人工智能导论第三次作业

1

$$\sum_{i=1}^n \log p(oldsymbol{x} \mid \mu, oldsymbol{\Sigma}) = -rac{n}{2} \log |2\pi oldsymbol{\Sigma}| - rac{1}{2} \sum_{i=1}^n \left(oldsymbol{x}_i - \mu
ight)^T oldsymbol{\Sigma}^{-1} \left(oldsymbol{x}_i - \mu
ight)$$

(1) \diamondsuit $abla_{\mu}J(\mu,\mathbf{\Sigma})=0$, 即:

$$\frac{\partial \sum_{i=1}^{n} \log p(\boldsymbol{x} \mid \mu, \boldsymbol{\Sigma})}{\partial \mu} = 0$$

$$\frac{\partial \left[-\frac{n}{2} \log |2\pi \boldsymbol{\Sigma}| - \frac{1}{2} \sum_{i=1}^{n} (\boldsymbol{x}_{i} - \mu)^{T} \boldsymbol{\Sigma}^{-1} (\boldsymbol{x}_{i} - \mu) \right]}{\partial \mu} = 0$$

$$\frac{\partial \left[\sum_{i=1}^{n} (\boldsymbol{x}_{i} - \mu)^{T} \boldsymbol{\Sigma}^{-1} (\boldsymbol{x}_{i} - \mu) \right]}{\partial \mu} = 0$$

$$\boldsymbol{\Sigma}^{-1} \sum_{i=1}^{n} (\boldsymbol{x}_{i} - \mu) = 0$$

$$\mu = \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{x}_{i}$$

故 $\hat{\mu}_{\mathrm{MLE}}=rac{1}{n}\sum_{i=1}^{n}m{x}_{i}$ 。另一方面,令 $abla_{\Sigma}J(\mu,m{\Sigma})=0$,即:

$$egin{aligned} rac{\partial \sum_{i=1}^n \log p(oldsymbol{x} \mid \mu, oldsymbol{\Sigma})}{\partial oldsymbol{\Sigma}} &= 0 \ rac{\partial \left[-rac{n}{2} \log |2\pi oldsymbol{\Sigma}| - rac{1}{2} \sum_{i=1}^n \left(oldsymbol{x}_i - \mu
ight)^T oldsymbol{\Sigma}^{-1} \left(oldsymbol{x}_i - \mu
ight)^T \left(oldsymbol{x}_i - \mu
ight)
ight]}{\partial oldsymbol{\Sigma}} &= 0 \ -rac{n}{2} oldsymbol{\Sigma}^{-1} + rac{1}{2} oldsymbol{\Sigma}^{-1} \left(\sum_{i=1}^n \left(oldsymbol{x}_i - \mu
ight)^T \left(oldsymbol{x}_i - \mu
ight)
ight) oldsymbol{\Sigma}^{-1} &= 0 \ n oldsymbol{\Sigma} &= \sum_{i=1}^n \left(oldsymbol{x}_i - \mu
ight) \left(oldsymbol{x}_i - \mu
ight)^T \ oldsymbol{\Sigma} &= rac{1}{n} \sum_{i=1}^n \left(oldsymbol{x}_i - \mu
ight) \left(oldsymbol{x}_i - \mu
ight)^T \end{aligned}$$

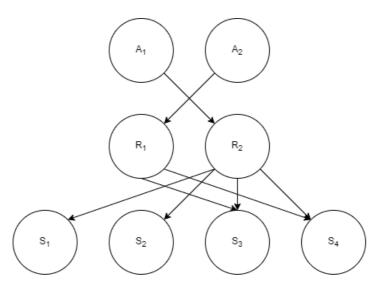
故
$$\hat{\Sigma}_{ ext{MLE}} = rac{1}{n} \sum_{i=1}^{n} \left(m{x}_i - \hat{\mu}_{ ext{MLE}}
ight) \left(m{x}_i - \hat{\mu}_{ ext{MLE}}
ight)^T$$
。

(2)证明:

$$\begin{split} \mathbf{E} \left[\mathbf{\hat{\Sigma}_{MLE}} \right] &= \mathbf{E} \left[\frac{1}{n} \sum_{i=1}^{n} \left(\mathbf{x}_{i} - \hat{\mu}_{\mathrm{MLE}} \right) \left(\mathbf{x}_{i} - \hat{\mu}_{\mathrm{MLE}} \right)^{T} \right] \\ &= \mathbf{E} \left[\frac{1}{n} \sum_{i=1}^{n} \left(x_{i}^{2} - 2x_{i} \hat{\mu}_{\mathrm{MLE}} + \hat{\mu}_{\mathrm{MLE}}^{2} \right) \right] \\ &= \mathbf{E} \left[\frac{1}{n} \sum_{i=1}^{n} \left(x_{i}^{2} \right) - \hat{\mu}_{\mathrm{MLE}}^{2} \right] \\ &= \mathbf{E} \left[\frac{1}{n} \sum_{i=1}^{n} \left(x_{i}^{2} \right) - \mu^{2} + \mu^{2} - \hat{\mu}_{\mathrm{MLE}}^{2} \right] \\ &= \mathbf{E} \left[\frac{1}{n} \sum_{i=1}^{n} \left(x_{i}^{2} \right) - \mu^{2} \right] - \mathbf{E} \left(\hat{\mu}_{\mathrm{MLE}}^{2} \right) - \mu^{2} \right] \\ &= \mathbf{\Sigma} - \left[\mathbf{E} \left(\hat{\mu}_{\mathrm{MLE}}^{2} \right) - \mathbf{E}^{2} \left(\hat{\mu}_{\mathrm{MLE}} \right) \right] \\ &= \mathbf{\Sigma} - \mathbf{Var} \left(\hat{\mu}_{\mathrm{MLE}} \right) \\ &= \mathbf{\Sigma} - \mathbf{Var} \left(\hat{\mu}_{\mathrm{MLE}} \right) \\ &= \mathbf{\Sigma} - \mathbf{Var} \left(\frac{1}{n} \sum_{i=1}^{n} x_{i} \right) \\ &= \mathbf{\Sigma} - \frac{1}{n^{2}} \sum_{i=1}^{n} \mathbf{Var} \left(x_{i} \right) \\ &= \mathbf{\Sigma} - \frac{1}{n} \mathbf{\Sigma} \\ &= \frac{n-1}{n} \mathbf{\Sigma} \end{split}$$

2

(1)



 $p(S_1, S_2, S_3, S_4, R_1, R_2, A_1, A_2) = p(S_1|R_2)p(S_2|R_2)p(S_3|R_1, R_2)p(S_4|R_1, R_2)p(R_1|A_2)p(R_2|A_1)p(A_1)p(A_2)$

(2) R_1, R_2

(3) 需要 2+2+4+4+2+2+1+1=18 个参数。

取消独立性假设: $2^8 - 1 = 255$ 个参数。

(4) 若有吸烟经历,则 R_1 的概率发生变化;若已知 $S_3=1$,则 R_1,R_2 的概率都发生变化。

(5)

$$\begin{split} &p(R_2)\\ &= \sum_{A_1,A_2,S_1,S_2,S_3,S_4,R_1} p(A_1,A_2,S_1,S_2,S_3,S_4,R_1,R_2)\\ &= \sum_{A_1,A_2,S_1,S_2,S_3,S_4,R_1} p(S_1|R_2)p(S_2|R_2)p(S_3|R_1,R_2)p(S_4|R_1,R_2)p(R_1|A_2)p(R_2|A_1)p(A_1)p(A_2)\\ &= \sum_{A_1,S_1,S_2,S_3,S_4,R_1} p(S_1|R_2)p(S_2|R_2)p(S_3|R_1,R_2)p(S_4|R_1,R_2)p(R_2|A_1)p(A_1) \sum_{A_2} p(R_1|A_2)p(A_2)\\ &= \sum_{A_1,S_1,S_2,S_3,S_4,R_1} p(S_1|R_2)p(S_2|R_2)p(S_3|R_1,R_2)p(S_4|R_1,R_2)p(R_2|A_1)p(A_1)\tau(R_1)\\ &= \sum_{S_1,S_2,S_3,S_4,R_1} p(S_1|R_2)p(S_2|R_2)p(S_3|R_1,R_2)p(S_4|R_1,R_2)\tau(R_1) \sum_{A_1} p(R_2|A_1)p(A_1)\\ &= \sum_{S_1,S_2,S_3,S_4,R_1} p(S_1|R_2)p(S_2|R_2)p(S_3|R_1,R_2)p(S_4|R_1,R_2)\tau(R_1) \sum_{A_1} p(R_2|A_1)p(A_1)\\ &= \sum_{S_1,S_2,S_3,S_4,R_1} p(S_1|R_2)p(S_2|R_2)p(S_3|R_1,R_2)p(S_4|R_1,R_2)\tau(R_1)\tau(R_2) \end{split}$$

$$egin{aligned} &p(R_2|A_1,A_2,S_1,S_2,S_3,S_4)\ &=rac{p(A_1,A_2,S_1,S_2,S_3,S_4,R_2)}{p(A_1,A_2,S_1,S_2,S_3,S_4)}\ &=rac{p(A_1)p(A_2)p(S_1|R_2)p(S_2|R_2)p(S_3|R_1,R_2)p(S_4|R_1,R_2)}{p(A_1)p(A_2)p(S_1|R_2)p(S_2|R_2)p(S_3|R_1,R_2)p(S_4|R_1,R_2)} \end{aligned}$$