Part I - Feb. 2000

I-1 S 2000

STRING : $y(x,t) = A \sin(kx)\sin(\omega t + \phi) \qquad 0 < x < L$

$$\ddot{y} = \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2} = v^2 \frac{\partial^2 y}{\partial x^2} \qquad v = \sqrt{\frac{T}{\mu}}$$

$$\omega = vk$$

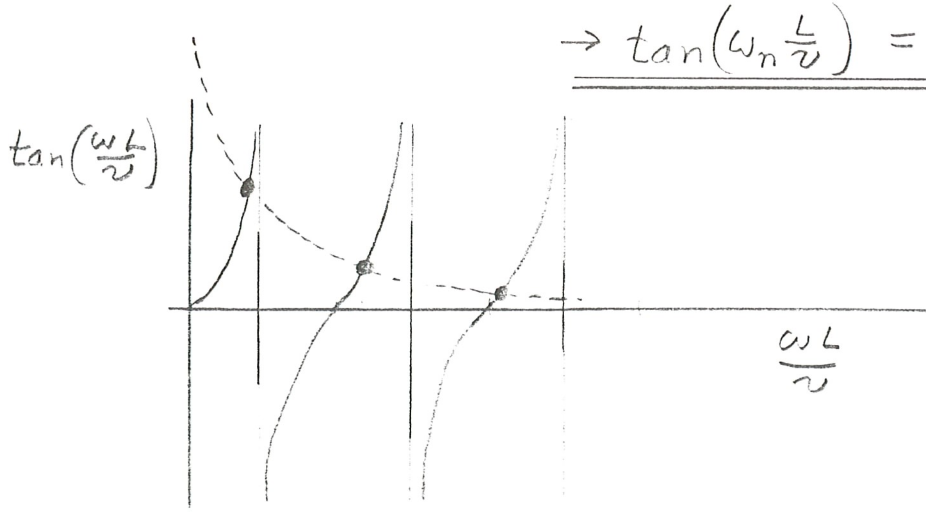
MASS : $m\ddot{y} = -2T\frac{\partial y}{\partial x}$ AT $x = L$ $\left\{ \begin{array}{l} \text{FOR SYMMETRIC NORMAL MODES} \\ \text{NOTE: FOR ANTISYMMETRIC} \\ \text{NORMAL MODES THE MASS} \\ \text{DOES NOT MOVE} \end{array} \right.$

CONSTRAINT : USE STRING SOLUTION AT $X = L$ IN MASS EQ.

$$-m\omega_n^2 A \sin(k_n L)\sin(\omega_n t + \phi) = -2TAk_n \cos(k_n L)\sin(\omega_n t + \phi)$$

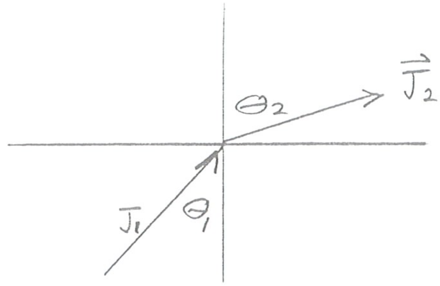
$$m\omega^2 \tan(k_n L) = 2Tk_n \rightarrow \tan(k_n L) = \frac{2Tk_n}{m\omega_n^2}$$

$$\rightarrow \underline{\underline{\tan\left(\omega_n \frac{L}{v}\right) = \frac{2T}{mv\omega_n}}}$$



I-2

S 2000



NORMAL COMPONENT OF $\vec{J}$ IS CONTINUOUS $\Rightarrow J_1 \cos\theta_1 = J_2 \cos\theta_2$

TANGENTIAL COMPONENT OF $\vec{E}$ IS CONTINUOUS, AND
$\vec{J} = S\vec{E}$ IN THE MEDIA $\Rightarrow (J_1/S_1)\sin\theta_1 = (J_2/S_2)\sin\theta_2$

a) DIVIDE THE 2 EQUATIONS TO FIND THE ANGLE

$$\frac{1}{S_1}\tan\theta_1 = \frac{1}{S_2}\tan\theta_2 \Rightarrow \underline{\underline{\tan\theta_2 = \left(\frac{S_2}{S_1}\right)\tan\theta_1}}$$

b) AT THE INTERFACE $E_{2\perp} - E_{1\perp} = \sigma/\epsilon_0$

$$\Rightarrow \sigma = \epsilon_0\left(\frac{J_2}{S_2}\cos\theta_2 - \frac{J_1}{S_1}\cos\theta_1\right)$$

$$= \underline{\underline{J_1\epsilon_0\left(\frac{1}{S_2} - \frac{1}{S_1}\right)\cos\theta_1}}$$

S2000

I-3. In mag. field $\vec{B} = \nabla \times \vec{A}$, $\vec{p}_{can} \to \vec{p}_{kin} + \frac{e}{c}\vec{A}$

K.E. is $\frac{p_{kin}^2}{2m} = \frac{|\vec{p}_{can} - \frac{e}{c}\vec{A}|^2}{2m} = \frac{|\vec{p} + \frac{e}{c}\vec{A}|^2}{2m}$

$\mathcal{H}$ can be evaluated by classical or Q.M. method

Classical. $\mathcal{H} = \frac{1}{2m}|\vec{p} + \frac{e}{c}\vec{A}|^2 - \frac{e^2}{r}$

$$= \frac{p^2}{2m} + \frac{e}{mc}\vec{p}\cdot\vec{A} + \frac{e^2}{2mc^2}A^2 - \frac{e^2}{r}$$

$$\vec{p}\cdot\vec{A} = \frac{1}{2}\vec{p}\cdot(\vec{r}\times\vec{B}) = -\frac{1}{2}r\times\vec{p}\cdot\vec{B} = -\frac{\hbar}{2}\vec{L}\cdot\vec{B}$$

$$A^2 = \frac{1}{4}(\vec{r}\times B\,\hat{z})^2 = \frac{1}{4}(x^2+y^2)B^2$$

$$\mathcal{H} = \frac{p^2}{2m} - \frac{e^2}{r} \underbrace{- \frac{e\hbar}{2mc}\vec{L}\cdot\vec{B}}_{\substack{-\mu_B \vec{L}\cdot\vec{B} \\ \text{paramagnetic} \\ \text{interaction}}} + \underbrace{\frac{1}{8}\frac{e^2}{mc^2}(x^2+y^2)B^2}_{\substack{\text{diamagnetic} \\ \text{interaction}}}$$

Q.M. $\vec{p}\cdot\vec{A}$ term is $\frac{e}{2mc}(\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p})$

$$\vec{p}\cdot\vec{A} + \vec{A}\cdot\vec{p} = \frac{1}{2}\vec{p}\cdot(\vec{r}\times\vec{B}) + \frac{1}{2}(\vec{r}\times\vec{B})\cdot\vec{p}$$

$= \frac{1}{2}(\vec{p}\times\vec{r})\cdot\vec{B} - \frac{1}{2}\vec{B}\cdot(\vec{r}\times\vec{p})$. But, $\vec{r}\times\vec{p} = -\vec{p}\times\vec{r}$ (cross product involves only terms like $p_x y$, $p_y x$) — and result is same as above.

I-4 S 2000

a) CONSERVATION OF ENERGY: $\frac{\vec{p}_i^{\,2}}{2m} + h\nu = \frac{\vec{p}_f^{\,2}}{2m} + \Delta E$

CONSERVATION OF MOMENTUM $\vec{p}_f = \vec{p}_i + \frac{h\nu}{c}\hat{n}$

$$\Rightarrow \vec{p}_f^{\,2} = \vec{p}_i^{\,2} + \frac{2h\nu}{c}\vec{p}_i \cdot \hat{n} + \left(\frac{h\nu}{c}\right)^2$$

THUS $h\nu = \frac{1}{2m}(\vec{p}_f^{\,2} - \vec{p}_i^{\,2}) + \Delta E$

$$\boxed{h\nu = \Delta E + \underbrace{\frac{h\nu}{mc}\vec{p}_i \cdot \vec{n}}_{\text{DOPPLER SHIFT}} + \underbrace{\frac{(h\nu)^2}{2mc^2}}_{\text{RECOIL SHIFT}}}$$

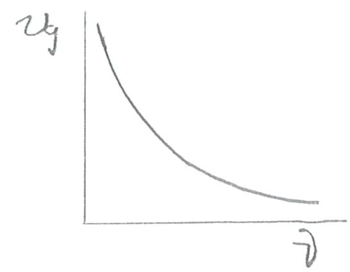
b) $h\nu_{ABSORPTION} = \Delta E + \frac{(h\nu)^2}{2mc^2}$

$h\nu_{EMISSION} = \Delta E - \frac{(h\nu)^2}{2mc^2}$

$h\nu_A - h\nu_E = 2 \times (\text{RECOIL SHIFT}) = \frac{(h\nu)^2}{mc^2}$

THE ENERGY OF THE ABSORBED PHOTON IS HIGHER THAN ΔE IN ORDER TO SUPPLY THE RECOIL ENERGY. THE ENERGY OF THE EMITTED PHOTON IS LOWER THAN ΔE BECAUSE SOME OF ΔE WENT INTO THE RECOIL OF THE ATOM.

II-1 S 2000



$\omega = \sqrt{gk}$ $\quad V_g = \frac{d\omega}{dk} = \frac{1}{2}\sqrt{\frac{g}{k}} = \frac{1}{2}\frac{g}{\omega} = \frac{g}{4\pi\nu}$

$t = \frac{d}{V}$ $\quad t_{(.2)} - t_{(.1)} = d\left(\frac{1}{V_{(.2)}} - \frac{1}{V_{(.1)}}\right)$

$$= \frac{4\pi d}{g}\left(\nu_{(.2)} - \nu_{(.1)}\right)$$

$$= \frac{4\pi d}{g}\left(\underbrace{(.2) - (.1)}_{(.1)}\right) = \frac{4\pi d}{g}\left((.1)\right) = t_{(.1)}$$

$\Rightarrow \underbrace{\left(t_{(.2)} - t_{(.1)}\right)}_{\text{6 HOURS}}$ = TRAVEL TIME AT 0.1 Hz = 6 HOURS

FOR $\nu = 0.1$ Hz $\quad$ EVENT TIME + $\underbrace{\text{TRAVEL TIME}}_{\text{6 HOURS}}$ = 6AM

$\Rightarrow$ <u>EVENT OCCURED AT MIDNIGHT</u>

NOTE: BECAUSE $V_p \propto V_g$ FOR THESE WAVES,

$$\left(V_p = \frac{\omega}{k} = \sqrt{\frac{g}{k}} = 2V_g\right),$$

THE SAME ANSWER FOR THE EVENT TIME WILL BE FOUND IF THE STUDENT MISTAKENLY USES V_p INSTEAD OF V_g TO DO THE COMPUTATION.

S2000

II-2. For electrostatic field in space, potential obeys Laplace's equation $\nabla^2 \varphi = 0$, and there can be no maximum or minimum. Hence there can be no stable position for a charge, (Earnshaw's theorem). - answer to part c.

a). $\varphi > 0$, P.E. $V = q\varphi < 0$. Hence the charge is bound in the system, Charge is unstable, will move in region bounded by potential surface $\varphi(\vec{r}) = \varphi(0)$ - (or possibly crash into a positive charge.)

b) $V > 0$. Charge will fly out of system with final K.E. $= q\varphi(0)$.

II − 3 S2000

IN GENERAL, ONE WOULD SOLVE THE WAVE EQ.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E - V = \text{A CONSTANT}$$

IN EACH OF THE 3 REGIONS, THEN MAKE SURE ψ AND $\frac{d\psi}{dx}$ WERE CONTINUOUS WHERE REGIONS TOUCHED. BUT HERE WE ARE TOLD THAT THE ENERGY EIGENVALUE E IS INDEPENDENT OF L. THIS CAN ONLY HAPPEN IF <u>ψ IS A CONSTANT IN THE CENTER REGION</u>. THIS IMPLIES THAT $E - V(r) = E - U = 0$ THERE. THUS, $E = U$.

IN THE LEFT WELL, WITH $x=0$ AT THE WALL,

$$\psi(x) = A \sin kx$$
$$\psi'(x) = kA \cos kx = 0 \text{ AT } x = a \Rightarrow k_n a = (n+\tfrac{1}{2})\pi$$

CHOOSE $n=0$ FOR THE GROUND STATE $\Rightarrow k_0 = \frac{\pi}{2a}$

THEN $\frac{\hbar^2}{2m}k_0^2 = \underline{\underline{\frac{\hbar^2}{2m}\left(\frac{\pi}{2a}\right)^2 = U}}$

II-4 S 2000

a) $Z = \frac{1}{N_1!} V^{N_1} z_1^{N_1} \frac{1}{N_2!} V^{N_2} z_2^{N_2} \frac{1}{N_3!} V^{N_3} z_3^{N_3}$

$F = -kT \ln Z \qquad P = -\frac{\partial F}{\partial V}\Big|_T$

$= kT \frac{1}{Z} \frac{\partial Z}{\partial V}\Big|_T$

$$\underline{\underline{P = (N_1 + N_2 + N_3) \frac{kT}{V}}}$$

b) FOR POINT PARTICLES (3 QUADRATIC TERMS IN $\mathcal{H}$)

$E_1 = \frac{3}{2} N_1 kT$

FOR LINEAR MOLECULES (5 QUADRATIC TERMS IN $\mathcal{H}$)

$E_2 = 5/2 \, N_2 \, kT$

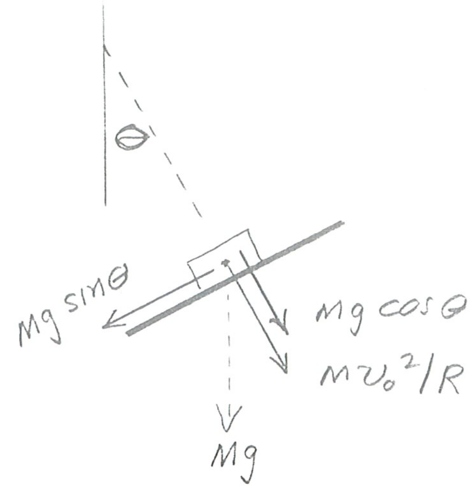
WHEN $\epsilon = |\vec{p}|c \qquad z_3 = \underbrace{4\pi \int_0^\infty e^{-pc\beta} p^2 dp}_{\frac{1}{\beta^3} \int_0^\infty e^{-x} x^2 dx}$

$\langle \epsilon_3 \rangle = -\frac{1}{z_3} \frac{\partial z_3}{\partial \beta}$

$= -\frac{1}{z_3} (-3) \frac{z_3}{\beta} = 3kT \Rightarrow E_3 = 3 N_3 kT$

$$\underline{\underline{E_{TOTAL} = \left(\frac{3}{2} N_1 + \frac{5}{2} N_2 + 3 N_3\right) kT}}$$

III -1 S 2000



N ≡ NORMAL FORCE BETWEEN CAR & TRACK $= \frac{Mv_0^2}{R} + Mg\cos\theta$

CAR SLIPS WHEN $Mg\sin\theta > \mu N = \mu M\left(\frac{v_0^2}{R} + g\cos\theta\right)$

$$\Rightarrow \mu = \frac{\sin\theta}{\left(\cos\theta + \frac{v_0^2}{gR}\right)}$$

S2000

III-2 $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{total}$ $\oint \vec{H}\cdot d\vec{l} = I_{free}$, $B = \mu H$

a) $2\pi R H = NI$, $B = \mu_0 \frac{NI}{2\pi R}$ $\Phi = \mu_0 \frac{NI}{2\pi R} A$

$$L = \frac{N\Phi}{I} = \frac{\mu_0 N^2 A}{2\pi R} = L_0$$

b) B_{tan} is discontinuous. B has different values in upper and lower halves.

$B_1 = \mu_1 \frac{NI}{2\pi R}$, $B_2 = \mu_2 \frac{NI}{2\pi R}$ $\Phi = \frac{(\mu_1 + \mu_2) A}{2} \frac{NI}{2\pi R}$

$$L = \frac{\mu_1 + \mu_2}{2} \frac{N^2 A}{2\pi R} = \frac{\mu_1 + \mu_2}{2\mu_0} L_0$$

c) $B_\perp$ continuous $\oint H\cdot d\vec{l} = \int \frac{B}{\mu}\cdot dl$

$$= B\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\pi R = NI.$$

$$B = \frac{NI}{\pi R} \frac{1}{\frac{1}{\mu_1} + \frac{1}{\mu_2}} = \left(\frac{\mu_0 NI}{2\pi R}\right)\left(\frac{2}{\frac{\mu_0}{\mu_1} + \frac{\mu_0}{\mu_2}}\right)$$

$$L = L_0 \left(\frac{2}{\frac{\mu_0}{\mu_1} + \frac{\mu_0}{\mu_2}}\right)$$

S2000

III-3 Because the system oscillates between K_0 and $\bar{K}_0$, these are not eigenstates of the system. Let the eigenstates be K_A and K_B. Initially, $\psi(0) = K_0$, so we expect $K_0 = \frac{1}{\sqrt{2}}(K_A + K_B)$.

By orthogonality, $\bar{K}_0 = \frac{1}{\sqrt{2}}(K_A - K_B)$

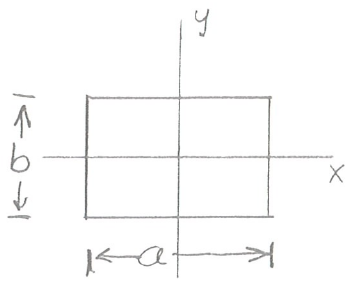
After time t,

$$\psi(t) = \frac{1}{\sqrt{2}}\left(K_A + e^{-i\frac{\Delta E}{\hbar}t} K_B\right) e^{-i\frac{E_A}{\hbar}t},$$

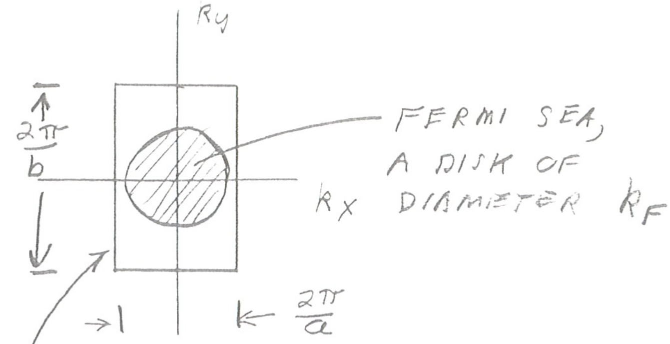
where $E_B = E_A + \Delta E$. Hence the system oscillates between K_0 and $\bar{K}_0$, converting from one to the other in time τ given by $\frac{\Delta E}{\hbar}\tau = \pi$.

III-4 S 2000

a)



PRIMITIVE UNIT CELL IN REAL SPACE



1ST BRILLOUIN ZONE IN k SPACE

DENSITY OF POINTS IN k SPACE $= \frac{A}{(2\pi)^2}$

DENSITY OF STATES IN k SPACE $= \frac{2A}{(2\pi)^2}$

$$\left(\pi k_F^2\right) \frac{A}{2\pi^2} = N_{ELECTRONS} = \frac{A}{ab}$$ IF 1 ELECTRON PER PRIMITIVE CELL

$$\Rightarrow k_F = \sqrt{\frac{2\pi}{ab}}$$

b) FERMI SEA OVERLAPS THE BOUNDARY OF THE 1ST BRILLOUIN ZONE IF

$$k_F > \frac{\pi}{a}$$

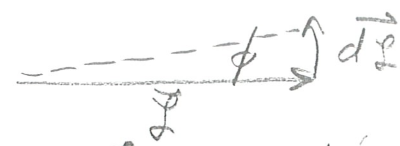
$$\frac{2\pi}{ab} > \frac{\pi^2}{a^2} \quad \Rightarrow \quad \underline{\frac{b}{a} < \frac{2}{\pi}}$$

IV - 1 S 2000

a) $\vec{\mathcal{L}}$ IS OUTWARD ALONG THE AXLE

$|\vec{\mathcal{L}}| = I_0 \omega_s$ $\qquad I_0(\text{DISK ABOUT AXIS}) = \frac{1}{2} M R^2$

$\vec{\tau} = MgL\,(\text{DOWNWARD}) = \frac{d\vec{\mathcal{L}}}{dt}$



$MgL = |\vec{\mathcal{L}}| \underbrace{\frac{d\phi}{dt}}_{\Omega_p} = \frac{MR^2\omega_s}{2}\,\Omega_p$ $\qquad \left|\frac{d\vec{\mathcal{L}}}{dt}\right| = |\vec{\mathcal{L}}| \frac{d\phi}{dt}$

$\Rightarrow \underline{\underline{\Omega_p = \frac{2gL}{R^2\omega_s}}}$

b) CONSERVE THE VERTICAL COMPONENT OF ANGULAR MOMENTUM, $\mathcal{L}_z$.

$\mathcal{L}_z|_{\text{INITIAL}} = \mathcal{L}_z|_{\text{FINAL}}$

$I_0 \omega_s \sin\theta = I_z \Omega_p \quad \Rightarrow \quad \underline{\sin\theta = \frac{I_z}{I_0}\frac{\Omega_p}{\omega_s}}$

NOW FIND I_z

PARALLEL AXIS THEOREM $\Rightarrow I_z = ML^2 + I_d$

$I_d \equiv$ MOMENT OF DISK ABOUT DIAMETER

FOR A PLATE $\perp$ TO Z AXIS $\quad I_z = I_x + I_y$

SO HERE $\quad \frac{1}{2}MR^2 = 2 I_d \Rightarrow I_d = MR^2/4$

FINALLY, $\underline{\underline{\sin\theta = \left(\frac{2L^2}{R^2} + \frac{1}{2}\right)\frac{\Omega_p}{\omega_s}}}$

IV-2 S 2000

WE ARE TOLD THE ELECTRIC FIELD IS RADIAL.

$\Rightarrow \vec{E}$ IS PARALLEL TO THE INTERFACIAL PLANE.

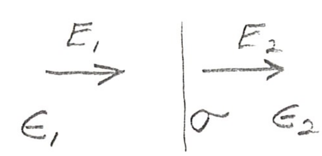
BUT PARALLEL $\vec{E}$ IS CONTINUOUS.

$\Rightarrow \vec{E}$ HAS THE SAME r DEPENDENCE EVERYWHERE

$$\vec{E}(\vec{r}) = E_0 \frac{R^2}{r^2} \hat{r} \quad \text{FOR} \quad r > R$$

$$V(R) = \int_R^\infty E(r)\,dr = E_0 R^2 \underbrace{\int_R^\infty \frac{dr}{r^2}}_{1/R} = E_0 R$$

NOW FIND E_0 USING THE RELATION BETWEEN THE NORMAL COMPONENT OF $\vec{E}$ AND THE SURFACE CHARGE σ



$$\epsilon_2 E_2 - \epsilon_1 E_1 = \sigma$$

IN THE SPHERE $\vec{E} = 0$ SO $\sigma = \epsilon_0 E_0$ ABOVE

$\kappa \epsilon_0 E_0$ BELOW

$$Q = 2\pi R^2 \epsilon_0 E_0 + 2\pi R^2 \kappa \epsilon_0 E_0 = 2\pi \epsilon_0 (1+\kappa) R^2 E_0$$

$$V(R) = E_0 R = \underline{\underline{\frac{Q}{2\pi \epsilon_0 (1+\kappa) R}}}$$

S2000

IV-3. The spin-orbit interaction is due to the interaction of the electron's magnetic moment $\vec{\mu}_e = -g_s \mu_B \vec{S}$ ($g_s = 2$, μ_B = Bohr magneton), with the motional magnetic field it experiences, $\vec{B}_m = \vec{E} \times \frac{\vec{v}}{c}$. Taking $\vec{E}(\vec{r})$ to be of the form $\vec{E}(\vec{r}) = + \frac{e Z(r)}{r^3} \vec{r}$, we have

$$\vec{B} = \frac{e Z(r)}{r^3} \vec{r} \times \frac{\vec{v}}{c} = \frac{e}{mc} \vec{r} \times m\vec{v} \frac{Z(r)}{r^3}$$

$$= \frac{e\hbar}{mc} \frac{Z(r)}{r^3} \vec{L}.$$

$$\mathcal{H}_{so} = -\mu_e \cdot \vec{B} = + g_s \mu_B \frac{e\hbar}{mc} \frac{Z(r)}{r^3} \vec{S} \cdot \vec{L}.$$

Hence, $X = g_s \mu_B \frac{e\hbar}{mc} \left\langle \frac{Z(r)}{r^3} \right\rangle$,

where the brackets indicate the expectation value for the appropriate wave function. (Can substitute $g_s = 2$. Also, $\frac{e\hbar}{\cdot} = \mu_B$).

IV-4 S2000

NUMBER OF STATES WITH $|\vec{k}| < k \equiv N(k)$

$$N(k) = \frac{V}{(2\pi)^3} \frac{4}{3} \pi k^3$$

IF $\epsilon = a|\vec{k}|^n$ THEN $k = \left(\frac{\epsilon}{a}\right)^{1/n}$

$$N(\epsilon) = \frac{V}{(2\pi)^3} \frac{4}{3} \pi \left(\frac{\epsilon}{a}\right)^{3/n} \propto \epsilon^{3/n}$$

$$D(\epsilon) = \frac{dN(\epsilon)}{d\epsilon} \propto \epsilon^{3/n - 1}$$

$$E = \int_0^\infty D(\epsilon)\, \epsilon\, \bar{n}(\epsilon)\, d\epsilon \qquad \bar{n}(\epsilon) = \frac{1}{e^{\epsilon/kT} - 1}$$

$$E \propto \int \frac{\left(\epsilon^{3/n - 1}\right) \epsilon\, d\epsilon}{e^{\epsilon/kT} - 1} \rightarrow \int \frac{\epsilon^{3/n}\, d\epsilon}{e^{\epsilon/kT} - 1}$$

$$= (kT)^{\frac{3}{n} + 1} \times \text{A CONSTANT}$$

$$C_V = \left.\frac{\partial E}{\partial T}\right|_V \propto (kT)^{3/n}$$

SO IF $C_V \propto T^3$, THEN $n = 1$

EXCITATIONS IN AN UNCHARGED, NON-MAGNETIC SUBSTANCE WITH $\epsilon \propto |k|$ ARE PHONONS (QUANTIZED SOUND WAVES).

VI-1 S2000

a) THE WEAK INTERACTION IS INVOLVED IN THIS REACTION, HENCE ITS VERY SMALL MAGNITUDE.

b) $$\ell = \frac{1}{\sigma n} = \frac{1}{10^{-45} \times 3.2 \times 10^{28} \times 2 \times 10^{2}}$$

$$= \frac{1}{6.4} \times 10^{15} \sim \underline{1.5 \times 10^{14} \text{ meters}}$$

c) NUCLEAR DIAMETER $\sim 10^{-13}$ cm $= 10^{-15}$ m

$$\ell = \frac{1}{\sigma n} \sim \frac{1}{10^{-45}(10^{+15})^{3}} = \underline{1 \text{ meter}}$$

V-2 S 2000

a) POTENTIAL ENERGY OF OBJECT OF MASS m A DISTANCE R FROM A POINT MASS M IS $PE = -GMm/R$.

ESCAPE WOULD REQUIRE A RADIAL VELOCITY SUCH THAT $KE = \frac{1}{2} m v^2 = -PE$

THE MAXIMUM v IS c, $\Rightarrow \frac{1}{2} c^2 = \frac{GM}{R}$

$$\underline{\underline{R = \frac{2GM}{c^2}}}$$

b) $dE = T\,dS - P\,\cancel{dV}^{\,0} \Rightarrow T = \left.\frac{\partial E}{\partial S}\right|_R$

$S = \alpha A = 4\pi\alpha R^2$ $\qquad \alpha = \left(\frac{k_B c^3}{4G\hbar}\right)$

EXPRESS R IN TERMS OF E

$E = Mc^2$ & $R = \frac{2GM}{c^2} \Rightarrow R = \frac{2GE}{c^4}$

$$S = 4\pi\alpha\left(\frac{2G}{c^4}\right)^2 E^2$$

$$dS = 8\pi\alpha\left(\frac{2G}{c^4}\right)^2 E\,dE$$

$$T = \frac{dE}{dS} = \frac{c^8}{32\pi\alpha G^2}\frac{1}{E} = \frac{c^6}{32\pi\alpha G^2}\frac{1}{M}$$

$$\underline{\underline{= \left(\frac{c^3\hbar}{8\pi k_B G}\right)\frac{1}{M}}}$$

V-3 S 2000

$|x| < a \qquad \psi = \alpha \cos(kx) \qquad \frac{\hbar^2 k^2}{2m} = U - E_0 = E_0$

$\psi(a) = \alpha \cos(ka)$

$x > a \qquad \psi = \alpha \cos(ka)\, e^{-(x-a)\kappa}$

$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -|E_0|\psi \rightarrow \frac{\hbar^2 \kappa^2}{2m} = E_0$

$\Rightarrow \kappa = k$

NOW EQUATE THE DERIVATIVES OF ψ AT $x = a$

$-\alpha k \sin(ka) = -\alpha \cancel{\kappa}^{k} \cos(ka)$

$\rightarrow \sin(ka) = \cos(ka) \Rightarrow ka = \frac{\pi}{4}$

$$U = 2E_0 = \frac{\hbar^2 k^2}{m} = \underline{\underline{\frac{\hbar^2}{m}\left(\frac{\pi}{4a}\right)^2}}$$

V-4 S 2000

IN GENERAL $dE = T\,dS - P\,dV$

$$= T\left(\frac{\partial S}{\partial T}\Big|_V dT + \frac{\partial S}{\partial V}\Big|_T dV\right) - P\,dV$$

$$\frac{\partial E}{\partial V}\Big|_{T,N} = \left(T\frac{\partial S}{\partial V}\Big|_T - P\right)$$

USE A MAXWELL RELATION TO FIND $\frac{\partial S}{\partial V}\Big|_T$:

$$F \equiv E - TS$$
$$dF = dE - T\,dS - S\,dT = -S\,dT - P\,dV$$

CROSS DERIVATIVES ARE EQUAL FOR AN EXACT DIFFERENTIAL

$$\frac{\partial S}{\partial V}\Big|_T = \frac{\partial P}{\partial T}\Big|_V$$

WHEN $P = \frac{NkT}{V-Nb}$ $\quad \frac{\partial S}{\partial V}\Big|_T = \frac{Nk}{V-Nb} = \frac{P}{T}$

THUS IN THIS CASE

$$\underline{\underline{\frac{\partial E}{\partial V}\Big|_{T,N} = (P-P) = 0}}$$