

Physics Qualifying Examination – Part I

7-Minute Questions

September 13, 2014

1. A sealed container is divided into two volumes by a moveable piston. There are N_A molecules on one side and N_B molecules on the other (all at the same temperature). Determine the ratio of volumes, V_A/V_B , in equilibrium by applying the second law of thermodynamics. Explicitly show the steps of your derivation.
2. Suppose an astrophysical body which is a distance L from Earth emits two light pulses at the same time: one with a peak frequency ω_1 and another with a peak frequency $\omega_2 > \omega_1$. Suppose the dispersion relationship of the light in the medium between the astrophysical body and the Earth is

$$k^2 c^2 = \omega^2 - (Q n_e)^2$$

where Q is a constant and n_e is the electron density which is approximated to be a constant. What is the difference of pulse arrival times on Earth?

3. Why is it that we are able to do accurate trajectory calculations using classical mechanics for a particle of mass 1 milligram (10^{-6} kg) moving freely along \hat{x} with speed 10^3 m/s? Include in your answer an estimate of the effect an initial uncertainty of 1 micron (10^{-6} m) in the lateral \hat{y} direction would have on the calculation.
4. A cylinder of uniform cross-section floats in a liquid of density ρ_f , with its axis perpendicular to the surface. A length h of the cylinder is submerged when the cylinder floats at rest. If the cylinder is displaced up or down slightly from the equilibrium position and then released, the cylinder will oscillate vertically. What is the period of this motion?
5. An atom is capable of existing in two states: a ground state of mass M and an excited state of mass $M + \Delta$. If the transition from ground to excited state proceeds by the absorption of a photon, what must be the photon frequency in the laboratory where the atom is initially at rest?

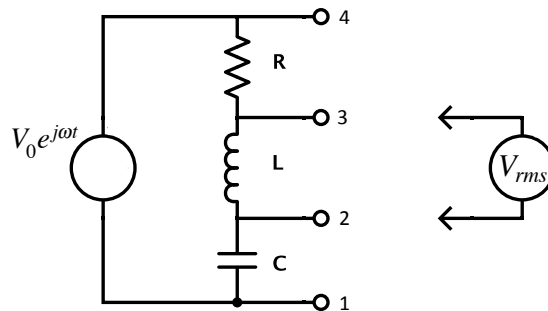
6. Find the minimum thickness of a soap film that gives constructive interference of reflected red light ($\lambda = 7000 \text{ \AA}$). The index of refraction for the film is 1.33. Assume a parallel beam of incident light directed at 30° from the normal.

7. For some frequency ω you measure the following r.m.s. voltages with a voltmeter.

$$V_{4-3} = 12 V_{\text{rms}}$$

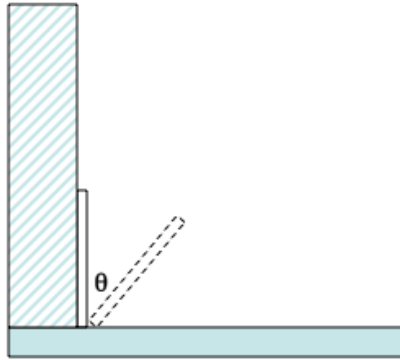
$$V_{3-2} = 2 V_{\text{rms}}$$

$$V_{2-1} = 7 V_{\text{rms}}$$



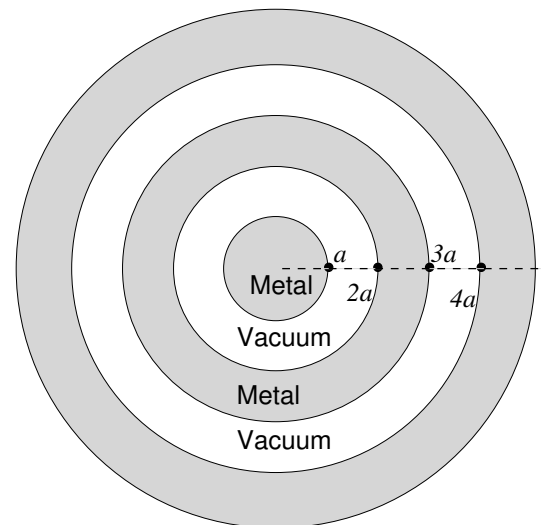
- What do you measure with this meter for V_{3-1} ?
 - What do you measure with this meter for V_{4-1} ?
 - In terms of L , C , and R , at what frequency ω_{max} will V_{4-3} be maximum, if V_0 is held constant?
 - What will you measure for V_{4-3} at ω_{max} ?
8. What is the Doppler width in an optical emission spectral line from an atom in a typical flame? Express your answer in Hertz.

9. A uniform rod initially stands on a horizontal smooth surface and tightly against a smooth vertical wall. Under gravity, the rod starts to fall directly away from the wall (i.e. the rod falls in a plane normal to the wall). At what angle does the bottom of the rod start to move away from the wall?



10. There are a series of three long concentric cylindrical conductors as shown. The innermost conductor is a solid rod and has a positive surface charge density of σ (C/m^2).

- With respect to r , the perpendicular distance from the axis of the rod, determine the electric field as a function of r from 0 to $4a$.
- What are the potential differences between a & $2a$, $2a$ & $3a$ and $3a$ & $4a$?
- What is the capacitance per unit length of this assembly if wires are connected respectively to the innermost and outermost metal pieces?



Physics Qualifying Examination – Part II

12-Minute Questions

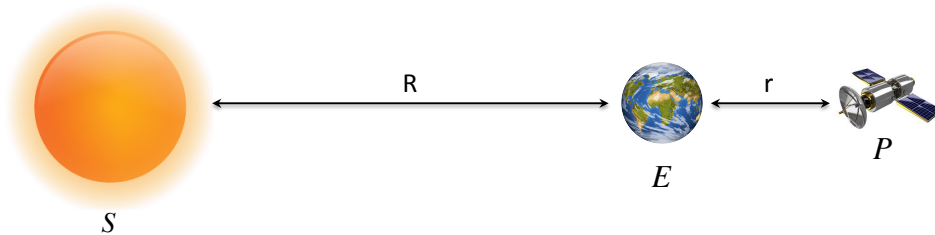
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- Suppose the world were 5-dimensional. Given N non-interacting spin- $\frac{1}{2}$ particles of mass m in a 5-dimensional box of volume $V_{5D} = L^5$ at temperature $T = 0$, calculate the Fermi energy E_F and the Fermi wavevector k_F . Note that the volume of a five-dimensional sphere of radius r is $\frac{8}{15}\pi^2 r^5$.

Hint: The density of states in k -space for a spin-zero particle in a 5-dimensional box is given by the following:

$$\rho d^5k = \frac{V_{5D}}{(2\pi)^5} d^5k.$$

- The Planck satellite is located at the Lagrange L2 point. In leading approximation, the arrangement is that the sun S , earth E and satellite P are aligned in a straight line and all perform circular motion with angular frequency ω about the center-of-mass of the earth-sun system.

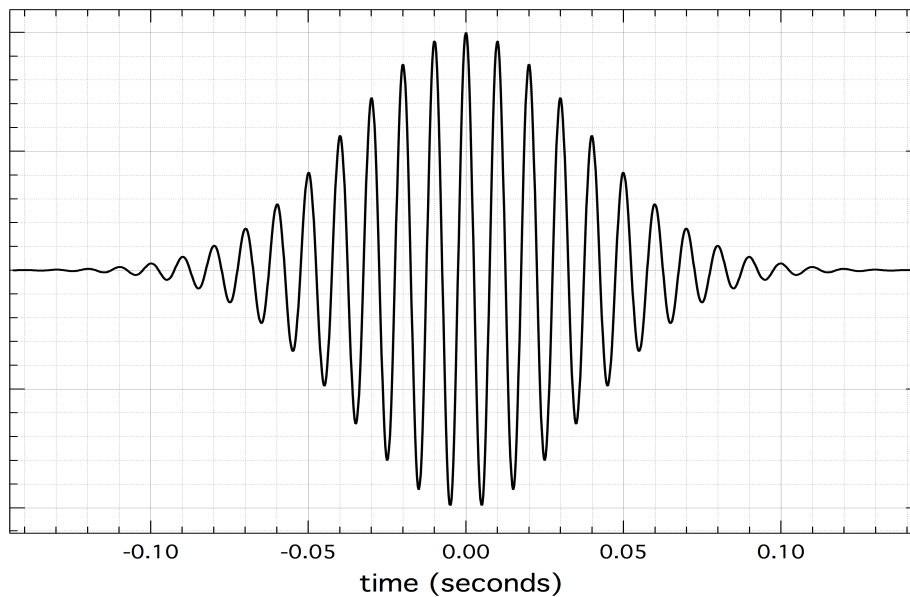


- Set up the force balance equation for the Planck satellite to maintain its position relative to the earth and sun, i.e., that its orbital frequency be the same as that of the earth.
 - With $M_S = 1.987 \times 10^{30}$ kg, $M_E = 5.97 \times 10^{24}$ kg, and $R_{ES} = 1.495 \times 10^{11}$ m, determine the distance r of the satellite from the earth.
- An electron is in the S_z basis spin state

$$\chi = A \begin{pmatrix} 4 \\ 3i \end{pmatrix}$$

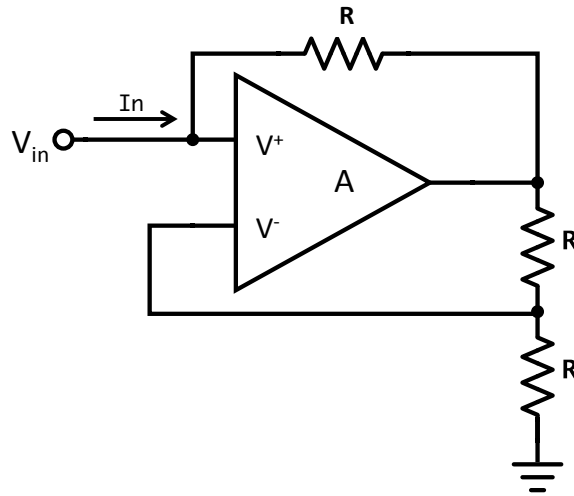
- Determine the normalization constant A .
- If you measured S_y for this electron, what values would you get, and what is the probability of each?
- Find $\langle S_x \rangle$ for this electron.
- Find $\langle S_z^2 \rangle$ for this electron.

4. For the wave packet shown below, sketch the amplitude distribution of the frequencies contained. In your sketch, estimate the full width at half-maximum and identify the central frequency.



5. Describe in detail two specific experimental techniques that would enable you to directly measure the superconducting energy gap in a conventional superconductor.
6. Using the procedure outlined below, calculate the smallest Bragg reflection angle 2θ for a lattice of 100 planes of cubic face-centered nickel using an x-ray energy of 8.00 keV. Nickel has a lattice constant of $a = 3.52 \text{ \AA}$.
- Derive the wavelength from the energy.
 - Determine the spacing d_{100} between two adjacent planes within the 100 planes in the nickel lattice.
 - Apply Bragg's diffraction condition.

7. For the circuit below, $V_{OUT} = A(V^+ - V^-)$, $A \rightarrow \infty$, and the current into V^+ and V^- is negligible. Find the input impedance $Z_{IN} \equiv V_{IN} / I_{IN}$.



8. The total power P (energy per time) radiated by an accelerated charge q has the functional dependencies

$$P \propto q^2 \dot{v}^2$$

- Use dimensional analysis to fill in the proportionality constant, including units, apart from a numerical factor of order 1.
- A proton ($q = 1.6 \times 10^{-19}$ Coul, $m = 1.67 \times 10^{-27}$ kg) is accelerated from rest in a uniform electric field \mathcal{E} through a distance ℓ ,

$$\ell = 1 \text{ m}, \mathcal{E} = 3 \times 10^6 \text{ V/m}$$

What fraction of the energy gained is lost by radiation?

9. You walk into a lab in which a detector is reporting the number of decays (per unit time) every second from a radioactive source. You notice three things: (i) the average number of decays reported is about 10 and is not changing over time, (ii) the majority of the number of decays reported are between 7 and 13, (iii) it is not that unusual to observe a number of decays larger than 13 or smaller than 7. A fellow graduate student offers the following: if the next reported decay is 10, he will pay you \$100; if not, you pay him \$20. On average would you win this bet? Why or why not?

10. A certain cryostat consists of concentric cylindrical inner and outer shells separated by a thin vacuum space (Fig. 1a). The dominant heat load on the liquid reservoir is radiation from the sides of the cryostat; the radiative load from the top and bottom can be neglected. The boil-off rate for liquid nitrogen (LN_2) in this cryostat is initially measured to be 1 L/day. The cryostat is subsequently modified with the installation of 10 concentric wraps of multilayer insulation in the vacuum space (Fig. 1b). The role of the insulation is to intercept the thermal radiation from the 300 K outer vessel, so that the inner vessel is looking out at a lower temperature. Assume that the various layers of insulation are thermally isolated from one another and from the inner and outer shells, and that the emissivities of all radiating surfaces (inner and outer shells and various layers of insulation) are equal.

What is the boil-off rate for LN_2 following installation of the multilayer insulation? What is the temperature of the innermost wrap of the insulation? LN_2 boils at 77 K at atmospheric pressure.

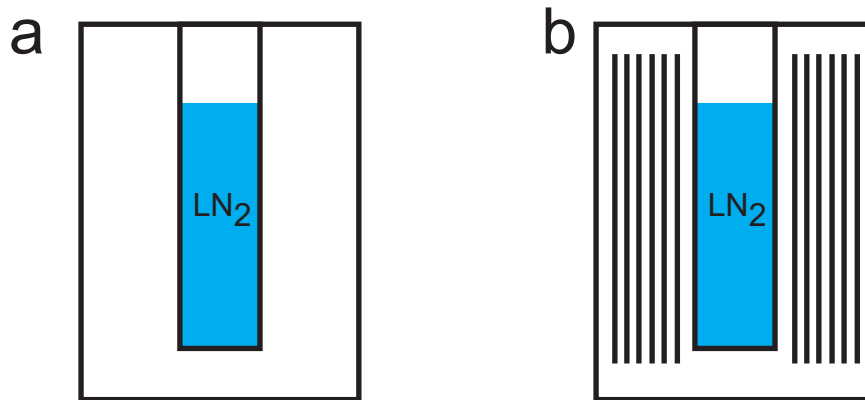


Figure 1: Cross-sectional view of double-walled vacuum cryostat for LN_2 before (a) and after (b) installation of multilayer insulation in the vacuum space.

11. a. If a given muon at rest lives 10^{-6} s, and its mass is $100 \text{ MeV}/c^2$, how energetic must this muon be to reach the Earth's surface if it is produced high in the atmosphere (say $\sim 10^4$ m up)?
 b. Suppose to a zeroth approximation that the Earth has a 1 gauss magnetic field pointing in the direction of its axis, extending out to 10^4 m. How much, and in what direction, is a muon of energy E normally incident at the equator deflected by the field?
12. Assume a uranium nucleus breaks up spontaneously into two roughly equal parts. Estimate the reduction in electrostatic energy of the nuclei. What is the relationship of this to the total change in energy? (Assume uniform charge distribution; nuclear radius $= 1.2 \times 10^{-13} A^{1/3} \text{ cm.}$)

13. a. Given the normalized wave functions listed below, calculate the dependence on atomic number Z of a lifetime for spontaneous decay of the $2p$ state in the hydrogen-like atoms (e.g. H, He^+ , Li^{++} , etc.).

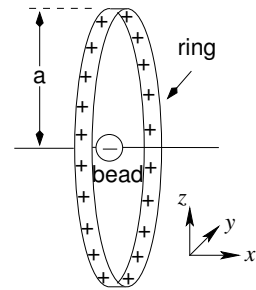
$$|1s(Zr)\rangle = \left(\frac{Z}{a_0}\right)^{\frac{3}{2}} 2e^{\frac{-Zr}{a_0}},$$

$$|2p(Zr)\rangle = \left(\frac{Z}{2a_0}\right)^{\frac{3}{2}} \frac{Zr}{a_0\sqrt{3}} e^{\frac{-Zr}{2a_0}},$$

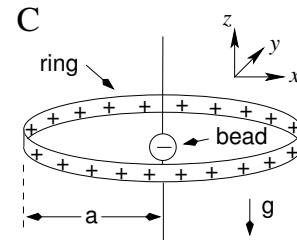
- b. What is the electron configuration, total spin S , total orbital angular momentum L , and the total angular momentum J for the ground state of atomic oxygen?

14. A small bead of mass m is constrained to move freely along the x -axis by a string. The bead has a net negative charge of $-Q$. An equal amount of positive charge, $+Q$, is uniformly distributed on a narrow, thin ring of radius a . The ring is situated on the yz plane with its center at the origin.

A&B



- a. How much work is needed to move the bead, initially in the plane of the ring (i.e., at $x = 0$), to infinity?
- b. If one displaces this bead just a small distance from the origin and releases it from rest then what angular frequency simple harmonic motion would you expect to observe?
- c. The system is now rotated 90° so that the string lies on the z -axis and gravity acts downward. In terms of the given parameters and any fundamental constants, what is the maximum mass that the bead can have so that it will remain at an equilibrium position near the ring of charge (i.e. it won't "fall" down the string)?



15. A charged particle of mass m and charge $q > 0$ undergoes gyro motion in a magnetic field

$$\mathbf{B} = \hat{\mathbf{z}}B_z \exp[-y^2/\Delta y^2].$$

Its velocity of gyration v_\perp is such that $\rho \ll \Delta y$, where ρ is the gyro radius. Working from the force a magnetic dipole experiences in a non-uniform field, determine the trajectory of guiding center motion starting from a point $(0, y_0, 0)$.