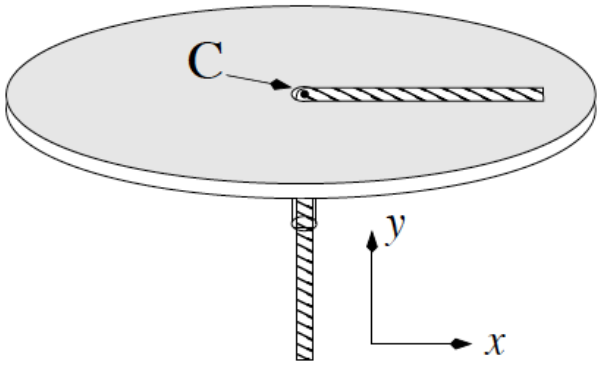
Physics Qualifying Examination – Part I 7-Minute Questions

February 7, 2015

1. A uniform rope of mass m and length L is at rest with half of its length lying on a horizontal table top and half hanging vertically in a hole through the table. Gravity acts downwards with an acceleration g. The rope is allowed to slide down. All surfaces are frictionless. How fast is it going when the last bit of rope just leaves the table top?



2. An RC circuit, with a capacitor in series with a resistor, has a resistance R=3000 Ω and a capacitance C=5.0 μF. This circuit is connected to an ideal dc battery with an *emf* V=5.0 V.

   a. If the capacitor is initially uncharged, then how long does it take to reach 80% of the maximum charge on the capacitor?
   b. Assuming this took T seconds, in terms of the given algebraic constants, R, C and/or V, how much energy was dissipated by the resistor? (Retain your answer in algebraic form.)

3. A long solenoid of cylindrical shape, having n turns of wire per unit length, carries a current I. Find the magnetic force acting on a small area, A, of the solenoid surface. Neglect magnetic permeability of the material inside the solenoid. Explain your answer.

4. In the electro-optic effect an electric field is applied to a crystal causing a slight change in the refractive index. To first order, the change in the refractive index is equal to the applied electric field times a constant of proportionality. The constant of proportionality is typically referred to as the electro-optic coefficient. Consider a nonlinear crystal which has an electro-optic coefficient of $R=10^{-10}$ m/V. The thickness of the crystal is d=2 mm.

   a. If a voltage of 4000 V is applied across the crystal, what would be the change in the refractive index?
   b. Suppose that the crystal is placed in one of the arms of a Michelson interferometer which uses an ultraviolet laser at a wavelength of 200 nm. How many bright-dark-bright fringe shifts would be observed as the applied voltage is increased from 0 V to 4000 V?

5. The astronomical unit is 150 million km. The Stefan-Boltzmann constant is 5.67×10^{-8} W m^{-2} K^{-4}. The radius of the Sun is 696,000 km. Make the following simplifying approximations: The Sun is a blackbody with a temperature of 5780 K. The Earth has no atmosphere. The Earth is a blackbody with zero reflectivity (albedo).

   a. Calculate the luminosity (total power) of the Sun.
   b. Calculate the solar constant (the solar power per unit area at Earth's distance from the Sun).
   c. Calculate the temperature of the Earth.

6. A cold sodium atom (^{23}Na), at rest, is isolated in a vacuum system on the Earth's surface. A laser, with λ = 589 nm (i.e., D-line or the 3s to 3p transition) shines on this atom from directly below. The atom absorbs photons and then reradiates that energy uniformly in all directions. Gravity, with an acceleration of 9.8 m/s^2, acts downward.

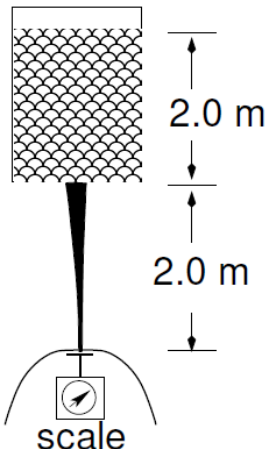
   In steady-state how many photons per second, to two significant digits, must it absorb and reradiate if it is, on average, to remain stationary?

7. In reference frame S, two subatomic particles (with rest mass 938 MeV/c^2), each moving at 0.500 c, approach each other head-on.

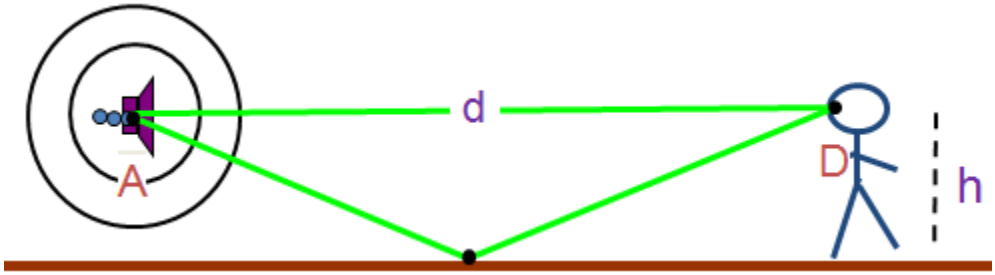
   a. Calculate the total kinetic energy of the two particles in frame S.
   b. Calculate the total kinetic energy of the particles as seen in reference frame S', which is moving with speed 0.500 c relative to S so that one of the particles is at rest.
   c. The two particles collide and stick together. What is the mass of the resulting object?

8. A barrel, as shown, is open at the top and has a small 1.0 cm radius hole in the bottom. Water inside the barrel ($\rho = 10^3$ kg/m^3) fills the barrel to a depth of 2.0 m. Water leaks out of the hole and falls a distance of 2.0 m. There it strikes a scale plate and flows out horizontally. Assume the acceleration due to gravity is 10 m/s^2.

   a. What is the radius of the water stream at the position just above the scale plate?
   b. What force, in Newtons, does the scale read? The weight of the water moving horizontally is negligible.



9. A speaker sits on a pedestal $h = 2.0$ m tall and emits a sine wave at 340 Hz (the speed of sound in air is 340 m/s). Only the direct sound wave and that which reflects off the ground at a position half-way between the speaker and the person (also 2.0 m tall) makes it to the person's ear. What is the furthest location (d) that a person will observe a maximum in the sound intensity?



10. Five electrons are confined in a cubic box of length a on a side. The electrons are in the lowest energy states.

    a. What is the ground state energy?
    b. What is the associated frequency of the lowest energy transition that will excite an electron?

Physics Qualifying Examination – Part II 12-Minute Questions

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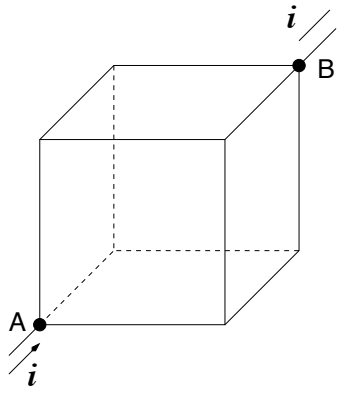
1. Twelve wires, each of resistance r, are connected to form the edges of a cube. Calculate the effective resistance R of this network across a body-diagonal of the cube (from A to B). Exploit the symmetry of the problem.



2. Consider the three spin-1 matrices

$$S_x = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},\ S_y = \frac{\hbar}{\sqrt{2}}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix},\ S_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

   a. What are the possible values one can get if we measure the spin along the x-axis?
   b. Suppose one obtains the largest possible value when one measures the spin along the x-axis. If one now measures the spin along the z-axis, what are the possible outcomes and what is the probability of each outcome?

3. A solid at temperature T is placed in an external magnetic field of strength $B = 3$ Tesla. The solid contains weakly interacting paramagnetic atoms of spin $1/2$. If the magnetic moment is equal to the Bohr magneton $\mu_B = 9.27 \times 10^{-24}$ Joules per Tesla, below what temperature must one cool the solid so that 75% of the atoms have their spin aligned anti-parallel to the external magnetic field?

4. Electrons $\left(e^-\right)$ and positrons $\left(e^+\right)$ have opposite charges but the same rest mass, m_e. Suppose a photon has energy $E_\gamma > 2m_e c^2$. Can the photon decay into an electron/positron pair? Show why or why not.

5. A parallel-plate capacitor with circular plates 20 cm in diameter is being charged, the displacement current density between the plates is uniform with a magnitude of 20 A/m^2.

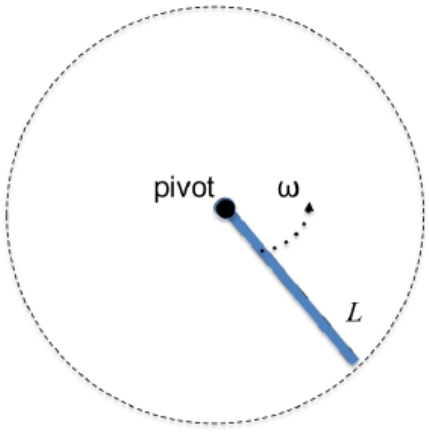
   a. Calculate the magnitude of the $\mathbf{B}$ field at a distance 5.0 cm radially from the axis symmetry of the region.
   b. Calculate dE/dt in this region.

6. A deuterium molecule consists of 2 electrons and 2 deuterons (the deuteron has a nuclear spin $i = 1$). Its lowest energy states have total electron spin $s = 0$, but the molecule can also rotate, with rotational angular momentum quantum numbers $n = 0,1,\ldots$ The resulting wavefunction for the molecule is a product of the rotational wavefunction $|nm_n\rangle = Y_{nm_n}(\theta,\phi)$ and the appropriately symmetrized nuclear spin wavefunction.

   a. What are the possible total nuclear spin quantum numbers for the molecule?
   b. For each of your answers to a., give the exchange symmetry of the nuclear spin wavefunction.
   c. Given that the rotational wavefunctions are multiplied by $(-1)^n$ upon exchange of the two nuclei, what must the nuclear spin quantum number(s) be for the states with n odd?

7. A uniform rope of mass *M* and length *L* is pivoted at one end and whirls with uniform angular velocity ω. Neglect gravity.

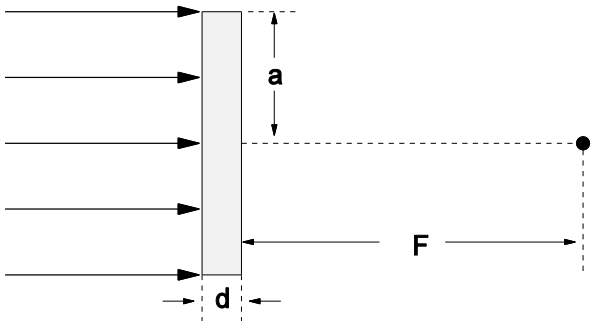
   a. What is the tension at the pivot (i.e. at $r = 0$)?
   b. What is the tension in the rope at a distance r from the pivot? (It may help to know that the tension at $r = L$ is 0.)
   c. If the Young's modulus for the rope is Y and the rope has cross-sectional area *A*, by how much does the rope stretch? Assume that the stretch is sufficiently small that the calculation need not be done self-consistently.



8. A hot air balloon uses a nylon envelope of surface mass density $\sigma = 0.1$ kg m^2 which, when inflated, is approximately a spherical shell of radius $R = 8.75$ m. The mass of the basket, propane fuel, and other solid flying machine elements total 329 kg. Suppose the environmental air at ground level has mass density $\rho_0 = 1.20$ kg m^{-3}, pressure $P_0 = 101$ kPa, and the temperature $T_0 = 20$ deg C. The ideal gas law applies. The air inside the inflated envelope has uniform temperature $T_b = 120$ deg C, the maximum safe operating temperature for nylon. The local acceleration of gravity is $g = 9.80$ m s^{-2}.

   a. Neglecting the contained air, what is the total unloaded mass (envelope plus other elements, no people), m_0, of the balloon?
   b. What is the net upwards-directed force in newtons acting on the unloaded balloon at ground level $(h = 0)$?
   c. How many people (each of mass 70 kg) may be lifted off the ground by the balloon?

9. The index of refraction of optical media can be varied by diffusing in impurities. If the index of refraction is increased then it is possible to make a lens from an optical component having a constant thickness. Given a disk of radius a and thickness d, determine the radial variation of the index of refraction $n(r)$ to order r^2 which will produce a converging lens with focal length F using the assumption that the radius $a \ll F$ and also that the lens is thin.



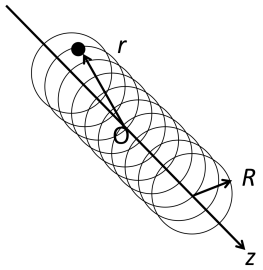
10. Describe briefly how a photomultiplier tube works and make a sketch. Indicate what materials are used and the typical operating condition (i.e., voltage and gain). Are such tubes able to distinguish between events in which one and two nearly identical photons are absorbed simultaneously? You should assume that the photomultiplier gain is a stochastic process with Poisson statistics. At each dynode assume the multiplication approximates a factor of 5.

11. A symmetric rigid body with $I_1 = I_2$ is undergoing force-free rotation about its x_1 principal axis with rotation rate ω_1. Analyze the stability of motion by applying a small perturbation such that

$$\vec{\omega}_1 = \omega_1 \hat{x}_1 + \lambda \hat{x}_2 + \mu \hat{x}_3,$$

where λ and μ are small quantities. Find the solution for λ and μ as functions of time.

12. A bead slides without friction on a wire shaped in a spiral of radius R with a pitch $\theta = \alpha z$, where z is the axial displacement and θ is the azimuthal angle. The bead is subject to a central force $F = -kr$ where r is the distance from an origin somewhere on the z axis to the bead. Solve for the motion.



13. A particle of mass M moves under the action of a force $F(x) = -\frac{F_0}{2}\left(e^{ax} - e^{-ax}\right)$ where $a > 0$. Sketch the potential energy and solve for the frequency of small oscillations around $x = 0$ for motion along the x-axis.

14. A uniform thin cylindrical tube of mass m, radius R and length (or height) L is rolling with angular velocity ω on a horizontal surface. Gravity, g, acts downward. The tube is an insulator and there is a net positive charge of Q distributed uniformly around the rim. There is also a uniform magnetic field of magnitude B which is perpendicular to the horizontal surface. The magnitude of the B-field is increasing smoothly with time. If the tube is long (i.e., so it does not tip) and there is a coefficient of static friction μ at the surface, at what B-field strength will the rolling tube begin to slip? (There are no electrostatic forces.)

15. State whether the following decays can occur by the strong interaction, the weak interaction, the electromagnetic interaction, or not at all, and explain why.

a. $\Sigma^+ \rightarrow p + \pi^0$

b. $\Sigma^0 \rightarrow \Lambda + \gamma$

c. $\Xi^0 \rightarrow n + \pi^0$

Name & Symbol	Charge	Z-comp. of isospin	Baryon number	Other non-zero quantum nos.	Mass/GeV
Down d	-e/3	-1/2	1/3	0	0.35
Up u	2e/3	+1/2	1/3	0	0.35
Strange s	-e/3	0	1/3	Strangeness -1	0.5
Charm c	2e/3	0	1/3	Charm +1	1.3-1.7
Bottom b	-e/3	0	1/3	Bottom +1	4.7-5.3
Top t	2e/3	0	1/3	Top +1	ca. 175

p uud n ddu $\pi^0 = u\bar{u}$ or $d\bar{d}$

Hyperons	Quark Content	Spin	Mass/MeV
Λ	uds	$\frac{1}{2}$	1115.68 ± 0.01
Σ^+	uus	$\frac{1}{2}$	1189.36 ± 0.06
Σ^-	dds	$\frac{1}{2}$	1197.34 ± 0.05
Σ^0	uds	$\frac{1}{2}$	1192.46 ± 0.08
Ξ^-	dss	$\frac{1}{2}$	1321.32 ± 0.13
Ξ^0	uss	$\frac{1}{2}$	1314.9 ± 0.6
Ω^-	sss	$\frac{3}{2}$	1672.45 ± 0.32