# Solutions: Part I
## Fall, 2000

I-1

$$\mathcal{H} = \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + ax^2 + by^2 + mgz$$

By equipartition theorem, each quadratic term contribute $\frac{1}{2}kT$ to total energy.

$$\overline{mgz} = \frac{\int_0^\infty mgz\, e^{-mgz/kT}\, dz}{\int_0^\infty e^{-mgz/kT}\, dz} = kT.$$

So, $E = N E_1 = N(\frac{5}{2}kT + kT) = \frac{7}{2}NkT.$

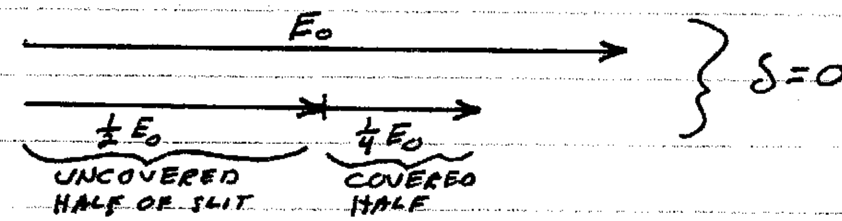
$$C = \frac{\partial E}{\partial T} = \frac{7}{2}Nk$$

I-2

USE A GRAPHICAL REPRESENTATION OF ADDING THE COMPLEX $\vec{E}$ FIELDS. LET $\delta$ BE THE PHASE DIFFERENCE ACROSS THE SLIT OR A PART OF IT.

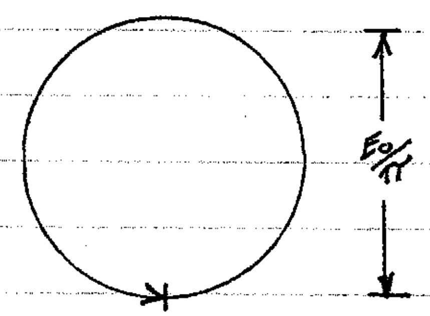
$\underline{X=0}$ BEFORE

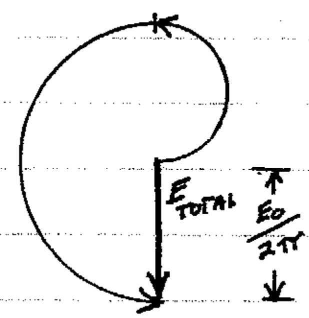
AFTER




$$\Rightarrow \frac{\langle I\rangle}{\langle I_0\rangle} = \frac{\left(\frac{3}{4}E_0\right)^2}{E_0^{\,2}} = \underline{\underline{\frac{9}{16}}}$$

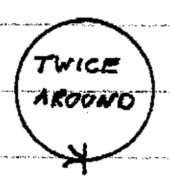
$\underline{X = X_0}$ BEFORE ($\delta = 2\pi$) AFTER ($\delta = \pi$ FOR EACH HALF)

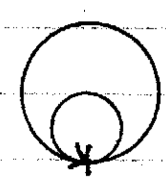



$$\Rightarrow \frac{\langle I\rangle}{\langle I_0\rangle} = \underline{\underline{\frac{1}{4\pi^2}}}$$

$\underline{X = 2X_0}$ BEFORE ($\delta = 4\pi$) AFTER ($\delta = 2\pi$ FOR EACH HALF)



$$\Rightarrow \frac{\langle I\rangle}{\langle I_0\rangle} = \underline{\underline{0}}$$

Tom Greytak

I-3

a) SINGLE PARTICLE STATES $\psi_{n_x n_y n_z} = \psi_{n_x}(x)\,\psi_{n_y}(y)\,\psi_{n_z}(z)$

2-PARTICLE STATES CORRESPONDING TO 2 LOWEST ENERGIES

$\psi_{000}(1)\,\psi_{000}(2)$ SYM. $E = 2\times(\frac{3}{2}\hbar\omega) = 3\hbar\omega$

$\frac{1}{\sqrt{2}}\left(\psi_{000}(1)\psi_{100}(2) + \psi_{000}(2)\psi_{100}(1)\right)$ SYM. $E = 4\hbar\omega$

$\frac{1}{\sqrt{2}}\left(\psi_{000}(1)\psi_{100}(2) - \psi_{000}(2)\psi_{100}(1)\right)$ ANTI. $E = 4\hbar\omega$

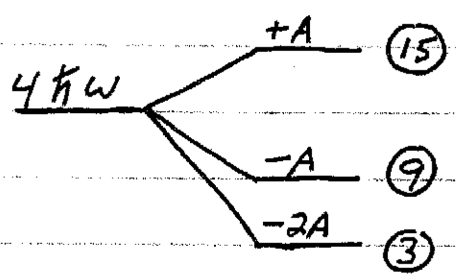
+ SIMILAR PAIRS FOR y + Z EXCITATION

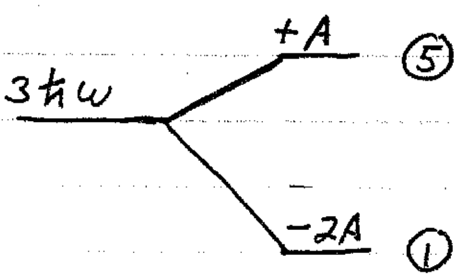
<u>SPIN</u> $1+1 \rightarrow 2, 1, 0$
$$\begin{cases} |S=2, m_S\rangle & \text{SYM.} \quad 5 \text{ STATES} \\ |S=1, m_S\rangle & \text{ANTI.} \quad 3 \text{ STATES} \\ |S=0, m_S=0\rangle & \text{SYM} \quad 1 \text{ STATE} \end{cases}$$

$4\hbar\omega$ ——— $\left.\begin{matrix}(3 \text{ ANTI. SPACE})\times(3 \text{ ANTI. SPIN}) = 9 \\ (3 \text{ SYM. SPACE})\times(6 \text{ SYM. SPIN}) = 18\end{matrix}\right\}$ 27

$3\hbar\omega$ ——— $(1 \text{ SYM. SPACE})\times(6 \text{ SYM. SPIN}) =$ 6

b) $$\vec{S}_1\cdot\vec{S}_2 = \frac{S_T(S_T+1) - 1(1+1) - 1(1+1)}{2} = \frac{S_T(S_T+1)}{2} - 2 \begin{cases} 1 & \text{IF } S_T=2 \\ -1 & \text{IF } S_T=1 \\ -2 & \text{IF } S_T=0 \end{cases}$$

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I-4

Electron has maximum momentum when neutrinos move in opposite direction

$\longleftarrow p_{\bar{\nu}_e}$

$\longleftarrow p_{\nu_\mu} \qquad \longrightarrow p_e$ in $\mu$ rest frame, $c=1$

Let $\vec{p}_\nu = \vec{p}_{\nu_e} + \vec{p}_{\nu_\mu}$. max when $\vec{p}_\nu = -\vec{p}_e$

$|\vec{p}_\nu| = |\vec{p}_e| \equiv p$

$$m_\mu = E_\nu + E_e \qquad E_\nu = p_\nu \quad (\nu \text{ massless})$$

$$m_\mu = p + (p^2 + m_e^2)^{1/2}$$

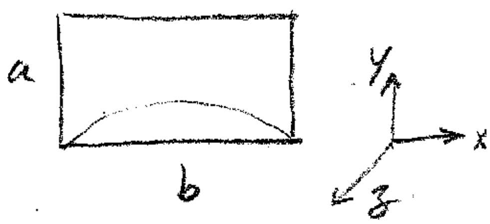
$$(m_\mu - p)^2 = p^2 + m_e^2 \qquad m_\mu^2 - 2m_\mu p + \cancel{p^2} = \cancel{p^2} + m_e^2$$

$$p = \frac{m_\mu^2 - m_e^2}{2m_\mu}$$

$$E_e = (p^2 + m_e^2)^{1/2} = \left[\frac{m_\mu^4 - 2m_\mu^2 m_e^2 + m_e^4}{4m_\mu^2} + m_e^2\right]^{1/2}$$

$$= \left[\frac{(m_\mu^2 + m_e^2)^2}{4m_\mu^2}\right]^{1/2} = \frac{m_\mu^2 + m_e^2}{2m_\mu}$$

II-1 (x II-2)




$$\nabla^2 \vec{E} = \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2}$$

transverse: $\vec{E} = E(x)\hat{y}\, f(z,t)$

take $\vec{E} = E_0 \hat{y} \sin \frac{x}{b}\pi \, e^{i(kz-\omega t)}$

wave equation $\rightarrow \left(\frac{\pi}{b}\right)^2 + k^2 = \left(\frac{\omega}{c}\right)^2$

$\omega = \sqrt{c^2 k^2 + \omega_0^2}$ $\quad \omega_0 = \frac{\pi}{b} c$ = min. frequency

$$v_p = \frac{\omega}{k} = \frac{1}{k}\sqrt{c^2k^2+\omega_0^2}$$

$$v_g = \frac{d\omega}{dk} = \frac{c^2 k}{\sqrt{c^2k^2+\omega_0^2}}$$

$$v_p = 2v_g \rightarrow \frac{1}{k}\sqrt{c^2k^2+\omega_0^2} = \frac{2c^2k}{\sqrt{c^2k^2+\omega_0^2}}$$

$$2c^2k^2 = c^2k^2 + \omega_0^2$$

$$\omega_0^2 = c^2k^2$$

$$\omega = \sqrt{2}\,\omega_0$$

II-2 a) $\frac{\Delta E}{E} = \frac{\Delta \Phi}{c^2} = \frac{gh}{c^2} \sim \frac{10 \times 30}{(3\times10^8)^2} = \frac{300}{9\times10^{16}} = 3\times10^{-15}$

b). $\Delta E = h\Delta\nu$, $\frac{\Delta E}{E} = \frac{\Delta\nu}{\nu}$

Doppler shift. $\frac{\Delta\nu}{\nu} = \frac{v}{c}$, $v = 3\times10^{-15}c$

$= 9\times10^{-7}$ m/sec.

$\sim 10^{-6}$ m/s

II - 3.

$$\mathcal{H} = K + V = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 + V$$

$$L = m r^2 \dot{\theta}$$

$$\mathcal{H} = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + V = \frac{1}{2} m \dot{r}^2 + V_{eff}$$

$$V_{eff} = V + \frac{L^2}{2mr^2}$$

$$\vec{F} = -\frac{A}{r^3}\hat{r}, \quad V = -\frac{A}{2r^2}$$

$$V_{eff} = -\frac{A}{2r^2} + \frac{L^2}{2mr^2}$$

For $\underline{L^2 = -mA}$, $V_{eff} = 0$, $\dot{r} = \text{constant}$

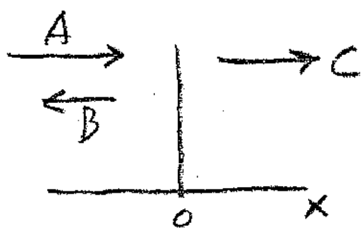
$$\frac{dr}{dt} = v, \quad \frac{d\theta}{dt} = \frac{L}{mr^2} \longrightarrow \frac{dr}{d\theta} = \frac{mr^2 v}{L}.$$

$$\frac{dr}{r^2} = \frac{mv}{L} d\theta \qquad \frac{1}{r_0} - \frac{1}{r} = \frac{mv}{L}(\theta - \theta_0)$$

take $\theta_0 = 0$. $\quad \frac{1}{r} = \frac{1}{r_0} - \frac{mv}{L}\theta$

$$r = \frac{1}{\frac{1}{r_0} - \frac{mv\theta}{L}} = \frac{r_0}{1 - \frac{mvr_0}{L}\theta}$$

II - 4.




$x<0 \quad \psi = Ae^{i(kx-\omega t)} + Be^{i(-kx-\omega t)}$

$x>0 \quad \psi = Ce^{i(kx-\omega t)}$

continuity at $x=0$: $A+B=C$.

$$\frac{\partial^2\psi}{\partial^2 x} + \frac{2m}{\hbar^2}(E-V(x))\psi = 0$$

$$\left.\frac{\partial\psi}{\partial x}\right|_{0-}^{0+} = -\frac{2m}{\hbar^2}\int_{0-}^{0+}(E-K\delta(x))\psi\,dx = \frac{2mK}{\hbar^2}C$$

$$ik(C-A+B) = \frac{2mK}{\hbar^2} \quad , \text{let } \alpha = -\frac{2m}{\hbar^2 k}K$$

$$C-A+B = i\alpha C$$

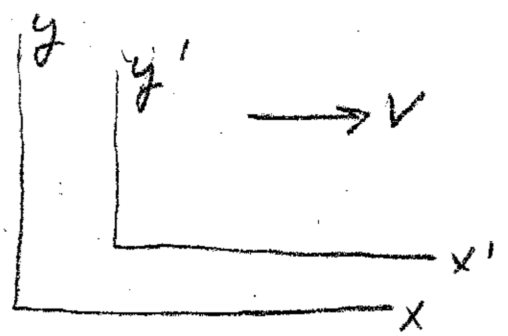
$$C(1-i\alpha) = A-B$$

$$\underline{C \qquad = A+B}$$

$$\frac{C}{A} = \frac{1}{1-i\frac{\alpha}{2}} \quad , \text{fraction transmitted: } \left|\frac{C}{A}\right|^2 = \frac{1}{1+\left(\frac{\alpha}{2}\right)^2}$$

$$\frac{B}{A} = \frac{C}{A}-1 = \frac{i\frac{\alpha}{2}}{1-i\frac{\alpha}{2}}, \quad \text{reflected } \left|\frac{B}{A}\right|^2 = \frac{\left(\frac{\alpha}{2}\right)^2}{1+\left(\frac{\alpha}{2}\right)^2}.$$

III-1.




relativistic addition of velocities, x direction

$$v_x = \frac{v_x' + V}{1 + \frac{v_x' V}{c^2}}$$

in x'-y' frame, $v_x' = \frac{c}{n}$

$$v_x = \frac{\frac{c}{n} + V}{1 + \frac{V}{nc}} \cong \left(\frac{c}{n} + V\right)\left(1 - \frac{V}{nc}\right)$$

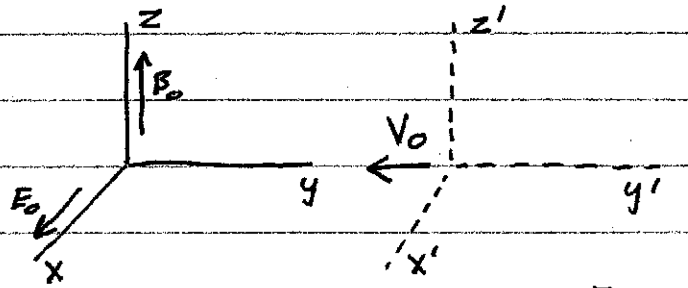
$$= \frac{c}{n} + V - \frac{V}{n^2} + O\left(\frac{V^2}{c}\right)$$

$$\cong \frac{c}{n} + V\left(1 - \frac{1}{n^2}\right)$$

## III-2

$$\vec{E} = E_0\hat{x}, \quad \vec{B} = B_0\hat{z}, \quad \vec{F} = Q\left[\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right] = Q\left[E_0\hat{x} + \frac{B_0}{c}\,\vec{v}\times\hat{z}\right]$$

CHANGE TO A MOVING COORDINATE SYSTEM TO ELIMINATE $\vec{E}$ TERM

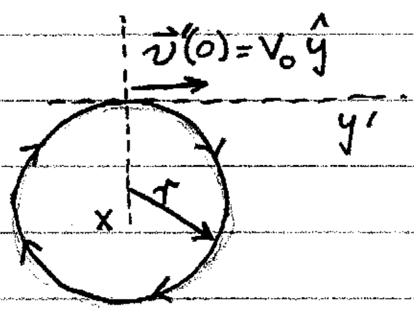



$$x = x', \quad z = z', \quad y = y' - V_0 t$$

$$v_x = v_x', \quad v_z = v_z', \quad v_y = v_y' - V_0$$

$$\vec{F}' = Q\left[\left(E_0 - \frac{V_0 B_0}{c}\right)\hat{x} + \frac{B_0}{c}\,\vec{v}'\times\hat{z}\right] = \frac{QB_0}{c}\left(\vec{v}'\times\hat{z}\right) \quad \left\{ \begin{array}{l} \text{AFTER SETTING} \\ V_0 = cE_0/B_0 \end{array}\right.$$

THIS GIVES UNIFORM CIRCULAR MOTION IN THE MOVING FRAME




$$\vec{F}' = Q\,\frac{B_0}{c}\,V_0\,\hat{x} \quad \text{INITIALLY}$$

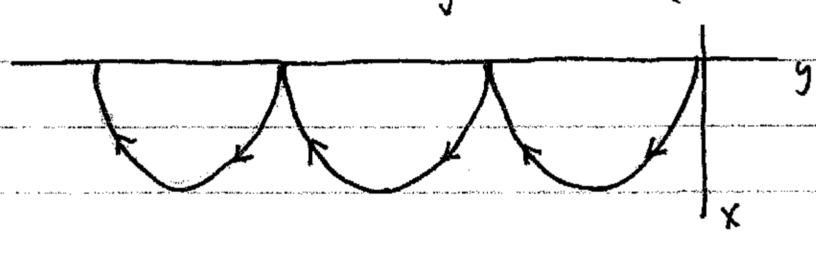
$$\Rightarrow m V_0^2/r = Q B_0 V_0/c$$

$$\Rightarrow r = \frac{mcV_0}{QB_0} = \underline{\underline{\frac{mc^2 E_0}{QB_0^2}}}$$

$$\omega = V_0/r = \underline{\underline{\frac{QB_0}{mc}}}$$

$$\underline{\underline{x = r\left(1 - \cos(\omega t)\right)}}$$

$$y' = r\sin(\omega t) \Rightarrow y = r\sin(\omega t) - V_0 t = \underline{\underline{r\left(\sin(\omega t) - \omega t\right)}}$$




Tom Greytak

III-3

Rotating coordinate transformation

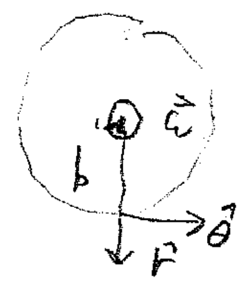
$$\vec{a}_{rot} = \vec{a} - 2\vec{\omega}\times\vec{v}_{rot} - \vec{\omega}\times(\vec{\omega}\times\vec{r})$$

a) $F' = \vec{F} - 2m(\vec{\omega}\times\vec{v}') - \vec{\omega}\times(\omega\times\vec{r})$

$$\xrightarrow[F=0]{} -2m(\vec{\omega}\times\vec{v}') - m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

$\vec{\omega}\times\vec{r} = \omega b\hat{\theta}$

$\vec{\omega}\times(\vec{\omega}\times\vec{r}) = -\omega^2 b\hat{r}$




b) i) $\vec{v}' = 0$

$\vec{F}' = -m(-\omega^2 b\hat{r}) = \underline{m\omega^2 b\hat{r}}$

ii) $\vec{v}' = -\omega b\hat{\theta}$ $\quad \vec{\omega}\times\vec{v}' = -\omega b(-\omega\hat{r})$

$\vec{F}' = -2m(\omega^2 b\hat{r}) + m\omega^2 b\hat{r} = \underline{-m\omega^2 b\hat{r}}$

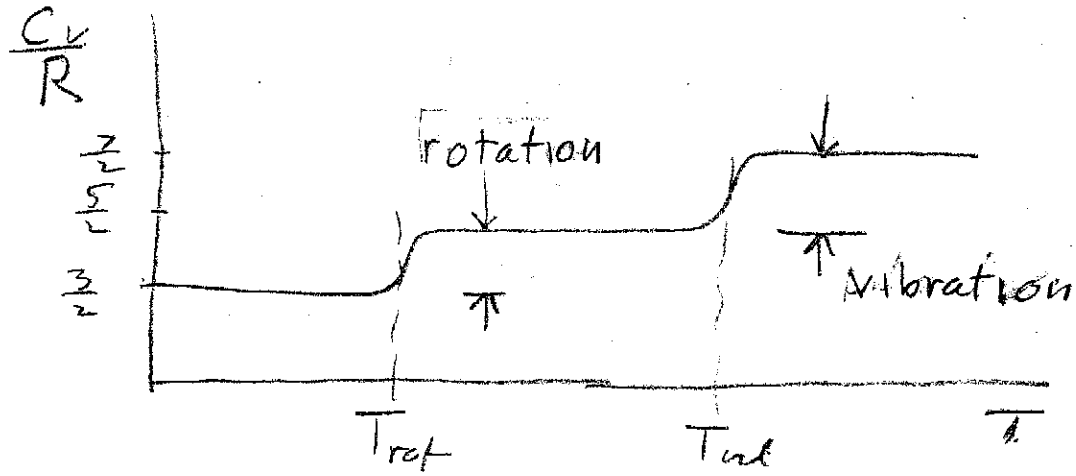
iii) $\vec{F}' = -m\omega^2 b\hat{r} + m\omega^2 b\hat{r} = 0.$

c) i) $v' = 0$, in lab, ball moves to right at speed $\omega b$

ii) $v' = \omega b$. in lab, ball at rest.

iii) $v' = \frac{\omega b}{2}$, in lab, ball moves to right at speed $\omega b/2$

III − 4:




$$E = \frac{p^2}{2m} + E_{rot} + E_{vib}$$

$\frac{p^2}{2m}$ contribute $3 \times \frac{1}{2} RT = \frac{3}{2} RT$

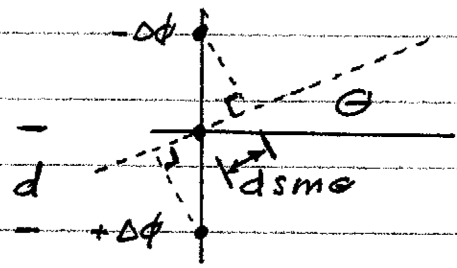
$$E_{rot} = \frac{L^2}{2I} = \frac{\hbar^2 \ell(\ell+1)}{2I} \quad \ell = 0, 1, 2. \quad (2 \text{ axes})$$

$E_{rot}$ freeze out when lowest state is not excited. $\frac{\hbar^2}{I} \sim k T_{rot}$.

$$E_{vib} = \left(n + \tfrac{1}{2}\right) \hbar \omega.$$

$E_{vib}$ freeze out when $k T_{vib} \sim \hbar \omega$

IV - 1

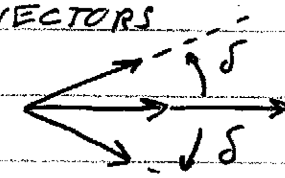



LET $\underline{\underline{\delta \equiv \Delta\phi + 2\pi\left(\frac{d}{\lambda}\right)\sin\theta}}$

a) ALGEBRA

$$E = 2E_0 + E_0 e^{-i\delta} + E_0 e^{i\delta}$$
$$= 2E_0 + 2E_0\cos\delta$$
$$= 2E_0(1+\cos\delta)$$

VECTORS

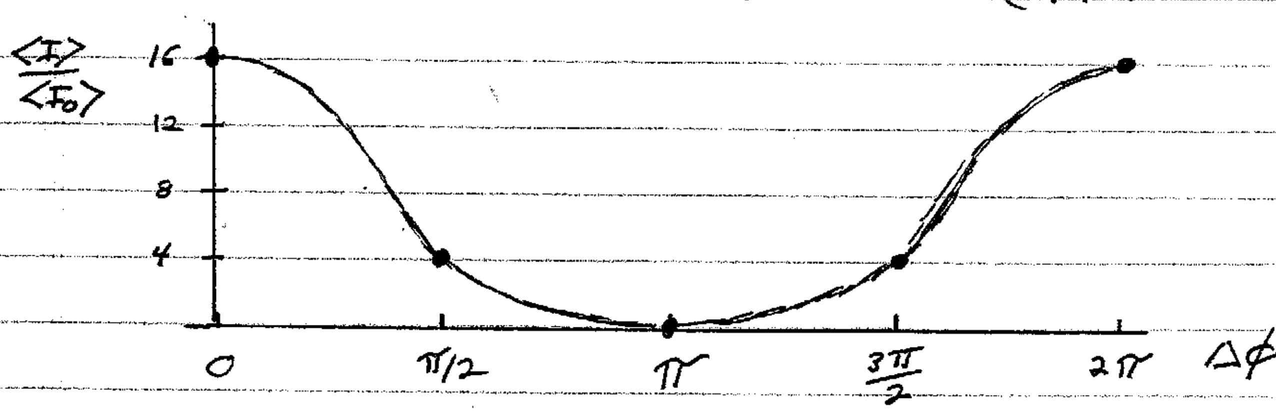



$$E = 2E_0(1+\cos\delta)$$

EITHER WAY GIVES $E = 4E_0\cos^2(\delta/2)$

$$\underline{\underline{\langle I\rangle = 16\langle I_0\rangle\cos^4(\delta/2)}}$$

b) WHEN $\theta = 0$ $\quad \langle I\rangle/\langle I_0\rangle = 16\cos^4(\Delta\phi/2)$




Tom Greytak

IV-2.

Interaction due to $\vec{E}$ is

$$V = -eEz$$

Matrix elements between S & P, $m=0$.
For these 2 levels.

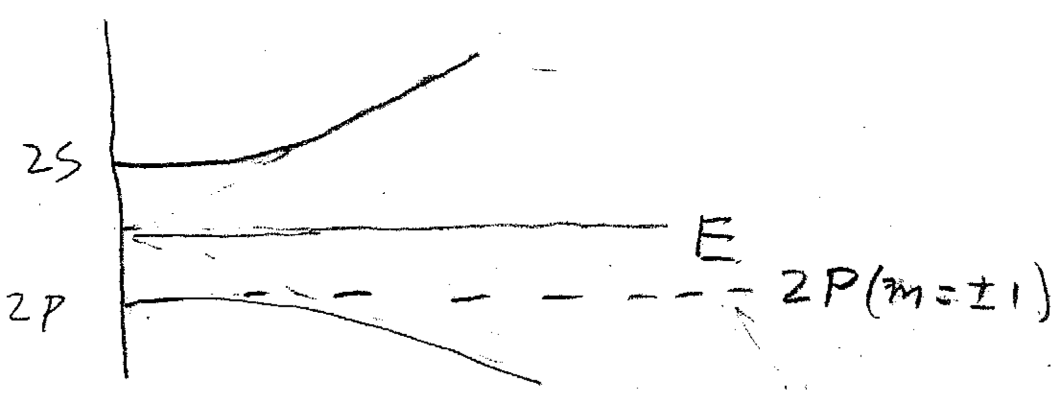
$$\mathcal{H} = \begin{pmatrix} 0 & CE \\ CE & \Delta \end{pmatrix}$$ where $C \equiv \langle 2S|-ez|2P(m=0$

Secular equation

$$\begin{vmatrix} -W & CE \\ CE & \Delta - W \end{vmatrix} = 0 \quad -\Delta W + W^2 - (CE)^2 = 0$$

$$W^2 - \Delta W - (CE)^2 = 0, \quad W = \frac{\Delta \pm \sqrt{\Delta^2 + 4(CE)^2}}{2}$$

$$W(E \to 0) = 0, \Delta.$$
$$W(E \to \infty) = \pm CE.$$




(Drawing also correct inverted.)

IV-3 Tom Greytak

TO FIRST ORDER THE TENSION IN THE LOWER STRING IS $mg$ AND IN THE UPPER IS $4mg$.

LOWER MASS EQ. $m\ddot{x}_1 = -\left(\frac{x_1 - x_2}{L}\right) mg$

$$\ddot{x}_1 + \frac{g}{L} x_1 - \frac{g}{L} x_2 = 0$$

UPPER MASS EQ. $3m\ddot{x}_2 = -\left(\frac{x_2}{L}\right) 4mg + \left(\frac{x_1 - x_2}{L}\right) mg$

$$-\frac{g}{L} x_1 + 3\ddot{x}_2 + 5\frac{g}{L} x_2 = 0$$

LOOK FOR HARMONIC SOLUTIONS AT $\omega$. DEFINE $\omega_0^2 \equiv g/L$.

$$(\omega_0^2 - \omega^2) X_1 - \omega_0^2 X_2 = 0$$
$$-\omega_0^2 X_1 + (5\omega_0^2 - 3\omega^2) X_2 = 0$$

SOLUTION WHEN $(\omega_0^2 - \omega^2)(5\omega_0^2 - 3\omega^2) - \omega_0^4 = 0$

$$3\omega^4 - 8\omega^2\omega_0^2 + 4\omega_0^4 = 0$$

$$(\omega/\omega_0)^2 = \frac{8 \pm \sqrt{64-48}}{6} = \frac{8 \pm 4}{6} = \underline{\underline{2 \text{ OR } 2/3}}$$

(2) $-\omega_0^2 X_1 - \omega_0^2 X_2 = 0 \Rightarrow X_2 = -X_1$

(2/3) $\frac{1}{3}\omega_0^2 X_1 - \omega_0^2 X_2 = 0 \Rightarrow X_2 = \frac{1}{3} X_1$

$x_1 = A \cos \omega_2 t + B \cos \omega_{2/3} t$

$x_1 = a$ AT $t=0 \Rightarrow A + B = a$

$\left\{\begin{array}{l}\text{USING FACT THAT} \\ \text{VELOCITIES ARE } 0 \\ \text{AT } t=0\end{array}\right.$

$x_2 = -A \cos \omega_2 t + \frac{B}{3} \cos \omega_{2/3} t$

$x_2 = 0$ AT $t=0 \Rightarrow A = B/3$

$\frac{4}{3}B = a \quad B = \frac{3}{4}a$

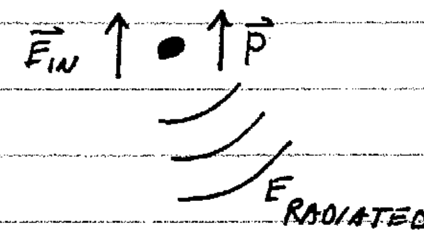
$A = \frac{1}{4}a$

$$\underline{\underline{x_1(t) = \frac{1}{4}a \cos \omega_2 t + \frac{3}{4} a \cos \omega_{2/3} t}}$$

IV-4

MODEL: THE ATMOSPHERE IS COMPOSED OF POLARIZABLE MOLECULES. THE FREQUENCY DEPENDENCE OF THE SCATTERING OF LIGHT BY THE MOLECULES FAVORS THE BLUE END OF THE SPECTRUM OVER THE RED.

$E_{INCIDENT} \quad \vec{E}_{IN} e^{i\omega t - \vec{k}\cdot\vec{r}}$




DIPOLE MOMENT $\quad \vec{P} \propto \vec{E}_{IN} e^{i\omega t}$

$E_{RADIATED} \propto \ddot{P} \propto \omega^2 E_{IN}$

$I_{RADIATED} \propto E^2_{RADIATED} \propto \omega^4 E^2_{IN} \quad \propto \omega^4 I_{INCIDENT}$

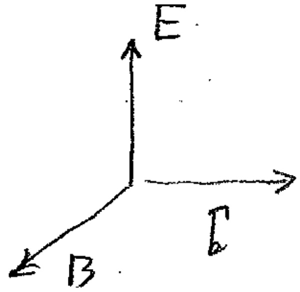
$\frac{I_{RADIATED}}{I_{INCIDENT}} \propto \omega^4 \propto \left(\frac{1}{\lambda}\right)^4$

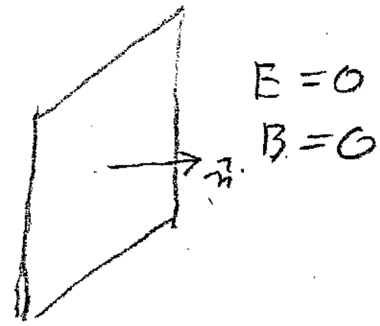
CALL $\lambda_{BLUE} \sim 4500 \mathring{A}$ AND $\lambda_{RED} \sim 6000 \mathring{A}$ $\Big\{$ THE ENDS OF THE VISABLE SPECTRUM

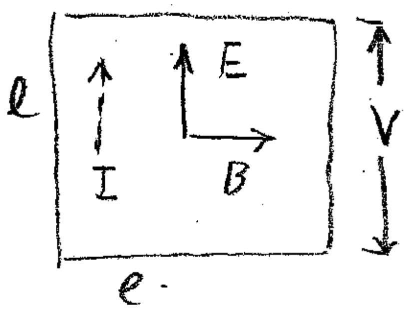
THEN $\quad \frac{I_{RAD.}(RED)}{I_{RAD.}(BLUE)} = \left(\frac{45}{60}\right)^4 = \left(\frac{3}{4}\right)^4 = \frac{81}{256} \sim \underline{\underline{1/3}}$

Tom Greytak

V-1




$V = El$

Ampere's Law - $Bl = \mu_0 I$

$$R = \frac{V}{I} = \frac{El}{Bl/\mu_0} = \mu_0 \frac{E}{B}$$

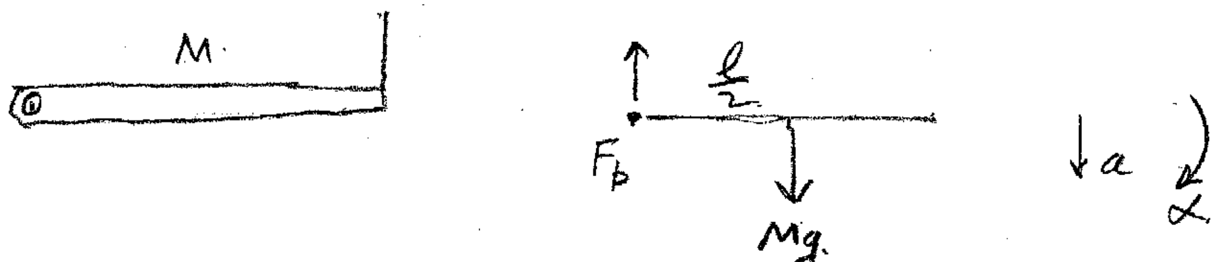
in free space, S.I., $\frac{E}{B} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

so, $R = \mu_0 \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$$\mu_0 = 4\pi \times 10^{-7}, \quad \varepsilon_0 = 8.85 \times 10^{-12}$$

$$R = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} \sim \sqrt{1.3 \times 10^{5}} \sim 370\,\Omega$$

V.-2.



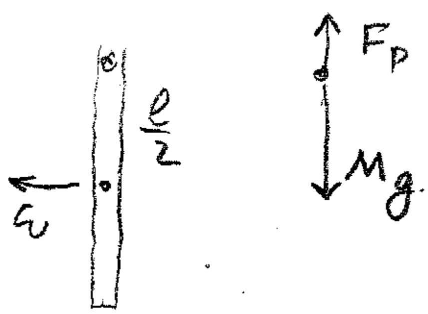

a) $Mg - F_p = Ma$

$Mg\frac{\ell}{2} = I_o\alpha$ $\qquad a = \alpha\frac{\ell}{2}\,,\quad I_o = \frac{1}{3}M\ell^2$

$$Mg\frac{\ell}{2} = \frac{1}{3}M\ell^2\frac{a}{\ell/2} = \frac{2}{3}M\ell a,$$

$$a = \frac{3}{4}g, \qquad F_p = M(g-a) = \frac{1}{4}Mg$$

b)




$$F_p - Mg = M\left(\frac{\ell}{2}\right)\omega^2$$

Energy: $\frac{1}{2}I_o\omega^2 = Mg\frac{\ell}{2}$

$$\omega^2 = \frac{Mg\ell}{I_o} = \frac{Mg\ell}{\frac{1}{3}M\ell^2} = 3\frac{g}{\ell}$$

$$F_p = Mg + M\frac{\ell}{2}\left(3\frac{g}{\ell}\right) = Mg\left(1+\frac{3}{2}\right) = \underline{\frac{5}{2}Mg}$$

V-3

USE V & P AS INDEPENDENT VARIABLES.

USE THE COMBINED 1ST + 2ND LAWS:

$$dU = T\,dS - P\,dV \Rightarrow dS = \frac{1}{T}dU + \frac{P}{T}dV$$

EXPAND dU

$$dS = \frac{1}{T}\left(\frac{\partial U}{\partial V}\Big|_P dV + \frac{\partial U}{\partial P}\Big|_V dP\right) + \frac{P}{T}dV$$

$$= \frac{1}{T}\left(AP^2 dV + 2APV\,dP\right) + \frac{P}{T}dV$$

$$= \frac{1}{T}\left(P(1+AP)dV + 2AV\,dP\right) = 0$$ FOR AN ADIABATIC PROCESS

$$\Rightarrow \underline{\underline{\frac{dP}{dV}\Big|_S = -\frac{1+AP}{2AV}}}$$

Tom Greytak

V-4.

General form $\Phi = \psi(x)\,\chi$ , $\chi$ = spin function (2-fold degenerate)

$\psi(x) = A_n e^{ik_n x}$

Continuity: $k_n \times 2\pi a = 2\pi n$.. $n = 0, \pm 1, \pm 2, ..$

$$k_n = \frac{n}{a}$$

$$A_n = \frac{1}{\sqrt{2\pi a}}$$

$$\psi_n(x) = \frac{1}{\sqrt{2\pi a}} e^{in\frac{x}{a}}$$

a) Lowest state: $n = 0$. $E_0 = \frac{\hbar^2 k_n^2}{2m} = 0$

next, $n = \pm 1$, $E_1 = \frac{\hbar^2}{2ma^2}$

$E_0$ is 2-fold degenerate, $E_1$ is 4-fold degenerate

b). $\mathcal{H}_B = -\vec{\mu}\cdot\vec{B} = -2\mu\,\vec{S}\cdot\vec{B}$ $\vec{S}\cdot\vec{B} = mB$, $m = \pm 1/2$

$E_0 \to 0 \pm \mu B$ 2 states, non degenerate

$E_1 \to \frac{\hbar^2}{2ma^2} \pm \mu B$

2 states, each 2-fold degenerate