STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Graduate Placement Exam, Part 1

Aug. 20, 2013 (1:00 pm - 5:30 pm)

General Instructions: This exam is for incoming graduate students who wish to demonstrate mastery in one or more areas of the graduate core curriculum, in order to skip one or more of the first-year courses. Do two of the three problems in either or both areas. Each problem is worth 20 points, and unless stated otherwise, all parts of each question have equal weight.

Each solution should typically take less than 45 minutes.

Use one exam book for each problem, and label it carefully with the problem topic and number and your name. Make sure to do every part of the problems you choose.

You may use a one page help sheet, a calculator, and with the proctor's approval, a foreign language dictionary. No other materials may be used.

Classical Mechanics 1

The Noether theorem gives a relation between a certain class of symmetries and conservation laws.

a. (*5 pts.*) State the contents of this theorem, and the conditions under which it holds. If a Lagrangian $L(x, \dot{x}, t)$ is invariant under $x \to x + \delta x$, what is the conserved object? Check explicitly that it is conserved.

b. (*5 pts.*) What is the relation between conserved generators $G(p, q)$ of transformations of p and q, and the Hamiltonian? If a Hamiltonian is unchanged under a transformation is the corresponding generator conserved?

c. (*5 pts.*) Consider the Lagrangian for a free relativistic point particle. Derive the generator of time translations. Is the Lagrangian invariant under time translations? Is the Hamiltonian invariant under time translations?

d. (*5 pts.*) Three of the following four equations are equivalent. Which is the equation that is not equivalent to the other three? Prove the equivalence of the other three.

$$\frac{dL}{dt} = 0, \quad \frac{\partial L}{\partial t} = 0, \quad \frac{dH}{dt} = 0, \quad \frac{\partial H}{\partial t} = 0$$

Solution

a. If a Lagrangian is invariant under a continuous symmetry transformation with a constant parameter, then there is a conserved quantity.

$\frac{\partial L}{\partial \dot{x}}\delta x$ is conserved: from $\delta L = \frac{\partial L}{\partial x}\delta x + \frac{\partial L}{\partial \dot{x}}\delta \dot{x} = 0$ it follows that

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\delta x\right) = \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{x}}\right)\delta x + \frac{\partial L}{\partial \dot{x}}\frac{d}{dt}\delta x = \frac{\partial L}{\partial x}\delta x + \frac{\partial L}{\partial \dot{x}}\delta \dot{x} = 0 \tag{1}$$

b.

$$\frac{d}{dt}G(p, q) = \{G, H\} \tag{2}$$

where $\{\ ,\ \}$ denotes Poisson brackets. If $\delta H = \{H, G\} = 0$, then $\frac{d}{dt}G = 0$ because $\frac{d}{dt}G = \{G, H\}$ and $\{G, H\} = -\{H, G\}$. The Hamiltonian is only conserved if it does not explicitly depend on t.

c.

$$L = -mc^2\sqrt{1-(\dot{x}^2/c^2)} \tag{3}$$

Under $\delta x = b\dot{x}$ with b a constant, $\delta L = b\frac{d}{dt}L$. Then

$$\delta L = \frac{\partial L}{\partial x}b\dot{x} + \frac{\partial L}{\partial \dot{x}}\frac{d}{dt}(b\dot{x}) = b\frac{d}{dt}L \tag{4}$$

Hence

$$\frac{\partial L}{\partial \dot{x}}b\dot{x} - bL = b(p\dot{x} - L) = bH \tag{5}$$

is conserved. H is the generator of time translations because

$$\delta x = \{x, bH\} = b(d/dp)H = b\dot{x} \tag{6}$$

and similarly for δp. The Lagrangian is not invariant under $\delta x = b\dot{x}$ since $\delta L = b\frac{d}{dt}L$. The Hamiltonian is invariant under $\delta x = b\dot{x}$ and $\delta p = b\dot{p}$ since $\delta H = \{H, bH\} = 0$.

d.

$$\frac{dH}{dt} = \{H, H\} + \frac{\partial H}{\partial t} = \frac{\partial H}{\partial t} \tag{7}$$

From $H = p\dot{q} - L$ we get

$$\frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t} \tag{8}$$

Hence $\frac{dL}{dt} = 0$ is not equivalent.

(As an example, if $L = T - V$ and $H = T + V$, with T and V not explicitly depending on t, then

$$\frac{\partial H}{\partial t} = \frac{\partial L}{\partial t} = 0 \quad \text{and} \quad \frac{dH}{dt} = 0 \quad \text{(energy conservation)} \tag{9}$$

but $\frac{dL}{dt} = \frac{d}{dt}(T - V)$ will not vanish in general.)

Classical Mechanics 2

A simple pendulum of mass M and length L is free to swing (angle ϕ) under gravity (in the y, vertical direction.) Its point of attachment P (massless) is free to move (without friction) on a line inclined by θ to the x (horizontal) direction. The displacement of P along that line is denoted r.

a. (*7 pts.*) What are the equations of motion (in the simplest suitable coordinates)? Check their Galilean invariance.

b. (*2 pts.*) What quantities are conserved?

c. (*7 pts.*) Suppose the variables $r = \dot{r} = \phi = \dot{\phi}$ are all equal to 0 at $t = 0$. Find the behavior of $r(t)$ and $\phi(t)$ to lowest non-vanishing order in t ($t \ll \sqrt{(L/g)}$).

d. (*4 pts.*) What is a chaotic system? Could this system be chaotic, and why or why not?

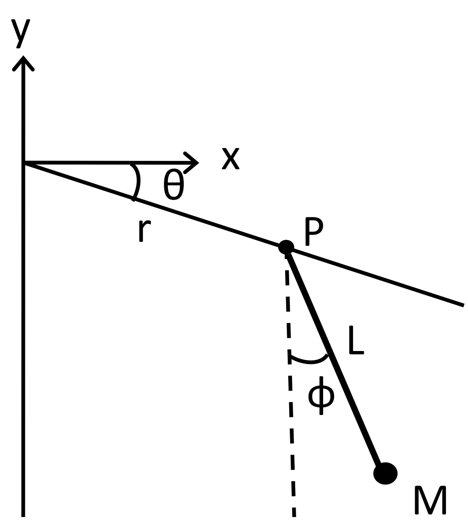

Figure 1:

Solution

a. The Lagrangian is

$$\mathcal{L} = T - V = \frac{M}{2}[\dot{x}^2 + \dot{y}^2] - Mgy \tag{1}$$

where

$$\begin{aligned} x &= r\cos\theta + L\sin\phi & \dot{x} &= \dot{r}\cos\theta + L\dot{\phi}\cos\phi \\ y &= -r\sin\theta - L\cos\phi & \dot{y} &= -\dot{r}\sin\theta + L\dot{\phi}\sin\phi. \end{aligned} \tag{2}$$

Therefore the Lagrangian is

$$\mathcal{L} = \frac{M}{2}[\dot{r}^2 + L^2\dot{\phi}^2 + 2L\dot{r}\dot{\phi}\cos(\theta+\phi)] + Mg[r\sin\theta + L\cos\phi]. \tag{3}$$

The equations of motion are

$$\frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{r}} = \frac{\partial\mathcal{L}}{\partial r} \quad \text{and} \quad \frac{d}{dt}\frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \frac{\partial\mathcal{L}}{\partial\phi}. \tag{4}$$

From these we get

$$\frac{d}{dt}[\dot{r} + L\dot{\phi}\cos(\theta+\phi)] = g\sin\theta \tag{5}$$

$$\frac{d}{dt}[L\dot{\phi} + \dot{r}\cos(\theta+\phi)] = -g\sin\phi - \dot{r}\dot{\phi}\sin(\theta+\phi) \tag{6}$$

The equations of motion can be written explicitly as

$$\ddot{r} + L\ddot{\phi}\cos(\theta+\phi) - L\dot{\phi}^2\sin(\theta+\phi) = g\sin\theta \tag{7}$$

$$L\ddot{\phi} + \ddot{r}\cos(\theta+\phi) = -g\sin\phi \tag{8}$$

Note that neither $\ddot{r}$ nor $\ddot{\phi}$ depends on r or $\dot{r}$. This just expresses the Galilean invariance of the system. Motions observed from a frame moving with speed $\dot{r}$ down the incline look the same as those seen from rest, except for a shift of r and $\dot{r}$. It is not actually necessary to write the explicit form (7) of the first equation, since the right side of the source equation (5), $g\sin\theta$, is a constant. Thus a first integral is immediate,

$$[\dot{r} + L\dot{\phi}\cos(\theta+\phi)] = [\dot{r}(0) + L\dot{\phi}(0)\cos(\theta+\phi(0))] + (g\sin\theta)t \tag{9}$$

The interpretation of this integral is that, if forces are resolved into components parallel and perpendicular to the incline, the only parallel force (because of no friction) is the constant $Mg\sin\theta$, so the mass has constant acceleration in this direction. Both r and ϕ obey complicated non-linear equations of motion, but their velocities couple in a simple way.

b. The only conserved quantity is the energy, $T + V$.

c. There are many ways. Most straightforward, perhaps, is to write eqs.(7,8) at $t = 0$,

$$\ddot{r}(0) + L\ddot{\phi}(0)\cos\theta = g\sin\theta \tag{10}$$
$$L\ddot{\phi}(0) + \ddot{r}(0)\cos\theta = 0 \tag{11}$$

These integrate to

$$r(t) + L\phi(t)\cos\theta = g\sin\theta\; t^2/2 + \text{higher order} \tag{12}$$
$$L\phi(t) + r(t)\cos\theta = 0\cdot t^2 + \text{higher order} \tag{13}$$

Solving these simultaneous linear equations,

$$r(t)\sin^2\theta \approx g\sin\theta\; t^2/2 \tag{14}$$
$$L\phi(t)\sin^2\theta \approx -g\sin\theta\cos\theta\; t^2/2, \tag{15}$$

or, $r(t) \approx gt^2/(2\sin\theta)$ and $L\phi(t) \approx -gt^2/(2\tan\theta)$. The behavior as the slope $\theta \to 0$ is curious. One might want to verify whether higher order terms can still be ignored. It is reassuring that these results give the correct limits as $\theta \to \pi/2$, that is, in the case of free fall.

d. A chaotic system is one that has "sensitive dependence on initial conditions." If there are as many integrals of the motion as there are degrees of freedom, then the system is "integrable" and not chaotic. This system has two degrees of freedom, but only one conservation law. This suggests that the motion occurs in the three-dimensional subspace of $(\dot{r},\ r\ \dot{\phi},\ \phi)$ of constant energy, and that it at least in some regions of this phase space, it might be chaotic. However, the fact that another first integral of the motion exists suggests otherwise. In fact, this system is integrable. The variable r can be eliminated from equations 7 and 8. This becomes a second order differential equation involving $\ddot{\phi}$, $\dot{\phi}$, and ϕ, which can be integrated to get $\phi(t)$. Then $r(t)$ can be found.

Classical Mechanics 3

A rocket moves in a straight line as observed in an inertial frame S. It accelerates by expelling material at a constant velocity u relative to the rocket. At time t the velocity of the rocket in frame S is v.

a. (*7 pts.*) In an infinitesimal time interval $\mathrm{d}t$ an amount of mass $\mathrm{d}m_e$ is expelled, and the rocket mass changes by $\mathrm{d}M$. In the instantaneous rest frame of the rocket-at-time-t, write the conservation of energy and momentum equations for the rocket from time t to time $t + \mathrm{d}t$.

b. (*6 pts.*) What is the change of velocity $\mathrm{d}v$ of the rocket measured in the inertial frame S in this time interval?

c. (*7 pts.*) Find the final velocity v of the rocket as a function of its final mass M, if it starts from rest with mass M_0.

Solution

a. In $\mathrm{d}t$ the rocket gains a small velocity v' in its instantaneous rest frame. To first order in small quantities, conservation of energy and momentum give, respectively

$$\mathrm{d}M + \frac{\mathrm{d}m_e}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = 0 \tag{1}$$

$$Mv' - \frac{\mathrm{d}m_e\ u}{\sqrt{1-\left(\frac{u}{c}\right)^2}} = 0 \tag{2}$$

b. The Lorentz transformation law gives for the new velocity in the inertial frame

$$v + \mathrm{d}v = \frac{v'+v}{1+\frac{vv'}{c^2}} \tag{3}$$

$$\simeq v + v'\left[1-\left(\frac{v}{c}\right)^2\right] \tag{4}$$

c. Substituting from the conservation laws gives the differential relation

$$\frac{\mathrm{d}v}{1-\left(\frac{v}{c}\right)^2} = \frac{-\mathrm{d}M\ u}{M} \tag{5}$$

Integrating

$$\int_0^v \frac{\mathrm{d}v/c}{1-\left(\frac{v}{c}\right)^2} = -\frac{u}{c}\int_{M_0}^{M}\frac{\mathrm{d}M}{M} \tag{6}$$

yields

$$\frac{1}{2}\ln\frac{1+v/c}{1-v/c} = \frac{u}{c}\ln\frac{M_0}{M} \tag{7}$$

$$\frac{1+v/c}{1-v/c} = \left(\frac{M_0}{M}\right)^{2u/c} \tag{8}$$

and thus

$$v = c\,\frac{M_0^\alpha - M^\alpha}{M_0^\alpha + M^\alpha}, \quad \text{where } \alpha \equiv \frac{2u}{c} \tag{9}$$

Electromagnetism 1

Each of the five questions below are independent. Please answer them in a couple of lines.

a. (*4 pts.*) Give a physical argument that $1/|\vec{x}-\vec{x}'|$ is a Green's function of the Laplace equation.

b. (*4 pts.*) Give two different gauge potentials for a constant magnetic field in the z-direction and show that they differ by a pure gauge.

c. (*4 pts.*) Show that $\vec{E} \perp \vec{B}$ for a plane wave radiation field.

d. (*4 pts.*) Show that Ohm's law $\vec{J} = \sigma\vec{E}$ gives rise to damped plane waves.

e. (*4 pts.*) Explain why a static Coulomb field does not contribute to radiation.

Solution

a. $E = -\boldsymbol{\nabla}\phi$ and the divergence of E yields the Poisson equation; the Green's function is the electric field at $\vec{x}'$ due to a unit point charge at $\vec{x}$.

b. $B_1 = Bx\hat{y}$ and $B_2 = -By\hat{x}$ and $B_{1k} - B_{2k} = \partial k_B xy$

c. Substitute plane wave $\sim \exp[i\vec{k}\vec{x} - i\omega t]$ in Maxwell equation. This gives $\vec{k} \times \vec{E} = (\omega/c)\vec{B}$.

d. We have the Maxwell Equation in Matter $\vec{\nabla} \times \vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\vec{D}}{dt}$. So σ becomes an imaginary frequency which gives a damped wave.

e. $E_{Couolomb} \sim 1/r^2$ so that the Poynting vector drops at least as $1/r^3$ and does not contribute to radiation.

Electromagnetism 2

A plane monochromatic electromagnetic wave is given by

$$\vec{E} = \Big(0, \quad a\sin\omega(t - \frac{x}{c}), \quad 0\Big)$$

$$\vec{B} = \Big(0, \quad 0, \quad a\sin\omega(t - \frac{x}{c})\Big) \tag{1}$$

a. (*7 pts.*) What are the electric fields $\vec{\bar{E}}$ and the magnetic fields $\vec{\bar{B}}$ as functions of the coordinates $\bar{x}, \bar{y}, \bar{z}, \bar{t}$ that an observer $\bar{O}$ sees who is moving in the direction of the positive x-axis with velocity v?

b. (*7 pts.*) The same for an observer who is moving in the direction of the positive y-axis. Check that the source-free Maxwell equations are satisfied.

c. (*6 pts.*) An observer which is at rest with respect to blackbody radiation, observes that it has temperature T. A second observer moves with relativistic velocity v with respect to the first observer. Show that the second observer still sees the spectrum of blackbody radiation if he looks at radiation that comes into his detector from the direction into which he is moving, but with blue-shifted temperature T'. What is T'?

Solution

a.

$$\begin{aligned}
\vec{E} &= \Big(0, \quad la\sin l\omega(\bar{t} - \frac{\bar{x}}{c}), \quad 0\Big) \\
\vec{B} &= \Big(0, \quad 0, \quad la\sin l\omega(\bar{t} - \frac{\bar{x}}{c})\Big), \quad \text{with } l = \sqrt{\frac{1-(v/c)}{1+(v/c)}} \text{ and} \\
\bar{t} &= \frac{t - (v/c^2)x}{\sqrt{1-(v^2/c^2)}}, \quad \bar{x} = \frac{x - vt}{\sqrt{1-(v^2/c^2)}}, \quad \bar{y} = y, \quad \bar{z} = z
\end{aligned} \tag{2}$$

b.

$$
\begin{aligned}
\vec{E} &= \left(\frac{v/c}{\sqrt{1-(v^2/c^2)}}a\sin\Omega, \quad a\sin\Omega, \quad 0\right) \\
\vec{B} &= \left(0, \quad ,0, \quad \frac{a\sin\Omega}{\sqrt{1-(v^2/c^2)}}\right), \quad \text{with } \Omega = \omega\left[\frac{\bar{t}+(v/c^2)\bar{y}}{\sqrt{1-(v^2/c^2)}} - \frac{\bar{x}}{c}\right] \\
\bar{t} &= \frac{t-(v/c^2)y}{\sqrt{1-(v^2/c^2)}}, \quad \bar{y} = \frac{y-vt}{\sqrt{1-(v^2/c^2)}}, \quad \bar{x} = x, \quad \bar{z} = z \\
\mathrm{d\vec{i}v}\,\vec{E} &= \left(\frac{-\omega(v/c^2)}{\sqrt{1-(v^2/c^2)}} + \frac{\omega(v/c^2)}{\sqrt{1-(v^2/c^2)}}\right)a\sin\Omega = 0 \\
\mathrm{d\vec{i}v}\,\vec{B} &\sim \frac{\partial}{\partial\bar{z}}\Omega = 0
\end{aligned}
\tag{3}
$$

Also curl$\vec{\bar{E}} = -\frac{1}{c}\frac{\partial\vec{\bar{B}}}{\partial t}$ and curl$\vec{\bar{B}} = \frac{1}{c}\frac{\partial\vec{\bar{E}}}{\partial t}$ hold.

c. The number of photons per unit volume in an angular cone $d\Omega$ between ω and $\omega + d\omega$ is according to Planck

$$
N(\omega) = \frac{\omega^2}{\pi^2 c^3(e^{\hbar\omega/kT}-1)}\left(\frac{d\Omega}{4\pi}\right) \tag{4}
$$

The frequencies which the moving observer sees are blue shifted, but the number of photons remains the same. So $\omega/T = \omega'/T'$, and with $\omega' = l^{-1}\omega$ we get $T' = l^{-1}T$.

Electromagnetism 3

a. (*5 pts.*) Consider the totally antisymmetric equation $\partial_{[\mu}F_{\nu\rho]} = \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0$. Show that it contains two of Maxwell's equations written in terms of electric and magnetic fields E_i and B_i. Next write down another equation containing again $F_{\mu\nu}$ and a current density $j^\mu(\vec{x},t)$, and show that it contains the other two Maxwell equations.

b. (*4 pts.*) What is the electromagnetic 4-vector current $J^\mu(t)$ for a relativistic point particle? Write down (but do not derive) the equation for the Lorentz force on a relativistic electron, beginning with $\frac{dp^i}{dt} = \dots$, and using fields E_i and B_i on the right-hand side. Rewrite this equation in a manifestly relativistic way, with $F_{\mu\nu}$ and J^μ on the right-hand side.

c. (*5 pts.*) Synchrotron radiation I. Consider a relativistic electron moving in the xy plane under the influence of a constant magnetic field in the z-direction. Prove that the electron moves in a circle, and derive the angular frequency Ω.

d. (*6 pts.*) Synchrotron radiation II. How does the power P radiated by a relativistic electron depend on its acceleration a and the electric charge e? (For example, does it depend on a^2 or a^3, and on e^2 or e^4 etc.) Derive a formula for P up to a numerical overall constant k, by using dimensional arguments to fix the power of c in this formula. (This argument does not fix the powers of $\gamma = (1 - v^2/c^2)^{-1/2}$.) Now use this formula to calculate the power P of the electron in part c.

Solution

a. $\partial_{[i}F_{jk]} = 0$ implies $F_{jk} = \partial_j A_k - \partial_k A_j$. Define $F_{ij} = \epsilon_{ijk}B_k$ and $F_{i0} = E_i$. Then $B_i = \frac{1}{2}\epsilon_{ijk}F^{jk}$ and $\text{div}\vec{B} = 0$, which is one of the Maxwell equations.

Next, $\partial_{[0}F_{jk]} = 0$ implies $\partial_0(\epsilon_{jkl}B_l) + \partial_j E_k - \partial_k E_j = 0$ which is the Maxwell equation $\text{curl}\vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$.

The other manifestly relativistic equation is $\partial_\mu F^{\mu\nu} = -\frac{4\pi}{c}j^\nu$. For $\nu = 0$ it yields $\partial_i F^{i0} = -\partial_i E_i = -\frac{4\pi}{c}j^0$, and with $j^0 = c\rho =$ charge density, we get a third Maxwell equation $\text{div}\vec{E} = 4\pi\rho$.

Finally, for $\nu = j$ we get $\partial_0 F^{0j} + \partial_k F^{kj} = \partial_0 E_j + \epsilon^{kjl}\partial_k B_l = -\frac{4\pi}{c}j_j$, which becomes the fourth Maxwell equation $\text{curl}\vec{B} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$

For a point particle,

$$J^\mu(t) = e\frac{dx^\mu(t)}{d\tau} \tag{1}$$

b. The Lorentz force equation reads $\frac{dp^i}{dt} = e\vec{E} + \frac{e}{c}\vec{v}\times\vec{B}$ where $p^i = m\frac{dx^i}{d\tau}$ with τ the proper time, so

$$\frac{dx^i}{d\tau} = \gamma\frac{dx^i}{dt}$$

with $\gamma = (1-v^2/c^2)^{-1/2}$. Further, $v^i = \frac{dx^i}{dt}$. To show that this equation is relativistically correct, we write it in manifestly relativistic form as

$$\frac{dp^\mu}{d\tau} = \frac{1}{c}F^\mu{}_\nu\, J^\nu$$

where $J^\nu = e\frac{dx^\nu(t)}{d\tau}$ and $F^\mu{}_\nu = F^\mu{}_\nu\,(x(t),t)$. For $\mu = i$ we get

$$\begin{aligned}\frac{dp^i}{d\tau} &= \frac{1}{c}F^i{}_0J^0 + \frac{1}{c}F^i{}_kJ^k \\ &= \frac{1}{c}E^iJ^0 + \frac{1}{c}\epsilon^{ikl}B_lJ^k\end{aligned}$$

Multiplying by $\frac{d\tau}{dt}$ yields

$$\frac{dp^i}{dt} = E^i + \epsilon^{ikl}\frac{e}{c}v_kB_l, \tag{2}$$

which is indeed the Lorentz force.

c. Multiplication of the Lorentz force $\frac{dp^i}{dt} = \frac{e}{c}\epsilon^{ijk}\frac{dx^j}{dt}B^k$ yields $p^i\frac{dp^i}{dt} = 0$. Hence $|\vec{p}|$ is constant. Then we get $(x^1 = x, x^2 = y)$

$$\frac{d^2x}{dt^2} = \left(\frac{eB}{\gamma m_e c}\right)\frac{dy}{dt} \quad \text{and} \quad \frac{d^2y}{dt^2} = -\left(\frac{eB}{\gamma m_e c}\right)\frac{dx}{dt}.$$

Taking the d/dt derivative and interating shows that the dependence of x and y on t is $\cos\Omega t$ and $\sin\Omega t$ where

$$\Omega = \frac{eB}{\gamma m_e c}.$$

d. Power is energy radiated per second. It is proportional to a^2 and q^2. Hence $P = k\gamma^m a^2q^2c^n$, where k is a numerical constant, and n is an integer to be determined by dimensional arguments. The factor γ^m cannot be determined by dimensional arguments alone.

Using that the dimension of $\frac{q^2}{l}$ is that of an energy, we get

$$[P] = \frac{q^2}{l}\frac{1}{t} = \left(\frac{l}{t^2}\right)^2\left(\frac{q^2}{l}l\right)[c^n] \quad \Rightarrow \quad [c^n] = \frac{t^3}{l^3} \quad \Rightarrow \quad n = -3$$

For synchrotron radiation, we find the acceleration $m\frac{d^2x^i}{dt^2} = \frac{1}{\gamma}\frac{e}{c}\left(\vec{v}\times\vec{B}\right)^i$. Hence the centripetal acceleration is $a = \frac{e}{m_e}\frac{v}{c}\frac{B}{\gamma}$ and the power is

$$P \sim \frac{e^4 B^2}{m_e^2 c^3}\gamma^{m-2}\left(\frac{v}{c}\right)^2$$