MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS

EDUCATION OFFICE
ROOM 4-352

PHONE (617)–253–4842
FAX (617)–258–8319

DOCTORAL GENERAL EXAMINATION

PART 1

SEPTEMBER 9, 1998

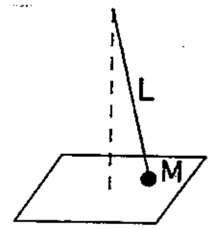
FIVE HOURS

1. This examination is divided into five sections, each consisting of four problems. Answer all the problems. Each problem is worth 5 points, thus the maximum score for the exam is 100.
2. Use a separate fold of paper for each problem. Write your name and the problem number (IV-3 for example) on each fold. A diagram or sketch as part of the answer is often useful, particularly when a problem asks for a quantitative response.
3. Read the problem carefully and do not do more work than is necessary. For example "give" and "sketch" do not mean "derive".
4. Calculators may be used but are not necessary.
5. No books, notes or reference materials may be used.

SECTION I

1. Quantum pendulum

For a real pendulum (3 dimensional space) of length L and mass M undergoing small amplitude oscillations, what is the energy of the ground state? Give the energies and degenericies of the next two energy levels.



2. Variation of atmospheric pressure with elevation

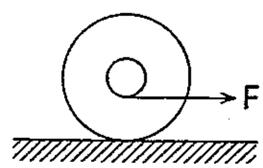
Find an expression for the atmospheric pressure, P, atop Mt. Everest (height h) relative to the sea level pressure, P_0. Model the atmosphere as an ideal gas with air molecules of average mass m, and assume that the temperature, T, does not vary with height.

3. Reflection from a plasma

Consider a plama containing n_0 electrons per unit volume which are free to move under the influence of an applied electric field $E(t) = E_0 \exp[-i\omega t]$. Compute the index of refraction of the plasma, and find the optical frequency below which incident EM waves will be completely reflected.

4. Mechanics

a) A yo-yo rests on a horizontal surface. A gentle horizontal pull, F, on the cord makes the yo-yo roll without slipping. Which way does it roll and why?



b) What causes tides? Why are there two tides on opposite sides of the earth? Are they of equal height?

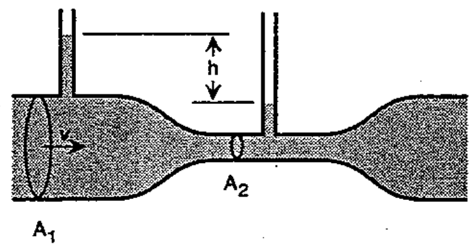
SECTION II

1. Weakness of the electromagnetic interaction

Estimate the interaction energy of two elementary particles of charge e and mass m separated by a distance d, determined by the constraint that the kinetic energy due to the momentum uncertainty be less than the rest energy. Express the ratio of this energy to the rest energy as a function of the fine structure constant α. Explain how this leads to the assertion that the electromagnetic interaction is weak.

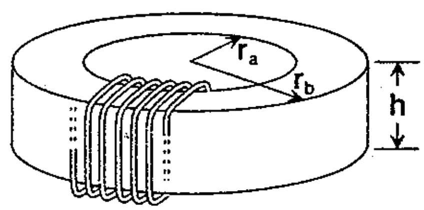
2. Flow in a pipe

The flow of water in a pipe is measured by the device shown (a Venturi meter). Find the flow speed v in terms of the cross-sectional areas A_1 and A_2, and h, the difference in height of the liquid levels in the two vertical tubes.



3. Inductance

N_0 turns of wire are wound uniformly on a square cross-section, toroidal, non-magnetic core with dimensions as shown. What is the (self) inductance of the device?



4. Relativity

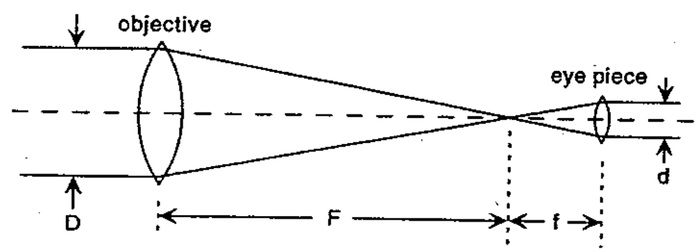
Two pi-mesons of momenta $\vec{p_1}$ and $\vec{p_2}$ are observed among the products of a high energy collision. These two mesons may have resulted from the decay of a short-lived particle of rest mass m_x, which was not necessarily at rest. If this were so, what is m_x in terms of m_π, $\vec{p_1}$ and $\vec{p_2}$?

SECTION III

1. Field ionization

Estimate the static electric field needed to ionize a hydrogen atom with principal quantum number n. If possible, express your answer in volts per centimeter.

2. Telescope



a) Find the magnification of the Keplerian telescope shown above. Express your answer in terms of the quantities shown in the figure.

b) Estimate the resolution of such a telescope.

c) Often in telescopes such as this, each of the two lenses is made up of two different optical materials. Explain why this is done.

3. Fermi gas

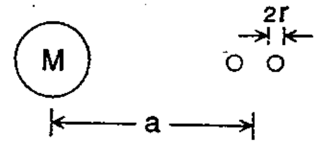
Consider a non-interacting gas of Fermi particles with number density $n = N/V$.

a) Using the uncertainty principle, estimate the Fermi energy.

b) Estimate the gas pressure at zero temperature.

4. Gravitational stability

Two small spherical objects, each of radius r and uniform density ρ are a distance a from a large mass M. Note that $r/a \ll 1$. Find the critical density ρ_c above which the two small objects will not be pulled apart by M.



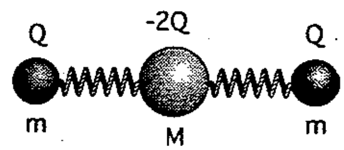
SECTION IV

1. Phase and group velocities

An electromagnetic wave of angular frequency ω propagates in a lossless neutral plasma whose index of refraction n is $(1-\omega_p^2/\omega^2)^{1/2}$ where ω_p is the plasma frequency. Prove that the product of the phase and group velocities equals c^2.

2. Normal modes

The dynamics of a linear molecule can be approximated by the model shown to the right. The charges and masses of the individual atoms are indicated in the figure and the two springs are identical.



a) Sketch the normal modes of motion along the molecular axis.

b) Which modes of this system could be excited by an external electromagnetic field?

3. Entropy change

A thermally insulated box is divided by a partition into two compartments, each having volume V. Initially, one compartment contains N ideal gas molecules at temperature T and the other compartment is evacuated. The partition is then removed and the gas expands to fill both compartments. Find the entropy change in this free expansion process.

4. Astronomy

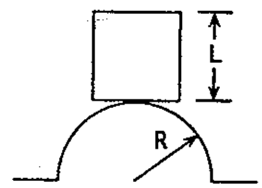
a) All stars, even the most distant ones, located 90^o in the sky from the plane of the earth's orbit appear to move in tiny circles in the sky on a yearly basis. What causes this phenomenon? Knowing that the earth is 1.50×10^{11} meters from the sun, what is the angular diameter of these circles?

b) A crude estimate of the surface temperature of a planet whose thermal properties resemble earth's can be obtained from the expression $T = 300r^{\delta}$ where T is in degrees Kelvin and r is the planet's distance from the sun in astronomical units. Determine the value of the exponent δ.

SECTION V

1. Mechanics

A cube of side L rocks without slipping on top of a semi-circular cylinder of radius R. When the cube is not tipped, its center of mass is directly above the center of the cylinder. Find the condition on L such that the situation is stable, i.e. the cube does not fall off.



2. Landé g-factor

Two angular momenta $\vec{I}$ and $\vec{J}$ couple to form a resultant angular momentum $\vec{F} \equiv \vec{I} + \vec{J}$. The interaction with an external magnetic field is $\mathcal{H} = -\vec{\mu} \cdot \vec{B}$. Only $\vec{J}$ has a magnetic moment: $\vec{\mu_J} = g_J \mu_0 \vec{J}$. At low fields the energy associated with the interaction of $\vec{F}$ with $\vec{B}$ is $E = -g_F \mu_0 \vec{F} \cdot \vec{B}$. Find an expression for g_F.

3. Nuclear and particle physics

a) A uranium-235 nucleus undergoes slow neutron fission into two intermediate-weight nuclear fission fragments.

(i) What mechanism or process accelerated these fragments?

(ii) Fission fragments are almost always radioactive. What kind of radioactivity is expected and why?

b) Parity

(i) What is the effect of the parity operator, i.e. what does $\mathbf{P}\psi(\vec{r})$ equal?

(ii) Give examples of interactions that do and do not conserve parity.

4. Electrostatics

A system is composed of a plane parallel capacitor with plates of area A separated by a small distance d carrying surface charges $\pm\sigma$ coulombs per unit area, and a removable dielectric slab of dielectric constant ϵ. Find the work necessary to remove the slab completely from between the plates. Must one pull the slab out, or must one restrain it from popping out during the process?