frac{d}{dt} v$$



$$M \frac{d}{dt} v = F_3 - F = F_3 - Av$$

$$= -\frac{I}{a^2}\frac{d}{dt} v - Av$$

$$\Rightarrow \boxed{\frac{d}{dt} v = -\frac{Av}{M + \frac{I}{a^2}}} \quad (1)$$

Loss of energy (Rate):

$$\frac{d\,(KE + V)}{dt} = -A v^2$$

$$= \frac{d}{dt}\left(\frac{1}{2}\left(M + \frac{I}{a^2}\right) v^2 + a\cos(\alpha) Mg\right)$$

$$= \left(M + \frac{I}{a^2}\right) v \left(-\frac{Av}{M + \frac{I}{a^2}}\right) + aMg\,(-\sin(\alpha))\,\dot{\alpha}$$

used (1) for $\dot{v}$

$$= -Av^2 - aMg\sin(\alpha)\,\dot{\alpha}$$

$$\Rightarrow \boxed{\dot{\alpha} = 0}$$

from (1)

$$\boxed{\begin{aligned} v_{(t)} &= v_0\, e^{-\frac{A}{M + \frac{I}{a^2}} t}, \quad \alpha = \text{const} \\ R_{(t)} &= \frac{v^2(t)}{g} \frac{\frac{3}{2}\sin(\alpha)}{\cos(\alpha)} + a\cos(\alpha) \end{aligned}}$$

$$I = \frac{1}{2} M a^2$$

a) E.m.f. induced in one loop: $\mathcal{E} = -\frac{1}{c}\frac{\partial \Phi}{\partial t} = \frac{i\omega}{c}AB = \frac{i\omega}{c}EA$

2pts Impedance of a loop: $Z = R + i(\omega L - \frac{1}{\omega C})$

Current: $I = \mathcal{E}/Z = \frac{i\omega E A}{c(R + i(\omega L - \frac{1}{\omega C}))}$

Dissipated power: $P = \langle \bar{I}\mathcal{E}\rangle_{\substack{\text{Average}\\ \text{over cycle}}} = \frac{\frac{1}{2}(\omega E A/c)^2 R}{R^2 + (\omega L - 1/\omega C)^2}$

Power balance: $\frac{dS}{dz} = -nP = -\frac{1}{a}S$,

concentration of loops $n = e^{-3}$; $\langle S\rangle_{ave} = \frac{c}{8\pi}E^2$

where $a = (R^2 + (\omega L - \frac{1}{\omega C})^2)c^3/(4\pi n\omega^2 A^2 R)$ is the length of power attenuation: $S(z) = S(0)e^{-z/a}$

$$\boxed{a = \frac{(R^2 + (\omega L - \frac{1}{\omega C})^2)c^3}{4\pi n\omega^2 A^2 R}}$$

Angular distribution of radiated power is that of a magnetic dipole radiation. The dipole is oriented along the $\hat{y}$ axis, and thus $\frac{dP}{d\Omega} \sim \sin^2\theta$, where θ is the polar angle measured from the $\hat{y}$ axis

$$\boxed{\frac{dP}{d\Omega} = \frac{3P^{rad}}{8\pi}\sin^2\theta \quad \text{where } P^{rad} = \frac{c}{3}k^4 I^2 A^2}$$

Check: $\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin^3\theta = 2\pi\int_{-1}^{1} dx\,(1-x^2) = 4\pi\,\frac{2}{3} = \frac{8\pi}{3}$

b) For a loop with a unit vector $\vec{n}_o$ normal to its plane, angular distribution of radiated power is:

$$\frac{dP(\vec{n})}{d\Omega} \sim \left(1-(\vec{n}\cdot\vec{n}_o)^2\right)(n_o\cdot n_B)^2$$

points to the observer

angular distribution of radiation of magnetic dipole

unit vector along $\vec{B} = \vec{n}_B B$

magnetic flux component

in components:

$$\frac{dP}{d\Omega}(\vec{n}) \sim \left(1-(n^x n_o^x + n^y n_o^y + n^3 n_o^3)^2\right)(n_o^y)^2$$

Averaging over orientations of $\vec{n}_o$:

(i) $\langle (n_o^y)^2\rangle = \frac{1}{3}$

(ii) $\langle (n_o^y)^4\rangle = \frac{1}{2}\int_0^1 (1-x^2)^2 dx = \frac{1}{2}\left(1-\frac{2}{3}+\frac{1}{5}\right) = \frac{4}{15}$

(iii) $\langle (n_o^x)^2 (n_o^y)^2\rangle = \frac{1}{4}\int_0^1 (1-x^2)x^2 dx = \frac{1}{4}\left(\frac{1}{3}-\frac{1}{5}\right) = \frac{1}{30}$

iv) $\langle (n_o^3)^2 (n_o^y)^2\rangle = \frac{1}{30}$ (like iii)

$$\left\langle \frac{dP}{d\Omega}(\vec{n})\right\rangle_{\substack{\text{averaged}\\ \text{over } \vec{n}_o}} \sim \left[\frac{1}{3} - \frac{4}{15}(n^y)^2 - \frac{1}{30}\left((n^x)^2+(n^3)^2\right)\right] = \frac{9}{30} - \frac{7}{30}(n^y)^2$$

Thus $\left\langle \frac{dP}{d\Omega}(n)\right\rangle \sim 2+7\left((n^x)^2+(n^3)^2\right) = \boxed{2+7\sin^2\theta}$

angle measured from the y-axis

c) For unpolarized wave, $\vec{B}$ is ~~unif~~ randomly oriented within the x-y plane. Thus, from part b),

$$\left\langle \frac{dP}{d\Omega}(\vec{n})\right\rangle_{n_o, B} = \left\langle 9 - 7(n\cdot n_B)^2\right\rangle_B = 9 - \frac{7}{2}\left((n^x)^2+(n^y)^2\right) = \boxed{\frac{11}{2}+\frac{7}{2}(n^3)^2}$$

or: $\boxed{\left\langle \frac{dP}{d\Omega}(n)\right\rangle_{n_o,B} \sim 11+7\cos^2\theta'}$

measured from the $\hat{z}$-axis

d) By uncertainty principle, the angle of divergence is $\delta\theta = \lambda/r_o$

e) This part can be interpreted in several different ways. All solutions are accepted as correct.
0 pts This part, however, hasn't been graded

f) Magnetic ~~moment~~ moment of one loop: $\vec{m} = \frac{1}{c} I A \hat{y} = \frac{\mathcal{E} A}{Z} \hat{y} = \frac{i\omega A^2}{cZ} \vec{B}$

2 pts Thus effective magnetic permeability: $\mu_{eff} = 1 + 4\pi n \frac{i\omega A^2}{c^2 Z(\omega)}$

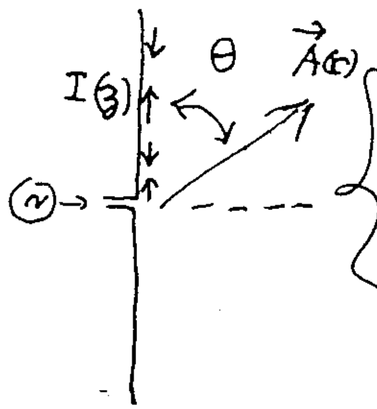
From wave equation in effective medium:

$$K^2 = \frac{\omega^2}{c^2} \underset{1}{\underset{\parallel}{\epsilon_{eff}}} \mu_{eff} \Rightarrow K = \frac{\omega}{c}\left(1 + 2\pi n \frac{i\omega A^2}{c^2 Z(\omega)}\right)$$

Thus, the attenuation length a is given by

$$\frac{1}{a} = 2\,\mathrm{Im}\,K = 4\pi n \frac{\omega^2 A^2 R}{c^3\left(R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right)}$$

$$\boxed{a = \frac{c^3}{4\pi n} \frac{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}{\omega^2 A^2 R}}$$



Current in the antenna:

$$\left\{\begin{aligned} & I(z) \sim \cos(kz+\varphi)e^{-i\omega t} \\ & I(z=\pm \ell/2)=0 \end{aligned}\right. \Rightarrow I(z) = I\sin\left(k\left(\frac{\ell}{2}-|z|\right)\right),$$

$$k = \omega/c$$

$$A(r) = \frac{\sin\theta}{rc}\int_{-\ell/2}^{\ell/2} I(z)\, e^{ikz\cos\theta}dz =$$

$$= \frac{2\sin\theta}{rc} I \int_0^{\ell/2} \underbrace{\sin\left(k\left(\frac{\ell}{2}-z\right)\right)\cos(kz\cos\theta)}_{\frac{1}{2}\left[\sin k\left(\frac{\ell}{2}-z(1-\cos\theta)\right)+\sin k\left(\frac{\ell}{2}-z(1+\cos\theta)\right)\right]} dz$$

$$= \frac{I\sin\theta}{rc}\left[\frac{\cos k\left(\frac{\ell}{2}-z(1-\cos\theta)\right)}{k(1-\cos\theta)} + \frac{\cos k\left(\frac{\ell}{2}-z(1+\cos\theta)\right)}{k(1+\cos\theta)}\right]\Bigg|_0^{\ell/2}$$

$$= \frac{2I}{krc\sin\theta}\left(\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\frac{k\ell}{2}\right)$$

$$\vec{B} = i\vec{k}\times\vec{A} \rightarrow \bar{S} = \underbrace{\left\langle\frac{c}{4\pi}B^2\right\rangle}_{\text{average over cycle}} = \frac{ck^2}{4\pi}\frac{1}{2}A^2$$

$$\boxed{\frac{dP}{d\Omega} = \bar{S}r^2 = \frac{I^2}{2\pi c\sin^2\theta}\left(\cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\frac{k\ell}{2}\right)^2}$$

a) $k\ell \ll 1 \rightarrow \cos\left(\frac{k\ell}{2}\cos\theta\right) - \cos\frac{k\ell}{2} = \frac{1}{2}\left(\frac{k\ell}{2}\right)^2(1-\cos^2\theta) = \frac{k^2\ell^2}{8}\sin^2\theta$

Taylor expand up to 2.d term

4 pts

$$\boxed{\frac{dP}{d\Omega} = \frac{I^2k^4\ell^4\sin^2\theta}{2^7\pi c}}$$

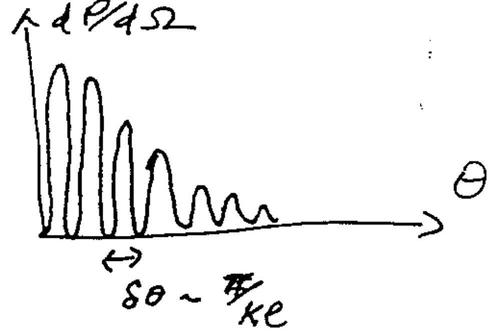
Feeding current $I(z=0) = \frac{Ik\ell}{2} = \omega Q$

$$I = \frac{2\omega C}{k\ell}V = \frac{2c}{\ell}CV$$

C — capacitance

b) When $k\ell \gg 1$, radiated power oscillates as function of θ:

3 pts



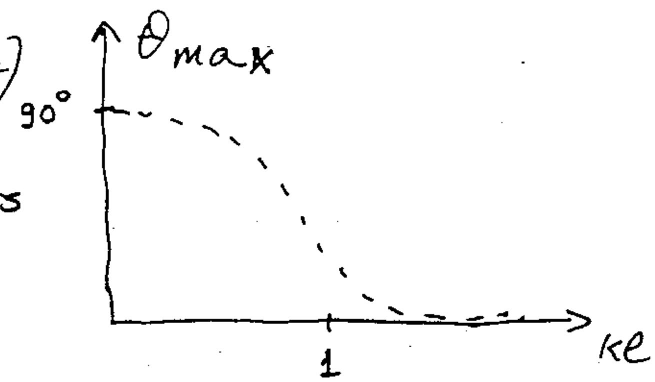
Averaging over this fast oscillation gives

$$\boxed{\left\langle \frac{dP}{d\Omega} \right\rangle \sim \frac{I^2}{\sin^2\theta}}$$

Feeding current $I(z=0) = I \sin\frac{k\ell}{2} = \frac{V}{Z}$ ← impedance

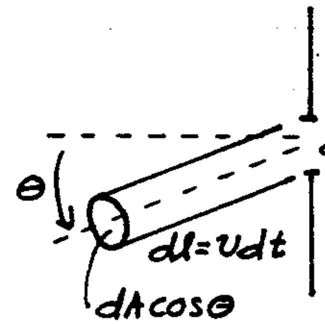
c)

3 pts



STATISTICAL MECHANICS I (SEPT. 97)

a)



$$v \equiv |\vec{v}|$$

$$dN = -n(dA\cos\theta)(v\,dt)$$
$$= -nv\cos\theta\, dA\,dt$$

$$dE = -n\left(\tfrac{1}{2}mv^2\right)(dA\cos\theta)(v\,dt)$$
$$= -\tfrac{1}{2}mnv^3\cos\theta\, dA\,dt$$

FIND $\dfrac{\langle dE/dt\rangle}{\langle dN/dt\rangle} = \dfrac{1}{2}m\dfrac{\langle v^3\rangle}{\langle v\rangle}$ { θ AND φ AVERAGES ARE THE SAME IN THE NUMERATOR AND DENOMINATOR

USE $p(v) \propto \dfrac{v^2}{\sigma^3}e^{-v^2/2\sigma^2}$ $v \geq 0$ { A SIMPLE DERIVATION OF THIS DISTRIBUTION IS GIVEN AT THE END OF THIS SOLUTION

$$\frac{\langle v^3\rangle}{\langle v\rangle} = \frac{\int_0^\infty v^5 e^{-v^2/2\sigma^2}\,dv}{\int_0^\infty v^3 e^{-v^2/2\sigma^2}\,dv}$$

$$\int_0^\infty v^{2n+1} e^{-v^2/2\sigma^2}\,dv = 2\sigma^2(2\sigma^2)^n \underbrace{\int_0^\infty \xi^n e^{-\xi}\,d\xi}_{n!}$$

$$\frac{\langle v^3\rangle}{\langle v\rangle} = \frac{(2\sigma^2)^2\,2}{(2\sigma^2)} = 4\sigma^2$$, NOW USE $\sigma = \sqrt{\frac{kT}{m}}$

$$\langle dE/dt\rangle / \langle dN/dt\rangle = 2m\sigma^2 = \underline{\underline{2kT}}$$

S.M. I

b) $E = \frac{3}{2} NkT$

$$dE = \underbrace{\tfrac{3}{2} Nk\,dT + \tfrac{3}{2} kT\,dN}_{\text{GENERAL EXPANSION}} = \underbrace{2kT\,dN}_{\text{FROM a)}}$$

$$\Rightarrow \frac{dT}{T} = \frac{1}{3}\frac{dN}{N} \Rightarrow \ln\frac{T}{T_0} = \frac{1}{3}\ln\frac{N}{N_0} \Rightarrow \underline{\underline{\frac{T}{T_0} = \left(\frac{N}{N_0}\right)^{1/3}}}$$

DERIVATION OF SPEED DISTRIBUTION:

$$p(v_x, v_y, v_z) = \left(\frac{1}{\sqrt{2\pi\langle v_x^2\rangle}}\right)^3 e^{-\frac{v_x^2+v_y^2+v_z^2}{2\langle v_x^2\rangle}}$$

FROM CANONICAL ENSEMBLE WITH $\langle v_x^2\rangle = \sqrt{\frac{kT}{m}} \equiv \sigma^2$

$P(v) \equiv$ probability $v_x^2 + v_y^2 + v_z^2 \leq v^2$

$$= \left(\frac{1}{\sqrt{2\pi\langle v_x^2\rangle}}\right)^2 \int_0^v e^{-\xi^2/2\langle v_x^2\rangle} 4\pi \xi^2 d\xi$$

USING $dv_x dv_y dv_z = v^2 \sin\theta \, d\theta d\phi$ AND DOING THE θ ϕ INTEGRAL TO GET 4π

$$p(v) = \frac{d}{dv} P(v) = \left(\frac{1}{\sqrt{2\pi\langle v_x^2\rangle}}\right)^3 4\pi v^2 e^{-v^2/2\langle v_x^2\rangle}$$

$$= \frac{2}{\sqrt{2\pi}} \frac{v^2}{\sigma^3} e^{-\frac{v^2}{2\sigma^2}} \quad v > 0$$

STATISTICAL MECHANICS II (SEPT. 97)

a) $\epsilon = c\hbar k$

$$\#(k) = \overset{\text{SPIN}}{2}\left(\tfrac{4}{3}\pi k^3\right)\frac{V}{(2\pi)^3}$$

$$d\#/dk = \frac{1}{\pi^2} k^2 V$$

$$d\#/d\epsilon \equiv D(\epsilon) = \left(\frac{1}{c\hbar}\right)^3 V \epsilon^2$$

$$\bar{n}_f = \frac{1}{e^{(\epsilon-\mu)/kT}+1} \qquad \bar{n}_b = \frac{1}{e^{(\epsilon-\mu)/kT}-1}$$

$$\bar{n} = \left(e^{(\epsilon-\mu)/kT} \pm 1\right)^{-1} \qquad \begin{Bmatrix} f \\ b \end{Bmatrix}$$

$$U = \sum_{\text{STATES}} \epsilon(\vec{k})\,\bar{n}(\epsilon) = \int_0^\infty \epsilon\, D(\epsilon)\,\bar{n}(\epsilon)\,d\epsilon$$

$$= \frac{1}{\pi^2}\left(\frac{1}{c\hbar}\right)^3 V \int_0^\infty \epsilon^3 \left(e^{(\epsilon-\mu)/kT} \pm 1\right)^{-1} d\epsilon$$

GIVEN

$$PV = kT \ln Z = kT \sum_{\text{STATES}} \ln\left(1 \pm e^{-(\epsilon-\mu)/kT}\right)^{\pm 1} \qquad \begin{Bmatrix} f \\ b \end{Bmatrix}$$

$$= \pm kT \int_0^\infty D(\epsilon) \ln\left(1 \pm e^{-(\epsilon-\mu)/kT}\right) d\epsilon$$

$$= \pm\left[kT\left(\frac{1}{\pi^2}\right)\left(\frac{1}{c\hbar}\right)^3 V\right] \int_0^\infty \epsilon^2 \ln\left(1 \pm e^{-(\epsilon-\mu)/kT}\right) d\epsilon$$

INTEGRATE BY PARTS

$$= (-)\left(\begin{smallmatrix}+\\-\end{smallmatrix}\right)\left(\begin{smallmatrix}+\\-\end{smallmatrix}\right)[\;] \int_0^\infty \frac{1}{3}\epsilon^3 \left(1 \pm e^{-(\epsilon-\mu)/kT}\right)^{-1} e^{-\frac{(\epsilon-\mu)}{kT}}\left(-\frac{1}{kT}\right) d\epsilon$$

$$+ 0$$

$$= \frac{1}{3}\frac{1}{kT}[\;] \int_0^\infty \epsilon^3 \left(e^{(\epsilon-\mu)/kT} \pm 1\right)^{-1} d\epsilon$$

$$\underline{\underline{= \frac{1}{3} U}}$$

S.M. II

b) $\sum_i \nu_i \mu_i = 0$ FOR AN EQUILIBRIUM REACTION

NOTE $\mu = 0$ FOR PHOTONS

$\rightarrow (1)\mu_{e_-} + (1)\mu_{e_+} + (1)\,0 = 0$

$\Rightarrow \mu_{e_-} + \mu_{e_+} = 0$, BUT $\mu_{FERMION} \geq 0 \Rightarrow \underline{\underline{\mu_{e_-} = \mu_{e_+} = 0}}$

FROM THE RESULT IN a) WITH $\mu_{e_-} = 0$ ONE HAS

$$\underline{\underline{U_{e_-}/V = \frac{1}{\pi^2}\left(\frac{1}{c\hbar}\right)^3 \int_0^\infty \epsilon^3 \frac{1}{(e^{\epsilon/kT}+1)}\, d\epsilon}}$$

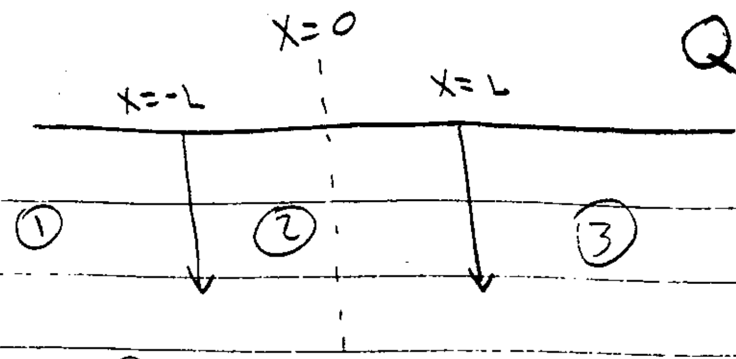
FOR PHOTON THERE IS ALSO A FACTOR OF 2 DEGENERACY DUE, THIS TIME, TO POLARIZATION.

$$U/V\Big|_{PHOTONS} = \frac{1}{\pi^2}\left(\frac{1}{c\hbar}\right)^3 \int_0^\infty \epsilon^3 \frac{1}{e^{\epsilon/kT}-1}\, d\epsilon$$

AND SINCE $P = \frac{1}{3} U/V$ IN EACH CASE

$$\underline{\underline{P_{e_-}/P_\gamma = \frac{\int_0^\infty \epsilon^3 (e^{\epsilon/kT}+1)^{-1} d\epsilon}{\int_0^\infty \epsilon^3 (e^{\epsilon/kT}-1)^{-1} d\epsilon}}}$$

Quantum Mechanics
Problem I



look for bound state

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]\psi(x) = E\psi(x)$$

at the δ function $\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi - \lambda\,\delta(x-L)\right]\psi = E\psi$

$$-\frac{\hbar^2}{2m}\left\{\frac{d\psi}{dx}\Big|_{L+\varepsilon} - \frac{d\psi}{dx}\Big|_{L-\varepsilon}\right\} - \lambda\psi(L) = 0$$

$$\frac{d\psi}{dx}\Big|_{L+\varepsilon} - \frac{d\psi}{dx}\Big|_{L-\varepsilon} = -\frac{2m\lambda}{\hbar^2}\psi(L)$$

3) $\psi = e^{-\alpha x}$ $\quad \alpha > 0$

2) $\psi = A[e^{\alpha x} + e^{-\alpha x}]$ $\quad$ function should be even

1) $\psi = e^{\alpha x}$

$$E = -\frac{\hbar^2}{2m}\alpha^2$$

Quantum Mechanics

Problem I

at L $\quad -\alpha e^{-\alpha L} - \{ A\alpha e^{\alpha L} - A\alpha e^{-\alpha L} \} = -\frac{2m\lambda}{\hbar^2} e^{-\alpha L}$

$$\alpha + A\alpha e^{2\alpha L} - A\alpha = \frac{2m\lambda}{\hbar^2}$$

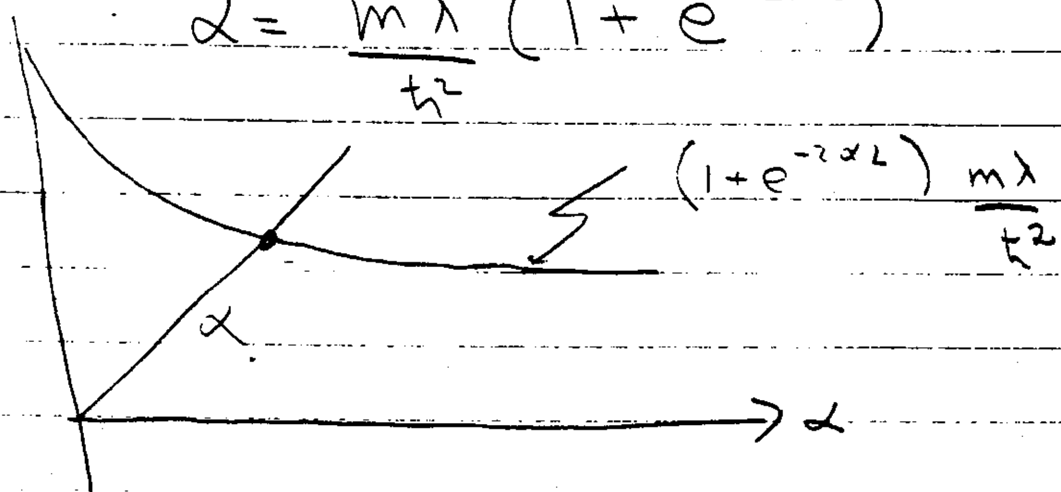
match at L $\quad e^{-\alpha L} = A[e^{\alpha L} + e^{-\alpha L}]$

$$1 = A[e^{2\alpha L} + 1]$$

$$\alpha \left\{ 1 + \frac{(e^{2\alpha L} - 1)}{(e^{2\alpha L} + 1)} \right\} = \frac{2m\lambda}{\hbar^2}$$

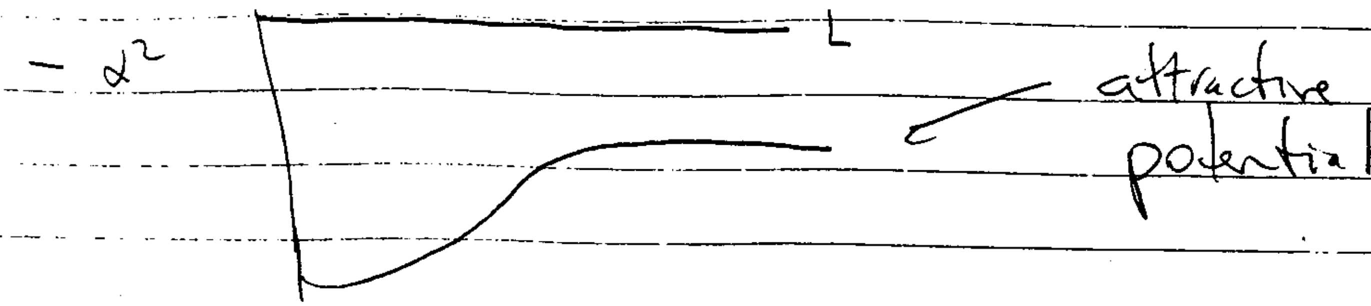
$$2e^{2\alpha L} = \frac{2m\lambda}{\hbar^2 \alpha}(e^{2\alpha L} + 1)$$

$$\alpha = \frac{m\lambda}{\hbar^2}(1 + e^{-2\alpha L})$$



L gets big α gets smaller

Problem I. 3

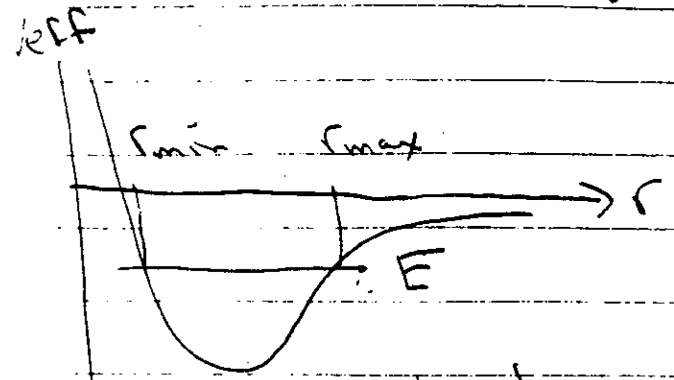


α approaches $\frac{m\lambda}{\hbar^2}$ as L gets big

Quantum Mechanics II

3 dimensional problem

1) $V(r) \to V(r) + \frac{\ell(\ell+1)\hbar^2}{2mr^2} = V_{eff}(r)$



WKB quantization

$$\frac{1}{\hbar}\int_{r_{min}}^{r_{max}} \left(2m\left(E - V_{eff}(r')\right)\right)^{1/2} dr' = (n + 1/2)\pi$$

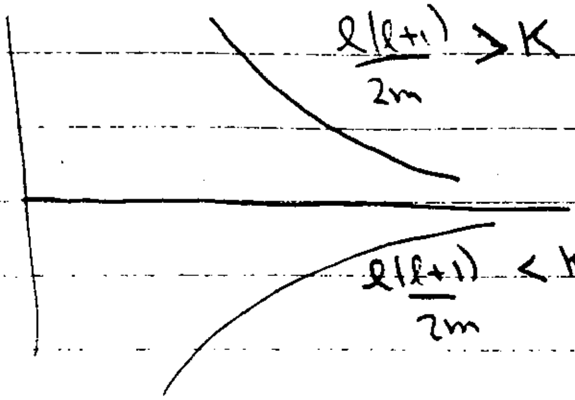
In a given $V(r)$ find the maximum E which gives (classically) bound motion in the potential $V(r)$. Find the associated r_{min} and r_{max}. Compute n with the above formula.

For ℓ there are $(2\ell+1)$ states with each energy so you need to multiply by $(2\ell+1)$

Quantum Mechanics II 2

b) For large r suppose $V(r) = -K/r^2$

$$V_{eff} = \frac{-K}{r^2} + \frac{\ell(\ell+1)\hbar^2}{2mr^2} \qquad K > 0$$



no band states

Band states

max E in the case when there are band states is 0 $\quad r_{max} = \infty$

$$\int^{\infty} \sqrt{\left(K - \frac{\ell(\ell+1)\hbar^2}{2m}\right)}\, \frac{dr'}{r'} = \infty$$

There are an ∞ #

Suppose $V(r) \sim \frac{-K}{r^{2-\varepsilon}}$: more slowly than $\frac{1}{r^2}$

for $\varepsilon > 0$

Now as $r \to \infty$ $V_{eff} \sim \frac{1}{r^{2-\varepsilon}}$

$$\int^{\infty} \sqrt{\frac{1}{(r')^{2-\varepsilon}}}\, dr' = \infty$$

# of band states