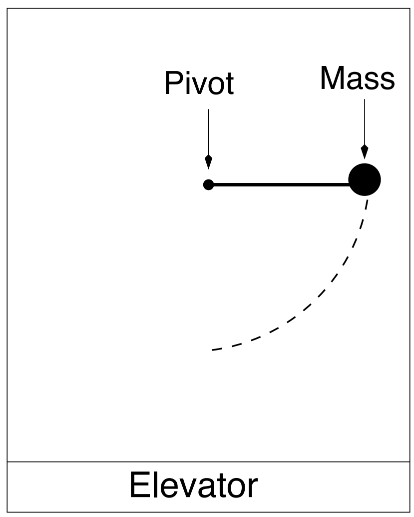
Physics Qualifying Examination – Part I   7-Minute Questions

September 10, 2011

1. A pendulum is comprised of a small mass, m, at the end of a rigid, massless rod of length, L. The horizontal, frictionless pivot is anchored to the wall of an elevator which is accelerating at a rate of a, in the upward direction. The downward acceleration due to gravity is g (g is stated as a positive value). If the pendulum is released from the horizontal position, what is the tension in the rod at the bottom of the pendulum's swing?



2. Ions are accelerated to high energies in solar flares, and create neutrons (rest mass 940 MeV) through collisions with particles in the solar atmosphere. Some of these neutrons escape and are detected at the Earth (1.5 x 10^{11} m from the Sun). Others undergo further nuclear reactions in the solar atmosphere, or simply β decay (mean lifetime 888 s). For a neutron kinetic energy of 1000 MeV, calculate the fraction of escaping neutrons that are expected to survive to reach the Earth.

3. An atomic ion consists of one electron and a nucleus of charge Ze and is in the ground energy level. The maximum λ which causes excitation is 1.005 nm. Find the value of Z.

4. Suppose that we have two lasers. The first has a center wavelength of 410 nm and has a wavelength spread of 1 nm. The second laser is centered at 780 nm with a spread of 0.01 nm. Assume that the power output of the lasers is the same and that both are lasing in a fundamental (TEM_{00}) Gaussian mode.

a. Which of these lasers has a longer coherence length? Estimate the coherence length of that laser.
b. Which of these lasers can focus to a smaller spot size using an ideal lens? Estimate the minimum spot size for a collimated 1 cm diameter beam on a 1 cm diameter F/10 lens.
c. Suppose that both of these lasers propagate through 10 km of air. Assuming that Rayleigh scattering is the dominant scattering mechanism, which laser would be attenuated more and why?

5. A microphone with an output impedance of 10,000 ohms is connected to an amplifier with an input resistance of 1,000 ohms. The voltage gain of the amplifier is 2,000 and its output impedance is 200 ohms. The amplifier is connected to a loudspeaker with a resistance of 8 ohms. The microphone produces an open circuit voltage of $0.2\ \mathrm{V}_{rms}$.

a. What power is generated by the voltage source in the microphone?
b. What *rms* voltage appears across the loudspeaker?
c. What power is dissipated in the loudspeaker?
d. What is the power gain of the system?
e. What is the power gain of the amplifier?

6. The Hamiltonian of a quantum-mechanical harmonic oscillator is given by

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2 .$$

Obtain the value of α for which the wave function

$$\Psi(x) = C\exp(-\alpha^2 x^2),$$

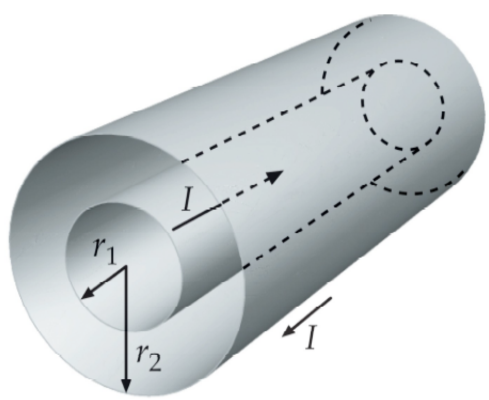
is a solution of the Schrödinger equation. Show the corresponding energy value from this state.

7. Ultrasound is used in medicine for diagnostic imaging. Short pulses of ultrasound are passed through the patient's body, and the echo reflected from a structure of interest is recorded. The distance to the structure can be determined from the time delay for the echo's return.

The speed of sound at these frequencies is about 1,500 m/s.

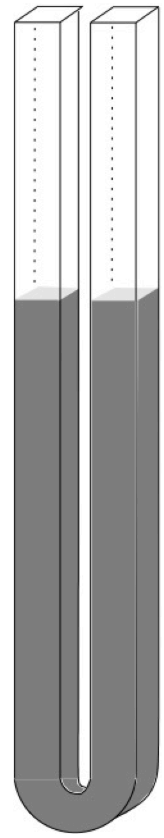
a. What is the wavelength of ultrasound with a frequency of 2.4 MHz?
b. What is the size of the structures that this ultrasound can reveal compared to a wavelength? Bigger, smaller, or about the same as the wavelength?
c. One can also measure the velocity of the blood using ultrasound using Doppler effect. If the ultrasound machine emits at 2.4 MHz, what is the frequency of the ultrasound reflected from blood moving in an artery toward the combined emitter/receiver at a speed of 0.50 m/s?

8. A coaxial cable consists of two very thin-walled conducting cylinders of radius r_1 and r_2, as shown in the figure below. The currents in the inner and outer cylinders are equal in magnitude but opposite in direction.



a. What is the magnetic field as a function of the radius, r, from the central axis of the cable for: $0 < r < r_1$, $r_1 < r < r_2$, and $r > r_2$?
b. Find the self-inductance of this cable arrangement per unit length.

9. One liter of water (density ρ = 1.00 g/cm^3) is poured into a uniform U-shape pipe with a 1.00 cm^2 cross sectional bore as shown in the figure at right (not to scale).



A slight pressure increase on just one side causes the water on that side to displace downward. This displacement is directly proportional to the pressure increase. Upon releasing this pressure you observe that the water exhibits simple harmonic motion.

Assuming that water acts as a perfect Newtonian fluid (incompressible and there is no viscosity or viscous drag), at what angular frequency does it oscillate?

10. This problem investigates the Planck blackbody spectrum in one dimension.

a. Consider a one-dimensional electromagnetic cavity of length L. Calculate $N(v)dv$, the number of modes in the frequency range $(v, v+dv)$.

b. The average energy in a single mode of frequency v is given by $\overline{E}(v) = \dfrac{hv}{e^{hv/kT} - 1}$.

   Use this to write down the one-dimensional blackbody spectrum. This is the energy stored in the frequency interval $(v, v+dv)$.

c. In three dimensions, Stefan found that the total power radiated by a blackbody was proportional to T^4. Derive the T dependence for one dimension.

Physics Qualifying Examination – Part II 12-Minute Questions

September 10, 2011

1. Discuss critically at least two pieces of observational evidence for a finite age of the visible universe.

2. A particle of mass m moves in one dimension according to the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V(x)\psi = E\psi \ .$$

The potential is given by $V(x)=\begin{cases}0, & |x|\le L/2\\ V_0, & |x|>L/2\end{cases}$

and V_0 is positive.

a. Give the ground state wavefunction and ground state energy when $V_0 \to \infty$.

b. How is the ground state wavefunction modified when V_0 is finite but very large: $V_0 \gg \hbar^2/2mL^2$? How far does this wavefunction "leak" into the forbidden region $|x|>L/2$?

3. Point-like masses m_1 and m_2 are connected by a massless spring of spring constant k and equilibrium length $r_0=0$ and fall freely in a uniform gravitational field.

a. Describe in words the motion of the center of mass and the internal motion about the center of mass in as much detail as you can.

b. Give the Lagrangian in the polar coordinates r and θ for the internal motion about the center of mass in terms of the reduced mass m.

c. Give the Euler-Lagrange equations for $\theta(t)$ and for $r(t)$.

d. Show that the equation of motion for $u=1/r^2$, where E is the total energy in the center of mass frame and l is the angular momentum, is:

$$\frac{d^2u}{d\theta^2}\frac{d^2u}{d\theta^2}+4u=\frac{4mE}{l^2}$$

e. Give the general solution to this equation and interpret.

4. A rocket (moving in the $\hat{x}$ direction exhausting "propellant" in the $-\hat{x}$ direction) uses light as a propellant. Assume a perfectly efficient conversion of rocket mass energy into photons. If the initial and final masses of the rocket are M_i and M_f, the final velocity v of the rocket relative to its initial rest frame can be expressed as

$$\frac{M_i}{M_f} = f(v)$$

where $f(v)$ is a function that does not involve the mass. Find $f(v)$.

5. Calculate the temperature T_c of the Bose-Einstein condensation of N spinless ($S = 0$) bosons in a three-dimensional cube of volume $V = L^3$. The energy dependence on momentum p is linear, $E = c|\vec{p}|$. Find the number of particles in the condensate at temperature $T < T_c$. Leave answers in the form of a dimensionless integral.

6. Two infinite parallel black sheets are spaced 1 cm apart. The left is maintained at temperature T_H and the right at temperature T_L.

a. Calculate the net radiated power per unit area transferred from left to right between the plates.

A third infinite black plate is placed halfway between the other two and its temperature is allowed to float.

b. By what factor is the net radiated power transfer between the original plates changed? Show your work.
c. What is the equilibrium temperature of the center plate?

The center plate is removed and then Argon gas at pressure p is introduced in the space between the plates.

d. Make a qualitative sketch of the behavior of the power per unit area transferred between the plates by gas conduction as a function of p, starting at $p = 0$ and extending to a sufficiently high pressure to illustrate the pressure behavior of conduction. Explain your reasoning.

7. For a single electron what is the minimum energy to place this particle in a 3D infinite square well (a cube) 0.1 nm on a side? Express your answer in eV.

a. Compare your answer to the electron binding energy of the hydrogen atom. Which is larger? Using no more than two sentences, give an intuitive explanation as to why this should be the case. Note that the cube has sides of length very close to twice the Bohr radius.
b. What is the minimum energy to place three electrons in the cube described in the original question? For this calculation, assume the electrons are non-interacting identical fermions. Express your answer in eV.

8. It is theoretically shown that there is an upper bound for the total cross section of any high-energy hadronic scattering, known as the Froissart-Martin bound, given by

$$\sigma_{tot}(s) \leq A_0 \ln^2 \frac{s}{s_0},$$

where A_0 is a constant characteristic cross section, s is the c.m. energy squared for the hadronic collision, and s_0 a reference energy scale, taken to be $s_0 = (1\ \mathrm{GeV})^2$.

[Here we have adopted the natural units with $\hbar = c = 1$, in which 1 GeV^{-2} $\approx$0.4 mb; 1 mb = 10^{-27} cm^2]

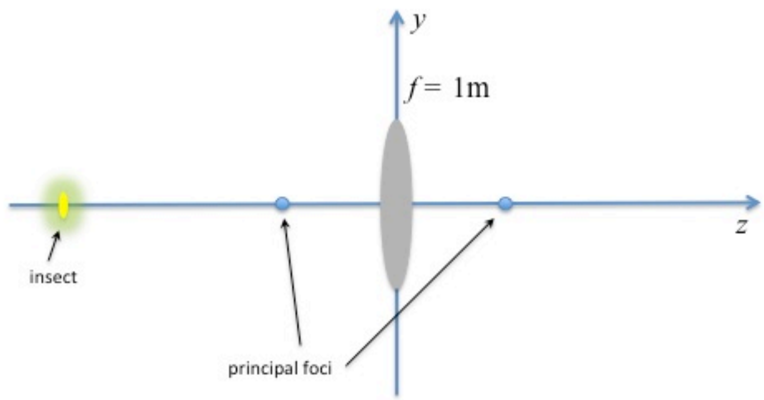
a. It has been measured at the Tevatron that the total hadronic scattering cross section at $\sqrt{s} = 1800$ GeV is 80 mb. Assuming that the Froissart-Martin bound is saturated, predict the total hadronic cross section with $\sqrt{s} = 14000$ GeV.
b. Assume the characteristic cross section A_0 is of a simple geometrical form πR^2, estimate the effective hadronic size R at the Tevatron energy in units of Fermi and compare the associated effective energy scale to the proton mass.

9. A particle of mass m moves in the two-dimensional potential

$$V = \frac{1}{2} k \sin^2(\sqrt{x^2 + y^2 - xy})$$

a. Write the Lagrangian for the system.
b. Write the Lagrangian appropriate for small oscillations about $x = y = 0$.
c. Calculate the normal frequencies.
d. Write the general small oscillation solution and sketch the normal modes.

10. A lens of focal length $f = 1$m is placed at the origin and is used to image a small glowing insect located 3 m from the lens and initially on the axis of the lens (z axis). The insect is flying with a velocity $\vec{v} = (0.1\hat{y} + 0.1\hat{z})m/s$ from its initial location at $x = 0,\ y = 0,$ and $z = -3\ m$.



Find the velocity of the image of the insect.

11. The Hamiltonian for a two-level quantum mechanical system can be written as follows:

$$H = a_0 1 + \sum_{i=1}^{3} a_i \sigma_i$$

in which $a_{0,1,2,3}$ are real numbers and σ_i are the Pauli matrices.

a. How are a_0 and a_i related to Tr H and Tr$(\sigma_i H)$?
b. Obtain Tr $(\sigma_i \sigma_j H)$.
c. Determine the energy eigenvalues of the system.

12. When taking a picture of the sun with a pin-hole camera, what diameter D of the pin hole yields the best resolution? The distance between the pin-hole and film is $l = 10$ cm. Assume visible light of $\lambda \sim 5000$ angstrom.

13. Two concentric metal spheres of radius a and b (with $b > a$) are separated by a weakly conducting medium of conductivity σ.

a. If they are maintained at a potential difference V, what current flows from one shell to the other?
b. Now, assume that two metal spheres of radius a are placed in sea water and separated by a distance $d \gg a$. The spheres are maintained at a voltage difference V and a current I is measured between them. Based on part (a) derive a formula for the conductivity of seawater in terms of $I,\ V,\ a$ and d.

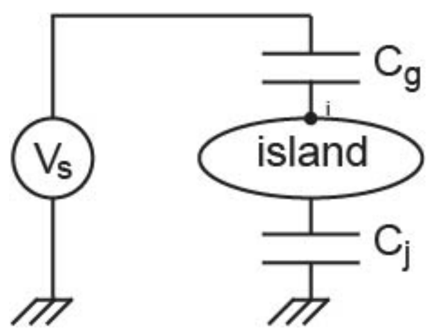
14. A charged particle with mass, m, and charge q is placed in a long solenoid which creates a strong magnetic field, $\mathbf{B}$. The particle's velocity is $\mathbf{v} = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp}$, using coordinates parallel and perpendicular to $\mathbf{B}$.

a. Derive the magnetic moment, $\mathbf{M}$, from the particle's motion perpendicular to the magnetic field.

Suppose the particle is instead in a toroidal solenoid with N uniformly spaced turns carrying current, I.

b. Derive the toroidal magnetic field, $\mathbf{B}$, inside the solenoid and the radial gradient in the field intensity.
c. Will the charged particle be confined in the toroidal solenoid? Justify your answer.

15. Single electron box. Consider the circuit shown below. Capacitor C_j connects a small metallic island (i.e. a small droplet of a perfect metal) to a charge reservoir (the circuit ground), while capacitor C_g connects the island to the voltage source V_s.



a. Calculate the voltage V_i on the metallic island assuming there is no excess charge on the island.
b. Now allow for the possibility that charges can tunnel between the metallic island and ground, so that there are n excess electrons on the island. Calculate the voltage V_i on the metallic island in this general case. Express V_i in terms of V_s, C_g, n, and the electron charge e.