

Physics Qualifying Examination – Part I 7-Minute Questions
February 16, 2013

1. Electrons in a vacuum tube screen are accelerated through a potential difference of 200 kV. Calculate their speed in units of c , where c is the speed of light. Assume the electrons start from rest.

2. A certain observable has the following matrix representation:

$$M = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Find the normalized eigenvectors of this observable and the corresponding eigenvalues. Is there any degeneracy?

3. Consider a rod of length L_0 oriented with an angle θ_0 relative to the x -axis in its rest frame. The rod moves with a relativistic velocity v along the x -axis relative to an observer.

- Determine the length, L , of the rod as measured by the observer.
- Determine the angle, θ , the rod makes relative to the x -axis in the frame of the observer.

4. Consider two speakers playing the same steady tone (in phase). Your ear is 3.0 m from one speaker, 2.7 m from the other. Assume the speed of sound to be 330 m/s.

- What frequencies do you hear weakly in this ear?
- What frequencies do you hear strongly in this ear?

5. An $80.0\,\Omega$ resistor, $200.0\,\text{mH}$ inductor, and a $0.150\,\mu\text{F}$ capacitor are connected in parallel across a $120\,\text{V}$ (rms) AC source operating at $60\,\text{Hz}$.

- What is the resonant frequency of this circuit?
- Calculate the rms current in the resistor, inductor, and capacitor.
- What rms current is delivered by the source?
- Does the current lead or lag the voltage? By what angle?

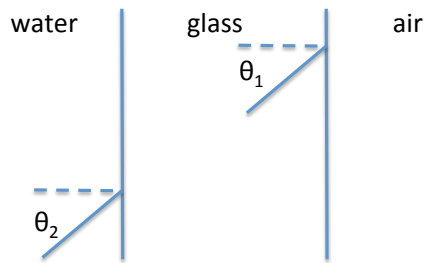
6. Assume a rocket expels its exhaust with a speed v_e relative to the rocket. If this rocket has a total initial mass M , and if it burns an amount of fuel of mass m , what is its final speed? Assume the rocket starts at rest in free space (i.e. no gravity or air friction).

7. Enumerate the eigenstates of J^2 and J_z where \vec{J} is the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ associated with the $(1s)(2p)$ configuration of the He atom. Draw an energy level diagram and label each level by the appropriate quantum numbers for J^2 and J_z . Determine whether electric dipole transitions from each of these levels in the $(1s)^2$ ground state is possible by checking the appropriate selection rules.

8. What is the typical atmospheric pressure in Denver, the “Mile High City”, in the summer?

9. A long solenoid is made of a superconducting wire carrying a current I_0 . The solenoid is slowly stretched so that its cross section does not change but its length changes from L_0 to L_1 . What is the new current in the solenoid assuming the solenoid is still tightly wound? Explain your answer.

10. The index of refraction n is 1 for air, 1.33 for water and 1.52 for glass. Consider the three-layer system shown in the diagram below.



- Calculate the critical angle, θ_1 , for total reflection at the glass-air interface for a ray of light originating in the glass.
- Calculate the critical angle, θ_2 , for total reflection for a ray of light originating in the water.

Physics Qualifying Examination – Part II 12-Minute Questions

February 16, 2013

1. A spin $1/2$ particle is in an eigenstate of the \hat{z} -spin operator, S_z , with the eigenvalue $\hbar/2$ at time $t = 0$. A uniform magnetic field $\vec{B} = B\hat{y}$ is applied so that the Hamiltonian for the particle is

$$H = -\gamma B S_y$$

where S_y is the \hat{y} -spin operator. Find the value(s) of the magnetic field $B > 0$ for which the probability is unity that a measurement of the \hat{z} -component of the spin will yield $-\hbar/2$ at time $t = T > 0$.

2. Consider an infinite plate of charge with surface charge per unit area σ at $z = a$ and another plate, with a hole of radius r at the origin of the xy plane, at $z = 0$ with surface charge per unit area of $-\sigma$. Compute the electric flux through the hole.

3. A particle of mass m moves under the influence of an attractive central force, $\vec{F}(\vec{r}) = -Ar^3\hat{r}$, where r is the distance from the origin of the force, \hat{r} is a unit vector in the direction of \vec{r} , and A is a constant. Initially $r = r_0$ and the particle moves with velocity v_0 in a tangential direction.

- Derive and sketch the effective potential of this system as a function of r . Indicate all important inflection points. Can the particle pass through the origin of this reference frame?
- Find the velocity v_0 required for this particle to move in a purely circular orbit at a radius r_0 .
- Obtain a formula for the frequency of small oscillations about this equilibrium radius. How does the period of these oscillations compare to the orbital period?

4. A solid conducting sphere of radius R rotates about the z -axis at an angular velocity $\vec{\omega} = \omega \hat{z}$. It is placed in a uniform magnetic field $\vec{B} = B \hat{z}$. In the steady state, the free electrons do not move with respect to the positive charge of the ions. The motion is nonrelativistic, so you may neglect the self magnetic field of the sphere. Take the origin of coordinates at the center of the sphere. Find the electric field and charge density in the steady state for all $|\vec{r}| < R$, where $\vec{r} = (x, y, z)$.

5. A small, planar loop of wire with twisted leads (figure a) will be used to measure the structure of the fringing magnetic field of a finite length solenoidal electromagnet. The loop has cross sectional area A . The current in the solenoid is pulsed on then off, as shown in figure b.

- Derive an equation for the voltage induced across the ends of the wire in terms of the rate of change of the magnetic field at the loop's location. Draw a graph of the waveform for this voltage.
- Draw a diagram of the simplest operational amplifier circuit using standard electronics components that can be used with the coil to produce a voltage proportional to the magnetic field intensity. Assume ideal characteristics for the operational amplifier and derive an equation for the circuit's output voltage when attached to the coil.
- Name the primary non-ideal, dc, property of operational amplifiers that will make this circuit difficult to apply in practice. Explain your answer.

Figure a

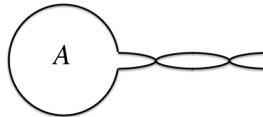
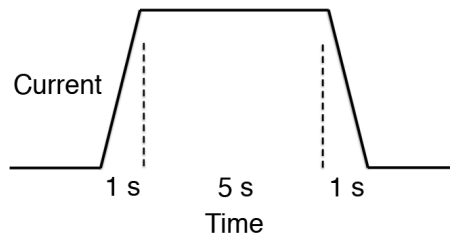


Figure b



6. A pion traveling at speed V , along the positive \hat{x} -direction in the lab frame, decays into a muon and an anti-neutrino $\pi^- \rightarrow \mu^- \bar{\nu}$. If, in the lab frame, the neutrino moves along the positive \hat{z} -direction, at what angle does the muon emerge? Assume the neutrino is massless and express your answer in terms of the pion and muon masses and the pion velocity V .

7. “Circular” hydrogenic atomic states $|\psi\rangle = |nlm\rangle$ with $|m| = l = n - 1$ can spontaneously decay via electric dipole transitions to lower energy states with $n' < n$. The lifetime, τ , for spontaneous decay varies with the transition matrix element d and the transition frequency ω as $\tau \propto 1/(d^2\omega^3)$.

- Given a value of $n > 2$ what are the possible values of n' that the circular state $|nlm\rangle$ can decay to?
- The radiative lifetime of a circular atomic state in a zero temperature environment is $\tau \propto n^p$. Find the exponent p for $n \gg 1$.
- The ratio of stimulated to spontaneous emission rates is equal to the occupation number of photons per mode of the field $n_{ph}(\omega)$. At high temperature T for a mode of frequency ω , $n_{ph} \propto k_B T / \hbar \omega$. The effective lifetime of the circular states is $\tau \propto n^q$. Find the exponent q for $n \gg 1$.

8. The Friedmann equation for a universe with curvature $k = 0$ (i.e. ‘flat’), can be written as:

$$H = \frac{1}{a} \frac{da}{dt} = H_0 (\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda)^{1/2}$$

where H is the Hubble expansion rate, H_0 is its current value, and a is the scale factor. Ω_m , Ω_r , Ω_Λ , are, respectively, the fractions of the critical density of matter, radiation, and the cosmological constant at the current time, t_0 . a is taken to be 1 at the present time, $a(t_0) = 1$.

For the case of a flat universe that contains only pressureless matter, ($\Omega_m = 1$, $\Omega_r = \Omega_\Lambda = 0$),

- Find $a(t)$.
- Determine the age, t_0 in terms of H_0 .

For the case of a flat universe that contains only a cosmological constant, ($\Omega_\Lambda = 1$, $\Omega_r = \Omega_m = 0$),

- Find $a(t)$.

9. A one-dimensional simple harmonic oscillator with spring constant k , mass m , and electric charge e , is in its ground state at $t = 0$. The oscillator is subject to an electric field for time T parallel to the motion of the oscillator, resulting in the following interaction Hamiltonian:

$$H'(t) = eE_0 x \sin \omega_0 t,$$

where x is the oscillator displacement, E_0 is the amplitude of the electric field, and ω_0 is tuned to be the natural frequency of the oscillator, $\omega_0^2 = k/m$.

- a. What is the probability, $P(T)$, in the lowest non-trivial order perturbation theory that the oscillator will be in any one of its excited states?
- b. What is the time behavior of $P(T)$ for $\omega_0 T \ll 1$?

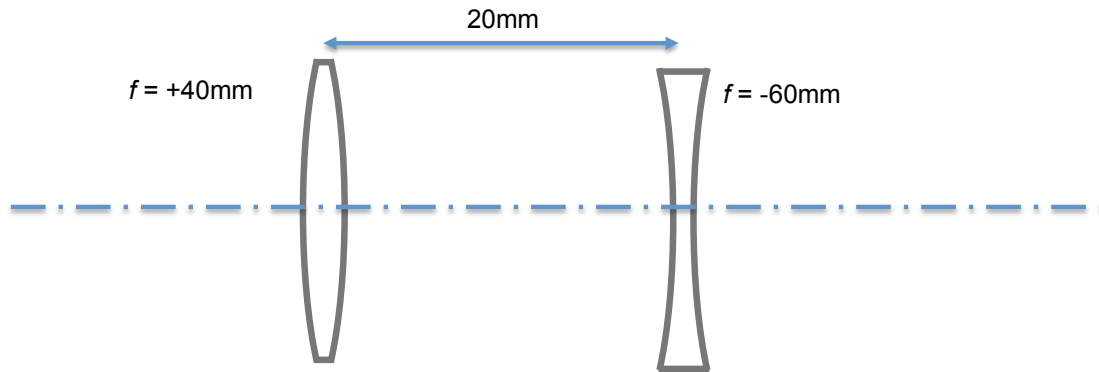
10. Consider a three-dimensional gas of non-interacting fermions with a particular dispersion $\varepsilon_k = (k - k_F)^2$ (k_F is the Fermi momentum). What is the temperature dependence of the specific heat C_V as T goes to zero? How will the result change if $k_F = 0$?

11. Consider an electron bound to a donor in indium antimonide, which has a dielectric constant $\varepsilon/\varepsilon_0 = 18$ and an effective electron mass $m^* = 0.015m_e$.

- a. Calculate the ionization energy.
- b. Calculate the radius of the ground state orbit.
- c. Estimate the minimum donor concentration at which the ground state orbits overlap to produce an impurity band.

12. The Clausius-Clapyron equation, $dP/dT = L/(T \cdot \Delta V)$, relates the slope of the coexistence line between two phases in the pressure vs. temperature plane to the latent heat of the phase transformation, L , and the volume difference of the two phases ΔV . The density of ice is 917 kg/m^3 , the density of water is 1000 kg/m^3 , the latent heat of fusion for water is 334 kJ/kg , the heat of vaporization is 2.27 MJ/kg , and the heat of sublimation is 2.833 MJ/kg . Sketch the phase diagram for H_2O (assuming that there are only 3 phases), and explicitly calculate the slope of the three phase boundaries in the vicinity of the triple point (0°C , 0.006 atm).

13. A simple zoom lens for a camera can be made from one converging lens and one diverging lens with focal lengths as shown below. The imaging plane of the camera is to the right of the diverging lens. Changing the separation of the lenses changes the effective focal length. The effective focal length is defined as the focal length of a single converging lens that produces an image that is the same size as that produced by the zoom lens.



- This zoom lens in the configuration above is pointed at a far-away object. In this configuration, what is the effective focal length of the zoom lens in the thin-lens approximation?
- If there are no constraints on the locations of the lenses except that the diverging lens is to the right of the converging lens, and that the camera imaging plane is to the right of the diverging lens, what are the maximum and minimum effective focal lengths that can be achieved with this zoom lens for a far-away object? Ignore the thickness of the lenses.

14. Wave plates are made from birefringent crystals cut to have orthogonal principal axes normal to the beam direction. For quartz, the refractive indices corresponding to these principal axes are $n_1 = 1.556$ and $n_2 = 1.547$ at a wavelength of $\lambda_0 = 532\text{ nm}$. Suppose you want to convert circularly polarized light of $\lambda_0 = 532\text{ nm}$ to linearly polarized light using a zero order quartz plate. Determine the thickness of the wave plate, and indicate the direction of linear polarization after the light passes through the quartz.

15. A body of mass m is hanging in a constant gravitational field from a massless pulley of radius R . One end of the thread is fixed to the ceiling, while the other end is run through a massive pulley (mass M , radius R , moment of inertia I) and connected to a spring of constant k , which is attached to the wall (as shown below). The thread cannot slip over the pulleys.

- Determine the Lagrangian of this system.
- From the Lagrangian, obtain the equation(s) of motion.
- Find an equilibrium position of the system.
- If the equilibrium is stable, find the period of small oscillations about the equilibrium position.

