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DEPARTMENT OF PHYSICS

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DOCTORAL GENERAL EXAMINATION

PART 1

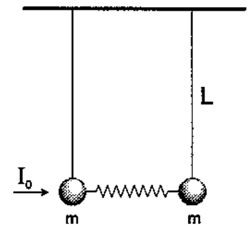
FEBRUARY 2, 1999

FIVE HOURS

1. This examination is divided into five sections, each consisting of four problems. Answer all the problems. Each problem is worth 5 points, thus the maximum score for the exam is 100.
2. Use a separate fold of paper for each problem. Write your name and the problem number (IV-3 for example) on each fold. A diagram or sketch as part of the answer is often useful, particularly when a problem asks for a quantitative response.
3. Read the problem carefully and do not do more work than is necessary. For example "give" and "sketch" do not mean "derive".
4. Calculators may be used but are not necessary.
5. No books, notes or reference materials may be used.

GROUP I

1. Coupled pendulums



A pair of identical, classical pendulums are coupled by a spring (spring constant K) that is relaxed when the pendulums are vertical. Determine, by any means you wish, the normal mode frequencies ω_1 and ω_2. An impulse I_0 is applied as shown at $t = 0$. Give an expression for the subsequent horizontal displacement of the left mass as a function of time.

2. Proton charge distribution

Suppose the proton has a uniform charge distribution extending to $r_p = 10^{-5} a_0$ where a_0 is the Bohr radius. Find the first order energy shift (magnitude and sign) of the ground state energy for hydrogen, relative to the model assuming a point proton. $\psi_g(\vec{r}) = (\pi a_0)^{-3/2} exp(-r/a_0)$.

3. Mass dependence in atomic systems

For each of the following systems with discrete energy levels, consider the excitation energy necessary to go from the ground state to the first excited state. Find the ratio of the excitation energies for the two different mass situations given.

a) An electron confined in a hard-walled cubic box to a proton in the same box.

b) Positronium to hydrogen. [Neglect the magnetic moments of the particles.]

c) The rotational energy levels of a nitrogen molecule composed of nuclei of mass 14 amu to a nitrogen molecule of mass 15 nuclei.

d) The vibrational energy levels of the molecules in c).

4. Classical radiation pressure

A plane electromagnetic wave is normally incident on a perfect conductor in vacuum. The wave is totally reflected without attenuation by the surface.

a) Determine the magnetic and electric field due to the wave at the surface of the conductor in terms of the peak electric field E_0 in the incoming wave.

b) Using Maxwell's equations, determine the surface current density on the conductor.

c) Find the ratio of the Lorentz pressure of the wave on the surface to the energy flux in the incoming wave.

GROUP II

1. Drag on a moving boat

A motorboat has mass M and velocity v_0 at the time its engine quits. If it then slows down under the influence of a drag force $-cv^2$, what is the subsequent distance traveled as a function of time?

2. Particle physics

a) What is the experimental evidence that the anti-neutrinos released in the decays $\pi^- \to \mu^- + \bar{\nu}_\mu$ and $N \to P + e + \bar{\nu}_e$ are of different kinds?

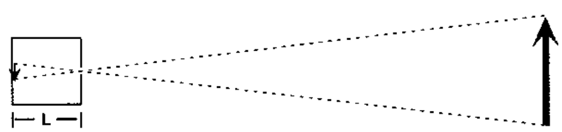
b) The decay channels $B_s^0 \to e^\pm + \mu^\mp$ are expected to have very low probability. Why?

c) In a careful measurement, the photons from the 2γ decay of positronium are found not to propagate in exactly opposite directions. What can cause this?

d) The long-lived K particle, K_l, decays primarily to three pi-mesons. About 1 in 1000 decays to two pi-mesons. What is the reason for this?

e) The particle and anti-particle, B_d and $\bar{B}_d$ may possibly decay at different rates. If this proves to be true, what would be the explanation?

3. Pinhole camera



A pinhole camera images a distant object onto its backplane, a distance L from the pinhole (diameter d). Estimate the optimum size of the pinhole aperture so as to achieve the best image, assuming monochromatic light of adequate intensity.

4. Newtonian cosmology

Using the observed Hubble law for small redshift, $v_r = Hr$, where v_r is the radial velocity, r the distance to the astrophysical object, and H the Hubble constant, establish the relation for the critical mass density of the universe by a simple Newtonian argument using energy. Assume an isotropic and homogeneous universe. [Hint: you may find it simpler to work with shells rather than spheres in this problem.]

GROUP III

1. Electric field of a charged hemisphere

A uniform distribution of electric charge is placed on a hemispherical shell. Prove that $\vec{E}$ over the plane surface bounded by the rim is $\perp$ to the plane.

2. Chemical potential

Consider a gas of N non-interacting Fermions in a potential $U(\vec{r})$ where the density of single particle states as function of the single particle energy ϵ is $D(\epsilon)$.

a) Write down the Fermi distribution, that is, the mean occupation number in thermal equilibrium of a single particle state of energy ϵ. Sketch its form. Identify the chemical potential. Explain how the value of chemical potential could be determined in terms of the properties of the system.

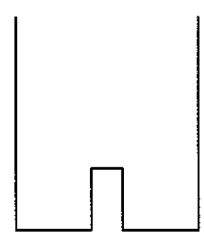
b) What simple property of the system determines whether the chemical potential increases, decreases, or remains unchanged from its $T = 0$ value as the temperature is raised by a small amount above absolute zero?

3. Clock in orbit

An atomic clock oscillates at frequency ν_0 in a laboratory on the earth. An identical clock passes directly overhead on a satellite in a circular orbit of radius R_s. Its frequency is measured by directing an electromagnetic wave of that frequency toward the lab on earth where it is received as a frequency ν_r. Find $\nu_r - \nu_0$ taking into account both the relativistic Doppler shift and the Einstein gravitational red shift. Note that in this geometry the first order Doppler shift is zero.

4. Ammonia inversion line

The figure indicates a model for the potential energy of the N in the molecule NH_3 along a line perpendicular the the plane of the three hydrogens. The N can tunnel through the barrier in the plane.



a) Sketch plausible eigenfunctions for the lowest two energy eigenstates of the nitrogen in this potential.

b) Assume the energy eigenvalues in a) are $E_0 \pm B$. Indicate which of these energies would go with each of your wavefunctions. Using the two wavefunctions, establish a time dependent state which has the highest probability of finding the nitrogen on the right at $t = 0$. Determine the time necessary for the particle to develop the highest probability of being found on the left.

GROUP IV

1. Rotating liquid under gravity

A bucket of water at the earth's surface rotates about its vertical symmetry axis at an angular velocity ω_0. Determine the shape of the water surface at equilibrium.

2. Relativistic collision

Two relativistic protons collide head on, the speed of each in the center-of-mass corresponding to $\gamma = 10$. Find the expression for the total energy of one proton in the other's rest frame. Evaluate it using the numerical value of γ.

3. Atomic fountain

In order to do precision spectroscopy on exotic atoms it is necessary to observe them for relatively long times at low density. This can be accomplished using an atomic fountain. Atoms from an atomic beam source are directed upward, come momentarily to rest in the observation region, then fall backward toward the earth.

a) What is the maximum height h attained by an atom leaving the source with a vertical speed v?

b) The probability density for the initial speed v in the beam is given by $p(v) = (2v^3/v_0^4)\exp[-(v/v_0)^2]$ where $v_0 \equiv \sqrt{2kT/m}$ is a characteristic speed. Find the probability density for the maximum height attained by an atom, $p(h)$.

4. Pulsar energy loss

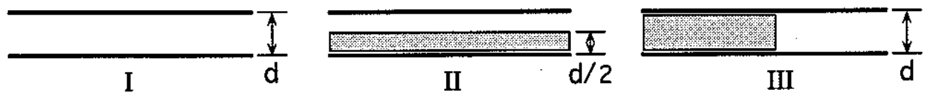
A pulsar is believed to be a rotating neutron star with a large magnetic dipole frozen into its body at an angle to the rotation axis. All the slower pulsars, those not being fed by accretion from a neighboring star, show a reduction in their rotation frequency with time which can be characterized by the expression $\dot{\omega} = -c\,\omega^n$. Assume that the neutron star is stiff in that its moment of inertia and field distribution do not depend on frequency.

a) Determine the value of the exponent n if the primary loss mechanism is magnetic dipole radiation. Show your reasoning; do not simply state the result.

b) Determine n if the loss mechanism were primarily gravitational radiation, as might be the case if a mass quadrupole is induced in the star by the huge magnetic fields. [The power radiated by gravitational moments has the same frequency dependence as electromagnetic radiation.]

GROUP V

1. Capacitors and dielectrics



Three plane capacitors have the same plate spacings, d, and plate areas, A. I, filled with a vacuum, has capacitance C_0. II contains a dielectric slab (constant k) of thickness $d/2$. III has a dielectric slab (constant k) filling the gap over half the area.

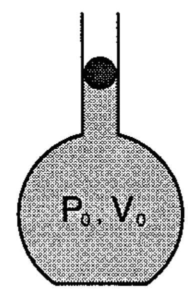
a) What is C_0 in terms of the given parameters?

b) What are the capacitances of II and III in terms of C_0 and k?

c) Capacitor I is charged up with $\pm Q_0$ and isolated. How much work must be done to insert the dielectrics to create situations II and III?

2. Gas oscillator

A container filled with an ideal gas has a precise cylindrical neck (inside diameter d) containing a close fitting ball of mass m which, at rest, compresses the gas to a volume V_0 at a pressure of P_0. When disturbed, the ball oscillates up and down. What is the period of small oscillations assuming adiabatic pressure changes in the gas and a ratio of specific heats given by $C_p/C_v \equiv \gamma$.



3. Bound state

A particle in an infinite one dimensional space is trapped in the only bound state, $\psi(x) = \sqrt{\alpha}\exp[-\alpha|x|]$, associated with an attractive delta function potential, $V(x) = -V_0\delta(x)$. The magnitude of the delta function suddenly changes from V_0 to fV_0 where the positive factor f may be greater or less than one. Find the probability that the particle is still bound after the change.

4. Classical precession of magnetic moments

a) Derive the classical expression for the precession frequency of a magnetic moment $\vec{\mu}$ in a constant external magnetic field $\vec{B}_0$. Show that the energy of the moment is a constant of the motion.

b) Show that by applying a time varying circularly polarized field B_{rf} at the precession frequency, and perpendicular to $\vec{B}_0$, it is possible to change the energy of the system. Determine the time it takes to go from a minimum to a maximum in the energy.