STONY BROOK UNIVERSITY

DEPARTMENT OF PHYSICS AND ASTRONOMY

Graduate Placement Exam Part 1, August 26, 2008

General Instructions: This exam is for incoming graduate students who wish to demonstrate mastery in one or more areas of the graduate core curriculum, in order to skip one or more of the first-year courses. Do two of the three problems in either or both areas.

Each solution should typically take on the order of 45 minutes.

Use one exam book for each problem, and label it carefully with the problem topic and number and your name. Make sure to do every part of the problems you choose.

You may use a one page help sheet, a calculator, and with the proctor's approval, a foreign language dictionary. No other materials may be used.

Classical Mechanics 1

A spaceship with mass m is in an elliptic orbit around an asteroid. The asteroid has mass M, homogeneous mass density ρ, and radius R.

a) If the ellipse has major semi-axis a and minor semi-axis b, derive a formula for the total energy E and the period T of the spaceship. Assume that $m \ll M$. Hint: use the two conservation laws at the two extremal points of the ellipse for E, and Kepler's area law for T.

b) The crew changes the orbit to a circular orbit at low altitude with radius R. Some crew members descend to the asteroid, and dig a tunnel along the diameter of the asteroid, and throw a rock with mass μ into the tunnel with the same speed v as the spaceship. How long does it take for the rock to move from one end of the asteroid to the other end? Of course, there is no friction.

c) The rock emerges from the other side of the asteroid with speed v. How long does it take before the rock enters again the tunnel? Hint: use a).

Classical Mechanics 2

A tall, thin, vertical chimney of length h begins to topple over, as a rigid unit, by pivoting from its base. When it has tipped by some angle θ_B, it breaks, due to the internal torque exceeding the strength of the material.

Determine the position x along the chimney where it will break, as a function of θ_B.

Classical Mechanics 3

a) Show that
$$T_k = \frac{dS}{dE} \tag{1}$$
with S the action of one period of a periodic orbit, T_k the period and E the energy.

b) Explain why the Kepler equations of motion, which are differential equations, can actually be solved algebraically.

c) Show that
$$Q = \log(1 + \cos p\sqrt{q}), \qquad P = 2(1 + \cos p\sqrt{q})\sqrt{q}\sin p \tag{2}$$
is a canonical transformation, by showing that it can be derived from the generating function
$$F = -(e^Q - 1)^2 \tan p. \tag{3}$$

d) What is the Poisson bracket of $x^2 + y^2$ and $L_z = xp_y - yp_x$?

Classical Electrodynamics 1

A sphere of radius R made of a dielectric with constant ϵ_2 is immersed in a dielectric fluid with constant ϵ_1 and with fixed electric field $\vec{E}_1 = E_0\hat{z}$ at large distance.

a) Evaluate the electric potential $V_1(\vec{x})$ outside the sphere and $V_2(\vec{x})$ inside the sphere

b) What is the induced polarization charge on the surface of the sphere?

Classical Electrodynamics 2

a) A charge Ze is immersed in a neutral plasma (Jellium) of mean electron density n_0 at temperature T. Derive the equation (commonly known as the screening equation or Debye-Huckel law) for potential as a function of distance from the embedded charge. (Reminder: if $\psi(\vec{r}) = \psi(r)$ only, then $\nabla^2\psi = \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)$.)

b) Solve the screening equation for the potential at large temperature T. Sketch a graph of the screening length versus T.

c) For $T = 10^6$ K and $n_0 = 10^{15}$ cm^{-3}, what is the screening length? (The Boltzmann constant $k_B = 1.381 \times 10^{-23}\ J/K$.)

Classical Electrodynamics 3

The radiation field of a charged particle q with velocity $\vec{v}$ and acceleration $\vec{a}$ is

$$\vec{E}_{rad} = \frac{q}{c^2}\frac{\hat{n}\times((\hat{n}-\vec{v}'/c)\times\vec{a}')}{(1-\hat{n}\cdot\vec{v}'/c)^3}\frac{1}{|\vec{x}-\vec{x}'|} \tag{4}$$

where the primes refer to the retarded time, $t' = t - |\vec{x}-\vec{x}'|/c$, and $\hat{n} = (\vec{x}-\vec{x}')/|x-x'|$.

a) Use $\vec{E}_{rad}$ to discuss the radiation pattern in the non-relativistic case. Draw the power radiated per solid angle in polar coordinates. What is the total power radiated?

b) Use $\vec{E}_{rad}$ to discuss the radiation pattern in the relativistic case for $\vec{v}'||\vec{a}'$. What is the power radiated per solid angle. Sketch a polar plot. At what polar angle $\Delta\theta$ does it peak?

c) What is $\Delta\theta$ for electrons of energy 1 GeV?