

Qualifying Examination - Part I

Time: 9:30-11:30 a.m.

Saturday, September 12, 2009

The examination is to be written on the single sheets that are provided; no more than one question is to be answered on a single sheet. Number each question. You are to answer all questions in Part I; however, if you do omit any questions, ***cross out those numbers on your title page.*** When you are finished, collect the answer sheets in order and place them together (with the title page on top) back in the envelope.

Place your code letter (from your title page) on the back of each sheet of paper.

Part I counts one-third (1/3) of the final grade.

Part II is in this same room at 1:00 p.m.

PHYSICAL CONSTANTS

Planck's constant	$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 4.14 \times 10^{-15} \text{ eV}\cdot\text{s}$
Vacuum speed of light	$c = 3.00 \times 10^8 \text{ m/sec}$
	$\hbar c = 197 \text{ MeV}\cdot\text{fm} = 1.97 \times 10^{-5} \text{ eV}\cdot\text{cm}$
Electron charge	$e = 1.60 \times 10^{-19} \text{ C} = 4.80 \times 10^{-10} \text{ esu}$
Boltzmann constant	$k = 1.38 \times 10^{-23} \text{ J/K} = 8.62 \times 10^{-5} \text{ eV/K}$
Gas constant	$R = 8.31 \text{ J/(mol}\cdot\text{K)}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$
Permittivity of free space	$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$
Electron mass	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV}/c^2$
Proton mass	$m_p = 1.67 \times 10^{-27} \text{ kg} = 938 \text{ MeV}/c^2$
Bohr radius of hydrogen	$a_B = 5.3 \times 10^{-11} \text{ m}$
Ionization energy of hydrogen	13.6 eV
Avogadro's number	$6.02 \times 10^{23} / \text{mole}$

Conversion Factors

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ erg}$
$1 \text{ m} = 10^{10} \text{ \AA} = 10^{15} \text{ fm} = 6.25 \times 10^{-4} \text{ miles}$
$1 \text{ atm} = 1.01 \times 10^5 \text{ N/m}^2 = 760 \text{ Torr}$
$1 \text{ cal} = 4.186 \text{ J}$

Divergence and curl in spherical coordinates

$$\nabla \cdot E = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta E_\theta) + \frac{1}{r \sin \theta} \frac{\partial E_\phi}{\partial \phi}$$

$$\begin{aligned} \nabla \times E = & r \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta E_\phi) - \frac{\partial E_\theta}{\partial \phi} \right] + \theta \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial E_r}{\partial \phi} - \frac{\partial (r E_\phi)}{\partial r} \right] \\ & + \phi \frac{1}{r} \left[\frac{\partial (r E_\theta)}{\partial r} - \frac{\partial E_r}{\partial \theta} \right] \end{aligned}$$

where r, θ, ϕ are the unit vectors associated with the spherical coordinates r, θ, ϕ .

Useful integrals

$$\int_0^\infty e^{-\alpha x^2} dx = \frac{1}{2} \sqrt{\frac{\pi}{\alpha}}$$

$$\int_0^\infty x e^{-\alpha x^2} dx = \frac{1}{2\alpha}$$

$$\int_0^\infty x^2 e^{-\alpha x^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{\alpha^3}}$$

$$\int_0^x \sin^2 x \, dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{(1-x)}; \quad -\sum_{n=1}^{\infty} (-)^n \frac{x^n}{n} = \ln(1+x)$$

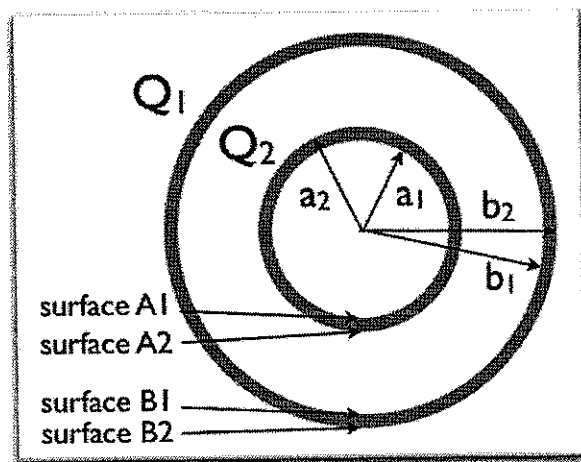
Physics Qualifier Part I—Fall 2009

7-minute questions

- 1) Write down the energy eigenvalues for the one-dimensional Schrodinger equation with $V(x) = \frac{1}{2}m\omega^2 x^2$ for $x > 0$ and ∞ for $x < 0$. Justify your answer.

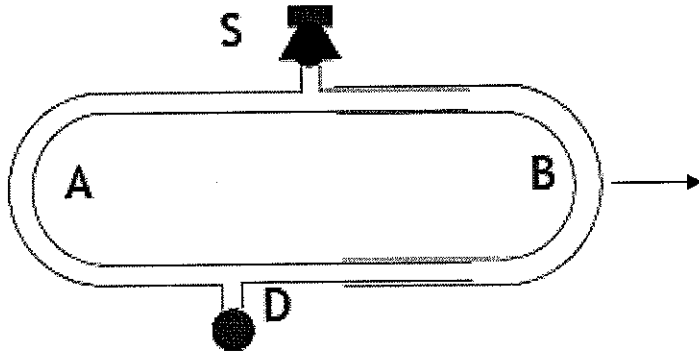
- 2) Consider two concentric metal spheres of finite thickness in a vacuum. The inner sphere has radii $a_1 < a_2$. The outer sphere has radii $b_1 < b_2$. A charge Q_2 is put on the inner sphere and a charge Q_1 on the outer sphere.

- a) Find the charge density on each of the four surfaces.
- b) If $Q_2 = -Q_1$ what is the mutual capacitance of the system?



- 3) This question is about heating the air in a house. The air can be treated as a diatomic ideal gas. The thermal expansion of the house volume can be neglected. The house is not airtight.
- a) Find the total internal energy of the air in the house as a function of pressure and volume.
- b) Now the furnace is turned on and the temperature rises (of course, that's why you did it, to achieve a comfortable temperature). How does the energy of the air in the house change as the temperature rises?
- c) What happens to the additional energy that the furnace puts into the air?

- 4) In the figure below is shown an acoustic interferometer. S is a diaphragm that vibrates under the influence of an electromagnet. D is a sound detector such as the ear or a microphone. Path SAD is fixed but path SBD can be varied in length. The interferometer contains air (the speed of sound is 340 m/s) and it is found that the sound intensity has a minimum value of 100 units at one position of B and continuously climbs to a maximum value of 900 units at a second position 1.65 cm from the first.

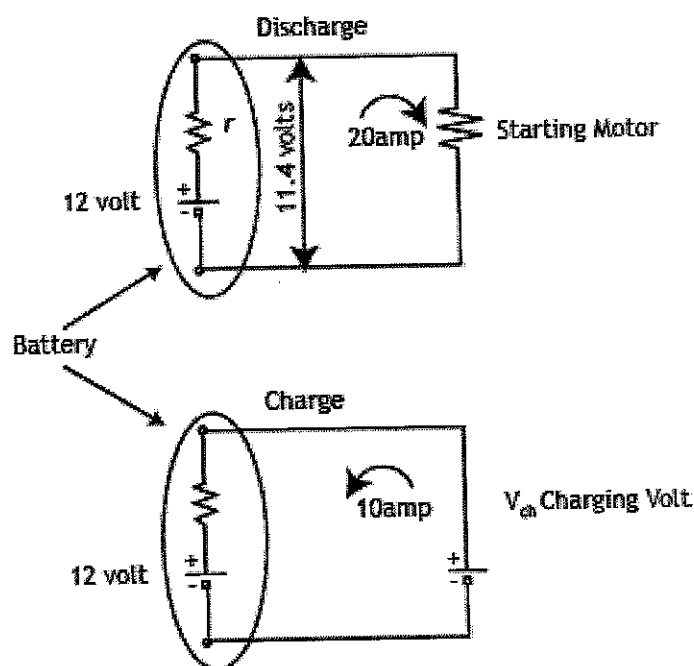


- a) Find frequency of the sound emitted from the sources
 - b) Find the relative amplitudes of the two waves arriving at the detector
- 5) Consider a particle moving in a one-dimension potential, with $b > 0, c > 0$,
- $$U(x) = \frac{b}{3}x^3 + \frac{c}{4}x^4$$
- a) Write down the equation of motion.
 - b) Calculate the position of the stable equilibrium point x_e
 - c) Linearize the equation of motion near the equilibrium point and find the frequency of small oscillations
- 6) A proton (mass $938 \text{ MeV}/c^2$) is in a 1-dimensional square well 100 nm in length, with walls that are infinitely high. The center of the well is at $x=0$.
- a) Is the proton relativistic?
 - b) What is the expression for the proton's allowed energy levels? Calculate numerically the energy of the ground level.
 - c) Calculate the expression of the expectation value of the position of the proton squared $\langle x^2 \rangle$ in the ground level.

$$\int u^2 \sin u \, du = 2u \sin u - (u^2 - 2) \cos u$$

- 7) An automobile battery has emf of 12 volt and an internal resistance r . The battery has a terminal voltage of 11.4 volts when the battery is delivering 20 amp to the car's starting motor.

- What is the resistance of the battery r ?
- How much power is supplied by the 12-volt emf of the battery when it is delivering 20 amp?
- How much of this power is delivered to the starter?
- What voltage V_{ch} must be put across the terminals of the battery to charge the battery with current 10 amp?



- The diameter of the sun subtends an angle of about half a degree of arc at the earth.
 - What is the focal length of a single converging lens that will give a real image of the sun that is 12 cm in diameter?
 - If the diameter of the lens is 3.0 cm, estimate the smallest distance between features on this image that can be resolved.
- A light beam of intensity I_0 and frequency f_0 is incident on a mirror along a direction z perpendicular to the mirror. The mirror moves with velocity v along the positive z direction.
 - What is the frequency of the reflected light?
 - What is the reflected intensity?
- Experiments show that for warm-blooded animals, the "metabolic rate" (the power P which is the average time-rate of release of energy as heat) is proportional to the $3/4$ power of the total mass M of the animal. Of course the total energy release during any time interval T is directly proportional to the mass. To what power of the mass should the pulse-rate (heart beats per minute) be proportional? Give a quantitative reason for your answer.

Qualifying Examination - Part II

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where \mathbf{r} , $\boldsymbol{\theta}$, $\boldsymbol{\phi}$ are the unit vectors associated with the spherical coordinates r , θ , ϕ .

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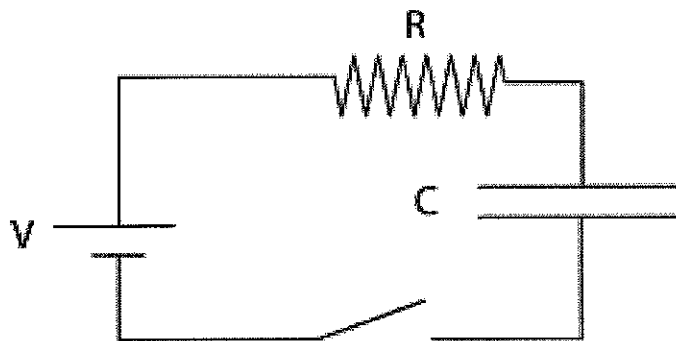
$$\int_0^x \sin^2 x \, dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{(1-x)}; \quad -\sum_{n=1}^{\infty} (-)^n \frac{x^n}{n} = \ln(1+x)$$

Physics Qualifier Part II—Fall 2009

12 minute questions

11) Consider an initially uncharged series RC circuit. At $t=0$ a battery of emf V is connected to the circuit. Show that the displacement current in the parallel plate air gap capacitor is equal to the charging current.



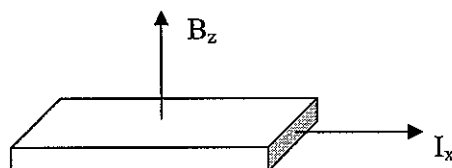
12) Consider a point charge q and an uncharged dielectric sphere of radius a , separated by $R \gg a$. If R is increased by a factor of 2, by what factor should q be increased to keep the force between them unchanged?

*9



$R \gg a$

13) A Hall probe is used to measure the magnetic field in a particle detector. The Hall probe consists of a piece of p-doped silicon with a carrier density $n = 10^{18} \text{ cm}^{-3}$ and the dimensions $\Delta x = 4 \text{ mm}$, $\Delta y = 2 \text{ mm}$, $\Delta z = 0.5 \text{ mm}$. A magnetic field B_z is applied in the z -direction and a current $I_x = 0.1 \text{ A}$ flows in the x -direction. Note that $1 \text{ T} = 1 \text{ Vs/m}^2$.



- Which quantity X is measured to determine the magnetic field B_z ?
- Write down the equilibrium condition that determines this quantity.
- Calculate the current density j_x from I_x and the carrier velocity v_x from j_x .
- Calculate the quantity X measured for a magnetic field $B_z = 1 \text{ T}$.

14) The ratio $R = (\sigma(e^+e^- \rightarrow \text{hadrons}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-))$ increases as the collision energy crosses thresholds for new quarks. In the quark model, R is proportional to the number of available quarks, weighted by the square of the quark charge. Assuming three colors, two charge $+2/3$ quarks, and three charge $-1/3$ quarks, calculate R in three regions:

- a) below charm threshold;
- b) above charm, but below bottom threshold;
- c) above bottom threshold.

15) A positron e^+ is moving straight with a kinetic energy, $K_{e^+} = m_e$, where m_e is the electron mass and K_{e^+} is given in the natural units $c = 1$. It annihilates with an electron e^- at rest, creating two photons. One of the outgoing photons (γ_1) emerges at a right angle to the incident positron direction in the lab frame.

- a) Find the energies of the outgoing photons E_1, E_2 in the lab frame.
- b) Find the direction (θ_2) of the second photon (γ_2) with respect to the incident positron direction in the lab frame.

16) A particle of mass M moving in one dimension finds itself in the potential

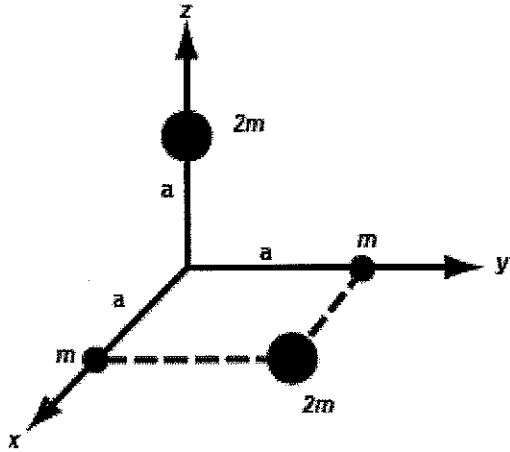
$$V(x) = \begin{cases} \infty, & x < 0 \\ Fx, & x > 0 \end{cases}$$

where $F > 0$. A good guess for the ground state wavefunction is

$$\psi(x) = \begin{cases} \sqrt{\frac{b^3}{2}} x \exp\left(-\frac{bx}{2}\right) & x > 0 \\ 0 & x < 0 \end{cases}$$

- a) What is the optimal value of b ? ($\int_0^\infty u^n e^{-u} du = n!$)
- b) Give an upper bound for the ground state energy.

17) A rigid system consists of four point masses located as shown:



- Obtain the moment of inertia tensor $I_{ij} = \sum_s m_s (r_s^2 \delta_{ij} - r_{si} r_{sj})$ of the system with respect to the origin in the coordinates x, y, z .
- Determine the principal moments of inertia for the system and give the equations that determine the principal axes.

18) In a θ pinch, azimuthal currents flow to produce a purely axial magnetic field (z directed) that depends upon radius in a cylindrical geometry. Consider a θ pinch plasma embedded in an externally applied axial magnetic field of $B_{r=a}=1$ Tesla, with a kinetic pressure profile

$$P(r) = 2n_0 k T_0 \left(1 - \frac{r^2}{a^2} \right)$$

with plasma radius $a = 1$ meter, central electron density $n_0 = 10^{20} \text{ m}^{-3}$ and an equal electron and ion temperature of 10 keV. Assume the plasma is in static equilibrium and can be described by a 1D radial force balance between the magnetic and kinetic pressures.

- Find the profile $B(r)$
- Give a numerical value for the magnetic field at $r = 0$.

19) An excited atom of Helium has one electron in state $n=1$, $l=0$ and the other in state $n=2$, $l=1$. By using the Bohr model, estimate the electric and magnetic forces on the outer electron. Which force is stronger, and by what factor?

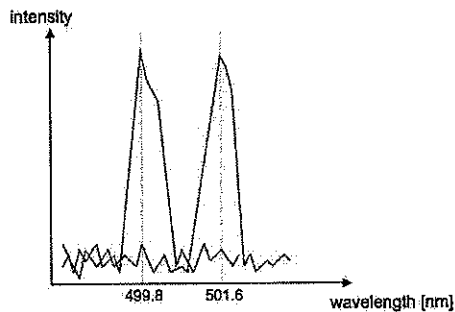
20) A block of copper of mass $M=0.1\text{kg}$ is assumed to have a constant temperature throughout its volume. The block of copper is in vacuum, but is connected to a bath of liquid nitrogen (at $T_R=77\text{ K}$) by a cylindrical copper rod of length $l=5\text{cm}$ and cross-sectional area $A=0.05\text{cm}^2$, but negligible heat capacity. The block of copper also has a heater attached to it with which heat can be added to the copper block. The thermal conductivity of copper is $k = 400\text{W}/(\text{m} \cdot \text{K})$, and its specific heat is $c_v = 384\text{J}/(\text{kg} \cdot \text{K})$.

- a) What is the equilibrium temperature T_1 of the block when the heater continuously puts 1 W of heat into the block?
- b) Suppose that the heater is now turned off. Find the temperature as a function of time, in terms of the variables given.

21) Answer the following parts (a) through (e) with no more than one or two sentences each, and a brief calculation, if necessary.

- a) Electrons in solids occupy a continuum of states known as bands. Which, if any, of the following three classes of materials have band gaps: metals? semiconductors? insulators?
- b) Silicon is a common material with a band-gap. Which of the three categories does it fall in (metal, semiconductor, or insulator)?
- c) Silicon has a band gap of about 1.1 eV . Is Silicon transparent to infrared light of wavelength two micrometers? Why?
- d) Is Silicon transparent to green light?
- e) Pure, undoped Silicon has a higher electrical resistance at 4.2 Kelvin than at 77 Kelvin . Why, and by about what factor?

22) The figure shows the intensity versus wavelength of light from interstellar gas on two opposite sides of galaxy M87. The gas orbits the core of the galaxy at a radius of $r = 100$ light years ($1 \text{ light year} = 9.46 \times 10^{15} \text{ m}$). It moves towards us on one side and away from us on the other side.

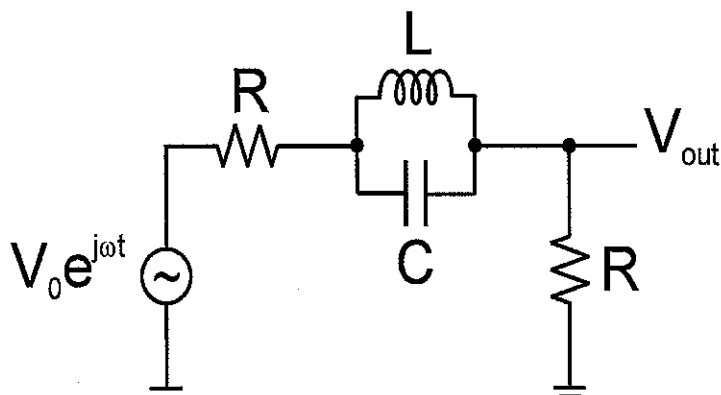


- a) What is the speed of the gas relative to us?
- b) What is the mass of the core of M87 in multiples of the Sun's mass,
 $M_{\text{sun}} = 1.99 \times 10^{31} \text{ kg}$?

23) A transmission diffraction grating has 20,000 lines and a line separation of 2500 nm. Light of wavelength 589 nm is normally incident, and fills the grating.

- a) Determine the angle relative to normal incidence of the first order spectral line. (Express in degrees)
- b) Determine the grating angular dispersion $d\theta/d\lambda$ (in degrees/nm) for first order.
- c) What is the smallest angular separation of two spectral lines that can just be resolved in first order? (Express in degrees)
- d) Determine the grating resolving power $\lambda/\Delta\lambda$.

24) Consider the following circuit.



- Sketch $|V_{out}/V_{in}|$ as a function of driving frequency ω , indicating the magnitude at very low frequencies, at the resonant frequency ω_0 of the circuit, and at very high frequencies.
- Find the phase shift as a function of frequency.

25) A classical gas in d dimensions has free energy

$$F = -cVT^{d+1}$$

at high temperatures. Here, c is some positive numerical constant of order 1.

- Show that this implies an equation of state relating energy density $e = E/V$ and entropy density $s = S/V$ of the form

$$e = c' s^\alpha$$

where c' is a numerical constant, and determine α in terms of d .

- The specific heat is defined by

$$C_V = T \left. \frac{dS}{dT} \right|_V.$$

Given the equation of state (2), what is the range of values of α for which the specific heat is negative?

- Is there a dimension d for which the specific heat of a classical gas with free energy (1) becomes negative? Explain.