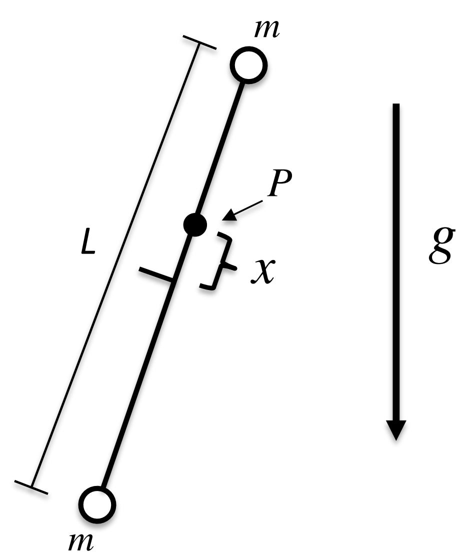
Physics Qualifying Examination – Part I 7-Minute Questions

September 14, 2013

1. A transformer is to be designed that will produce a 12 volt rms voltage in the secondary when a 60 Hz, 120 volt rms voltage is applied to the primary. An iron core with cross-section 2 cm^2 is available. What is the minimum number of primary and secondary turns that you could use if the iron saturates at a field of 1.5 T?

2. Two small spheres, each of mass m, are joined by a massless rigid rod of length L. This system, pivots freely about a point P that is located a distance x from the center of the rod. There is in a uniform downward gravitational field of strength g.

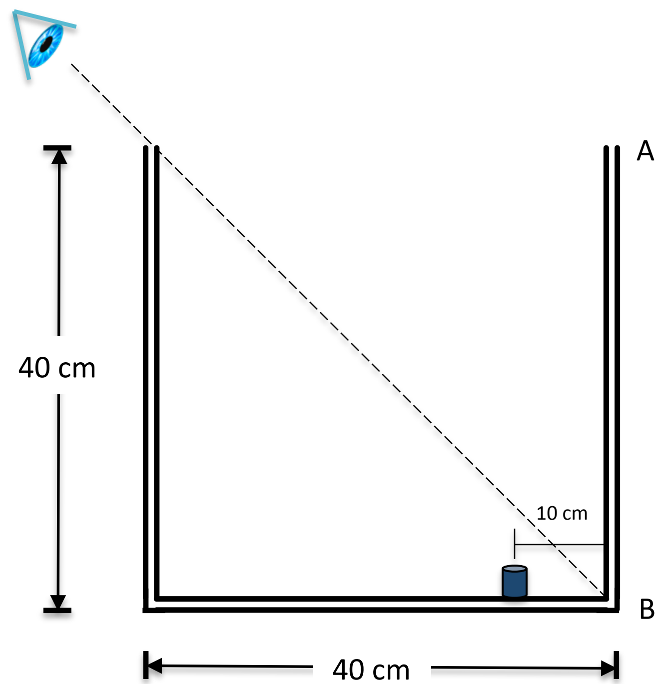
   a. Find the angular frequency of small oscillations about the vertical.
   b. For what value of x would the period of the oscillation be a minimum?



3. A string vibrates according to the equation $y(x,t) = A(\sin kx)(\cos \omega t)$ where $A = 0.5$ cm, $k = \frac{\pi}{3}$ cm^{-1}, and $\omega = 40\pi s^{-1}$.

   a. Find the amplitude and speed of the component travelling waves whose superposition can give rise to this vibration.
   b. What is the distance between nodes?
   c. What is the total energy of the vibration in one loop (one half wavelength) if the mass density of the string is 0.20 g/m?

4. A cubical vessel (40 cm on a side) with opaque walls is located so an observer sees just (all) of wall A-B. What minimum depth of water (index of refraction, $n = 1.3$) must be poured into the vessel for the observer to see an object on the bottom 10 cm from corner B?

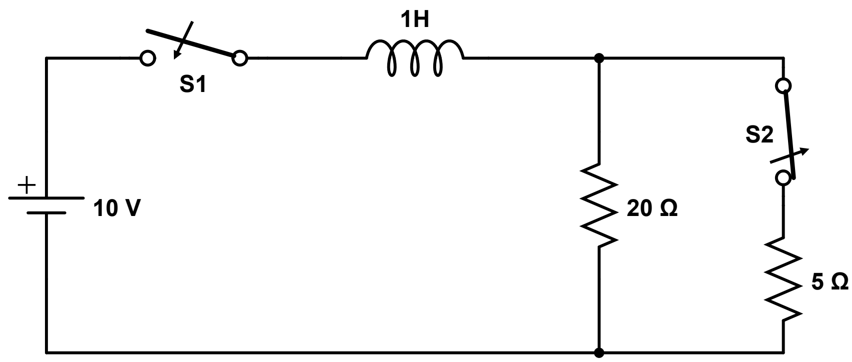


5. One mole of an ideal monatomic gas, each atom having mass m, is maintained at the temperature T.

   a. What is the rms speed of the atoms?
   b. Find the net heat into the gas during an isothermal expansion from initial volume V_1 to final volume V_2.

6. Given the ionization potential for hydrogen, 13.6 eV, calculate the energy (change) and the wavelength of emitted light for the transition from $n = 3$ to $n = 2$ in singly ionized helium.

7. In the circuit shown, switch S_1 is initially open and switch S_2 is initially closed. At time $t = 0$, S_1 is closed and at time $t = 0.25$ s, S_2 is opened.



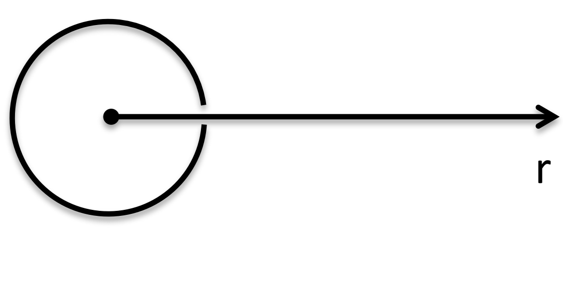
   a. Find the voltage across the inductor, V_L: (i) just after switch S_1 is closed, (ii) just before switch S_2 is opened, and (iii) at time $t = \infty$.
   b. Make a sketch of V_L as a function of time with numerically labeled axes.

8. A neutron star with radius $R = 10^6$ cm and a mass of $M = 2\times 10^{33}$ g is accreting material from space.

   a. Calculate the energy gained by a proton (in units of its rest mass energy $m_p c^2$) as it falls onto the surface of the neutron star from ∞ ($v = 0$ at $r = \infty$).
   b. Assuming that all the kinetic energy of the accreted material is radiated by the neutron star, calculate the luminosity L for an accretion rate of $\dfrac{dM}{dt} = 10^6$ g/s.

9. An atom consists of filled shells plus two equivalent ($n = \ell$) *d* electrons. Find the number of quantum states allowed by the Pauli principle.

10. A thin conducting spherical shell of radius a has a net charge Q. If there is a small hole in the shell as shown, find the variation of electric field and potential along the path r.



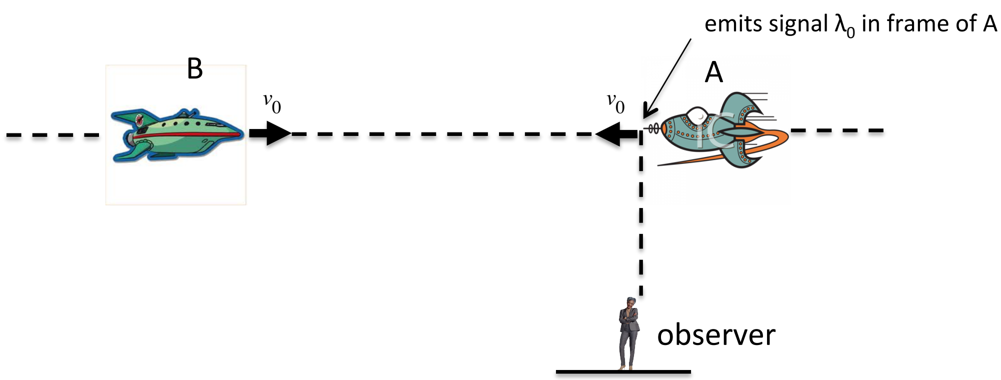
Physics Qualifying Examination – Part II 12-Minute Questions

September 14, 2013

1. Spacecraft A moves past a stationary observer. At the moment shown, the spacecraft speed is $v_0 = \frac{2}{3}c$ relative to the observer's frame and the craft emits a light signal with wavelength λ_0 in its own frame.

   a. When the signal is received by the observer, its wavelength is found to be $\lambda = 500$ nm. Determine λ_0.

   b. Spacecraft B approaches spacecraft A head on, also having $v_0 = \frac{2}{3}c$ in the observer's frame. What wavelength will be measured on spacecraft B for the light signal it receives from spacecraft A?



2. The normalized energy eigenfunctions for the two lowest states of the simple harmonic oscillator $(H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2)$ are

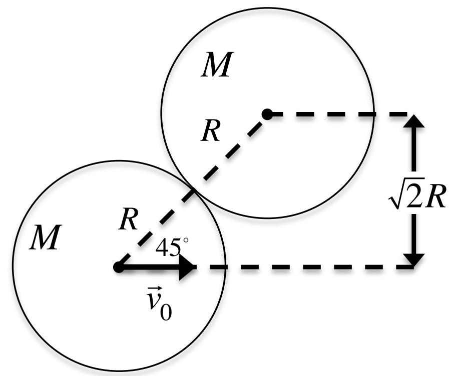
$$\psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\alpha x^2/2} \text{ and } \psi_1(x) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} \sqrt{2\alpha}\, x\, e^{-\alpha x^2/2}$$

where $\alpha = m\omega/\hbar$. Suppose that we prepare a state that at time $t = 0$ has the wave function $\psi(x) = C\left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}}\left[1 + i\sqrt{2\alpha}x\right]e^{-\alpha x^2/2}$.

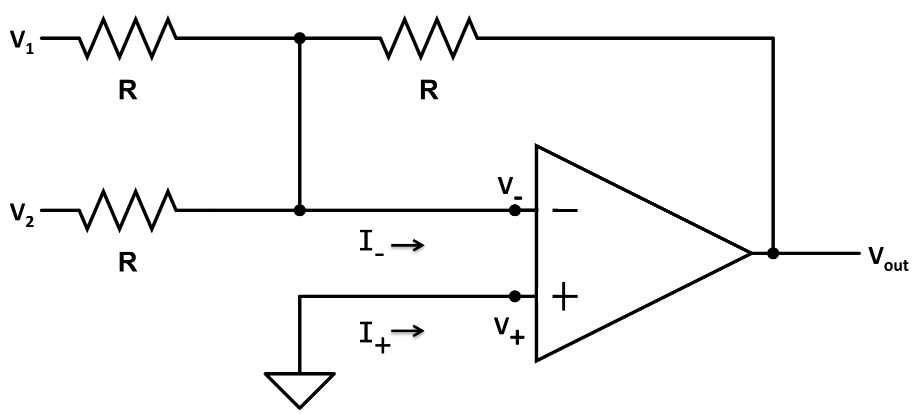
   a. Find the normalization constant C.
   b. What is the expectation value of the energy for this state?
   c. Find an expression for the full time-dependent wave function.
   d. Find the expectation value of the position as a function of time.

3. Sphere A moving to the right in free space with velocity v_0 collides with sphere B, which is initially at rest. Each sphere has mass M and radius R. They collide with an impact parameter $\sqrt{2}R$, as shown. The two spheres stick together immediately on contact and then move as a rigid body. (All answers are to be given in terms of M, R, and v_0. The moment of inertia for a sphere rotating about its axis in free space is $\frac{2}{5}MR^2$.

   a. Find the velocity of the center of mass before and after the collision.
   b. Find the moment of inertia about the central point after the collision.
   c. Find the angular velocity after the collision.
   d. Find the total kinetic energy (translation & rotation) before the collision.
   e. Find the total kinetic energy after the collision.



4. For an ideal operational amplifier in the external feedback circuit shown



   a. What is the ideal value of $(V_+ - V_-)$? (as the amplifier gain goes to infinity)
   b. What are the ideal values of I_+ and I_-?
   c. Find V_{out} as a function of V_1 and V_2.

5. The attenuation coefficient K of 663 keV gamma rays in aluminum is to be measured. The aluminum attenuator is 10.00 ± 0.01 cm thick. The clock, which controls the counter, is accurate to 1 part in 10^6. A total of 8642 counts is recorded in one minute without the attenuator, and a total of 1121 counts is recorded in one minute with the attenuator in place. Background counts are negligible.

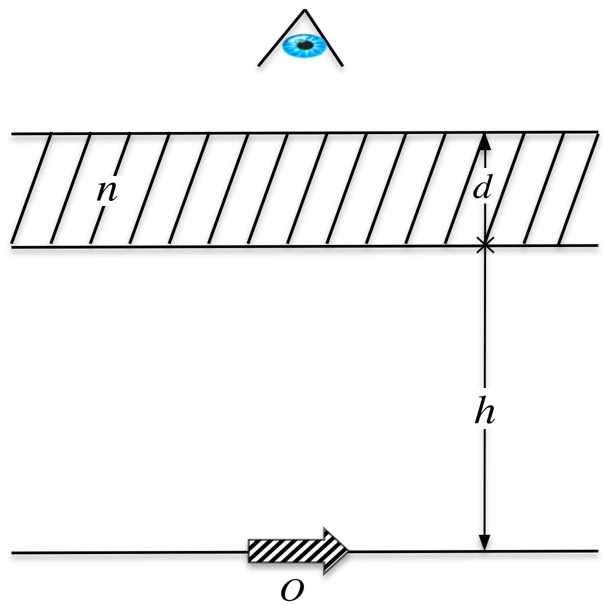
   a. Find K.
   b. Find ΔK, the one standard deviation uncertainty in K.

6. The radioactive gas radon ($^{222}_{86}\mathrm{Rn}$, half life 4 days) may be a hazard in some houses. It results from the decay of radium ($^{226}_{88}\mathrm{Ra}$, 1600 years) and decays into polonium ($^{218}_{84}\mathrm{Po}$, 3 minutes).

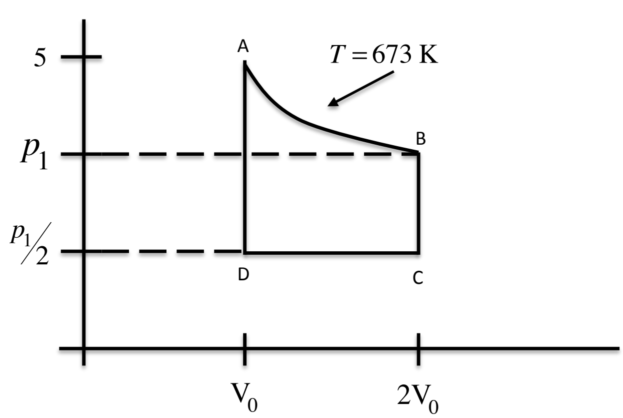
   a. What particles are emitted in these decays?
   b. Sketch the potential energy function for the emitted particles in and near the daughter nucleus, and explain its essential features.
   c. Why are the lifetimes very long (on a nuclear scale)?
   d. $^{226}_{88}\mathrm{Ra}$ can also decay into $^{212}_{82}\mathrm{Pb}$ with the emission of a single particle X. What is X?
   e. Which of the modes (A) $^{226}\mathrm{Ra} \to ^{222}\mathrm{Rn} \to ^{218}\mathrm{Po} \to ^{214}\mathrm{Pb}$ or (B) $^{226}\mathrm{Ra} \to ^{212}\mathrm{Pb}$ is more likely and why?

7. Consider an electron-proton plasma with equal temperatures $T_e = T_p = T$, no magnetic field, in a uniform gravitational field $\vec{g}$ in the $-z$ direction. If the electrons and protons were neutral, their densities would be given by the Boltzmann law $n_{e,p} \propto \exp(-m_{e,p}\, gz/kT)$. Then the scale height would be quite different for electrons and protons. However, this would give rise to huge electric fields that would tend to move protons up and electrons down. Use the Boltzmann distribution law to find an electrical potential $\phi(z)$ such that $n_p(z) = n_e(z)$.

8. As shown in the figure, a glass plate of thickness d and index of refraction n is placed at height h above an object O. As viewed from above, find the distance of the image measured from the top surface of the glass. Assume that we are dealing with small angles and that the index of refraction outside of the glass is unity.



9. One mole of an ideal gas is carried around the thermodynamic cycle shown. Starting at A, where the pressure is 5 atmospheres, there is an isothermal expansion to B, a drop in pressure at constant volume to C, an isobaric compression to the original volume at D, and return to A again at a constant volume. Calculate p_1, V_0, T_c, and T_D, and the work done by the gas in the cycle.



10. Are the following transitions in carbon allowed or forbidden for electric dipole transitions? If they are forbidden, explain why.

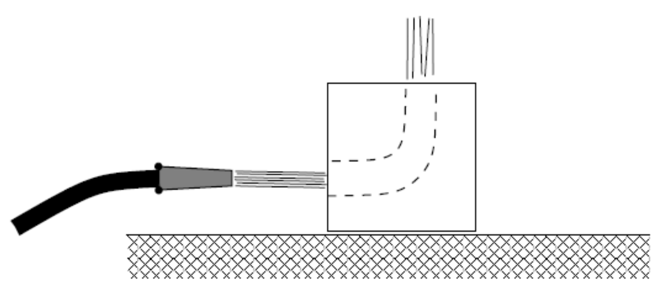
   a. $1s^2\,2s^2\,2p\,3p\;{}^1P_1 \rightarrow 1s^2\,2s^2\,2p^2\;{}^1S_0$
   b. $1s^2\,2s^2\,2p\,5p\;{}^1D_2 \rightarrow 1s^2\,2s^2\,2p\,3s\;{}^3P_1$
   c. $1s^2\,2s^2\,2p\,5d\;{}^3P_0 \rightarrow 1s^2\,2s^2\,2p\,3p\;{}^3P_0$
   d. $1s^2\,2s^2\,2p\,5d\;{}^3P_2 \rightarrow 1s^2\,2s^2\,2p\,3p\;{}^3P_1$
   e. $1s^2\,2s^2\,2p\,5d\;{}^3P_2 \rightarrow 1s^2\,2s^2\,2p\,3p\;{}^3P_0$

11. Robert Hanbury-Brown and Richard Twiss developed a famous experiment during the late 1950's to measure the angular diameter of a star. They observed correlated intensity fluctuations of green ($\lambda = 500$ nm) light from a particular star using two independent telescopes with detectors. The correlations were observed when the telescopes were separated by less than 80 m and vanished when the telescopes were 80 m apart. Find the angular diameter of the star.

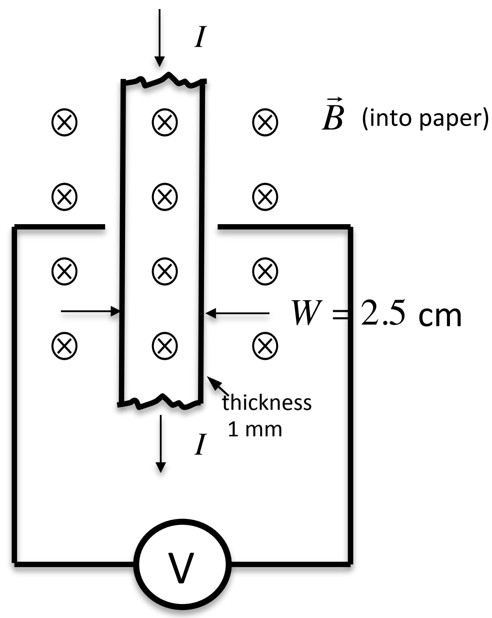
12. An infinitely long cylinder of radius a contains a uniform electric charge density ρ_0. The cylinder is rotated at a rate Ω about its symmetry axis along the z direction. Determine the direction and magnitude of the magnetic field everywhere in space.

13. A block of mass M, at rest on a frictionless surface, contains a curved channel. The channel is designed so that when a horizontal jet of fluid is aimed at it, the fluid is redirected 90° and shoots straight up and out of the top of the block. The flow of the liquid jet is Q kg/s and has a velocity v m/s. Gravity acts only in the vertical direction.

   a. In terms of M, Q, and v, what is the time dependent speed of the block, $v_b(t)$?
   b. How much power, $P(t)$, is being used in accelerating the block as a function of time?



14. Suppose that a current $I = 10A$ is flowing through a strip of conducting material that is 2.5 cm wide and 1 mm thick (thickness measured into paper). A magnetic field $B = 1.5$ T is applied as shown. The potential difference across the strip is measured and found to be 75 µV. From this information deduce the number of charge carriers per unit volume in the material.



15. A "perfect" spherical blackbody of mass density ρ, specific heat C, and radius r is placed into a totally dark, empty space. Assuming that the initial temperature is T_1, how long does it take to cool to a lower temperature T_2?