Chapter Four

Portfolio Optimization: Applications in Quantum Computing
投资组合优化：在量子计算中的应用

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4.1 Introduction
简介

The traditional Markowitz mean-variance model of portfolio selection has provided the framework for portfolio asset selection for decades (Markowitz, 1952, Fabozzi et al., 2013). In this framework, an investor or a portfolio manager wishes to minimize the so-called *risk* with a particular asset mix. The measurement of risk in a portfolio, for this model, is rooted in the association of risk with the measure of variance, which is a measure of the degree of change or variability of the data compared with its expected value—its mean. With multiple assets, the expression of variance of the portfolio includes the covariance—how much the assets vary together. The mathematical expression is quadratic and the optimal selection of assets becomes a quadratic optimization problem. This is formulated as a nonlinear programming problem, which can be solved by suitable application of various operations research techniques (Hillier and Lieberman, 2010).

传统的Markowitz均值-方差模型下的投资组合选择提供了一个框架

，在此框架下可以找到可以持续几十年 的投资组合（Markowitz, 1952, Fabozzi et al.，2013）。凭借这一框架，投资者或投资经理希望用一种特别的投资组合把所谓的风险最小化。对于这个模型而言，在一个投资组合中的风险的度量，，是植根于关联风险的方差的度量，它将变化或变异的数据与其预期值的均值相比。多重资产下，投资组合的方差的表述包含着描述多资产共同变动的协方差的概念。数学表达式为二次的，资产的最优选择也成为二次函数下的最优化问题。它被转化为一个非线性规划问题，并可以使用各种合适的操作技术来解决（Hillier and Lieberman，2010）。

The efficient frontier is a curve, an area, or a surface that traces out the risk versus return values resulting from various combinations of assets 有效边界是一个曲线、一个区域或者一个表面，它指出了不同投资组合下的风险与收益的相互作用。

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(Elton et al., 2007). As the number of asset combinations increases—in both the number of assets and their weightings in the portfolio—the selection of the optimal portfolio quickly becomes difficult. The optimization techniques traditionally used to solve this problem are so-called *classical* optimization techniques that rely on mathematically well-defined gradient-based or descent/directional indications that must be well defined and constrained (Gilli and Schumann, 2012).

(Elton et al.， 2007)。 随着资产组合资产数量增加——在数量和权重两个方面的投资——选择最优投资组合很快变得更加困难。传统上来解决这个问题的优化技术是所谓的依靠数学上被良好定义和约束的可微分的函数或者梯度下降/有向标志（Gilli and Schumann， 2012）。

More modern heuristic optimization techniques, such as stochastic local search, simulated annealing, threshold accepting, tabu search, genetic algorithms, particle swarm, and ant colony optimization provide a fast alternative to these classical optimization techniques (Gilli et al., 2011). These heuristic techniques provide ways to approach problems that are too hard to solve classically. Many classical and heuristic techniques will typically yield solutions that involve some combination of all of the assets under consideration. How can we make an appropriate selection of a subset of assets? In other words, in what way can we make a decision to select *m* assets out of a universe of *n*?

更多的现代探索性（启发式的）优化技术，如随机局部搜索（stochastic local search），模拟退火（ simulated annealing），阈值接收算法（threshold accepting），禁忌搜索算法（tabu search）、遗传算法（genetic algorithms）、粒子群算法（particle swarm）、蚁群优化（ant colony optimization）提供了一种快速替代这些经典的优化技术（Gilli et al.，2011）。这些新技术解决了以往难以解决的经典复杂问题。许多经典和启发式技术通常会产生这样的解决方案，它包含着将所有的资产考虑在内的不同投资组合。我们如何找到一个适合投资组合？换句话说，我们怎样来从n种投资组合中找出一个合适的？

With suitable crafting, this can be approached as a quadratic unconstrained binary optimization (QUBO) problem. This crafting is core to, and discussed further in, this chapter. The problem is quadratic because of mixed terms in the signature equation: $V = X^T QX$. The final model is unconstrained because it is built without additional restricting equations. It is binary in nature because it is written as a binary decision problem (yes/no; include/exclude) of whether to include a particular stock in the final portfolio. The problem is one of optimization because the goal is to

seek the best—optimal—combination of stocks.
辅以合适的方法，这可以逼近为二次无约束的二进制优化（quadratic uncon strained binary optimization (QUBO)）问题。这在本章中是核心并且将会被深入讨论。问题是二次的因为签名方程（signature equation）的混合条件：V = XT QX。最后的模型是不受约束的，因为它的建立没有额外的限制方程。它二进制的性质是因为它被写作为一个二进制的决策问题（是/不）（0/1）（包括/排除）某一特定股票在最终投资组合中的。问题是一个优化，因为目标是寻找最佳的股票投资组合。

Specifically, this is a *combinatorial* optimization problem because the formulation of the objective will seek a discrete set of assets represented in graph form: this problem can be investigated using a graph-theoretic approach (Boros et al., 2008; Jallo and Budai, 2010, Papadimitriou and Steiglitz, 1998).
具体而言，这是一个组合优化问题，因为制定的目标将用图表形式演示搜寻到的一组离散的资产投资分配集：这个问题可以用图论的方法解决（Boros et al.，2008；Jallo and Budai, 2010, Papadimitriou and Steiglitz, 1998).

Iterating through all of the combinations of assets and weightings can be computationally intensive. Heuristics provide some improvement over classical techniques but could either run too long, yield suboptimal results, or not return a solution at all. One paradigm of solving this type of problem is quantum computing (Choi, 2010; D-Wave, 2013b, 2013c). This paradigm holds promise toward many challenging computational problems that are either difficult to solve or not possible with current techniques.
遍历所有资产、权重组合可以说是计算密集型的。探索性的新方法对传统的古典方法有所改善，但是也不能运行太长的未达最佳标准的结果或是根本不可能得到一个解决方案。解决这类问题的一个范例是量子计算（Choi, 2010; D-Wave, 2013b, 2013c）。 这一范式使人们看到应对许多具有挑战性的计算性问题或目前的技术不可能解决问题的希望。

The problem of making a binary (include/exclude) selection of assets for a portfolio can be solved in a rudimentary way using the maximum independent set (MIS) graph-theoretic approach to solving the QUBO in a quantum computing paradigm (D-Wave, 2013b). Can this technique be applied or extended to solve a weighted binary asset selection problem in a Markowitz mean-variance framework?
做一个二进制（包括/排除）资产组合选择的问题可以以一种基本的方式，使用最大独立集（maximum independent set (MIS)）图论的方法来解决量子计算模式下二次无约束的二进制优化（QUBO）问题（D-Wave, 2013b）。那么，这一技术是否能应用或扩展到解决Markowitz的均值-方差框架下的加权资产选择问题？

Overall, this investigation will help bridge the quantum computing paradigm with financial engineering research topics. It will provide insight into the problem domain of the quantum computer linked with associated financial engineering concepts. The fundamental goal of this chapter is to demonstrate how the framework can be used to solve financial problems. It will discuss the limitations of the environment with respect to financial modeling considerations. This will be accomplished by presenting the formulation of financial portfolio optimization in the context of this hardware paradigm. This chapter presents and discusses existing sample work (D-Wave, 2013b).
总体而言，本研究报告将有助于克服伴随金融工程研究课题的量子计算模式所遇到的障碍。并且，它将提供与金融工程概念相联系的量子计算问题领域的深入见解。本章的基本目标是演示这个框架如何在解决金融问题中应用。并将讨论有关金融建模考虑的社会环境限制。这将通过提出硬件范式情况下的金融投资组合优化来完成。本章提出并讨论了现有的样本工作（D-Wave，2013B）。

The research question covers three areas that are reflected in the literature: classical mean-variance portfolio theory; general operations research theory with specific consideration given to combinatorial optimization topics; and the hardware realization of adiabatic quantum computation.
研究的问题包括三个方面，反映在以下相关文献资料中：经典均值-方差组合理论；给以特定考虑的组合优化问题下的一般操作研究理论绝热（等焓adiabatic ）量子计算的硬件实现。

The remainder of the chapter delves into a survey of background literature, the models used, experimental methodology, results obtained, discussion, and conclusion. The background literature section presents a variety of research and applied papers on the underlying topics. The model section relates the portfolio optimization representation to the graph-theoretic domain and into the underlying Ising problem domain of the target hardware. The methodology section presents the information relevant to the implementation undertaken in this study. The results section presents what came out of the investigation. The discussion section relates the results to the underlying domain, along with limitations and future areas for investigation. The conclusion finalizes the chapter.

本章的其余部分将深入探讨的背景文献、模型、实验方法、结果、相关讨论以及结论。背景文献部分，介绍了众多相关研究和应用论文的基本主题。模型部分，把涉及的投资组合优化模型表示与图论领域问题相联系，并且深入到目标硬件的基础伊辛（Ising）模型。方法部分，介绍了有关这项研究的具体实现的相关信息。结果部分，阐述了从研究调查中得到的内容。讨论部分，涉及基本结果，以及对调查的局限性和未来发展趋势等的思辨。结论部分，总结整个第四章节。

4.2 Background 背景

Considering the hybrid nature of topics in this chapter, the available literature is deep and varied, crossing several disciplines. The following is a survey of material that weaves a path through the background subjects of classical mean-variance portfolio theory, general operations research

考虑到本章主题的性质比较杂糅，牵扯的知识覆盖面广，涵盖多个学科。以下是一些相关材料，给出了一个学习路径，从经典的均值-方差投资组合理论，到一般操作的研究

theory, combinatorial optimization topics, and the hardware realization of adiabatic quantum computation.
理论，组合优化问题，再到绝热量子计算的硬件实现。

4.2.1 PORTFOLIOS AND OPTIMIZATION 投资组合优化

As related at the beginning of the “Introduction” section, the Markowitz paper on mean-variance portfolio theory historically laid the foundation for variance–covariance portfolio modeling over the last several decades (Markowitz, 1952, Fabozzi et al., 2013). Meanwhile, portfolio optimization has evolved from that work with different objective representations or constraints. For example, more recent models consider a conditional value-at-risk (VaR) objective with constraints (Krokhmal et al., 2001).
有关“介绍部分”开始的相关问题部分，均值方差投资组合理论历史性地为几十年来的方差-协方差投资组合模型奠定了基础（Markowitz, 1952, Fabozzi et al., 2013）。同时，投资组合优化理论已经从有着不同的目标或约束的工作中发展演进到现在。例如，近年来更多的模型考虑条件性风险价值（value- at-risk，VaR）以及带约束的目标优化（Krokhmal et al., 2001）。

There is a lot of literature in the broad areas of portfolio theory, portfolio optimization, and the general topic of optimization theory and techniques. In particular, in the optimization arena, a recent review illustrates a lot of research activity over the 1998–2008 timeframe (Floudas and Gournaris, 2009). Similarly, finance-oriented discussions are presented in the study by Gilli et al., 2011 with specific consideration of heuristic models in finance with Gilli and Schumann, 2012.
在投资组合理论、投资组合优化理论和最优化理论与技术的广泛领域，有大量的文献资料。特别是，在投资组合优化领域，最近的一项调查说明了从1998到2008年间有很多的相关研究（Floudas and Gournaris, 2009）。同样，以财务（金融）为主导(finance-oriented)的讨论被Gilli等人于2011提出，其中特别考虑启发探索式模型在金融中的运用。

Other (non–mean-variance) methods include heuristic optimization with differential evolution under consideration of risk preferences and loss aversion (Maringer, 2006). Risk parity, equal weighting, and minimum variance are discussed in the study by Chaves et al., 2010.
其他（非均值-方差non－mean-variance）的方法包括风险偏好和厌恶损失（risk preferences and loss aversion）考虑下利用（

Differential Evolution）差分进化算法得到的探索性I启发式）优化（Maringer，2006）。风险平价（Risk parity)，相等的权重（equal weighting），和最小方差（minimum variance）在查韦斯等人2010年的研究中被讨论。

Some very particular formulations of the mean-variance problem are solved with modern and classical optimization techniques: using Lagrangian relaxation (Shaw et al., 2008); a hybrid Grey Relational Analysis approach (Huang et al., 2011); and a genetic algorithm formulation (Soleimani et al., 2009), as just a few examples.

一些非常特别的均值-方差规划问题被使用现代的经典的最优化方法解决。使用拉格朗日松弛法（Lagrangian relaxation）（Shaw et al.， 2008）；一种混合的灰色关联分析方法研究（Huang et al.，2011）；与遗传算法的制定（“et al.，2009），这些仅仅作为几个例子。

Underlying all of these approaches is the desire to find improved methods to solve the optimization problem. Many of the problems are difficult to solve because their model exhibits several local optima or have discontinuities in the function expression (Gilli and Schumann, 2012). Reflecting back on the base mean-variance formulation of portfolio optimization is a problem that is quadratic in nature with squared terms of variance and mixed covariance terms that can exhibit some of these characteristics.

所有这些方法都是为了找到更好的方法来解决优化问题。许多的问题是难以解决的问题，因为他们的模型具有多个局部最优或函数表达式中存在间断点（Gilli and Schumann, 2012）。回顾基于的投资组合优化的基础上的均值方差模型是一个问题，它本质上是二次性质的方差和能表达一些这类性质的混合协方差。

Finding the best overall solution out of these multiple local optima is the goal of global optimization. A sampling of current research papers specific to the global optimization of this quadratic formulation includes

从这些局部最优解中找到最佳的整体解决方案是全局优化的目标。对最近针对二次函数规划的论文进行抽取，包括：

equilibrium search techniques (Pardalos et al., 2008), barrier function formulations (Dang and Xu, 2000), an interior-point algorithm approach (Akrotirianakis and Rustem, 2005), and an unconstrained max-flow approach (Boros et al., 2008).
平衡的搜索技术（equilibrium search techniques ）（Pardalos et al. ， 2008 ）， 屏障性功能规划（ barrier function formulations ） （ Dang and Xu, 2000 ）， 内点算法（ an interior-point algorithm approach) (Akrotirianakis and Rustem, 2005），和一个无约束的最大流方法(an unconstrained max-flow approach)（Boros et al., 2008）。

Some of the research considers constraints in the formulation whereas some are written in unconstrained representation. Constrained problems may sometimes be rewritten as unconstrained formulations using various techniques (Hillier and Lieberman, 2010; Gilli et al., 2011). The unconstrained representations are used in this investigation.
一些研究在有约束的条件下考虑规划问题而另外一些把问题用无约束的方式表示约束问题有时会被使用不同的方法改写为无约束的规划问题（eberman, 2010; Gilli et al., 2011). 在这个研究中许多无约束的表达被使用。

A link between MISs, cliques, and stock market data is discussed in the study by Boginski et al. (2005) where the cross-correlations between stocks are studied over time. The thesis by Jallo and Budai (2010) further elaborates on market graphs related to pure stock returns, liquidity-weighted returns, and volume measures.
管理信息系统，派系以及股票市场数据之间的联系在Boginski等人的研究（2005）中被讨论，同时在这一方面，股票之间的交叉相关性研究随着时间的推移广泛深入。Jallo和Budai (2010)的论文进一步阐述了市场分析图（market graphs），关于纯股票收益（pure stock returns)，流动性加权回报率(liquidity-weighted returns)以及物量计算(volume measures)相关问题。

The article by Charpin and Lacaze (2007) presents a binary optimization on portfolios that provides a method to constrain a portfolio to a specific size, that is, cardinality, and also that determines an optimal portfolio satisfying minimum weight conditions. They solved the problem with branch-and-bound optimization in the Lingo or CPLEX commercial application environments. The binary nature of the problem is similar to what is considered in this chapter, but the model has a different form and this chapter’s model formulation is quickly translated to graph-theoretic representation. The paper by Bertsimas et al. (1999) uses mixed-integer programming to perform a constrained optimization using classical models that have binary selection characteristics.

本文由Charpin和Lacaze （2007）提出了一种将投资组合限制到一个特别的范围的方式，即提供一种基数（cardinality：Cardinality defines the numeric relationships between occurrences of the entities on either end of the relationship line），同时也确定满足最小权重（minimum weight）条件的最优投资组合。他们在LINGO或商业应用环境（CPLEX：commercial application environments）使用分支定界方法解决了这个问题。问题的二元性在本章中被广泛考虑，但模型有一个不同的形式，本章的模型公式很快就会被转化为图论的表示。Bertsimas等人在1999年的论文使用混合整数模型，来构造一个经典的具有二进制选择特性的约束优化。

4.2.2 ALGORITHMIC COMPLEXITY 算法的复杂性

The topic of algorithmic complexity is an important motivation to, and marker of, the investigation of new techniques to solving these problems. Since the core of this chapter is the practical implementation of a new approach to solving hard problems, algorithmic complexity deserves at least a cursory overview. Algorithms, or sequences of steps in a well-defined computational procedure, are used to specify the way of solving various problems. Algorithms are used at some point in the description, implementation, or analysis of all of the methods cited in this chapter. To pick one algorithm over another, a measure of efficiency is used to compare their performance. This is typically the measurement of how quickly a particular algorithm finds an answer given a certain input size.

算法复杂度是一个衡量新技术解决问题的一个重要的指标。既然本章的核心是解决困难问题的新方法的具体实现，算法复杂度至少应有一个粗略的介绍。算法，或者说用系统的方法描述解决问题的策略机制，被用来解决特定的问题。在本章的所有方法的介绍、实现、分析过程中都会使用到算法。要在几个算法中权衡选取最优算法，期中一个最重要 的指标是算法的效率。算法的效率主要是看一个算法能在多长时间内，给出一个输入的特定问题的答案。

A few good references for this topic include Cormen et al. (2009), Kozen (1992), Papadimitriou and Steiglitz (1998).
对于这个问题，你可以去参考Cormen et al. (2009), Kozen (1992), Papadimitriou 以及 Steiglitz (1998)等人的著作。

Algorithms may be put into three classes: P, NP, and NP-complete. Problems that can be solved in polynomial time are in class P. Polynomial time solutions are found in a time proportional to the size of the problem input raised to some constant power. Problems that can only be verified, but not solved, in polynomial time are called nondeterministic polynomial, class NP. Finally, problems that do not have an algorithmic way of finding an exact solution are in the class NP-complete and are considered the hardest. Typically, algorithms applied to NP-complete problems find good enough solutions or approximations rather than the best or optimal solution. There is another class called NP-hard, which for the purpose of this chapter, means problems that are at least as hard as the hardest problems in NP. However, NP-hard problems do not need to be verifiable in polynomial time and, thus, do not actually need to be members of class NP.
算法可大体分为三类：P, NP和NP-complete. 能在多项式时间（polynomial time）内解决的问题被归类到P类算法。 这种算法的时间复杂度与输入数据量密切相关，能算出确切的功率。在多项式时间内，只能验证的问题但不能解决的问题，被称为NP类。最后，没有找到一个确切的算法的来解决的问题被归在NP-complete类，它也被认为是最难的。通常情况下，把算法应用到NP-完全问题中所找到的解决方案，只能说是足够好或者近似足够好，而非最好的或最佳的解决方案。还有另外一类，被称为NP-Hard，这正是这一章的目的。这类问题是指，它至少和NP类问题一样难。然而，NP-Hard类问题，不需要在多项式级别时间内可验证，因此，实际上它并不需要是NP类的成员。

This is important because some of the models considered in the chapter are in the classes NP-complete or NP-hard and the D-Wave system can be applied to these problems.
这一点很重要，因为本章的一些模型被认为能应用到NP—complete或者NP-hard以及D—Weve量子计算机系统中。

4.2.3 PERFORMANCE
性能

As mentioned in Section 4.2.2, the performance categorization of algorithms allows us to measure usefulness, among other things. One of the

reasons for investigating the D-Wave system is that it holds promise to help solve problems that are too complex, too slow or do not yield solutions under classical methods. In other words, it promises to yield performance improvements over the current paradigm.
在4.2.2节提到过，算法的性能使得我们可以测量的算法的实用性。研究D-Wave（量子计算）系统的原因是它的初衷是帮助解决那些过于复杂，计算太慢的问题或者提高经典算法下的计算速度。换句话说，它使得现有的算法能从中得到性能改进而获益。

Therefore, it would be useful to know how the D-Wave system compares to current, classical systems used to solve particular problems. The D-Wave system has been experimentally compared with three conventional software solvers IBM ILOG CPLEX, METSlib TABU search, and akmaxsat. Three problems were investigated from the NP-hard problem domain: QUBO, weighted maximum 2-satisfiability (W2SAT), and quadratic assignment. Various problem sizes and implementation situations are used in the comparison. Problem solution quality and success are also compared, in addition to timing comparisons. The results do indicate situations in which the D-Wave equipment may not be ideal or yield improvement. However, the results show that the hardware implementation
因此，了解D-Weve系统（量子计算）与当前的传统计算系统在解决特定问题时的区别就显得尤其重要。D-Wave系统已被实验性地同三种常规性软件相比，他们是IBM ILOG CPLEX，METSlib TABU search以及akmaxsat。NP-Hard领域的三个问题被研究：QUBO，weighted maximum 2-satisfiability (W2SAT)和quadratic assignment。不同规模的问题以及不同的实施情况在比较中被使用。问题在解决的质量、成功与否以及时间复杂度等方面同样被比较。结果表明D-Wave或许并不是一种理想的提高现有效率的办法。然然，结果表明，在硬件实现方面

can be several thousand times faster, up to even 10,000 times faster, than current implementations (McGeoch and Wang, 2013).
比起当前的速度，D-Wave可以将速度提升至当前的几千倍甚至10000倍以上。

4.2.4 ISING MODEL
伊辛模型（ISING MODEL ）

The Ising model was originally constructed to help understand the behavior of magnetic materials. Two main terms in the model represent the collection of individual field strengths at each molecule and the collection of interactions among neighboring molecules. The Ising representation discussed in Boixo (Boixo et al., 2012) and, in the form used here, in Choi (2008) is the energy function@@
伊辛模型最初是用来帮助理解磁性材料的行为。在模型中的2个主要术语代表着集合中的每个分子和相邻分子之间的相互作用以及集合中的单个电场强度。伊辛表示了boixo（boixo et al.，2012），在这里使用的形式，在Choi（2008）是能量函数@ @

where the $s_i \in \{-1, +1\}$ represents the spins of the molecules in a system that is in an applied magnetic field, h_i is the strength of a magnetic field at molecule i, and J_{ij} represents the strength of interaction between the neighboring molecule's spins i and j.
在si∈{ 1 }－1，+代表一个系统，在外加磁场的分子旋转情况下，hi是在分子磁场的强度，Jij代表相邻分子之间的自旋相互作用的强度I和J。

It was later realized that this simple model could be applied to many other situations in which consideration is given to a collection of individual properties and their interactions. As stated in the study by Bian and colleagues (Bian et al., 2010), there were apparently more than 12,000 papers published between 1969 and 1997 that used the Ising representation. Presumably, that number may be much higher now.
后来人们认识到这个简单的模型能够被用到许多其他的条件中，这类问题的一般特点为，问题中给出独立的可分元素的集合以及他们之间的相互关系。经过Bian以及他的同事们的研究下，超过12000篇涉及Ising问题的论文在1969到1997年间相继发表。想必，现在这一数字会更高。

This Ising model shares a similar structure to the QUBO problem, which can be used to represent the portfolio optimization problem. Moreover, the D-Wave equipment is a hardware implementation of an optimization engine designed to solve the Ising problem, to which the QUBO form can be translated (D-Wave, 2013b, 2013c; Bian et al., 2010).
这是模型共享一个类似QUBO问题的结构，可以用来表示投资组合优化问题。与此同时，D-Wave设备给Ising问题的解决提供硬件支持(D-Wave, 2013b, 2013c; Bian et al., 2010)。

4.2.5 ADIABATIC QUANTUM COMPUTING 绝热量子计算（ADIABATIC QUANTUM COMPUTING ）

Adiabatic quantum computing is a new computing paradigm that has potential to yield satisfactory results when used to solve some of the NP-complete or NP-hard problems. The quantum computing aspect speaks to the use of special purpose-built analog environments built to take advantage of quantum physics properties of the core materials. This is in contrast to our current digital computing paradigm. Adiabatic, in this sense, means that the underlying material’s quantum nature is kept in its ground state as
绝热量子计算是一种新的计算模式，，它被用来解决一些NP-complete或NP-hard问题时，具有产生令人满意的结果的潜力。在量子计算方面它使用特殊的环境来充分发挥量子核心材料的特性与优点。这与我们目前的数字计算模式有很大的不同。绝热，在这个意义上来讲，意味着底层材料的量子性质被保持在它的基态

the system evolves toward the solution. This contrasts with the classical cooling usage of the term that is a temperature-based annealing process (Farhi et al., 2000).
当系统解决问题的过程中。与此形成鲜明对比的术语是退火过程（Farhi et al.，2000）。

However, one of the heuristic methods that can be applied to solve difficult optimization problems is simulated annealing, which is based on that classical physical cooling process mentioned in the previous paragraph.
然而，一个启发式的可以应用到解决困难的优化问题的方法——模拟退火，基于在过去的经典物理中的冷却过程，被纳入到公众的视线。

In both cases, the general idea is to iteratively evolve a problem by the introduction of a random component, through either temperature or quantum processes, to help improve the solution. In this sense, quantum annealing is conceptually similar and uses entropy of the quantum process to explore the objective. In the environment discussed in this chapter, the quantum annealing is realized by the D-Wave hardware system (Farhi et al., 2000; Bian et al., 2010; Hillier and Lieberman, 2010; Gilli et al., 2011).
在这两种情况下，一种一般的想法是通过引入一个随机的组件，通过任一温度或量子过程，来帮助解决问题。在这个意义上，量子退火是概念上类似的，并使用量子的熵变化过程（entropy of the quantum process ）来探索待解问题。在本章中讨论的环境中，量子退火（quantum annealing ）被D-Wave硬件系统实现（Farhi et al.， 2000； Bian et al.， 2010； Hillier and Lieberman, 2010； Gilli et al.，2011）。