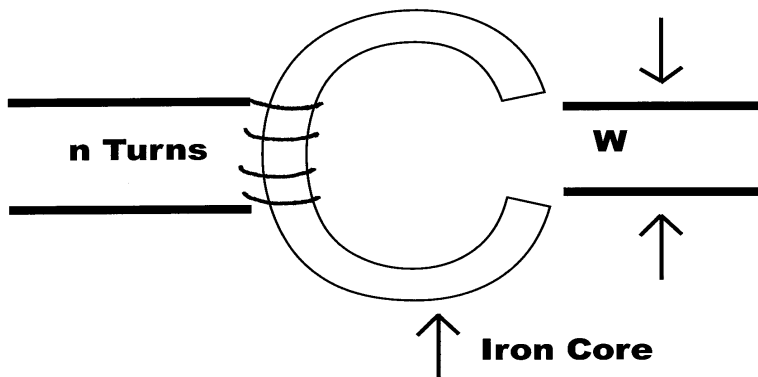1. Calculate the magnetic induction B in the gap of a C-shaped iron core electromagnet wound with n turns of a wire carrying current I as shown in the figure. The length of the path of the field lines in the iron core is L and the width of the gap is W, with W <<L. Express B as a function of I and the magnetic field in the iron, H.

n Turns

W

Iron Core

2. A hot-air balloon has a volume of about 2000 m^3 . Assume that the air in this volume has been heated to a temperature of 50°C, that the surrounding air is at STP, and that the air is made up only of nitrogen gas.

a) What is the maximum mass that this balloon can lift?
b) Roughly how much heat is required to get the air to a temperature of 50°C? Assume that during the heating process all the heat goes into air that stays in the balloon.

3. A hollow sphere and uniform solid sphere of equal mass m and radius R roll down an inclined plane from the same height H without slipping. Each is moving horizontally as it leaves the ramp. The hollow sphere hits the ground at a distance L as shown. Find the range L' of the solid sphere in terms of L. The moment of inertia for the spherical shell is $(2/3)(mR^2)$ and for the uniform sphere is $(2/5)(mR^2)$.

Uniform sphere

Hollow sphere

L

L'
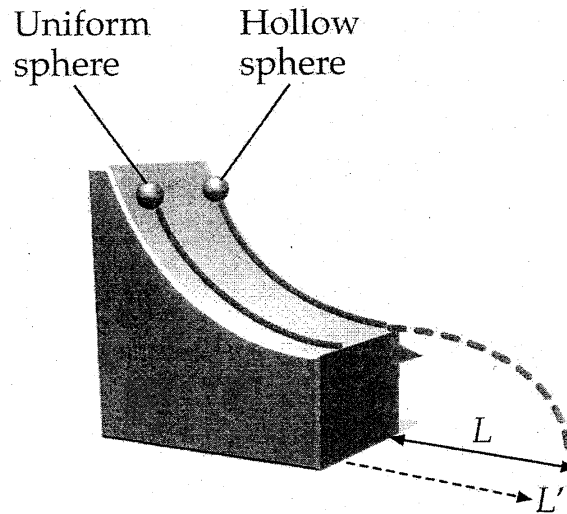
4. A system consists of two solid bodies with the constant and identical volume heat capacities C_V and the initial temperatures $T_1 \neq T_2$. The bodies are brought in contact with each other, so that thermal equilibrium between them is established. Assume that the system is isolated and that the volumes of the bodies do not change. Does the entropy of the system increase, decrease, or stay the same in this process? Explain your answer.

5. Consider two different one-dimensional quantum potential wells of the same depth. The width of well I is 0.1 nm and that of well II is 0.05 nm. If there is a particle with the mass of an electron in well I and a particle with the mass of a proton in well II, which particle has the lower ground state energy? Neglect tunneling into the barriers. Justify your answer.

6. Consider a single electron at a corner of a cubical volume of space. What fraction of the total flux of the electric field passes through each of the faces of the cube?

7. A thin film has an index of refraction of 1.4 and it is surrounded by air. White light illuminates it along the surface normal. Analyzing the resulting reflected light, it is observed that the missing wavelengths are 618.0 nm and 515.0 nm in the visible portion of the spectrum.
a) Derive the film thickness.
b) Calculate the wavelength of any one wavelength that is a maximum in the reflected interference pattern.

8. Find the electric field inside a uniformly charged solid sphere with total charge Q and radius R. Calculate the net force that the southern hemisphere exerts on the northern hemisphere.

9. A 10 microhenry inductor with 3Ω resistance is connected to the 120 volt (RMS), 60 Hz ac power line.
a) What value of capacitor should be placed in parallel with this inductor to minimize the current drawn from the power line?
b) What is the value of the RMS current drawn for the power line by this combination of components?

10. An object is placed 12.0 cm in front of a diverging lens of focal length 6.0 cm.
a) How far is the image from the lens?
b) Is the image in front of or behind the lens?
c) Is the image real or virtual?
d) What is the size of the image relative to the object?
e) Is the image erect or inverted?
f) A human being stands on the opposite side of the lens from the object. Where should the person focus his or her eyes to see the object?

11. Consider a uniform half cylinder rocking on a plane with mass M, radius r and length L as shown in the figure. We want to study the motion for angles $\phi < \pi/2$. Assume that the center of mass is at $a = 4r/3\pi$ and that the moment of inertia with respect to the axis of the object that passes through the center of mass of the system is $Mr^2/2 - Ma^2$.

a) Determine the Lagrange function of the system as a function of the coordinate ϕ.
b) Derive the equation of motion.
c) Linearize the equation of motion for small $\phi \ll 1$ and calculate the frequency of harmonic oscillation.
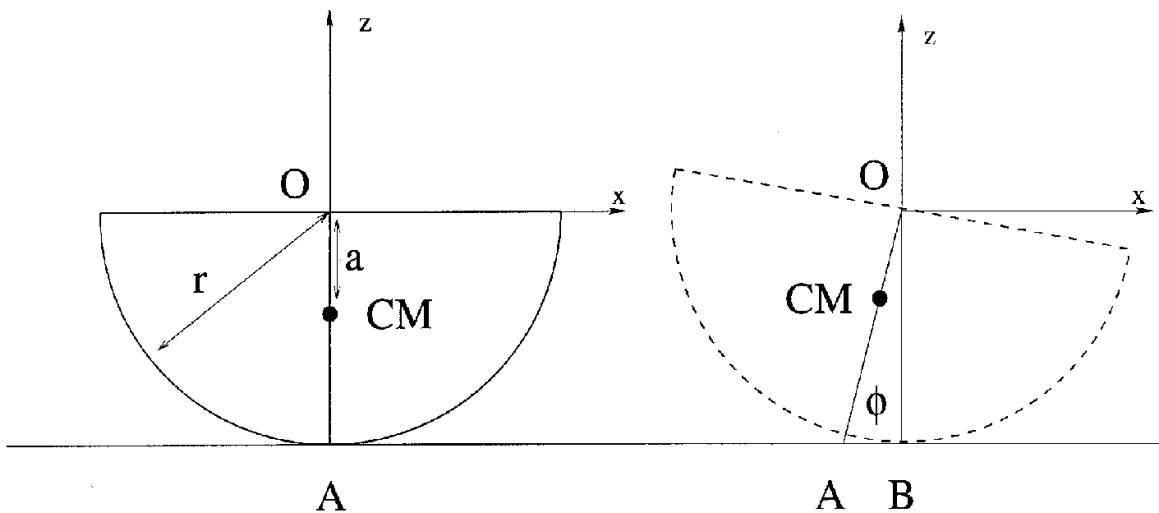
12. The tension in a wire of length L and cross sectional area is increased quasi-statically and isothermally from H_i to H_f. The wire has isothermal Young's modulus Y which is defined as:

$$Y = (L/A) \, \partial H/\partial L|_T .$$

Find an expression for the work done for a very small change in length, such that the cross-sectional area and the isothermal Young's modulus remain nearly constant.

13. Positronium is a hydrogen-like bound state made up of an electron and a positron of equal mass m_e .

a) Estimate the binding energy of the ground state (n= 1) and the Lyman-alpha (2p → 1s) transition wavelength for positronium.

b) The lifetime for a hydrogen atom decay from 2p to 1s is 1.6 ns. Estimate the lifetime for the same decay in positronium. Hint: The decay rate for this electron dipole transition scales as $\Gamma \propto \omega^3 a_p^2$, where ω is the energy difference between the two states and a_p is the Bohr radius in positronium.

14. Using ultra-relativistic dynamics for gas molecules, that is, $\varepsilon(p) = cp$, find, for a dilute gas:
a) the equilibrium distribution function;
b) the free energy;
c) the equation of state.

Hint: $\int_0^\infty x^2 e^{-x} dx = 2$

15. A rocket travels to a star that is a distance D away from Earth. The passengers measure that their acceleration (a') is constant.
a) Write the Lorentz transformation between the rocket and the Earth (star) reference frames.
b) Calculate the relation between velocity (dx/dt) and acceleration (d^2x/dt^2).
c) Calculate the total time in the rocket frame in terms of (a') and the final velocity of the rocket v_{final} .

16. A particle of mass M is confined by a central potential V(r) with ground state wavefunction:

$\Psi(r) = N \exp(-c/r - \beta r)$

a) Sketch a graph of this function.

b) Use Schrödingers' equation to find the potential.
Hint: In three dimensions the square radial momentum is:

$$p_r^2 = \frac{-\hbar^2}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}$$

c) What is the value of the energy of this state?

17. A Maxwellian gas is in a box of negligible mass that rests on a frictionless plane. The gas consists of particles of mass m, at number-density n and temperature T, so that its velocity distribution function is:

$$f = n \left(\frac{m}{2\pi kT} \right)^{3/2} \exp\left(\frac{-mv^2}{2kT} \right)$$

A beam of particles moving in a horizontal direction is then injected into the box. The beam has the same volume as the box, and also contains particles at density n and mass m. But its temperature is zero and the particles are all moving to the right at speed v_o. Through collisions the beam energy is shared between all the particles in the box.
a) Find the final velocity.
b) Find the final temperature.
c) Find the velocity distribution function of the gas in the box in the final steady state.

18. a) You have a radioactive source which together with the room background produces approximately (A) counts per second in your detector. The room background rate alone is approximately (B) counts per second. If you have a fixed total time available, what is the optimal division between time counting the source and time counting the background to minimize the statistical uncertainties in your determination of the net source rate C = A-B? Clearly state your assumptions and show your work.

b) Now assume that the background rate is negligible, and you have a second source that produces a rate of about B counts per second in the detector. How should you divide the counting time between sources A and B to provide the most precise determination of the ratio of source strengths, C = A/B ?

19. An electron at the surface of liquid helium has a potential energy:

$$V(z) = \infty \qquad\qquad z < 0$$
$$\frac{-e^2}{16\pi\,\varepsilon_0}\left(\frac{K-1}{K+1}\right)\frac{1}{z} \qquad z > 0$$

where the dielectric constant of liquid helium is $K = 1+ (5.7 \text{ x } 10^{-2})$. Find the ground state energy in meV of the electron. Assume that the motion parallel to the surface has zero momentum.
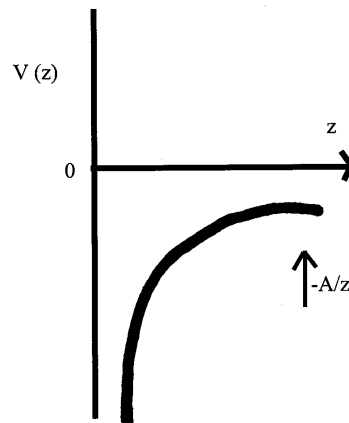
20. A bead of mass m moves without friction on a table with a circular orbit of radius r as shown in figure. It is connected to a string that passes through a hole in the center of the table. Someone pulls down on the string such that the string remains taut and the circular orbit's radius is halved. The bead initially has energy E_o. After the radius is halved, how much work was done?
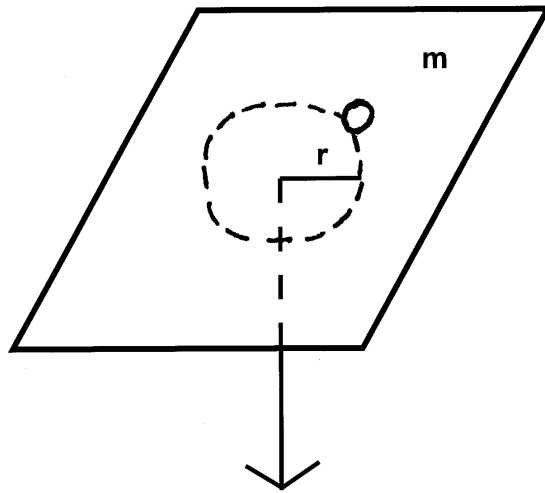
21. Galactic rotation curves are obtained by measuring the orbital velocity V(r) of the stars orbiting the galactic center. A typical curve of the measurements out to the luminous edge of the galaxy is shown below.
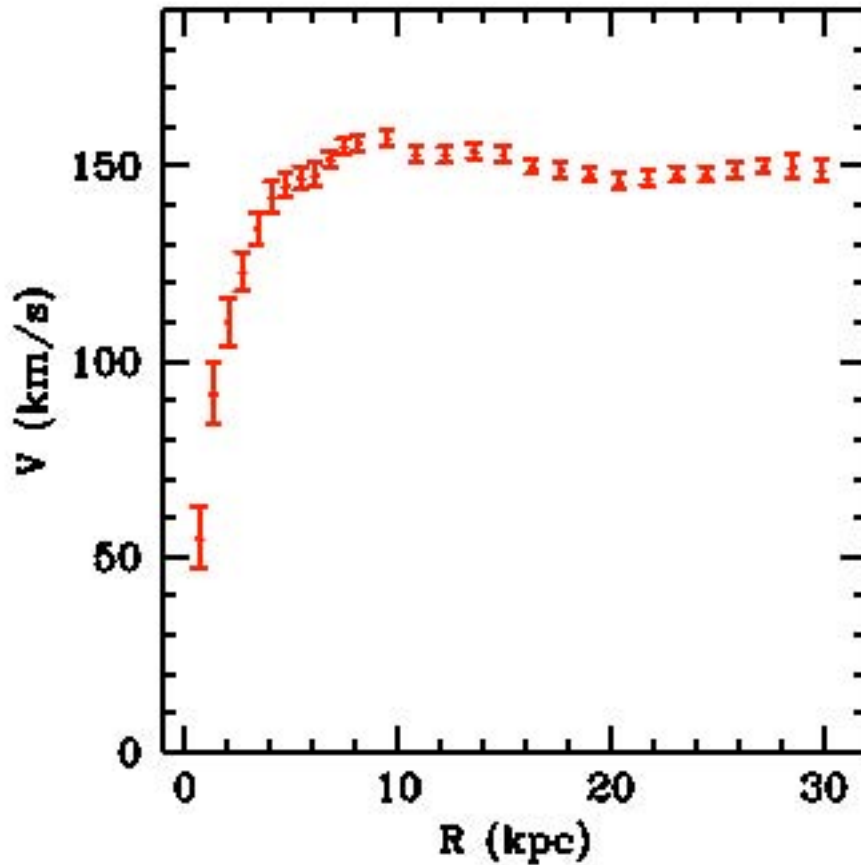
a) If the mass contained inside radius R is M(R), calculate the expected V(R).
b) For a spiral galaxy like ours, explain how the measured rotation curve differs from your expectations if the radius of the galaxy is 30 kpc.
c) Use the rotation curve to estimate the mass density ρ (R) for R > 10 kpc in terms of R, V(R) and the gravitational constant G.
d) If the mass density continued to obey this form for R > 30 kpc, what would the mass of the galaxy be?

22. Consider a gas phase laser.

(a) Explain, both mathematically and physically, why a system having two energy levels in thermal equilibrium cannot function as a laser medium.
(b) If a system has three energy levels and is not in thermal equilibrium, explain the condition for functioning as a laser medium. A quantitative analysis means you need to draw the level scheme you are analyzing and write down and solve the relevant rate equations.

23. Consider the ^{23}Na atom with electron configuration $(1s)^2(2s)^2(2p)^6 (3p)^1$ and nuclear spin I = 3/2.
a) Draw a diagram to show the multiplet structure (fine structure) of the 3p levels due to the electron spin. Label each level by the appropriate quantum numbers and express them in spectroscopic notation.
b) Now show the hyperfine structure of each of your fine structure levels due to the nuclear spin. Label each hyperfine structure level by the appropriate quantum numbers.

24. Two hadrons exchange a virtual meson of rest mass μ and momentum p. Use the relation:

$$\Delta E \, \Delta t \approx \hbar$$

to estimate the value of p for a maximum range R of the resulting interaction. Work relativistically, using v(p) and E(p).

25. The supernova observed in 1987 was located about D= 1.7 x 10^{21} meters from Earth. The Kamiokande detector in Japan observed 10 neutrinos in a tank of one kiloton water within an interval of Δt= 1 second. The average neutrino energy was 10 MeV and the energies of the ten neutrinos observed varied from 5 to 20 MeV. The experimental signature of each neutrino interaction included recoil nucleon as well as a charged lepton.
a) What interactions were most likely responsible for the detection of neutrinos?
b) Estimate an upper limit for the neutrino mass from the observation.

26. A neutral pion is moving in the x-direction and is observed to decay into two photons. One of the photons travels in the x-direction and the other in the negative x-direction. The difference in energy between these two photons is measured to be equal to the rest energy of the pion. Calculate the speed of the pion just before its decay, in units of the speed of light.

27. A Cherenkov detector, consisting of a transparent gas with index of refraction n= 1.0008, is used to distinguish between charged pions and kaons by measuring the Cherenkov angle. Given a beam of kaons (mass m_K = 494 MeV/c^2) and pions (mass m_π = 140 MeV/c^2) of momentum p:

a) Show that the difference in angle $\Delta\Theta$ for the 2 particle types is given approximately by:

$$\Theta \, \Delta\Theta \approx (1/\, n^2\,) \, (m_K{}^2 - m_\pi{}^2)/\, 2p^2$$

b) Assuming that the angular resolution allows the separation of ultrarelativistic pions and kaons for $\Delta\Theta > 0.5$ mrad, show that this detector can distinguish the 2 particle types up to a momentum of p = 75 GeV. Hint: Work in the assumption of small angles $\sin\Theta \approx \Theta$.