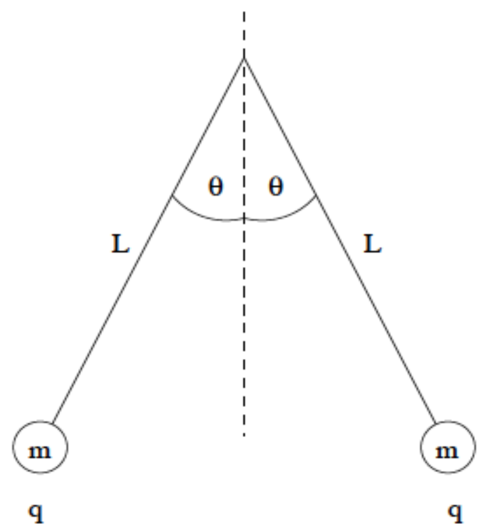
Physics Qualifying Examination – Part I 7-Minute Questions

February 19, 2011

1. Two small spheres of mass m, each with positive charge of magnitude q, are suspended from a common point by massless threads of length L. Each thread makes an angle θ with the vertical as shown in the figure. The magnitude of the electric field from a point charge q is kq/r^2. Find an expression for the charge q at equilibrium in terms of L, m g, θ and the Coulomb constant k.

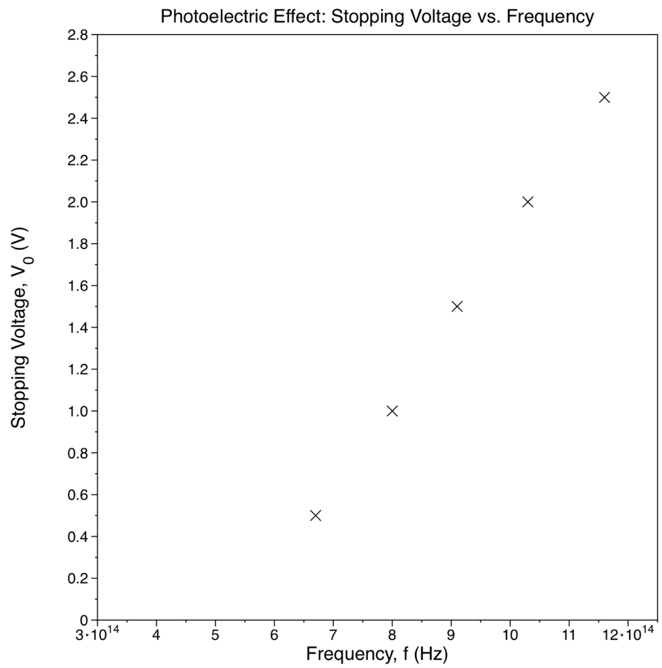


2. Draw a diagram to show the energy levels associated with $n = 3,\ l = 2$ and with $n = 3,\ l = 1$ for the sodium atom. Label the levels by the appropriate quantum numbers and by the spectroscopic notation. You should neglect nuclear spin, but you must consider electron spin. Show all the electric-dipole-allowed radiative transitions among these levels and cite the relevant selection rules.

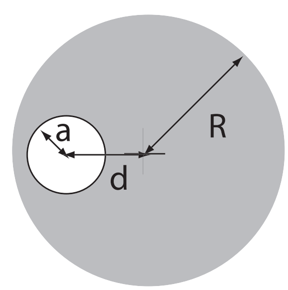
3. A series RLC circuit, driven with an emf ε_{rms} =120 V at frequency f_d = 60 Hz, contains a resistor R = 200 Ω, an inductive impedance X_L = 80 Ω, and a capacitive impedance X_C = 150 Ω.

a. What is the average rate P_{ave} at which energy is dissipated in the resistor?
b. What capacitance is needed to maximize the average rate P_{ave} if the other parameters of the circuit are not changed? What is the maximum value of P_{ave}?

4. The graph below shows a plot of the stopping voltage as a function of the frequency of light incident on a metal surface. Based on the data shown, what is the work function of the metal in units of eV?

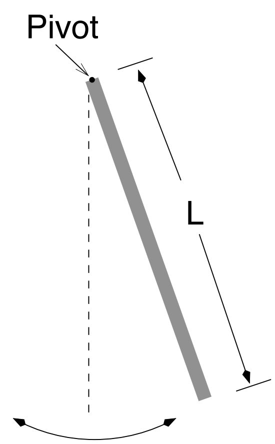


5. A very long cylindrical conducting wire of radius R has a uniform current density J running through it. Inside the conductor, a cylindrical cavity of radius a has been drilled which runs the length of the wire for water cooling. It is displaced from the center of the wire by a distance d, as shown in the figure. Find the magnetic field in the cavity.



6. A pion at rest decays into a muon plus a neutrino. Assume the neutrino is massless. The pion rest mass is 139 MeV/c^2 and the muon rest mass is 105 MeV/c^2. Find the kinetic energy of the muon.

7. A uniform thin rod of length L and mass m is suspended from one end. Derive the angular frequency of the harmonic motion (assume small amplitude) for this pendulum.



8. A mixture of ice and water at an initial temperature of 0° C completely fills a closed container that has the shape of a cube 10 cm on each edge. In the water are 10 ice cubes, each of which measures 2 cm x 2 cm x 2 cm. The container is made of plastic that has a thickness of 4 mm and a thermal conductivity of 0.2 W/m·K. The container is located in a room where the air temperature is 27° C. Assume that the thermal conductivities of water and ice are large enough that the temperature is uniform in the mixture, and assume that ice and water both have a density of 1 g/cm^3. The latent heat of melting for water is 334 J/g and the specific heat of water is 4.187 J/g·K.

a. Approximately how long will it take the ice cubes to melt?
b. After the ice cubes have completely melted, how long will it take the water temperature to rise to 10° C?

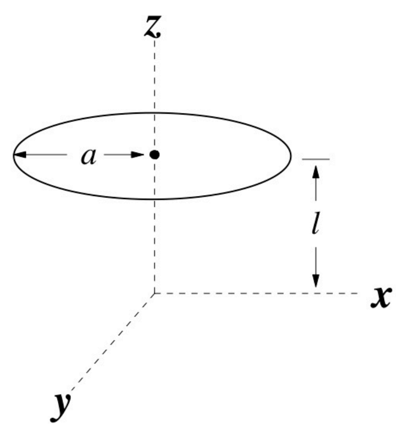
9. Suppose that we have a laser that operates at a wavelength of λ=1 micron. The power of the laser is 10 microwatts.

a. How many photons per second is the laser emitting?
b. Suppose that we have an ideal detector (no noise) that detects the photons from the laser and records the data in 1 picosecond time intervals. What is the average number of photons that the detector will detect for every one picosecond?
c. Find the approximate fractional "sample to sample" fluctuation in the detected number of photons in 1 picosecond samples.
d. Find the approximate fractional "sample to sample" fluctuation in the detected number of photons in 1 microsecond samples.

10. A magnetic field

$$\mathbf{B}(r,z) = -\frac{B_0 r}{2L}\hat{\mathbf{r}} + \frac{B_0 z}{L}\hat{\mathbf{z}}$$

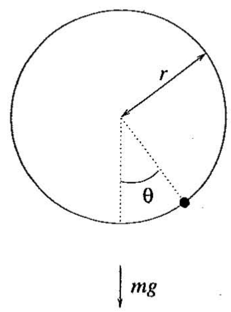
is suddenly switched on from zero so that it reaches its full value in a very short time τ. The hoop is positioned with its center at $z = l$, $r = 0$, and its axis aligned with the z axis. Determine the force in the z direction on a conducting circular hoop of radius a and resistance R, during the short time of turn on.



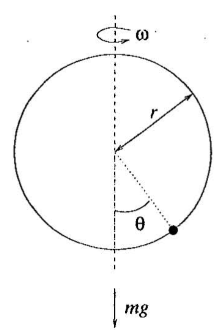
Physics Qualifying Examination – Part II 12-Minute Questions

February 19, 2011

1. Consider a point particle with mass m confined to move along a frictionless ring of radius r in the presence of a uniform gravitational field whose strength is g.



a. Using θ and $\dot{\theta}$ as dynamical variables, write down an explicit expression for the kinetic energy and the potential energy of the particle. Show that $\dot{\theta}=0$, $\theta=0$ is the lowest energy configuration of this system.
b. Now consider rotating this ring along the vertical axis with angular velocity ω. Find the critical ω_c so that for $\omega>\omega_c, \theta=0$ is no longer the minimum energy configuration. Find θ which minimizes the energy as a function of ω for $\omega>\omega_c$.



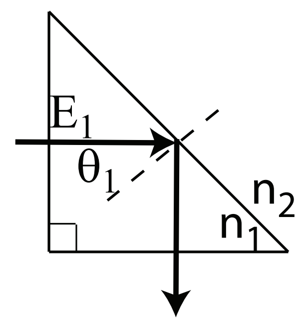
2. An "impedance" of a transmission line is the square root of the ratio of the inductance per unit length to the capacitance per unit length. For a coaxial cable with inner radius r and outer radius b (with no dielectric medium), compute the impedance. A possibly useful integral:

$$\int_0^\infty dx \frac{1}{(1+x^2)^{3/2}} = 1$$

3. Two identical non-interacting spin ½ particles move in a one-dimentional box which spans from $x = 0$ to $x = L$. The potential is zero inside the box and infinite at the walls. The normalized energy eigenstates are $\psi_n = (2/L)^{1/2}\sin(n\pi x/L)$.

a. If single-particle spin eigenstates (up and down) are denoted by $|\uparrow\rangle \equiv u$ and $|\downarrow\rangle \equiv d$, construct all two-particle spin states.
b. What are the ground state energy and eigenfunction(s) including both the spin and spatial components?
c. What are the 1st excited state energy and eigenfunction(s) including both the spin and spatial components?

4. Light is internally reflected on the hypotenuse of the prism as shown in the figure. The incidence angle is $\theta_1 = 45^\circ$. The refractive index of the prism is n_1 and outside the prism $n_2 = 1$. Find the condition on n_1 such that 100% of the light energy is reflected (total internal reflection).



Assume there is a nonlinear medium outside the prism with index of refraction $n_2 = 1 + \kappa |E_2|^2$ where E_2 is the electric field amplitude. For the above geometry ($\theta_1 = 45^\circ$ and light polarized perpendicular to the plane of the drawing, find the values of κ for which there is no longer total internal reflection. Assume $\kappa |E_2|^2 \ll 1$. **Your answer should depend on n_1 and the incident field amplitude E_1 only.** You may use the result of the Fresnel transmission coefficient: $t = \frac{E_2}{E_1} = 2n_1\cos\theta_1/(n_1\cos\theta_1 + n_2\cos\theta_2)$.

5. A point particle of mass M_1 is incident with the velocity v_0 and impact parameter s on a sphere of radius R and mass M_2 that is initially at rest. Assume that the two masses attract each other according to Newton's law of gravitation. Determine the cross section for collisions where mass M_1 actually hits the sphere. Hint: The problem can be solved by making use of conservation laws.

6. In magnetic resonance imaging (MRI) of the human body, the energy level of the proton in the hydrogen atoms of water splits in a magnetic field.

a. Draw the energy level diagram with and without a magnetic field. How many levels are there? What are their quantum numbers?
b. Write down the energy U of a proton with magnetic moment μ in a magnetic field $\boldsymbol{B}$.
c. Calculate the level splitting $\Delta\boldsymbol{E}$ (in eV) for the magnetic field $B = 1\ \mathrm{T}$.
d. Is the splitting large or small compared to the thermal energy at room temperature?
e. Give the probabilities $p_{\pm}$ for the magnetic moment of the proton being parallel and anti-parallel to the B-field (both the general formula and numerical values).
f. How big is the difference ($p_+ - p_-$), which is related to the magnetization? (Hint: use a Taylor expansion of the exponential.)

The magnetic moment of the proton is $\mu_p = 1.41 \times 10^{-26}\ \mathrm{J/T}$.

7. The pressure of the radiation from a black body is given by $p = AT^4$ where $A = \pi^2 k_B^4 / 45(\hbar c)^3$ is a constant. Use this relation to obtain the energy of the black body radiation $E(T,V) = \varepsilon(T)V$ where V is the volume.

a. From the first law of thermodynamics obtain $\varepsilon = T\left(\dfrac{\partial S}{\partial V}\right)_T - p$
b. Derive the Maxwell relation for $\left(\dfrac{\partial S}{\partial V}\right)_T$ from the Helmholz Free Energy $(F = E - TS)$.
c. Using the above relations, evaluate $\varepsilon(T)$.

8. The Hamiltonian for a rigidly rotating system is

$$H = AL^2 + BL_z + CL_x .$$

For $A, B \gg C$, find the energy eigenvalues to leading nonvanishing order in C.

9. An electron is trapped in a quantum dot with the aim of preparing its spin state at time $t_i = 0$ and measuring the spin state at a later time t_f. A time-independent magnetic field B is applied in the z-direction. Thus the Hamiltonian is:

$$H = -\mu B \sigma_z ,$$

where μ is the magnetic moment and the Pauli spin matrices are:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The coordinates are chosen so that the up-spin wavefunction is given by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and the down-spin wavefunction is given by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. The electron is prepared at $t = 0$ in a state such that a measurement of the spin component along the x-direction yields the result $+\frac{\hbar}{2}$ with 100% probability. This means that the expression for the spin wavefunction at $t = 0$ is

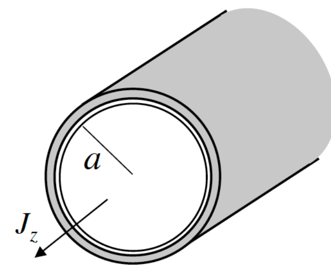
$$\psi(t=0) = \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

a. What is the spin wavefunction at the measurement time t_f?

b. Find the probability that the result is $+\frac{\hbar}{2}$ if the $x-$component of the spin is measured at $t = t_f$.

10. A fully ionized (hot, low density) electron-proton plasma has a temperature T and density $n_e = n_i = n$. Estimate how long (in time) an electron having thermal velocity travels before its trajectory is distorted by a large angle collision with a proton.

11. Consider a long cylindrical plasma column inside a perfectly conducting shell. The plasma is described by MHD force balance, $\rho d\mathbf{v} / dt = \mathbf{J} \times \mathbf{B} - \nabla p$, and Ohm's law $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$. The plasma is separated from the conducting shell by a thin vacuum layer. The plasma pressure vanishes at the plasma radius a.

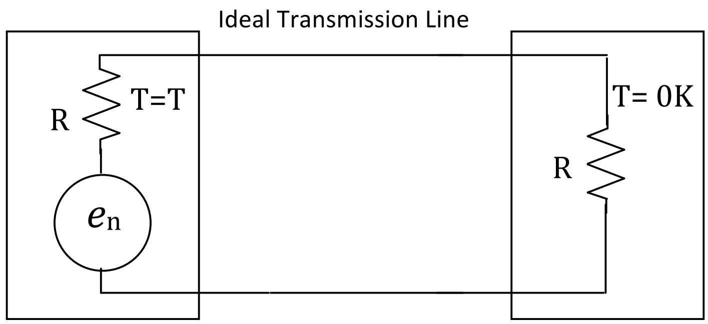


a. Assume a uniform axial current density is established in the plasma. In steady state, what is the equilibrium plasma pressure, p, at the cylindrical origin $r = 0$ in terms of the current density, J_z, and the plasma radius, a?
b. If the plasma has small but finite electrical resistivity, η, give an example of how the plasma current might be maintained in steady state.

12. A non-relativistic rocket is launched from rest into vertical motion from the Earth's surface. The exhaust gas speed relative to the rocket is u, the initial mass is $m_0 = m_p + m_f$ where m_f is the fuel mass and m_p the payload mass. The gas mass flow rate as a function of time is $-\dot{m}(t) \geq 0$. The acceleration of gravity is g. The vertical coordinate is $y(t)$ and the speed is $\mathbf{v}(t)$. Neglect drag and assume u and g are constant.

a. Give the equation of motion describing the acceleration of the rocket at any time for $t > 0$.
b. Integrate to show that $\mathrm{v}(t) = u \ln \left(\dfrac{m_0}{m(t)} \right) - gt$.
c. Find the mass flow rate which gives a constant acceleration value a and express i) the time and ii) altitude at burnout (when the fuel is gone) and iii) the maximum altitude at which the payload comes to rest. Express in terms of a and the other parameters given.

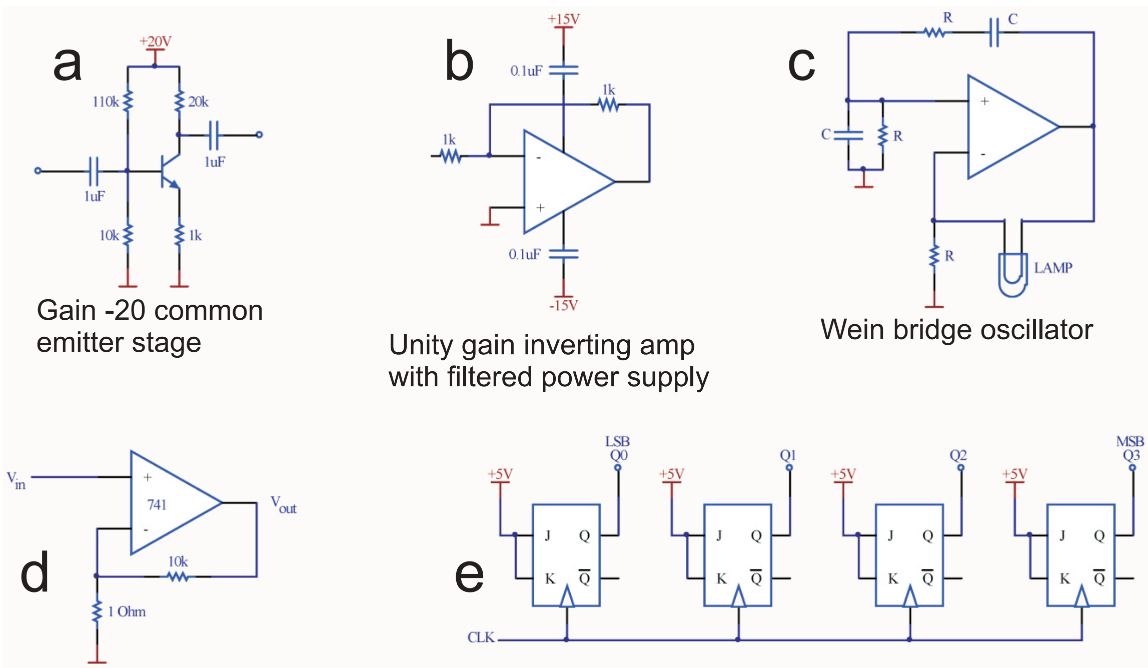
13. Two identical ideal resistors are connected by an ideal transmission line that transmits all frequencies without loss. The resistor on the right is maintained at absolute zero temperature. The one on the left is maintained at temperature T. The resistors are a perfect match to the transmission line at all frequencies and absorb all the power entering from the transmission line, reflecting none.



A resistor at finite temperature T generates a randomly varying thermal voltage called "Johnson noise". The r.m.s. value of this is conventionally given as $V_{r.m.s.} = \sqrt{4k_B TR\Delta f}$, where k_B is Boltzmann's constant and Δf is the bandwidth of the measurement, or as a spectral density, $e_n = \sqrt{4k_B TR}$ volts/sqrt(Hz), assumed to be independent of frequency, or "white". You can model this as a noise-free resistor in series with a voltage source equal to the Johnson noise, as shown at the left.

a. Given this magnitude for the Johnson noise, calculate the net power transferred per unit bandwidth (1 Hz) from the resistor on the left to the one on the right.
b. If you integrate this power to arbitrarily high frequencies, you will get arbitrarily large powers transferred. Why doesn't this really happen?
c. Johnson noise is usually discussed in connection with electronic circuits. Why is it usually presented as being independent of frequency?
d. Name a 3-d analogy of this 1-d thermal noise.

14. Indicate why the 5 following circuits will fail to work as advertised. (3 points per correct answer up to a maximum of 10 points.)



Broadband op amp stage with gain 10^4

4 bit synchronous counter

15. A horizontal circular ring laser gyroscope operates as a Sagnac interferometer at the North Pole. The cavity is made from a large number (>>10) of mirrors used at near grazing incidence. The radius of the ring is 1 m and the laser wavelength is 1 micron. Find the beat frequency in Hz due to the Earth's rotation. (Hint: A simple analysis can be based on the assumption that this laser has clockwise and counter-clockwise modes lasing at the exact same frequency and producing a stable pattern of nodes and anti-nodes when viewed from an inertial (non-rotating) frame.)