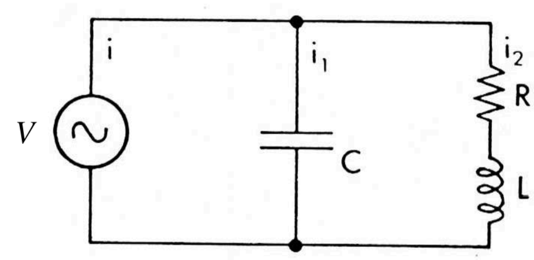
Physics Qualifying Examination – Part I 7-Minute Questions

February 18, 2012

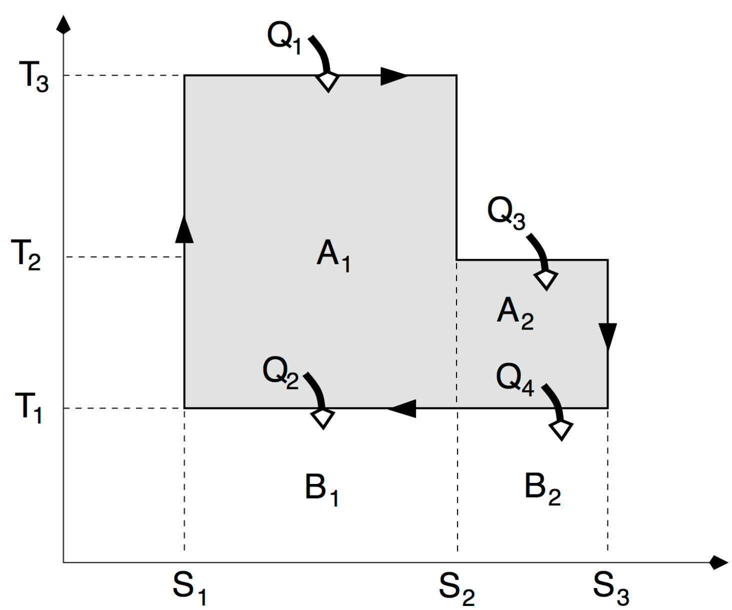
1. In the parallel RLC circuit shown, $R = 800.0\ \Omega$, $L = 160.0\ \text{mH}$ and $C = 0.0600\ \mu F$. The source has $V_0 = 20.0\ \text{V}$ and $f = 2400.0\ \text{Hz}$.



$V = V_0 \sin \omega t$

a. Find the peak current in branch 1.
b. Find the peak current in branch 2.
c. Find the phase constant (and sign) for branch 1.
d. Find the phase constant (and sign) for branch 2.
e. At what frequency (in Hz) would this circuit be resonant?

2. Consider an engine whose cycle of operation is represented in the T-S plane below. Let A_1 and A_2 denote the areas of the shaded rectangles between S_1, S_2 and S_2, S_3 respectively. B_1 and B_2 denote the respective areas below T_1. The Q's refer to heat flow.



a. Show that an engine that runs a cycle encompassing A_1 is the equivalent of a Carnot engine.
b. Show that an engine that runs a cycle encompassing A_1 and A_2 is not as efficient as a Carnot engine just operating between T_1 and T_3.

3. A moving rod is observed to have a length of 2.00 m and to be oriented at an angle of 30.0° with respect to the direction of motion. The rod has a speed of 0.995 times the speed of light.

a. What is the proper length of the rod in its rest frame?
b. What is the orientation angle in its rest frame?

4. A perfectly rigid diatomic molecule with a moment of inertia I and a permanent dipole moment p is fixed in place at the origin and constrained to only rotate about the z-axis (i.e. in the x-y plane). There is a uniform time-dependent electric field E in the x- direction, which is a small perturbation. The Hamiltonian for the system is

$$H = -\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\phi^2} - pE\cos\phi$$

Here ϕ is the angle with respect to the x-axis.

a. First set $E = 0$. What are the energy eigenvalues and degeneracies of the molecular spectrum?
b. If light polarized in the x- direction is incident on the molecule, what frequencies will be absorbed?

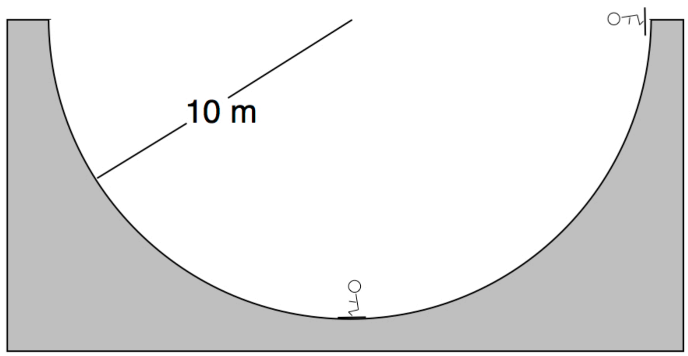
5. Larmor's formula states that the power radiated by charge q with acceleration a is

$$P_{rad} = \frac{\mu_0 q^2 a^2}{6\pi c}.$$

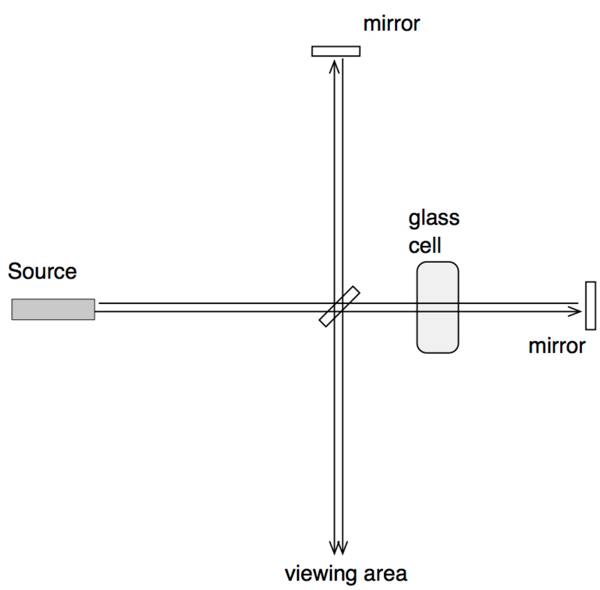
Starlight, scattering from a planet's atmosphere, involves accelerating charges that determine the color of the sky as observed from the planet's surface. For a planet like the Earth, the atmosphere is mostly molecules for which polarization $p=\alpha E$, where α is a constant.

a. Explain, with words and equations, why the sky is blue and the setting (or rising) sun is red.
b. Now imagine a fictitious planet in which the scattering of light in the atmosphere is dominated by Thomson scattering from free electrons (a plasma), with a star that has a spectrum identical to our Sun's. What colors would the sky and setting sun be on this planet? Again, use equations to justify your answer.

6. A 50 kg skier starts from rest at the top of a 10 m radius half-pipe and skis straight downward. Find the skier's effective weight at the bottom of the half-pipe. (Neglect the height of the skier and assume that $g = 10$ ms^2.)



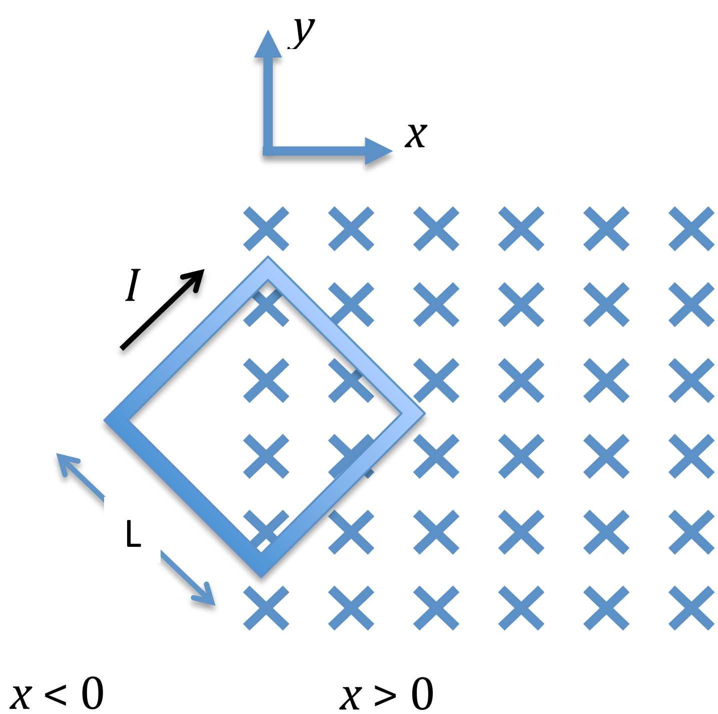
7. Consider a Michelson interferometer that operates at a wavelength of 600 nm. One of the arms of the interferometer has a 2.0 cm long glass cell. To begin, the air is pumped out of the cell and the mirrors of the interferometer are adjusted to produce a bright spot at the center of the interference pattern. Then a valve is opened and air is slowly admitted into the cell. How many bright dark bright fringe shifts are observed as the cell fills with air? The index of refraction of air at 1.0 atm pressure near room temperature is 1.00028.



8. Draw a diagram to show all energy states associated with the 2^3P spectroscopic term (L=1 and S=1) of the (1s)(2s) configuration of the He atom taking spin-orbit coupling into consideration. Label each state by the appropriate quantum numbers for this case. Identify the degeneracy, if any, of each state.

9. A light beam of intensity I_0, travelling in the $\hat{z}$-direction and linearly polarized along the $\hat{x}$-direction, encounters a sequence of N linear polarizers, oriented at angles $\theta_n = \frac{n\pi}{2N}, n = 1 \ldots N$ with respect to the $\hat{x}$- direction. What is the intensity of the light beam after the Nth polarizer? Give numerical results for $N = 1, 2, 17$, and ∞. What is the direction of the final polarization?

10. A current I flows in a square conducting loop of side L and mass m. A magnetic field points into the page as shown. It has strength B for all $x > 0$ and strength 0 for all $x < 0$. The loop is held fixed with its center at exactly $x = 0$. At time $t = 0$ the loop is released. How fast is the loop moving a long time after it is released? Assume that the current I flowing in the loop is a constant, independent of time. That is, the current does not decay, and the induced current is negligible.



Physics Qualifying Examination – Part II 12-Minute Questions

February 18, 2012

1. The density of states for a free electron gas, including the factor of 2 for spin degeneracy, is $\frac{dN}{dE} = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2}$, where V is the volume and m is the electron mass.

    a. Assuming a metal with atomic density n and one conduction electron per atom, calculate the Fermi energy E_F at temperature $T = 0$ under the assumption that the conduction electrons are free.
    b. Calculate the total kinetic energy of the conduction electrons at $T = 0$. Express your answer in terms of n, V, and E_F.

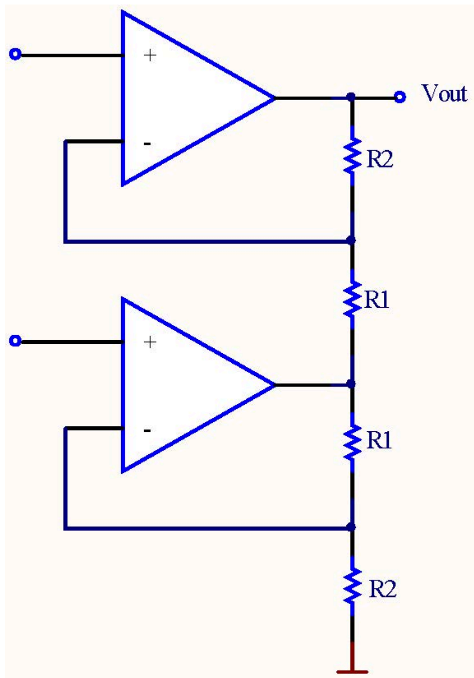
2. A particle of mass m is bound by the linear potential $U = kr$, where k is a constant and $r \geq 0$.

    a. Find the energy and angular momentum for a circular orbit of radius r_0 about the origin.
    b. What is the frequency of this circular motion?
    c. If the particle is slightly disturbed from this circular motion, what will be the frequency of small oscillations?

3. A magnetic dipole with the moment $\vec{\mu}$ is fixed at the origin. Determine the force on an electric dipole $\vec{p}$ at position $\vec{r}$ moving with velocity $\vec{v}$. Formal expressions involving derivatives are acceptable. Also, determine how you would expect the force $\vec{F}$ to scale at large $|\vec{r}|$.

4. A magnetic system is represented by N pairs of interacting spins $s = 1/2$ with the interaction represented by the Hamiltonian $H = A\vec{s}_1 \bullet \vec{s}_2$. Calculate the specific heat $C(T)$ for the magnetic system as a function of temperature T. Sketch a plot for $C(T)$ and find low temperature and high temperature asymptotes for $C(T)$. Disregard any other contributions to the specific heat.

5. Consider the differential amplifier circuit shown below. What is the common mode gain? What is the differential gain? Recall that the inputs of the operational amplifiers draw negligible current and that negative feedback will maintain equal voltages at the inverting and non-inverting inputs of each operational amplifier.



6. Consider a quantum mechanical system with two states, $|\alpha\rangle$ and $|\beta\rangle$. In this orthonormal basis of states, the Hamiltonian is given by the matrix

$$\begin{pmatrix} W & V \\ V & -W \end{pmatrix}$$

a. Obtain the exact energy eigenvalues.
b. Consider the Hamiltonian as $H = H_W + H_V$, where

$$H_W = \begin{pmatrix} W & 0 \\ 0 & -W \end{pmatrix} \text{ and } H_V = \begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix}.$$

Assuming that $|V| \ll |W|$, obtain the energy eigenvalues to second order in perturbation V.

c. Demonstrate that your results in (a) and (b) agree to the second order in V.

7. Suppose two particles of masses m_1 and m_2 with relativistic velocities are incident upon each other with 3-momenta $\vec{p}_1 = (q_1, 0, 0)$ and $\vec{p}_2 = (q_2, 0, 0)$ in the lab frame, eventually leading to a collision. What is the relativistic boost velocity needed to go to the center of mass frame?

8. You have a paramagnetic salt crystal that contains 1 mole of spin ½ atoms that you will use as a refrigerant. Briefly explain you reasoning for all parts.

a. You begin with the crystal held at a temperature of 4 K and apply a sufficiently strong magnetic field that aligns essentially all of the spins parallel to B. Is the entropy of the spin system large or small?
b. You thermally isolate the crystal and slowly reduce the B field until the temperature reaches 0.05 K. You can assume (with some realism) that over this temperature range the heat capacity of the crystal lattice is negligible, and the lattice and spin temperatures remain in equilibrium. How much has the entropy of the entire crystal system changed? The spin system?
c. There is a parasitic heat load of 1 microwatt on the crystal. You gradually reduce B to keep the temperature constant at 0.05 K. After time $t_{\max}$ the field reaches zero. Making the (somewhat unrealistic) assumption that there are no interactions between the spins and no other source of B field, the spins are independent and have no preferred orientation. What is the entropy of the spin system? You can make use of Stirling's approximation for the log of N factorial: $\ln(N!) \approx N\ln(N) - N$.
d. What is the time $t_{\max}$ in hours that you can maintain the system at 50 mK with a heat load of 1μW?

9. A galaxy of mass m is inside a homogeneous and isotropic universe of mass density ρ.

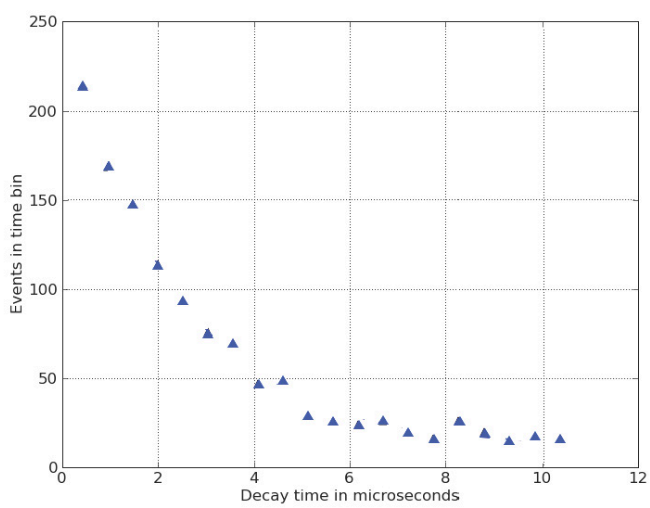
a. Find the radial acceleration of the galaxy as a function of R, where R is the distance of the galaxy to the center of the distribution.
b. Obtain an expression for dR/dt by assuming that the total energy of the galaxy vanishes.
P.S. You have now derived Friedman's equations for a flat universe.

10. A satellite of mass m is put into a circular orbit at a distance r_0 above the center of the Earth. A viscous force resulting from the thin upper atmosphere has a magnitude $F_v = Av^{\alpha}$, where v is the velocity of the satellite. For a particular value of α, we observe that the rate of change in the radial distance r given by $dr/dt = -C$, where C is a positive constant, sufficiently small so that the loss of energy per orbit is small compared to the total kinetic energy.

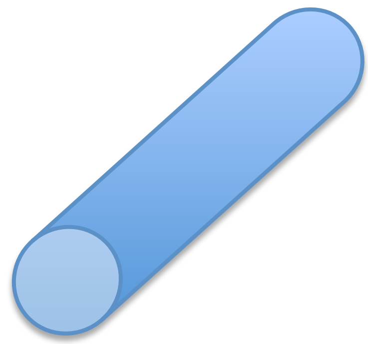
a. Find α.
b. Obtain an expression for A in terms of G, C, M_E, and m, where M_E is the mass of the Earth.

11. Muons are created by cosmic ray interactions in the upper atmosphere. Depending on their charge they can decay into an electron or positron, and two neutrinos. Some of these muons reach sea level such that their properties can be measured in the laboratory. To measure their lifetime, muons are stopped in a large block of scintillator, where they subsequently decay into an electron or positron, and two neutrinos. A short light pulse is produced by the stopping muon which is detected and amplified by a photomultiplier tube. When the muon decays a second pulse is produced by the emitted electron or positron. The signals from the photomultiplier are fed into an electronic circuit which determines the time delay between the two pulses.

a. Why do muons decay? What forces are involved in the decay and in the interaction of muons with matter?
b. The figure below shows the measured distribution of muon lifetimes.
   i. Write down the equation that describes the distribution of measured lifetimes.
   ii. Why does the distribution not approach zero for very long lifetimes?
   iii. Describe how to extract the muon lifetime from the measured distribution.
   iv. From the figure, estimate the muon lifetime. Sketch the expected distribution in the figure.
c. Is the measured lifetime the same for positive and negative muons? Explain.



12. An infinitely long cylinder of radius R is uniformly charged, with positive charge per unit volume of ρ.



a. Find the electric field $\vec{E}$ everywhere as a function of the distance r from the axis of the cylinder.
b. Find the electric potential V everywhere as a function of r. Define $V = 0$ at the surface of the cylinder.
c. Sketch E and V as functions of r, from $0 \le r \le 3R$, showing the values of each at $r = 0$, R and $3R$.

13. Consider Yukawa scattering off the potential $V(r) = g^2 \frac{\exp(-\mu r)}{r}$, where g and μ are constants ($\mu \ge 0$). In the first Born approximation, the scattering amplitude of a particle with mass m and a momentum change $\boldsymbol{q}$ is given by

$$f(\boldsymbol{q}) = \frac{2g^2 m}{\hbar^2} \frac{1}{q^2 + \mu^2}.$$

a. Express the momentum change q in terms of the scattering angle θ and the energy E of the incident particle and schematically sketch dependence of the differential cross-section $d\sigma / d\Omega = |f(\theta)|^2$ as a function of θ for $\mu = \sqrt{2Em} / \hbar$ and $\mu = 0$.
b. Find the dependence on energy E of the total cross section
$$\sigma = 2\pi \int_0^\pi \sin\theta |f(\theta)|^2 d\theta.$$
c. What is the value of σ in the limit of Coulomb scattering (i.e. $\mu = 0$)? Explain.

14. A fully ionized electron-proton plasma with temperature T = 10^4 K is placed in a uniform magnetic field B = 1 T. Assuming that ion-ion collision rate is about 10^6 s^{-1}, estimate the diffusion coefficient of the ions across the uniform magnetic field.

15. Collimated light of wavelength 500 nm is normally incident on a spherical mirror with a radius of curvature $R = 1$ m and a diameter D. Find D so that the blurring of the focal spot due to diffraction is equal to the blurring due to spherical aberration in the paraxial focal frame.

(Hint: Use a Taylor series expansion to compare spherical and parabolic surfaces.)