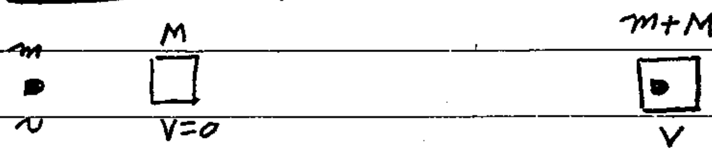
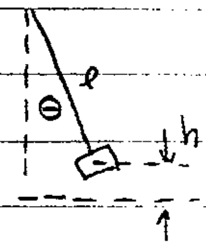
I-1



MOMENTUM IS CONSERVED IN THE COLLISION, BUT ENERGY IS NOT.

$$mv = (m+M)V \rightarrow V = \left(\frac{m}{m+M}\right)v$$

CONSERVE ENERGY DURING OSCILLATION



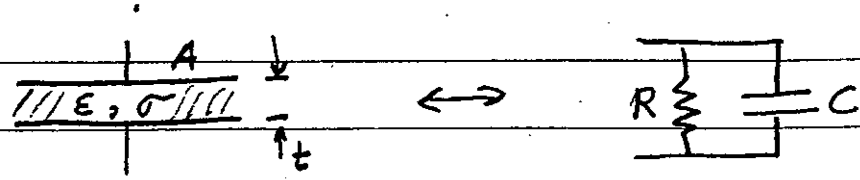
$$\tfrac{1}{2}(m+M)V^2 = (m+M)gh = (m+M)g\ell(1-\cos\theta)$$

$$\tfrac{1}{2}\left(\frac{m}{m+M}\right)^2 v^2 = g\ell(1-\cos\theta) \rightarrow v = \left(\frac{m+M}{m}\right)\sqrt{2g\ell(1-\cos\theta)}$$

$$\omega = \sqrt{\frac{g}{\ell}} = \frac{2\pi}{\tau} \Rightarrow \sqrt{\ell} = \frac{\tau}{2\pi}\sqrt{g}$$

$$\underline{\underline{v = \left(\frac{m+M}{m}\right)\frac{g\tau}{2\pi}\sqrt{2(1-\cos\theta)}}}$$

I-2

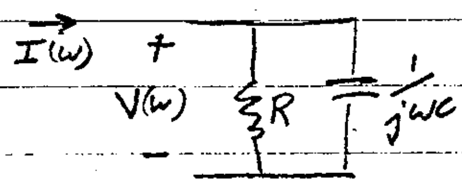


$$C = \frac{\varepsilon A}{t} \qquad R = \frac{1}{\sigma}\frac{t}{A}$$

a) A CHARGE INITIALLY PLACED ON THE CAPACITOR AT time = 0 WILL DECAY EXPONENTIALLY IN THIS RC CIRCUIT $Q(t) = Q_0 e^{-t/\tau}$ $t > 0$

THE CHARACTERISTIC TIME IS $\underline{\underline{\tau = RC = \varepsilon/\sigma}}$

b) FIND COMPLEX AC IMPEDANCE $Z(\omega)$



$$Z(\omega) = \frac{(R)\left(\frac{1}{j\omega C}\right)}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}$$

$$\underline{\underline{Z(\omega) = \frac{R}{1 + j\omega\tau}}}$$

DEVICE BEHAVES AS A RESISTOR AT LOW FREQUENCIES $Z \to R$ WHEN $\omega \ll 1/\tau$

DEVICE BEHAVES AS A CAPACITOR AT HIGH FREQUENCIES $Z \to \frac{1}{j\omega C}$ $\omega \gg 1/\tau$

I-3

$$\hat{\mathcal{H}} = \frac{\hat{p}^2}{2m} + V(\hat{x}) = \frac{\hat{p}^2}{2m} + A\hat{x}$$

IN MOMENTUM SPACE $\hat{p} \to p \quad \hat{x} \to i\hbar \frac{d}{dp}$

$$\hat{\mathcal{H}} \psi_E(p) = E\psi_E(p) \to \left(\frac{p^2}{2m} + iA\hbar \frac{d}{dp}\right)\psi_E(p) = E\psi_E(p)$$

$$\Rightarrow \frac{d\psi_E}{\psi} = -i\frac{1}{A\hbar}\left(E - \frac{p^2}{2m}\right)dp$$

$$\Rightarrow \underline{\underline{\psi_E(p) \propto e^{-\frac{i}{\hbar A}\left(Ep - \frac{1}{6m}p^3\right)}}}$$

$$\text{prob}(p) \propto |\psi_E(p)|^2 = \text{CONSTANT FOR ALL } p$$

[NOTE; THE CLASSICAL MOTION IS UNBOUNDED IN REAL SPACE. THE PARTICLE TURNS AROUND AT $X = E/A$ AND ACCELERATES FOR EVER TOWARD $X = -\infty$]

I-4

BEFORE $\xrightarrow{h\nu}$ AFTER $\overset{e^+}{\rightarrow}$ $\underset{e^-}{\rightarrow}$

EXPRESS THE PROCESS IN TERMS OF 4-VECTOR:

PHOTON $\left(\frac{h\nu}{c}, i\frac{h\nu}{c}\right)$

ELECTRON + POSITRON $\left(P_{e^+}, i\frac{E_{e^+}}{c}\right) + \left(P_{e^-}, i\frac{E_{e^-}}{c}\right)$

IT IS POSSIBLE TO CONSERVE LINEAR MOMENTUM:

$$h\nu/c = 2P_{e^\pm} \text{ — Why?}$$

IT IS POSSIBLE TO CONSERVE ENERGY:

$$h\nu = 2E_{e^\pm}$$

IT IS NOT POSSIBLE TO MAKE THE INVARIENT LENGTH OF THE 4-VECTOR THE SAME BEFORE AND AFTER:

BEFORE $\vec{p}\cdot\vec{p} = 0$

AFTER $\vec{p}\cdot\vec{p} = -2m_0^2c^2$

II-1

$$\vec{F} = m\ddot{\vec{x}} \Rightarrow -k(x_m - x_e) - mg = m\ddot{x}_m$$

$$x_m = Re\left(X_m(\omega)\, e^{i\omega t}\right), \quad x_e = Re\left(X_e(\omega)\, e^{i\omega t}\right)$$

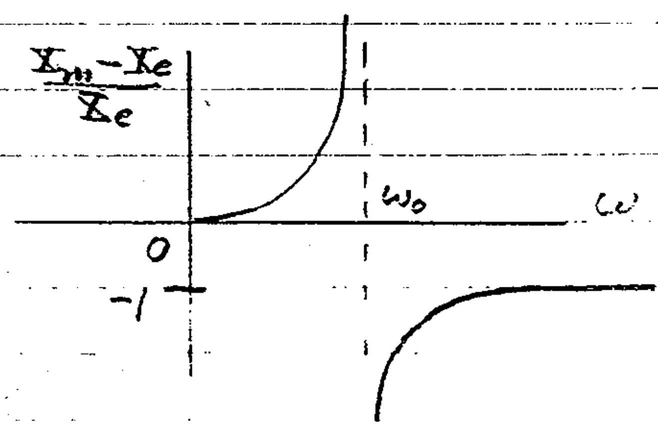
$$-k(X_m - X_e) = -m\omega^2 X_m$$

$\left\{ \begin{array}{l} -mg \text{ ONLY CONTRIBUTES} \\ \text{TO AN } \omega = 0 \text{ OFF-SET} \\ \text{TO } x_m - x_e \end{array} \right.$

$$(k - m\omega^2) X_m = k X_e$$

$$X_m(\omega)/X_e(\omega) = \frac{k}{k - m\omega^2} = \frac{k/m}{k/m - \omega^2} = \frac{\omega_0^2}{\omega_0^2 - \omega^2}$$

$$\frac{X_m(\omega) - X_e(\omega)}{X_e(\omega)} = \frac{\omega_0^2}{\omega_0^2 - \omega^2} - 1 = \underline{\underline{\frac{\omega^2}{\omega_0^2 - \omega^2}}}$$



$X_m - X_e$ FOLLOWS X_e DIRECTLY (NO ω DEPENDENT CONSTANT OF PROPORTIONALITY) WHEN $\underline{\underline{\omega >> \omega_0}}$

II-2

THE MOST GENERAL STATE OF THE ELECTRON SPIN IS REPRESENTED BY $\begin{bmatrix} \alpha \\ \beta + i\gamma \end{bmatrix}$

WHERE α, β & γ ARE REAL AND $\alpha^2 + \beta^2 + \gamma^2 = 1$

WANT $\langle S_z \rangle = \frac{1}{2} \lfloor \alpha, \beta - i\gamma \rfloor \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta + i\gamma \end{bmatrix}$

$$= \frac{1}{2} \lfloor \alpha, \beta - i\gamma \rfloor \begin{bmatrix} \alpha \\ -\beta - i\gamma \end{bmatrix} = \frac{1}{2}(\alpha^2 - \beta^2 - \gamma^2) = \underline{\frac{1}{2}(2\alpha^2 - 1)}$$

HAVE $\langle S_x \rangle = \frac{1}{2} \lfloor \alpha, \beta - i\gamma \rfloor \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta + i\gamma \end{bmatrix}$

$$= \frac{1}{2} \lfloor \alpha, \beta - i\gamma \rfloor \begin{bmatrix} \beta + i\gamma \\ \alpha \end{bmatrix} = \frac{1}{2}(\alpha\beta + i\gamma^2 + \alpha\beta - i\gamma^2) = \underline{\alpha\beta}$$

ALSO HAVE $\langle S_y \rangle = \frac{1}{2} \lfloor \alpha, \beta - i\gamma \rfloor \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta + i\gamma \end{bmatrix}$

$$= \frac{1}{2} \lfloor \alpha, \beta - i\gamma \rfloor \begin{bmatrix} i\beta - \gamma \\ -i\alpha \end{bmatrix} = \frac{1}{2}(i\alpha\beta - \alpha\gamma - i\alpha\beta - \alpha\gamma) = \underline{-\alpha\gamma}$$

SUM SQUARES OF LAST TWO RESULTS

$$\langle S_x \rangle^2 + \langle S_y \rangle^2 = \alpha^2\beta^2 + \alpha^2\gamma^2 = \alpha^2(\beta^2 + \gamma^2) = \alpha^2(1 - \alpha^2) \equiv C$$

$$\alpha^4 - \alpha^2 + C = 0 \Rightarrow \alpha^2 = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 - 4C^2}$$

$$\langle S_z \rangle = \alpha^2 - \frac{1}{2} = \underline{\underline{\pm \frac{1}{2}\sqrt{1 - 4\left(\langle S_x \rangle^2 + \langle S_y \rangle^2\right)}}}$$

CHECK: IF EITHER $\langle S_x \rangle$ OR $\langle S_y \rangle = \pm\frac{1}{2}$, THEN $\langle S_z \rangle = 0$

II-3

$$\vec{E} = \hat{y}\, E_0 \cos(k_x x - \omega t)\, e^{-\alpha z}$$

a) $\nabla^2 \vec{E} = \frac{1}{c^2} \ddot{\vec{E}} \Rightarrow (-k_x^2 + \alpha^2)\vec{E} = -\left(\frac{\omega^2}{c^2}\right)\vec{E}$

$$\underline{\underline{\alpha = \sqrt{k_x^2 - \frac{\omega^2}{c^2}}}}$$

b) $\nabla \times \vec{E} = \alpha E_0 \cos(k_x x - \omega t) e^{-\alpha z}\, \hat{x}$

$\quad - k_x E_0 \sin(k_x x - \omega t) e^{-\alpha z}\, \hat{z}$

$\quad = -\frac{\partial B}{\partial t}$, NOW INTEGRATE TO FIND $\vec{B}$

$$\underline{\underline{\vec{B} = \left(\frac{\alpha}{\omega}\right) E_0 \sin(k_x x - \omega t) e^{-\alpha z}\, \hat{x} + \left(\frac{k_x}{\omega}\right) E_0 \cos(k_x x - \omega t) e^{-\alpha z}\, \hat{z}}}$$

II-4

$$Z_N = (Z_{ONE})^N \qquad \text{FOR SIMILAR, INDEPENDENT SYSTEMS}$$

$$Z_{ONE} = \sum_{STATES} e^{-\epsilon_{STATE}/kT} = 2 + 3e^{-\Delta/kT}$$

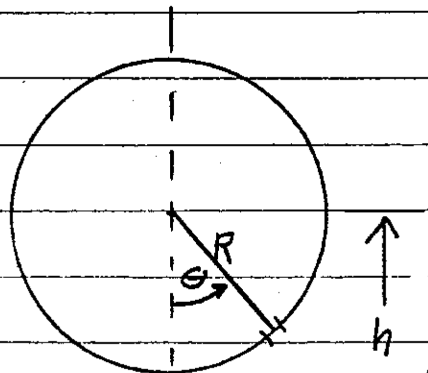
$$Z_N = \left(2 + 3e^{-\Delta/kT}\right)^N$$

$$F = -kT \ln Z_N = -NkT \ln\left(2 + 3e^{-\Delta/kT}\right)$$

$$S = -\left.\frac{\partial F}{\partial T}\right|_V = Nk \ln\left(2 + 3e^{-\Delta/kT}\right) + Nk \frac{3(\Delta/kT)\, e^{-\Delta/kT}}{2 + 3e^{-\Delta/kT}}$$

III-1

ISOLATE THE BALLOON AND SUM THE FORCES DUE TO ATMOSPHERIC PRESSURE ON ITS SURFACE.



$$P(\theta) = P_0 e^{-(h - R\cos\theta)/h_0}$$

$$= P_0 e^{-h/h_0} e^{+\frac{R}{h_0}\cos\theta}$$ SMALL <<1

$$\approx P_0 e^{-h/h_0}\left(1+\frac{R}{h_0}\cos\theta\right)$$

RADIUS OF RING

VERTICAL COMPONENT

$$F_z = \int_0^\pi 2\pi (R\cos\theta)(\cos\theta)\, P(\theta)\, R\,d\theta$$

THIS $\int \to 0$

$$\approx 2\pi P_0 R^2 e^{-h/h_0}\int_0^\pi \sin\theta\cos\theta\left(1+\frac{R}{h_0}\cos\theta\right)d\theta$$

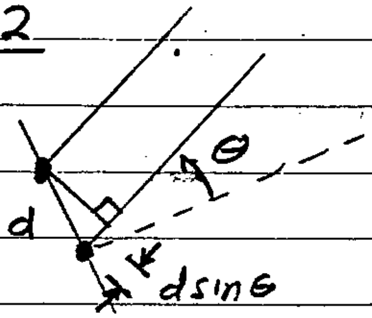
$$= 2\pi \frac{P_0 R^3}{h_0} e^{-h/h_0} \underbrace{\int_0^\pi \cos^2\theta \sin\theta\, d\theta}_{-\frac{1}{3}\left[\cos^3\theta\right]_0^\pi = \frac{2}{3}}$$

$$F_z = \frac{4\pi}{3} R^3 \frac{P_0}{h_0} e^{-h/h_0} = \frac{V P_0}{h_0} e^{-h/h_0}$$ $V \equiv$ VOLUME OF BALLOON

AT EQUILIBRIUM HEIGHT $F_z = Mg = \frac{V P_0}{h_0} e^{-h/h_0}$

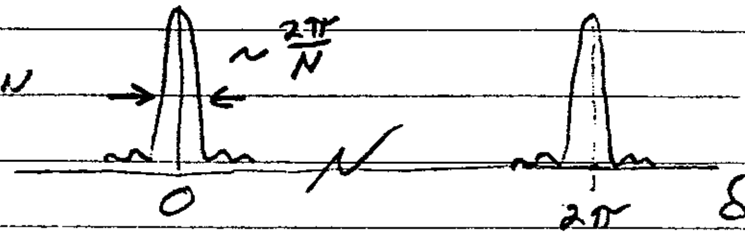
$$-h/h_0 = \ln \frac{M g h_0}{V P_0} \qquad \underline{\underline{h = h_0 \ln \frac{V P_0}{M g h_0}}}$$

III-2



$$\delta_{\text{BETWEEN RADIATORS}} = \phi + 2\pi \frac{d \sin\theta}{\lambda}$$

INTERFERENCE PATTERN FOR N RADIATORS



a) $\frac{2\pi}{N} = \cancelto{0}{\phi} + 2\pi \frac{d}{\lambda} \cancelto{\sim\Delta\theta}{\sin\Delta\theta} \rightarrow 2\pi \frac{d}{\lambda} \Delta\theta$

$$\Delta\theta_{FWHM} \approx \frac{\lambda}{d}\frac{1}{N} = \frac{1}{50} = \underline{\underline{2 \times 10^{-2} \text{ RADIANS} \sim 1.1^\circ}}$$

b) STAY ON INTERFERENCE MAXIMUM ($\delta = 0$) WHILE VARYING ϕ & θ

$$0 = \partial\phi + 2\pi \frac{d}{\lambda} \partial(\sin\theta)\Big|_{\theta \text{ SMALL}} = \partial\phi + 2\pi \frac{d}{\lambda} \partial\theta$$

$$\Rightarrow \frac{d\theta}{d\phi}\Big|_\delta = -\frac{1}{2\pi}\frac{\lambda}{d} = \underline{\underline{-\frac{1}{\pi}}}$$

NOTE: SIGN DEPEN... ON HOW ϕ IS DEFINED RELAT... THE DIRECTION OF $\vec{\ }$

III-3

TOTAL POWER RADIATED BY THE SUN (WHEN $\alpha = 1$)

1) $$4\pi r_s^2 \sigma T_s^4$$ $\sigma \equiv$ STEPHAN-BOLTZMANN CONSTANT

POWER INTERCEPTED BY A SPHERICAL DUST PARTICLE (AND ABSORBED WHEN $\alpha = 1$)

2) $$\left(4\pi r_s^2 \sigma T_s^4\right)\left(\frac{\pi a^2}{4\pi R^2}\right)$$

POWER RADIATED AWAY BY PARTICLE (WHEN $\alpha = 1$)

3) $$4\pi a^2 \sigma T_D^4$$

IN STEADY STATE 2) = 3)

$$\left(\cancel{4\pi} r_s^2 \cancel{\sigma} T_s^4\right)\left(\frac{\cancel{\pi a^2}}{4\cancel{\pi} R^2}\right) = \cancel{4\pi} \cancel{a^2} \cancel{\sigma} T_D^4$$

$$T_D^4 = \left(\frac{r_s}{2R}\right)^2 T_s^4$$

$$T_D = \sqrt{\frac{r_s}{2R}}\, T_s = \left(\frac{1}{20}\right) 6000 = \underline{300\,K}$$

III-4

AT THE SOURCE $\nu_S \equiv \nu_0$

IN THE MIRROR FRAME $\nu_M = \gamma\left(1-\frac{v}{c}\right)\nu_S$
$= \gamma\left(1-\frac{v}{c}\right)\nu_0$

IN THE DETECTOR FRAME

$$\nu_D = \nu_M\,\gamma\left(1-\frac{v}{c}\right) = \nu_0\,\gamma^2\left(1-v/c\right)^2$$

USE $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$

$$= \nu_0\,\frac{(1-v/c)^2}{1-(v/c)^2}$$

$$= \underline{\underline{\nu_0\left(\frac{1-v/c}{1+v/c}\right)}} \rightarrow \nu_0\left(1-\frac{2v}{c}\right) \quad \text{FOR } v \ll c$$

IV -1

K.E. WHEN SHELL EXITS GUN = mal { SIMILAR TO DROPPING A MASS IN A UNIFORM GRAVIT FIELD

ALSO $= \frac{1}{2}m\left(v_x^2(0) + v_z^2(0)\right)$

BUT AT 45° $v_x(0) = v_z$, SO $\underline{v_x^2(0) = v_z^2(0) = al}$

LET t_F BE THE TIME OF FLIGHT, $d = v_x(0)\, t_F$

$$\underline{t_F = \frac{d}{v_x(0)}}$$

IN THE VERTICAL DIRECTION $v_z(t) = v_z(0) - gt$

TOP OF TRAJECTORY REACHED AT $t = \frac{1}{2}t_F$, SO

$$0 = v_z(0) - \frac{1}{2}g\,t_F \Rightarrow \underline{t_F = \frac{2v_z(0)}{g}}$$

EQUATE EXPRESSIONS FOR t_F

$$\frac{d}{v_x(0)} = \frac{2v_z(0)}{g} \Rightarrow d = \frac{2}{g}v_z(0)\,v_x(0) = \frac{2al}{g}$$

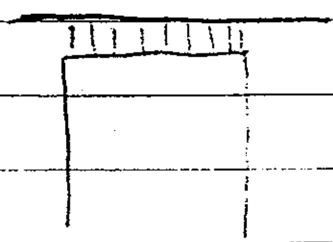
SOLVE TO GET $\underline{\underline{a = \frac{1}{2}\left(\frac{d}{l}\right)g}} = \frac{1}{2}\frac{10^4}{4} = 1.25 \times 10^3\, g$

IV - 2

INSIDE A PERMANENT MAGNET $B >> H$ SO
$\vec{B} \approx 4\pi \vec{M}$.

SINCE THE PLATE IS HIGHLY PERMEABLE, THE MAGNET WILL SEE ITS IMAGE IN THE PLATE WITH A SIMILAR MAGNETIZATION.

IMAGINE PULLING THE MAGNET A SMALL DISTANCE δ FROM THE PLATE. SINCE B IS CONTINUOUS NORMAL TO AN INTERFACE THE $\vec{B}$ IN THE GAP WILL BE THE SAME AS THE $\vec{B}$ IN THE MAGNET



ΔU DUE TO SEPARATION δ

$$= \frac{1}{8\pi} B^2 A \delta$$

$$\text{FORCE} = \frac{\Delta U}{\delta} = \frac{1}{8\pi} B^2 A = 2\pi M^2 A$$

BALANCE AGAINST GRAVITY $\quad 2\pi M^2 A = mg$

$$m = 2\pi M^2 A / g = 2\pi \times 10^6 \times 1 / 0.98 \times 10^3 \sim 6 \times 10^3 \text{ g}$$

$$\underline{\underline{= 6 \text{ Kg}}}$$

IV-3

THE DIPOLE MOMENT OF A STATE IS $\vec{d} = e\langle\psi|\vec{r}|\psi\rangle$. THIS IS CLEARLY ODD AND CHANGES SIGN UNDER $\vec{r} \to -\vec{r}$. THUS A SPONTANEOUS DIPOLE MOMENT FOR THE NEUTRON VIOLATES PARITY SYMMETRY $\mathbb{P}$.

THE LARGEST POSSIBLE SEPARATION OF ELEMENTARY CHARGES e IN A NEUTRON IS THE SIZE $a \approx 10^{-15}$ meters, LEADING TO $d \sim 10^{-34}$ IN SI UNITS.

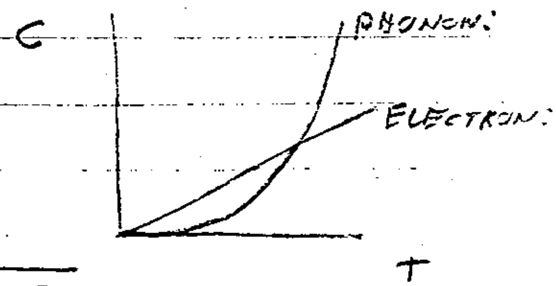
IV-4

RECALL THAT SIMPLE BUT PHYSICALLY SOUND ARGUMENTS SHOW THAT AT LOW TEMPERATURE: THE PHONON CONTRIBUTION TO THE HEAT CAPACITY IS PROPORTIONAL TO T^3 AND THE ELECTRONIC HEAT CAPACITY IS PROPORTIONAL TO T.

THEN ON DIMENSIONAL GROUNDS

$$C_{PHONON} \sim Nk\left(\frac{kT}{\epsilon_{DEBYE}}\right)^3 = Nk\left(\frac{T}{T_{DEBYE}}\right)^3$$

$$C_{ELECTRON} \sim Nk\,\frac{kT}{\epsilon_{FERMI}} = Nk\left(\frac{T}{T_{FERMI}}\right)$$



EQUATE THE TWO EXPRESSIONS TO FIND THE CROSS-OVER TEMP.

$$\left(\frac{T}{T_D}\right)^3 = \frac{T}{T_F} \Rightarrow T = \sqrt{\frac{T_D^{\,3}}{T_F}}$$

$$= \sqrt{\frac{10^6}{10^5}} \sim 3\,K$$

ELECTRONIC HEAT CAPACITY DOMINATES WHEN $\underline{T < 3K}$

V-1

$$p^2c^2 + m_o^2c^4 = E^2 \qquad \text{BUT} \quad p = \gamma m_o v$$

$$\text{SO} \quad \underbrace{\left(\gamma^2 (v/c)^2 + 1\right)}_{\frac{1}{1-(v/c)^2}} m_o^2 c^4 = E^2$$

$$1-(v/c)^2 = \left(\frac{m_oc^2}{E}\right)^2 \rightarrow v/c = \left(1 - \left(\frac{m_oc^2}{E}\right)^2\right)^{1/2}$$

$$\underline{v/c \approx 1 - \frac{1}{2}\left(\frac{m_oc^2}{E}\right)^2} \quad \text{WHEN } E \gg m_oc^2$$

$$t = D/v$$

$$\Delta t = t_1 - t_2 = D\left(\frac{1}{v_1} - \frac{1}{v_2}\right) = \frac{D}{v_1v_2}(v_2 - v_1) \approx \frac{D}{c^2}(v_2 - v_1)$$

$$\rightarrow \frac{D}{c}\left(\frac{v_2}{c} - \frac{v_1}{c}\right) = \frac{D}{2c}(m_oc^2)^2 \underbrace{\left(\frac{1}{E_2^2} - \frac{1}{E_1^2}\right)}_{\frac{E_1^2 - E_2^2}{E_1^2E_2^2}}$$

$$\Rightarrow (m_oc^2) = \left(\frac{2c\Delta t}{D}\right)^{1/2} \frac{E_1E_2}{\sqrt{E_1^2 - E_2^2}}$$

$$= \underbrace{\left(\frac{2\times 3}{1.5\times 10^5 \times \sim 3\times 10^7}\right)^{1/2}}_{\sim 10^{-6}} \underbrace{\frac{10\times 40}{\sqrt{40^2 - 10^2}}}_{\sim 10\ \text{MeV}}$$

$$\underline{\approx 10\ \text{eV}}$$

V-2

THE FIRST TERM IS DUE TO THE STRONG ATTRACTIONS BETWEEN THE NUCLEI SATURATED AT THEIR MINIMUM SEPARATION. a_1 IS LIKE A BINDING ENERGY PER PARTICLE AND HENCE THIS TERM IN THE BINDING ENERGY IS PROPORTIONAL TO THE TOTAL NUMBER OF NUCLEONS.

THE SECOND TERM IS A SURFACE TENSION REPRESENTING THE ABSENCE OF SOME BONDING (DUE TO FEWER NEIGHBORS) FOR PARTICLES ON THE SURFACE. IT IS PROPORTIONAL TO R^2 WHERE THE NUCLEAR RADIUS GROWS AS $A^{1/3}$. DUE TO THEIR SIMILAR ORIGIN IN STRONG INTERACTIONS THE FIRST TWO COEFFICIENTS ARE OF THE SAME ORDER OF MAGNITUDE.

THE THIRD TERM REPRESENTS THE COULOMB REPULSION BETWEEN PROTONS WHICH GROWS AS Q^2/R WHERE THE NET CHARGE Q IS SIMPLY PROPORTIONAL TO Z. THE COULOMB FORCE IS SIGNIFICANTLY WEAKER THAN THE STRONG FORCE, HENCE THE SMALLER VALUE OF THE COEFFICIENT a_3.

V-3

$$\hat{\mathcal{H}} = \frac{\hat{L}_z^2}{2MR^2} \qquad \hat{L}_z = -i\hbar\frac{\partial}{\partial\phi} \quad \Rightarrow \psi_m(\phi) = \frac{1}{\sqrt{2\pi}}e^{im\phi}$$

$$E_m = \frac{m^2\hbar^2}{2MR^2} \qquad m = 0, \pm 1, \pm 2, \text{----}$$

$E_0 = 0$, THE GROUND STATE IS NON-DEGENERATE

$$\Delta E_0 = \langle 0|\mathcal{H}_1|0\rangle + \sum_{m\neq 0}\frac{|\langle 0|\mathcal{H}_1|m\rangle|^2}{E_0 - E_m}$$

$$\mathcal{H}_1 = -\vec{p}\cdot\vec{\mathcal{E}} = -Q\mathcal{E}_0 x = -Q\mathcal{E}_0 R\cos\phi$$

$$\langle 0|\mathcal{H}_1|0\rangle = 0$$

$$\langle 0|\mathcal{H}_1|\pm 1\rangle = -Q\mathcal{E}_0 R\,\frac{1}{2\pi}\underbrace{\int_0^{2\pi}\cos^2\phi\,d\phi}_{\pi} = -\tfrac{1}{2}Q\mathcal{E}_0 R$$

$$\langle 0|\mathcal{H}_1|\,|m|>1\rangle = 0$$

$$\Delta E_0 = -\tfrac{1}{2}(Q\mathcal{E}_0 R)^2 \big/ (\hbar^2/2MR^2)$$

TWO IDENTICAL TERMS IN THE SUM

V-4

GIVEN $\mathcal{F} = (K_1 + K_2 T)(L - L_0)$

$C_L = AT^3$

$$dS = \frac{\partial S}{\partial T}\Big|_L dT + \frac{\partial S}{\partial L}\Big|_T dL$$

FIND $\frac{\partial S}{\partial T}\Big|_L$: $C_L \equiv \frac{đQ}{dT}\Big|_L$ BUT $đQ = T\,dS$

$$\Rightarrow \frac{\partial S}{\partial T}\Big|_L = \frac{C_L}{T} = AT^2$$

FIND $\frac{\partial S}{\partial L}\Big|_T$: $dU = T\,dS + \mathcal{F}\,dL$, $F \equiv U - TS$

$$dF = -S\,dT + \mathcal{F}\,dL$$

$$\Rightarrow \frac{\partial S}{\partial L}\Big|_T = -\frac{\partial \mathcal{F}}{\partial T}\Big|_L$$ A MAXWELL RELATION

$$= -K_2(L - L_0)$$

SO $dS = AT^2\,dT - K_2(L - L_0)\,dL$

$$S(T,L) = \frac{1}{3}AT^3 + f(L)$$

$$\frac{\partial S}{\partial L}\Big|_T = -K_2(L - L_0) = f'(L)$$

$$\Rightarrow f(L) = -\frac{1}{2}(L - L_0)^2 + \text{CONSTANT}$$

$$\underline{\underline{S(T,L) = \frac{1}{3}AT^3 - \frac{1}{2}K_2(L - L_0)^2 + \text{CONSTANT}}}$$