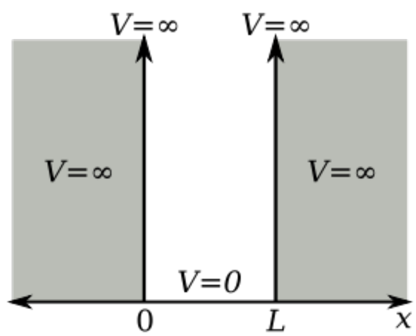
Physics Qualifying Examination – Part I 7-Minute Questions

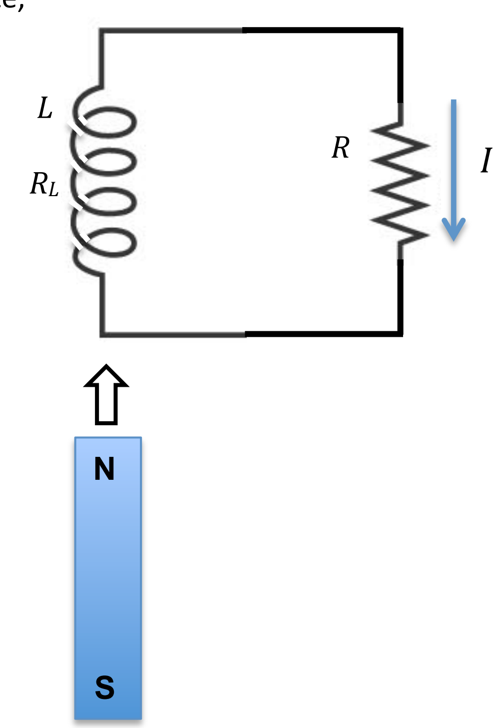
September 15, 2012

1. Evaluate $\langle p \rangle$ and $\sqrt{\langle \Delta p^2 \rangle}$ for the momentum of a one-dimensional particle in the ground state if it is confined between two rigid walls: $V = 0$ for $0 < x < L$ and $V = \infty$ otherwise.



2. The figure shows an inductor of inductance L and resistance R_L in series with a resistor of resistance R. A permanent magnet is positioned as shown. The wire of the inductor is wound counter-clockwise looking from the position of the magnet. The magnet is moved in the direction shown by the block arrow so that the flux upward, Φ, through the inductor is increasing at a constant rate, $d\Phi / dt$.

   a. Is the current I positive or negative? (Current flowing through the resistor is defined as positive in the direction of the arrow.) Explain.
   b. Find the magnitude of the current.



3. Voltage in a house in Madison (rms voltage 110.0 V and frequency 60.0 Hz) drives a variable resistor set at 50.0 Ω, a variable inductor set at 10.0 mH and a fixed capacitance of 20.0 μF (all in series).

a. FInd the power drawn by the circuit as described above.
b. Find the power drawn if the resistance is halved and the inductance is unchanged.
c. Using a resistance of 25 Ω, find the inductance needed to keep the power drawn the same as in part a).

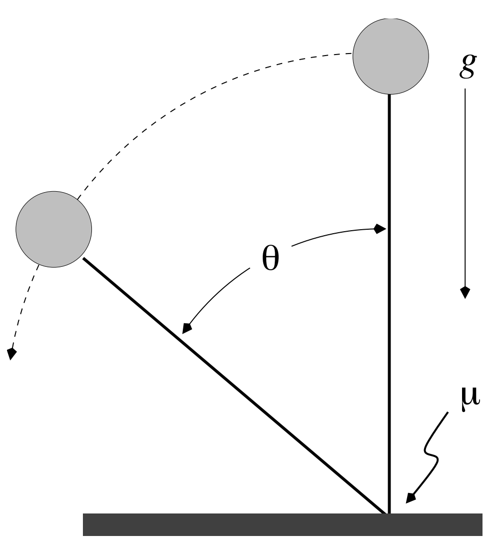
4. Using the third law of thermodynamics and the Maxwell thermodynamic relations, show that

$$(\partial P / \partial T)_V \to 0$$

as $T \to 0$.

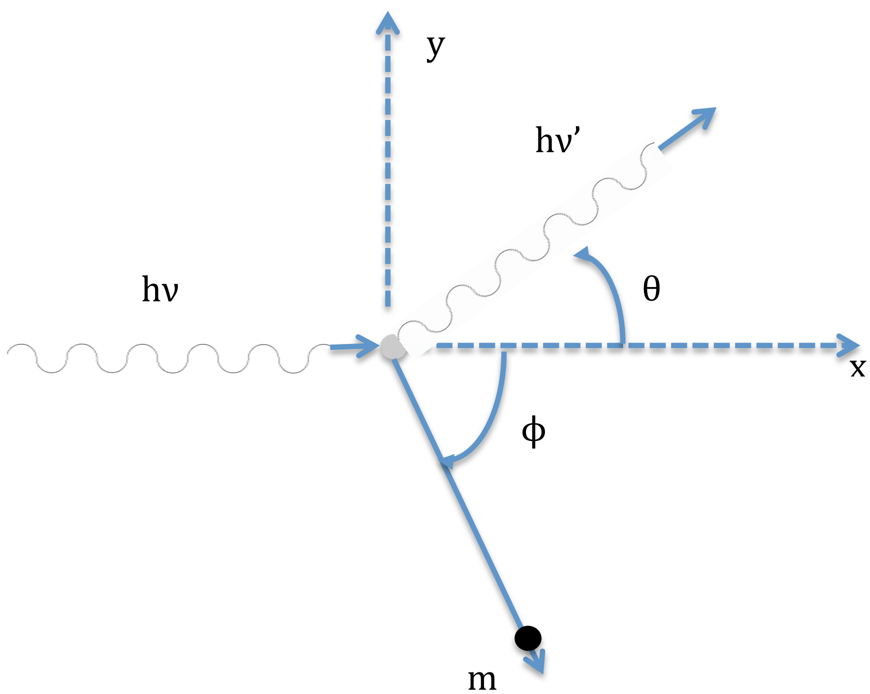
5. The figure shows an assembly consisting of a small ball connected to a rigid massless rod. The rod sits vertically at rest on a horizontal surface with a static coefficient of friction μ. Initially the ball is at rest and perfectly balanced above the rod. However it is an unstable equilibrium and so, eventually the mass begins to fall along the curved arc as shown.

a. Find the angle θ that makes the tension in the rod go to zero.
b. Find the minimum value of μ that is necessary for that to happen.



6. A photon traveling along the x-axis scatters from an electron, which is initially at rest in the lab (i.e. Compton scattering). The incoming photon has energy $h\nu$. The scattered photon has energy $h\nu'$ and scatters at an angle of θ with respect to the x-axis. The electron, with mass m, recoils with velocity V at an angle of ϕ with respect to the x-axis.

a. Write expressions for the 4-momenta of the photon and electron before and after the collision.
b. For $V = \frac{4}{5}c$, find the change in the energy of the photon $(\Delta E = h\nu' - h\nu)$.
   Express your answer in terms of the rest mass energy of the electron.



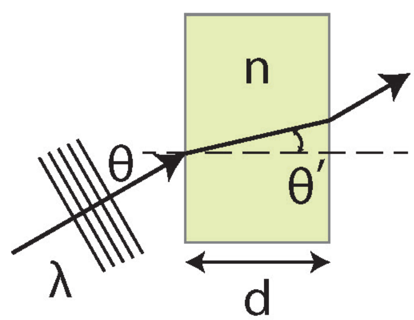
7. Plutonium was once thought to be produced only artificially, but it is now known that a very small amount of ^{244}Pu, which has a half-life of 80.0 million years, is probably primordial.

a. Consider 1.000 g of ^{244}Pu in a sample of rock when the Earth formed. Find the number of ^{244}Pu atoms in the sample at the time of formation.
b. Find the initial number of decays per second.
c. The Earth is approximately 4.55 billion years old. Find the number of ^{244}Pu atoms remaining at the present time.

8. The 21.1 cm hydrogen line was first detected by Ewen and Purcell in 1951. This line appears to have a wavelength of 21.3 cm when it reaches the Earth from a distant hydrogen source.

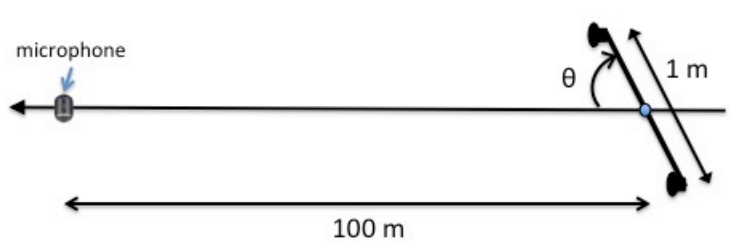
a. Find the rest frequency of this line.
b. Find the radial velocity of the hydrogen source.

9. Consider the propagation of a plane wave of wavelength λ through an etalon plate of index n and thickness d as shown in the figure.



a. Find the wavelengths for which the transmission is a maximum at normal incidence.
b. Find the wavelengths for which the transmission is a maximum when the beam is incident at angle θ from the surface normal.

10. Two acoustic speakers are positioned at the ends of a 1.0 m bar, which is free to pivot about its center. The two acoustic speakers produce a 500 Hz sound; they are in phase and each emits the same power. Take the speed of sound in air to be 350 m/s. A microphone is 100 m away as shown.



a. Find the orientations (θ) of the bar at which the microphone detects the minimum **and** maximum in sound intensities.
b. Find the ratio of the maximum intensity to that of just one of the speakers.

Physics Qualifying Examination – Part II 12-Minute Questions

September 15, 2012

1. An insulating sphere of radius R has a uniform volume charge density ρ.

   a. Find the magnitude and direction of the electric field at a distance r from the center of the sphere.

Now consider two overlapping spheres of radii R_1 and R_2 with uniform volume charge densities ρ and $-\rho$ and with their centers at a distance d from each other $(d < R_1 + R_2)$.

   b. Find the magnitude and the direction of the electric field in the region of overlap, in terms of ρ, d, R_1 and R_2.

2. Consider a two-neutrino system in which the mass eigenstates are labeled as v_1 and v_3 with energies E_1 and E_3, while the flavor eigenstates are v_e and v_τ. The masses of v_1 and v_3 are m_1 and m_3 respectively, and the mixing angle between the two flavor states is θ_{13}.

   a. Given that the survival probability for v_e is $P_{v_e \to v_e} = 1 - \sin^2(2\theta_{13})\sin^2\left(\frac{(E_3 - E_1)t}{2\hbar}\right)$, show that, in the ultra-relativistic limit, if pure v_e is produced at the source, the probability of v_e to be found at distance L from the source can be expressed as $P_{v_e \to v_e} = 1 - \sin^2(2\theta_{13})\sin^2(1.27\frac{\Delta m_{13}^2 L}{E})$, where $\Delta m_{13}^2 = m_1^2 - m_3^2$ in eV^2, L is the baseline distance in km, and E is the neutrino energy in GeV.
   b. Now apply the above result to the Daya Bay experiment where pure electron anti-neutrinos are produced at the reactors and detectors are placed at a distance L to observe electron anti-neutrinos that survive. For $\Delta m_{13}^2 \sim 2.5 \times 10^{-3}$ eV^2 and $E \sim 1.8$ MeV, what is the best value of L?

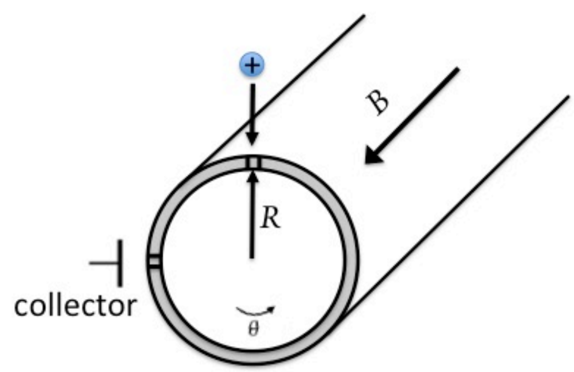
3. Find the classical heat capacity per mole at constant volume of an insulating solid in the high temperature limit (assume the oscillations of the atoms in the solid are small and harmonic).

4. Consider a counting experiment such as counting the number of decays from a radioactive source. In the absence of a source, the following background counts are recorded.

time	counts
first minute	25
second minute	35
third minute	30
fourth minute	23
fifth minute	27

a. Do these data seem reasonable? Substantiate your conclusions.
b. Find the standard deviation of the mean.
c. A radioactive source is then placed next to the detector. A 10-minute measurement of the radioactive source results in a statistical uncertainty of 2.8% in the source plus background ($S+B$) count rate. Find the additional time necessary to reduce the statistical uncertainty to 1.0%.
d. In the limit that the signal S is much greater than the background, by what factor is the significance increased if the signal is doubled?

5. A weak beam of singly-ionized atoms is injected radially into a long cylindrical chamber. A solenoid is wrapped tightly on the chamber to create a uniform axial magnetic field. The radius of the chamber is $R=1$ m, and the magnetic field strength is $B=1$ T.



a. A collector, located 90° from the direction of injection, is used to detect the ions. Find the kinetic energy in eV of the collected ions.

Suppose the chamber is filled with a hydrogen plasma that has an electrostatic potential profile $\phi(r)$, with a maximum value of 1 kV relative to the grounded chamber.

b. An injected cesium ion with the same kinetic energy as part (a) becomes doubly-ionized through a charge-exchange collision in the plasma. Does the ion strike the chamber wall above or below the collector? Justify your answer.
c. If the charge-exchange collison occurs at $r=0.5$ m, find the energy of the doubly-ionized cesium atom at the wall.

6. This problem models accretion which is a fundamental process in astrophysics. Consider a particle of mass $m \ll M$ which comes under the graviational influence of a mass M. Assume the particle approaches from infinity with the velocity V and an impact parameter b.

a. Find the orbital energy that must be dissipated in order for m to be captured into <u>any</u> bound orbit around M.

Suppose m is captured into a bound circular orbit around M conserving its angular momentum.

b. Find the radius R of this orbit.
c. Find how much energy must be dissipated in order for m to be captured into this bound circular orbit around M.

7. A solid blackbody object in a vacuum with a mass of 0.10 kg, and a surface area $1\times10^{-3}\,\mathrm{m}^2$ is at rest. A 200 watt laser shines on the object.

a. Find the speed of the object after 10 minutes of illumination.
b. Find the temperature at steady state.

8. A mass m is suspended from a fixed point by a spring of force constant k and equilibrium length r_0 (without the mass). The mass is free to move in a vertical plane. Let x describe the horizontal coordinate and y be the upward vertical coordinate relative to the suspension point.

a. Find the Lagrangian for the system using the specified x-y Cartesian coordinates.
b. From the Lagrangian, show that the equations of motion are

$$\ddot{x} = -\omega_k^2 x(1-\frac{r_0}{r}),\ \ddot{y} = -g-\omega_k^2 y(1-\frac{r_0}{r})$$

where $r=\sqrt{x^2+y^2}$, $\omega_k^2 = k/m$ and g is the acceleration of gravity.
c. Find the pairs of coordinates (x_{eq}, y_{eq}) for which the mass will be in equilibrium.

9. A Fabry-Perot-like cavity consists of a 98% reflecting mirror and a 100% reflecting mirror, separated by a distance L. Laser light of frequency ν and intensity I_0 is incident on this cavity, normal to the partially reflecting mirror. Gas is introduced into the cavity.

a. Find the index change required to vary the intra-cavity intensity from maximum to minimum.
b. Find the value of the maximum intensity within the cavity.

10. The walls of a 100 liter vacuum chamber will outgas water at a rate of 2.5 x 10^{-4} torr liter/second.

a. Turbomolecular vacuum pumps are available with ratings between 50 liters/s and 1000 liters/s in increments of 50 liters/s. Find the size pump (in liters/s) that yields an equilibrium pressure of 1.0 x 10^{-6} torr.
b. If the pump is attached to a tube of 100 cm in length and 3 cm in diameter, find the equilibrium pressure in the chamber.

Hint: conductance, S, for a tube of diameter d, length L is: $S = \frac{12d^3}{L} \frac{\text{liters}}{\text{s cm}^2}$.

11. A simple model for the Hamiltonian of a diatomic molecule ("pseudo-hydrogen") is

$$H_{ph} = \begin{pmatrix} \varepsilon & -t \\ -t & \varepsilon \end{pmatrix},$$

where ε is the atomic energy and $-t/\hbar$ is the rate at which an electron tunnels between the two atoms. The parameter t is positive ($t > 0$). This Hamiltonian is expressed in the basis $\{\psi_1, \psi_2\}$, where ψ_1 is the atomic wavefunction for an electron on atom 1 and ψ_2 is the atomic wavefunction for an electron on atom 2.

a. Find the energy levels and eigenfunctions for the pseudo-hydrogen molecule.

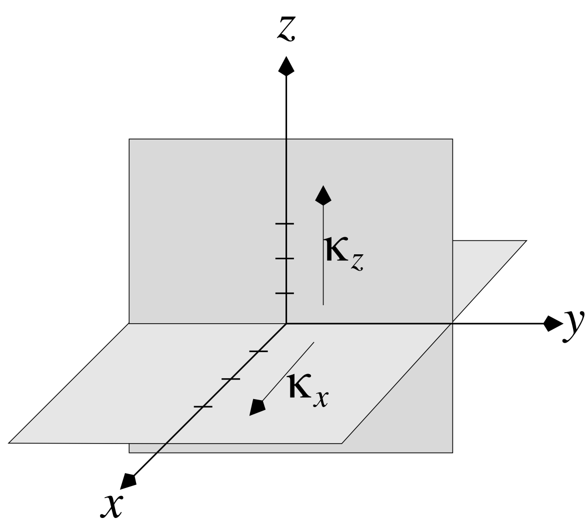
A similar model for pseudo-methane, a central atom connected to four others, is

$$H_{pm} = \begin{pmatrix} \varepsilon & -t & -t & -t & -t \\ -t & \varepsilon & 0 & 0 & 0 \\ -t & 0 & \varepsilon & 0 & 0 \\ -t & 0 & 0 & \varepsilon & 0 \\ -t & 0 & 0 & 0 & \varepsilon \end{pmatrix}.$$

b. Find the ground state energy level for pseudo-methane.

12. The red color of a He-Ne laser is due to the transition from an energy level of the $1s^2 2s^2 2p^5 5s$ configuration with $J = 1$ into an energy level of the $1s^2 2s^2 2p^5 4p$ configuration $J = 2$. Apply Russel-Saunders (LS) coupling scheme to the $1s^2 2s^2 2p^5 4p$ configuration by treating $2p^5$ as a core with $\ell_1 = 1,\ s_1 = 1/2$ and treating $4p$ as an electron with $\ell_2 = 1,\ s_2 = 1/2$. Determine how many energy levels are within the $1s^2 2s^2 2p^5 4p$ configuration and give the values of $L,\ S,$ and J for each level.

13. The xy plane carries a uniform surface current flowing in the x direction with a surface current density of $\kappa_x = 10$ A/m. The yz plane carries a uniform surface current flowing in the z direction, also with a surface current density of $\kappa_z = 10$ A/m. Observe the figure below.



a. Determine the magnetic field everywhere in space.
b. An electron located instantaneously at $x = -3$ cm, $y = 0$, and $z = 2$ cm is moving in the x direction. Find the largest velocity possible so that the electron remains confined to the quadrant $z > 0,\ x < 0$.

14. The characteristics of the n-channel junction field-effect transistor (JFET) 2N3819 are shown below.

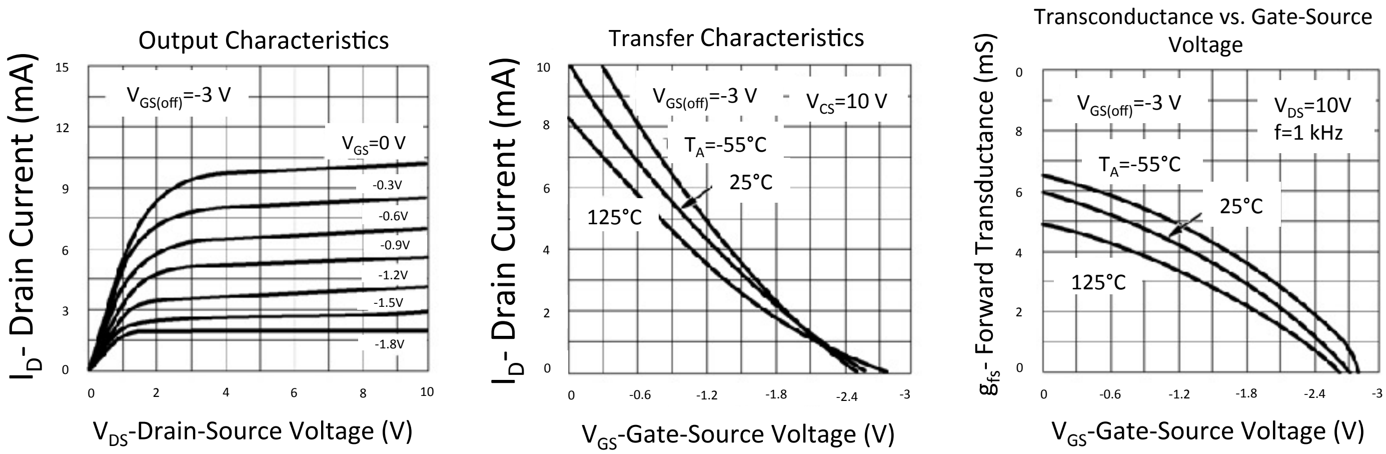


Figure 1: Characteristics of n-channel JFET 2N3819.

a. You are to use the 2N3819 to design a common-source amplifier with a gain of -10 (see electrical schematic in figure 2 below). Take $V_{dd} = 20$ V and assume that C_{in}, C_s and C_{out} are large enough that they can be considered shorts at relevant signal frequencies. Indicate appropriate values for R_g, R_d and R_s.

   Hints:
   - Choose R_g so that the transistor gate is grounded at dc and so that the amplifier stage has an acceptably large input impedance $\sim 1\ \text{M}\Omega$.
   - Next choose R_s to establish a quasistatic operating point with gate-source voltage ≈ -1 V (refer to the center plot of I_d vs. V_{gs}).
   - For this operating point, determine the forward transconductance from the plot of g_{fs} vs. V_{gs}.
   - Using this value of g_{fs} choose R_d to achieve the desired gain.

b. What are the quasistatic voltages at the gate, source, and drain of the transistor? Use the left-hand plot of the transistor output characteristics to verify that the transistor is biased in the active region.

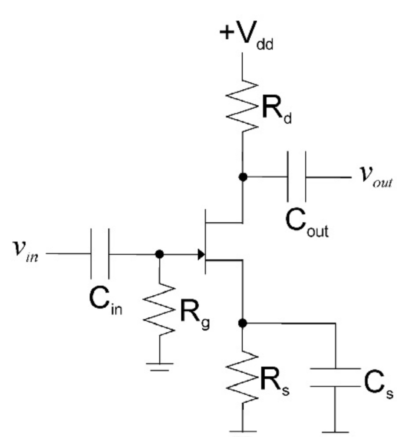


Figure 2: JFET amp circuit schematic.

15. In the lab inertial frame, there is a time-invariant magnetic field. At the spatial point $\vec{x} = (x_1, 0, 0)$ in this frame, the field has a value $\vec{B}(t, x_1, 0, 0) = B_0 \hat{y}$. Suppose there is a second inertial frame $(t', \vec{x}')$ belonging to an observer named Einstein that is moving at constant velocity $v\hat{x}$ with respect to the lab frame. Einstein's frame and the lab frame have the same spatial origin at $t = t' = 0$.

Find the magnetic field in Einstein's frame at time $t' = 0$ and at the point $(x', y', z') = (x_1 / \gamma, 0, 0)$ where $\gamma \equiv 1/\sqrt{1-(v/c)^2}$.

You may work in units with $\varepsilon_0 = \mu_0 = 1$ in which the electromagnetic field strength tensor is given by

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}.$$