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DEPARTMENT OF PHYSICS

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DOCTORAL GENERAL EXAMINATION

PART 1

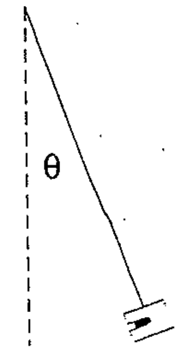
SEPTEMBER 8, 1999

FIVE HOURS

1. This examination is divided into five sections, each consisting of four problems. Answer <u>all</u> the problems. Each problem is worth 5 points, thus the maximum score for the exam is 100.
2. Use a separate <u>fold</u> of paper for each problem. Write your name and the problem number (IV-3 for example) on each fold. A diagram or sketch as part of the answer is often useful, particularly when a problem asks for a quantitative response.
3. Read the problem carefully and do not do more work than is necessary. For example "give" and "sketch" do not mean "derive".
4. Calculators may be used but are not necessary.
5. No books, notes or reference materials may be used.

GROUP I

1. Ballistic Pendulum



The velocity of a bullet can be measured by firing it into a pendulum mass (in which it becomes embeded) and observing the subsequent amplitude of motion. Let M represent the mass of the pendulum, m the mass of the bullet, g the acceleration of gravity, and τ the period of oscillation. Find the speed of the bullet in terms of these quantities and the maximum angle θ achieved by the pendulum.

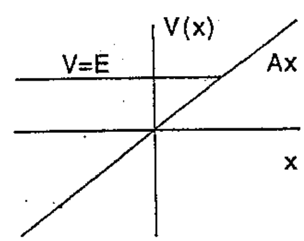
2. Leaking Capacitor

A parallel plate capacitor of area A and separation t is filled with a dielectric that has a dielectric constant ϵ and a conductivity σ.

a) Determine the characteristic time it takes for charge to leak from one plate to the other.

b) Determine the AC electrical impedance of the leaky capacitor as a function of frequency. In what range of frequencies is it possible to think of the device as a resistor and in what range is it better modeled as a capacitor?

3. Particle in a Constant Force Field



A particle moving in one dimension is subject to a constant force arising from a potential $V(x) = Ax$. It is in an energy eigenstate with energy E. Find the wavefunction in momentum space, $\psi_E(p_x)$. Show that all possible values of the momentum are equally probable.

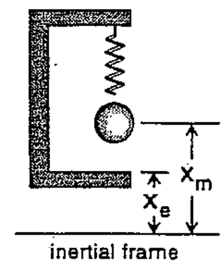
4. Particle Decay

Show from a relativistic invariance argument that the decay of a single photon into an electron and a positron is forbidden.

GROUP II

1. Vertical Seismometer

A simple vertical seismometer can be made with a mass and a spring: one end of the spring is attached to the Earth and the other to the mass. Measurements are made of the motion of the mass relative the the earth, $(x_m - x_e)$.



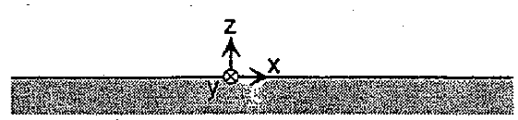
a) Determine the ratio of the relative displacement to the displacement of the Earth as a function of the frequency of the motion, $(x_m(\omega) - x_e(\omega))/x_e(\omega)$.

b) In what frequency range does the seismometer sense the displacement of the Earth directly?

2. Electron Beam

Measurements on a coherent beam of electrons indicate that the mean value of the spin in the x direction is $< S_x >$. Independent measurements indicate that the mean value of the spin in the y direction is $< S_y >$. What would be the mean value of the spin in the z direction, $< S_z >$, for this beam?

3. Evanescent Wave



The plane $z = 0$ forms the boundary between a dielectric material (such as glass) and a vacuum. In the vacuum region $z > 0$ there is an electric field linearly polarized in the y direction, propagating in the x direction, and decreasing exponentially in the z direction: $\vec{E} = \hat{y} E_0 \cos(k_x x - \omega t) \exp(-\alpha z)$.

a) Find α in terms of k_x, ω and any necessary physical constants.

b) Find $\vec{B}$ in the region $z > 0$.

4. Defects in a Solid

A crystalline solid contains N similar, immobile, statistically independent defects. Each defect has 5 possible states ψ_1, ψ_2, ψ_3, ψ_4, and ψ_5 with energies $\epsilon_1 = \epsilon_2 = 0$, and $\epsilon_3 = \epsilon_4 = \epsilon_5 = \Delta$. Find the defect contribution to the entropy of the crystal as a function of Δ and T.

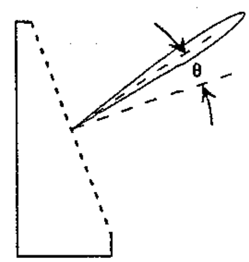
GROUP III

1. Balloon in the Earth's Atmosphere

A spherical balloon of fixed radius R and total fixed mass M is released into the atmosphere. Assume that the pressure in the atmosphere varies with height as $P(h) = P_0 \exp(-h/h_0)$ where the scale height of the atmosphere $h_0 >> R$. Find an expression for the equilibrium height attained by the balloon.

2. Phased Array Radar

The height-finding antenna of an airport's ground control approach radar consists of a linear array of 100 dipoles 1.6 cm apart radiating at a wavelength of 3.2 cm. The phase difference ϕ of the electric current fed to successive dipoles can be changed to sweep the direction of the transmitted beam.



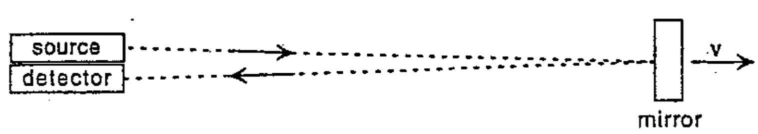
a) What is the approximate angular full width at half height of the beam when it is perpendicular to the array ($\theta = 0$)?

b) What is $d\theta/d\phi$ near $\theta = 0$?

3. Solar System Dust

The solar system is filled with dust, the so called zodiacal cloud. Determine the equilibrium temperature of the dust as a function of its distance R from the sun. Assume the sun has a surface temperature of 6000 K and an effective radius of r_s. You may also assume that the absorptivity of both the sun and the dust can be approximated as $\alpha(\nu, T) = 1$. What is the dust temperature at the distance of the Earth where $R \approx 200\, r_s$?

4. Doppler Effect



A beam of monochromatic light at frequency ν_0 leaves a source at rest in the laboratory, is reflected at normal incidence from a mirror moving away from the source with velocity v, and is then detected by a device at rest in the lab. What is the frequency of the detected light? Do not assume that the mirror is moving slowly compared to the speed of light, but do show that your general result reduces to the classical result when $v << c$.

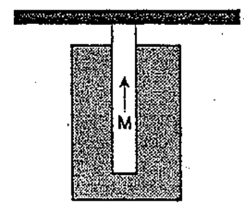
GROUP IV

1. Artillery Shell

The maximum range of a 4 meter long gun (achieved when the gun is elevated at 45 degrees) is 10 kilometers. What is the acceleration of the shell in the gun as a multiple of the acceleration of gravity, g? Assume the acceleration is constant in the gun barrel and that air resistance can be neglected.

2. Magnetic Attachment

A permenant magnet embeded in a non-magnetic block is used to attach the block to a highly permeable horizontal steel plate. The long, thin magnet has a uniform cross-section of 1 cm^2 and a uniform magnetization M of 10^3 cgs units (dyne-cm^{-2}-$gauss^{-1}$). How large a mass, block plus magnet, can be supported against gravity in this way.



3. Electronic Dipole Moment

What fundamental symmetry is violated if the neutron is found to have an electronic dipole moment? Using dimensional arguments, estimate a physical upper limit to the magnitude of any possible electronic dipole moment.

4. Heat Capacity of Solids

A solid has a Debye temperature $T_D \approx 10^2$ K, and a Fermi temperature $T_F \approx 10^5$ K. Estimate the temperature interval where the electronic heat capacity is larger than the phonon heat capacity.

GROUP V

1. Neutrino Mass

Neutrinos were detected from the supernova in the Large Magellanic Cloud in 1987. The cloud is about 1.6 $\times 10^5$ light years from the Earth. The neutrinos, varying in energy between 10 and 40 Mev, arrived over a time of 3 seconds. From the energy range and width of the arrival time interval, estimate an upper limit for the mass of the neutrino. Begin by finding an expression for v/c as a function of the total energy E in the limit $E \gg m_0c^2$.

2. Liquid Drop Model

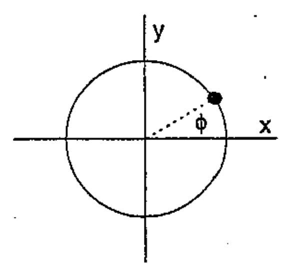
In a simplified liquid-drop model of a nucleus, the binding energy E is given in terms of the number of nucleons A and the number of protons Z as

$$E(A, Z) \approx a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3}$$

with $a_1 \approx$14 Mev, $a_2 \approx$ 13 Mev, and $a_3 \approx$ 0.58 Mev. Explain the origin of each of the terms in the above expression.

3. Particle on a Ring

A particle of mass M and charge Q is constrained to move on a circle of radius R in the $x - y$ plane. Its Hamiltonian is $\hat{H} = \hat{L}_z^2/2MR^2$. Find the shift of the ground state energy through terms of order $\mathcal{E}_0^2$ when the system is perturbed by a uniform electric field $\vec{\mathcal{E}} = \mathcal{E}_0 \vec{1}_x$ pointing in the x direction.



4. Elastic Rod

The tension $\mathcal{F}$ in an elastic rod depends on the temperature T and length L as $\mathcal{F} = (K_1 + K_2 T)(L - L_0)$. The heat capacity of the rod at constant length is given by $C_L = AT^3$. In these expressions K_1, K_2, L_0 and A are constants. Use a Maxwell relation to help you find the entropy of the rod as a function of its temperature and length, $S(T, L)$. [Note that the differential of work done on the rod is given by $đW = \mathcal{F} dL$.]