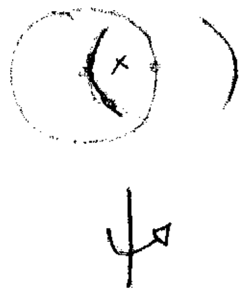
SPRING 2001

Section 1
Problem 1



x marks center of mass, CM
Each orbit is an ellipse with focus at CM.
If radius is b, distance from centre of circle to CM is b/2
But this ought to be ae where a is semi major (if unprojected).
Semi major axis a projects to b
therefore $\frac{ae}{a} = \frac{b/2}{b} \Rightarrow e = \frac{1}{2}$.

1-2 SHO Hamiltonian - raising + lowering ops.

$$\frac{p^2}{2m} + kx^2 = H$$

$$\left(\frac{p}{\sqrt{2m}} + i\sqrt{k}x\right)\left(\frac{p}{\sqrt{2m}} - i\sqrt{k}x\right) = a^+ a \approx H$$

$$\left.\begin{aligned} a^+ &= \frac{p}{2m} + i\sqrt{k}x \\ a &= \frac{p}{2m} - i\sqrt{k}x \end{aligned}\right\} \Rightarrow a^+ - a = 2i\sqrt{k}\,x$$

Some constant

$$a^+|n\rangle = \kappa\,|n+1\rangle \qquad a|0\rangle = 0$$

$$a|n\rangle = \kappa'\,|n-1\rangle$$

Then $\langle n'|x|n\rangle \propto \langle n'|a^+ - a|n\rangle$

$\neq 0$ if $n' \pm 1 = n$ if $n', n \neq 0$

$n' = 0,\ n = 1$

$n' = 1,\ n = 1$

$\Delta E = \hbar\omega$

1.3 $m\ddot{x} = -mg + A\rho C_D \dot{x}$

$m\dot{v} = -mg + A\rho C_D v$

Solution $v(t) = \alpha e^{-\lambda t} + \beta$, $\beta = v_{term}$

$\dot{v} = -\lambda\alpha e^{-\lambda t}$ $\quad\quad \gamma = A\rho C_D$

Then $-m\lambda\alpha e^{-\lambda t} = -mg + \gamma\alpha e^{-\lambda t} + \gamma\beta$

Two condition $-m\lambda \cancel{\alpha e^{-\lambda t}} = \gamma\cancel{\alpha e^{-\lambda t}}$

$\lambda = \gamma/m = A\rho C_D/m$

and

$-mg + \gamma\beta = 0 \Rightarrow \beta = \frac{mg}{\gamma} = \frac{mg}{A\rho C_D}$

For a person: $A \sim 1 m^2$
$m \sim 100 kg$
$g \sim 10 m/s^2$
$C_D \sim 10 m/s$

$\rho = \frac{14 g}{22.4 \times 10^3 cm^3} = 0.6 \times 10^{-3} \frac{g}{cm^3} \times \frac{1 kg}{10^3 g} \times \frac{10^6 cm^3}{1 m^3} = 0.6 \frac{kg}{m^3}$

$\gamma = 1 m^2 \times 0.6 \frac{kg}{m^3} \times 10 \frac{m}{s} = 6 \ kg/s$

$\beta = \frac{100 kg \times 10 m/s^2}{6 \ kg/s} = 0.16 \times 10^3 m/s = 160 m/s$

$\frac{1}{\lambda} = \frac{100 kg}{6 \ k/s} = 16 s$

1-4 $\psi_0 = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ gives $\mathcal{H}\psi = E_0 n \psi$

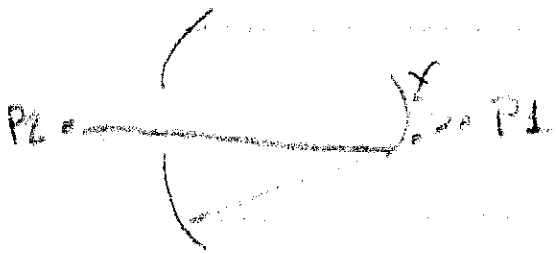
An orthogonal ev. must also be an ev. $\rightarrow$

$\psi_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ \vdots \end{pmatrix}$, $\psi_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \\ \vdots \end{pmatrix}$, ...

Eigenvalues are nE_0 and 0. Degeneracy of nE_0 is 1, 0 is $n-1$

Section 2

Problem 1 Change parabolic to paraboloidal; add word perfect before focus, twice



Hyperboloid. All paths to P1 are same length. All paths to P2 should likewise be same length. The second mirror adds a line segment P_2X and subtracts P_1X from the path length $\overline{P_2X} - \overline{P_1X}$ = constant. But a hyperbola is the locus of points the difference of whose distances to two points is a constant. Rotate about symmetry axis & get a hyperboloid.

2-2 $$Z = \sum_{l=0}^{\infty} (2l+1)\, e^{-l(l+1)\varepsilon/kT}$$

$$\varepsilon = \hbar\omega$$

Internal energy $U = kT^2 \dfrac{\partial Z}{\partial T}$

$$Z \to \int_0^\infty 2l\, e^{-l^2\varepsilon/kT}\, dl \qquad \text{for } l \text{ large}$$

$$x = \frac{l^2\varepsilon}{kT} \to ; \quad dx = \frac{2l\varepsilon}{kT}\, dl$$

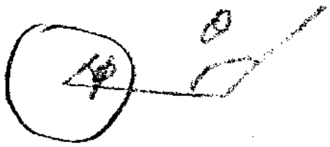
Then $$Z = \int_0^\infty e^{-x} \frac{kT}{\varepsilon}\, dx$$

$$= -\frac{kT}{\varepsilon} e^{-x}\Big|_0^\infty = \frac{kT}{\varepsilon}$$

$$U = kT^2 \frac{\partial Z}{\partial T} = kT^2 \frac{k}{\varepsilon} = \frac{k^2T^2}{\varepsilon}$$

$$C_v = \frac{\partial U}{\partial T} = \frac{2k^2T}{\varepsilon}$$

2-3 $\mathcal{H} = \frac{p_\theta^2}{2m} + \frac{p_\phi^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{1}{a^2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{b^2}\frac{\partial^2}{\partial\phi^2}\right)$



$\psi \propto e^{il\phi} e^{in\theta}$

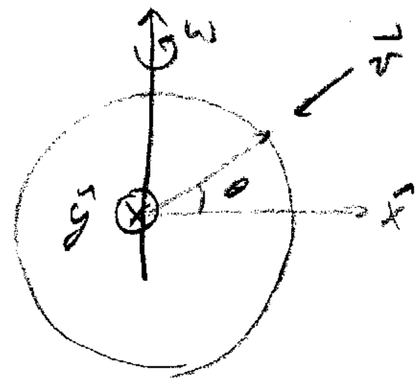
$$\mathcal{H}\psi = -\frac{\hbar^2}{2m}\left(\left(\frac{-n^2}{a^2}\right) + \left(\frac{-l^2}{b^2}\right)\right)\psi = E\psi$$

$$\frac{\hbar^2}{2m}\left(\frac{n^2}{a^2} + \frac{l^2}{b^2}\right) = E_{nl}$$

All states have infinite degeneracy

$\psi \propto e^{il\phi+\phi_0} e^{in\theta+\theta_0}$

2-4 $F_{cor} = -2m\vec{\omega} \times \vec{v}$



$\vec{v} = v\cos\theta\,\hat{x} + v\sin\theta\,\hat{y}$

$\vec{\omega} = \omega\hat{z}$

$$\vec{\omega} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v\cos\theta & 0 & v\sin\theta \\ 0 & 0 & \omega \end{vmatrix}$$

$$= 0\hat{x} + -\omega v\cos\theta\,\hat{y} + 0\hat{z} = -\omega v\cos\theta\,\hat{y}$$

$\vec{F}_{cor} = +2m\omega v\cos\theta\,\hat{y}$ $\quad(\hat{y} = \text{east})$

Fall time: $d = \frac{1}{2}gt_0^2 \Rightarrow t_0 = \sqrt{\frac{2d}{g}} = \sqrt{\frac{440\text{ m}}{10\text{ m/s}}} = 6.7\text{ s}$

Then, $\cancel{m}\dot{v}_y = 2\cancel{m}\omega v_r\cos\theta$

$$v_r = -gt \Rightarrow v_y(t) = \cancel{2}\omega\cos\theta\left(\frac{-gt^2}{\cancel{2}}\right)$$

$$\Rightarrow y(t) = -\frac{\omega g t^3}{3}\cos\theta$$

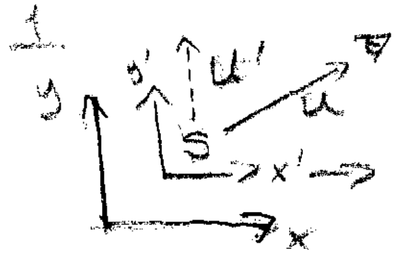
$$\Rightarrow y_0 = -\frac{\omega g}{3}\left(\frac{2d}{g}\right)^{3/2}\cos\theta$$

$$= -\left(\frac{7.3\times10^{-5}/\text{s}}{3}\right)\left(10\frac{\text{m}}{\text{s}^2}\right)(6.7\text{ s}^3) = 5\times10^{-3}\text{ m}$$

$$= 5\text{ mm}$$

Section 3

1



$x = \gamma(x' - vt')$ + or - not sure try -

$t = \gamma\left(t' - \frac{vx'}{c^2}\right)$

$u' =$ $y = y'$

$u' = \frac{dy'}{dt'} = c$

$u = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y}$

$=$

$$\left(\text{tangent of angle}\right) = \frac{dy/dt}{dx/dt} = \frac{dy/dt'}{dx/dt'}$$

$$= \frac{c}{\gamma v} = \boxed{\frac{1}{\gamma\beta} = \tan\theta}$$

$\cot\theta = \beta\gamma$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \beta^2\gamma^2 = \csc^2\theta$

$\gamma^2 = \csc^2\theta$ $\boxed{\sin\theta = \frac{1}{\gamma}}$

$1 + \frac{\beta^2}{1-\beta^2} = \frac{1}{1-\beta^2} = \gamma^2$

3-2 Particle on a sphere

$$\mathcal{H}:\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi$$

$$\nabla^2 = \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2} = L^2$$

$$\rightarrow -\frac{\hbar^2}{2m}L^2\psi = E\psi \qquad \psi = \delta(r-a)Y_{lm}(\theta,\phi)$$

Eigenfunctions: $L^2 Y(\theta,\phi) = \lambda Y(\theta,\phi)$

$$\lambda = l(l+1)$$

Then, $+\frac{\hbar^2}{2ma^2}(l+1)l\ \psi = E\psi$

$$\Rightarrow E_l = \frac{+\hbar^2 l(l+1)}{2ma^2} \qquad \frac{\text{kg m}^2}{\text{s}^2}\ \frac{\cancel{\text{kg}}\ \cancel{\text{m}^2}}{\cancel{\text{s}^2}}\ \frac{\text{s}^2}{\cancel{\text{m}^2}\ \cancel{\text{kg}}}$$

$$\varepsilon = \frac{\hbar^2}{2ma^2} \qquad E_l = 0\varepsilon, 2\varepsilon, 6\varepsilon, \ldots$$

$$P = \frac{-1}{4\pi a^2}\frac{\partial E_l}{\partial a} = -\frac{1}{4\pi a^2}\left(\frac{+\hbar^2 l(l+1)}{2m}\right)\left(-\frac{2}{a^3}\right)$$

$$= \frac{\hbar^2 l(l+1)}{4\pi m a^5} \qquad \text{Units } \frac{\hbar^2}{\underset{\varepsilon}{ma^2}}\ \frac{1}{a^3}\ \frac{F\cdot a}{a^3} = \frac{F}{a^2}$$

<u>Stark effect</u> $V = E_0 z = E_0 r\cos\theta = E_0 a\cos\theta$

$$r\cos\theta = \sqrt{\frac{4\pi}{3}}\, Y_{10}\, k$$

$$\Delta E = \int \psi_{lm}^{*}\, V\, \psi_{lm}\, d\Omega$$

$$= \left[\int \psi_{lm}\, Y_{10}\, \psi_{lm}\, d\Omega\right]\left(\frac{4\pi}{3}\right) E_0$$

$$\propto \begin{pmatrix} l & 1 & l \\ -m & 0 & m \end{pmatrix} \begin{pmatrix} l & 1 & l \\ 0 & 0 & 0 \end{pmatrix}$$

Clebsch vanishes unless

$-m + 0 + m = 0$ ✓

$l + 1 + l$ is even

3-3 $$Z = \sum_{n=0}^{N} e^{-n\varepsilon/kT}$$

Sum the series

$$Z = 1 + \sum_{n=1}^{N} e^{-n\varepsilon/kT} = 1 + e^{-\varepsilon/kT} \sum_{n=0}^{N-1} e^{-n\varepsilon/kT}$$

$$= 1 + e^{-\varepsilon/kT}\left(Z - e^{-N\varepsilon/kT}\right) = 1 + e^{-\varepsilon/kT} Z - e^{-(N+1)\varepsilon/kT}$$

$$Z(1 - e^{-\varepsilon/kT}) = 1 - e^{-(N+1)\varepsilon/kT}$$

$$Z = \frac{1 - e^{-(N+1)\varepsilon/kT}}{1 - e^{-\varepsilon/kT}}$$

Internal energy $U = kT^2 \dfrac{\partial Z}{\partial T}$

$$\langle s \rangle = \frac{U}{\varepsilon} = \frac{kT^2}{\varepsilon} \frac{\partial Z}{\partial T}$$

Closed links $= N - \langle s \rangle$

3-4

$$\Phi = \pi a^2 B = \frac{\pi a^2 B_0 x}{x_0}$$

$$\mathcal{E} = -\frac{1}{c}\frac{\partial \Phi}{\partial t} = \frac{\pi a^2 B_0}{x_0}\frac{dx}{dt} = \frac{\pi a^2 B_0}{x_0} v$$

$$I = \frac{\mathcal{E}}{R} \quad ; \quad P = I\mathcal{E} = \frac{\mathcal{E}^2}{R} = Fv = m\dot{v}v$$

$$\Rightarrow m\dot{v}\cancel{v} = -\left(\frac{\pi a^2 B_0}{x^0}\right)^2 v^{\cancel{2}}$$

$$\dot{v} = -\frac{1}{m}\gamma v \qquad \gamma = \left(\frac{\pi a^2 B_0}{x_0}\right)^2$$

$$v(t) = \alpha e^{-\lambda t} + \beta \quad \text{and} \quad v(0) = \alpha + \beta = v_0$$

$$v(\infty) = 0 = \beta$$

$$\Rightarrow v(t) = v_0 e^{-\lambda t} \quad ; \quad \dot{v} = -\lambda v_0 e^{-\lambda t}$$

Then,

$$-\lambda \cancel{v_0 e^{-\lambda t}} = -\frac{1}{m}\gamma \cancel{v_0 e^{-\lambda t}} \Rightarrow \lambda = \frac{\gamma}{m}$$

Total distance is then

$$x = \int_0^\infty v(t)\,dt = -\frac{m v_0}{\gamma} e^{-\gamma/m}\Big|_0^\infty = \frac{m v_0}{\gamma} = m v_0 \left(\frac{x_0}{\pi a^2 B_0}\right)^2$$

4-1 Without GR,

$$\frac{\cancel{m} v^2}{\cancel{a}} = \frac{GM\cancel{m}}{a^{\cancel{2}}} \quad \text{and} \quad v = a\dot{\theta}$$

Orbit period is $\tau = \dfrac{2\pi a}{v} = \dfrac{2\pi a}{a\dot{\theta}} = \dfrac{2\pi}{\dot{\theta}}$

$$\rightarrow \dot{\theta} = \frac{2\pi}{\tau} \; ; \quad \text{Then} \quad a^2\dot{\theta}^2 = \frac{4\pi^2 a^2}{\tau^2} = \frac{GM}{a}$$

$$\tau^2 = \frac{4\pi^2 a^3}{GM}$$

With GR

$$\frac{\cancel{m}\, a^{\cancel{2}}\, 4\pi^2}{\cancel{a}\, \tau_G^2} = \frac{GM\cancel{m}}{(a - GM/c^2)^2}$$

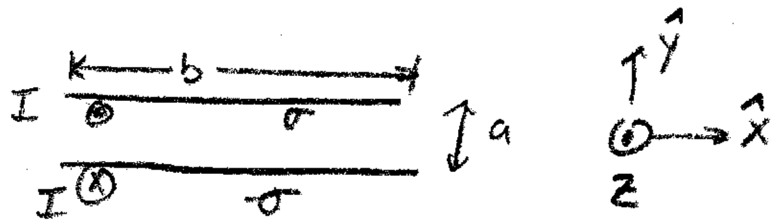
$$\tau_G^2 = \frac{4\pi^2 (a - GM/c^2)^2 a}{GM}$$

$$= \frac{4\pi a^3}{GM}\left(1 - \frac{GM}{ac^2}\right)^2 = \tau^2\left(1 - \frac{2GM}{ac^2}\right)$$

$$\frac{\tau_G}{\tau} = \left(1 - \frac{2GM}{ac^2}\right)^{1/2} = 1 - \frac{GM}{ac^2}$$

Period is <u>shorter</u> by a factor GM/ac^2

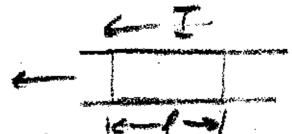
4-2



$$\vec{E} = -4\pi\sigma\hat{y} \rightarrow V = 4\pi\sigma a$$

A length l carries charge $\sigma l b$, then

$$C = \frac{\cancel{\sigma} l b}{4\pi\cancel{\sigma} a} \rightarrow C_0 = \frac{C}{l} = \frac{b}{4\pi a}$$



$$\vec{B} = \frac{4\pi}{c}\frac{I}{b}\hat{x} \rightarrow \Phi = l a \frac{4\pi I}{cb} = \frac{4\pi I l a}{cb}$$

$$\mathcal{E} = -L\dot{I} = -\frac{1}{c}\frac{d\Phi}{dt} \Rightarrow L\cancel{\dot{I}} = \frac{1}{c}\frac{4\pi l a}{cb}\cancel{\dot{I}}$$

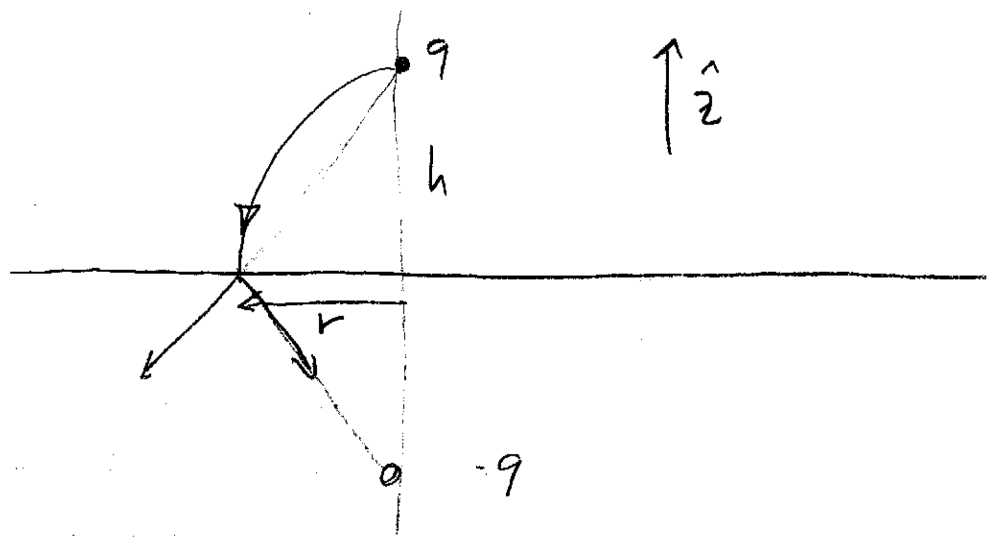
$$L_0 = L/l = \frac{4\pi a}{bc^2}$$

$$Z = \sqrt{\frac{L_0}{C_0}} = \sqrt{\frac{4\pi a}{bc^2}\,\frac{4\pi a}{b}} = \frac{4\pi}{c}\frac{a}{b}$$

$$Z = \frac{12}{3\times10^{10}}\,\frac{0.5}{5}\,\frac{s}{cm} = \frac{1.2}{3}\times10^{-10}\frac{s}{cm} = 0.4\times10^{-10}\,s/cm$$

$$= 40\,\Omega$$

4-3



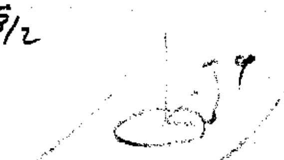
Brute force with an image charge

at r, $\vec{E} \perp$ to plate

$$E_z = \left[\frac{-q}{r^2+h^2} \frac{h}{(r^2+h^2)^{1/2}} - \frac{q}{r^2+h^2} \frac{h}{(r^2+h^2)^{1/2}} \right] \text{ on plate}$$

Surface charge density (w/o image charge) is then

$$E = 4\pi\sigma \Rightarrow \sigma = \frac{-qh}{2\pi(h^2+r^2)^{3/2}}$$



Total induced charge is

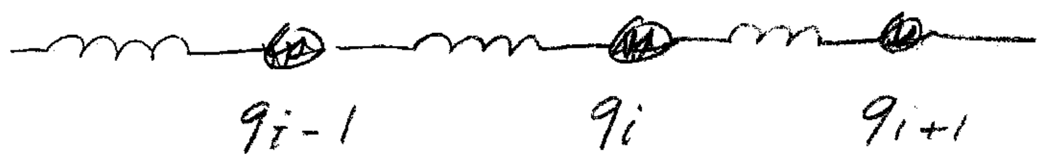
$$q_{ind} = \int_0^\infty r\,dr \int_0^{2\pi} d\varphi \, \frac{-qh}{2\pi(h^2+r^2)^{3/2}}$$

$$= -qh \int_0^\infty \frac{r\,dr}{(h^2+r^2)^{3/2}} \qquad \begin{array}{l} u = h^2+r^2 \\ du = 2r\,dr \end{array}$$

$$q_{ind} = -qh\,\frac{1}{2}\int_{h^2}^{\infty}\frac{du}{u^{3/2}} = -\frac{qh}{\cancel{2}}\,\frac{\cancel{2}}{u^{1/2}}\Bigg|_{h^2}^{\infty} = -q$$

The are more elegant ways to do this problem.

4-4



$$m\ddot{q}_i - k(q_{i+1} - q_i) - k(q_i - q_{i-1}) = 0$$

$$m\ddot{q}_i - k(q_{i+1} - 2q_i + q_{i-1}) = 0$$

General solution $\quad q_i = a_i e^{i\omega t}$

Then, $\ddot{q}_i = -\omega^2 a_i e^{i\omega t}$

and $-m\omega^2 a_i - k(a_{i+1} - 2a_i + a_{i-1}) = 0$

Trail solution for coefficients

$$a_j = a e^{i(j\gamma - \delta)}$$

$$-m\omega^2 \cancel{a} e^{i(j\gamma-\delta)} - k\left(\cancel{a} e^{i((j+1)\gamma-\delta)} - 2\cancel{a} e^{i(j\gamma-\delta)} + \cancel{a} e^{i((j-1)\gamma-\delta)}\right) = 0$$

$e^{-i\delta}$ factors out,

$$\cancel{e^{ij\gamma}}\left(-\omega^2 m - k\left(e^{i\gamma} - 2 + e^{-i\gamma}\right)\right) = 0$$

$$-\omega^2 m + 2k - 2k\cos\gamma = 0$$

$$m\omega^2 = 2K(1-\cos\gamma)$$

$$\left[\sin^2 x = \frac{1}{4}\left(e^{2xi} - 2 + e^{-2ix}\right) = \frac{1}{4}(2-2\cos 2x)\right.$$

$$\left. = \frac{1}{2}(1-\cos 2x)\right]$$

$$\Rightarrow m\omega^2 = 4K\sin^2 \gamma/2$$

$$\omega = 2\sqrt{\frac{K}{m}}\sin \gamma/2$$

Boundary condition: $a_0 = 0 = a_N$

$$a_0 = a e^{-i\delta} = 0 \Rightarrow \delta = \pi/2$$

$$a_N = 0 = a e^{i(N\gamma - \pi/2)} \Rightarrow N\gamma - \pi/2 = (2n+1)\pi/2$$

$$N\gamma - \pi/2 = \frac{n\pi}{2} + \pi/2$$

$$N\gamma = \frac{n\pi}{2} + \pi$$

$$\gamma = \frac{\frac{\pi}{2}(n+2)}{N}$$

$$\Rightarrow \omega = 2\sqrt{\frac{K}{m}}\sin\frac{\pi(n+2)}{4N}$$

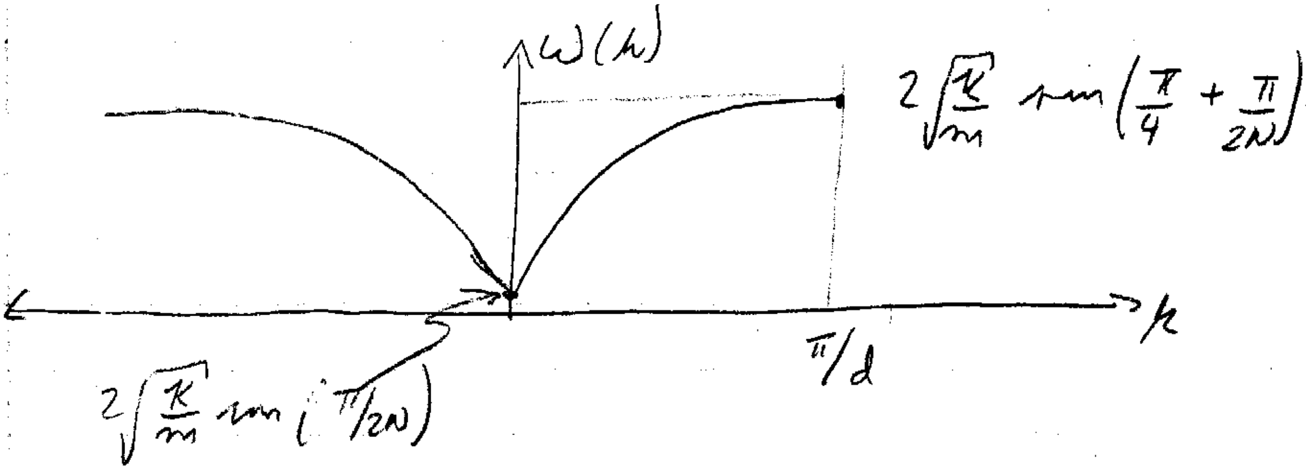
The wavelength $\lambda = 2Nd/n$

$$\Rightarrow k = 2\pi/\lambda = \frac{2\pi n}{2Nd} = \frac{\pi n}{Nd}$$

Then $\omega = 2\sqrt{\frac{K}{m}}\sin\left[\frac{\pi n}{4N} + \frac{\pi}{2N}\right]$

$$\omega = 2\sqrt{\frac{K}{m}}\sin\left(\frac{kd}{4} + \frac{\pi}{2N}\right)$$

Then, $v = \omega h = \dfrac{2Nd}{\pi n}\sqrt{\dfrac{K}{m}}\sin\left(\dfrac{kd}{4} + \dfrac{\pi}{2N}\right)$



5-1
Ans.

$$\beta_2 = v_2/c \ , \ \gamma_2 = \frac{1}{\sqrt{1-\beta_2^2}}$$

$$p_2 = m\beta_2\gamma_2 c$$

$$E_2 = \gamma_2 m c^2$$

F_1 is rest frame of 1,

$$E_2' = \gamma_1 E_2 + \gamma_1\beta_1 c\, p_2$$

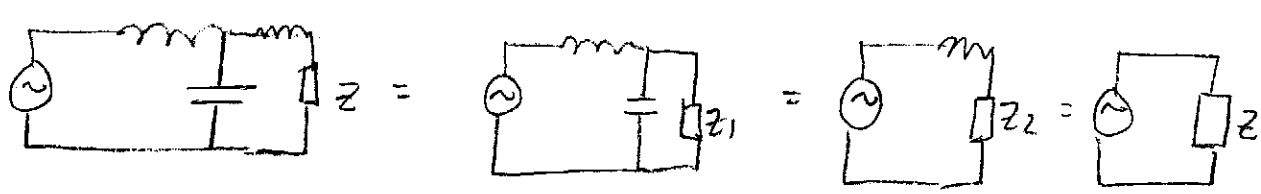
$$= \gamma_1\gamma_2 m c^2 + \gamma_1\beta_1 c\, m\beta_2\gamma_2 c$$

$$= mc^2\gamma_1\gamma_2(1+\beta_1\beta_2)$$

$$= \frac{mc^2(1+v_1/c\, v_2/c)}{\sqrt{(1-v_1^2/c^2)(1-v_2^2/c^2)}}$$

$$\beta_2' = \frac{\beta_1+\beta_2}{1-\beta_1\beta_2}$$

5-2



$$Z_1 = i\omega L + Z$$

$$Z_2 = \left(i\omega C + \frac{1}{Z_1}\right)^{-1} = \left(\frac{i\omega C Z_1 + 1}{Z_1}\right)^{-1} = \frac{Z_1}{1 + i\omega C Z_1}$$

$$Z = i\omega L + Z_2 = i\omega L + \frac{i\omega L + Z}{1 + i\omega C(i\omega L + Z)}$$

$$(Z - i\omega L)(1 - \omega^2 LC + i\omega C Z) = i\omega L + Z$$

$$i\cancel{\omega} C Z^2 + Z\left(1 - \omega^2 LC + \omega^2 LC - 1\right) - 2i\cancel{\omega} L + i\omega^{\cancel{2}} L^2 C = 0$$

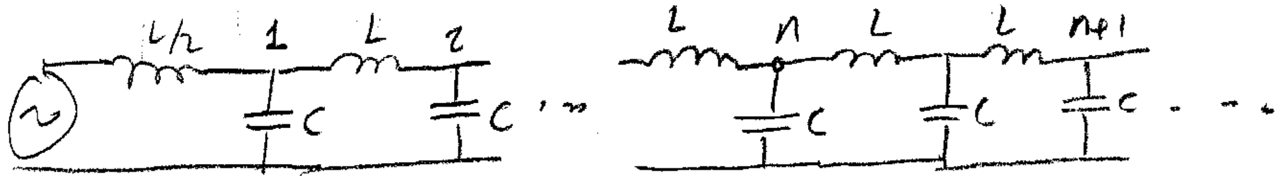
$$Z^2 = \frac{2iL - i\omega^2 L^2 C}{iC} = \frac{L}{C}\left(2 - \omega^2 LC\right)$$

$$Z = \sqrt{\frac{L}{C}\left(2 - \omega^2 LC\right)}$$

Wave propagate if $2 - \omega^2 LC > 0 \Rightarrow 2 > \omega^2 LC$

$$\sqrt{\frac{2}{LC}} > \omega$$

For a long chain, treat n, the step number, as a continuous variable



Then, Q_n is the charge on capacitor n and

$$\dot{Q}_n = -I_{n+1} + I_n \rightarrow \frac{\partial I(n)}{\partial n} = \frac{\partial Q(n)}{\partial t} = \frac{1}{C}\frac{\partial V}{\partial t} \quad (1)$$

$$V_{n+1} - V_n = \frac{L\, I_{n+1} + L\, I_n}{2} \rightarrow \frac{\partial V}{\partial n} = L\frac{\partial I(n)}{\partial t} \quad (2)$$

Then

$$\left.\begin{aligned} \frac{1}{C}\frac{\partial^2 V}{\partial t^2} &= \frac{\partial^2 I}{\partial t\,\partial n} \quad (1) \\ \frac{\partial^2 V}{\partial n^2} &= L\frac{\partial^2 I}{\partial t\,\partial n} \quad (2) \end{aligned}\right\} \quad \frac{1}{LC}\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 V}{\partial n^2} = 0$$

has solution $V = f(n \pm ct) = f(u)$, $u = n \pm ct$

$I = g(u)$ and

$$\frac{1}{LC} c^2 f'' - f'' = 0 \Rightarrow c^2 = LC \Rightarrow c = \sqrt{LC}$$

and $g = f$. Can also do problem discretely by using a series.

5-3 a) $i^i = (e^{i\pi/2})^i = e^{-\pi/2}$

$\frac{3.14}{2} = 1.57$

b) $\int_{-\infty}^{\infty} \delta(f(x))\, dx = \int_{-\infty}^{\infty} \delta(u) \frac{du}{f'(x_0)} = \frac{1}{f'(x_0)} = \frac{2\sqrt{b}}{c}$

Let $f(x) = u$, $du = \frac{df}{dx} dx$

If $f(x) = a + \sqrt{b+cx}$ — $0 = a + \sqrt{b+cx}$

$\frac{df}{dx} = \frac{1}{2} \frac{1}{\sqrt{b+cx}} c = \frac{c}{2\sqrt{b+cx}}$ $a^2 = b + cx$

$\frac{a^2-b}{c} = x_0$

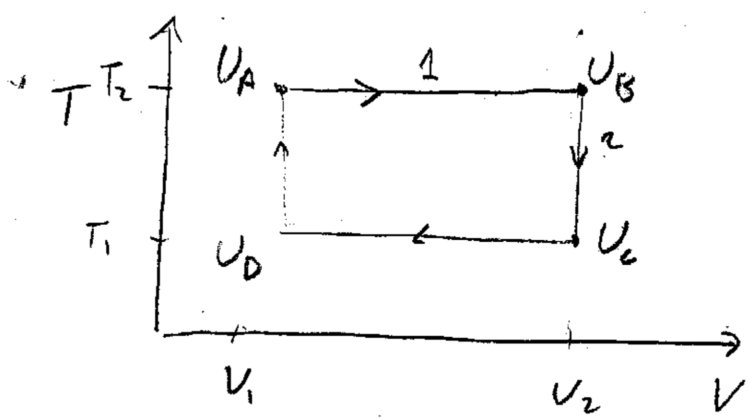
$f'(x_0) = \frac{c}{2\sqrt{b + a^2 - b}}$

$= \frac{2|a|}{|c|}$

5-4 Cycle

1. Expand gas from V_1 to V_2 at T_2.
2. Cool at V_2 from T_2 to T_1
3. Compress gas from V_2 to V_1 at T_1
4. Warm gas from T_1 to T_2 at V_1.

$$dU = k_B T\, ds - p\, dV$$



During 1, $Q_1 - W_1 = U_B - U_A$

2, $dW = 0$ $\quad dU = U_C - U_B = Q_2 = \lambda Q_1$

3, $Q_3 - W_3 = U_D - U_C$

4, $dW = 0$ $\quad dU = U_A - U_D = Q_4$

The total work out is $W_1 + W_3 = W$

$Q_1 = -Q_3$ for an ideal gas

$$W_1 + W_3 = Q_1 + U_A \underset{=0}{-} U_B + Q_3 + U_C - U_D$$

$$= (Q_1 + Q_3) + Q_4 + Q_2$$

$$\left.\begin{aligned} Q_1 &= \int k T_2 \, dS = k T_2 \, \Delta S \\ Q_3 &= -k T_1 \, \Delta S \end{aligned}\right\} \quad \frac{Q_1}{T_2} = -\frac{Q_3}{T_1}$$

$$W = Q_1 \left(1 - T_1/T_2\right)$$

Total heat taken in is $Q_1 + Q_3 = Q_1 + \lambda Q_1$

$$\Rightarrow \eta = \frac{(1 - T_1/T_2)}{(1+\lambda)}$$