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Welfare and fairness in free-to-play video games

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ABSTRACT

Are free-to-play (F2P) video games reducing consumers' welfare? How do changes in the game balance affect players? Despite being the most relevant business model in the video game industry, no consensus exists on the welfare implications of free-to-play (F2P) video games. This work proposes a theoretical economic model that explains why companies prefer F2P to pay-to-play (P2P) in the long term and how game fairness is determined in the short term. In the long term, the consequences of player welfare depend on the quality and network effects of the video game. In the short term, the impact on welfare depends mainly on the relationships among in-game items. Thus, welfare implications are likely to change as new items are added/removed. Finally, we test the theoretical models by estimating the fairness level in three F2P video games and show preliminary evidence that this business model would have reduced welfare.

1. Introduction

Free-to-play (F2P) is the ruling business model of the videogame industry. In 2017, F2P generated $82 billion worldwide (75.6% of the industry) Batchelor (2017), and some of those games earn up to $2.4 million daily, Davidovici-Nora (2013). In contrast to pay-to-play (P2P) video games, in which players pay for acquiring the game, players in F2P video games have access to the full game for free but can pay small amounts (microtransactions) for digital items to boost their skills, improve matchmaking, or provide cosmetic differentiation.

The F2P business model is based on incentivizing the purchase of virtual items, and players who buy these items are premium players. Because some of these items generate imbalances between players, some developers claim that this strategy is "killing the video game industry" because it is creating a "pay-to-win" culture that degrades the game experience for free players, leading many of them to quit the game because of the inherent imbalance between free and premium players, Helms (2019). In contrast, other developers claim that F2P video games "democratize" the video game industry because they expand the set of people who can experience video games; moreover, the inequality between premium and free players could foster competition among players to join the upper elite - the same logic as that of an "arms race," GamerzUnite (2013).

From companies' points of view, a trade-off exists between free and premium players. Premium players pay and want a significant imbalance, and the opposite is true for free players, who form the base of the game. Without free players, premium players cannot exploit the imbalance and utility they obtain when purchasing those items. Therefore, balance is essential, and differences between premium and free players can differ but should not be excessive Rong et al. (2018). Despite F2P's relevance in the industry, the contrasting opinions regarding its benefits for players, and the clear trade-off that firms face between free and premium players, it is surprising the lack of theoretical and empirical evidence. When is F2P more profitable than traditional P2P? When does F2P benefit or hurt players? How do video game companies adjust their imbalances/biases[1] between players? Although these questions have yet to be answered, this work aims to shed light on them.

To do so, we first present a model that explains why F2P has become popular. The model represents the strategic decision making of choosing between F2P or P2P and its long term consequences for players. We show that changing from a P2P to an F2P business model is always profitable and welfare-enhancing in multiplayer games with moderate network effects but that doing so might reduce welfare if the video game is single-player (weak network effects) or exclusively relies on multiplayer settings (strong network effects). We then present a second model that represents the short-term decision making of an already established F2P video game and explains how the optimal balance is set. This second model shows that the optimal balance is an economic decision and not a simple design feature. Interestingly, the optimal balance depends on the complement/substitute relationships among all items. Therefore, the

[1] Throughout the paper, we use "premium bias," "fairness," and "imbalance" interchangeably.

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inclusion or removal of items, updates, and patches are expected to affect balance and welfare. However, the net effect of these changes depends on the specific demand of each game, which might explain the current conflict between developers regarding whether the F2P strategy is socially desirable. These results not only assist in reducing the set of F2P video games that could raise authorities' concerns but also provide developers with testable hypotheses that help them design video games.

Finally, this work complements the theoretical modeling by estimating the fairness level of War Thunder, World of Tanks, and World of Warships -three popular F2P video games. This work finds that all these games present some degree of welfare-reducing fairness. Interestingly, fairness levels vary after the inclusion of new items, as suggested by the theory and preliminary evidence of Rong et al. (2018).

The remainder of this paper is organized as follows. Section 2 presents a summary of the theories that might imperfectly explain the F2P phenomenon from an economic perspective. Section 3 addresses the first research questions: When is F2P more profitable than P2P? How does this profitability affect players' welfare? Section 4 considers how video game companies set their imbalance/bias and the effects on player welfare. Section 5 builds on previous models to test the hypotheses drawn from the theoretical models by estimating an empirical model using data from three popular F2P video games. Finally, Section 6 concludes and offers directions for further research.

2. Alternative theories to explain the F2P phenomenon

A key characteristic of the F2P business model is that no work explicitly addresses it despite the fact that video games have been largely studied in the literature; see Evans et al. (2008) and Shapiro and Varian (2013). In fact, the closest we can get to a formal study of the F2P is to observe the features that are also present in other markets or models. For example, the complementary relationship between the video game and premium items, as well as the different pricing between these two types of goods, are fundamental features of the F2P business model but also key characteristics of other markets. In this sense, the F2P phenomenon can be explained using different theories and models. The closest could be the theory of complementary goods. The idea is that the extent to which a firm is willing to sacrifice profits on good i to boost those on good j depends on the markup on good j, and vice versa. Although simple, this basic framework allows us to capture most key features of F2P video games, but the standard framework requires some modifications (including network effects or the role of imbalances, that is, nonprice competition). However, even with these modifications, this approach does not explain how or why this business model has become relevant.

Another key feature of F2P is that players acquire the game (for free) and then pay for in-game items. This situation might resemble other business models with two-part tariffs, such as razors and blades or printers and ink Jing (2007) and Cheng and Tang (2010). Intuitively, this extreme see-saw pricing with free goods resembles the *damaged good strategy*; see Belleflamme and Peitz (2015). In software markets, this strategy implies the development of a full-featured version of software and another low-quality version by disabling a subset of features. As Belleflamme and Peitz point out, this strategy explains why the view and print versions of Adobe Reader and Microsoft Office are free and seems to explain why F2P video games are free. Nonetheless, although this body of literature provides a satisfactory answer to why prices are lower for the basic version and allows us to explain why this business model has become prevalent, it assumes that the free version is a "decaffeinated" version of the premium one. In F2P video games, the full game is free, players can enjoy it without restrictions, and the premium advantages might not be obvious, as in the case of better matchmaking, see Davidovici-Nora (2014).[2] Additionally, the damaged goods strategy assumes that both products are imperfect substitutes, and consumers choose the preferred version. However, in reality, premium items complement the basic good (video games). Therefore, this strategy provides no explanation or description of the complementary relationship between the items.

The possibility of asymmetric pricing is also common in the add-ons and ancillary goods literature, which also relies on the complementary relationship between basic goods and some items; see Verboven (1999) or Ellison (2005).[3] However, this body of literature normally requires some type of incomplete information assumption, which might not be a realistic assumption for F2P video games. F2P is a known genre in which players expect to find in-game items that are sold at positive prices.[4] Moreover, this literature did not consider the presence of network effects, which are a fundamental feature of some F2P video games. Finally, companies have developed both F2P video games and more traditional P2P video games. Such a movement from one business model to another requires the ancillary literature to recognize significant changes in marginal costs, which might not represent the F2P case; see Gomes and Tirole (2018). In addition, a conflict exists between those who support this business model and those who believe that it is killing the industry. This contradiction has not yet been clarified in the literature.

The presence of network effects and complementarities between different items is also linked to the literature on systems competition Katz and Shapiro (1994). Intuitively, video games appear to fulfill the definition of a system. Complementarities exist between F2P games and in-game items - similar to hardware and software items - and network effects and cross-subsidization exist between items. However, interesting differences also exist. For example, a key assumption of this body of literature is that components have little or no value in isolation; see Economides and Salop (1992). In F2P games, that is only true for premium items, but the game is playable and enjoyable alone, in many cases, without others playing it. Finally, the video game industry is a classical two-sided market Rochet and Tirole (2006) and Rong et al. (2019). Thus, intuitively, this body of literature could also explain the F2P business model. However, a fundamental limitation is that it normally considers only indirect and not direct network effects. Cases that assume both types of network effects quickly become more complex and intractable.

In summary, alternative theories address the F2P business model but do not explain why F2P has become popular, the impact on players' welfare, or how video game companies choose their fairness levels. In the following sections, we address this research gap.

3. Optimality of the free-to-play strategy

We begin by addressing why this business model has become relevant. To do so, we consider a market in which a monopoly must decide whether to sell a P2P or an F2P game. The P2P is one item with fixed content (quality level), and the F2P might encompass two (or more) items with different contents (or quality levels)–one that is free and represents the basic game, and the rest represents the basic game plus

[2] See Davidovici-Nora (2013) for a brief history of the F2P business model. Also see Evans et al. (2008) for a brief history of videogaming.

[3] This work is also related to the aftermarket literature, which is quite focused on antitrust issues. The"see-saw" relationship between basic and complementary good's prices is also common in this literature; see Carlton and Waldman (2010).

[4] Other more technical reasons also raise concerns. For example, a common assumption of this body of literature is that the relative markup is increasing in consumers' willingness to pay for quality (or content, in our case); see Lal and Matutes (1994), Verboven (1999), or Gomes and Tirole (2018). However, this phenomenon might be an undesirable feature in F2P video games, in which prices tend to increase less than linearly as players unlock new content in the game.

the premium items.

The set of feasible contents/qualities is represented by a closed interval $[s_L, s_H]$, where $0 < s_L < s_H$. Intuitively, s_L might represent the minimum playable game quality or minimum content that players are willing to accept. In contrast, s_H might represent the highest possible quality that can be implemented given the game mechanics or the maximum possible content that can be created for that game. For example, s_L could be the basic game, and s_H could be the basic game plus all add-ons, expansions, and power packs that can be implemented in the game. Without loss of generality and for simplicity, we assume no fixed or marginal costs.

Many F2P are accessible through platforms such as Epic Games, Steam, or Uplay. On these platforms, players can only hold a copy of each game. Thus, we assume that each player consumes zero or one unit of the game. For simplicity, I assume that each player values the game content/quality differently, θ_i, and players are uniformly distributed on $[-\Theta, 1]$, where $\Theta > 0$ with density 1. Therefore, the size of the player market is $\Theta + 1$. This assumption guarantees tractability at the same time that it captures the idea that people might perceive the game differently. For some people, a game can be a masterpiece; however, for others, it might just be a bunch of pixels. Additionally, players prefer to play popular games; therefore, in addition to, the quality/content, the game might have a network value ($\gamma \in [0,1]$). If Q represents total game sales, then the network value of a player with a game of quality s is γsQ. This assumption implicitly assumes that complementarity exists between quality and network effects. If two games are equally popular but one has more content or higher quality, it will generate more utility for users, ceteris paribus. Simultaneously, this formulation allows us to consider the possibility of games with large network effects and low quality, and vice versa. Therefore, when consuming/playing a game, the i-player's utility is $\theta_i s + \gamma sQ - p$, where p is the price of the game with quality s.

3.1. Pay-to-play game

A monopoly that develops a P2P video game sets the same quality/content for all players. Typically, in P2P video games, the content and quality rarely change over time and tend to be set before the game is published. Therefore, all players benefit from the same items in the game. Let $\widehat{\theta}_i$ be the indifferent player between playing and not playing the game.

$$\widehat{\theta}_i s + \gamma sQ - p = 0 \tag{1}$$

Given the assumption about the uniform distribution of users, we know that all players with θ higher than $\widehat{\theta}_i$ purchase the game. Solving Eq. (1) for $\widehat{\theta}_i$ and substituting $Q = 1 - \widehat{\theta}_i$ provides us with total sales:

$$Q = \frac{s-p}{(1-\gamma)s} \tag{2}$$

Given a quality level for the game, the firm sets a price (p) that maximizes its profits ($\Pi = pQ$). Formally,

$$\max_p \Pi = pQ = p\left(\frac{s-p}{(1-\gamma)s}\right) \tag{3}$$

In this framework, the optimal price is given by the following first-order condition:

$$p^* = s/2 \tag{4}$$

Substituting Eq. (4) into Eqs. (2) and (3) provides the equilibrium demand and profits.

$$Q^* = \frac{1}{(1-\gamma)2} \tag{5}$$

$$\Pi^* = \frac{s}{(1-\gamma)4} \tag{6}$$

Finally, we compute consumer surplus in equilibrium as the difference between the maximum price that each agent is willing to pay and the price that they pay. In other words, consumer surplus is computed as the area under the demand curve, as in Belleflamme and Peitz (2015).

$$CS^* = \int_{p^*}^{s(p)} Q\partial p = \int_{s/2}^{s} \frac{s-p}{(1-\gamma)s}\partial p = \frac{s}{8(1-\gamma)} \tag{7}$$

Prices and consumer welfare increase with quality. However, all players consume the same product; therefore, no "premium bias" or imbalance exists among players.

3.2. Free-to-play game

Normally, players can buy multiple in-game items. However, we argue that only *one* premium bias exists. The next lemma formalizes this point.

Lemma 1. *When players value the content (or game quality) differently, a profit-maximizing F2P video game only sets two quality levels - premium and free-to-play – independent of the number of in-game items in the game.*

Proof. See Annex A□

Therefore, let us assume that there are two goods: the free game and the free game with a premium account. We identify these goods with subscripts 1 and 2, respectively. We also assume that both items have different content (quality) levels s_1 and s_2 and are sold at different prices p_1 and p_2, where $p_1 = 0$. Following the previous model, let $\widehat{\theta}_1$ be the indifferent player between playing only the free game and not playing.

$$\widehat{\theta}_1 s_1 + \gamma s_1 Q - p_1 = 0 \tag{8}$$

and let $\widehat{\theta}_2$ be the indifferent player between playing the free game or buying the premium account with the free game.

$$\widehat{\theta}_2 s_2 + \gamma s_2 Q - p_2 = \widehat{\theta}_1 s_1 + \gamma s_1 Q - p_1 \tag{9}$$

By solving both expressions with respect to $\widehat{\theta}_1$ and $\widehat{\theta}_2$, we have

$$\theta_2 = \frac{p_2 - p_1}{s_2 - s_1} - \frac{\gamma(s_1 - p_1)}{(1-\gamma)s_1} \tag{10}$$

$$\theta_1 = \frac{p_1 - \gamma s_1}{(1-\gamma)s_1} \tag{11}$$

Previously, we assumed that players were uniformly distributed with respect to θ; thus, we know that those with $\theta > \theta_1$ play the game and those with $\theta > \theta_2$ buy the premium account. Given that $\theta_2 > \theta_1$, showing that those with $\theta \in [1, \theta_2)$ consume the free game with a premium account, and those with $\theta \in [\theta_2, \theta_1)$ consume only the free game, is direct. Therefore, the firm wants to maximize the profits that it can extract from these demands. Formally,

$$\max_{p_1, p_2} \Pi = p_1(\theta_2 - \theta_1) + p_2(1 - \theta_2) \tag{12}$$

We assume that $0 < (1-\gamma)s_2$ to guarantee that the first-order conditions lead to a global maximum. Given this assumption and solving the first-order conditions as we previously did, we can compute the optimal prices:

$$p_2 = \frac{s_2 - s_1}{2} + p_1 + \frac{(s_2 - s_1)\gamma(s_1 - p_1)}{2(1-\gamma)s_1} \tag{13}$$

$$p_1 = \frac{p_2[2s_1 - \gamma(s_1 + s_2)]}{(1-\gamma)2s_2} \tag{14}$$

Given that an F2P game is characterized by a basic free game and a premium version, we assume $p_1 = 0$ and $s_2 > s_1$ and substitute in Eq. (14). After some algebraic manipulations, we have the relationship between the qualities of the basic video game and the video game with all premium items[5]:

$$s_1^* = \frac{\gamma s_2}{2-\gamma} \tag{15}$$

Note that premium items have more content (or better quality) than free items ($s_2 > s_1$) because they either have better performance or are perceived as higher quality (e.g., premium status). Thus, the higher the imbalance, the higher s_2 and the lower the ratio $\frac{s_1}{s_2}$. This expression is increasing in the network effect parameter (γ), which points out that the fairness (imbalance) is increasing (decreasing) in network effects. Therefore, multiplayer games are the most affected ones. In single-player games, players do not care about how other people play the game because they do not interact, $\gamma = 0$; in this case, the model converges to the P2P. In this situation, the equilibrium premium price, demand, profits, and consumer surplus are

$$p_2^* = \frac{s_2}{2-\gamma} \tag{16}$$

$$Q^* = q_2 + q_1 = \frac{1}{2(1-\gamma)} + \frac{1}{2(1-\gamma)} = \frac{1}{1-\gamma} \tag{17}$$

$$\Pi^* = \frac{s_2}{2(1-\gamma)(2-\gamma)} \tag{18}$$

$$\begin{aligned} CS^* &= \int_{p_2^*}^{s_2(p_2)} (1-\theta_2)\partial p_2 + \int_{p_1^*}^{s_1(p_1)} (\theta_2-\theta_1)\partial p_1 \\ &= \frac{[\gamma(\gamma-2)+(1-\gamma)]^2 s_2}{2(2-\gamma)^2(1-\gamma)^2(1-2\gamma)} + \frac{\gamma s_2}{2(2-\gamma)^2(1-2\gamma)} \end{aligned} \tag{19}$$

Profits, welfare, and prices depend on network effects and imbalance, and F2P demand depends only on network effects. However, a fundamental result is the comparison between Eqs. (6) and (18), which provides us with an answer to the current popularity of F2P games, summarized in the following proposition.

Proposition 1. *When players value content (or game quality) differently, profits are larger with F2P than P2P. Thus, it is always optimal for companies to move from P2P to the F2P.*

3.3. Welfare analysis

However, to determine whether this business model hurts players, we must compare Eqs. (7) and (19). The net effect on the players depends on the difference between s and s_2.

Figure 1 illustrates how welfare behaves as a function of s (quality/content) and γ (network effects). Assuming that the company produces a game with the same quality independent of the business model[6], from a social point of view, F2P is preferred when network effects are moderate - between 0.33 and 0.5. In the remaining cases, a P2P scheme is socially preferred. This result explains the current discussion in the video game industry regarding the positive/negative effects of this strategy on consumer welfare.

Lemma 2. *In games in which network effects are not relevant, a P2P model is socially desired. However, if the network effects are moderate, the F2P model is socially optimal.*

The claim that F2P degrades the game experience for some players might be partially true, particularly for multiplayer games, in which network effects are expected to be a dominant force. In such cases, the welfare gains of premium players may not compensate for the welfare losses of the free players. However, it is also true that F2P might expand the set of people who can experience video games, possibly increasing welfare. This might be the case in some multiplayer games that do not necessarily rely on a multiplayer setting but also on other single-player functionalities, such as playing against "bots" (artificial intelligence). However, this analysis assumes that the price and "bias" (represented by the difference in quality/content) cannot be changed simultaneously. That may be a realistic assumption in some cases for which creating content or changing the quality level requires significant effort or when companies address game development and make a strategic decision about the business model but does not explain the case in which F2P games that continuously updated. In the next section, we address this case.

4. Imbalance and the optimal fairness

To address how F2P video games set their fairness levels in the short term, let us consider a market with two F2P video games, $i = 1{,}2$, that face demand Q_i for the basic game (excluding the premium items) and are sold at price P_i. In each game, players can buy N digital items at price $v_j^i j = 1{,}2, \ldots, N$. Therefore, N different demands exist for each premium item q_j^i. We assume that the larger the imbalance (δ), the more attractive the items are because those who hold them have some advantages, $\frac{\partial q}{\partial \delta} > 0$; the less attractive it is to play the basic game because those who do not hold them are disadvantaged, $\frac{\partial Q}{\partial \delta} < 0$; and the higher the prices, the lower the demand, $\frac{\partial q}{\partial v} < 0, \frac{\partial Q}{\partial P} < 0$. Additionally, we assume demands are well-behaved.

Note that premium players are the only source of revenue, and their demand depends positively on the imbalance. However, the attractiveness of premium items depends on free players who dislike the imbalance. Therefore, the impact of an increase in the imbalance on profits (and welfare) is unclear a priori. Finally, for presentation purposes, I temporarily assume that producing the game and in-game items is costly, c_i and d_j^i, respectively.[7] In addition, implementing such an imbalance has a twice-differentiable cost, $F(\delta)$, that intuitively, represents the cost of equilibrating each match; the larger the imbalance, the more complicated is to balance a match with premium and free players. For example, an increase in imbalance (a reduction in fairness) might require dedicating more computing power to balance the matches, which is costly. Thus, a video game must set the prices for premium items and a fairness level that maximizes its profits. Formally, the decision problem for a video game company is

$$\max_{P_i, \boldsymbol{v}, \delta_i} \Pi(P, \boldsymbol{v}, \delta) = (P_i - c_i)Q_i(P, \delta) + \sum_{j=1}^{n} \left(v_j^i - d_j^i\right) q_j^i(Q_i, v_j, \delta_i) - F(\delta_i) \tag{20}$$

The first term represents sales of the basic game. If the game is free, this term is zero; however, we make that substitution later. The second term represents the revenue from selling N premium items, and the last term

[5] Making the substitution here implies that we can derive the condition that makes $p_1 = 0$ optimal. Given this condition, the firm does not have an incentive to set a positive price for the basic game. Instead, if we assume $p_1 = 0$ before the maximization, we impose $p_1 = 0$ exogenously, which does not guarantee that $p_1 = 0$ is optimal a posteriori.

[6] This assumption does not impose unrealistic behavior. If we consider that companies set the quality of the game in a previous phase, the backward induction algorithm shows that video game companies choose the highest possible quality for their games; therefore $s = s_2 = s_H$.

[7] Note that we assume no marginal cost in the previous model. This assumption reflects the traditional idea that, in digital markets, producing additional units of the item is costless. However, the assumption of positive marginal costs reflects the idea that producing additional units might imply a larger demand that must be accommodated with new servers or by employing more development time in adjusting the imbalance.

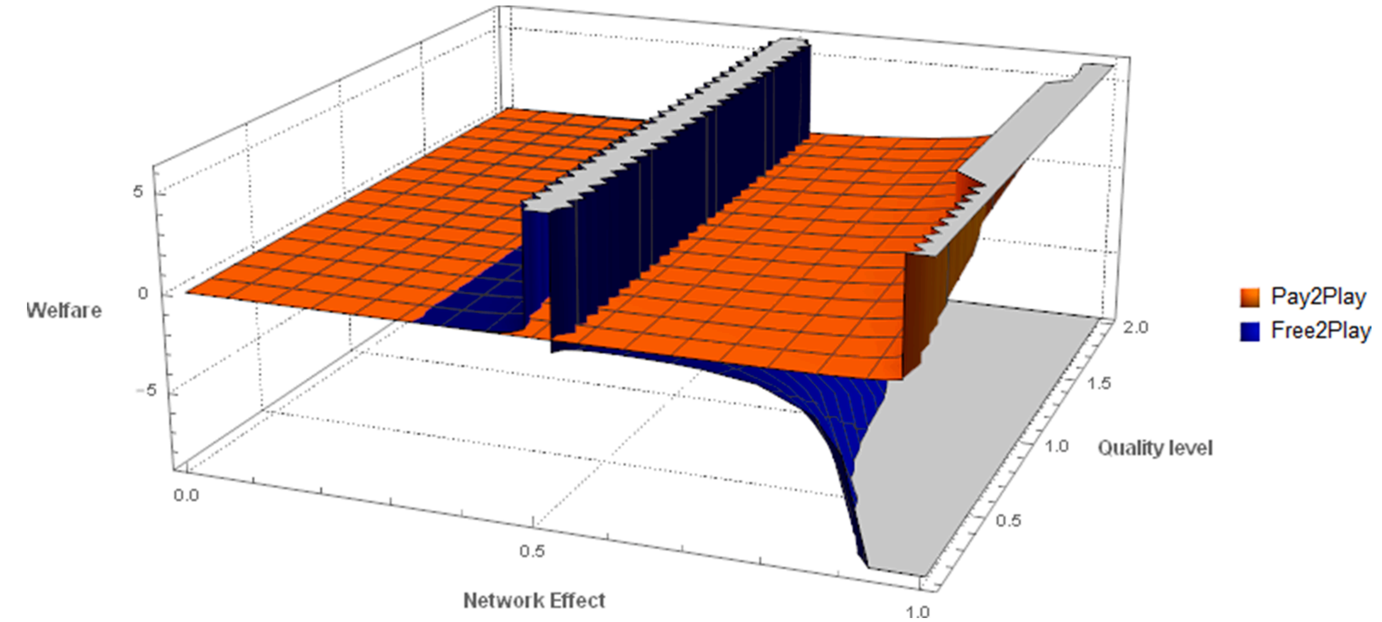

Fig. 1. Welfare levels by s and γ.

represents the cost of balancing the game. Solving for first-order conditions (FOC) and rearranging allows for an analysis of the link among markups, demand elasticities, and bias.[8]

$$\frac{P_i - c_i}{P_i} = \frac{1}{2|\epsilon_i|} - \sum_{j=1}^{n}\left(v_j^i - d_j^i\right)\left(\frac{\partial q_j^i}{\partial Q_i}\frac{1}{P_i}\right) \tag{21}$$

$$\frac{v_j^i - d_j^i}{v_j^i} = \frac{1}{|\epsilon_j|} + \sum_{k \neq j}\frac{v_k^i - d_k^i}{v_j^i q_j^i}\frac{q_k^i \epsilon_{k,j}}{|\epsilon_j|} \tag{22}$$

$$\frac{P_i - c_i}{P_i} = \frac{F_i^{'}\delta_i}{2P_iQ_i\epsilon_{i,\delta}} - \sum_{j=1}^{n}\left(v_j^i - d_j^i\right)\left(\frac{\partial q_j^i}{\partial Q_i}\frac{1}{P_i} + \frac{q_j^i\epsilon_{j,\delta}}{2P_iQ_i\epsilon_{i,\delta}}\right) \tag{23}$$

The element on the left-hand side represents a measure of market power, known as the Lerner index. If this element tends to one, it converges to a monopoly model. Conversely, if the value is zero, it converges to a perfectly competitive market. The right-hand side elements of Eqs. (21) and (23) are the inverse of the price elasticity and the adjustment of the markups that come from bias or other products (resembling the Dorfmann-Steiner theorem of the industrial organization literature). These last elements are the most interesting. To determine why, we rearrange Eq. (21) when the basic game is free and there are no marginal costs in producing additional units of the free game ($P_i = c_i = 0$). Then, we have the following:

$$\sum_{j=1}^{n}\left(v_j^i - d_j^i\right)\left(\frac{\partial q_j^i}{\partial Q_i}\right) = 0 \tag{24}$$

This expression highlights the necessary condition of optimality for F2P video games in the short term.[9] Although this expression might directly lead to the idea that $\frac{\partial q_j^i}{\partial Q_i} = 0$, which implies that in-game items and the free video game are independent products, for some items, an increase in free players might increase their demand, whereas the opposite is true in other cases, but they cancel each other out in equilibrium.[10]

Lemma 3. *In equilibrium, an F2P game balances items that benefit only premium players with those that benefit all players. Formally, the marginal revenue generated by premium items must equal the loss in revenue generated by nonpremium items.*

In other words, the complement and substitute items must be equilibrated. The complement items ($\frac{\partial q}{\partial Q} > 0$) might represent an imbalance that provides an advantage over the free players. The larger the number of free players, the higher the chances of competing against them and profiting from the imbalance. In contrast, substitute items ($\frac{\partial q}{\partial Q} < 0$) might represent the case of items that cannot be used against other players or even free items that compete with premium ones, such as gifts. For example, common for F2P games is to provide free items on holidays or on the completion of some challenges. These items might reduce the demand for premium items[11].

To characterize the optimal imbalance, let us equal Eq. (23) to Eq. (21) and substitute Eq. (22) assuming that $d_j^i = 0$:

$$\delta_i^* = \frac{1}{F_i^{'}}\sum_{j=1}^{n}\frac{\theta_{k\neq j}\epsilon_{j,\delta}}{|\epsilon_j|\left(|\epsilon_j| - 1\right)} \tag{25}$$

where $\theta_{k\neq j} = \sum_{\forall k\neq j}\pi_k\epsilon_{k,j}$, which represents the profits of all in-game items (π_k) weighted by their demand elasticity ($\epsilon_{k,j}$). Intuitively, the optimal imbalance is a weighted average of the influence of N premium items, which is determined by three effects. The first is the *cost effect* ($\frac{1}{F_i^{'}}$). When implementing imbalances between free and premium players, matchmaking becomes more complex. If the game is not correctly balanced, free players are more likely to quit the game because premium players can easily beat them. In contrast, if premium players cannot

[8] See Annex C for the first-order conditions.

[9] Nonetheless, it implicitly assumes that F2P video games compete in prices and $P_i = 0$ is optimal, which might not be true. In many cases, giving away the game could be considered exogenously given in the short term because it is a long term decision. To the best of my knowledge, no evidence exists of video games changing their business model from F2P to P2P; however, some games have recently changed from the classical P2P to the F2P business model, such as Counter-Strike: Global Offensive (Valve). In these cases, thinking that Eq. (24) holds and $P_i = 0$ is optimal in the short term might be reasonable.

[10] Formally, $\sum_{j=1}^{k}(v_j^i - d_j^i)\underbrace{\left(\frac{\partial q_j^i}{\partial Q_i}\right)}_{-} = \sum_{j=k+1}^{n}(v_j^i - d_j^i)\underbrace{\left(\frac{\partial q_j^i}{\partial Q_i}\right)}_{+}$. Note that, $v_j^i = d_j^i$ is not a realistic approach because the marginal costs of digital items tend to zero, but prices are nonnegative.

[11] For some items, free and premium demands may be independent ($\frac{\partial q}{\partial Q} = 0$).

exploit their premium status, they do not buy premium items. Therefore, the game must correctly balance who competes with whom and the computational cost of balancing players.

The second type of effect is the *elasticity effects* ($\epsilon_{j,\delta}$, and $|\epsilon_j|$), which are item-specific. All premium items were influenced by bias and price elasticity. The higher the bias elasticity ($\epsilon_{j,\delta}$), the higher the influence of the imbalance. F2P can be intuitively compared with persuasive advertising models, given that increases in imbalance can "persuade" players to adopt the item.[12] However, its influence depends on the item-specific price elasticity. The intuition for this is as follows. A price-inelastic item[13] in a video game could be necessary to beat it. Therefore, to the extent that it necessary to play, imbalances should not be created. These items are used in player-versus-environment gameplays or single-player modes. In contrast, price-elastic items are those that contribute to the imbalance used in player-versus-player gameplays.

The third element is the *complementary/substitutability effect* ($\theta_{k\neq j}$). This effect depends on whether the remaining items are substitutes, complements, or independent items. Depending on the role played by each in-game item in the game, they might contribute to an increase (complement items) or decrease (substitute items) in the imbalance. Interestingly, if all of the items are independent, the optimal imbalance is zero. Therefore, a free-to-play business model requires at least two nonindependent premium items.

Proposition 2. *A profit-maximizing F2P video game only sets two quality levels: premium and free-to-play. However, to do so, at least two non-independent premium items are required.*

Equation (25) also highlights that premium items contribute additively to this imbalance. This feature is essential because F2P video games tend to include new content over time, which might influence the degree of fairness. For example, F2P video games such as War Thunder or League of Legends, include new items every three or four months[14],[15]. The inclusion of those new items tends to be followed by numerous changes in other items to maintain "fair gameplay." In other words, to maintain the balance of the game. Rong et al. (2018) showed that developers are well aware of these effects, and that *when a new game role is introduced, due to its strong attributes, it will create an imbalance when fighting with original roles... [Therefore, the developers] need to further update the games and fix that problem.* In addition, Eq. (25) is additively separable in premium items, allowing us to study the changes in imbalance when new items are included.

Apart from the direct effects of these three elements on the imbalance, other indirect effects are also worth pointing out. On the one hand, free players are not directly considered when setting optimal fairness. Given that free players are not sources of revenue, their interests are generally ignored. Nonetheless, premium players need them to exploit their premium status.

Corollary 1. *The interest of free players is only considered to the extent they are needed by premium players*

On the other hand, note that $\theta_{k\neq j}$ depends on item-specific demands, which depend on the basic game's demand and fairness and the fairness levels of other games. Intuitively, this imbalance can be considered as a market share shifter. If one firm increases its imbalance, it does not increase total demand but shifts demand toward that firm. The larger the number of competitors, the larger the share to steal from competitors; thus, the elasticity with respect to the imbalance might be greater.

Corollary 2. *Fairness is expected to increase as the number of competitors increases. A greater imbalance is also expected in market segments with a low number of substitute video games.*

Finally, from Eq. (25), whether a decrease in fairness harms consumers is unclear. Free players enjoy free games but suffer from structural imbalance. In contrast, premium players enjoy an imbalance but must pay for it. In the following section, we address this issue.

4.1. Welfare effects of the imbalance

The main dispute in the video game industry today is whether the F2P has revitalized or it is killing the industry. In Section 3, we highlight the impact of F2P video games on welfare relative to the P2P business model. However, we do not focus on how changes in premium bias influence welfare. Now, the question is whether an increase in the imbalance could harm consumers of F2P video games. To address this case and complement our previous findings, let us define the welfare of this market, which we identify as consumer surplus.

$$W(\overline{P},\overline{v},\overline{\delta}) = \sum_{i=1,2}\int_{P_i}^{r_i(\delta_i)} Q_i(\cdot)\partial P_i + \sum_{i=1,2}\left(\sum_{j=1}^{n}\int_{v_j^i}^{s(\delta_i)} q_j^i \partial v_j^i\right) \tag{26}$$

The first item is the welfare that the free game creates, and the second is the welfare that in-game items generate. The main interest lies in knowing how the deviation from equilibrium values affects players' welfare. If an increase in the imbalance increases welfare, this implies that the equilibrium imbalance of the market is suboptimal from a social point of view. For simplicity and without loss of generality, let us assume a monopolistic solution. Considering that the duopolistic version does not provide us with extra insights, it only points out wealth transfer to the second platform when there is a deviation from the equilibrium. Formally,

$$\left.\frac{\partial W(\overline{P},\overline{v},\overline{\delta})}{\partial \delta_i}\right|_{\delta_i=\delta_i^m} = -Q_i\frac{\partial P_i}{\partial \delta_i} + \int_{P_i}^{r_i}\frac{\partial Q_i}{\partial \delta_i}\partial P_i + \sum_{j=1}^{n}\left(\int_{v_j}^{s_j}\frac{\partial q_j^i}{\partial \delta_i}\partial v_j^i - q_j^i\frac{\partial v_j^i}{\partial \delta_i}\right) \tag{27}$$

Because F2P games verify $P_i = 0$, $\frac{\partial P_i}{\partial \delta_i} = \frac{\partial P_{-i}}{\partial \delta_i} = 0$, Eq. (27) becomes,

$$\left.\frac{\partial W(\overline{P},\overline{v},\overline{\delta})}{\partial \delta_i}\right|_{\delta_i=\delta_i^m} = \underbrace{\int_{P_i}^{r_i}\frac{\partial Q_i}{\partial \delta_i}\partial P_i}_{-} - \underbrace{\sum\nolimits_{j=1}^{n} q_j^i\frac{\partial v_j^i}{\partial \delta_i}}_{+/-} + \underbrace{\sum\nolimits_{j=1}^{n}\int_{v_j}^{s_j}\frac{\partial q_j^i}{\partial \delta_i}\partial v_j^i}_{+} \tag{28}$$

A priori, the net change in welfare when the imbalance increases is unclear. However, this change encompasses three effects. The first is negative, representing the loss of free players welfare. Ceteris paribus, an increase in imbalance makes the game less enjoyable for free players. In contrast, the last term represents the increase in welfare for premium players that enjoy a higher imbalance. Intuitively, the opposite is true. Finally, the second element has an undefined sign that depends on the variation in digital item prices resulting from an increase in the imbalance, $\frac{\partial v_j^i}{\partial \delta_i}$. In fact, finding demand functions that generate positive and negative signs in Eq. (28) is possible. This result sheds light on why no consensus exists regarding the benefits or harms of F2P gaming. Different games might face different demands, which justifies the divergence of opinions regarding their effects on welfare.

Currently, public authorities have expressed concerns regarding consumer policy in the digital age, especially around video games and their new business practices OECD (2019). This analysis shows that a key evaluation of F2P video games must first measure how large the "premium bias" or imbalance is and then determine whether it is increasing or reducing consumer welfare.

Proposition 3. *Neither F2P is socially undesirable nor are all P2P games preferred from a social point of view. It depends on the network effects and*

[12] If elasticities are not constant, a priori, it is not possible to address the consequences in the imbalance. Hence, for simplicity and without loss of generality, we assume constant elasticities to derive the intuitions. The results are available on request.

[13] Intuitively, one whose demand does not change "too much" when prices change.

[14] https://warthunder.com/en/game/changelog/

[15] https://na.leagueoflegends.com/en-us/news/game-updates/

contents (qualities), but the introduction of new items or updates might modify the welfare effects, which depend on the specific market demand.

From the public authorities' point of view, it is interesting that any evaluation of the welfare impact of these games must focus on three aspects. First, the sensitivity of free players' demand to the premium imbalance ($\frac{\partial Q_i}{\partial \delta_i}$). Second, the sensitivity of premium players' demand to the premium imbalance ($\frac{\partial q_j^i}{\partial \delta_i}$). Third, how much do premium prices change with any change in the premium imbalance ($\frac{\partial v_j^i}{\partial \delta_i}$). Once these three points are addressed, it is possible to determine whether the F2P increases or decreases welfare in a specific market. In some cases, prices might remain constant when the premium bias changes. Such cases only require computing how the free and premium players' demands react to changes in the premium bias. However, another way of addressing this issue is to directly study players' demand. In Annex B, we explore two demand functions that lead to opposite conclusions regarding the welfare effects of the F2P business model.

5. Estimation of imbalances in free-to-play videogames

Previous models provide us with tools to address how companies set a premium bias and its impact on welfare. In this section, we follow a quantitative approach, similar to that of Rong et al. (2019) to determine how to measure the premium bias and its consequences. Let us focus on three F2P video games: World of Tanks (Wargaming Group Limited), World of Warship (Wargaming Group Limited) and War Thunder (Gaijin Entertainment). In these games, players fight in two different teams against each other to control strategic points.[16] The team that controls the majority of those points for the longest period or survives the fight wins. Before entering a match, players can choose between different vehicles.[17] Some are free, but others can only be played if a player first buys them – the premium vehicles. Based on our previous insights, premium vehicles should perform better than their free counterparts. If a premium bias exists, it should enhance performance and/or improve matchmaking.[18] Additionally, the theory predicts that premium bias changes over time if new items are added. Thus, we expect to find a positive premium bias that varies with the launch of expansion packs or the inclusion of new items.

To measure the degree of imbalance, we could use players data; however, their use might raise concerns regarding selection bias. The more a user plays a game, the better he or she gets at playing it, and the higher the chances become of buying a premium item. Thus, disentangling unobserved skill effects from the premium item effect might be difficult. Using item-level data, we can divide items by their tiers, which vary as a function of the time spent in the game. In this way, we do not have to worry about the players' skill levels because they are equalized when considering the average worldwide performance of each item.

This bias can be addressed in several ways. The first choice is to look at the kill/death ratio (K/D) of each vehicle, which measures individual victories against defeats. If premium vehicles have a higher kill/death ratio, it would imply they are enhancing players' performance. However, this measure also depends on players' skills. The other option is to use the win ratio, which measures how many matches (battles) a vehicle has won versus how many it has lost. This variable is less related to the skill level and has far more to do with the state of balance/matchmaking because it considers not only the vehicle's individual performance but also that of the team. If premium vehicles tend to have a higher win ratio, it would imply that they profit from better balancing. The following sections focus on this measure.

Claim 1. *If a premium bias exists, the win rate and K/D ratio might contain such an effect.*

5.1. Analysis of databases

These games have different gameplays, realisms, and modes. However, an essential difference is data availability. The game with the most available data was World of Tanks. In this sense, three different data sources were collected that provided a more accurate picture of premium biases–one for the mobile version (World of Tanks Blitz) and two for the PC version (World of Tanks Charts and Noobmeter). In the case of World of Warships, only two databases (World of Warships Numbers and World of Warships Stats) were found. In the case of War Thunder, only a dataset is available (Thunderskill).[19]

In all cases, individual item or "vehicle" data were collected between August and October 2019. The first extraction was performed on August 31, 2019, the second on September 30, 2019, and the third on October 31, 2019. In all cases, extraction was automatically performed by web scraping.[20] All of the tables presented in this section are those extracted on August 31, 2019. The rest of the tables are available on request but do not provide different insights than those presented here.

Originally, the War Thunder (WT), World of Tanks Blitz (WoTB), and Noobmeter databases did not differentiate between premium and free vehicles. This information was introduced by using developers' information about premium vehicles.[21] In the case of WoTB, because no information was available on the type of vehicle, the World of Tanks (WoT) Charts database was used to fill this gap. The WoT Charts and World of Warships Numbers (WoW) database present all of the required information. Lastly, the World of Warships (WoW) Stats lacked information about the nation and type of vehicle; the WoW Numbers database was used to fill this gap.

Although each database slightly differs in the variables it considers, in general, they have the same set of variables. Table 1 shows the variables common to all databases. Variables exclusive to some databases were not considered because they did not lead to further insights or showed no improvement with respect to the analysis presented here.[22]If we compare the databases, the first difference is the number of "battles." Vehicles in WT and WoTB present significantly fewer battles than those in WoT and WoWS. Each game has a different gameplay, which was observed for different K/D ratios. The WT shows a much larger K/D ratio than the rest of the games. This result is not surprising given that the WT was developed by a different company than the other games. Nonetheless, independent of the game, the proportion of premium vehicles tended to be between 20% and 34%.

To continue the analysis, addressing whether statistically significant differences exist between the performances of premium and free

[16] https://worldoftanks.eu, https://worldofwarships.eu and https://warthunder.com/ [Last access: October, 2021]

[17] Heavy, medium or light tanks, self-propelled artillery and tank destroyers in World of Tanks. Heavy, medium or light tanks, rocket launchers, tank destroyers, and anti-aircraft vehicles in War Thunder. Destroyers, cruisers, battleships, and carriers in World of Warships.

[18] In this case, better matchmaking implies that premium users tend to be matched against other players with some disadvantages (e.g., lower skill level, worse vehicle) when assigning each player to one of the teams

[19] This dataset is available in an open repository; see Sanchez-Cartas (2020). It encompasses data for three types of gameplay. However, only the "arcade" mode has enough information to carry out an analysis. Therefore, throughout this paper, we consider only this type of gameplay.

[20] War Thunder from: https://thunderskill.com/en, World of Tank from: https://www.blitzstars.com/, https://wotcharts.eu/MainPage, and https://www.noobmeter.com/, and World of Warship from: https://wows-numbers.com/, and https://wowstats.org/ships/ [Last access: October, 2021]

[21] https://wiki.warthunder.com/Category:Premium_ground_vehicles, and https://wiki.wargaming.net/en/Premium_Tanks [Last access: October, 2021]

[22] For example, the WoT database has information regarding hits per battle, wins, kills, and death per minute. However, the information contained in such variables is highly correlated with the kill / death ratio or the win ratio.

Table 1
Averages, standard deviation, and percentage of vehicles in each category by database. August, 2019. Battles >250.

Video Game - Variable	War Thunder n=371	World of Tanks (Noobmeter) n=520	World of Tanks (WoTCharts) n=612	World of Tanks: Blitz n=181	World of Warships (WoWNumbers) n=362	World of Warships (WowStats) n=347
Battles[a]	1,900.6 (2,292.4)	292,394.2 (591,960.7)	5,549,887 (6,985,997)	473 (241.6)	1,889,821 (2,261,519)	980,390 (1,169,947)
Kills / Deaths[a]	2.22 (1.2)	-	0.87 (0.23)	0.78 (0.12)[c]	1.52 (0.98)	0.85 (0.19)
Win Rate[a]	55.08% (9.73)	52.76% (3.2)	51.34% (2.27)	51.02% (2.9)	51.11% (2.47)	53.24% (2.74)
Premium / Free[b]	20.75%	31.54%	30.39%	34.25%	22.93%	23.91%
Tier 1[b]	11.26%	2.12%	1.80%	-	1.93%	2.02%
Tier 2[b]	14.66%	9.04%	8.01%	-	5.80%	5.76%
Tier 3[b]	23.56%	10.38%	9.15%	2.76%	7.73%	7.78%
Tier 4[b]	20.42%	9.04%	7.84%	3.31%	8.01%	8.07%
Tier 5[b]	14.14%	11.54%	9.97%	10.50%	11.60%	11.24%
Tier 6[b]	13.09%	11.54%	11.27%	13.26%	13.54%	13.83%
Tier 7[b]	-	11.35%	10.95%	19.34%	14.36%	14.70%
Tier 8[b]	1.83%	19.23%	20.59%	23.20%	17.96%	18.16%
Tier 9[b]	-	6.35%	8.99%	13.26%	9.94%	9.80%
Tier 10[b]	1.05%	9.42%	11.44%	14.36%	9.12%	8.65%
Heavy / Air Carrier[b]	14.82%	20.19%	20.26%	31.49%	9.67%	10.09%
Light / Cruiser[b]	14.56%	23.27%	22.55%	9.39%	34.53%	35.16%
Medium / Battleship[b]	39.35%	29.03%	28.76%	34.25%	24.31%	25.36%
Destroyer[b]	19.95%	19.03%	19.93%	23.76%	31.49%	29.39%
SPG[b]	0.53%	8.5%	8.5%	1.1%	-	-
SPAA[b]	7.55%	-	-	-	-	-

[a] Average (Standard deviation).
[b] Proportion in the sample.
[c] Per battle.

vehicles is essential. Table 2 shows that statistically significant differences exist in terms of the win rate and K/D ratio. In only one case of WoWS, was there no statistically significant differences in terms of the K/D ratio; however, this finding might be the result of the large standard deviation of the sample that considers old vehicles no longer in service.

5.2. Regression analysis

As previously stated, the dependent variable is the win ratio.[23] I assume that "winning" varies with premium status, tier and type of vehicle, K/D ratio, and other covariates. If the game was completely balanced, none of those variables would be statistically significant. Moreover, on average, all vehicles would have the same win ratio. However, this assumption might only be true when the number of battles is high. Therefore, all vehicles are assumed to converge over time to what is called the "intrinsic win-rate," which is given as the mean probability of winning a random battle, evened out over all maps, team composition, skill levels, and any other random factors that might be involved.

Assuming that winning is a latent Bernoulli process, the question is how many battles must be considered to ensure that the intrinsic win rate is being observed. This question is equivalent to asking how many times a coin needs to be flipped to know whether the outcome is fair. If the maximum allowed error is 0.01 and at a 99.99% confidence level, only cases with more than 40.000 battles should be considered. In all cases, these requirements exist, except for the WT and WoTB cases. In these cases, only if vehicles with more than 250 battles are considered, vehicles have 1.700 battles on average, which is equivalent to assuming a maximum allowed error of 0.025 at a 95% confidence level.[24]

Following the theoretical model in Section 4, the variation in the win ratio should depend on whether items (vehicles) are premium, their complementary/substitute role in the game (their type), their sensitivity to the premium bias (their tier[25]), and whether the bias came from better matchmaking or skills enhancement. Therefore, the following model is proposed to test the hypotheses:

$$WR_j = c + \beta_1 P + X_j \Gamma + \beta_2 P * K/D + \epsilon_j \tag{29}$$

where WR is the win ratio of vehicle j and X_j is a 4 x K matrix that contains individual characteristics (tier, type, number of battles, and K/D ratio of vehicle j. In some specific databases, other controls might be included) with Γ as the corresponding 4 x K matrix of parameters. P is a vector that takes the value of 1 if the vehicle is premium and 0 otherwise, with β_1 as the corresponding parameter. $P * K/D$ is a vector that controls the interaction between premium vehicles and the K/D ratio, with β_2 as the corresponding parameter. Finally, ϵ_j denotes the error term.

Equation (29) is linear in its unknown parameters and, thus, is amenable to estimation using standard methods. Tables 3 and 4 present

[23] Only in the Noobmeter database is the logarithm of the OP rating used instead. The OP rating is computed as the ratio of the win ratio of the vehicle with respect to that of the players. Intuitively, this ratio corrects deviations caused by different skill levels, which we correct in the rest of the estimations by including the K/D as an independent variable; however, in this case, no information on the K/D ratio was available. The OP rating imperfectly corrects the deviations generated by different skill levels but, this approach is the best one in this database to attempt to infer the premium bias. See: http://blog.noobmeter.com/2013/03/february-march-tank-winrates-and-op.html [Last access: October, 2021]

[24] There is neither an empirical nor theoretical reason to choose 250 battles. Other numbers can be chosen as well, and the overall conclusions do not change. Nonetheless, the larger the number, the less noise around the true winning ratio.

[25] The assumption here is that the premium bias works similar to a doping substance. The more professional or skilled the player, the higher the effect of the doping.

Table 2
Mann-Whitney U tests on equality of means. August, 2019.

Averages	War Thunder			World of Tanks: Blitz			World of Warships Wownumbers		
Variables	Premium	No Premium	p-value	Premium	No Premium	p-value	Premium	No Premium	p-value
Battles	1,520.1	2,000.2	0.01	410.56	505.51	0.00	1,120,945.3	2,118,554.44	0.00
Kill/Death ratio	1.75	1.35	0.00	0.85	0.75	0.00	1.38	1.56	0.26
Win rate	62.7	53.1	0.00	53.12	49.93	0.00	52.51	50.69	0.00
Averages	World of Tanks Noobmeter			World of Tanks: Wotcharts			World of Warships Wowstas		
Variables	Premium	No Premium	p-value	Premium	No Premium	p-value	Premium	No Premium	p-value
Battles	311,378.1	283,648.9	0.19	2,980,762.3	6,671,617.8	0.00	621,840.8	1,093,116.1	0.01
Kill/Death ratio	6.45[a]	6.37[a]	0.00	0.98	0.82	0.00	0.91	0.83	0.00
Win rate	53	52	0.00	53.25	50.78	0.00	54.98	52.69	0.00

[a] OP Rating instead of Kill/death ratio.

a battery of OLS regressions based on Eq. (29). The dependent variable is the win ratio of the vehicles, except in regressions (4), (5), and (6).[26]

The premium bias can be estimated in different ways, and the different regressions prove this statement. Regressions (1) to (6) and (10) to (12) present positive coefficients; in contrast, regressions (7) to (9) and (13) to (18) present negative coefficients. Although this finding might seem contradictory, it is not. In the first case, the interaction with the K/D ratio was not considered, either because that information was not available (regressions (4) to (6)) or because it was not significant at the 90% confidence level. Nonetheless, in all cases, we reject the null hypothesis of no premium bias. The key point is that bias is related to the K/D ratio. In other words, bias exists in both matchmaking and vehicle performance. Therefore, evidence exists of a premium bias that might arise from enhancing performance items (vehicles) and/or better matchmaking.

Proposition 4. *Premium bias can be of three types: better team matches, individual performance boosts, or a combination of these. Evidence of matchmaking bias was found in WarThunder and individual performance bias in World of Warships. In World of Tanks, evidence of both was found.*

Interestingly, in all cases, a positive relationship exists between the K/D ratio and win rate. The more a vehicle contributes to defeating the opponent, the greater the chance of winning. The same logic applies to the survival and spot ratios. Additionally, as the theory predicts, the type, number of battles, and tier of the vehicles used are relevant variables. All ships in regressions (13) to (18) perform equally or better than the omitted air carriers. Effects similar to those of the tanks are found. All vehicles tend to perform equally or better than anti-aircraft vehicles (13) or self-propelled artillery (46), except for (7) to (12). In (10) to (12), the exception can be explained because two variables (spots and survival) are included that might incorporate such information. For (7)(9), light tanks simply perform worse than self-propelled artillery. Finally, it would be interesting to observe the role of the number of battles and tiers. They share common information about the players skills and knowledge. This feature also makes sense from a theoretical point of view, given that item demand and its elasticity with respect to premium bias are related, which would explain why the sign of the tier is negative when the number of battles is not considered, see (3), and why the sign of the parameter for the number of battles is negative in (2), (4), (5), and (6). By contrast, in (1), the sign is positive for tiers 8 and 10; however, such unusual behavior might also be a consequence of an insufficient number of battles for those vehicles (tier 8 has only six observations).

5.3. Estimation of premium bias

Table 5 presents the average premium bias among the databases by computing the marginal effects of the premium covariate. First, worth noting is the difference between the estimation in the Noobmeter database and the other two WoT databases. The difference relies on the dependent variable used in the Noobmeter case, which is the logarithm of the OP rating. Therefore, it is important to define a standard measure to allow comparisons between games and databases. If not, any difference in premium bias is susceptible to over/underestimation. In this case, the Noobmeter database confirms the presence of a premium bias; however, comparing it with other games is not recommended because of the different dependent variables. However, no reason exists to choose one over the other. This remains an area of future work in not only video game analysis but also similar environments.

The results are expressed as an increase in percentage points over the average win ratio. Interestingly, in the WT video game, premium vehicles perform significantly better than their counterparts in WoT. For example, in August 2019, premium vehicles in WT had a 3.23% extra probability of winning a random match. In contrast, a similar item in WoT had less than a 1% extra probability on the same dates. Not surprisingly, the premium biases are small because, in the end, the video game must maintain a balance between free and premium players. A large premium bias likely expels free players and creates a pay-to-win reputation that might prevent other players from coming in.

Second, Eq. (25) shows that given the additive influence of items in the bias, the inclusion of new items in the game changes that bias. The estimation of premium bias in Table 5 shows this effect.[27] WT decreases its premium bias over time, whereas WoT does not seem to change, and WoW only shows a change in October. Considering August as the reference, the variations in premium bias are consistent with the introduction of new items in those months (see Table 6). When comparing databases, the WoT biases were stable, and only the October estimates of WoW showed a decline.

Proposition 5. *The estimated bias oscillates between 3% and 0.1%, depending on the game. However, bias changes over time because of the introduction/removal of items, as predicted by theory. Therefore, any premium bias estimation must consider whether the set of items is stable.*

Nonetheless, these results should be considered with caution, especially in regressions (4), (5), (6), (13), (14), and (15), in which the independent variables cannot explain more than 50% of the variations in

[26] In all cases, the specification is tested using a Ramsey RESET test and a Pregibon link test. In all cases, the null hypothesis of no additional omitted independent and significant variables cannot be rejected at 95%, except in (7), (8), (9), (15), and (18), in which the Ramsey RESET test cannot reject the null hypothesis. Similarly, in regressions (4), (5), (6) (7), (8), (9), (15), (16), (17), and (18) standard errors (reported in parentheses) are corrected for heteroskedasticity, and the White-Huber procedure is used to correct for correlations of error terms across observations. In the rest of the cases, the null hypothesis of homoskedasticity was not rejected. Lastly, in all cases, the variance inflation factors (VIF) are smaller than 10.

[27] Rong et al. (2018) also found this effect when studying the Chinese video game market.

Table 3
Regression analysis. Vehicles with more than 250 contests played. 1/2.

	(1) WarThunder. Aug	(2) WarThunder. Sep	(3) WarThunder. Oct	(4) Noobmeter. Aug[a,b]	(5) Noobmeter. Sep[a,b]	(6) Noobmeter. Oct[a,b]	(7) WotCharts. Aug	(8) WotCharts. Sep	(9) WotCharts. Oct
Premium	3.238*** (3.75)	2.074** (2.05)	1.617* (1.76)	0.0989*** (10.10)	0.0774*** (9.30)	0.0992*** (9.90)	-0.0203*** (-2.69)	-0.0233*** (-3.22)	-0.0225*** (-3.16)
KD ratio	16.12*** (15.70)	13.73*** (12.17)	4.043*** (12.50)	-	-	-	0.0669*** (14.89)	0.0630*** (13.42)	0.0666*** (14.58)
Premium=0 × KD ratio				-	-	-	0 (.)	0 (.)	0 (.)
Premium=1 × KD ratio				-	-	-	0.0307*** (3.65)	0.0355*** (4.15)	0.0333*** (4.16)
(Tank) Medium	4.914*** (3.86)	7.499*** (5.27)	8.322*** (6.56)	0.0398** (2.37)	0.0653*** (4.10)	0.0612*** (4.06)	0.00699*** (4.87)	0.00540*** (3.95)	0.00688*** (4.81)
(Tank) Heavy	2.636* (1.75)	6.010*** (3.57)	6.524*** (4.41)	0.0635*** (4.00)	0.0801*** (5.25)	0.0689*** (4.32)	0.00989*** (5.73)	0.00828*** (4.98)	0.00968*** (5.64)
(Tank) Destroyer	3.319** (2.46)	6.073*** (3.98)	6.083*** (4.57)	0.0265 (1.52)	0.0410*** (2.61)	0.0327** (2.35)	-0.00419** (-2.41)	-0.00475*** (-2.83)	-0.00447** (-2.58)
(Tank) Light	8.637*** (6.15)	10.17*** (6.44)	12.38*** (8.55)	0.0280 (1.46)	0.0552*** (3.04)	0.0279* (1.89)	0.0146*** (8.53)	0.0127*** (7.66)	0.0145*** (8.60)
(Tank) Artillery	1.524 (0.36)	9.295** (2.03)	9.418*** (2.68)	-	-	-	-	-	-
Tier=1	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)
Tier=2	-2.470* (-1.83)	-5.440*** (-3.52)	-1.859 (-1.37)	-0.148*** (-3.39)	0.0104 (0.25)	-0.0141 (-0.41)	0.00840*** (3.05)	0.00754*** (2.84)	0.00814*** (3.08)
Tier=3	-1.637 (-1.22)	-4.029*** (-2.69)	0.122 (0.09)	-0.141*** (-3.54)	-0.0327 (-0.79)	-0.0538 (-1.58)	0.0173*** (6.62)	0.0156*** (6.17)	0.0166*** (6.52)
Tier=4	-1.975 (-1.49)	-4.551*** (-3.06)	-1.789 (-1.33)	-0.188*** (-4.73)	-0.0777* (-1.82)	-0.0616* (-1.79)	0.0208*** (7.19)	0.0189*** (6.73)	0.0203*** (7.05)
Tier=5	-1.230 (-0.86)	-4.685*** (-2.94)	-1.050 (-0.73)	-0.236*** (-6.09)	-0.121*** (-2.86)	-0.0742** (-2.27)	0.0180*** (6.33)	0.0156*** (5.78)	0.0168*** (6.10)
Tier=6	-2.021 (-1.45)	-5.380*** (-3.45)	-2.140 (-1.51)	-0.255*** (-6.76)	-0.134*** (-3.21)	-0.0980*** (-3.07)	0.0188*** (7.32)	0.0158*** (6.36)	0.0186*** (7.44)
Tier=7	-	-	-	-0.265*** (-6.95)	-0.134*** (-3.19)	-0.0936*** (-2.67)	0.0181*** (7.50)	0.0146*** (6.28)	0.0180*** (7.68)
Tier=8	3.978 (1.42)	-	-	-0.333*** (-8.70)	-0.196*** (-4.64)	-0.168*** (-5.16)	0.0182*** (7.00)	0.0136*** (5.41)	0.0185*** (7.37)
Tier=9	-	-	-	-0.318*** (-8.34)	-0.200*** (-4.72)	-0.168*** (-5.18)	0.0161*** (6.56)	0.0112*** (4.91)	0.0174*** (7.31)
Tier=10	6.148* (1.89)	-	-	-0.300*** (-7.93)	-0.170*** (-4.03)	-0.126*** (-3.86)	0.00852*** (3.08)	0.00324 (1.23)	0.0116*** (4.18)
Total Battles	0.000218 (1.52)	0.000336* (1.81)	0.000368** (2.16)	-	-	-	-3.63e-10*** (-5.03)	-1.53e-10*** (-4.72)	-8.72e-10*** (-4.89)
Survival	-	-	-	-	-	-	-	-	-
Spots	-	-	-	-	-	-	-	-	-
(Ships) Battleship	-	-	-	-	-	-	-	-	-
(Ships) Cruiser	-	-	-	-	-	-	-	-	-
(Ships) Destroyer	-	-	-	-	-	-	-	-	-
(Ships)~ Avg. Planes Destr.	-	-	-	-	-	-	-	-	-
Constant	28.06*** (14.09)	31.28*** (13.73)	37.56*** (22.82)	6.576*** (162.08)	6.447*** (147.16)	6.408*** (192.13)	0.434*** (87.48)	0.438*** (88.24)	0.434*** (88.43)
Observations	371	338	352	520	517	497	612	615	618
R^2	0.665	0.593	0.589	0.444	0.497	0.349	0.756	0.741	0.781

Only significant variables printed. (-) represents that no data was available, blank spaces non significant variables. *t* statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

[a] Log(OP Rating) instead of Win Rate ratio. [b] Kills per Battle instead of Kill/death ratio
[b] Sqrt of Kill/death ratio

the win ratio. Although the explanatory power in such cases is not sufficient, the ranking of bias does not vary excessively when making cross-comparisons between the databases.

Finally, we use these estimations to address whether the premium bias levels reduce or increase welfare (in comparison with a classical P2P game). To do so, we rely on predictions made by the theoretical model described in Section 3. The idea of this exercise is not to propose a guide for estimating the welfare effects of F2P video games but to show that it is possible to do so using publicly available data. Other theoretical or empirical models might better suit this case; however, for the sake of simplicity, let us assume that our theoretical long-term model is a good description of the long-term business model of WT, WoT, and WoWS. Rearranging Eq. (15), the premium bias can be represented as a ratio between s_1 and s_2, and as a function of γ:

$$\frac{s_1}{s_2} = \frac{\gamma}{2-\gamma} \approx \delta \tag{30}$$

Using our database, estimating the ratio $\frac{s_1}{s_2}$ implies that the win rate represents the quality levels of free (s_1) and premium items (s_2). Therefore, using the results presented in Table 5 and Eqs. (7) and (19), we can estimate the parameter γ. In this way, we determine whether these games are welfare-enhancing in this specific model. Figure 2 depicts the network effects of each game at each time point. All video games are multiplayer online video games; therefore, finding a positive and large network effects makes sense, as shown in Fig. 2. In all cases,

Table 4
Regression analysis. Vehicles with more than 250 contests played. 2/2.

	(10) WoTBlitz. Aug[a]	(11) WoTBlitz. Sep[a]	(12) WoTBlitz. Oct[a]	(13) WoWNumbers. Aug	(14) WoWNumbers. Sep	(15) WoWNumbers. Oct	(16) WoWStats. Aug[a]	(17) WoWStats. Sep[a]	(18) WoWStats. Oct[a]
Premium	0.00386** (2.56)	0.00392*** (2.62)	0.00333** (2.26)	-0.0461*** (-5.72)	-0.0452*** (-5.60)	-0.00816 (-0.70)	-0.0167 (-1.57)	-0.0167 (-1.57)	-0.0106 (-1.06)
KD ratio	0.137*** (15.18)	0.137*** (15.25)	0.140*** (15.63)	0.0140*** (6.97)	0.0142*** (7.14)	0.0168*** (5.29)	0.118*** (15.18)	0.118*** (15.18)	0.124*** (17.09)
Premium=0 × KD ratio				0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)
Premium=1 × KD ratio				0.0422*** (7.45)	0.0414*** (7.33)	0.0132 (1.56)	0.0277** (2.21)	0.0277** (2.21)	0.0202* (1.73)
(Tank) Medium				-	-	-	-	-	-
(Tank) Heavy				-	-	-	-	-	-
(Tank) Destroyer				-	-	-	-	-	-
(Tank) Light				-	-	-	-	-	-
(Tank) Artillery	-	-	-	-	-	-	-	-	-
Tier=1	-	-	-	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)	0 (.)
Tier=2	-	-	-	0.00284 (0.35)	0.00332 (0.42)	0.00222 (0.34)	0.0155*** (4.49)	0.0155*** (4.49)	0.0175*** (5.72)
Tier=3	0 (.)	0 (.)	0 (.)	0.0169** (2.15)	0.0171** (2.22)	0.0192*** (3.92)	0.0143*** (5.02)	0.0143*** (5.02)	0.0150*** (5.76)
Tier=4	0.00366 (0.73)	0.00379 (0.76)	0.00482 (0.96)	0.0353*** (4.43)	0.0364*** (4.66)	0.0382*** (7.39)	0.0232*** (6.46)	0.0232*** (6.46)	0.0233*** (7.04)
Tier=5	0.0159*** (3.42)	0.0161*** (3.47)	0.0176*** (3.77)	0.0315*** (4.04)	0.0331*** (4.36)	0.0335*** (7.48)	0.0226*** (6.83)	0.0226*** (6.83)	0.0240*** (7.56)
Tier=6	0.0177*** (3.68)	0.0178*** (3.72)	0.0187*** (3.86)	0.0376*** (4.88)	0.0385*** (5.13)	0.0362*** (8.23)	0.0308*** (9.36)	0.0308*** (9.36)	0.0312*** (10.04)
Tier=7	0.00762 (1.65)	0.00796* (1.73)	0.00952** (2.06)	0.0319*** (4.18)	0.0326*** (4.38)	0.0320*** (7.00)	0.0255*** (7.67)	0.0255*** (7.67)	0.0266*** (8.44)
Tier=8	0.00642 (1.29)	0.00662 (1.33)	0.00832* (1.67)	0.0322*** (4.25)	0.0326*** (4.40)	0.0289*** (5.91)	0.0315*** (8.14)	0.0315*** (8.14)	0.0321*** (8.64)
Tier=9	0.0116** (2.27)	0.0118** (2.31)	0.0134*** (2.62)	0.0325*** (4.15)	0.0327*** (4.29)	0.0303*** (6.27)	0.0288*** (6.34)	0.0288*** (6.34)	0.0281*** (6.89)
Tier=10	-0.0103** (-2.16)	-0.0102** (-2.14)	-0.00873* (-1.83)	0.0241*** (3.07)	0.0265*** (3.48)	0.0219*** (3.76)	0.0104** (2.29)	0.0104** (2.29)	0.00934** (2.21)
Total Battles	-0.0000111*** (-4.16)	-0.0000112*** (-4.22)	-0.0000110*** (-4.24)	-2.15e-09*** (-4.53)	-2.35e-09*** (-5.18)	-2.07e-09*** (-5.23)	-4.26e-09*** (-8.22)	-4.26e-09*** (-8.22)	-3.77e-09*** (-8.23)
Survival	0.314*** (15.29)	0.315*** (15.51)	0.317*** (15.46)	-	-	-	-	-	-
Spots	0.0271*** (12.65)	0.0272*** (12.73)	0.0277*** (12.97)	-	-	-	-	-	-
(Ships) Battleship	-	-	-	0.0350*** (5.51)	0.0342*** (5.56)	0.0457*** (4.54)	0.00392 (0.60)	0.00392 (0.60)	0.00341 (0.56)
(Ships) Cruiser	-	-	-	0.0400*** (6.02)	0.0385*** (5.97)	0.0473*** (4.47)	0.0134** (2.01)	0.0134** (2.01)	0.0133** (2.15)
(Ships) Destroyer	-	-	-	0.0352*** (5.16)	0.0344*** (5.21)	0.0452*** (4.12)	0.0112 (1.55)	0.0112 (1.55)	0.0109 (1.61)
(Ships)~ Avg. Planes Destr.	-	-	-	-0.000928* (-1.88)	-0.00108** (-2.26)	-0.000960 (-1.23)	-0.000179 (-0.21)	-0.000179 (-0.21)	-0.000350 (-0.42)
Constant	0.267*** (26.70)	0.267*** (26.75)	0.262*** (26.37)	0.431*** (38.55)	0.432*** (39.43)	0.419*** (27.93)	0.401*** (38.39)	0.401*** (38.39)	0.396*** (41.52)
Observations	181	182	184	362	369	375	347	347	350
R^2	0.938	0.939	0.938	0.487	0.495	0.444	0.757	0.757	0.777

Only significant variables printed. (-) represents that no data was available, blank spaces non significant variables. *t* statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

[a] *a* Log(OP Rating) instead of Win Rate ratio. *a* Kills per Battle instead of Kill/death ratio Sqrt of Kill/death ratio

video games are in the welfare-reducing region, as defined in Fig. 1. In other words, the current evidence points out that such video games might have reduced players welfare by choosing a F2P business model. To analyze the sensitivity of these network effects to changes in covariates, a sensitivity analysis was conducted. In Table 7, we analyze the effect of changes in the K/D ratio, which is the most influential covariate. For simplicity, we only depict August, but the intuitions are the same for the other cases, which are available on request. In all cases, the network effects remained higher than 0.75, even when considering extreme scenarios in which premium vehicles defeat, on average, three opponents before being defeated (K/D ratio = 3).

Nonetheless, we should be cautious with this result because it is based on strong assumptions regarding the behavior of players and firms. In this sense, these preliminary results highlight the possibility that such video games might have reduced players' welfare by choosing an F2P business model instead of a P2P model.

Table 5
Estimation Premium bias by database. Percentage points.

Percentage points. August	WarThunder	WoT.Noobmeter[a]	WoTCharts	WoTBlitz	WoWNumber	WoWStats
Avg. K/D	1.44	-	0.865	0.782	1.52	0.852
Estimated Premium Bias	3.23***	0.099***	0.63***	0.38***	1.81***	0.69***
Estimated 95% Interval	[1.53-4.93]	[0.08-0.12]	[0.36-0.9]	[0.09-0.68]	[1.29-2.33]	[0.29-1.09]
Percentage points. September	WarThunder	WoT.Noobmeter[a]	WoTCharts	WoTBlitz	WoWNumber	WoWStats
Avg. K/D	1.44	-	0.827	0.782	1.53	0.852
Estimated Premium Bias	2.07**	0.077***	0.6***	0.39***	1.79***	0.69***
Estimated 95% Interval	[0.09-4.05]	[0.06-0.094]	[0.34-0.87]	[0.1-0.7]	[1.23-2.3]	[0.29-1.09]
Percentage points. October	WarThunder	WoT.Noobmeter[a]	WoTCharts	WoTBlitz	WoWNumber	WoWStats
Avg. K/D	2.18	-	0.864	0.78	1.51	0.849
Estimated Premium Bias	1.62*	0.099***	0.64***	0.33**	1.18***	0.66***
Estimated 95% Interval	[-0.19-3.43]	[0.08-0.12]	[0.37-0.9]	[0.004-0.6]	[0.63-1.7]	[0.28-1.04]
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$						

[a] Log(OP Rating) instead of Win Rate ratio.

Table 6
Log-changes between August and October 2019.

Updates	War Thunder	World of Tanks	World of Warships
August	-	Patch 1.6	Patch 0.87
September	Patch 1.91	-	-
October	Patch 1.93	-	Patch 0.89

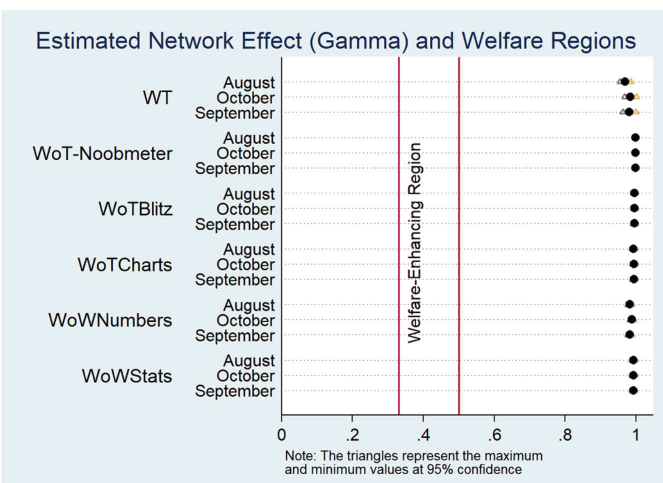

Fig. 2. Estimation of network effects and the influence on welfare by the database.

Proposition 6. *Using the model in Section 3, the estimation of bias in World of Tanks, War Thunder, and World of Warships is compatible with a reduction in player welfare compared with the alternative P2P business model.*

These results do not imply that multiplayer video games reduce welfare, but that multiplayer F2P video games might not create as much welfare as P2P games. In other words, given current game designs and mechanics, welfare would be higher if instead of F2P games, they were P2P games. As long as the firm's objective is only to maximize profits, this gap remains because firms prefer to create F2P video games. In this sense, the role of video game platforms (e.g., Steam, Epic Store, GoG.com) is essential because they can enforce policies aimed at increasing users' welfare. For example, the inclusion of single-player mechanics or any type of content that reduces the prevalence of network effects reduces the gap, as shown in Fig. 1. However, it might increase the bias because the single-player mode could attract free players who are less willing to endure a greater imbalance, allowing the video game to establish a greater imbalance in the multiplayer mode. However, this approach is not the only one. Policies aimed at increasing game quality certainly increase welfare. In the case of F2P, it moves up the blue area in Fig. 1. In the case of P2P, increased quality makes it more desirable as a business model, Eq. (6). Ichi.io's quality control could be an example of a welfare-enhancing policy, as the specialized media pointed out Bycer (2017). Given that both F2P and P2P business models benefit from higher levels of quality, it is natural that digital stores (e.g., Steam and Epic Games) have recently introduced policies aimed at increasing game quality.[28],[29] Arguably, the introduction of such policies might have reduced this gap. Nonetheless, the extent to which these policies have influenced welfare remains an open research question, as is which business model (P2P or F2P) results in higher quality.

6. Conclusions

Free-to-play (F2P) is the most important business model in the global video game industry, accounting for more than 75% of all revenue. Despite its relevance, no study has addressed why F2P has become so widely popular as a business model, when it benefits or hurts players, or how F2P video games balance the interests of free and premium players. Some developers claim that this strategy is "killing the video game industry" because it is creating a "pay-to-win" culture that degrades the game experience for free players, leading many to quit the game because

Table 7
Sensitivity analysis of network effects by K/D ratio. August.

Influence KD ratio on γ per video game	WarThunder	WoT.Noobmeter[a]	WoTCharts	WoTBlitz	WoWNumber	WoWStats
K/D ratio = 1	1.03	-	0.99	0.96	1	0.99
K/D ratio = 2	0.90	-	0.96	0.85	0.96	0.96
K/D ratio = 3	0.79	-	0.93	0.76	0.92	0.94

[a] Noobmeter does not consider the K/D ratio and among the covariates, Premium is the one that has the major influence.

[28] https://www.thegamer.com/epic-games-store-crappy-games-says-tim-sweeney/
[29] https://www.ccn.com/valve-quality-control-hot-trash-games/

of the inherent imbalance between free users and those who pay. In contrast, other developers claim that F2P video games "democratize" the video game industry because they expand the set of people that can experience video games. This work addresses firms' motivations to choose this new business model over the old pay-to-play (P2P) model and how such a decision might impact players' welfare. We show that it is always profitable to move from P2P to F2P in the long term, and that multiplayer F2P video games might reduce welfare if network effects are strong. The opposite is true when network effects are moderate.

We then address how video games set prices for in-game items and the imbalance between free and premium players. We show that optimal imbalance takes into account balancing costs, elasticities of demand, and complementarity/substitutability relationships among premium items. Interestingly, free players' interests are ignored, except to the extent that premium players need them to exploit the imbalance. In this case, stating that all increases in such an imbalance (fairness reductions) decrease consumers welfare is not possible because it depends on the specific market demand. This result provides an additional explanation for the current controversy among players and developers about whether this business model is "killing the industry." The business model choice depends on the game. Thus, these results not only help to narrow the set of F2P that might raise policymakers concerns but also provide developers with testable hypotheses that help them design video games. Moreover, this work opens the door for future research on whether popular games such as Fortnite are increasing or reducing welfare.

Given the current lack of frameworks to estimate these imbalances, this work provides us with a testable hypothesis regarding the factors that might influence the imbalance. We conclude the paper by showing how we can estimate premium bias in three popular free-to-play video games (War Thunder, World of Tanks, and World of Warships). We find that War Thunder has the largest premium imbalance on average, but it declines over time, followed by World of Warships and World of Tanks. By comparing these imbalances with the theoretical welfare-reducing threshold, all of them seem to be in the welfare-reducing region. Additionally, observing how some of those fairness levels change over time while others remain fixed is interesting, which could be attributed to the inclusion of new items, as the theory predicts.

Although this work advances the study of the strategy of F2P video games, research on this strategy remains underdeveloped Rong et al. (2018). This work aims to spark the discussion around a market that has not been studied in-depth and in which there are many potential research areas, such as the optimality and profitability of matchmaking player systems and the taxation of transactions in F2P video games. Therefore, future research could focus on the relationship between imbalances and prices that incentivize people to keep playing, the relationship between fairness and market imperfections, and how these imbalances could be used to avoid overcrowding or foster adoption. Finally, the role of gender is a promising area that has not been explored. This topic has been shown to be relevant in online games and might also influence welfare measures, Liu (2016).

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Appendix A. Annex

A1. Annex A. Proof Lemma 1

We begin by assuming that the firm can produce K-1 items with different premium biases, $s_2, s_3, \ldots, s_K$, at K-1 different prices, $p_2, p_3, \ldots, p_k$, where s_1 is the basic game at price $p_1 = 0$. Let $\widehat{\theta}_1$ be the indifferent player between playing and not playing the F2P game.

$$\widehat{\theta}_1 s_1 + \gamma s_1 Q - p_1 = 0 \tag{31}$$

For $2 \leq k \leq K$, let $\widehat{\theta}_k$ be the indifferent player between s_k and s_{k-1},

$$\widehat{\theta}_k s_k + \gamma s_k Q - p_k = \widehat{\theta}_k s_{k-1} + \gamma s_{k-1} Q - p_{k-1} \tag{32}$$

By solving with respect to both expressions $\widehat{\theta}_1$ and $\widehat{\theta}_k$, we have

$$\theta_k = \frac{p_k - p_{k-1}}{s_k - s_{k-1}} - \frac{\gamma(s_1 - p_1)}{(1-\gamma)s_1} \tag{33}$$

$$\theta_1 = \frac{p_1 - \gamma s_1}{(1-\gamma)s_1} \tag{34}$$

which represents the indifferent players between consuming the game or not ($\widehat{\theta}_1$) and consuming item k or $k-1$ ($\widehat{\theta}_k$). Following the uniform distribution assumption and solving the previous expressions, we can compute demand as $Q = 1 - \theta_1$. Therefore,

$$Q = 1 - \theta_1 = \frac{s_1 - p_1}{(t-\gamma)s_1} \tag{35}$$

Given the qualities of the K-1 premium items and the free game, the firm sets prices for all items to maximize profits. Formally,

$$\max_{p_1,\ldots,p_k} \Pi = \sum_{k=1}^{K-1} p_k(\theta_{k+1} - \theta_k) + p_K(1 - \theta_{K-1}) \tag{36}$$

We assume $0 < (1-\gamma)s_2$ to guarantee that the first-order conditions lead to a global maximum. Under this assumption, the first-order conditions can be written as:

$$\theta_k = \frac{1-\gamma Q}{2} \tag{37}$$

$$\theta_k = \theta_{k+1} \tag{38}$$

$$2(\theta_2 - \theta_1) = \frac{\gamma p_K}{(1-\gamma)s_1} > 0 \tag{39}$$

Equations (37) and (38) directly prove that the monopolist's optimal strategy is to offer an F2P game and only one premium product.[30] The intuition behind this result is that, although the company can choose to produce many different digital items with different contents or qualities, the optimal option is to choose only two categories. This result supports the idea of a single premium imbalance or bias and does not exclude the possibility of having N different items influencing the imbalance.

A2. Annex B. A closer look at how fairness affects long-term welfare

Let us assume that demand for free players is constant. Intuitively, this implies that, independent of the premium imbalance, a constant number of free players will be willing to play. This assumption does not imply that free players do not care about the imbalance but, instead, that there will always be the same number of free players at any level of imbalance.[31] Therefore, we can focus on the premium players' demands. In this case, we only need to know whether any increase in the premium imbalance leads to an increase in the monopoly price of these items. This case is also interesting because because the F2P is socially preferred if the monopolist does not increase its prices after an increase in premium bias.[32] We focus on two demand functions. The first represents the case of a *persuasive imbalance* that assumes that the premium imbalance increases players' willingness to pay and, thereby, the demand. The second represents the case of *imbalance discovery* that assumes that, initially, all players are unaware of the presence of the imbalance. Only through experiencing the game do some of them discover.

Persuasive Imbalance

Take a continuum of players with mass equal to one. Each player consumes either one or no unit of a premium item. Premium players are heterogeneous in how they value premium imbalance, denoted by μ. Let us assume that μ is uniformly distributed over the interval [0,1]. A persuasive imbalance implies that it "inflates" the intrinsic valuation of the premium items. In other words, a player is willing to pay $g(\delta)\mu$ for a premium item, where $g(0) = 1$ and $g^{'}(\delta) > 0$. Therefore, at price v, the players willing to buy the premium items are those for which $g(\delta)\mu \geq v$, which yields a demand of

$$q(,v,\delta) = 1 - \frac{v}{g(\delta)}$$

In this case, the optimal monopoly price is $v_m(\delta) \ = \frac{g(\delta)+c(\delta)}{2}$. Clearly $v_m^{'}(\delta) > 0$. A persuasive imbalance increases prices. Therefore, we have opposite signs in Eq. (28). However, in this specific example, the net effect can be shown to be only negative if $C^{'}(\delta) > g(\delta)q_\delta$, where $q_\delta \ = \frac{vg^{'}(\delta)}{g(\delta)^2}$. This assumption is likely to hold in cases in which the imbalance is already high and matching free and premium players is complex. Therefore, when the imbalance is persuasive and the pool of free players is constant, the monopoly imbalance is socially excessive if the marginal cost is sufficiently large.

Imbalance Discovery

Suppose that there are N players in the market with the same individual decreasing demand function $q(.,v)$ for premium items. All players are unaware of this imbalance. Only through experiencing the game do some of them discover. The monopolist sets an imbalance δ. This level can only be experienced by some players. Thus, we assume that each player has the same probability of experiencing an imbalance. Intuitively, some players might realize that an imbalance exists when playing against others, whereas other players do not realize such an imbalance. Thus, we assumed that an imbalance occurs with probability $\left(1-\frac{1}{N}\right)^{\delta}$, which can be approximated by $e^{-\delta/N}$ when N is large. Thus, the demand is

$$q(,v,\delta) = N\left(1-e^{-\delta/N}\right)d(v) \equiv G(\delta)d(v)$$

It follows that $\frac{\partial q}{\partial v} = G(\delta)d^{'}(v) < 0$, $\frac{\partial q}{\partial \delta} = G^{'}(\delta)d(v) > 0$, and $\frac{\partial q}{\partial \delta v} = G^{'}(\delta)d^{'}(v) < 0$. Thus, we have $\frac{\partial^2 \Pi}{\partial v_j^i \partial \delta_i} \ = \ - \ C'' Q_\delta Q_v$. As $^{'} Q_\delta Q_v < 0$, we thus have that $sign\{v^{'}(\delta)\} \ = sign\{c''(\delta)\}$. Therefore, we can conclude that when an imbalance is discovered and the pool of free players is constant, the monopoly imbalance is socially insufficient if the marginal cost is constant or decreasing.

A3. Annex C. Prices and optimal fairness. First-order conditions.

In Section 4, video games face the maximization problem defined in Eq. (20). The first-order conditions that lead to Eqs. (21), (22), and (23) can be derived by differentiating Eq. (20) with respect to each parameter.

[30] This result can be also found in Jing (2007), which used a more general framework.
[31] This assumption is acknowledged to be quite strong, and it is only considered for illustrative purposes.
[32] In this case, the monopolist supplies "too little" premium bias from a social point of view.

$$\frac{\partial\Pi_i}{\partial P_i} = Q_i + (P_i - c_i)\left(\frac{\partial Q_i}{\partial P_i} + \frac{\partial Q_i}{\partial P_{-i}}\frac{\partial P_{-i}}{\partial P_i}\right) + \sum_{j=1}^{n}\left(v_j^i - d_j^i\right)\frac{\partial q_j^i}{\partial Q_i}\left(\frac{\partial Q_i}{\partial P_i} + \frac{\partial Q_i}{\partial P_{-i}}\frac{\partial P_{-i}}{\partial P_i}\right) = 0 \tag{40}$$

$$\frac{\partial\Pi_i}{\partial v_k^i} = q_k^i + \left(v_k^i - d_k^i\right)\frac{\partial q_k^i}{\partial v_k^i} + \sum_{j\neq k}\left(v_j^i - d_j^i\right)\frac{\partial q_j^i}{\partial v_k^i} = 0 \tag{41}$$

$$\frac{\partial\Pi_i}{\partial\delta_i} = (P_i - c_i)\left(\frac{\partial Q_i}{\partial\delta_i} + \frac{\partial Q_i}{\partial\delta_{-i}}\frac{\partial\delta_{-i}}{\partial\delta_i}\right) + \sum_{j=1}^{n}\left(v_j^i - d_j^i\right)\left(\frac{\partial q_j^i}{\partial Q_i}\left[\frac{\partial Q_i}{\partial\delta_i} + \frac{\partial Q_i}{\partial\delta_{-i}}\frac{\partial\delta_{-i}}{\partial\delta_i}\right] + \frac{\partial q_j^i}{\partial\delta_i}\right) - \frac{\partial F(\delta_i)}{\partial\delta_i} = 0 \tag{42}$$

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