NUMBER SYSTEM

Lesson 1

By Prasad Weerathunga

INTRODUCTION TO NUMBERING SYSTEMS

- We are all familiar with the decimal number system (Base 10). Some other number systems that we will work with are:

  - **Binary → Base 2**
  - **Octal → Base 8**
  - **Hexadecimal → Base 16**

Characteristics of Numbering Systems

1) The digits are consecutive.
2) The number of digits is equal to the size of the base.
3) Zero is always the first digit.
4) The base number is never a digit.
5) When 1 is added to the largest digit, a sum of zero and a carry of one results.
6) Numeric values are determined by the implicit positional values of the digits.

The Decimal Number System

Name

- "decem" (Latin) => ten

Characteristics

- Ten symbols
  - **0 1 2 3 4 5 6 7 8 9**
- Positional
  - **2945 ≠ 2495**
  - **2945 = $(2*10^3) + (9*10^2) + (4*10^1) + (5*10^0)$**

Why?

(Most) people use the decimal number system

The Binary Number System

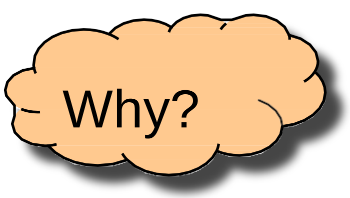
Name

- “binarius” (Latin) => two

Characteristics

- Two symbols
  - **0 1**
- Positional
  - $\mathbf{1010_B \neq 1100_B}$

Most (digital) computers use the binary number system

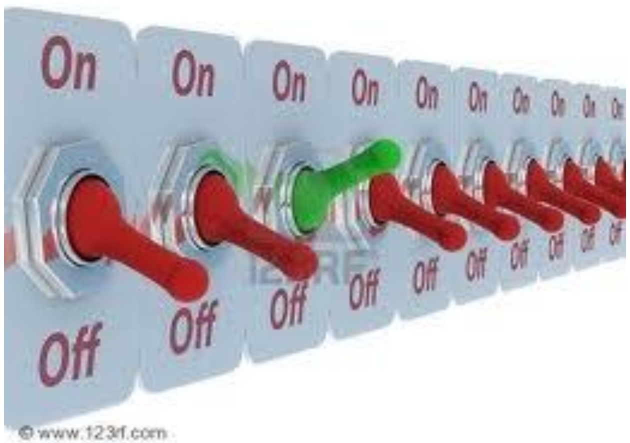


Terminology

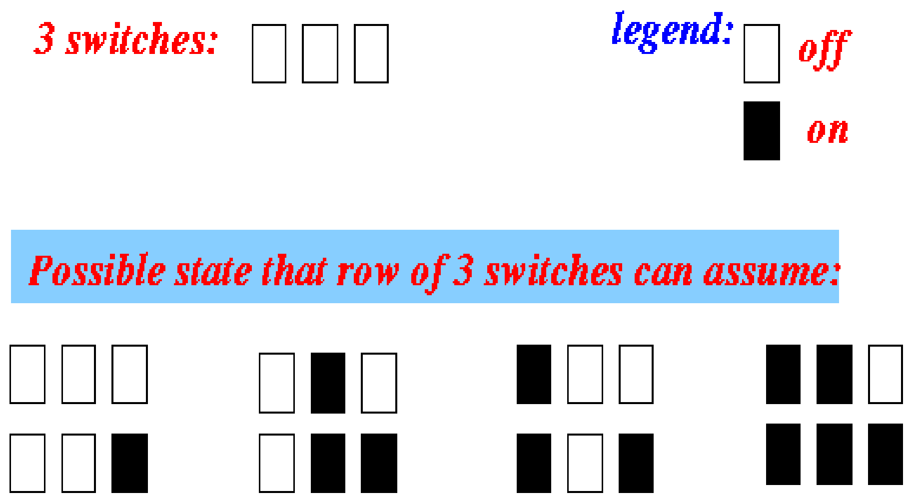
- **Bit**: a binary digit
- **Byte**: (typically) 8 bits

Memory cell used by a computer

- One switch can be in one of 2 states
- A row of n switches:



can be in one of 2^n states !



A row of 3 switches can be in one of $2^3 = 8$ states.
The 8 possible states are given in the figure above.

Memory cell used by a computer

- We saw how information can be represented by *number* by using a code (agreement)
- Recall: we can use numbers to represent marital status information:

- 0 = single
- 1 = married
- 2 = divorced
- 3 = widowed

BINARY NUMBER SYSTEM

- Also called the **"Base 2 system"**
- The binary number system is used to model the series of electrical signals computers use to represent information
- 0 represents the no voltage or an **off state**
- 1 represents the presence of voltage or an

  on state

Binary Numbering Scale

Base 2 Number	Base 10 Equivalent	Power	Positional Value
000	0	2^0	1
001	1	2^1	2
010	2	2^2	4
011	3	2^3	8
100	4	2^4	16
101	5	2^5	32
110	6	2^6	64
111	7	2^7	128

Decimal to Binary Conversion

- The easiest way to convert a decimal number to its binary equivalent is to use the Division Algorithm
- This method repeatedly divides a decimal number by 2 and records the quotient and remainder
  - The remainder digits (a sequence of zeros and ones) form the binary equivalent in least significant to most significant digit sequence

Decimal to Binary Conversion

Convert 67 to its binary equivalent:

$\mathbf{67_{10} = x_2}$

Step 1: 67 / 2 = 33 R 1 — **Divide 67 by 2. Record quotient in next row**

Step 2: 33 / 2 = 16 R 1 — **Again divide by 2; record quotient in next row**

Step 3: 16 / 2 = 8 R 0 — **Repeat again**

Step 4: 8 / 2 = 4 R 0 — **Repeat again**

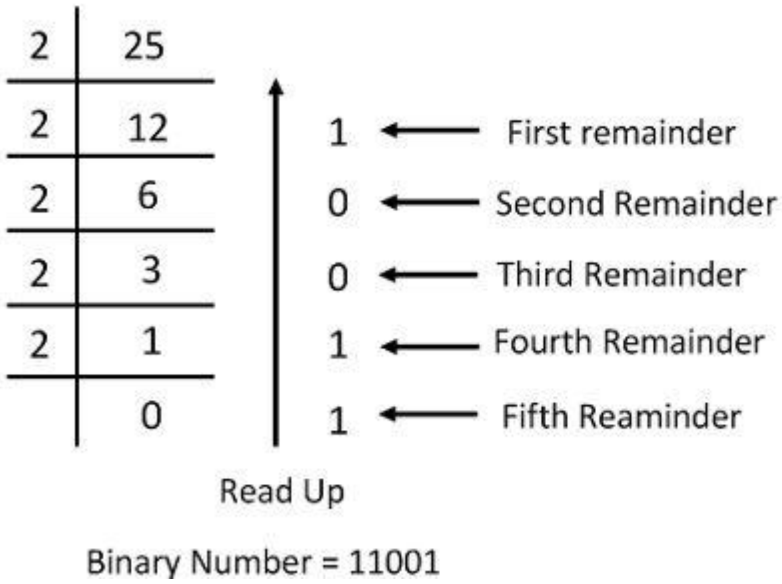
Step 5: 4 / 2 = 2 R 0 — **Repeat again**

Step 6: 2 / 2 = 1 R 0 — **Repeat again**

Step 7: 1 / 2 = 0 R 1 — **STOP when quotient equals 0**

$$\boxed{1\ 0\ 0\ 0\ 0\ 1\ 1_2}$$

Decimal to Binary Conversion



- Example

Find what is binary value of following decimals?

a) 75
b) 133
c) 1573

Binary Number Addition

Addition

```
                1
   3          0011B
+ 10        + 1010B
  --          ----
  13          1101B
```

```
               11
   7          0111B
+ 10        + 1010B
  --          ----
   1         10001B
```

Start at right column

Proceed leftward

Carry 1 when necessary

Results are mod 2^4

Binary Number Subtraction

Subtraction

```
                12
              0202
   10         1010B
-   7       - 0111B
   --         ----
    3         0011B
```

```
              2
    3         0011B
- 10        - 1010B
   --         ----
    9         1001B
```

Start at right column

Proceed leftward

Borrow 2 when necessary

Results are mod 2^4

- Example

Find addition of following binary numbers?

a) $101_2 + 110_2$

b) $10101_2 + 11100_2$

c) $111011_2 + 110110_2$

Find substraction of following binary numbers?

a) $11111_2 - 10011_2 = 1100$

b) $101100_2 - 100110_2 = 110$

c) $111111_2 - 100111_2 = 11000$

Binary to Decimal Conversion

- The easiest method for converting a binary number to its decimal equivalent is to use the ***Multiplication Algorithm***
- Multiply the binary digits by increasing powers of two, starting from the right
- Then, to find the decimal number equivalent, sum those products

Binary to Decimal Conversion

Convert $(10101101)_2$ to its decimal equivalent:

Binary	1 0 1 0 1 1 0 1
	x x x x x x x x
Positional Values	2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0
Products	128 + 32 + 8 + 4 + 1

$$\boxed{173_{10}}$$

Binary to Decimal Conversion

$1100110_2 = 1\times2^6 + 1\times2^5 + 0\times2^4 + 0*2^3 + 1*2^2 + 1\times2^1 + 0*2^0 = 64 + 32 + 4 + 2 = 102.$

- Example

Find what is Decimal value of following binary numbers?

a) 10110 =22

b) 100101 =37

c) 110101 =53

$$(10110)_2 = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 22_{10}$$

$$(100101)_2 = (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 37_{10}$$

$$(110101)_2 = (1 \times 2^5) + (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 53_{10}$$

OCTAL NUMBER SYSTEM

- Also known as the Base 8 System
- Uses digits 0 - 7
- Readily converts to binary
- Groups of three (binary) digits can be used to represent each octal digit
- Also uses multiplication and division algorithms for conversion to and from base 10

Decimal-Octal Equivalence

Decimal	Octal	Decimal	Octal	Decimal	Octal
0	0	16	20	32	40
1	1	17	21	33	41
2	2	18	22	34	42
3	3	19	23	35	43
4	4	20	24	36	44
5	5	21	25	37	45
6	6	22	26	38	46
7	7	23	27	39	47
8	10	24	30	40	50
9	11	25	31	41	51
10	12	26	32	42	52
11	13	27	33	43	53
12	14	28	34	44	54
13	15	29	35	45	55
14	16	30	36	46	56
15	17	31	37	47	57
				…	…

Decimal-Octal Conversion

Octal to decimal: expand using positional notation

$$\begin{aligned} 37_O &= (3*8^1) + (7*8^0) \\ &= \quad 24 \quad + \quad 7 \\ &= \quad 31 \end{aligned}$$

Decimal to octal: use the shortcut

```
31 / 8 = 3 R 7
 3 / 8 = 0 R 3
```

- Example

Find what is Octal value of following decimals?

a) 756_{10}

b) 1351_{10}

c) 352_{10}

Find what is decimal value of following Octals?

a) 351_8

b) 2136_8

c) 157_8

Division by 8	Quotient	Remainder (Digit)
(756)/8	94	4
(94)/8	11	6
(11)/8	1	3
(1)/8	0	1

Division by 8	Quotient	Remainder (Digit)
(1351)/8	168	7
(168)/8	21	0
(21)/8	2	5
(2)/8	0	2

Division by 8	Quotient	Remainder (Digit)
(352)/8	44	0
(44)/8	5	4
(5)/8	0	5

$$351 = (3 \times 8^2) + (5 \times 8^1) + (1 \times 8^0) = 233$$

$$2136 = (2 \times 8^3) + (1 \times 8^2) + (3 \times 8^1) + (6 \times 8^0) = 1118$$

$$157 = (1 \times 8^2) + (5 \times 8^1) + (7 \times 8^0) = 111$$

Octal Number Addition

Addition

```
  2036₈         4210₈
+ 5762₈       + 2671₈
  ----          ----
 10020₈        7101₈
```

- Example

Find addition of following Octal numbers?

a) $5376_8 + 3657_8 = 11255_8$

b) $435_8 + 342_8 = 777_8$

c) $471_8 + 242_8 = 733_8$

Octal Number Subtraction

Subtraction

```
  375₈          471₈
- 126₈        - 536₈
 ----          ----
  652₈          242₈
```

- Example

Find addition of following Octal numbers?

a) 1463_8 - 1102_8 =361

b) 25616_8 - 17520_8 =6076

c) 15677_8 - 7260_8 =6417

HEXADECIMAL NUMBER SYSTEM

- Base 16 system
- Uses digits 0-9 &

letters A,B,C,D,E,F

- Groups of four bits represent each base 16 digit

Decimal	Hexadecimal
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

Decimal to Hexadecimal Conversion

Convert 830_{10} to its hexadecimal equivalent:

$830 / 16 = 51 \text{ R}14$ = E in Hex

$51 / 16 = 3 \text{ R}3$

$3 / 16 = 0 \text{ R}3$

$\boxed{33E_{16}}$

Hexadecimal to Decimal Conversion

Convert $3B4F_{16}$ to its decimal equivalent:

Hex Digits

3 B 4 F

x x x x

Positional Values

16^3 16^2 16^1 16^0

Products

12288 +2816 + 64 +15

$\boxed{15{,}183_{10}}$

- Example

Find what is Hexadecimal value of following decimals?

a) 751_{10}

b) 5067_{10}

c) 936_{10}

Find what is decimal value of following Hexadecimals?

a) $A3F_{16}$

b) $29FC_{16}$

c) $8C6_{16}$

Division by 16	Quotient	Remainder (Digit)
(751)/16	46	15
(46)/16	2	14
(2)/16	0	2

$= (2EF)_{16}$

Division by 16	Quotient	Remainder (Digit)
(5067)/16	316	11
(316)/16	19	12
(19)/16	1	3
(1)/16	0	1

$= (13CB)_{16}$

Division by 16	Quotient	Remainder (Digit)
(936)/16	58	8
(58)/16	3	10
(3)/16	0	3

$= (3A8)_{16}$

$$(A3F)_{16} = (10 \times 16^2) + (3 \times 16^1) + (15 \times 16^0) = (2623)_{10}$$

$$(29FC)_{16} = (2 \times 16^3) + (9 \times 16^2) + (15 \times 16^1) + (12 \times 16^0) = (10748)_{10}$$

$$(8C6)_{16} = (8 \times 16^2) + (12 \times 16^1) + (6 \times 16^0) = (2246)_{10}$$

Hexadecimal Number Addition

Addition

```
  A34₁₆        25D3₁₆
+ 8E9₁₆      + C85F₁₆
----          ----
 131D₁₆        EE32₁₆
```

- Example

Find addition of following Octal numbers?

a) $9D3_{16} + 2EF_{16}$

b) $A301_{16} + 489_{16}$

c) $EAB58_{16} + B287_{16}$

Hexadecimal Number Subtraction

Subtraction

```
  B3CD₁₆        85AD₁₆
+ 849A₁₆      + 5EOF₁₆
 ----          ----
  2F33₁₆        279E₁₆
```

- Example

Find addition of following Octal numbers?

a) $F2CD3_{16}$ - $846C_{16}$

b) 824_{16} - $A35_{16}$

c) $A2CD_{16}$ - $9BFC_{16}$

Binary-Hexadecimal Conversion

Observation: $16^1 = 2^4$

- Every 1 hexadecimal digit corresponds to 4 binary digits

Binary to hexadecimal

```
1010000100111101
   B  A 1   3   DH
```

Digit count in binary number not a multiple of 4 =>
pad with zeros on left

Hexadecimal to binary

```
   A    1   3   DH
1010000100111101
B
```

Discard leading zeros from binary number if appropriate

Binary-Hexadecimal Conversion

- The easiest method for converting binary to hexadecimal is to use a substitution code
- Each hex number converts to 4 binary digits

Substitution Code

0000 = 0	0100 = 4	1000 = 8	1100 = C
0001 = 1	0101 = 5	1001 = 9	1101 = D
0010 = 2	0110 = 6	1010 = A	1110 = E
0011 = 3	0111 = 7	1011 = B	1111 = F

Binary-Octal Conversion

Observation: $8^1 = 2^3$

- Every 1 octal digit corresponds to 3 binary digits

Binary to octal

```
001010000100111101B
 1  2  0  4  7  5O
```

Digit count in binary number not a multiple of 3 =>
pad with zeros on left

Octal to binary

```
 1  2  0  4  7  5O
001010000100111101B
```

Discard leading zeros from binary number if appropriate

THANK YOU